2

Seismic Bearing Capacity of Shallow Foundations

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1. Introduction

The seismic risk mitigation is one of the greatest challenges of the Civil Engineering and an important contribution toward this challenge can be given by the Geotechnical Earthquake Engineering. Lesson learned by recent destructive earthquakes (January 2010 Port-au-Prince region of Haiti and March 2011 Tohoku Japan), confirms that local soil conditions can play a significant role on earthquake ground motions.
Earthquake-induced damage in Port-au-Prince was devastating and widespread. Yet, there were clearly areas of the city where little to no damage occurred, and areas of the city where an overwhelming majority of the buildings were severely damaged or destroyed.
These types of damage patterns are common in earthquakes, and a wide number of factors need to be considered in order to conclusively piece together the causes.
For a given earthquake, these factors include, but are not limited to: (*a*) relative distance from the fault rupture plane, (*b*) construction type and quality, (*c*) local soil conditions (i.e. strength/stiffness of the soil foundation, depth to bedrock, impedance contrasts, geology), (*d*) topography (topographic and basin effects), and (*e*) near fault effects (rupture directivity, fling step, hanging wall effects, polarity effects, etc.). Often several of these factors work together and it can be difficult to identify the primary cause of damage.
Design of foundations in seismic areas needs special considerations compared to the static case. The inadequate performance of structures during recent earthquakes has motivated researchers to revise existing methods and to develop new methods for seismic-resistant design. This includes new design concepts, such as, performance-based design (PBD) (Priestley et al., 2005) and new measures of the structure performance based on energy concepts and damage indexes (Park et al., 1987; Moustafa, 2011).
Similarly, the widespread damage and inadequate performance of code-designed structures during the 1994 Northridge (California) and the 1995 Kobe (Japan) earthquakes have prompted seismologists and engineers of the essential importance of characterizing and modelling near-field ground motions with impulsive nature (Moustafa & Takewaki, 2010).
For foundations of structures built in seismic areas, the demands to sustain load and deformation during an earthquake will probably be the most severe in their design life.
As stressed by Hudson (1981) the soil-structure interaction is a crucial point for the evaluation of the seismic response of structures.

Due to seismic loading, foundations may experience a reduction in bearing capacity and increase in settlement. Two sources of loading must be taken into consideration: "inertial" loading caused by the lateral forces imposed on the superstructure, and "kinematic" loading caused by the ground movements developed during the earthquake.
Part 5 of Eurocode 8 (2003) states that foundations shall be designed for the following two loading conditions :

a. inertia forces on the superstructure transmitted on the foundations in the form of axial, horizontal forces and moment ;
b. soil deformations arising from the passage of seismic waves.

In the last years the seismic action has increased in many National Codes according to recent records which show values up to 0.8 g for very destructive earthquakes. The upgrading of the seismic action requires accurate analyses taking into account all the boundary conditions including the presence of surcharges, sloping ground, depth factors and so on.
With the aim to investigate the influence of these factors on the seismic stability of a shallow foundation, a model based on the limit equilibrium method has been developed.
Many analytical and numerical solutions are today available to evaluate seismic bearing capacity of shallow foundations, and cover area such as the limit equilibrium method, limit analysis, methods of characteristics, finite element analysis and other areas for the computation of the seismic bearing capacity factors required for the design of a foundation. Nevertheless, pseudo-static approaches are more attractive because they are simple, when compared to difficult and more complex dynamic analyses.
Thus, a pseudo-static model to account for reduction in bearing capacity due to earthquake loading is presented. In this model the loading condition consists in normal and tangential forces on the foundation and inertial forces into the soil. An upper bound solution of the limit load of the shallow foundation is found.
Results of the proposed analysis are given in terms of the ratios between seismic and static bearing capacity factors N_c^*/N_c , N_q^*/N_q and N_γ^*/N_γ. Results are also compared with those deduced by other authors using different methods of analysis.

2. Method of analysis

The prediction of the bearing capacity of a shallow foundation is a very important problem in Geotechnical Engineering, and in the last decades solutions using limit analysis, slip-line, limit equilibrium and, recently, numerical methods (i.e. finite element and difference finite methods) have been developed.
The problem of static bearing capacity of shallow foundations has been extensively studied in the past by Terzaghi (1943), Meyerhoff (1963), Vesic (1973) and many others. The ultimate load that the foundation soil can sustain is expressed by the linear combination of the three bearing capacity factors N_c , N_q and N_γ which depend uniquely on the friction angle of the soil. Further solutions for the bearing capacity were given successively in a more general form, taking into account, by means of corrective factors, of the shape of the foundation, of the load and ground inclination and of the depth and inclination of the bearing surface.
In all these studies, the bearing capacity evaluation is based on the assumption that a failure surface can develop beneath the foundation, according to the well known failure surfaces given by the limit equilibrium method or by the limit analysis.

Most foundation failures during earthquakes occur due to liquefaction phenomena, even if failures due to reduction in bearing capacity have been observed during Naigata earthquake (1964) Japan and Izmit earthquake (1999) in Turkey (Day, 2002).
Liquefiable soils are categorized by all seismic codes as extreme ground conditions, where, following a positive identification of this hazard, the construction of shallow footings is essentially allowed only after proper soil treatment. More specifically, liquefaction-induced shear strength degradation of the foundation subsoil may result in post-shaking static bearing capacity failure, while excessive seismic settlements may also accumulate. However, the accurate estimation of the degraded bearing capacity and the associated dynamic settlements could potentially ensure a viable performance-based design of shallow footings.
Richards et al. (1993) observed seismic settlements of foundations on partially saturated dense or compacted soils. These settlements were not associated with liquefaction or densification and could be easily explained in terms of seismic bearing capacity reduction.
In fact, the inertial forces applied on the foundation and in the soil mass reduce the static bearing capacity. Thus, many authors have investigated the seismic bearing capacity giving results in terms of the ratio of the seismic to the static bearing capacity factors N_c^*/N_c , N_q^*/N_q and N_γ^*/N_γ .
The pseudo-static approach is being used to determine bearing capacity of the foundations subjected to seismic loads in non-liquefying soils, considering also the depth effects for an embedded footing and the effect of a sloping ground located at some distance from the footing. Dynamic nature of the load and other factors which affect the dynamic response are not being accounted for.
Ground factors and bearing capacity ratios N_c^*/N_c , N_q^*/N_q and N_γ^*/N_γ are presented as a function of the friction angle of soil ϕ', of the ratio H/B between the embedment depth H and the width of the footing B, of a slope angle β and of the ratio d/B being d the distance from the edge of the slope. The inertial and kinematic effects due to seismic loading have been analyzed in the evaluation of the seismic bearing capacity.

2.1 Limit equilibrium analysis

The method of analysis is based on the limit equilibrium technique. The failure mechanism, as shown in Figure 1, is a circular surface which from the foundation propagates until the ground surface is reached (Castelli & Motta, 2010; 2011).
A similar model was proposed by Castelli & Motta (2003) for a bearing capacity analysis of a strip footing resting on a soil of limited depth.
The seismic forces are considered as pseudo-static forces acting both on the footing and on the soil below the footing. The ultimate load can be found by a moment equilibrium respect to the centre of the circular surface.
Referring to Figure 1 a moment equilibrium can be written and the mobilizing moment is :

$$M_{mob} = \sum_{i=1}^{n_{tot}} W_i (1-k_v) b_{wi} + q_{\lim} \sum_{i=1}^{n_1} \Delta x_i b_{qli} + q_{\lim} \sum_{i=1}^{n_1} k_{h1} \Delta x_i b_{qlhi} + \sum_{i=1}^{n_{tot}} k_{h2} W_i b_{whi} \tag{1}$$

The resisting moment given by the shear strength S_i acting along the base of the slices is :

$$M_{res} = R\sum_{i=1}^{n_{tot}} S_i = R\sum_{i=1}^{n_{tot}} c\Delta x_i \,/\, \cos\alpha_i + R\sum_{i=1}^{n_{tot}} N_i \tan\phi_i \tag{2}$$

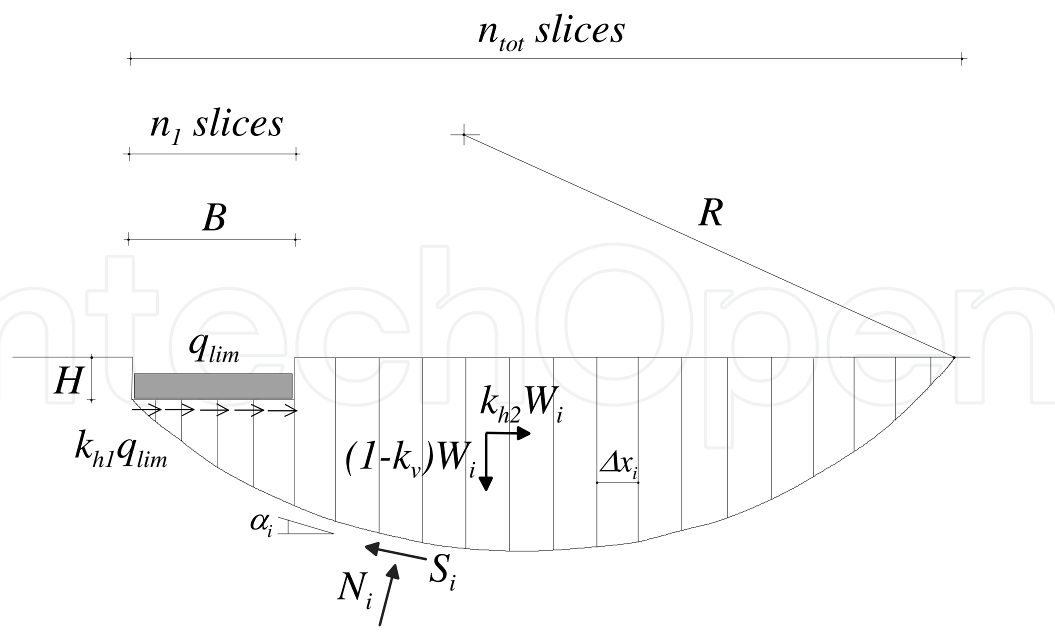


Fig. 1. Failure mechanism and applied forces adopted in the analysis

being :

$$S_i = c\Delta x_i / \cos\alpha_i + N_i \tan\phi_i \tag{3}$$

The force N_i resultant of the normal stress distribution acting at the base of the slice can be derived by the Bishop's method of slices (1955) with an equilibrium equation in the vertical direction, so one obtains (see Figure 1), for the slices under the footing where i = 1 to n_1 :

$$N_i = \frac{q_{\lim}\Delta x_i + W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} \tag{4}$$

and for the remaining slices ($n_1 +1 \leq i \leq n_{tot}$) :

$$N_i = \frac{W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} \tag{5}$$

Thus :

$$M_{res} = R\sum_{i=1}^{ntot} c\Delta x_i / \cos\alpha_i + \begin{array}{l} R\sum_{i=1}^{n_1} \dfrac{q_{\lim}\Delta x_i + W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} \cdot \tan\phi + \\ + \sum_{i=n1+1}^{ntot} \dfrac{W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} R \tan\phi \end{array} \tag{6}$$

where :

- q_{lim} = vertical limit load acting on the footing;
- c = soil cohesion;
- Δx_i = width of the ith slice;
- W_i = weight of the ith slice;
- R = radius of the circular failure surface;

- α_i = angle of the base of the i[th] slice;
- n_1 = number of slices under the footing;
- n_{tot} = total number of slices;
- k_{h1} = horizontal seismic coefficient for the limit load;
- k_{h2} = horizontal seismic coefficient for the soil mass;
- k_v = vertical seismic coefficient for the soil mass;
- b_{wi} = distance of the weight W_i of the i[th] slice to the centre of the circular failure surface;
- b_{whi} = distance of the inertia force $k_{h2}W_i$ of the i[th] slice to the centre of the circular failure surface;
- b_{qli} = distance of the limit load q_{lim} acting on the i[th] slice to the centre of the circular failure surface;
- b_{qhli} = distance of the shear limit force $k_{h1}q_{lim}$ acting on the i[th] slice to the centre of the circular failure surface.

Substituting the following terms :

$$a_1 = R\sum_{i=1}^{n_{tot}} c\Delta x_i \,/\, \cos\alpha_i$$

$$a_2 = R\tan\phi\sum_{i=1}^{n_1} \frac{\Delta x_i}{\cos\alpha_i + \sin\alpha_i \tan\phi}$$

$$a_3 = R\tan\phi\sum_{i=1}^{n_1} \frac{W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi}$$

$$a_4 = R\tan\phi\sum_{i=n_1+1}^{n_{tot}} \frac{W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi}$$

$$a_5 = \sum_{i=1}^{n_{tot}} W_i(1-k_v)b_{wi}$$

$$a_6 = \sum_{i=1}^{n_{tot}} k_{h2}W_i b_{whi}$$

$$a_7 = \sum_{i=1}^{n_1} \Delta x_i b_{qli}$$

$$a_8 = \sum_{i=1}^{n_1} k_{h1}\Delta x_i b_{qlhi}$$

and equating $M_{mob} = M_{res}$ the limit load is given by :

$$q_{\lim} = \frac{a_5 + a_6 - a_1 - a_3 - a_4}{a_2 - a_7 - a_8} \qquad (7)$$

Even if the failure mechanism adopted is quite simple, it allows to investigate a variety of loading and geometric conditions that could have been troublesome using other failure mechanisms and results are in a very good agreement with those obtained by other authors.
In fact, referring to the kinematic effect due to the inertia of the soil mass on the seismic bearing capacity, Figure 2 shows a comparison between the results of the present study (for k_v = 0), those produced by the method proposed by Paolucci & Pecker (1997) and those found by Cascone et al. (2004) with the method of characteristics.

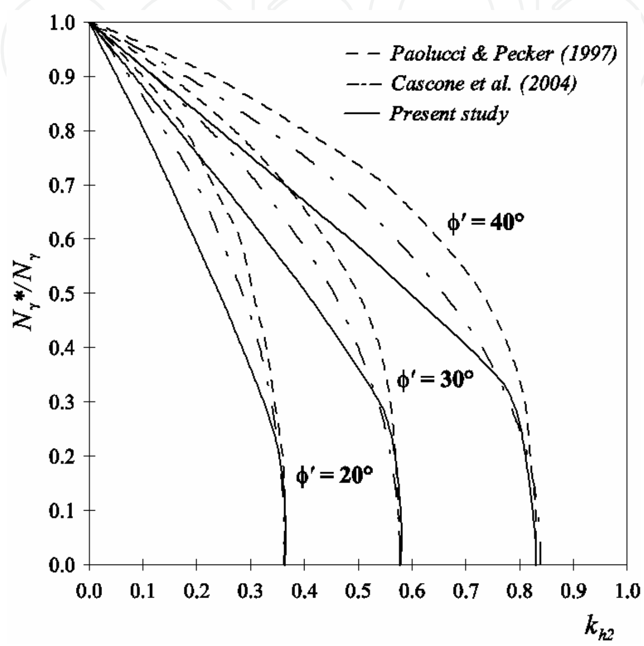


Fig. 2. Seismic ratios as a function of the soil mass inertia

The reduction of the bearing capacity is presented in terms of the ratio N_γ^*/N_γ as a function of the seismic coefficient k_{h2} in the soil mass.
Despite of the different methods, the results obtained are in good agreement. However, for low values of k_{h2} , the limit equilibrium approach seems to give the greatest reduction thus it is on the safe side.

3. Parametric analysis

To investigate the influence of the depth factor on the seismic stability of a shallow foundation, the model proposed has been applied and an upper bound solution of the limit load is found. Results of the analysis are given in terms of the ratios between seismic and static bearing capacity factors N_c^*/N_c , N_q^*/N_q and N_γ^*/N_γ .
Ground factors and bearing capacity ratios are presented as a function of the friction angle of soil ϕ' and of the ratio H/B between the embedment depth H and the width of the footing B. The inertial and kinematic effects due to seismic loading have been analyzed in the evaluation of the seismic bearing capacity.
For a shallow foundation resting on a cohesionless soil, with horizontal ground surface and in absence of surcharge, the limit load can be expressed by :

$$q_{\lim} = 1/2\, B\gamma N_\gamma i_{\gamma_i} i_{\gamma_k} d_\gamma \tag{8}$$

where:

- B = width of the footing;
- γ = unit weight of soil;
- N_γ = bearing capacity factor;
- $i_{\gamma i}$ = load inclination factor due to the inertia of the structure;
- $i_{\gamma k}$ = reduction factor due to the inertia of the soil mass (*kinematic interaction factor*);
- d_γ= depth factor.

The load inclination factor related to the inertia of the structure ($i_{\gamma i}$) has been discussed by some authors (Pecker & Salencon, 1991; Budhu & Al-Karni, 1993; Dormieux & Pecker, 1995; Paolucci & Pecker, 1997; Fishmann et al., 2003), while less information are available on the depth factor (d_γ) and on the reduction factor due to the inertia of the soil mass ($i_{\gamma k}$ = kinematic interaction factor).

Conventionally, the depth factor (d_γ) is assumed equal to unit (Brinch Hansen, 1970). Nevertheless, in an analysis in which the effects due to the inertia of the soil mass are taken into consideration, it is also necessary to take into account the inertia of the soil mass corresponding to the embedment depth H of the footing.

3.1 Depth factor evaluation

In static conditions the depth factor d_γ has been evaluated by a parametric analysis, for both drained (ϕ' = 20°, 30°, 40°) and undrained conditions (ϕ = 0), varying the ratio H/B between the embedment depth H and the width of the footing B.

In the present analysis, the depth factor d_γ for drained conditions is defined as the ratio between the bearing capacity factors N_γ' of a shallow foundation with embedment H and the conventional bearing capacity N_γ of a shallow foundation with an embedment equal to 0 :

$$d_\gamma = N_\gamma ' / N_\gamma \tag{9}$$

Similarly, for undrained conditions the depth factor d_γ° is defined as the difference between the bearing capacity factors $N_\gamma^\circ{}'$ of a shallow foundation with an embedment H and the conventional bearing capacity N_γ° of a shallow foundation with an embedment equal to 0 :

$$d_\gamma^\circ = (N_\gamma^\circ{}' - N_\gamma^\circ) = N_\gamma^\circ{}' \tag{10}$$

being N_γ° in undrained conditions, as known, equal to 0.

With reference to equation (10) in Table 1 are reported, as an example, the values of the depth factor d_γ° for the undrained conditions.

H/B	d_γ° *(eq.10)*
0	0
0.25	0.562
0.5	1.25
0.75	2.062
1.0	3.0

Table 1. Values of the depth factor d_γ° for the undrained conditions

In Figure 3 are reported the values of the depth factor d_γ for drained conditions versus H/B. For the values of the friction angle of soil ϕ' taken into consideration, curves shown approximately a linear trend, thus it is possible to express the depth factor d_γ as a linear function of the ratio H/B according to the equation :

$$d_\gamma = 1 + [(0.85H / B)\cot g\varphi^{'}] \tag{11}$$

For undrained conditions the results obtained (Figure 4) can be conveniently expressed by the following linear equation :

$$d_\gamma^{\circ} = (2H / B) - 0.25 \tag{12}$$

that, obviously, is valid for $H/B > 0.125$.

3.2 Kinematic interaction factor evaluation

The kinematic interaction factor $i_{\gamma k}$ has been evaluated by a parametric analysis only for drained conditions, varying the friction angle of soil in the range ϕ' = 20°, 30° and 40° and the horizontal seismic coefficient for the soil mass k_{h2} between 0.1 up to 0.3.

The kinematic interaction factor $i_{\gamma k}$ is defined in this study as the ratio between the bearing capacity factor N_γ^* derived for a given value of the horizontal seismic coefficient k_{h2}, and the conventional bearing capacity factor N_γ :

$$i_{\gamma k} = N_\gamma^{\ *} / N_\gamma \tag{13}$$

The numerical analyses have been carried out assuming a vertical seismic coefficient for the soil mass k_v equal to ½ k_{h2} . In Figure 5 are reported the values of the kinematic interaction factor $i_{\gamma k}$ obtained for the soil friction angles taken into consideration.

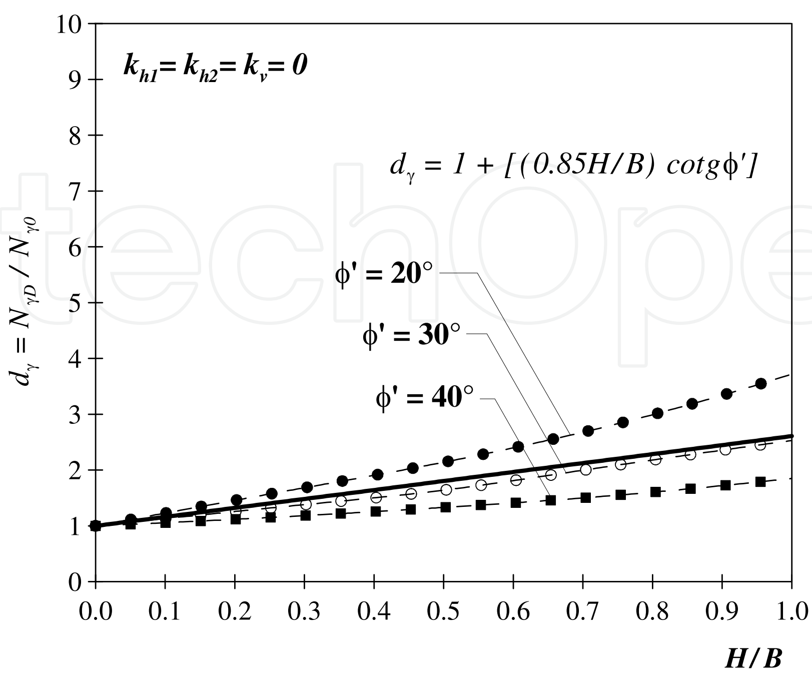


Fig. 3. Depth factor d_γ for drained conditions

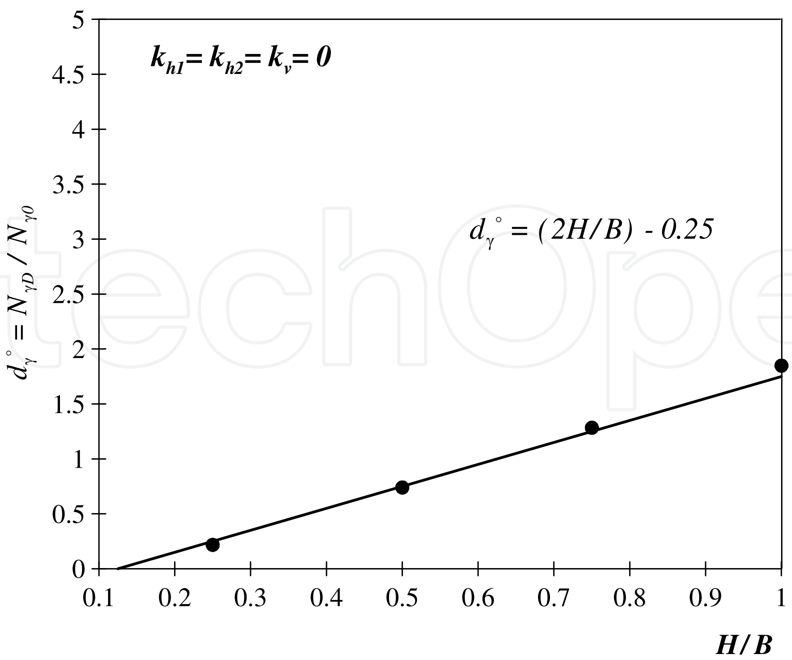


Fig. 4. Depth factor d_γ° for undrained conditions

Curves shown approximately a linear trend, thus it is possible to express the kinematic interaction factor $i_{\gamma k}$ as a linear function of k_{h2} by the following equation :

$$i_{\gamma k} = 1 - k_{h2} \cot g\phi' \tag{14}$$

It is simple to verify that for $k_{h2} = tan\ \phi'$ the kinematic interaction factor $i_{\gamma k}$ is equal to 0.

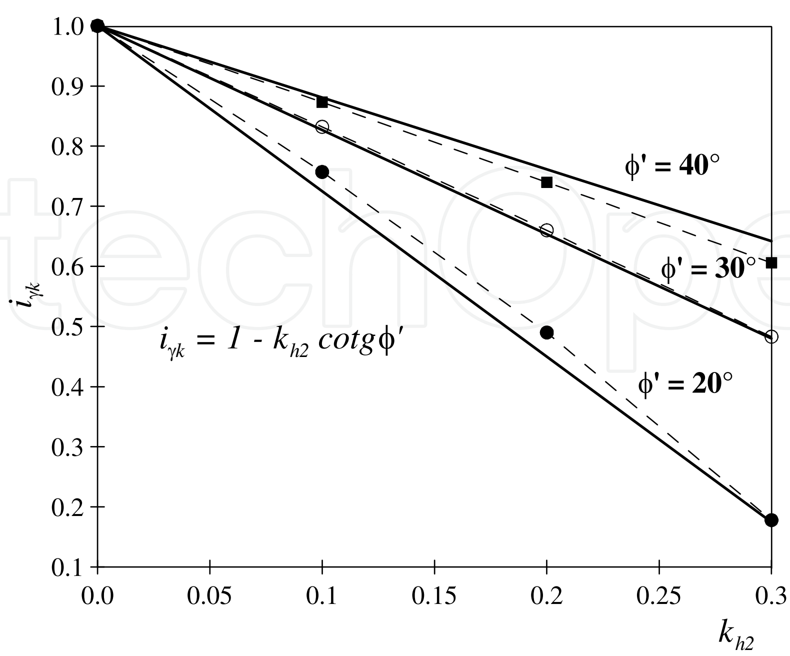


Fig. 5. Values of the kinematic interaction factor $i_{\gamma k}$

4. Bearing capacity of strip footings near slopes

When a shallow foundation is placed near the edge of a sloping ground the bearing capacity may be reduced both in static (De Buhan & Gaernier, 1988; Saran et al., 1989; Narita & Yamaguchi, 1990; Shields et al., 1990; Jao et al., 2001) and seismic conditions (Sawada et al., 1994; Pecker, 1996; Sarma & Iossifelis, 1990; Sarma & Chen, 1996; Kumar & Rao, 2003).
In this case, the failure mechanism is influenced by the distance *d* of the foundation from the edge of the sloping ground (Figure 6). If the shallow foundation is far enough from the edge, the failure mechanism will be not affected by the slope.
In the last decades extensive studies have been made for two dimensional problems of a strip footing resting on inclined slope surface so that different methods of analysis are available.
In these studies, the bearing capacity evaluation is based on the assumption that a failure surface can develop beneath the foundation, according to a general shear failure surfaces given by the limit equilibrium method or by kinematic mechanisms.
General shear failure is characterized by the existence of a well-defined failure pattern (Terzaghi, 1943), which consists of a continuous slip surface from one edge of the footing to the horizontal or inclined ground surface. Referring to Figure 6, the mobilizing moment is :

$$\begin{aligned} M_{mob} &= \sum_{i=1}^{n_{tot}} W_i(1-k_v)b_{wi} + q_{\lim}\sum_{i=1}^{n_1}\Delta x_i b_{qli} + \sum_{i=n_1+1}^{n_{tot}} q_v(1-k_v)b_{qvi}\Delta x_i + \\ &+ q_{\lim}\sum_{i=1}^{n_1} k_{h1}\Delta x_i b_{qlhi} + \sum_{i=1}^{n_{tot}} k_{h2}W_i b_{whi} + \sum_{i=n_1+1}^{n_{tot}} k_{h3}q_v b_{qhi} \end{aligned} \quad (15)$$

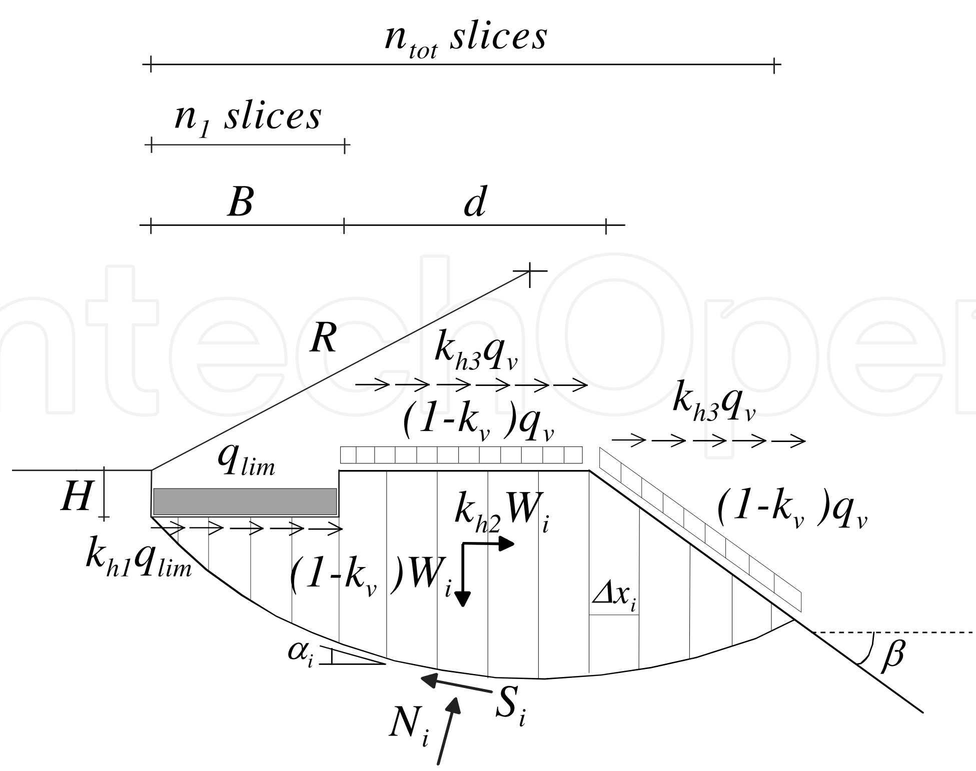


Fig. 6. Strip footings near slopes: failure mechanism and applied forces

The resisting moment M_{res} and the shear strength S_i acting along the base of the slices are expressed by equation (2) and (3) respectively.
For the slices $i = 1$ to n_1 the force N_i resultant of the normal stress distribution acting at the base of the slice, derived by the Bishop's method of slices (1955) with an equilibrium equation in the vertical direction, is given by equation (4), while for the remaining slices ($n_1 + 1 \leq i \leq n_{tot}$) is :

$$N_i = \frac{q_v(1-k_v)\Delta x_i + W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} \tag{16}$$

Thus :

$$\begin{aligned} M_{res} = R\sum_{i=1}^{n_{tot}} c\Delta x_i / \cos\alpha_i + R\sum_{i=1}^{n_1} \frac{q_{\lim}\Delta x_i + W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} \tan\varphi + \\ + \sum_{i=n_1+1}^{n_{tot}} \frac{q_v(1-k_v)\Delta x_i + W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} R\tan\phi \end{aligned} \tag{17}$$

where :

- q_{lim} = vertical limit load acting on the footing;
- c = soil cohesion;
- ϕ' = friction angle of soil;
- Δx_i = width of the ith slice;
- W_i = weight of the ith slice;
- q_v = vertical surcharge;
- R = radius of the circular failure surface;
- α_i = angle of the base of the ith slice;
- n_1 = number of slices under the footing;
- n_{tot} = total number of slices;
- k_{h1} = horizontal seismic coefficient for the limit load;
- k_{h2} = horizontal seismic coefficient for the soil mass;
- k_{h3} = horizontal seismic coefficient for the surcharge;
- k_v = vertical seismic coefficient for the soil mass and the surcharge;
- b_{wi} = distance of the weight W_i of the ith slice to the centre of the circular failure surface;
- b_{whi} = distance of the inertia force $k_{h2}W_i$ of the ith slice to the centre of the circular failure surface;
- b_{qvi} = distance of the surcharge force $q_v\Delta x_i$ of the ith slice to the centre of the circular failure surface;
- b_{qhi} = distance of the horizontal surcharge force $k_{h3}q_v\Delta x_i$ of the ith slice to the centre of the circular failure surface;
- b_{qli} = distance of the limit load q_{lim} acting on the ith slice to the centre of the circular failure surface;
- b_{qhli} = distance of the shear limit force $k_{h1}q_{lim}$ acting on the ith slice to the centre of the circular failure surface.

Substituting the following terms :

$$a_1 = R\sum_{i=1}^{n_{tot}} c\Delta x_i / \cos\alpha_i \tag{18}$$

$$a_2 = R\tan\phi \sum_{i=1}^{n_1} \frac{\Delta x_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} \tag{19}$$

$$a_3 = R\tan\phi \sum_{i=1}^{n_1} \frac{W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} \tag{20}$$

$$a_4 = R\tan\phi \sum_{i=n_1+1}^{n_{tot}} \frac{q_v(1-k_v)\Delta x_i + W_i(1-k_v) - c\Delta x_i \tan\alpha_i}{\cos\alpha_i + \sin\alpha_i \tan\phi} \tag{21}$$

$$a_5 = \sum_{i=1}^{n_{tot}} W_i(1-k_v)b_{wi} \tag{22}$$

$$a_6 = \sum_{i=1}^{n_{tot}} k_{h2} W_i b_{whi} \tag{23}$$

$$a_7 = \sum_{i=n_1+1}^{n_{tot}} q_v(1-k_v)\Delta x_i b_{qvi} \tag{24}$$

$$a_8 = \sum_{i=n_1+1}^{n_{tot}} k_{h3} q_v \Delta x_i b_{qhi} \tag{25}$$

$$a_9 = \sum_{i=1}^{n_1} \Delta x_i b_{qli} \tag{26}$$

$$a_{10} = \sum_{i=1}^{n_1} k_{h1} \Delta x_i b_{qlhi} \tag{27}$$

and equating $M_{mob} = M_{res}$ the limit load is given by :

$$q_{\lim} = \frac{a_5 + a_6 + a_7 + a_8 - a_1 - a_3 - a_4}{a_2 - a_9 - a_{10}} \tag{28}$$

For example, referring to the ground slope factor g_γ taking into account the effect of the sloping ground surface, in Figure 7 the values derived assuming the distance d equal to zero and the slope angle $\beta > 0$ (angle that the ground surface makes with the horizontal), have been evaluated for three different friction angles (ϕ = 20°, 30° and 40°) and compared with those obtained by the well known Brinch Hansen's solution (1970).
The angle β is positive when the ground slopes down and away from the footing. According to Brinch Hansen (1970) we have :

$$g_\gamma = (1 - 0.5\tan\beta)^5 \tag{29}$$

while in the present study we obtain :

$$g_\gamma = (1 - 0.5\tan\beta)^{4.5} \tag{30}$$

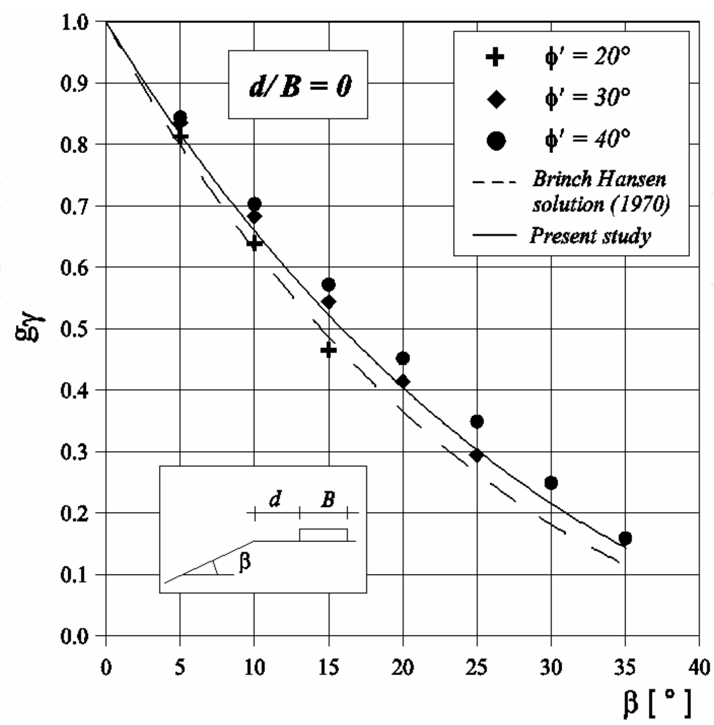


Fig. 7. Values of the ground factors g_γ and comparison with Brinch Hansen's solution (1970)

4.1 Seismic analysis

The recommendations of Eurocode 8 - Part 5 (2003) state that in the calculation of the bearing capacity of a shallow foundation one should include the load inclination and eccentricity arising from the inertia forces of the structure, as well as the possible effects of the inertia in the soil.

Thus, the seismic analysis was carried out considering the following seismic coefficients: k_{h1} = 0.1 and 0.2 for the inertia of the structure; k_{h2} = 0.2 and 0.4 for the inertia of the soil mass.

The value of k_{h1} was chosen lower than k_{h2} because the Eurocode 8 (2003) allows to reduce the seismic action by a behaviour factor associated with the ductility classification of the structures.

This consideration takes to the conclusion that the kinematic effect, and the consequent reduction in bearing capacity due to the soil inertia, cannot be neglected and in some circumstances it's reduction could be more significant than the reduction due to the inertia of the structure (Cascone et al., 2006). In this study the seismic coefficient k_{h3} of the surcharge was assumed equal to k_{h1}.

The friction angle of soil was chosen in the range 0° up to 40°, while the angle of the slope near the footing was varied in the range 5° to 35°. In the seismic analysis the angle of the sloping ground is affected by further limitations, because simple equilibrium considerations, for a cohesionless soil (c' = 0), take to the following :

$$\tan\beta < \frac{(1-k_v)\tan\phi' - k_{h,i}}{1 - k_v + k_{h,i}\tan\phi'} \quad (i = 2, 3) \tag{31}$$

or in a simpler form :

$$\beta < \phi' - \theta \tag{32}$$

where :

$$\theta = \tan^{-1}\left[\frac{k_{h,i}}{1-k_v}\right] \text{ (i = 2, 3)} \tag{33}$$

In the following, Table 2 shows some limit values of β, for $k_v = 0$ and $\phi' = 20°$, 30° and 40°.

ϕ'	k_{h2} , k_{h3}			
	0.1	0.2	0.3	0.4
20°	14.29°	8.69°	3.30°	-
30°	24.29°	18.69°	13.30°	8.19°
40°	34.29°	28.69°	23.30°	18.19°

Table 2. Limit values of β for a vertical seismic coefficient $k_v = 0$

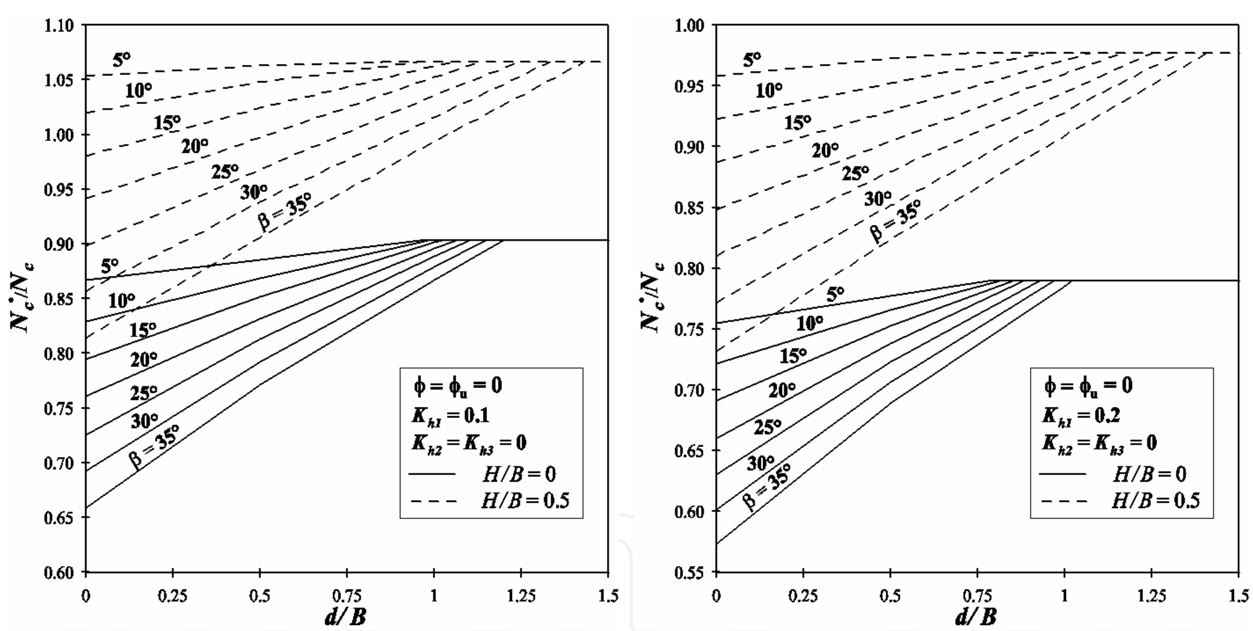
In Figures 8 to 16 the results of the parametric analysis are shown in a synthetic form. The seismic bearing capacity ratios N_c^*/N_c , N_q^*/N_q , N_γ^*/N_γ are represented as a function of d/B for various slope angles and for different values of the friction angle of soil.
The threshold distance (d_t) at which the sloping ground does not affect anymore the bearing capacity mainly increases with the increasing of the angle of friction and secondarily with the increasing of the seismic coefficient and with the increasing of the slope angle β.
The embedment depth of the footing does not play a significant role on the threshold distance, however it may produce a considerable increasing of the bearing capacity.
Referring to the N_c^*/N_c ratios, we can observe values of the normalized threshold distances varying between about $d_t/B = 1$, for an undrained analysis ($\phi_u = 0°$), and $d_t/B = 5$ for $\phi' = 40°$.
For the N_q^*/N_q ratios, we determined values of the normalized threshold distances varying between about $d_t/B = 2$, for $\phi' = 20°$ and about $d_t/B = 4$ for $\phi' = 40°$.
Finally for the N_γ^*/N_γ ratios, we determined values of the normalized threshold distances varying between about $d_t/B = 1.5$ for $\phi' = 20°$ and about $d_t/B = 4$ for $\phi' = 40°$.
No significant difference in the threshold distance was found when the inertia of the structure or the inertia of the soil mass is considered.
Furthermore the combined effects of soil and structure inertia can be taken into account by using the superposition of the effects principle.
In this case, at the same way as found by Paolucci & Pecker (1997) and Cascone et al. (2004), the bearing capacity of the soil self weight under both the seismic loading due to the coefficients k_{h1} and k_{h2} , can be evaluated through the following equation :

$$q_{\lim} = \frac{1}{2}B\gamma N_{\gamma_e} \approx \frac{1}{2}B\gamma N_\gamma e_{\gamma_i} e_{\gamma_k} \tag{34}$$

where :

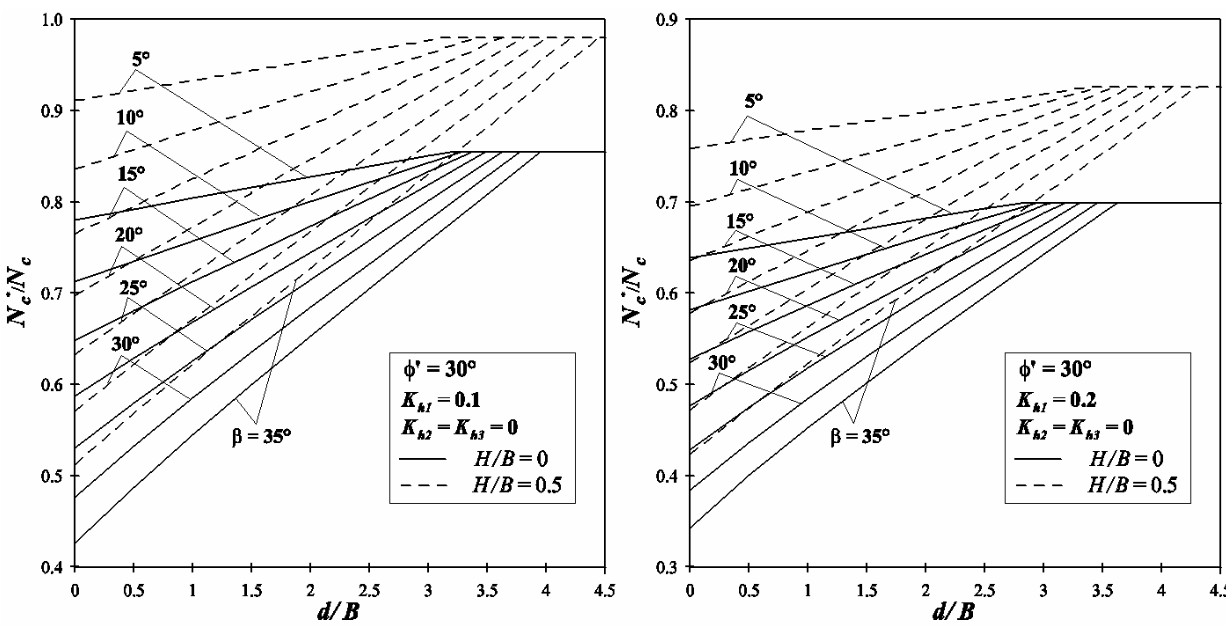
- $N_{\gamma e}$ = bearing capacity factor reduced by both acting the coefficients k_{h1} and k_{h2} ;
- N_γ = static bearing capacity factor;
- $e_{\gamma i} = N_{\gamma 1}^*/N_\gamma$ bearing capacity ratio for structure inertia only ($k_{h1} > 0$, $k_{h2} = k_{h3} = 0$);
- $e_{\gamma k} = N_{\gamma 2}^*/N_\gamma$ bearing capacity ratio for soil mass inertia only ($k_{h2} > 0$, $k_{h1} = k_{h3} = 0$).

Figure 17 shows a comparison between the $N_{\gamma e}/N_\gamma$ ratio and the product $e_{\gamma i} \cdot e_{\gamma k}$ for, as an example, k_{h1} = 0.1, 0.2 and 0.3 and k_{h2} = 0.1, 0.2 and 0.3.
In particular, Figure 17 shows that when the seismic coefficients k_{h1} and k_{h2} are small, there is not a significant difference between the $N_{\gamma e}/N_\gamma$ ratio and the product $e_{\gamma i} \cdot e_{\gamma k}$.
On the contrary, when the seismic coefficients are high enough to produce a great reduction of the limit load, one can find a great difference in using the $N_{\gamma e}/N_\gamma$ ratio instead of the product $e_{\gamma i} \cdot e_{\gamma k}$.
As example, for ϕ' = 20°, k_{h1} = 0.1 and k_{h2} = 0.1, we have :
$N_{\gamma e}/N_\gamma$ = 0.565 and $e_{\gamma i} \cdot e_{\gamma k}$ = 0.587
while for ϕ' = 20°, k_{h1} = 0.3 and k_{h2} = 0.3, we have :
$N_{\gamma e}/N_\gamma$ = 0.05 and $e_{\gamma i} \cdot e_{\gamma k}$ = 0.096.



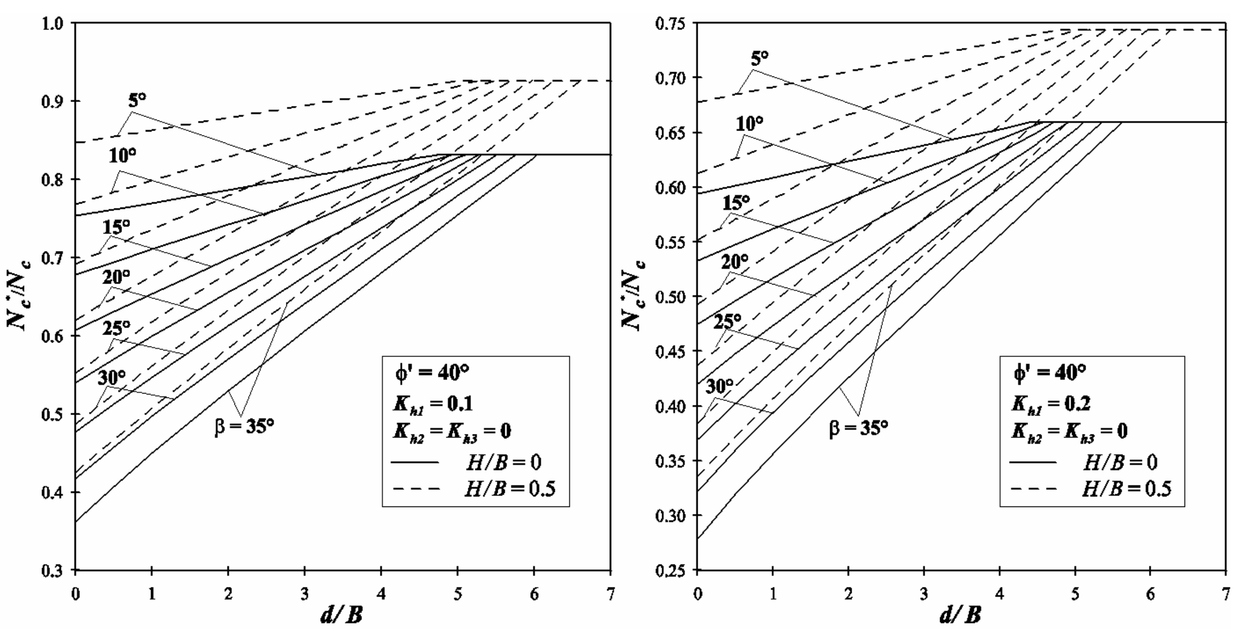
a b

Fig. 8. N_c^*/N_c ratios as a function of the normalized *d/B* slope distance (undrained analysis $\phi = \phi_u = 0$) for k_{h1} = 0.1 (*a*) and k_{h1} = 0.2 (*b*)



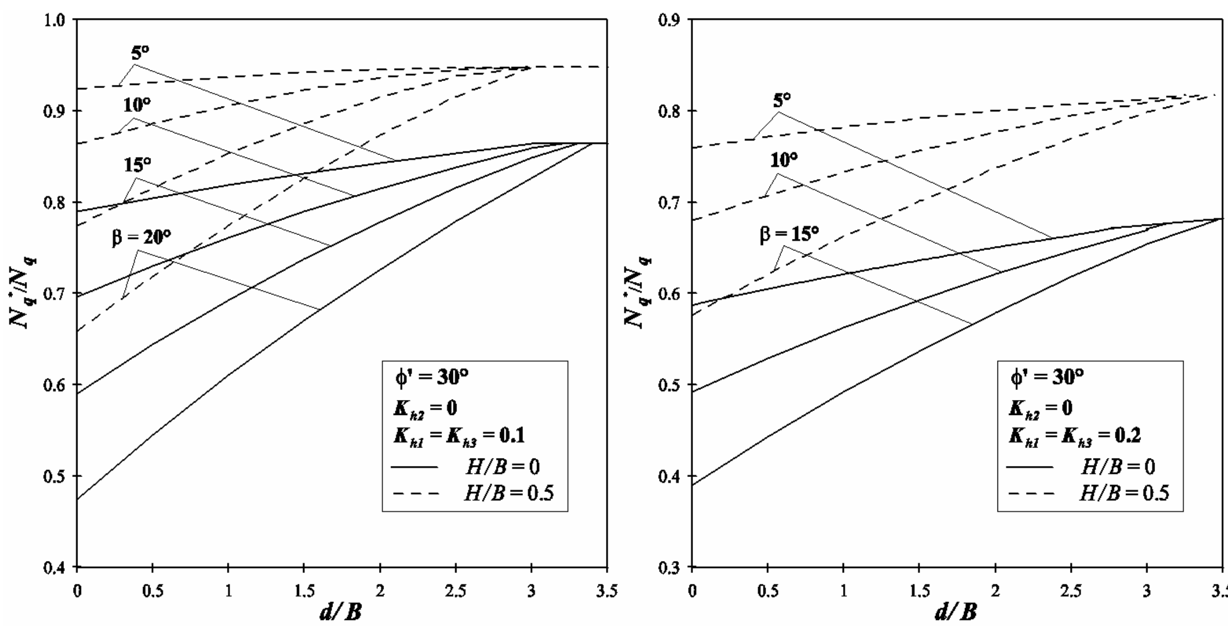
a b

Fig. 9. N_c^*/N_c ratios as a function of the normalized *d/B* slope distance when $\phi' = 30°$ and $k_{h1} = 0.1$ (*a*) and $k_{h1} = 0.2$ (*b*)



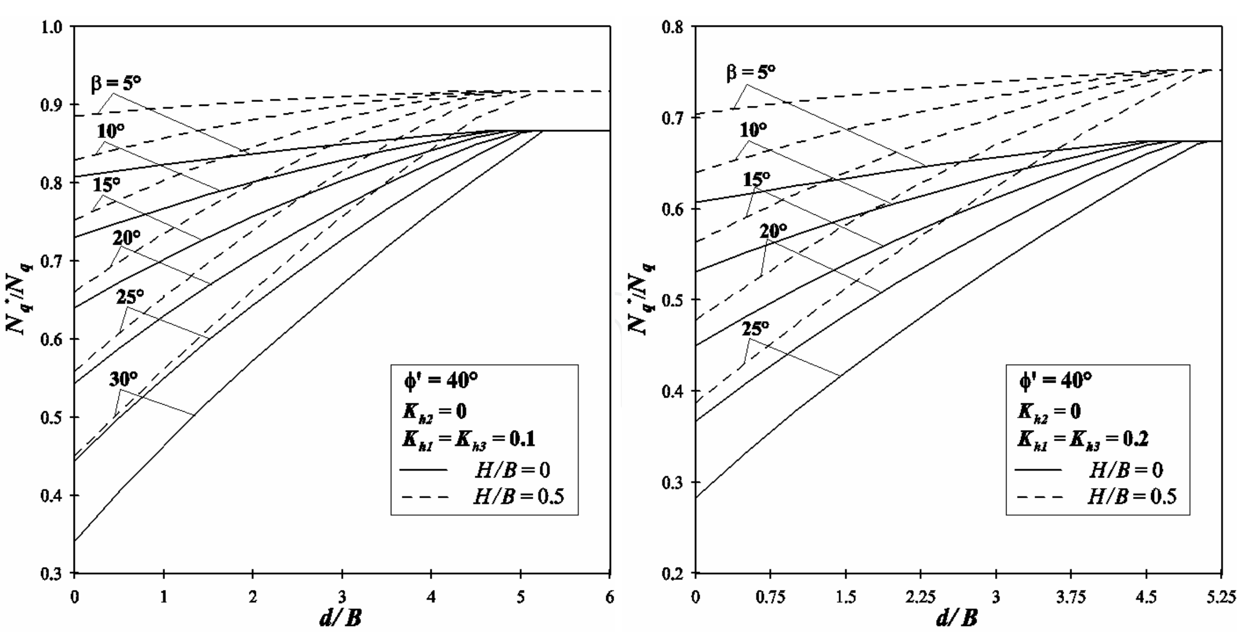
a b

Fig. 10. N_c^*/N_c ratios as a function of the normalized *d/B* slope distance when $\phi' = 40°$ and $k_{h1} = 0.1$ (*a*) and $k_{h1} = 0.2$ (*b*)



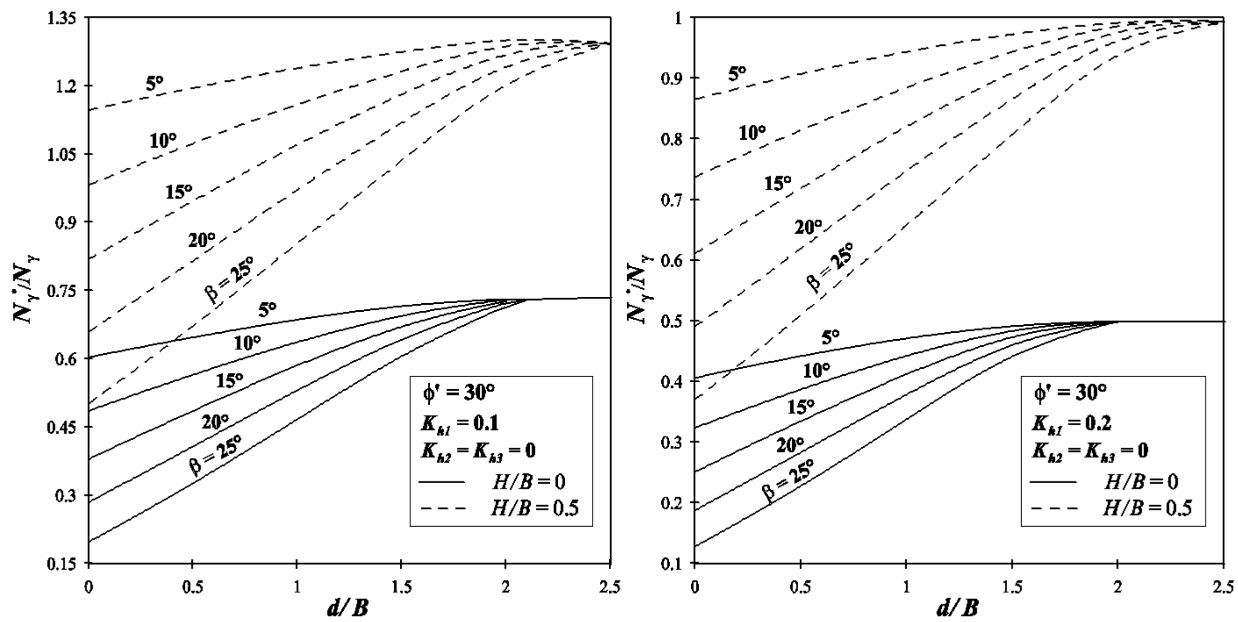
a b

Fig. 11. N_q^*/N_q ratios as a function of the normalized *d/B* slope distance when $\phi' = 30°$ and $k_{h1} = k_{h3} = 0.1$ (*a*) and $k_{h1} = k_{h3} = 0.2$ (*b*)



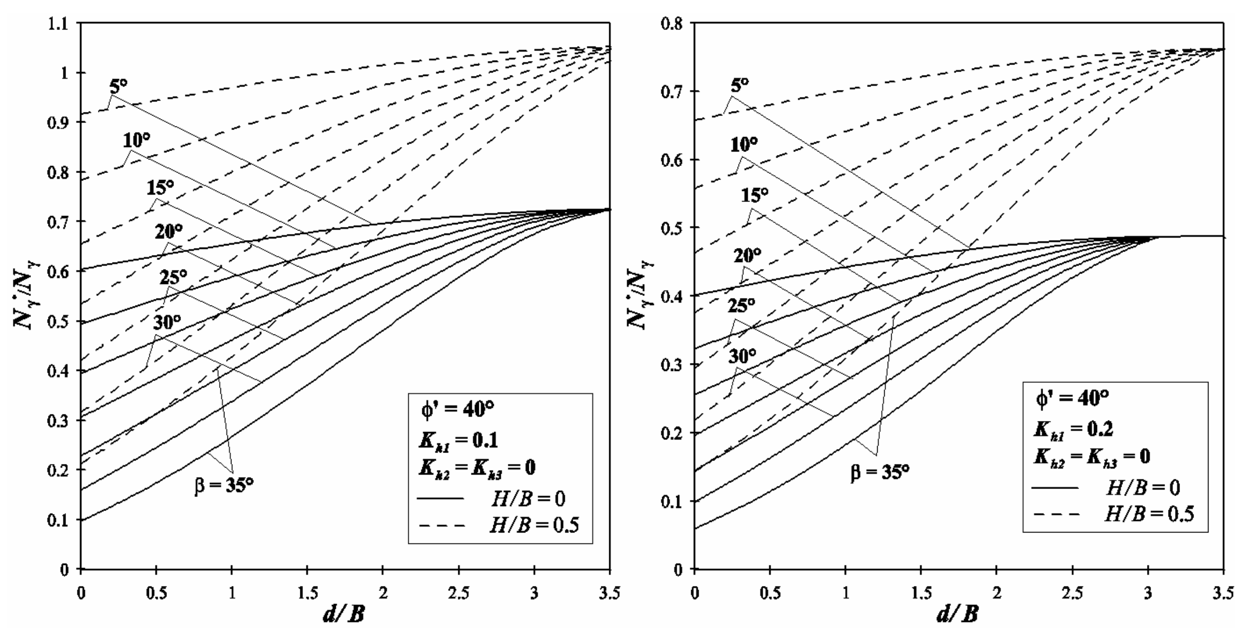
a b

Fig. 12. N_q^*/N_q ratios as a function of the normalized *d/B* slope distance when $\phi' = 40°$ and $k_{h1} = k_{h3} = 0.1$ (*a*) and $k_{h1} = k_{h3} = 0.2$ (*b*)



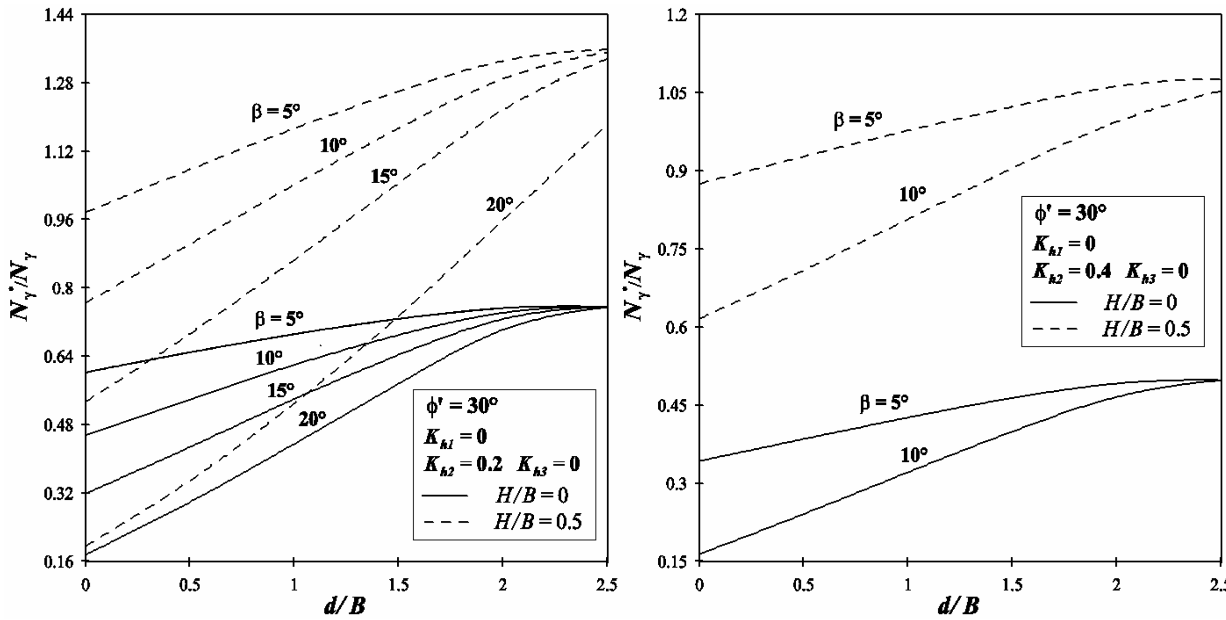
a b

Fig. 13. N_γ^*/N_γ ratios for structural inertia as a function of the normalized *d/B* slope distance when ϕ' = 30° and k_{h1} = 0.1 (*a*) and k_{h1} = 0.2 (*b*)



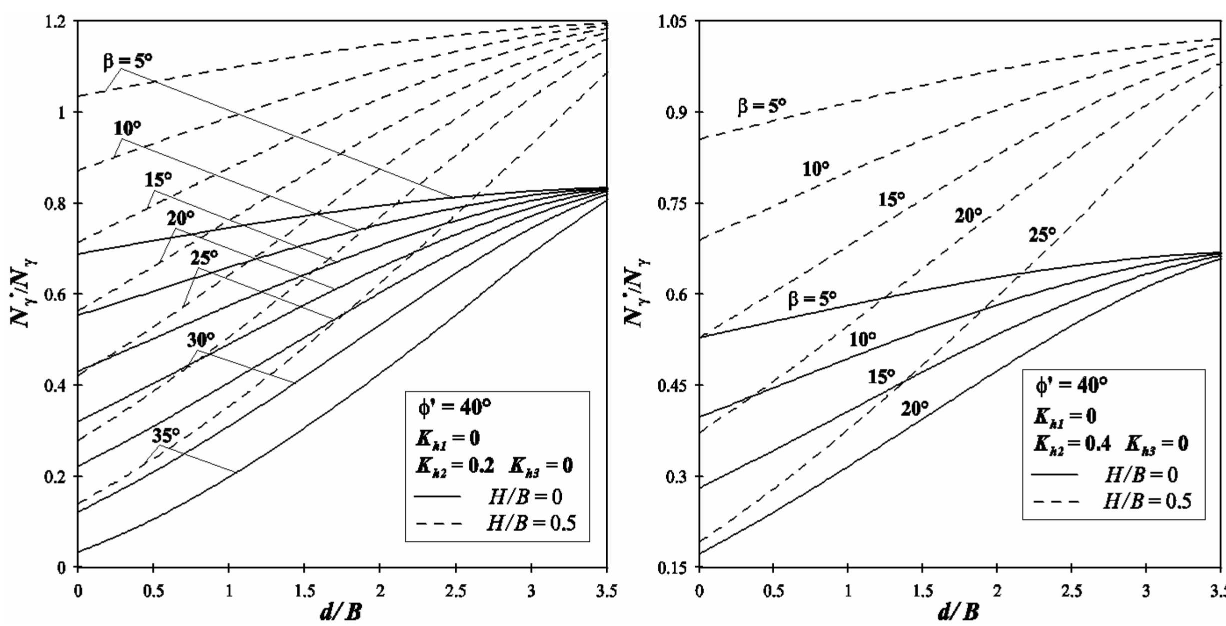
a b

Fig. 14. N_γ^*/N_γ ratios for structural inertia as a function of the normalized *d/B* slope distance when ϕ' = 40° and k_{h1} = 0.1 (*a*) and k_{h1} = 0.2 (*b*)



a b

Fig. 15. N_γ^*/N_γ ratios for soil inertia as a function of the normalized *d/B* slope distance when ϕ' = 30° and k_{h2} = 0.2 (*a*) and k_{h2} = 0.4 (*b*)



a b

Fig. 16. N_γ^*/N_γ ratios for soil inertia as a function of the normalized *d/B* slope distance when ϕ' = 40° and k_{h2} = 0.2 (*a*) and k_{h2} = 0.4 (*b*)

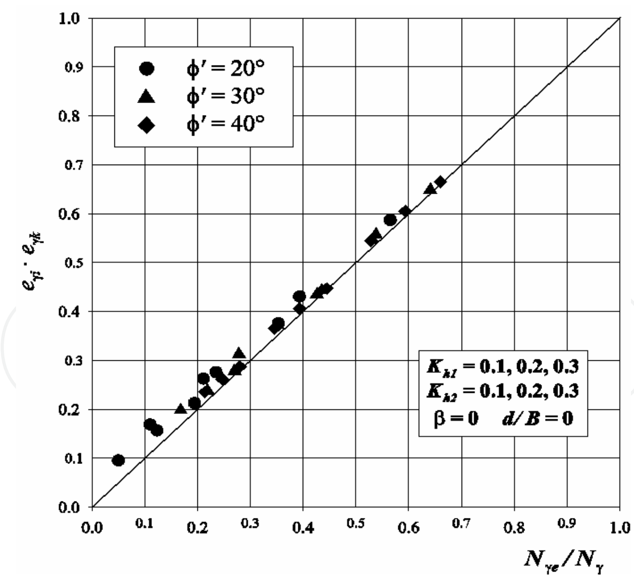


Fig. 17. Comparison between the ratio $N_{\gamma e}/N_{\gamma}$ and the product $e_{\gamma 1} \cdot e_{\gamma 2}$

This happens because, when the limit load approaches to zero, the critical surface associated to the simultaneous presence of both the inertial and the kinematic effects is significantly different from that deduced when one considers separately the inertial and the kinematic effects. In this case the superposition of the effects principle may lead to an unconservative design, being the product $e_{\gamma i} \cdot e_{\gamma k}$ significantly greater than the $N_{\gamma e}/N_{\gamma}$ ratio.

5. Conclusions

The design of shallow foundations subject to different static loadings has been an important area of research for geotechnical engineers. The devastating effects of recent earthquakes on shallow foundations has increased the complexity of the problem. Consequently, it is useful to obtain closed-form solutions for the earthquake resistant design of foundations.
Many analytical and numerical solutions are available for the computation of the seismic bearing capacity factors required for the design of shallow foundations.
In the present study the seismic bearing capacity of shallows foundation has been evaluated with the limit equilibrium method.
Numerical analysis shows that, by considering pseudo-static seismic forces, design solutions can be found for the computing of seismic bearing capacity factors for shallow foundations embedded in both horizontal and sloping ground.
Seismic bearing capacity factors with respect to cohesion, surcharge and unit weight components have been computed for a wide range of variation in parameters such as soil friction angle (ϕ'), horizontal and vertical seismic coefficients (k_h and k_v).
An "upper bound" approach of the limit load was adopted to evaluate the seismic reduction factors to take into account the embedment depth of the footing ($i_{\gamma i}$) and the inertia of the soil mass ($i_{\gamma k}$), as well as, the bearing capacity ratio for structure inertia only ($e_{\gamma i}$) and the bearing capacity ratio for soil mass inertia only ($e_{\gamma k}$).
Some considerations can be formulated:

- In the evaluation of the bearing capacity due to the soil weight it has been observed that the depth of the embedment depth may play a significant role especially for low values of the friction angle.

- In some cases the seismic reduction in the bearing capacity for the soil inertia (*kinematic effect*) cannot be ignored, being of about the same amount of that produced by the inertia of the structure.
- The bearing capacity factors decrease appreciably with increases in both k_h and k_v. Bearing capacity decreases as ground inclination β increases and as the embedment depth H increases.
- The superposition of the effects principle can be applied to determine the reduced bearing capacity caused by both the seismic actions. However, when the seismic reduction is great, due to high seismic coefficients k_{h1} and k_{h2}, the superposition of the effects principle may lead to an unconservative design.
- By the simple limit equilibrium method modified bearing capacity factors and simple relations have been proposed which can be used for the practical design of shallow foundations embedded in both horizontal and sloping soil. In many cases the solutions obtained compare well with the previous static results and available results for the seismic conditions.

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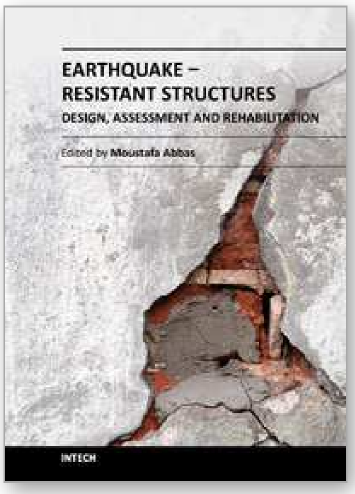

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This book deals with earthquake-resistant structures, such as, buildings, bridges and liquid storage tanks. It contains twenty chapters covering several interesting research topics written by researchers and experts in the field of earthquake engineering. The book covers seismic-resistance design of masonry and reinforced concrete structures to be constructed as well as safety assessment, strengthening and rehabilitation of existing structures against earthquake loads. It also includes three chapters on electromagnetic sensing techniques for health assessment of structures, post earthquake assessment of steel buildings in fire environment and response of underground pipes to blast loads. The book provides the state-of-the-art on recent progress in earthquake-resistant structures. It should be useful to graduate students, researchers and practicing structural engineers.