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Reflection and Transmission of a Plane TE-Wave at a Lossy, Saturating, Nonlinear Dielectric Film

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1. Introduction

Reflection and transmission of transverse-electric (TE) electromagnetic waves at a single nonlinear homogeneous, isotropic, nonmagnetic layer situated between two homogeneous, semi-infinite media has been the subject of intense theoretical and experimental investigations in recent years. In particular, the Kerr-like nonlinear dielectric film has been the focus of a number of studies in nonlinear optics (Chen & Mills, 1987; 1988; Leung, 1985; 1988; Peschel, 1988; Schürmann & Schmoldt, 1993).

Exact analytical solutions have been obtained for the scattering of plane TE-waves at Kerr-nonlinear films (Leung, 1989; Schürmann et al., 2001). As far as exact analytical solutions were considered in these articles absorption was excluded, at most it was treated numerically (Gordillo-Vázquez & Pecharromán, 2003; Schürmann & Schmoldt, 1996; Yuen & Yu, 1997).

As Chen and Mills have pointed out it is a nontrivial extension of the usual scattering theory to include absorption (Chen & Mills, 1988) and it seems (to the best of our knowledge) that the problem was not solved till now. In the following we consider a nonlinear lossy dielectric film with spatially varying saturating permittivity. In Section 2 we reduce Maxwell's equations to a Volterra integral equation (14) for the intensity of the electric field $E(y)$ and give a solution in form of a uniform convergent sequence of iterate functions. Using these solutions we determine the phase function $\vartheta(y)$ of the electric field, and, evaluating the boundary conditions in Section 3, we derive analytical expressions for reflectance, transmittance, absorptance, and phase shifts on reflection and transmission and present some numerical results in Section 4.

It should be emphasized, that the contraction principle (that is used in this work) (Zeidler, 1995) includes the proof of the existence of the exact bounded solution of the problem and additionally yields approximate analytical solutions by iterations. Furthermore, the rate of convergence of the iterative procedure and the error estimate can be evaluated (Zeidler, 1995). Thus this approach is useful for physical applications.

The present approach can be applied to a linear homogeneous, isotropic, nonmagnetic layer with absorption. In this case the problem is reduced to a linear Volterra integral equation that can be solved by iterations without any restrictions. The lossless linear permittivity as well as the Kerr-like permittivity can be treated as particular cases of the approach.

Referring to figure 1 we consider a dielectric film between two linear semi-infinite media (substrate and cladding). All media are assumed to be homogeneous in $x-$ and $z-$ direction, isotropic, and non-magnetic. The film is assumed to be absorbing and characterized by a complex valued permittivity function $\varepsilon_f(y)$.

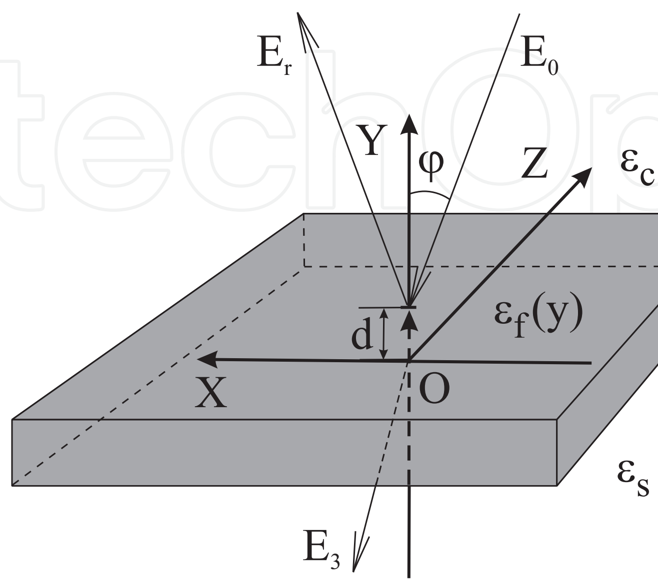

Fig. 1. Configuration considered in this paper. A plane wave is incident to a nonlinear slab (situated between two linear media) to be reflected and transmitted.

A plane wave of frequency ω_0 and intensity E_0^2, with electric vector $\mathbf{E_0}$ parallel to the z-axis (TE) is incident on the film of thickness d. Since the geometry is independent of the z-coordinate and because of the supposed TE-polarization fields are parallel to the z-axis ($\mathbf{E} = (0, 0, E_z)$). We look for solution $\mathbf{E}$ of Maxwell's equations

$$rot\mathbf{H} = -i\omega_0\varepsilon\mathbf{E}$$

$$rot\mathbf{E} = i\omega_0\mu_0\mathbf{H}$$

that satisfy the boundary conditions (continuity of E_z and $\partial E_z/\partial y$ at interfaces $y \equiv 0$ and $y \equiv d$) and where (due to TE-polarization) $\mathbf{H} = (H_x, H_y, 0)$. Due to the requirement of the translational invariance in x-direction and partly satisfying the boundary conditions the fields tentatively are written as ($\hat{z}$ denotes the unit vector in z-direction)

$$\mathbf{E}(x,y,t) = \begin{cases} \hat{z}\left[E_0 e^{i(px - q_c\cdot(y-d) - \omega_0 t)} + \right. \\ \quad \left. E_r e^{i(px + q_c\cdot(y-d) - \omega_0 t)}\right], & y > d, \\ \hat{z}\left[E(y) e^{i(px + \vartheta(y) - \omega_0 t)}\right], & 0 < y < d, \\ \hat{z}\left[E_3 e^{i(px - q_s y - \omega_0 t)}\right], & y < 0, \end{cases} \tag{1}$$

where $E(y)$, $p = \sqrt{\varepsilon_c}k_0 \sin\varphi$, $k_0 = \omega_0\sqrt{\varepsilon_0\mu_0}$, q_c, q_s, and $\vartheta(y)$ are real and $E_r = |E_r|\exp(i\delta_r)$ and $E_3 = |E_3|\exp(i\delta_t)$ are independent of y.

We assume a permittivity $\varepsilon(y)$ of the three layer system modeled by

$$\frac{\varepsilon(y)}{\varepsilon_0} = \begin{cases} \varepsilon_c,\ y > d, \\ \varepsilon_f(y) = \varepsilon_f^0 + \varepsilon_R(y) + i\varepsilon_I(y) + \frac{aE^2(y)}{1+arE^2(y)},\ 0 < y < d, \\ \varepsilon_s,\ y < 0, \end{cases} \tag{2}$$

with real constants $\varepsilon_c, \varepsilon_s, \varepsilon_f^0, a \geq 0, r \geq 0$ and real-valued continuously differentiable functions $\varepsilon_R(y), \varepsilon_I(y)$ on $[0, d]$. A particular nonlinearity in (2) of cubic type ($r = 0$) can be met in the context of a Kerr-like nonlinear dielectric film, while the case when $r > 0$ corresponds to the saturation model in optics (see (Bang et al., 2002; Berge et al., 2003; Dreischuh et al., 1999; Kartashov et al., 2003)).

The problem to be solved is to find a solution of Maxwell's equations subject to (1) and (2). With respect to the physical significance of (1) and (2) some remarks may be appropriate. Though ansatz (1) widely has been used previously (Chen & Mills, 1987; 1988; Leung, 1985; 1988; Peschel, 1988; Schürmann & Schmoldt, 1993) it should be noted that it is based on the assumption that the time-dependence of the optical response of the nonlinear film is described by one frequency ω_0. Phase matching is, e.g., assumed to be absent so that small amplitudes of higher harmonics can be neglected. The permittivity function (2) also represents an approximation. The dipole moment per unite volume and hence the permittivity is not simply controlled by the instant value of the electric (macroscopic) field at the point (x, y, z), due to the time lag of the medium's response. Further more the response is nonlocal in space. - The model permittivity (2) does not incorporate these features. Nevertheless, experimental observations (cf., e.g. (Peschel, 1988)) indicate that (2) has physical significance (with $\varepsilon_R = \varepsilon_I = r = 0$). Finally, Maxwell's equations, even for an isotropic material, imply that all field components are coupled if the permittivity is nonlinear. The decomposition into TE-and TM-polarization is an assumption motivated by mathematical simplicity. To apply the results below to experiments it is necessary to make sure that TE-polarization is maintained.

2. Nonlinear Volterra integral equation

By inserting (1) and (2) into Maxwell's equations we obtain the nonlinear Helmholtz equations, valid in each of the three media ($j = s, f, c$),

$$\frac{\partial^2 \tilde{E}_j(x,y)}{\partial x^2} + \frac{\partial^2 \tilde{E}_j(x,y)}{\partial y^2} + k_0^2 \frac{\varepsilon(y)}{\varepsilon_0} \tilde{E}_j(x,y) = 0, \quad j = s, f, c, \tag{3}$$

where $\tilde{E}_j(x, y)$ denotes the time-independent part of $\mathbf{E}(x, y, t)$.

Scaling x, y, z, p, q_c, q_s by k_0 and using the definition of $\frac{\varepsilon}{\varepsilon_0}$ in equation (2), equation (3) reads

$$\frac{\partial^2 \tilde{E}_j(x,y)}{\partial x^2} + \frac{\partial^2 \tilde{E}_j(x,y)}{\partial y^2} + \varepsilon_j(y)\tilde{E}_j(x,y) = 0, \quad j = s, f, c, \tag{4}$$

where the same symbols have been used for unscaled and scaled quantities. Using ansatz (1) in equation (4) we get for the semi-infinite media

$$q_j^2 = \varepsilon_j - p^2, \qquad j = s, c. \tag{5}$$

For the film ($j = f$), we obtain, omitting tildes,

$$\frac{d^2E(y)}{dy^2} - E(y)\left(\frac{d\vartheta(y)}{dy}\right)^2 + \left(\varepsilon_f^0 + \varepsilon_R(y) - p^2 + \frac{aE^2(y)}{1+arE^2(y)}\right)E(y) = 0 \tag{6}$$

and

$$E(y)\frac{d^2\vartheta(y)}{dy^2} + 2\frac{d\vartheta(y)}{dy}\frac{dE(y)}{dy} + \varepsilon_I(y)E(y) = 0. \tag{7}$$

Equation (7) can be integrated leading to

$$E^2(y)\frac{d\vartheta(y)}{dy} = c_1 - \int_0^y \varepsilon_I(\tau)E^2(\tau)d\tau, \tag{8}$$

where c_1 is a constant that is determined by means of the boundary conditions:

$$c_1 = E^2(0)\frac{d\vartheta(0)}{dy} = -q_s E^2(0) \tag{9}$$

Insertion of $d\vartheta(y)/dy$ according to equation (3) leads to

$$\frac{d^2E(y)}{dy^2} + (q_f^2(y) - p^2)E(y) + \frac{aE^3(y)}{1+arE^2(y)} - \frac{(c_1 - \int\limits_0^y \varepsilon_I(t)E^2(t)dt)^2}{E^3(y)} = 0, \tag{10}$$

with

$$q_f^2(y) = \varepsilon_f^0 + \varepsilon_R(y). \tag{11}$$

As for real permittivity, real q_s (transmission) implies $c_1 \neq 0$.

Setting $I(y) = aE^2(y), a \neq 0$, multiplying equation (10) by $4E^3(y)$, and differentiating the result with respect to y we obtain

$$\begin{aligned}\frac{d^3I(y)}{dy^3} + 4\frac{d\left((q_f^2(y) - p^2)I(y)\right)}{dy} &= 2\frac{d(q_f^2(y))}{dy}I(y) \\ &\quad - \frac{2I(y)\frac{dI(y)}{dy}(3+2rI(y))}{(1+rI(y))^2} \\ &\quad -4\varepsilon_I(y)(ac_1 - \int\limits_0^y \varepsilon_I(t)I(t)dt).\end{aligned} \tag{12}$$

Equation (12) can be integrated with respect to $I(y)$ to yield

$$\begin{aligned}\frac{d^2I(y)}{dy^2} + 4\kappa^2I(y) &= -4\varepsilon_R(y)I(y) + 2\int_0^y \frac{d\varepsilon_R(t)}{dt}I(t)dt \\ &\quad -\frac{2}{r^2}\left(2rI(y) + \frac{1}{1+rI(y)} - \ln(1+rI(y))\right) \\ &\quad +4\int_0^y \varepsilon_I(t)\left(\int_0^t \varepsilon_I(z)I(z)dz\right)dt - 4ac_1\int_0^y \varepsilon_I(t)dt + c_2,\end{aligned} \tag{13}$$

where $\kappa^2 = \varepsilon_f^0 - p^2$ and c_2 is a constant of integration.

In the case $a = 0$ (it corresponds to the linear case with absorption in Eq. (2)) we obtain the following analog of (13)

$$\frac{d^2 I(y)}{dy^2} + 4\kappa^2 I(y) = -4\varepsilon_R(y)I(y) + 2\int_0^y \frac{d\varepsilon_R(t)}{dt} I(t)dt$$
$$+4\int_0^y \varepsilon_I(t)\left(\int_0^t \varepsilon_I(z)I(z)dz\right)dt - 4c_1\int_0^y \varepsilon_I(t)dt + c_2, \tag{14}$$

where $I(y)$ denotes $E^2(y)$. Later on for the case $a = 0$ under $I(y)$ we always understand $E^2(y)$.

The homogeneous equation $d^2I(y)/dy^2 + 4\kappa^2 I(y) = 0$ which corresponds to Eq. (13) has the solution that satisfies the boundary conditions at $y = 0$

$$\tilde{I}_0(y) = a|E_3|^2 \cos(2\kappa y), \tag{15}$$

so that the general solution of equation (13) reads (Stakgold, 1967)

$$I(y) = \tilde{I}_0(y) + \int_0^y dt \frac{\sin 2\kappa(y-t)}{2\kappa} \cdot$$
$$\left(-4\varepsilon_R(t)I(t) + 2\int_0^t \frac{d\varepsilon_R(\tau)}{d\tau} I(\tau)d\tau\right.$$
$$-\frac{2}{r^2}\left(2rI(t) + \frac{1}{1+rI(t)} - \ln(1+rI(t))\right)$$
$$\left.+4\int_0^t \varepsilon_I(\tau)\left(\int_0^\tau \varepsilon_I(z)I(z)dz\right)d\tau - 4ac_1\int_0^t \varepsilon_I(\tau)d\tau + c_2\right), \tag{16}$$

where the constant c_2 must be determined by the boundary conditions.

In the case $a = 0$ the general solution of equation (14) reads

$$I(y) = \tilde{I}_0(y) + \int_0^y dt \frac{\sin 2\kappa(y-t)}{2\kappa} \cdot$$
$$\left(-4\varepsilon_R(t)I(t) + 2\int_0^t \frac{d\varepsilon_R(\tau)}{d\tau} I(\tau)d\tau\right.$$
$$\left.+4\int_0^t \varepsilon_I(\tau)\left(\int_0^\tau \varepsilon_I(z)I(z)dz\right)d\tau - 4c_1\int_0^t \varepsilon_I(\tau)d\tau + c_2\right) \tag{17}$$

with $\tilde{I}_0(y) = |E_3|^2 \cos(2\kappa y)$.

The Volterra equations (16), (17) are equivalent to equation (3) for $0 < y < d$ for $a \neq 0$ and $a = 0$, respectively. According to equations (16) and (17) $I(y)$ and $\tilde{I}_0(y)$ satisfy the boundary conditions at $y = 0$. Evaluating some of the integrals on the right-hand side, equations (16) and (17) can be written as

$$
\begin{aligned}
I(y) = I_0(y) &+ \frac{1}{\kappa^2}\int_0^y \sin^2\kappa(y-\tau)\frac{d\varepsilon_R(\tau)}{d\tau}I(\tau)d\tau \\
&-\frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\varepsilon_R(\tau)I(\tau)d\tau \\
&-\frac{2}{r\kappa}\int_0^y \sin 2\kappa(y-\tau)I(\tau)d\tau \\
&-\frac{1}{\kappa r^2}\int_0^y \sin 2\kappa(y-\tau)\frac{1}{1+rI(\tau)}d\tau \\
&+\frac{1}{\kappa r^2}\int_0^y \sin 2\kappa(y-\tau)\ln(1+rI(\tau))d\tau \\
&+4\int_0^y \varepsilon_I(\tau)\left(\int_\tau^y \frac{\sin 2\kappa(y-t)}{2\kappa}\left(\int_\tau^t \varepsilon_I(z)dz\right)dt\right)I(\tau)d\tau,
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
I(y) = I_0(y) &+ \frac{1}{\kappa^2}\int_0^y \sin^2\kappa(y-\tau)\frac{d\varepsilon_R(\tau)}{d\tau}I(\tau)d\tau \\
&-\frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\varepsilon_R(\tau)I(\tau)d\tau \\
&+4\int_0^y \varepsilon_I(\tau)\left(\int_\tau^y \frac{\sin 2\kappa(y-t)}{2\kappa}\left(\int_\tau^t \varepsilon_I(z)dz\right)dt\right)I(\tau)d\tau,
\end{aligned}
\tag{19}
$$

respectively, with [on the evaluation of c_2 see Appendix B] (in the case $a \neq 0$)

$$
\begin{aligned}
I_0(y) &= \tilde{I}_0(y) + \frac{c_2\sin^2\kappa y}{2\kappa^2} \\
&-4ac_1\int_0^y \frac{\sin 2\kappa(y-t)}{2\kappa}\int_0^t \varepsilon_I(z)dzdt,
\end{aligned}
\tag{20}
$$

$$
ac_1 = -q_s I(0),
\tag{21}
$$

$$
\begin{aligned}
c_2 &= 2I(0)(q_s^2 + q_f^2(0) - p^2) - \frac{2I^2(0)}{1+rI(0)} \\
&+\frac{2}{r^2}\left(2rI(0) + \frac{1}{1+rI(0)} - \ln(1+rI(0))\right),
\end{aligned}
\tag{22}
$$

where $\tilde{I}_0(y)$ is given by equation (15), and with (in the case $a = 0$)

$$
\begin{aligned}
I_0(y) &= \tilde{I}_0(y) + \frac{c_2\sin^2\kappa y}{2\kappa^2} \\
&-4c_1\int_0^y \frac{\sin 2\kappa(y-t)}{2\kappa}\int_0^t \varepsilon_I(z)dzdt,
\end{aligned}
\tag{23}
$$

$$
c_1 = -q_s I(0),
\tag{24}
$$

$$
c_2 = 2I(0)(q_s^2 + q_f^2(0) - p^2),
\tag{25}
$$

where $\tilde{I}_0(y) = |E_3|^2 \cos(2\kappa y)$.

Iteration of the nonlinear integral equations (16) and (17) leads to a sequence of functions $I_j(y), 0 < y < d$. Subject to certain conditions it can be shown that the limit

$$I(y) = \lim_{j\to\infty} I_j(y)$$

exists uniformly in $0 < y < d$ and represents the unique solution of (16) and (17). The error of approximations can be expressed in terms of the parameters of the problem (see (68) and (69) in Appendix A).

Iterating (16) and (17) once by inserting $I_0(y)$ according to (20) and (23), the first iteration $I_1(y)$ reads ($a \neq 0$)

$$\begin{aligned} I_1(y) = I_0(y) &+ \frac{1}{\kappa^2}\int_0^y \sin^2\kappa(y-\tau)\frac{d\varepsilon_R(\tau)}{d\tau} I_0(\tau)d\tau \\ &- \frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\varepsilon_R(\tau)I_0(\tau)d\tau \\ &- \frac{2}{r\kappa}\int_0^y \sin 2\kappa(y-\tau)I_0(\tau)d\tau \\ &- \frac{1}{\kappa r^2}\int_0^y \sin 2\kappa(y-\tau)\frac{1}{1+rI_0(\tau)}d\tau \\ &+ \frac{1}{\kappa r^2}\int_0^y \sin 2\kappa(y-\tau)\ln(1+rI_0(\tau))d\tau \\ &+ 4\int_0^y \varepsilon_I(\tau)\left(\int_\tau^y \frac{\sin 2\kappa(y-t)}{2\kappa}\left(\int_\tau^t \varepsilon_I(z)dz\right)dt\right)I_0(\tau)d\tau, \end{aligned} \tag{26}$$

and ($a = 0$)

$$\begin{aligned} I_1(y) = I_0(y) &+ \frac{1}{\kappa^2}\int_0^y \sin^2\kappa(y-\tau)\frac{d\varepsilon_R(\tau)}{d\tau} I_0(\tau)d\tau \\ &- \frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\varepsilon_R(\tau)I_0(\tau)d\tau \\ &+ 4\int_0^y \varepsilon_I(\tau)\left(\int_\tau^y \frac{\sin 2\kappa(y-t)}{2\kappa}\left(\int_\tau^t \varepsilon_I(z)dz\right)dt\right)I_0(\tau)d\tau. \end{aligned} \tag{27}$$

$I_1(y)$ is used for numerical evaluation of the physical quantities defined in the following section.

3. Reflectance, transmittance, absorptance, and phase shifts

Conservation of energy requires that absorptance A, transmittance T, and reflectance R are related by

$$A = 1 - R - T, \tag{28}$$

with

$$T = \frac{q_s}{q_c}\frac{I(0)}{aE_0^2}, \quad T = \frac{q_s}{q_c}\frac{I(0)}{E_0^2}, \tag{29}$$

$$R = \frac{|E_r|^2}{E_0^2}, \tag{30}$$

for $a \neq 0$ and $a = 0$, respectively.

Due to the continuity conditions at $y = d$

$$E_0 + |E_r|e^{i\delta_r} = E(d)e^{i\vartheta(d)} \tag{31}$$

$$2E_0 e^{-i\vartheta(d)} = \frac{i}{q_c}\frac{dE(y)}{dy}\mid_{y=d} + E(d)(1 - \frac{1}{q_c}\frac{d\vartheta(y)}{dy}\mid_{y=d}) \tag{32}$$

reflectance, transmittance, absorptance and the phase shift on reflection, δ_r, and on transmission, δ_t, can be determined. Combination of equations (31) and (32) yields (for $a \neq 0$)

$$aE_0^2 = \frac{1}{4}\left\{\frac{\left(\frac{dI(y)}{dy}\mid_{y=d}\right)^2}{4q_c^2 I(d)} + I(d)\left(1 + \frac{q_s I(0) + \int\limits_0^d \varepsilon_I(\tau)I(\tau)d\tau}{q_c I(d)}\right)^2\right\}, \tag{33}$$

$$a|E_r|^2 = \frac{1}{4}\left\{\frac{\left(\frac{dI(y)}{dy}\mid_{y=d}\right)^2}{4q_c^2 I(d)} + I(d)\left(1 - \frac{q_s I(0) + \int\limits_0^d \varepsilon_I(\tau)I(\tau)d\tau}{q_c I(d)}\right)^2\right\}, \tag{34}$$

and (for $a = 0$)

$$E_0^2 = \frac{1}{4}\left\{\frac{\left(\frac{dI(y)}{dy}\mid_{y=d}\right)^2}{4q_c^2 I(d)} + I(d)\left(1 + \frac{q_s I(0) + \int\limits_0^d \varepsilon_I(\tau)I(\tau)d\tau}{q_c I(d)}\right)^2\right\}, \tag{35}$$

$$|E_r|^2 = \frac{1}{4}\left\{\frac{\left(\frac{dI(y)}{dy}\mid_{y=d}\right)^2}{4q_c^2 I(d)} + I(d)\left(1 - \frac{q_s I(0) + \int\limits_0^d \varepsilon_I(\tau)I(\tau)d\tau}{q_c I(d)}\right)^2\right\}. \tag{36}$$

Inserting equations (33)-(36) into equation (28) and using equations (29), (33) we obtain

$$A = \frac{1}{q_c a E_0^2}\int\limits_0^d \varepsilon_I(\tau)I(\tau)d\tau, \quad A = \frac{1}{q_c E_0^2}\int\limits_0^d \varepsilon_I(\tau)I(\tau)d\tau, \tag{37}$$

for $a \neq 0$ and $a = 0$, respectively. The continuity conditions (31), (32) and equations (29), (33) and (35) imply

$$\delta_r = -\arcsin \frac{\frac{dI(y)}{dy}\big|_{y=d}}{4q_c a E_0^2 \sqrt{1-T-A}}, \quad \delta_r = -\arcsin \frac{\frac{dI(y)}{dy}\big|_{y=d}}{4q_c E_0^2 \sqrt{1-T-A}} \tag{38}$$

for the phase shift on reflection (for $a \neq 0$ and $a = 0$, respectively), and

$$\delta_t = \vartheta(0) = \int_0^d \frac{q_s I(0) + q_c a E_0^2 \widetilde{A}(\tau)}{I(\tau)} d\tau + \arcsin\left(-\frac{\frac{dI(y)}{dy}\big|_{y=d}}{4q_c\sqrt{aE_0^2 I(d)}} \right),$$

$$\delta_t = \vartheta(0) = \int_0^d \frac{q_s I(0) + q_c E_0^2 \widetilde{A}(\tau)}{I(\tau)} d\tau + \arcsin\left(-\frac{\frac{dI(y)}{dy}\big|_{y=d}}{4q_c\sqrt{E_0^2 I(d)}} \right), \tag{39}$$

with

$$\widetilde{A}(\tau) := \frac{1}{q_c a E_0^2} \int_0^\tau \varepsilon_I(u) I(u)\, du, \quad \widetilde{A}(\tau) := \frac{1}{q_c E_0^2} \int_0^\tau \varepsilon_I(u) I(u)\, du, \tag{40}$$

for the phase shift on transmission (for $a \neq 0$ and $a = 0$, respectively).

4. Numerical evaluations

A numerical evaluation of the foregoing quantities is straightforward. It is useful to apply a parametric-plot routine using the first approximation $I_1(y)$. If the parameters of the problem $(a, r, \varepsilon_R, \varepsilon_I, \varepsilon_s, \varepsilon_f^0, \varepsilon_c, p, d, \omega_0)$ satisfy the convergence conditions (48) and (49) (see Appendix A) the results obtained for $I_1(y)$ are in good agreement with the purely numerical solution of equation (13) and (14) (cf. figure 3).

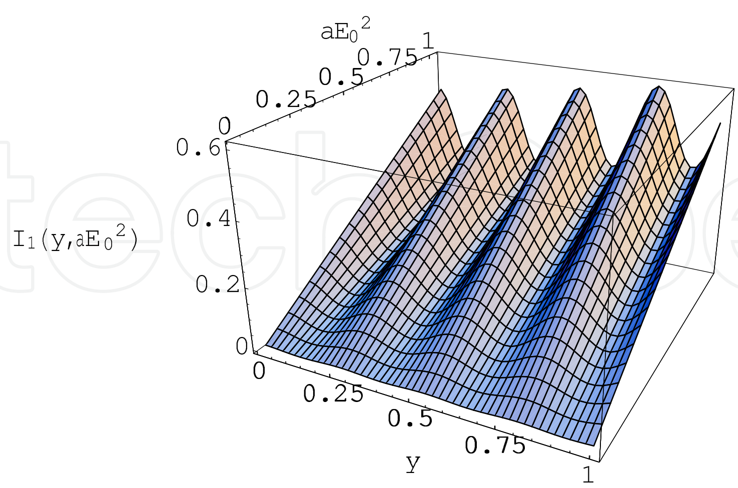

Fig. 2. Dependence of the field intensity $I_1(y, aE_0^2)$ inside the slab on the transverse coordinate y and aE_0^2 for
$r = 1000$, $\varepsilon_I = 0.1$, $\varepsilon_c = 1$, $\varepsilon_s = 1.7$, $\varepsilon_f^0 = 3.5$, $\varphi = 1.107$, $d = 1$, $\gamma = 0.033$, $b = 0.1$.

For the numerical evaluations the following steps can be performed:

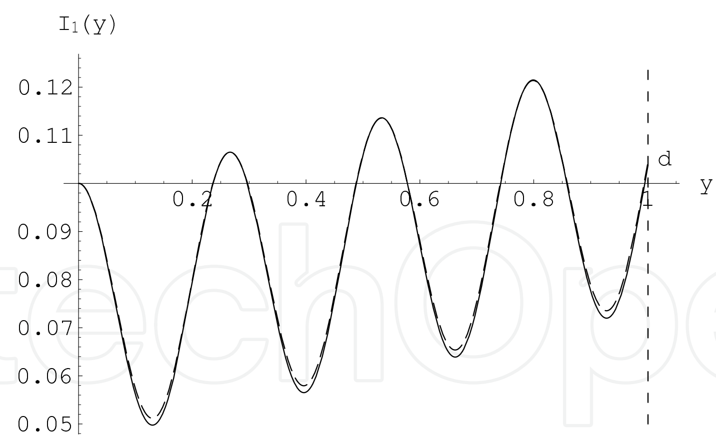

Fig. 3. Dependence of the field intensity $I_1(y, aE_0^2)$ inside the slab on the transverse coordinate y for $a|E_3|^2 = 0.1$. The other parameters are as in figure 2. Solid curve corresponds to the first iteration of equation (26) and dashed curve to the numerical solution of the system of differential equations (6), (7).

(i) Prescribe the parameters of the problem such that (48) and (49) are satisfied.

(ii) Prescribe a certain upper bound (accuracy) of the right-hand side R_j (see (66) in Appendix A) and perform a parametric plot of R_j (with $I(0)$ as parameter) with j=1. If R_1 is smaller (or equal) than (to) the prescribed accuracy for all aE_0^2 (or E_0^2) of a certain interval, accept $I_1(y)$ as a suitable approximation.

(iii) If R_1 exceeds the prescribed accuracy calculate $I_2(y)$ according to (50) and check again according to step (ii) or enlarge the accuracy so that R_1 is smaller (or equal) than (to) the prescribed accuracy.

The reason for the satisfactory agreement between the exact numerical solution and the first approximation $I_1(y)$ (cf. figure 3) is due to the foregoing explanation.

If $a|E_3|^2$ (or $|E_3|^2$) is fixed (as in the numerical example below), and thus aE_0^2 (or E_0^2) according to (33) (or (35)), the inequality (68) (or (69)) can be used to optimize the iteration approach with respect to another free parameter, e.g., d or r or p, as indicated.

Using the first approximation the phase function can be evaluated according to equation (8) as (for all values of a)

$$\vartheta_1(y) = \vartheta_1(d) - q_s I(0) \int_d^y \frac{d\tau}{I_1(\tau)} - \int_d^y \frac{d\tau}{I_1(\tau)} \int_0^\tau \varepsilon_I(\xi) I_1(\xi) d\xi, \tag{41}$$

where

$$\sin\vartheta_1(d) = -\frac{\frac{dI_1(y)}{dy}\big|_{y=d}}{4q_c\sqrt{aE_0^2 I_1(d)}},$$
$$\sin\vartheta_1(d) = -\frac{\frac{dI_1(y)}{dy}\big|_{y=d}}{4q_c\sqrt{E_0^2 I_1(d)}} \tag{42}$$

for $a \neq 0$ and $a = 0$, respectively.

Thus, the approximate solution of the problem is represented by equations (26) or (27) and (41). The appropriate parameter is $I(0) = aE^2(0)$ or $I(0) = E^2(0)$, since E_0 in equation (42) can be expressed in terms of $I(0)$ as shown in (33) or (35).

For illustration we assume a permittivity according to ($a \neq 0$)

$$\varepsilon_f(y) = \varepsilon_f^0 + \varepsilon_R(y) + i\varepsilon_I + \frac{I(y)}{1 + rI(y)}, \tag{43}$$

with

$$\varepsilon_R(y) = \gamma \cos^2 \frac{by}{d}, \tag{44}$$

where $\varepsilon_f^0, \gamma, b, d, r$ are real constants. For simplicity, ε_I is also assumed to be constant. Results for the first iterate solution $I_1(y, aE_0^2)$ are depicted in figures 2, 3. Using $I_1(y, aE_0^2)$, the phase function $\vartheta_1(y, aE_0^2)$, absorptance $A_1(d, aE_0^2)$ and phase shift on reflection $\delta_{r1}(y, aE_0^2)$ are shown in figures 4, 5 and 6, respectively. The left hand side of condition (48) is 0.572 for the parameters selected in this example. Results for R, T and the phase shift on transmission can be obtained similarly.

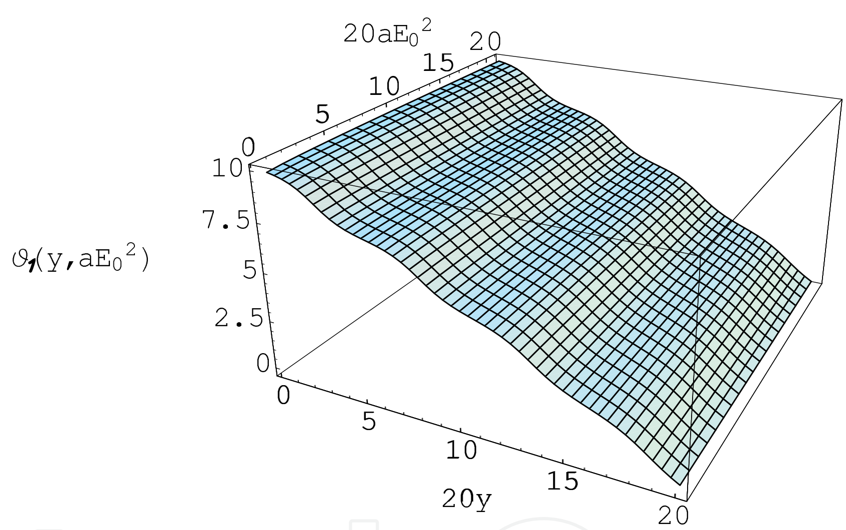

Fig. 4. Phase function $\vartheta_1(y, aE_0^2)$ according to equation (41) inside the slab. Parameters as in figure 3.

5. Summary

Based on known mathematics we have proposed an iterative approach to the scattering of a plane TE-polarized optical wave at a dielectric film with permittivities modeled by a complex continuously differentiable function of the transverse coordinate.

The result is an approximate analytical expression for the field intensity inside the film that can be used to express the physical relevant quantities (reflectivity, transmissivity, absorptance, and phase shifts). Comparison with exact numerical solutions shows satisfactory agreement.

It seems appropriate to explain the benefits of the present approach:

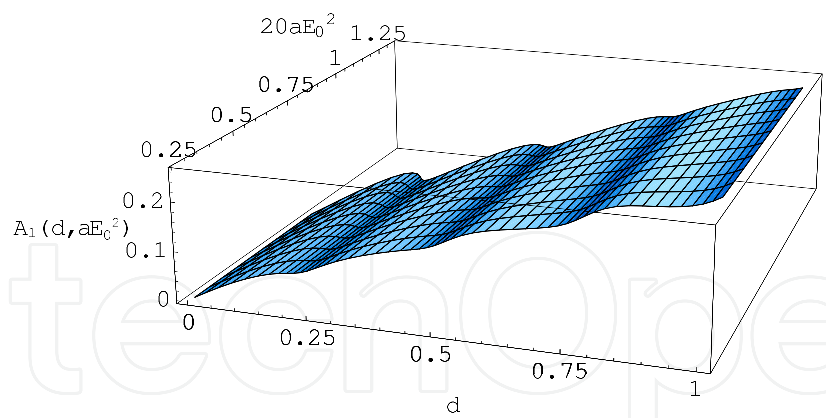

Fig. 5. Absorptance A_1 depending on the layer thickness d and on the incident field intensity aE_0^2 for the same parameters as in figure 3.

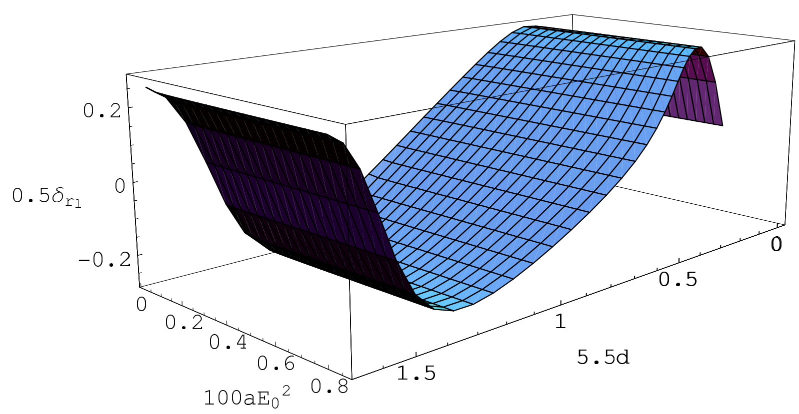

Fig. 6. Phase shift on reflection δ_{r1} depending on the layer thickness d and on the incident field intensity aE_0^2 for the same parameters as in figure 3.

(i) The approach yields (approximate) solutions in cases where the usual methods (cf. Refs. (Chen & Mills, 1987; 1988; Leung, 1985; 1988; Peschel, 1988; Schürmann & Schmoldt, 1993)) fail or could not be applied till now.

(ii) The quality of the approximate solutions can be estimated in dependence on the parameters of the problem.

On the other hand the conditions of convergence explicitly depend on the permittivity functions in question and thus have to be derived for every permittivity anew (cf. (Serov et al., 2004; 2010) and (65)).

6. Acknowledgment

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7. Appendix

7.1 Appendix A

We introduce in the Banach space $C[0,d]$ bounded integral operators $N_1, N_2, N_3, N_4, N_5, N_6$ by

$$\begin{gathered}
N_1(I) = \frac{1}{\kappa^2}\int_0^y \sin^2\kappa(y-\tau)\frac{d\varepsilon_R(\tau)}{d\tau}I(\tau)d\tau,\\
N_2(I) = -\frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\varepsilon_R(\tau)I(\tau)d\tau,\\
N_3(I) = -\frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)I(\tau)d\tau,\\
N_4(I) = -\frac{1}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\frac{1}{1+rI(\tau)}d\tau,\\
N_5(I) = \frac{1}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\ln(1+rI(\tau))d\tau,\\
N_6(I) = 4\int_0^y \varepsilon_I(s)\psi(y,s)I(s)ds,
\end{gathered} \tag{45}$$

where

$$\psi(y,s) = \int_s^y \frac{\sin 2\kappa(y-t)}{2\kappa}\left(\int_s^t \varepsilon_I(\tau)d\tau\right)dt, \tag{46}$$

with the values $\|N_1\|, \|N_2\|, \|N_3\|, \|N_4\|, \|N_5\|, \|N_6\|, \|I_0\|$ which are defined as

$$\begin{gathered}
\|N_1\| = \frac{1}{\kappa^2}\max_{0\le y\le d}\int_0^y |\sin^2\kappa(y-\tau)|\cdot|\frac{d\varepsilon_R(\tau)}{d\tau}|d\tau,\\
\|N_2\| = \frac{2}{\kappa}\max_{0\le y\le d}\int_0^y |\sin 2\kappa(y-\tau)|\cdot|\varepsilon_R(\tau)|d\tau,\\
\|N_3\| = \frac{2}{\kappa}\max_{0\le y\le d}\int_0^y |\sin 2\kappa(y-\tau)|d\tau,\\
\|N_4\| = \frac{1}{\kappa}\max_{0\le y\le d}\int_0^y |\sin 2\kappa(y-\tau)|d\tau,\\
\|N_5\| = \frac{1}{\kappa}\max_{0\le y\le d}\int_0^y |\sin 2\kappa(y-\tau)|d\tau = \|N_4\|,\\
\|N_6\| = 4\max_{0\le y\le d}\int_0^y |\varepsilon_I(z)|\cdot|\psi(y,z)|dz,\\
\|I_0\| = \max_{0\le y\le d}|I_0|.
\end{gathered} \tag{47}$$

We are in the position now to show that if

$$\|N_1\| + \|N_2\| + \|N_6\| + \frac{\|N_3\| + 2\|N_4\|}{r} < 1 \tag{48}$$

and

$$\frac{\|I_0\| + \frac{1}{r^2}\|N_4\|}{1 - (\|N_1\| + \|N_2\| + \|N_6\| + \frac{\|N_3\|+\|N_4\|}{r})} < \rho, \tag{49}$$

then in any ball $S_\rho(0)$ there exists a unique solution of the nonlinear integral equation (18) and this solution can be obtained as a uniform limit

$$I(y) = \lim_{j\to\infty} I_j(y)$$

of the iterations of (18). Indeed, let us introduce the iterations of (18) as follows:

$$\begin{aligned} I_j(y) = I_0(y) &+ \frac{1}{\kappa^2}\int_0^y \sin^2\kappa(y-\tau)\frac{d\varepsilon_R(\tau)}{d\tau} I_{j-1}(\tau)d\tau \\ &- \frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\varepsilon_R(\tau)I_{j-1}(\tau)d\tau \\ &- \frac{2}{r\kappa}\int_0^y \sin 2\kappa(y-\tau)I_{j-1}(\tau)d\tau \\ &- \frac{1}{\kappa r^2}\int_0^y \sin 2\kappa(y-\tau)\frac{1}{1+rI_{j-1}(\tau)}d\tau \\ &+ \frac{1}{\kappa r^2}\int_0^y \sin 2\kappa(y-\tau)\ln(1+rI_{j-1}(\tau))d\tau \\ &+ 4\int_0^y \varepsilon_I(\tau)\left(\int_\tau^y \frac{\sin2\kappa(y-t)}{2\kappa}\left(\int_\tau^t \varepsilon_I(z)dz\right)dt\right) I_{j-1}(\tau)d\tau, \end{aligned} \tag{50}$$

where $j = 1, 2, ...,$ and $I_0(y)$ is given by equation (20). In order to prove that the sequence (50) is uniformly convergent to the solution of (16) it suffices to check that all conditions of the Banach Fixed-Point Theorem (see (Zeidler, 1995)) are fulfilled.

We consider the nonlinear operator F as

$$F(I) := I_0(y) + N_1(I) + N_2(I) + \frac{1}{r}N_3(I) + \frac{1}{r^2}N_4(I) + \frac{1}{r^2}N_5(I) + N_6(I). \tag{51}$$

Then equation (18) can be rewritten in operator form

$$I(y) = F(I)(y). \tag{52}$$

We consider ρ such that $\|I\| = \max_{0\le y\le d} I(y) \le \rho$. First we must check whether this operator F maps the ball $S_\rho(0)$ to itself. Indeed, if $I(y) \in S_\rho(0)$ then

$$\begin{aligned} \|F(I)\| &\le \|I_0\| + \|N_1\|\cdot\|I\| + \|N_2\|\cdot\|I\| + \|N_6\|\cdot\|I\| + \frac{1}{r}\|N_3\|\cdot\|I\| \\ &\quad + \frac{1}{r^2}\|N_4\|\cdot\frac{1}{1+r\min\limits_{0\le y\le d} I(y)} + \frac{1}{r^2}\|N_4\|\cdot r\|I\| \\ &\le \|I_0\| + \|N_1\|\cdot\rho + \|N_2\|\cdot\rho + \|N_6\|\cdot\rho + \frac{1}{r}\|N_3\|\cdot\rho + \frac{1}{r^2}\|N_4\| \\ &\quad + \frac{1}{r}\|N_4\|\cdot\rho. \end{aligned} \tag{53}$$

Thus, the following inequality must be valid

$$\|I_0\| + \frac{1}{r^2}\|N_4\| + (\|N_1\| + \|N_2\| + \|N_6\| + \frac{1}{r}(\|N_3\| + \|N_4\|))\cdot\rho < \rho. \tag{54}$$

This inequality holds if

$$\frac{\|I_0\| + \frac{1}{r^2}\|N_4\|}{1 - (\|N_1\| + \|N_2\| + \|N_6\| + \frac{\|N_3\|+\|N_4\|}{r})} < \rho, \tag{55}$$

and thus if

$$\|N_1\| + \|N_2\| + \|N_6\| + \frac{\|N_3\| + \|N_4\|}{r} < 1. \tag{56}$$

It means that for this value of ρ continuous map F transfers ball $S_\rho(0)$ in itself. Hence, equation (16) has at least one solution inside $S_\rho(0)$. For uniqueness of this solution it remains to prove that F is contractive (see (Zeidler, 1995)). To prove the contraction of F we consider

$$\begin{aligned} F(I_1) - F(I_2) &= N_1(I_1 - I_2) + N_2(I_1 - I_2) + N_6(I_1 - I_2) \\ &+\frac{1}{r}N_3(I_1 - I_2) + \frac{1}{r^2}(N_4(I_1) - N_4(I_2)) + \frac{1}{r^2}(N_5(I_1) - N_5(I_2)). \end{aligned} \tag{57}$$

Hence

$$\begin{aligned} \|F(I_1) - F(I_2)\| \le \|N_1\|\|I_1 - I_2\| + \|N_2\|\|I_1 - I_2\| + \|N_6\|\|I_1 - I_2\| \\ +\|N_3\|\|\frac{1}{r}(I_1 - I_2)\| + \|\frac{1}{r^2}(N_4(I_1) - N_4(I_2))\| \\ +\|\frac{1}{r^2}(N_5(I_1) - N_5(I_2))\|. \end{aligned} \tag{58}$$

The following estimations hold

$$\begin{aligned} (i) \quad & \|\tfrac{1}{r^2}(N_4(I_1) - N_4(I_2))\| \le \max_{0\le y\le d} \tfrac{1}{\kappa r^2} \textstyle\int_0^y |\sin 2\kappa(y-\tau)| \cdot \\ & \left|\tfrac{1}{1+rI_1(\tau)} - \tfrac{1}{1+rI_2(\tau)}\right| d\tau = \\ & \max_{0\le y\le d} \tfrac{1}{\kappa r^2} \textstyle\int_0^y |\sin 2\kappa(y-\tau)| \cdot \left|\tfrac{rI_2(\tau)-rI_1(\tau)}{(1+rI_1(\tau))(1+rI_2(\tau))}\right| d\tau \\ & \le \tfrac{1}{r} \max_{0\le y\le d} \tfrac{1}{\kappa} \textstyle\int_0^y |\sin 2\kappa(y-\tau)| d\tau \cdot \|I_1 - I_2\|, \end{aligned}$$

hence

$$\|\frac{1}{r^2}(N_4(I_1) - N_4(I_2))\| \le \frac{\|N_4\|}{r} \cdot \|I_1 - I_2\|. \tag{59}$$

$$\begin{aligned} (ii) \quad \|\frac{1}{r^2}(N_5(I_1) - N_5(I_2))\| \le \max_{0\le y\le d} \frac{1}{\kappa r^2} \int_0^y |\sin 2\kappa(y-\tau)| \cdot \\ |\ln(1 + rI_1(\tau)) - \ln(1 + rI_2(\tau))|\, d\tau. \end{aligned} \tag{60}$$

Using

$$\begin{aligned} |\ln(1 + rI_1) - \ln(1 + rI_2)| &= \left|\ln\frac{1 + rI_1}{1 + rI_2}\right| \\ &= \left|\ln\left(1 + \frac{r(I_1 - I_2)}{1 + rI_2}\right)\right| \le r\|I_1 - I_2\|, \end{aligned} \tag{61}$$

equation (60) yields

$$\|\frac{1}{r^2}(N_5(I_1) - N_5(I_2))\| \leq \frac{1}{r^2} \cdot \|N_4\| \cdot r \cdot \|I_1 - I_2\| = \frac{\|N_4\|}{r}\|I_1 - I_2\|. \tag{62}$$

Thus, from equation (58), one obtains

$$\begin{aligned}\|F(I_1) - F(I_2)\| \leq (\|N_1\| + \|N_2\| + \|N_6\| + \frac{\|N_3\|}{r} + \frac{\|N_4\|}{r} + \frac{\|N_4\|}{r}) \\ \cdot\|I_1 - I_2\| \\ = (\|N_1\| + \|N_2\| + \frac{\|N_3\| + 2\|N_4\|}{r}) \cdot \|I_1 - I_2\|,\end{aligned} \tag{63}$$

so that F is contractive if

$$\|N_1\| + \|N_2\| + \|N_6\| + \frac{\|N_3\| + 2\|N_4\|}{r} < 1. \tag{64}$$

Thus, the uniform convergence follows.

If we denote by m the left-hand side of the inequality (48) the solution $I(y)$ of (16) can be approximated by the iterations $I_j(y)$ as follows (see (Zeidler, 1995)):

$$\begin{aligned}\|I - I_j\| \leq \frac{m^j}{1-m}\|I_1 - I_0\| \\ \leq \frac{m^j}{1-m}\left(\frac{1}{\kappa r^2}\max_{0\leq y\leq d}\int_0^y |\sin 2\kappa(y-\tau)|d\tau + m\|I_0\|\right) \\ \leq \frac{m^j}{1-m}\left(\frac{d^2}{r^2} + m\|I_0\|\right),\end{aligned} \tag{65}$$

where $j = 0, 1, 2, ...$ and I_0 is defined in (20).

Let us remark that for the sufficient condition (48) to hold parameters must be chosen such that (48) holds even if r is small (Equation (18) represents the exact solution $I(y)$ if (48) and (49) are satisfied). $I(y)$ can be approximated by the first iteration $I_1(y)$ with the error $\frac{d^2}{r^2}\frac{m}{1-m} + \frac{m^2}{1-m}\|I_0\|$, where m denotes the left-hand side of (48). Condition (48) must hold for a particular $r > 0$. In the limit $r \to 0$ equation (18) transforms to equation

$$\begin{aligned}I(y) = I_0(y) + \frac{1}{\kappa^2}\int_0^y \sin^2\kappa(y-\tau)\frac{d\varepsilon_R(\tau)}{d\tau}I(\tau)d\tau \\ -\frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\varepsilon_R(\tau)I(\tau)d\tau - \frac{3}{2\kappa}\int_0^y \sin 2\kappa(y-\tau)I^2(\tau)d\tau \\ +4\int_0^y \varepsilon_I(\tau)\left(\int_\tau^y \frac{\sin 2\kappa(y-t)}{2\kappa}\left(\int_\tau^t \varepsilon_I(z)dz\right)dt\right)I(\tau)d\tau,\end{aligned} \tag{66}$$

where $I_0(y)$ is the same as (23) with the constant c_2 which is equal to

$$c_2 = I_0(0)(q_s^2 + q_f^2(0) - p^2) - 2I^2(0).$$

Equation (66) is equivalent to (in the case of lossless medium) (41) in (Serov et al., 2004). Equation (66) is uniquely solvable in the ball of radius ρ if the following conditions are satisfied (they are consistent with the corresponding conditions from (Serov et al., 2004; 2010)):

$$m + 3d^2\rho < 1, \quad m + d\sqrt{6\|I_0\|} < 1,$$

where $m = \|N_1\| + \|N_2\| + \|N_c\|$ and the radius ρ is chosen so that

$$\rho \geq \frac{1 - m - \sqrt{(1-m)^2 - 6d^2\|I_0\|}}{3d^2}.$$

In order to obtain a condition of the type (48) for all $0 < r < 1$ (uniformly) combination of N_3, N_4, N_5 and part of I_0 within the estimations is necessary. It seems impossible to obtain a condition of the type (48) uniformly with respect to all nonnegative r. It is possible only to obtain such kind of condition uniformly for $0 < r < 1$ or for $1 < r < \infty$ independently. In this respect, some mathematical complications arise that are not the main point of this paper.

Estimation of $\|I_0\|$ (cf. Appendix C) gives

$$\|I_0\| \leq I(0) + \frac{1}{2}|c_2|d^2 + \frac{2}{3}a|c_1|\|\varepsilon_I\|d^3,$$

$$\|I_0\| \leq I(0) + \frac{1}{2}|c_2|d^2 + \frac{2}{3}|c_1|\|\varepsilon_I\|d^3, \tag{67}$$

where constants c_1 and c_2 are defined by (21) and (22) for $a \neq 0$, and by (24) and (25) for $a = 0$, respectively. Combining (65) and (67) we obtain the error of approximation (for $a \neq 0$)

$$R_j := \|I - I_j\| \leq \frac{d^2}{r^2} \cdot \frac{m^j}{1-m} + \frac{m^{j+1}}{1-m}\left(I(0) + \frac{1}{2}|c_2|d^2 + \frac{2}{3}a|c_1|\|\varepsilon_I\|d^3\right), \tag{68}$$

where $j = 0, 1, 2, ...$

Since for linear case ($a = 0$) equation (17) is the linear Volterra integral equation this equation has always a unique solution and the following error of approximation holds:

$$\|I - I_j\| \leq \frac{(\|I_0\|md)^{j+1}}{(j+1)!}e^{\|I_0\|md}, \tag{69}$$

where $j = 0, 1, 2, ...$, $\|I_0\|$ is estimated in (67), $m = \|N_1\| + \|N_2\| + \|N_6\|$ and I_j are defined by

$$\begin{aligned} I_j(y) = I_0(y) &+ \frac{1}{\kappa^2}\int_0^y \sin^2\kappa(y-\tau)\frac{d\varepsilon_R(\tau)}{d\tau}I_{j-1}(\tau)d\tau \\ &- \frac{2}{\kappa}\int_0^y \sin 2\kappa(y-\tau)\varepsilon_R(\tau)I_{j-1}(\tau)d\tau \\ +4\int_0^y \varepsilon_I(\tau)&\left(\int_\tau^y \frac{\sin 2\kappa(y-t)}{2\kappa}\left(\int_\tau^t \varepsilon_I(z)dz\right)dt\right)I_{j-1}(\tau)d\tau. \end{aligned} \tag{70}$$

7.2 Appendix B

The constant of integration c_2 is determined by equations (13) and (14) for $a \neq 0$ and for $a = 0$, respectively with $y = 0$ as

$$c_2 = \left.\frac{d^2 I(y)}{dy^2}\right|_{y=0} + 4(q_f^2(0) - p^2)I(0) + \frac{2}{r^2}\left(2rI(0) + \frac{1}{1+rI(0)} - \ln(1 + rI(0))\right), \tag{71}$$

$$c_2 = \left.\frac{d^2 I(y)}{dy^2}\right|_{y=0} + 4(q_f^2(0) - p^2)I(0). \tag{72}$$

According to equation (10), the second derivative of the field intensity $I(y)$ at $y = 0$ is given by (for $a \neq 0$)

$$\left.\frac{d^2 I(y)}{dy^2}\right|_{y=0} = 2q_s^2 I(0) - 2(q_f^2(0) - p^2)I(0) - \frac{2I^2(0)}{1+rI(0)}, \tag{73}$$

and (for $a = 0$)

$$\left.\frac{d^2 I(y)}{dy^2}\right|_{y=0} = 2q_s^2 I(0) - 2(q_f^2(0) - p^2)I(0), \tag{74}$$

leading to, taking into account boundary conditions, $E(0) = E_3 e^{-i\vartheta(0)}$ and $\frac{dE(y)}{dy}\ |_{y=0} = 0$,

$$c_2 = 2q_s^2 I(0) + 2(q_f^2(0) - p^2)I(0) - \frac{2I^2(0)}{1+rI(0)} + \frac{2}{r^2}\left(2rI(0) + \frac{1}{1+rI(0)} - \ln(1 + rI(0))\right), \tag{75}$$

$$c_2 = 2q_s^2 I(0) + 2(q_f^2(0) - p^2)I(0). \tag{76}$$

for $a \neq 0$ and $a = 0$, respectively.

7.3 Appendix C

With $\varepsilon_R(x) \in C^1[0, d]$ and $\varepsilon_I(x) \in C[0, d]$ one obtains

$$\|N_1\| = \frac{1}{\kappa^2} \max_{0 \le y \le d} \int_0^y |\sin^2 \kappa(y - \tau)| \cdot |\varepsilon'_R(\tau)| d\tau \le \max_{0 \le y \le d} \int_0^y (y - \tau)^2 d\tau \cdot \|\varepsilon'_R\| = \frac{1}{3} d^3 \|\varepsilon'_R\|, \tag{77}$$

$$\|N_2\| = \frac{2}{\kappa} \max_{0 \le y \le d} \int_0^y |\sin 2\kappa(y - \tau)| \cdot |\varepsilon_R(\tau)| d\tau \le 4 \max_{0 \le y \le d} \int_0^y (y - \tau) d\tau \cdot \|\varepsilon_R\| = 2d^2 \|\varepsilon_R\|, \tag{78}$$

$$\|N_3\| = \frac{2}{\kappa} \max_{0\le y\le d} \int_0^y |\sin 2\kappa(y-\tau)| d\tau \le 4 \max_{0\le y\le d} \int_0^y (y-\tau) d\tau = 2d^2, \tag{79}$$

$$\begin{aligned} \|N_4\| = \|N_5\| &= \frac{1}{\kappa} \max_{0\le y\le d} \int_0^y |\sin 2\kappa(y-\tau)| d\tau \\ &\le 2 \max_{0\le y\le d} \int_0^y (y-\tau) d\tau = d^2, \end{aligned} \tag{80}$$

$$\begin{aligned} \|N_6\| &= 4 \max_{0\le y\le d} \int_0^y |\varepsilon_I(z)| \cdot |\psi(y,z)| dz \\ &\le 4\|\varepsilon_I\| \max_{0\le y\le d} \int_0^y \int_z^y \frac{|\sin 2\kappa(y-\tau)|}{2\kappa} \int_z^t |\varepsilon_I(\tau)| d\tau dt dz \\ &\le 4\|\varepsilon_I\|^2 \max_{0\le y\le d} \int_0^y \int_z^y (y-t)(t-z) dt dz = \frac{d^4}{6}\|\varepsilon_I\|^2, \end{aligned} \tag{81}$$

$$\begin{aligned} \|I_0\| &\le \max_{0\le y\le d} a|E_3|^2 \cdot |\cos 2\kappa y| + |c_2| \max_{0\le y\le d} \frac{|\sin^2 \kappa y|}{2\kappa^2} \\ &+4a|c_1| \max_{0\le y\le d} \int_0^y \frac{|\sin 2\kappa(y-t)|}{2\kappa} \int_0^t |\varepsilon_I(\tau)| d\tau dt \\ &\le a|E_3|^2 + \frac{1}{2}|c_2|d^2 + 4a|c_1| \cdot \|\varepsilon_I\| \max_{0\le y\le d} \int_0^y (y-t) t dt \\ &= a|E_3|^2 + \frac{1}{2}|c_2|d^2 + \frac{2}{3}a|c_1| \cdot \|\varepsilon_I\| d^3, \end{aligned} \tag{82}$$

where $a \neq 0$ and ε'_R denotes the first derivative of ε_R.

For $a = 0$ the estimate of $\|I_0\|$ (82) transforms to the following one

$$\|I_0\| \le |E_3|^2 + \frac{1}{2}|c_2|d^2 + \frac{2}{3}|c_1| \cdot \|\varepsilon_I\| d^3, \tag{83}$$

with c_1 and c_2 from (24) and (25).

For ε_R, given by (44), we obtain

$$\|\varepsilon_R\| \le \gamma, \qquad \|\varepsilon'_R\| \le \begin{cases} \frac{2\gamma b^2}{d}, \ 2b \le 1, \\ \frac{\gamma b}{d}, \ 2b > 1. \end{cases} \tag{84}$$

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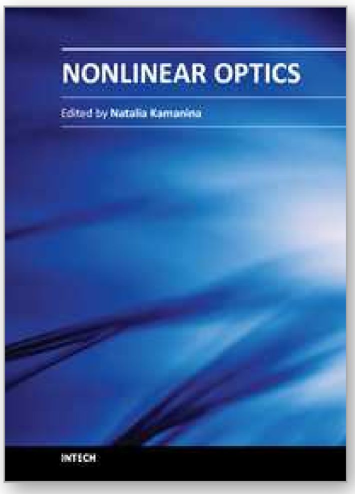

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Rapid development of optoelectronic devices and laser techniques poses an important task of creating and studying, from one side, the structures capable of effectively converting, modulating, and recording optical data in a wide range of radiation energy densities and frequencies, from another side, the new schemes and approaches capable to activate and simulate the modern features. It is well known that nonlinear optical phenomena and nonlinear optical materials have the promising place to resolve these complicated technical tasks. The advanced idea, approach, and information described in this book will be fruitful for the readers to find a sustainable solution in a fundamental study and in the industry approach. The book can be useful for the students, post-graduate students, engineers, researchers and technical officers of optoelectronic universities and companies.

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