6

Robust Decentralized PID Controller Design

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1. Introduction

Robust stability of uncertain dynamic systems has major importance when real world system models are considered. A realistic approach has to consider uncertainties of various kinds in the system model. Uncertainties due to inherent modelling/identification inaccuracies in any physical plant model specify a certain uncertainty domain, e.g. as a set of linearized models obtained in different working points of the plant considered. Thus, a basic required property of the system is its stability within the whole uncertainty domain denoted as robust stability. Robust control theory provides analysis and synthesis approaches and tools applicable for various kinds of processes, including multi input – multi output (MIMO) dynamic systems. To reduce multivariable control problem complexity, MIMO systems are often considered as interconnection of a finite number of subsystems. This approach enables to employ decentralized control structure with subsystems having their local control loops. Compared with centralized MIMO controller systems, decentralized control structure brings about certain performance deterioration, however weighted against by important benefits, such as design simplicity, hardware, operation and reliability improvement. Robustness is one of attractive qualities of a decentralized control scheme, since such control structure can be inherently resistant to a wide range of uncertainties both in subsystems and interconnections. Considerable effort has been made to enhance robustness in decentralized control structure and decentralized control design schemes and various approaches have been developed in this field both in time and frequency domains (Gyurkovics & Takacs, 2000; Zečevič & Šiljak, 2004; Stankovič et al., 2007).
Recently, the algebraic approach has gained considerable interest in robust control, (Boyd et al., 1994; Crusius & Trofino, 1999; de Oliveira et al., 1999; Ming Ge et al., 2002; Grman et al., 2005; Henrion et al., 2002). Algebraic approach is based on the fact that many different problems in control reduce to an equivalent linear algebra problem (Skelton et al., 1998). By algebraic approach, robust control problem is formulated in algebraic framework and solved as an optimization problem, preferably in the form of Linear Matrix Inequalities (LMI). LMI techniques enable to solve a large set of convex problems in polynomial time (see Boyd et al., 1994). This approach is directly applicable when control problems for linear uncertain systems with a convex uncertainty domain are solved. Still, many important control problems even for linear systems have been proven as NP hard, including structured linear control problems such as decentralized control and simultaneous static output feedback (SOF) designs. In these cases the prescribed structure of control feedback matrix (block diagonal for decentralized control) results in nonconvex problem formulation. There

are basically two approaches to solve the respective nonconvex control problem: 1) to reformulate the problem as LMI using certain convex relaxations (e.g. deOliveira et al., 2000; Rosinová & Veselý, 2003) or, alternatively, adopt an iterative procedure; 2) to formulate and solve the bilinear matrix inequalities (BMI) respective to robust control design problem. A nice review and basic characteristics of LMI and BMI in various control problems can be found in (Van Antwerp & Braatz, 2000).

To reduce the problem size in decentralized control design for large scale systems, the diagonal dominance or block diagonal dominance concept can be adopted. Recently, the so called Equivalent Subsystems Method has been developed for decentralized control in frequency domain, (Kozáková & Veselý, 2009). The main concept of the Equivalent Subsystems Method, originally developed as a Nyquist based frequency domain decentralized controller design technique, is the so called equivalent subsystem; equivalent subsystems are generated by shaping Nyquist plot of each decoupled subsystem using any selected characteristic locus of the matrix of interactions. The point of this approach consists in that local controllers of equivalent subsystems can be independently tuned for stability and required performance specified in terms of a suitable (preferably frequency domain) performance measure (e.g. degree of stability, phase margin, bandwidth), so that the resulting decentralized controller guarantees equivalent performance of the full system.

When designing decentralized control, besides robust stability, performance requirements have to be considered. Performance objectives can be of two basic types: a) achieving required performance in different subsystems; or b) achieving plant-wide desired performance. In this chapter two alternative approaches belonging to the latter group are presented, based on recent research results on robust decentralized PID controller design in the frequency and time domains.

The present chapter further extends the robust decentralized PID controller design techniques from (Kozáková et al., 2009; 2010; 2011; Rosinová et al., 2003; Rosinová & Veselý, 2007; 2011), bringing novel robust control design approaches. The results are illustrated on the case study dealing with robust decentralized controller design for the quadruple tank process. This laboratory process recently presented in (Johansson, 2000; Johansson et al., 1999) is an illustrative two input - two output laboratory plant for studying multivariable dynamic systems for both minimum and nonminimum-phase configurations.

The first presented approach is based on formulation and solution of BMI or LMI for uncertain linear polytopic system to design robust controller in the state space. In the time domain, we introduce the augmented model for closed-loop linear uncertain system with PID controller; this model is in general form, comprising both continuous- and discrete-time cases. For both cases, a general robust stability condition is formulated; the particular design procedures differ only in parameterization of augmented model matrices. A decentralized control design strategy is adopted, where robust PID control design approach is applied for structured - block diagonal controller matrices respective to decentralized controller.

The second approach is based on the Nyquist-type decentralized control design technique for uncertain MIMO systems described by a transfer function matrix. The decentralized controller is designed on subsystem level using the recently developed Equivalent Subsystem Method (Kozáková et al., 2009). Application of this method in the design for robust stability and nominal performance can be found e.g. in (Kozáková & Veselý, 2009) within a two-stage design scheme: 1. design of decentralized controller for nominal performance; 2. controller redesign with modified performance requirements to meet the

robust stability conditions. A direct "one-shot" robust DC design methodology based on integration of robust stability conditions in the Equivalent Subsystems Method enables to design local controllers of equivalent subsystems with regard to robust stability of the full system. The frequency domain approach is applicable for both continuous- and discrete-time PID controller designs.

2. Motivation: Case study - Quadruple tank process

This section aims at description, and analysis of two input - two output process from literature, which will be later used to demonstrate our proposed methods for decentralized PID controller design. The quadruple-tank process shown in Fig.1 has been introduced in (Johansson et al., 1999; Johansson, 2000) to provide a case study to analyze both minimum and nonminimum phase MIMO systems on the same plant. The aim is to control the level in the lower two tanks using two pumps. The inputs v_1 and v_2 are pump 1 and 2 flows respectively, the controlled outputs y_1 and y_2 are levels in lower tanks 1 and 2 respectively.

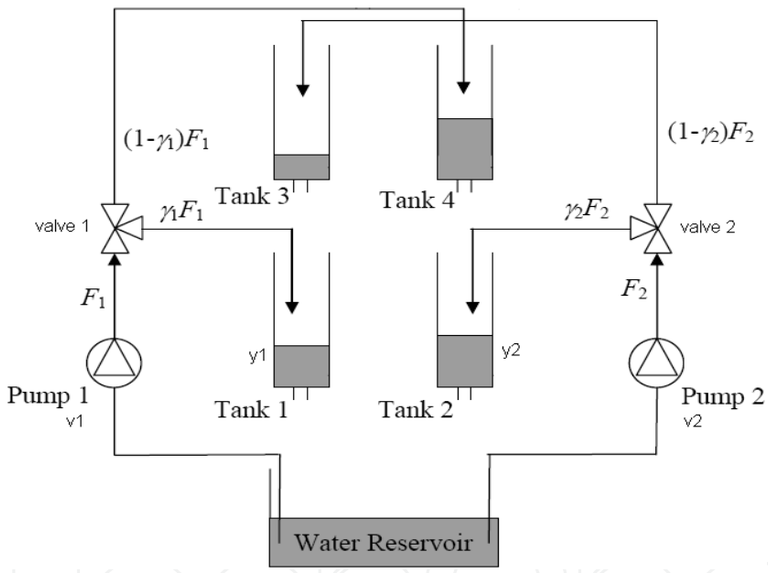


Fig. 1. Quadruple tank process scheme.

The nonlinear model of the four tanks can be described by state equations

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1
\end{aligned} \tag{1}$$

where A_i is cross-section of tank i, a_i is cross-section of the outlet hole of tank i, h_i is water level in tank i, g is acceleration of gravity, the flow corresponding to pump i is $k_i v_i$. Parameter γ_1 denotes position of the valve dividing the pump 1 flow into the lower tank 1 and related upper tank 4 and similarly γ_2 divides flow from pump 2 to the tanks 2 and 3. The flow to tank 1 is $\gamma_1 k_1 v_1$ and to tank 4 it is $(1-\gamma_1)k_1 v_1$, analogically for the tanks 2 and 3. The nonlinear model (1) can be linearized around the working point given by the water levels in tanks $h_{10}, h_{20}, h_{30}, h_{40}$. The deviation state space model was considered with $x_i = h_i - h_{i0}$ and the respective control variables $u_i = v_i - v_{i0}$. The linearized state space model for quadruple tank (1) is then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_3 \\ \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{-1}{T_1} & \frac{A_3}{T_3 A_1} & 0 & 0 \\ 0 & \frac{-1}{T_3} & 0 & 0 \\ 0 & 0 & \frac{-1}{T_2} & \frac{A_4}{T_4 A_2} \\ 0 & 0 & 0 & \frac{-1}{T_4} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{bmatrix} + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ 0 & \frac{\gamma_2 k_2}{A_1} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{2}$$

where $T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_{i0}}{g}}$, $i = 1,...,4$.

The argument t has been omitted; the state variables corresponding to levels in tanks 2 and 3 have been interchanged in state vector so that subsystems respective to input u_1 from pump 1 (tanks 1 and 3) and u_2 from pump 2 (tanks 2 and 4) are more apparent. This decomposition into two subsystems is used for decentralized control design.

The respective transfer function matrix having inputs v_1 and v_2 and outputs y_1 and y_2 is

$$G(s) = \begin{bmatrix} \frac{c_1\gamma_1}{T_1 s + 1} & \frac{c_1(1-\gamma_2)}{(T_3 s+1)(T_1 s+1)} \\ \frac{c_2(1-\gamma_1)}{(T_4 s+1)(T_2 s+1)} & \frac{c_2\gamma_2}{T_2 s+1} \end{bmatrix} \tag{3}$$

where $c_i = \frac{T_i k_i}{A_i}\sqrt{\frac{2h_{i0}}{g}}$, $i = 1,2$.

The plant can be shifted from minimum to nonminimum phase configuration and vice versa simply by changing a valve controlling the flow ratios γ_1 and γ_2 between lower and upper tanks. The minimum-phase configuration corresponds to $1 < \gamma_1 + \gamma_2 < 2$ and the nonminimum-phase one to $0 < \gamma_1 + \gamma_2 < 1$.

2.1 Decentralized control of quadruple tank – problem formulation and pairing selection

The basic control aim for quadruple tank is to reach the given level in the lower two tanks, i.e. prescribed values of y_1 and y_2 by controlling input flows v_1 and v_2 delivered by two

pumps. To achieve this aim, the decentralized control structure is employed, with two control loops respective to output values y_1 and y_2.
Decentralized control design consists of several steps, the crucial ones for controller design are

- choice of appropriate pairing of inputs to outputs;
- structural stability test respective to chosen pairing;
- robust decentralized controller design.

We consider the standard approach for the former two steps presented below; in Sections 3 and 4 we concentrate on the last step – robust decentralized control design.

Pairing and structural stability

Frequently used index to assess input-output pairing is the Relative Gain Array (RGA) index, see e.g. (Ogunnaike & Ray, 1994), (Skogestad & Postletwhaite, 2009), computed as

$$RGA(s) = G(s).*[G(s)^T]^{-1} \tag{4}$$

where G(s) is a square transfer function matrix of the linearized system.
Individual subsystems are then specified by the chosen pairing and their transfer functions are placed in the diagonal of the transfer function matrix. To check structural stabilizability using the chosen control configuration, the Niederlinski index is applied:

$$NI = \frac{\det(G(0))}{\Pi(diag(G(0)))} \tag{5}$$

If $NI < 0$, the system cannot be stabilized using the chosen pairing and the pairing must be modified.
In our case study, the steady state RGA(0) is considered to choose appropriate pairing with the respective RGA elements positive and closest possible to 1.

$$RGA(0) = G(0).*\left[G(0)^{-1}\right]^T = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} \tag{6}$$

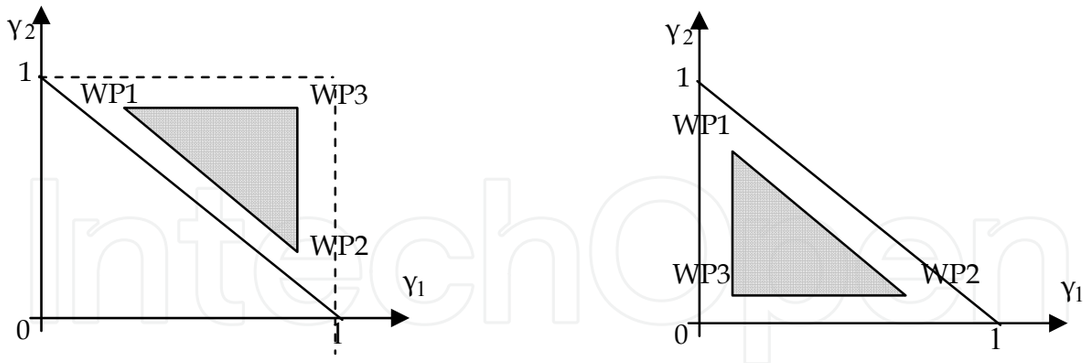
where $\lambda = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1}$ depends on valve parameters γ_1 and γ_2 exclusively. The diagonal elements λ are positive for $1 < \gamma_1 + \gamma_2 < 2$ (minimum phase system) and the respective pairing is $v_1 - y_1, v_2 - y_2$. For $0 < \gamma_1 + \gamma_2 < 1$ (nonminimm phase system), the opposite pairing $v_1 - y_2, v_2 - y_1$ is indicated. This result is approved by Niederlinski index.

2.2 Quadruple tank process – uncertainty domain

For quadruple tank system (1), we consider the uncertainty to be a change of valve position, i.e. change of γ_1 and γ_2, uncertainty domain is specified by three working points.

In minimum phase region:
WP1: γ_1 = 0.4, γ_2 = 0.8; WP2: γ_1 = 0.8, γ_2 = 0.4
WP3: γ_1 = 0.8, γ_2 = 0.8 (7)

In nonminimum phase region:
WP1: γ_1 = 0.1, γ_2 = 0.3; WP2: γ_1 = 0.3, γ_2 = 0.1
WP3: γ_1 = 0.1, γ_2 = 0.1 (8)



a) minimum phase configuration b) nonminimum phase configuration

Fig. 2. Uncertainty domain specified by working points

3. Robust decentralized PID controller design in the time domain

In this section, robust decentralized controller in time domain is designed based on robust stability conditions formulated and solved as linear (or bilinear) matrix inequalities. To include performance evaluation, the quadratic performance index is used. Decentralized robust control problem is formulated in general framework for augmented system, including the model of controlled system as well as controller dynamics. The robust stability conditions from literature are recalled, using D-stability concept which enables unified formulation for continuous-time and discrete-time cases. Our modification of these results includes derivative term of PID controller as well as a term for guaranteed cost. Thus, the decentralized control design procedure is presented in the general form comprising both continuous and discrete-time system models.

Notation: for a symmetric square matrix X, $X > 0$ denotes positive definiteness; * in matrices denotes the respective transposed term to make the matrix symmetric, 0 in matrices denotes zero block of the corresponding dimensions, I_n denotes identity matrix of dimensions nxn; dimension index is often omitted, when the dimension is clear from the context. Argument t denotes either continuous time for continuous-time, or sampled time for discrete-time system models; we intentionally use the same symbol t for both cases to underline that the formulation of developed results is general, applicable for both cases.

3.1 Preliminaries and problem formulation

3.1.1 Decentralized control of uncertain system, guaranteed cost control

Consider a linearized model of interconnected system, where subsystems with polytopic uncertainty are assumed, described by

$$\mathrm{S_i}:\ \delta x_i(t) = A_i(\alpha)x_i(t) + B_i(\alpha)u_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N}(A_{ij}(\alpha)x_j(t) + B_{ij}(\alpha)u_j(t))$$

$$y_i(t) = C_i x_i(t)\ ;\ \mathrm{i=1,...,N} \tag{9}$$

where $\delta x(t) = \dot{x}(t)$ for continuous-time system model; $\delta x(t) = x(t+1)$ for discrete-time system model; $x_i(t) \in R^{n_i}$, $u_i(t) \in R^{m_i}$, $y_i(t) \in R^{p_i}$ are the subsystem state, control and output vectors respectively, $\sum_{i=1}^{N} n_i = n$, $\sum_{i=1}^{N} m_i = m$, $\sum_{i=1}^{N} p_i = p$; C_i are matrices with corresponding dimensions. Uncertain model matrices $A_i(\alpha)$, $B_i(\alpha)$, $A_{ij}(\alpha)$, $B_{ij}(\alpha)$ are from polytopic uncertainty domains

$$A_i(\alpha) \in \left\{ \sum_{k=1}^{K} \alpha_k \tilde{A}_{ik}, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \geq 0 \right\}, B_i(\alpha) \in \left\{ \sum_{k=1}^{K} \alpha_k \tilde{B}_{ik}, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \geq 0 \right\},$$

$$A_{ij}(\alpha) \in \left\{ \sum_{k=1}^{K} \alpha_k \tilde{A}_{ijk}, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \geq 0 \right\} B_{ij}(\alpha) \in \left\{ \sum_{k=1}^{K} \alpha_k \tilde{B}_{ijk}, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \geq 0 \right\}. \quad (10)$$

The whole interconnected system model in the compact form is

$$\text{S: } \delta x(t) = A(\alpha)x(t) + B(\alpha)u(t)$$

$$y(t) = C_d x(t) \quad (11)$$

uncertain system matrix $A(\alpha) = \left(A_d(\alpha) + A_m(\alpha)\right)$, $B(\alpha) = \left(B_d(\alpha) + B_m(\alpha)\right)$ and

$$A(\alpha) \in \left\{ \sum_{k=1}^{K} \alpha_k \tilde{A}_{(k)}, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \geq 0 \right\}, B(\alpha) \in \left\{ \sum_{k=1}^{K} \alpha_k \tilde{B}_{(k)}, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \geq 0 \right\} \quad (12)$$

where $\tilde{A}_{(k)}$ has diagonal blocks $\tilde{A}_{ik}$ and off-diagonal blocks $\tilde{A}_{ijk}$, $\tilde{B}_{(k)}$ has diagonal blocks $\tilde{B}_{ik}$ and off-diagonal blocks $\tilde{B}_{ijk}$ respective to (10); and

$x(t) = (x_1^T\ x_2^T \ldots x_N^T)$, $u(t) = (u_1^T\ u_2^T \ldots u_N^T)$, $y(t) = (y_1^T\ y_2^T \ldots y_N^T)$ are state, control and output vectors of the overall system S;

$A_d(\alpha) = diag\{A_1(\alpha), \ldots, A_N(\alpha)\}$, $B_d(\alpha) = diag\{B_1(\alpha), \ldots, B_N(\alpha)\}$, $C_d = diag\{C_1, \ldots, C_N\}$ are overall system matrices of corresponding dimensions respective to the subsystems, matrices $A_m(\alpha)$, $B_m(\alpha)$ correspond to interconnections.

A closed loop system performance is assessed considering the guaranteed cost notion; the quadratic cost function known from LQ theory is used.

$$J_c = \int_0^{\infty} [x(t)^T Q x(t) + u(t)^T R u(t)] dt \quad \text{for a continuous-time and}$$

$$J_d = \sum_{k=0}^{\infty} [x(t)^T Q x(t) + u(t)^T R u(t)] \text{ for a discrete-time systems} \quad (13)$$

where $Q \in R^{n\times n}, R \in R^{m\times m}$ are symmetric positive semidefinite and positive definite block diagonal matrices respectively, with block dimensions respective to the subsystems. The concept of guaranteed cost control is used in a standard way: let there exist a control law $u(t)$ and a constant J_0 such that

$$J \le J_0 \tag{14}$$

holds for the closed loop system (9). Then the respective control $u(t)$ is called the *guaranteed cost control* and the value J_0 is the *guaranteed cost*.

Decentralized Control Problem

The control design aim is to find decentralized control law $u_i(x_i(t))$, or $u_i(y_i(t))$, i=1,…,N, i.e. the overall system is controlled using local control loops for subsystems, such that uncertain dynamic system (11) is robustly stable in uncertainty domain (12) with guaranteed cost.

Basically, control design problem will be transformed into the output feedback form: $u_i(t) = F_i y_i(t)$, employing augmented system model to include controller dynamics, as it is using PID controller.

3.1.2 Augmented system model for continuous and discrete-time PID controller

The augmented system model including PID controller dynamics is developed in this section in general form appropriate both for continuous and discrete-time PID controllers. Firstly, recall PID control algorithms for both cases.

Control algorithm for continuous-time PID is

$$u(t) = K_P e(t) + K_I \int_0^t e(t)dt + K_D \dot{e}(t) \tag{15}$$

where $e(t) = y(t) - w(t)$ is control error, $w(k)$ is reference value (negative feedback sign is included in matrices K_P, K_I, K_D); K_P, K_I, K_D are controller parameter matrices (for SISO system they are scalars) to be designed.

Generally, different output variables can be considered for proportional, integral and derivative controller terms, for better readability we assume that all outputs enter all three controller terms. We further assume that the reference value is constant, $w(k) = w$ and that the system states in model (11) correspond to the deviations from working point (these assumptions correspond to step change of reference value). Then the control law (15) can be rewritten as

$$u(t) = K_P y(t) + K_I \int_0^t y(t)dt + K_D \dot{y}(t)\,. \tag{16}$$

Integral term can be included into the state vector in the common way defining the auxiliary state $z = \int_0^t y(t)$, i.e. $\dot{z}(t) = y(t) = C_d x(t)$ and PID controller algorithm is

$$u(t) = K_P C_d x(t) + K_I C_d z(t) + K_D C_d \dot{x}(t) . \tag{17}$$

Then the closed-loop system (11) with PID controller (17) can be described by augmented model

$$\dot{x}_n = \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A(\alpha) & 0 \\ C_d & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} u =$$

$$= \begin{bmatrix} A(\alpha) & 0 \\ C_d & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} \begin{bmatrix} K_P C_d & K_I C_d \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} \begin{bmatrix} K_D C_d & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix}$$

or

$$\left(\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} B(\alpha) K_D C_d & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A(\alpha) & 0 \\ C_d & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} \begin{bmatrix} K_P C_d & K_I C_d \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix}$$

which in a compact form yields

$$M_d(\alpha) \dot{x}_n = A_C(\alpha) x_n \tag{18}$$

where

$$\begin{aligned} M_d(\alpha) &= \left(\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} \begin{bmatrix} K_D & 0 \end{bmatrix} \begin{bmatrix} C_d & 0 \\ 0 & C_d \end{bmatrix} \right), \\ A_C(\alpha) &= \begin{bmatrix} A(\alpha) & 0 \\ C_d & 0 \end{bmatrix} + \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} \begin{bmatrix} K_P & K_I \end{bmatrix} \begin{bmatrix} C_d & 0 \\ 0 & C_d \end{bmatrix} \end{aligned} \tag{19}$$

argument t is omitted for brevity.
A discrete-time PID (often denoted as PSD) controller is described by control algorithm

$$u(t) = k_P e(t) + k_I \sum_{i=0}^{t} e(i) + k_D [e(t) - e(t-1)] \tag{20}$$

where $u(t)$, $e(t) = y(t) - w(t)$, $w(t)$ are discrete time counterparts to the continuous time signals; k_P, k_I, k_D are controller parameter matrices to be designed. By analogy with continuous time case, for constant $w(t)$ we write

$$u(t) = k_P y(t) + k_I \sum_{i=0}^{t} y(i) + k_D [y(t) - y(t-1)] \tag{21}$$

State space description of PID controller can be derived in the following way. The dynamics of PID controller (21) requires two state variables, since besides $\sum_{i=0}^{t} y(i)$, also y(t-1) is needed. One possible choice of controller state variables is: $z(t) = [z_1^T(t) \;\; z_2^T(t)]^T$, $z_1(t) = \sum_{i=0}^{t-2} y(i)$, $z_2(t) = \sum_{i=0}^{t-1} y(i)$, then $y(t-1) = z_2(t) - z_1(t)$. Rewriting (21) as

$$\begin{aligned} u(t) &= k_P y(t) + k_I \sum_{i=0}^{t-1} y(i) + k_I y(t) + k_D [y(t) - y(t-1)] \\ &= (k_P + k_I + k_D) y(t) + k_I z_2(t) - k_D (z_2(t) - z_1(t)) \end{aligned} \tag{21}$$

we obtain the respective description of the discrete-time PID controller in state space as

$$\begin{aligned} z(t+1) &= \begin{bmatrix} 0 & I \\ 0 & I \end{bmatrix} z(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} y(t) = A_R z(t) + B_R y(t) \\ u(t) &= \begin{bmatrix} k_D & k_I - k_D \end{bmatrix} z(t) + (k_P + k_I + k_D) y(t) = \\ &= C_R z(t) + D_R y(t) \end{aligned} \tag{22}$$

where z(t) is controller dynamics state vector, $z(t) \in R^{2p}$.

The respective augmented model for discrete-time version of system (11) with PID controller is

$$x_n(t+1) = \begin{bmatrix} x(t+1) \\ z(t+1) \end{bmatrix} = \begin{bmatrix} A(\alpha) & 0 \\ B_R C_d & A_R \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} \begin{bmatrix} (D_R C_d & C_R \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \tag{23}$$

where $A_R \in R^{2p \times 2p}, A_R = \begin{bmatrix} 0 & I \\ 0 & I \end{bmatrix}$, $B_R \in R^{2p \times p}, B_R = \begin{bmatrix} 0 \\ I \end{bmatrix}$, $C_R \in R^{m \times 2p}, C_R = \begin{bmatrix} k_D & k_I - k_D \end{bmatrix}$, $D_R = k_P + k_I + k_D$.

Analogically as in continuous time case, the augmented system (23) can be rewritten in a compact form as

$$x_n(t+1) = A_C(\alpha) x_n(t) \tag{24}$$

where

$$A_C(\alpha) = \begin{bmatrix} A(\alpha) & 0 \\ B_R C_d & A_R \end{bmatrix} + \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} \begin{bmatrix} D_R & C_R \end{bmatrix} \begin{bmatrix} C_d & 0 \\ 0 & I_{2p} \end{bmatrix}. \tag{25}$$

Summarizing the augmented closed loop system models (18), (19) and (24), (25) for continuous and discrete-time PID controllers respectively, we can finally, using denotation $\delta x(t)$, introduced in (9), rewrite both of them in general form

$$M_d(\alpha) \delta x_n(t) = A_C(\alpha) x_n(t) \tag{26}$$

where $M_d(\alpha)$ is assumed to be invertible,

$A_C(\alpha) = A_{aug}(\alpha) + \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix} \begin{bmatrix} F_1 & F_2 \end{bmatrix} C_{aug} = A_{aug}(\alpha) + B_{aug}(\alpha) F C_{aug}$ and

for a continuous PID: $A_{aug}(\alpha) = \begin{bmatrix} A(\alpha) & 0 \\ C_d & 0 \end{bmatrix}$, $C_{aug} = \begin{bmatrix} C_d & 0 \\ 0 & C_d \end{bmatrix}$ and

$$M_d(\alpha) = \left(\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} B(\alpha) K_D C_d & 0 \\ 0 & 0 \end{bmatrix} \right); \tag{27a}$$

for a discrete-time PID: $A_{aug}(\alpha)=\begin{bmatrix} A(\alpha) & 0 \\ B_R C_d & A_R \end{bmatrix}$, $C_{aug}=\begin{bmatrix} C_d & 0 \\ 0 & I_{2p} \end{bmatrix}$ and $M_d(\alpha)=I$. (27b)

PID controller parameters are:

$$F=\begin{bmatrix} F_1 & F_2 \end{bmatrix}=\begin{bmatrix} K_P & K_I \end{bmatrix} \text{ and } K_D \text{ included in } M_d(\alpha) \text{ for a continuous-time case;} \quad (28a)$$

$$F=\begin{bmatrix} F_1 & F_2 \end{bmatrix};\ F_1=k_P+k_I+k_D,\ F_2=\begin{bmatrix} k_D & k_I-k_D \end{bmatrix} \text{ for a discrete-time case.} \quad (28b)$$

In a decentralized PID controller design, controller gain matrices are restricted to block diagonal structure respective to subsystem dimensions.
The presented general closed loop augmented system polytopic model (26) is advantageously used in next developments.

3.1.3 Robust stability

In this section we recall several recent results on robust stability for linear uncertain systems with polytopic model. These results are formulated as robust stability conditions in LMI form. Let us start with basic notions concerning Lyapunov stability and D-stability concept (Peaucelle et al., 2000; Henrion et al., 2002), used to receive the robust stability conditions in more general form.
Definition 3.1 (D-stability)
Consider the D-domain in the complex plain defined as

$$D=\{s \text{ is complex number}: \begin{bmatrix} 1 \\ s \end{bmatrix}^{*} \begin{bmatrix} r_{11} & r_{12} \\ r_{12}^{*} & r_{22} \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} < 0\} \quad (29)$$

Linear system is D-stable if and only if all its poles lie in the D-domain.
(For simplicity, we use in Def. 3.1 scalar values of parameters r_{ij}, in general, the stability domain can be defined using matrix values of parameters r_{ij} with the respective dimensions.) The standard choice of r_{ij} is $r_{11} = 0$, $r_{12} = 1$, $r_{22} = 0$ for a continuous-time system; $r_{11} = -1$, $r_{12} = 0$, $r_{22} = 1$ for a discrete-time system, corresponding to open left half plane and unit circle respectively.
The D-stability concept enables to formulate robust stability condition for uncertain polytopic system in general way, (deOliveira et al., 1999; Peaucelle et al., 2000). The following robust stability condition is based on the existence of Lyapunov function $V(t)=x(t)P(\alpha)x(t)$ for linear uncertain polytopic system

$$\delta x(t)=A(\alpha)x(t) \quad (30)$$

where $A(\alpha)$ is from uncertainty domain (12).

Definition 3.2 (Robust stability)

Uncertain system (30) is *robustly D-stable* in the convex uncertainty domain (12) if and only if there exists a matrix $P(\alpha)=P(\alpha)^T>0$ such that

$$r_{12}P(\alpha)A(\alpha)+r_{12}^{*}A^T(\alpha)P(\alpha)+r_{11}P(\alpha)+r_{22}A^T(\alpha)P(\alpha)A(\alpha)<0 \quad (31)$$

For one Lyapunov function for the whole uncertainty domain, i.e. $P(\alpha) = P > 0$, the *quadratic D-stability* is guaranteed by (31). Generally, robust stability condition (31) with parameter dependent matrix $P(\alpha)$ is less conservative (provides bigger stability domain for $A(\alpha)$ than quadratic stability one), however stability is guaranteed only for relatively slow changes of system parameters within uncertainty domain (12) (in comparison with system dynamics). On the other hand, quadratic stability guards against arbitrary quick changes of system parameters within uncertainty domain (12) at the expense of sufficient, relatively strong, stability condition; which can be overly conservative for the case of slow parameter changes.
We consider the parameter dependent Lyapunov function (PDLF) defined as

$$V(t) = x(t)P(\alpha)x(t) \tag{32}$$

$$P(\alpha) = \sum_{k=1}^{K} \alpha_k P_k \text{ where } P_k = P_k^T > 0 \tag{33}$$

PDLF given by (32), (33) enables to transform robust stability condition (31) for uncertain linear polytopic system (9), (10) into the set of N Linear Matrix Inequalities (LMIs). Several respective sufficient robust stability conditions have been developed in the literature, e.g. (deOliveira et al., 1999; Peaucelle et al., 2000; Henrion et al., 2002). Recall the sufficient robust *D*-stability condition proposed in (Peaucelle et al., 2000), which to the authors best knowledge belongs to the least conservative (Grman et al., 2005).

Lemma 3.1

If there exist matrices $H \in R^{nxn}, G \in R^{nxn}$ and K symmetric positive definite matrices $P_k \in R^{nxn}$ such that for all k = 1,…, K:

$$\begin{bmatrix} r_{11}P_k + \tilde{A}_{(k)}^T H^T + H\tilde{A}_{(k)} & r_{12}P_k - H + \tilde{A}_{(k)}^T G \\ r_{12}^* P_k - H^T + G^T \tilde{A}_{(k)} & r_{22}P_k - (G + G^T) \end{bmatrix} < 0 \tag{34}$$

then uncertain system (30) is robustly D-stable in uncertainty domain (12).
Note that matrices H and G are not restricted to any special form; they were included to relax the conservatism of the sufficient condition. Robust stability condition for more general dynamic system model (26), including also the term for guaranteed cost will be presented in the next section.

3.2 Robust decentralized PID controller design

In this section, the robust decentralized PID controller is designed, based on robust stability condition developed in our recent papers, (Rosinová & Veselý, 2007; Veselý & Rosinová, 2011). Robust stability condition with guaranteed cost for closed loop uncertain system (26) is provided in the next theorem.

Theorem 3.1

Consider uncertain linear system (26) with cost function (13). If there exist symmetric matrix $P(\alpha) > 0$ and matrices H, G and F of the respective dimensions such that

$$\begin{bmatrix} r_{11}P(\alpha) + A_C(\alpha)^T H^T + HA_C(\alpha) + Q + C^T F^T RFC & r_{12}P(\alpha) - HM_d(\alpha) + A_C(\alpha)^T G \\ r_{12}^* P(\alpha) - M_d(\alpha)^T H^T + G^T A_C(\alpha) & r_{22}P_k - M_d(\alpha)G - G^T M_d(\alpha)^T \end{bmatrix} < 0 \quad (35)$$

then the system (26) is robustly *D*-stable with guaranteed cost: $J \le J_0 = x^T(0)P(\alpha)x(0)$.

Proof. The proof is analogical to the one presented in (Rosinová & Veselý, 2007) for the continuous-time PID. Firstly, we formulate the sufficient stability condition for uncertain system (26) using the respective Lyapunov function. The assumption that $M_d(\alpha)$ is invertible, enables us to rewrite (26) as $\delta x(t) = M_d^{-1}(\alpha)A(\alpha)x(t)$ and use parameter dependent Lyapunov function (32) to write robust stability condition.

Denote $\delta V(t) = \dot{V}(t)$ for a continuous-time system, $\delta V(t) = V(t+1) - V(t)$ for a discrete-time system. Then the sufficient D-stability condition (31) can be rewritten in the following form (known from LQ theory, for details see e.g. Rosinová et al., 2003)

$$\begin{aligned} &r_{12}P(\alpha)M_d^{-1}(\alpha)A(\alpha) + r_{12}^* A^T(\alpha)\left(M_d^{-1}(\alpha)\right)^T P(\alpha) + r_{11}P(\alpha) + \\ &r_{22}A^T(\alpha)\left(M_d^{-1}(\alpha)\right)^T P(\alpha)M_d^{-1}(\alpha)A(\alpha) + Q + C_d^T F^T RFC_d < 0 \end{aligned} \quad (36)$$

where the term $Q + C_d^T F^T RFC_d$ has been appended to $\delta V(t)$ to consider the guaranteed cost. To prove Theorem 3.1, it is sufficient to prove that (35) implies (36). This can be shown applying congruence transformation on (35):

$$\begin{bmatrix} I & A_C^T(\alpha)\left(M_d^{-1}(\alpha)\right)^T \end{bmatrix} \{left\,hand\,side\,of\,(35)\} \begin{bmatrix} I \\ \left(M_d^{-1}(\alpha)\right)A_C(\alpha) \end{bmatrix} < 0 \quad (37)$$

which immediately yields (36).

It is important to note that robust stability condition (35) is linear with respect to parameter α. Therefore, for convex polytopic uncertainty domain (12) and PDLF (33), matrix inequality (35) is equivalent to the set of matrix inequalities respective to the polytope vertices, as summarized in Corollary 3.1.

Corollary 3.1

Uncertain linear system (26) with cost function (13) is robustly *D*-stable with parameter dependent Lyapunov function (32), (33) and guaranteed cost $J \le J_0 = x^T(0)P(\alpha)x(0)$ if the following matrix inequalities hold

$$\begin{bmatrix} r_{11}P_k + A_{Ck}^T H^T + HA_{Ck} + Q + C^T F^T RFC & r_{12}P_k - HM_{dk} + A_{Ck}^T G \\ r_{12}^* P_k - M_{dk}^T H^T + G^T A_{Ck} & r_{22}P_k - M_{dk}G - G^T M_{dk}^T \end{bmatrix} < 0, \text{ k=1,...,K} \quad (38)$$

where $A_C(\alpha) = A_{aug}(\alpha) + B_{aug}(\alpha)FC_{aug} \in \left\{ \sum_{k=1}^{K} \alpha_k A_{Ck}, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \ge 0 \right\}$,

$A_{Ck} = A_{aug\,k} + B_{aug\,k}FC_{aug}$, and $A_{aug\,k}$, $B_{aug\,k}$ correspond to the k-th vertex of uncertainty domain of the overall system (10), (12);

$M_d(\alpha) \in \left\{ \sum_{k=1}^{K} \alpha_k M_{dk}, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \geq 0 \right\}$, M_{dk} is for PID controller given by (27a) or (27b), and $B(\alpha)$ is given by (12).

Robust stability condition (38) is LMI for stability analysis, for controller synthesis it is in the BMI form. Therefore, (38) can be used for robust controller design either directly – using appropriate BMI solver (Henrion et al., 2005) or using some convexifying approach, (for discrete-time case see e.g. (Crusius & Trofino, 1999; deOliveira et al., 1999)). We have relatively good experience with the following simple convexified LMI procedure for static output feedback discrete-time controller design, which is directly applicable for discrete-time PID controller design problem formulated by (26), (27b), (28b).

The controller gain block diagonal matrix F is obtained by solving LMIs (39) for unknown matrices F, M, G and P_k of appropriate dimensions, the P_k being block diagonal symmetric, and M, G block diagonal with block dimensions conforming to subsystem dimensions. This convexifying approach does not allow including a term corresponding to performance index, therefore the resulting control guarantees only robust stability within considered uncertainty domain.

$$\begin{bmatrix} -P_k & A_{aug\,k}G + B_{aug\,k}KC_{aug} \\ G^T A_{aug\,k}^T + C^T K^T B_{aug\,k}^T & -G - G^T + P_k \end{bmatrix} < 0, \quad , \text{ k=1,...,K}$$

$$MC_{aug} = C_{aug}G \tag{39}$$

$$F = KM^{-1}$$

F is the corresponding output feedback gain matrix.

The main advantage of the use of LMI (39) for controller design is its simplicity. The major drawbacks are, that the performance index cannot be considered, and that due to convexifying constraint ($MC_{aug} = C_{aug}G$), it need not provide a solution even in a case when feasible solution is received through BMI (38). (This is the case in our example in Section 3.3, in nonminimum phase configuration.)

To conclude this section we summarize the described decentralized PID controller design procedure, assuming that the state space model is in the form of (9) with polytopic uncertainty domain given by (10), where columns of control input matrix B are arranged respectively to chosen pairing.

Design procedure for decentralized PID design in time domain

Step 1. Formulate the augmented state space model (26) for given system and chosen type of PID controller.

Step 2. Compute decentralized PID controller parameters using one of design alternatives:

- LMI alternative for discrete-time case – guarantees robust stability: solve LMI (39) for unknown block diagonal matrices F, M, G and $P_k>0$, of appropriate dimensions; PID controller parameters are given by F respectively to (28b).
- BMI alternative – guarantees robust stability and guaranteed cost for quadratic performance index (13): solve BMI (38) for unknown block diagonal matrices F, $P_k>0$ and matrices G, H, of appropriate dimensions, PID controller parameters are given by F and M_{dk} respectively to (28) and (27), M_{dk} is for PID controller given by (27).

3.3 Decentralized PID controller design for the Quadruple tank process

We consider quadruple tank linearized model (2) with parameters:

$$A_1 = A_3 = 30[cm^2];\;\; A_2 = A_4 = 35\,[cm^2];$$

$$a_1 = a_3 = 0.0977\,[cm^2];\;\; a_2 = a_4 = 0.0785\,[cm^2]\,;$$

$$h_{10} = h_{20} = 20\,[cm];\, h_{30} = 2.75\,[cm];\, h_{40} = 2.22\,[cm]\,;$$

$$g = 981\,[cm/s^2];\quad k_1 = 1.790;\, k_2 = 1.827\,.$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_3\\ \dot{x}_2\\ \dot{x}_4\end{bmatrix} = \left[\begin{array}{cc:cc} -0.0161 & 0.0435 & 0 & 0\\ 0 & -0.0435 & 0 & 0\\ \hdashline 0 & 0 & -0.0111 & 0.0333\\ 0 & 0 & 0 & -0.0333\end{array}\right]\cdot\begin{bmatrix}x_1\\ x_3\\ x_2\\ x_4\end{bmatrix} + \left[\begin{array}{cc} 0.0596\gamma_1 & 0\\ 0 & 0.0595(1-\gamma_2)\\ \hdashline 0 & 0.0522\gamma_2\\ 0.052(1-\gamma_1) & 0\end{array}\right]\cdot\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \left[\begin{array}{cc:cc} 1 & 0 & 0 & 0\\ \hdashline 0 & 0 & 1 & 0\end{array}\right]\begin{bmatrix}x_1\\ x_2\\ \hdashline x_3\\ x_4\end{bmatrix}$$

Subsystems are indicated via the splitting dashed lines. Polytope vertices respective to working points (7) or (8) for minimum phase or nonminimum phase configurations respectively determine the corresponding uncertainty domains indicated in Fig.2. State space model has been discretized with sampling period $T_s = 5[s]$ (sampling period was chosen with respect to the process dynamics).

Minimum phase configuration

In the minimum phase case, robust decentralized controller is designed for chosen pairing $v_1 - y_1$, $v_2 - y_2$ (see Section 2.1) using alternatively solution of LMI (39) or BMI (38) for decentralized discrete-time PI controller design. The resulting controller parameters are in Tab.1, the respective simulation results are illustrated and compared on step responses in one tested point from uncertainty domain, in Fig. 3.

Design approach	1st subsyst. controller	2nd subsyst. controller
LMI (39)	$\frac{5.862 - 2.602z^{-1}}{1 - z^{-1}}$	$\frac{6.45 - 2.578z^{-1}}{1 - z^{-1}}$
BMI (38) Q=0.01*I, R=5*I	$\frac{1.3002 - 1.0351z^{-1}}{1 - z^{-1}}$	$\frac{1.3833 - 1.1361z^{-1}}{1 - z^{-1}}$

Table 1. Decentralized PID controller parameters – minimum phase case

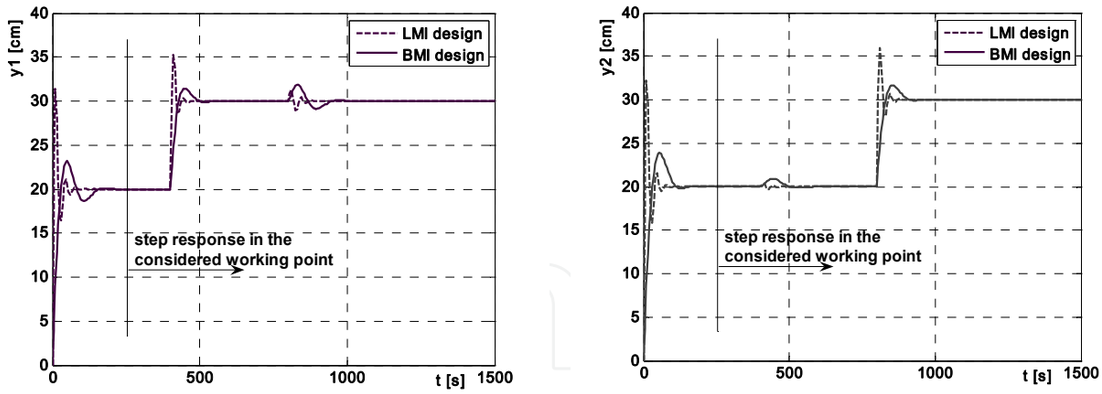


Fig. 3. Step response of y_1 and y_2 to setpoint step changes: w_1 in 400s and w_2 in 800s; comparison of LMI and BMI design results from Tab.1

Obviously, the results for the BMI solution including performance index outperform the ones obtained using simpler LMI approach.

Nonminimum phase configuration

In the nonminimum phase case, robust decentralized controller is designed for chosen pairing $v_1 - y_2$, $v_2 - y_1$ (see Section 2.1) using a solution of BMI (38) for decentralized discrete-time PI controller design, (in this case LMI procedure (39) does not provide a feasible solution). The resulting controller parameters are in Tab.2, the respective simulation results are illustrated on step responses in one tested point from uncertainty domain, in Fig. 4.

Design approach	1st subsyst. controller	2nd subsyst. controller
BMI (38) Q=0.01*I, R=5*I	$\frac{0.5371 - 0.5099z^{-1}}{1 - z^{-1}}$	$\frac{0.7221 - 0.6941z^{-1}}{1 - z^{-1}}$

Table 2. Decentralized PID controller parameters – nonminimum phase case

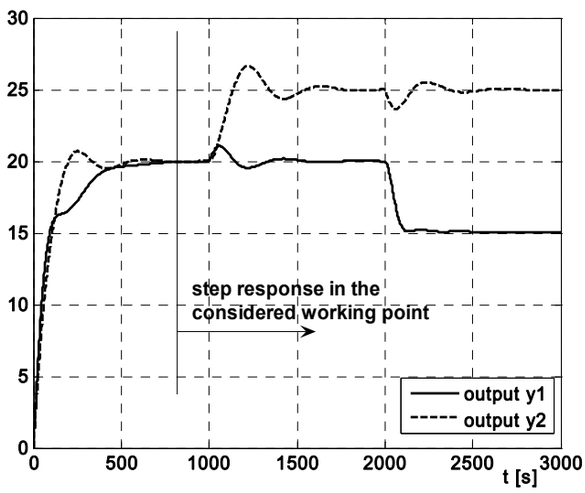


Fig. 4. Step response of y_1 and y_2 to setpoint step changes: w_1 (for y_2) in 1000s and w_2 (for y_1) in 2000s

Comparison of simulation results for minimum and nonminimum phase cases shows the deteriorating influence of nonminimum phase on settling time.

4. Robust decentralized PID controller design in the frequency domain

This section deals with an original frequency domain robust decentralized controller design methodology applicable for uncertain systems described by a set of transfer function matrices. The design methodology is based on the Equivalent Subsystems Method (ESM) - a frequency domain decentralized controller design technique to guarantee stability and specified performance of multivariable systems and is applicable for both continuous- and discrete-time controller designs (Kozáková et al., 2009). In contrast to the two stage robust decentralized controller design method based on the M-Δ structure stability conditions (Kozáková & Veselý, 2009), the recent innovation (Kozáková et al., 2011) consists in that robust stability conditions are directly integrated into the ESM, thus providing a one-step (direct) robust decentralized controller design for robust stability and plant-wide performance.

4.1 Preliminaries and problem formulation

Consider a MIMO system described by a transfer function matrix $G(s) \in R^{m\times m}$ and a controller $R(s) \in R^{m\times m}$ in the standard feedback configuration according to Fig. 5,

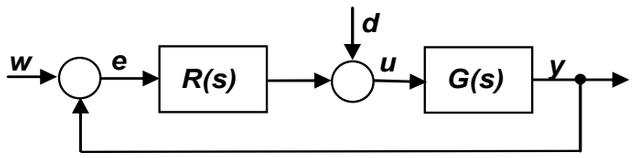


Fig. 5. Standard feedback configuration

where w, u, y, e, d are respectively vectors of reference, control, output, control error and disturbance of compatible dimensions. Necessary and sufficient conditions for closed-loop stability are given by the Generalized Nyquist Stability Theorem applied to the closed-loop characteristic polynomial

$$\det F(s) = \det[I + Q(s)] \tag{40}$$

where $Q(s) = G(s)R(s) \in R^{m\times m}$ is the open-loop transfer function matrix.

Characteristic functions of $Q(s)$ are the set of m algebraic functions $q_i(s)$, $i = 1,...,m$ defined as follows:

$$\det[q_i(s)I_m - Q(s)] = 0 \quad i = 1,...,m \tag{41}$$

Characteristic loci (CL) are the set of loci in the complex plane traced out by the characteristic functions of Q(s), $s = j\omega$.

Theorem 4.1 (Generalized Nyquist Stability Theorem)

The closed-loop system in Fig. 1 is stable if and only if

$$\text{a. } \det F(s) \neq 0 \qquad \forall s$$

$$\text{b. } N[0, \det F(s)] = \sum_{i=1}^{m} N\{0, [1 + q_i(s)]\} = n_q \tag{42}$$

where $F(s) = (I + Q(s))$ and n_q is the number of unstable poles of Q(s).
Let the uncertain plant be given as a set Π of N transfer function matrices

$$\Pi = \{G^k(s)\},\ k = 1, 2, ..., N \quad \text{where} \quad G^k(s) = \left\{G_{ij}^k(s)\right\}_{m \times m} \tag{43}$$

The simplest uncertainty model is the unstructured perturbation. A set of unstructured perturbations D_U is defined as

$$D_U := \{E(j\omega) : \sigma_{\max}[E(j\omega)] \leq \ell(\omega),\ \ \ell(\omega) = \max_k\ \sigma_{\max}[E(j\omega)]\} \tag{44}$$

where $\ell(\omega)$ is a scalar weight on the norm-bounded perturbation $\Delta(s) \in R^{m \times m}$, $\sigma_{\max}[\Delta(j\omega)] \leq 1$ over given frequency range, $\sigma_{\max}(\cdot)$ is the maximum singular value of (.), hence

$$E(j\omega) = \ell(\omega)\Delta(j\omega) \tag{45}$$

Using unstructured perturbation, the set Π can be generated by either additive (E_a), multiplicative input (E_i) or multiplicative output (E_o) uncertainties, or their inverse counterparts (Skogestad & Postlethwaite, 2009) thus specifying pertinent uncertainty regions. In the sequel, just additive (a) and multiplicative output (o) perturbations will be considered; results for other uncertainty types can be obtained by analogy.
Denote $G(s)$ any member of a set of possible plants $\Pi_k,\ k = a, i$; $G_0(s)$ the nominal model used to design the controller, and $\ell_k(\omega)$ the scalar weight on a normalized perturbation. The sets Π_k generated by the two considered uncertainty forms are:
Additive uncertainty:

$$\begin{aligned} &\Pi_a := \{G(s) : G(s) = G_0(s) + E_a(s),\ E_a(j\omega) \leq \ell_a(\omega)\Delta(j\omega)\} \\ &\ell_a(\omega) = \max_k\ \sigma_{\max}[G^k(j\omega) - G_0(j\omega)],\ k = 1, 2, \ldots, N \end{aligned} \tag{46}$$

Multiplicative output uncertainty:

$$\begin{aligned} &\Pi_o := \{G(s) : G(s) = [I + E_o(s)]G_0(s),\ \Delta(j\omega) \leq \ell_o(j\omega)\Delta(j\omega)\} \\ &\ell_o(\omega) = \max_k\ \sigma_{\max}\{[G^k(j\omega) - G_0(j\omega)]G_0^{-1}(j\omega)\},\ k = 1, 2, \ldots, N \end{aligned} \tag{47}$$

Standard feedback configuration with uncertain plant modelled using any unstructured uncertainty form can be recast into the $M - \Delta$ structure (for additive perturbation see Fig. 6) where M(s) is the nominal model and $\Delta(s)$ is the norm-bounded complex perturbation.

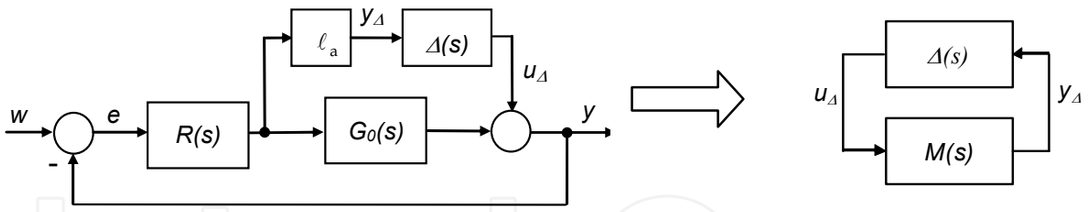


Fig. 6. Standard feedback configuration with additive perturbation (left) recast into the $M-\Delta$ structure (right)

According to the general robust stability condition (Skogestad & Postlethwaite, 2009), if both the nominal closed-loop system M(s) and the perturbations $\Delta(s)$ are stable, the $M-\Delta$ system in Fig. 2 is stable for all perturbations $\Delta(s):\sigma_{\max}(\Delta)\le 1$ if and only if

$$\sigma_{\max}[M(j\omega)]<1\ ,\ \forall\omega \tag{48}$$

For individual uncertainty forms $M(s)=\ell_k M_k(s),\ k=a,o$ the corresponding matrices $M_k(s)$ are given by (49) and (50), respectively (disregarding negative signs which do not affect resulting robustness condition). The nominal model $G_0(s)$ is usually obtained as a model of mean parameter values.

$$M(s)=\ell_a(s)R(s)[I+G_0(s)R(s)]^{-1}=\ell_a(s)M_a(s) \tag{49}$$

$$M(s)=\ell_o(s)G_0(s)R(s)[I+G_0(s)R(s)]^{-1}=\ell_o(s)M_o(s) \tag{50}$$

4.1.1 Problem formulation

Consider an uncertain system that consists of m subsystems and is given as a set of N transfer function matrices obtained in N working points of plant operation. Let the uncertain system be described by a nominal model $G_0(s)$ and any unstructured uncertainty form (46), (47). Consider the following splitting of $G_0(s)$:

$$G_0(s)=G_d(s)+G_m(s) \tag{51}$$

where

$$G_d(s)=diag\{G_i(s)\}_{m\times m},\quad \det G_d(s)\neq 0 \tag{52}$$

$$G_m(s)=G_0(s)-G_d(s) \tag{53}$$

A decentralized controller

$$R(s)=diag\{R_i(s)\}_{m\times m}\quad \det R(s)\neq 0 \tag{54}$$

is to be designed to guarantee stability over the whole operating range of the plant specified by (46) or (47) (robust stability) and a specified plant-wide performance (nominal performance).

To solve this problem, a frequency domain robust decentralized controller design technique has been developed (Kozáková and Veselý, 2009; Kozáková et. al., 2011); the core of it is the Equivalent Subsystems Method (ESM).

4.2 Decentralized controller design for performance: Equivalent Subsystems Method

The Equivalent Subsystems Method (ESM) is a Nyquist-based technique to design decentralized controller for stability and specified plant-wide performance. According to it, local controllers $R_i(s), i = 1,...,m$ are designed independently for so-called equivalent subsystems obtained from frequency responses of decoupled subsystems by shaping each of them using one of m characteristic loci of the interactions matrix $G_m(s)$. If local controllers are independently tuned for specified degree-of-stability of equivalent subsystems, the resulting decentralized controller guarantees the same degree-of-stability plant-wide (Kozáková et al., 2009). Unlike standard robust approaches, the proposed technique considers full nominal model of mean parameter values, thus reducing conservatism of resulting robust stability conditions. In the context of robust decentralized controller design, the Equivalent Subsystems Method is directly applicable to design DC for the nominal model (Fig. 3).

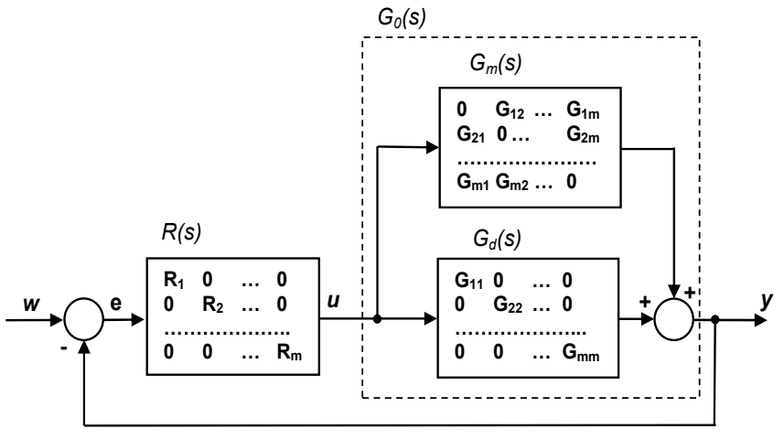


Fig. 7. Standard feedback loop under decentralized controller

The key idea behind the method is factorization of the closed-loop characteristic polynomial (40) in terms of the nominal system (51) under the decentralized controller (54). Then

$$\det F(s) = \det[R^{-1}(s) + G_d(s) + G_m(s)] \det R(s) \tag{55}$$

Denote the sum of the diagonal matrices in (55) as

$$R^{-1}(s) + G_d(s) = P(s) \tag{56}$$

where $P(s) = diag\{p_i(s)\}_{m\times m}$.

In order to "counterbalance" interactions $G_m(s)$, consider the closed-loop being at the limit of instability and choose the diagonal matrix $P(s) = p_k(s)I$ to have identical entries $p_k(s)$; then by similarity with (41) the bracketed term in (55) defines the k-th of the m

characteristic functions of $[-G_m(s)]$ (the set of characteristic functions are denoted $g_i(s), i = 1,2,...,m$); thus

$$\det[P(s) + G_m(s)] = \det[p_k I + G_m] = \prod_{i=1}^{m}[-g_k(s) + g_i(s)] = 0, \;\; k = 1,2,...m \tag{57}$$

With respect to stability, the interactions matrix $G_m(s)$ can thus be replaced by [-P(s)] yielding the important relationship

$$\begin{aligned} &\det[I + G(s)R(s)] = \det\{I + [G_d(s) + G_m(s)]R(s)\} = \\ &= \det[R^{-1}(s) + G_d(s) - P(s)]\det R(s) = \det[I + G^{eq}(s)R(s)] \end{aligned} \tag{58}$$

where

$$G^{eq}(s) = diag\{G_i^{eq}(s)\}_{m\times m} \tag{59}$$

is a diagonal matrix of m equivalent subsystems generated as follows

$$G_{ik}^{eq}(s) = G_i(s) + g_k(s), \qquad i = 1,2,\ldots,m \tag{60}$$

As all matrices are diagonal, on subsystems level (58) breaks down into m equivalent closed-loop characteristic polynomials (CLCP)

$$CLCP_i^{eq}(s) = 1 + R_i(s)G_i^{eq}(s) \qquad i = 1,2,...,m \tag{61}$$

Considering (58)-(61), stability conditions stated in the Generalized Nyquist Stability Theorem modify as follows:

Corollary 4.1

The closed-loop in Fig. 3 comprising the system (51) and the decentralized controller (54) is stable if and only if there exists a diagonal matrix $P(s) = p_k(s)I(s)$ such that

$$\begin{aligned} &1.\;\; \det[p_k(s)I + G_m] = 0, \;\; \text{for a fixed} \;\; k \in \{1,...,m\} \\ &2.\;\; \text{all equivalent characteristic polynomials (61) have roots with } \operatorname{Re}\{s\} < 0\,; \\ &3.\;\; N[0, \det F(s)] = n_q \end{aligned} \tag{62}$$

where N[0,g(s)] is number of anticlockwise encirclements of the complex plane origin by the Nyquist plot of g(s); n_q is number of open loop poles with $\operatorname{Re}\{s\} > 0$.

The decentralized controller design technique for nominal stability resulting from *Corollary 4.1* enables to independently design stabilizing local controllers for individual single input-single output equivalent subsystems using any standard frequency-domain design method, e.g. (Bucz et al., 2010; Drahos, 2000). In the originally developed ESM version (Kozáková et al., 2009) it was proved that local controllers tuned for a specified feasible degree-of-stability of equivalent subsystems constitute the decentralized controller guaranteeing the same

degree-of-stability plant-wide. To design local controllers of equivalent subsystems, the general conditions in *Corollary 4.1* allow using any frequency domain performance measure that can appropriately be interpreted for the full system. In the next subsection, the plant wide performance is specified in terms of maximum overshoot which is closely related to phase margins of equivalent subsystems.

4.2.1 Decentralized controller design for guaranteed maximum overshoot and specified settling time

The ESM can be applied to design decentralized controller to guarantee specified maximum overshoot of output variables of the multivariable system. The design procedure evolves from the known relationship between the phase margin (PM) and the maximum peak of the complementary sensitivity (Skogestad & Postlethwaite, 2009)

$$PM \geq 2\arcsin\left(\frac{1}{2M_T}\right) \geq \frac{1}{M_T}[rad] \tag{63}$$

where

$$M_T = \sigma_{\max}[T(j\omega)] \tag{64}$$

is the maximum peak of the complementary sensitivity T(s) defined as

$$T(s) = G(s)R(s)[I + G(s)R(s)]^{-1} \tag{65}$$

Relation between the maximum overshoot $\eta_{\max}$ and M_T is given by (Bucz et al., 2010)

$$\eta_{\max} \leq \frac{1.18M_T - |T(0)|}{|T(0)|}100[\%] \tag{66}$$

According to the ESM philosophy, local controllers are designed using frequency domain methods; if PID controller is considered, the most appropriate ones are e.g. the Bode diagram design or the Neymark D-partition method. If using the Bode diagram design, in addition to $\eta_{\max}$ it is also possible to specify the required settling time t_s related with the closed-loop bandwidth frequency ω_0 defined as the gain crossover frequency. The following relations between t_s and ω_0 are useful (Reinisch, 1974).

$$t_s \approx \frac{3}{\omega_0} \text{ for } M_T \in (1.3; 1.5)$$

$$\frac{\pi}{t_s} < \omega_0 < \frac{4\pi}{t_s} \tag{67}$$

In general, a larger bandwidth corresponds to a smaller rise time, since high frequency signals are more easily passed on to the outputs. If the bandwidth is small, the time response will generally be slow and the system will usually be more robust.

Design procedure:

1. Generating frequency responses of equivalent subsystems.
2. Specification of performance requirements in terms of $\eta_{\max}$, t_s and M_T using (66), (67).
3. Specification of a minimum phase margin PM for equivalent subsystems using (63).
4. Local controller design for specified PM in equivalent subsystems using appropriate frequency domain method.
5. Verification of achieved performance by evaluating frequency domain performance measure and via simulation.

4.3 Decentralized controller design for robust stability using the Equivalent Subsystems Method

In the context of robust control approach, the ESM method in its original version is inherently appropriate to design decentralized controller guaranteeing stability and specified performance of the nominal model (nominal stability, nominal performance). If, in addition, the decentralized controller has to guarantee closed-loop stability over the whole operating range of the plant specified by the chosen uncertainty description (robust stability), the ESM can be used either within a two-stage design procedure or a direct design procedure for robust stability and nominal performance.

1. Two stage robust decentralized controller design for robust stability and nominal performance

In the first stage, the decentralized controller for the nominal system is designed using ESM, afterwards, fulfilment of the M-Δ stability condition (48) is examined; if satisfied, the design procedure stops, otherwise in the second stage the controller parameters are modified to satisfy robust stability conditions in the tightest possible way, or local controllers are redesigned using modified performance requirements (Kozáková & Veselý, 2009).

2. Direct decentralized controller design for robust stability and nominal performance

By direct integration of robust stability condition (48) in the ESM, a "one-shot" design of local controllers for both nominal performance and robust stability can be carried out. In case of decentralized controller design for guaranteed maximum overshoot and specified settling time, the upper bound for the maximum peak of the nominal complementary sensitivity over the given frequency range

$$M_T = \max_{\omega}\{\sigma_{max}[T_0(j\omega)]\} \qquad T_0(s) = G_0(s)R(s)[I + G_0(s)R(s)]^{-1} \tag{68}$$

can be obtained using the singular value properties in manipulations of the M-Δ condition (48) considering (49) or (50). The following bounds for the nominal complementary sensitivity have been derived:

$$\sigma_{\max}[T_0(j\omega)] < \frac{\sigma_{\min}[G_0(j\omega)]}{|\ell_a(\omega)|} = L_A(\omega) \qquad \forall\omega \text{ additive uncertainty} \tag{69}$$

$$\sigma_{\max}[T_0(j\omega)] < \frac{1}{|\ell_o(\omega)|} = L_O(\omega) \qquad \forall\omega \quad \text{multiplicative output uncertainty} \tag{70}$$

Expressions on the r.h.s. of (69) and (70) do not depend on a particular controller and can be evaluated prior to designing the controller. In this way, if

$$M_T = \max_{\omega}\{\sigma_{\max}[T_0(j\omega)]\} \tag{71}$$

is used in the Design procedure, the resulting decentralized controller will simultaneously guarantee achieving the required maximum overshoot of all output variables (nominal performance) and stability over the whole operating range of the plant specified by selected working points (robust stability).

4.4 Discrete-time robust decentralized controller design using the Equivalent Subsystems Method

Controllers for continuous-time plants are mostly implemented as discrete-time controllers. A common approach to discrete-time controller design is the continuous controller redesign i.e. conversion of the already designed continuous controller into its discrete counterpart. This approach, however, is only an approximate scheme; performance under these controllers deteriorates with increasing sampling period. This drawback may be improved by modifying the continuous controller design before it is discretized which can often allow significantly larger sampling periods (Lewis, 1992). Then, the ESM design methodology can be applied in a similar way as in the continuous-time case using discrete characteristic loci, discrete Nyquist plots and discrete Bode diagrams of equivalent subsystems. Local controllers designed as continuous-time ones are subsequently converted into their discrete-time counterparts. Closed-loop performance under a discrete-time controller is verified using simulations and the discrete-time maximum singular value of the sensitivity $\sigma_M[S(z)]$ where

$$S(z) = [I + G(z)R(z)]^{-1},\ z = e^{j\omega T_s} \tag{72}$$

The maximum singular value $\max_{\omega} \sigma_{max}[S(e^{j\omega T})]$ plotted as function of frequency ω should be small at low frequencies where feedback is effective, and approach 1 at high frequencies, as the system is strictly proper, having a peak larger than 1 around the crossover frequency. The peak is unavoidable for real systems. Bandwidth frequency is defined as frequency where $\sigma_M[S(e^{j\omega T_s})]$ crosses 0.7 from below (Skogestad & Postlethwaite, 2009). Similarly, a discretized version of robust stability conditions (69), (70) based on (46) and (47) is applied.

4.4.1 Design of continuous controllers for discretization

The crucial step for the discrete controller design is proper choice of the sampling time T. Then, frequency response of the discretized system matches the one of the continuous time system up to a certain frequency $\omega < \omega_S / 2$, and the discrete controller can be obtained by converting the continuous–time controller designed from the discrete frequency responses to its discrete-time counterpart.

The sampling period T is to be selected according to the Shannon-Kotelnikov sampling theorem, or using common rules of thumb, e.g. as ~ 1/10 of the settling time of the plant step response, or from control system bandwidth according to the relation

$$20 < \frac{\omega_s}{\omega_0} < 40 \tag{72}$$

where ω_s is sampling frequency, and ω_0 is control system bandwidth, i.e. the maximum frequency at which the system output still tracks and input sinusoid in a satisfactory manner (Lian et al., 2002). A proper choice of sampling period is crucial for achievable bandwidth and feasibility of the required phase margin. Given a discrete-time transfer function $G(z)$, the frequency response can be studied by plotting Nyquist or Bode plots of $G(z)\big|_{z=e^{j\omega T}}$. The discrete-time robust controller design for maximum overshoot and settling time is described in the next Section.

4.5 Decentralized discrete-time PID Controller design for the Quadruple tank process

In the frequency domain, the direct robust decentralized PID design procedure has been applied for the transfer function matrix (3) identified in three working points within the minimum and nonminimum phase regions (7) and (8), respectively. In both cases the nominal model is a mean value parameter model.

Minimum phase configuration

From three plant models (3) evaluated in working points taken from the minimum phase uncertainty region as specified in (7), the resulting continuous-time nominal model is

$$G_0(s) = \begin{bmatrix} \dfrac{2.4667}{62s+1} & \dfrac{1.2333}{(23s+1)(62s+1)} \\ \dfrac{1.5667}{(30s+1)(90s+1)} & \dfrac{3.1333}{90s+1} \end{bmatrix} \tag{73}$$

All three transfer function matrices were discretized using the sampling period $T_S = 30s$ chosen as approx. 1/10 of the settling time of plant step responses in Fig. 8.

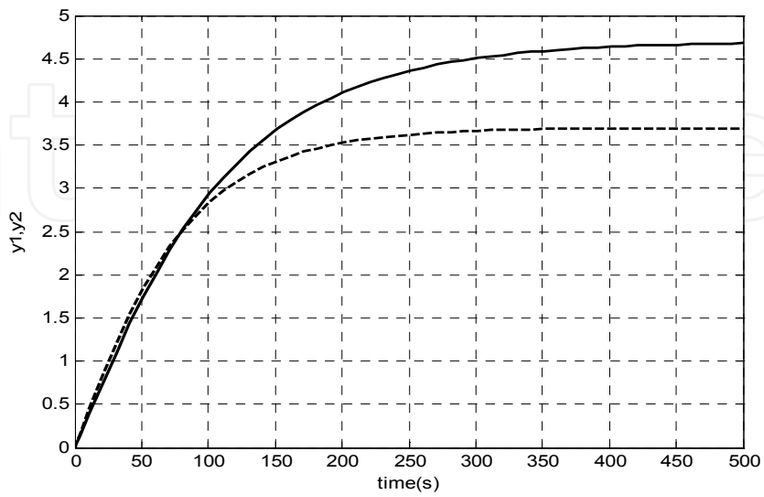


Fig. 8. Step response of the quadruple tank process

Discrete-time transfer function matrix of the nominal plant is

$$G(z)=\begin{bmatrix} \dfrac{0.9462z^{-1}}{1-0.6164z^{-1}} & \dfrac{0.2221z^{-1}+0.1226z^{-2}}{1-0.8877z^{-1}+0.1673z^{-2}} \\ \dfrac{0.1710z^{-1}+0.1097z^{-2}}{1-1.0840z^{-1}+0.2636z^{-2}} & \dfrac{0.8882z^{-1}}{1-0.7165z^{-1}} \end{bmatrix} \tag{74}$$

From the discretized transfer function matrices and the nominal model (74), upper bounds for $\sigma_{\max}[T_0(j\omega)]$ were evaluated according to (69) and (70).

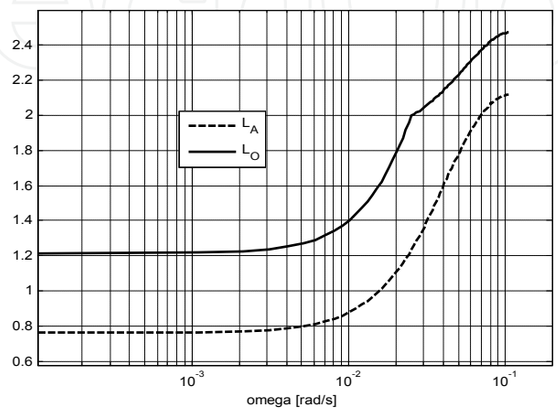


Fig. 9. Upper bounds for $\sigma_{\max}[T_0(j\omega)]$ evaluated according to (69) and (70)

Inspection of Fig. 9 reveals, that $M_{T\_A}=\min_{\omega} L_A=0.77<1$ is not feasible for the local controller design (closed-loop design magnitude less than 1 does not guarantee proper setpoint tracking, even at ω=0); hence $M_T<M_{T\_O}=\min_{\omega} L_O=1.22$ has been considered in the sequel.

Characteristic loci $g_1(z)$, $g_2(z)$ of $G_m(z)$ were calculated; $g_2(z)$ was selected to generate the equivalent subsystems according to (60). Bode plots of resulting equivalent subsystems are shown in Fig. 10.

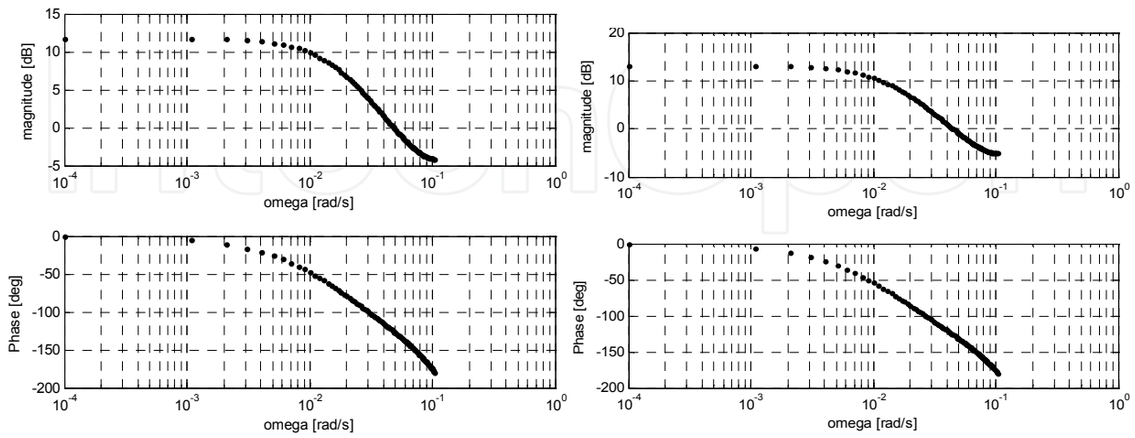


Fig. 10. Discrete Bode plots of equivalent subsystems generated by $g_2(z)$: $G_{12}^{eq}(z)$ (left), $G_{22}^{eq}(z)$ (right) (min. phase case)

Relevant parameters read form discrete Bode plots of uncompensated equivalent subsystems in Fig. 10 are summarized in Tab. 3.

Equivalent subsystem	PM	Crossover frequency
$G_{12}^{eq}(z)$	53.9^0	0.048 rad/ s^{-1}
$G_{22}^{eq}(z)$	58.35^0	0.0448 rad/s^{-1}

Table 3. Relevant parameters of equivalent subsystems generated by $g_2(z)$

For both equivalent subsystems the required settling time and maximum overshoot were chosen with respect to plant dynamics: $t_s = 600s$, $M_T = 1.05$ corresponding to $\eta_{\max} = 5\%$. Related values of other design parameters obtained from (63) and (67) respectively are: $PM_{\min} = 56.87^0$ and required crossover frequency $\omega_0 = 0.0131$. The required phase margin $PM_{req} > PM_{\min}$ was chosen $PM_{req} = 65^0$. To design local controllers, Bode design procedure (Kuo, 2003) has been applied independently for each equivalent subsystem to achieve the required phase margin: $PM(\omega_0)$ is found on the magnitude Bode plot; if $PM(\omega_0) > PM_{req}$, a PI controller $G_{PI}(s) = K_P + \frac{K_I}{s}$ is designed. If $PM(\omega_0) < PM_{req}$, a PD controller $G_{PD}(s) = 1 + K_D s$ is designed first, to provide $PM_{req}(\omega_0)$, and subsequently a PI controller is designed. The resulting PID controller is obtained in the series form $G_{PID}(s) = (K_P + \frac{K_I}{s})(1 + K_D s)$. Achieved design results are summarized in Tab. 4.

Eq. subsyst.	$R_i(s)$	$R_i(z)$	$PM_{achieved}$	$\omega_{0\_achieved}$
$G_{12}^{eq}(z)$	$R_1(s) = 0.1988 + \frac{0.0039}{s}$	$R_1(z) = \frac{0.199 - 0.082z^{-1}}{1 - z^{-1}}$	58.35^0	0.0122 rad/s^{-1}
$G_{22}^{eq}(z)$	$R_2(s) = 0.2212 + \frac{0.0034}{s}$	$R_2(z) = \frac{0.221 - 0.119z^{-1}}{1 - z^{-1}}$	65.70	0.0121 rad/s^{-1}

Table 4. Design results and achieved frequency domain performance measures (minimum phase configuration)

Design results in Tab. 4 along with Bode plots of compensated equivalent subsystems in Fig.11 prove achieving required design parameters. Closed-loop step responses are in Fig. 12.

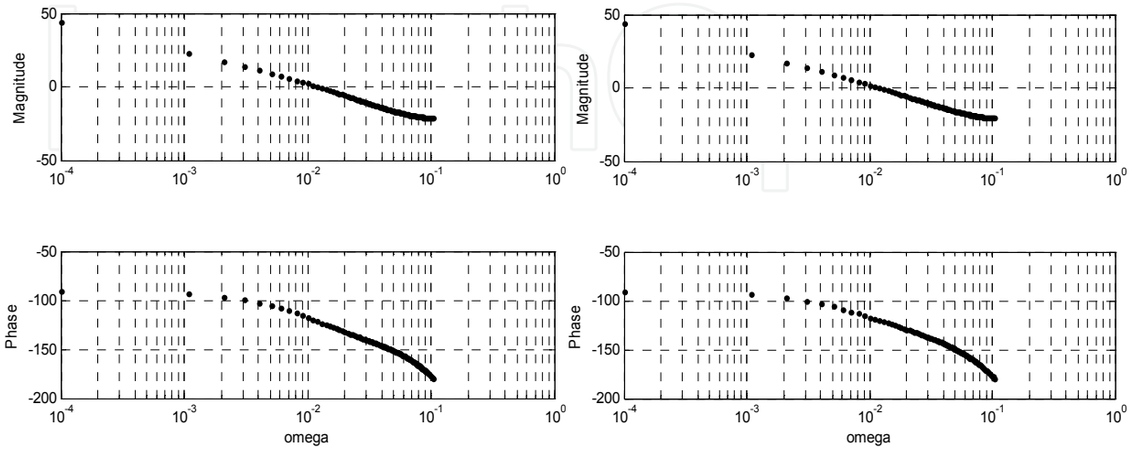


Fig. 11. Discrete Bode plots of equivalent subsystems under designed PI controllers: $G_{12}^{eq}(z)$ (left), $G_{22}^{eq}(z)$ (right)

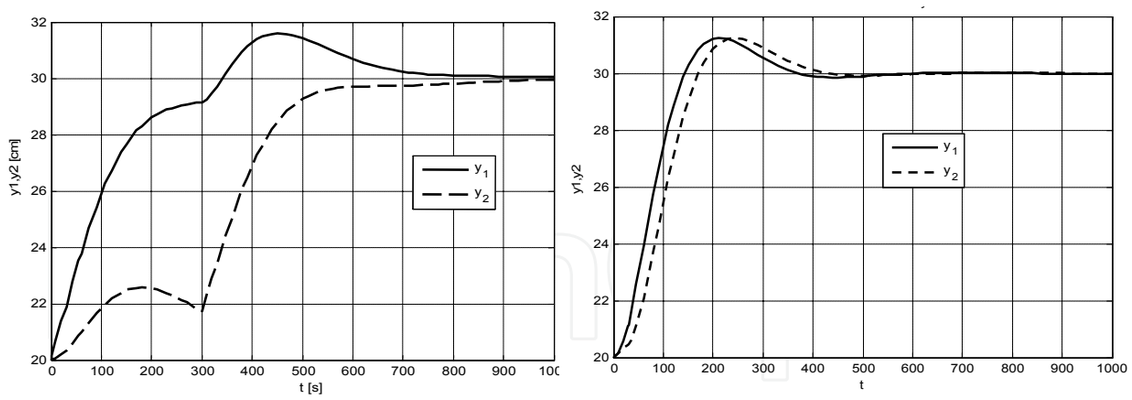


Fig. 12. Nominal closed-loop step responses of the quadruple tank process (reference steps 0.1m occurred at t=0s at the input of the 1st subsystem, and at t=300s and t=10s, respectively, at the input of the 2nd subsystem). Maximum overshoot and settling time (600s) were kept in both cases.

Nominal closed-loop stability was verified both by calculating closed-loop poles and using the Generalized Nyquist encirclement criterion (Fig. 13).

$$\text{Roots\_of\_CLCP} = \{\, 0.7019 \pm 0.2572i, 0.8313,\ 0.7167,\ 0.7165,\ 0.6164,\ 0.3720,\ 0.2637 \,\}$$

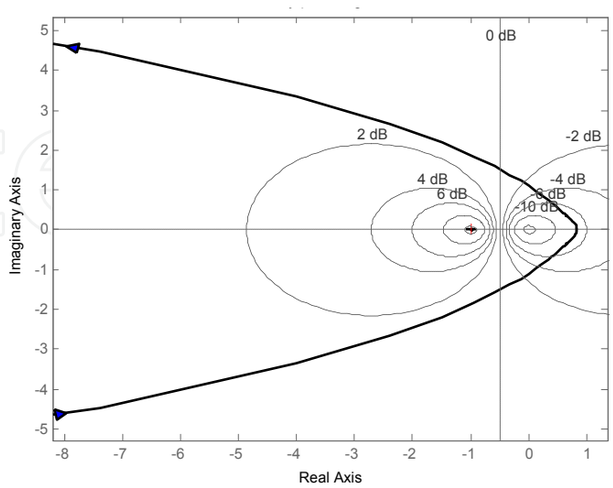


Fig. 13. Stability test using the Nyquist plot of $\det[I + G(z)R(z)]$

Achieved nominal performance was verified via plotting sensitivity magnitude plot in Fig. 14. Sensitivity peak $\max_{\omega}\{\sigma_M[S(j\omega)]\} < 2$ around the crossover frequency proves good closed-loop performance.

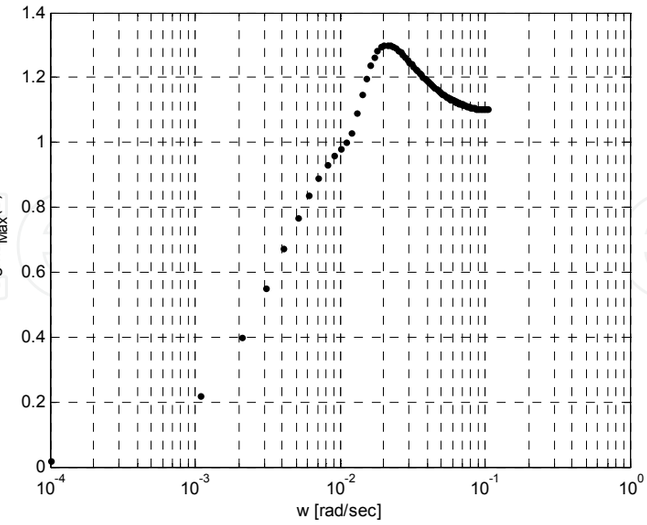


Fig. 14. $\sigma_M[S(z)]_{z=e^{j\omega T}}$ - versus –frequency plot

Fulfilment of robust stability condition (70) is examined in Fig. 15. The closed-loop system is stable over the whole minimum phase region (7).

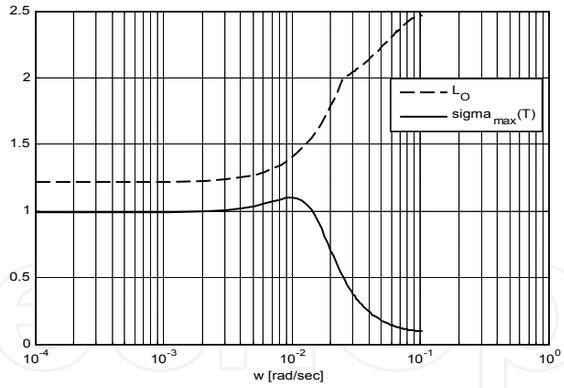


Fig. 15. Verification of the robust stability condition $\sigma_{\max}[T_0(j\omega)] < \dfrac{1}{|\ell_o(\omega)|} = L_O(\omega)$

Non-minimum phase configuration

To design robust decentralized PI controller for the non-minimum phase configuration, the continuous-time nominal model was evaluated for γ_1, γ_2 taken from the non-minimum phase uncertainty region (8) and interchanged columns of the transfer function matrix (due to opposite pairing as suggested in Section 2):

$$G_0(s) = \begin{bmatrix} \dfrac{3.0830}{(23s+1)(62s+1)} & \dfrac{0.6167}{62s+1} \\ \dfrac{0.7833}{90s+1} & \dfrac{3.9170}{(30s+1)(90s+1)} \end{bmatrix} \tag{73}$$

Discrete-time transfer function matrix of the nominal plant obtained for $T_S = 30s$ is

$$G(z) = \begin{bmatrix} \dfrac{0.5554z^{-1} + 0.3065z^{-2}}{1 - 0.8877z^{-1} + 0.1673z^{-2}} & \dfrac{0.2366z^{-1}}{1 - 0.6164z^{-1}} \\ \dfrac{0.2220z^{-1}}{1 - 0.7165z^{-1}} & \dfrac{0.4275z^{-1} + 0.2743z^{-2}}{1 - 1.0840z^{-1} + 0.2636z^{-2}} \end{bmatrix} \tag{74}$$

Upper bounds for $\sigma_{\max}[T_0(j\omega)]$ evaluated according to (69) and (70) are in Fig. 16.

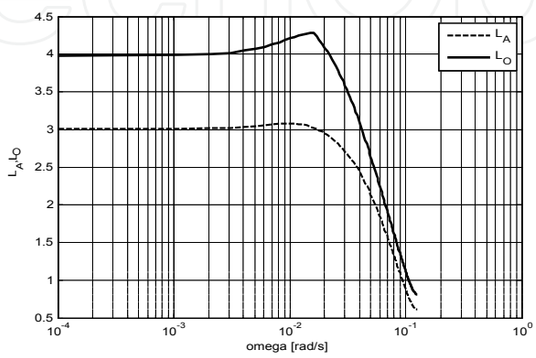


Fig. 16. Upper bounds for $\sigma_{\max}[T_0(j\omega)]$ evaluated according to (69) and (70)

Obviously, proper setpoint tracking can be guaranteed for both uncertainty types, just on a limited frequency range. Hence, $M_T = 1.05$ and multiplicative output uncertainty will be considered in the sequel.

Bode plots of equivalent subsystems generated using $g_2(z)$ are shown in Fig. 17, and their relevant parameters are summarized in Tab. 5.

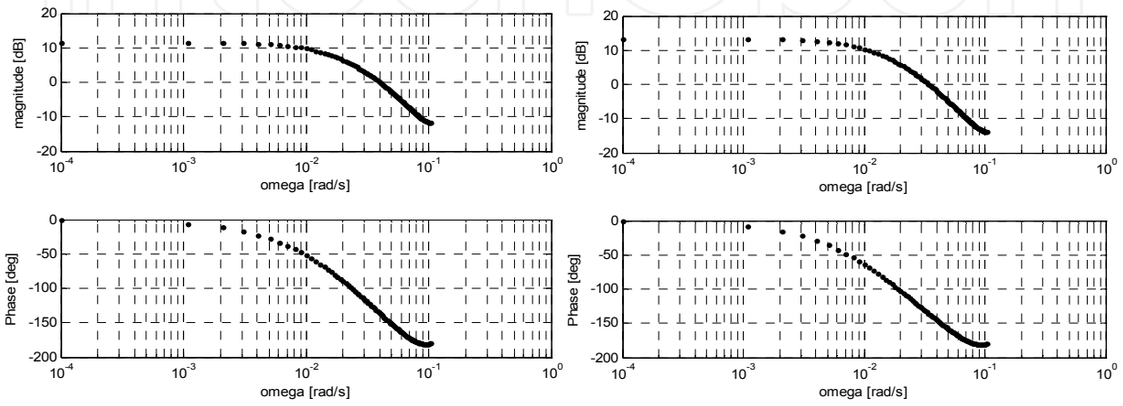


Fig. 17. Discrete Bode plots of equivalent subsystems generated by g2(z): $G_{12}^{eq}(z)$ (left), $G_{22}^{eq}(z)$ (right) (non-minimum phase case)

Equivalent subsystem	PM	Crossover frequency
$G_{12}^{eq}(z)$	43.81^0	0.040rad/s^{-1}
$G_{22}^{eq}(z)$	44.04^0	0.0344 rad/s^{-1}

Table 5. Relevant parameters of equivalent subsystems generated by g2(z).

For both equivalent subsystems the required settling time and maximum overshoot were chosen the same as in the minimum phase case: $t_s = 600s$, $M_T = 1.05$ corresponding to $\eta_{\max} = 5\%$. Related values of other design parameters are: $PM_{\min} = 56.87^0$ and required crossover frequency $\omega_0 = 0.0131$. The required phase margin $PM_{req} > PM_{\min}$ was chosen $PM_{req} = 60^0$. Achieved design results are summarized in Tab. 6 and Bode plots of compensated equivalent subsystems in Fig.18 prove achieving required design parameters.

Eq. subsyst.	$R_i(s)$	$R_i(z)$	$PM_{achieved}$	$\omega_{0\_achieved}$
$G_{12}^{eq}(z)$	$R_1(s) = 0.2083 + \frac{0.0039}{s}$	$R_1(z) = \frac{0.2083 - 0.0923z^{-1}}{1 - z^{-1}}$	54.17^0	0.0122 rad/s^{-1}
$G_{22}^{eq}(z)$	$R_2(s) = 0.2376 + \frac{0.0030}{s}$	$R_2(z) = \frac{0.2376 - 0.1832z^{-1}}{1 - z^{-1}}$	56.89	0.0120 rad/s^{-1}

Table 6. Design results and achieved frequency domain performance measures for the non-minimum phase case

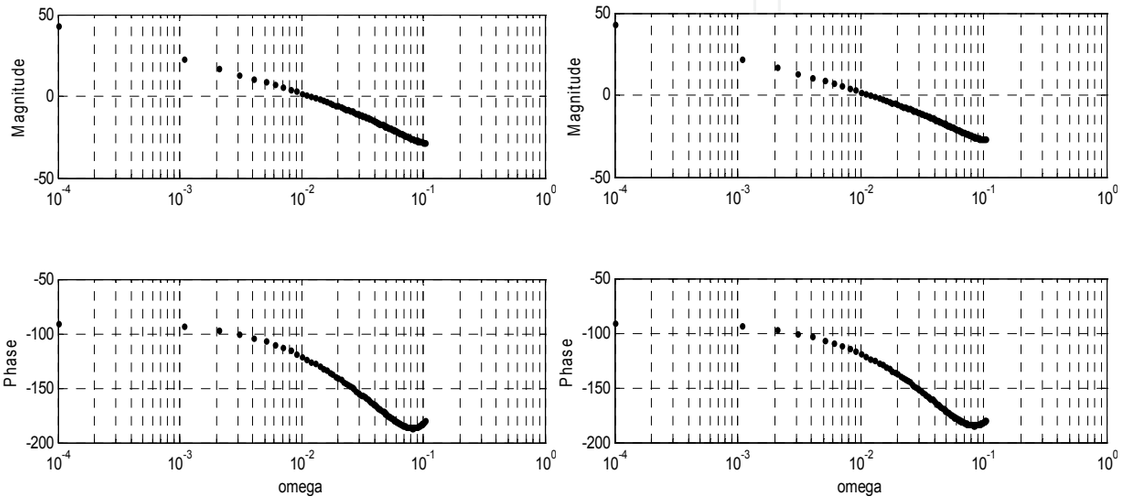


Fig. 18. Discrete Bode plots of equivalent subsystems under designed PI controllers: $G_{12}^{eq}(z)$ (left), $G_{22}^{eq}(z)$ (right)

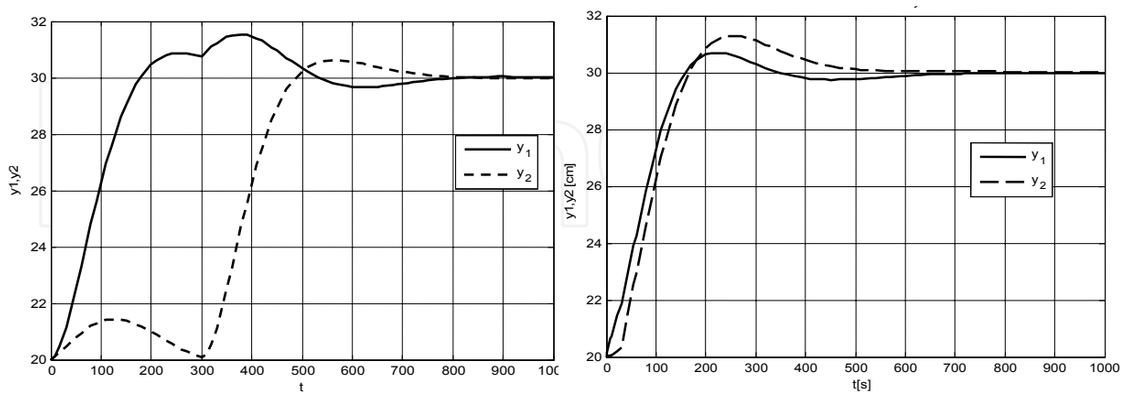


Fig. 19. Nominal closed-loop step responses of the quadruple tank system in non-minimum phase configuration (reference steps 0.1m occurred at t=0s at the input of the 1st subsystem, and at t=300s and t=10s, respectively, at the input of the 2nd subsystem). Maximum overshoot and settling time (600s) were kept in both cases.

Nominal closed-loop poles verify nominal stability.

$$\text{Roots\_of\_CLCP} = \{\, 0.6768 \;\pm\; 0.2761\text{i},\; 0.7335 \;\pm 0.2262\text{i},\; 0.7165,\; 0.6164,\; 0.5876,\; 0.3313 \;\}$$

The sensitivity magnitude plot in Fig. 20 with the peak $\max_{\omega}\{\sigma_M[S(j\omega)]\} < 2$ around the crossover frequency proves good closed-loop nominal performance.

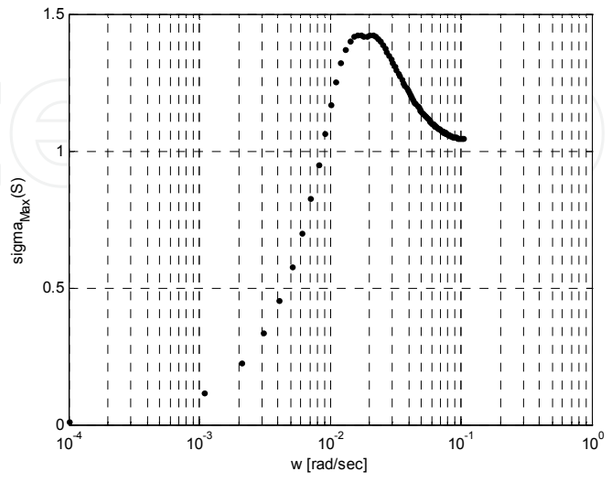


Fig. 20. $\sigma_M[S(z)]_{z=e^{j\omega T}}$ - versus –frequency plot

Fulfilment of robust stability condition (70) is examined in Fig. 21. The closed-loop system is stable over the whole non-minimum phase region (8).

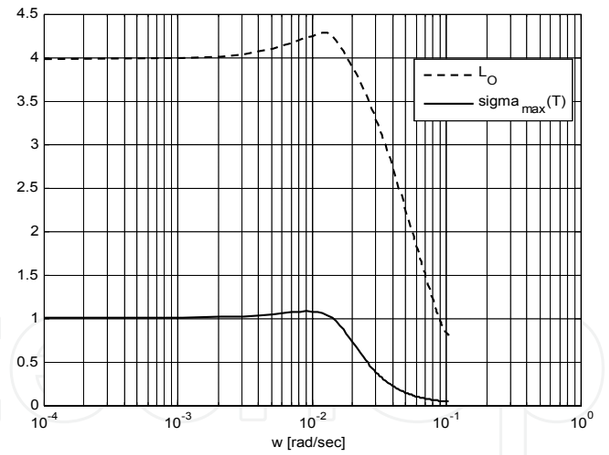


Fig. 21. Verification of the robust stability condition $\sigma_{\max}[T_0(j\omega)] < \dfrac{1}{|\ell_o(\omega)|} = L_O(\omega)$

5. Conclusion

The robust decentralized PID controller design procedures have been developed both in frequency and time domains. The proposed controller design schemes are based on different principles, with the same control aim: to achieve robust stability and specified performance. The comparative study of both approaches is presented on robust decentralized discrete-

time PID controller design for quadruple-tank process model, for minimum and nonminimum phase configurations. Both proposed approaches provide promising results verified by simulation on nonlinear process model.

6. Acknowledgment

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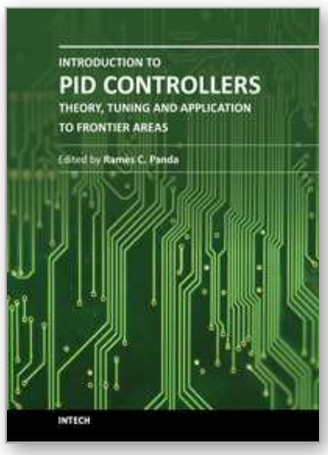

Introduction to PID Controllers - Theory, Tuning and Application to Frontier Areas

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This book discusses the theory, application, and practice of PID control technology. It is designed for engineers, researchers, students of process control, and industry professionals. It will also be of interest for those seeking an overview of the subject of green automation who need to procure single loop and multi-loop PID controllers and who aim for an exceptional, stable, and robust closed-loop performance through process automation. Process modeling, controller design, and analyses using conventional and heuristic schemes are explained through different applications here. The readers should have primary knowledge of transfer functions, poles, zeros, regulation concepts, and background. The following sections are covered: The Theory of PID Controllers and their Design Methods, Tuning Criteria, Multivariable Systems: Automatic Tuning and Adaptation, Intelligent PID Control, Discrete, Intelligent PID Controller, Fractional Order PID Controllers, Extended Applications of PID, and Practical Applications. A wide variety of researchers and engineers seeking methods of designing and analyzing controllers will create a heavy demand for this book: interdisciplinary researchers, real time process developers, control engineers, instrument technicians, and many more entities that are recognizing the value of shifting to PID controller procurement.

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