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2

The Physical Nature of Wave/Particle Duality

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1. Introduction

1.1 Waves and particles in quantum mechanics

In spite of the fact that the extraordinary progress of experimental techniques make us able to manipulate at will systems made of any small and well defined number of atoms, electrons and photons - making therefore possible the actual performance of the *gedankenexperimente* that Einstein and Bohr had imagined to support their opposite views on the physical properties of the wavelike/particlelike objects (*quantons*) of the quantum world - it does not seem that, after more than eighty years, a unanimous consensus has been reached in the physicist's community on how to understand their "strange" properties.
Unfortunately, we cannot know whether Feynman would still insist in maintaining his famous sentence "It is fair to say that nobody understands quantum mechanics". We can only discuss if, almost thirty years after his death, some progress towards this goal has been made. I believe that this is the case. I will show in fact that, by following the suggestions of Feynman himself, some clarification of the old puzzles can be achieved. This chapter therefore by no means is intended to provide an impartial review of the present status of the question but is focused on the exposure of the results of more than twenty years of research of my group in Rome, which in my opinion provide a possible way of connecting together at the same time the random nature of the events at the atomic level of reality and the completeness of their probabilistic representation by the principles of Quantum Mechanics.

1.2 The two slits experiment

In order to introduce the reader to the issues at stake I will briefly recall the essence of the debate between Bohr and Einstein which took place after the Fifth Solvay Conference (1927) where for the first time the different independent formulations of the new theory were presented by Heisenberg, Dirac, Born and Schrödinger, together with their common interpretation by Bohr - the socalled “Copenhagen interpretation” of Quantum Mechanics - which won since then a practically unanimous acceptance by the community.
This acceptance remained unquestioned for thirty years until when the books by Max Jammer (Jammer a1966, b1974) presented again to the new generation of physicists the ambiguities which still remained unsolved, and stimulated a renewed interest on those conceptual foundations of the theory which had been set aside under the impact of the the extraordinary experimental and theoretical boom of physics triggered at the end of World War 2 by the opening of the Nuclear Era.

The central issue of the debate, according to Jammer's reconstruction (Jammer b1974 p.127) , was "whether the existing quantum mechanical description of microphysical phenomena should and could be carried further to provide a more detailed account, as Einstein suggested, or whether it already exhausted all possibilities of accounting for observable phenomena, as Bohr maintained. To decide on this issue, Bohr and Einstein agreed on the necessity of reexamining more closely those thought-experiments by which Heisenberg vindicated the indeterminacy relations and by which Bohr illustrated the mutual exclusion of simultaneous space-time and causal descriptions."

The thought experiment which both agreed to discuss was the diffraction of a beam of particles of momentum p impinging perpendicularly on a screen D with two slits S_1 and S_2 at a distace d from each other. Each particle, which passes through, falls, deviating at random from its initial direction, on a photographic plate P located after the screen. When a sufficiently high number of particles has been detected, a distribution of diffraction fringes typical of a wave with a central maximum and adjacent minima and less pronounced maxima appears. Each particle is detected locally, but seems to propagate as a wave.

Its wavelike nature is expressed by the Bragg's relation connecting the wavelength λ of the wave in terms of the the distance d between the slits and the angle φ subtended by the central diffraction maximum ($\lambda = \varphi d$). On the other side its particlelike nature is expressed by its momentum p which is connected to the the wavelength by the de Broglie's relation ($p = h/\lambda$).

Since it is not possible to detect through which slit the particle is passed, its position x on D is uncertain by $\Delta x = d$. For the same reason, the momentum acquired by the particle in deviating from its initial direction normal to D is uncertain by $\Delta p = \varphi p$

From these relations the Heisenberg uncertainty relation

$$\Delta x = h/\Delta p$$

follows. Incidentally, the same phenomenon occurs with only one slit, with d now indicating the slit's width.

For Bohr eq. (1) holds for each individual particle. The particle's position x and its momentum p are, in his words, "complementary" variables. They cannot have simultaneously well defined sharp values. In the interaction with the classical instrument made of the screen D and the photographic plate P, each particle of the beam acquires a blunt value x affected by an uncertainty Δx and a blunt value p affected by an uncertainty Δp. The product of the uncertainties however, can never be less than the limit set by (1). Initially, before impinging on the instrument, each particle was in a state with a well defined sharp value of the momentum and a totally non localized position in space. At the end, after having been trapped in the photographic plate, each particle has acquired a well defined sharp value of its position in space, and has lost a well defined value of the momentum. The essence of the argument is that only by interacting with a suitable classical object one side of the quantum world acquires a real existence, at the expense of the complementary side becoming unseizable.

For Einstein instead Quantum Mechanics is only a statistical theory which does not fully describe reality *as it is*. The uncertainties, according to him, reflect only our uncomplete knowledge. He postulates the existence of "hidden variables" of still unknown nature, and concentrates his efforts on proving that Quantum Mechnics is "incomplete". In fact - he argues - if D is not fixed but is left free to move, one could identify the slit through which

the particle has passed by measuring the recoil of the screen produced by the momentum exchange with the particle deviated from its straight path. Both the position and the momentum of the particle could in this way be measured, violating the Heisenberg limit.
This does not work, however - replicates Bohr (Bohr 1948) - because the detection of "which slit" changes the diffraction pattern. In fact, he argues, if, by detecting the recoil of the screen one determines through which slit the particle has passed, the position in space of D becomes delocalized by a quantity ε in such a way that the resulting maxima and minima of the possible two-slit diffraction patterns superimpose and cancel each other. The original diffraction pattern with D fixed becomes the diffraction pattern of the single slit through which the particle is passed. Δx is reduced to the width of the slit and the uncertainty Δp is correspondingly increased. Heisenberg's relation for the particle still holds.
"It is not relevant - Bohr wrote many years later (Bohr 1958a) in a report of his debate with Einstein - that experiments involving an accurate control of the momentum or energy transfer from atomic particles to heavy bodies like diaphragms and shutters would be very difficult to perform, if practicable at all. It is only decisive that, in contrast to the proper measuring instruments, these bodies, together with the particles, would, in such a case constitute the system to which the quantum mechanical formalism has to be applied."
On the other hand, Bohr insists to stress the classical nature of the instrument (Bohr 1958b): "The entire formalism is to be considered as a tool for deriving predictions of definite statistical character, as regards information obtainable under experimental conditions described in classical terms.[..] The argument is simply that by the word "experiment" we refer to a situation where we can tell others what we have learned, and that, therefore, the account of the experimental arrangement and the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics."
It is therefore clear that for Bohr the *proper measuring instruments* on the one side must be treated as classical objects, but on the other one that the parts of the apparatus used for the determination of the localization in space time of particles and the energy-momentum transfer between particle and apparatus must be submitted to the quantum limitations. We will come back in a moment to this question in order to prove that this ambiguity can be understood in the framework of an interpretation of Quantum Mechnics in which both Einstein's purpose of saving the objectivity of the properties of macroscopic objects and Bohr's denial of the possibility of attributing to the objects at the atomic level independent properties, are recognized.

1.3 The EPR paradox

The second phase of the debate sees a change in Einstein's strategy of proving that the description of reality given by Quantum Mechanics is incomplete. This phase is based on the formulation of the EPR (Einstein, Podolski, Rosen) paradox (Einstein et al 1935).. I will briefly sketch its main argument, even if it is not essential for the further development of the argument of this Chapter.
This is how the authors formulate the basic assumption of their argument: "If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."
Consider a system of two particles in a state in which the relative distance $x_1 - x_2 = a$ and their total momentum $p_1+p_2 = p$ are fixed. This is possible because these quantities are not

complementary. Then EPR argue as follows. By measuring the position x_1 of the first particle it is possible, *without interfering directly with the second particle*, determine its position $x_2 = a+x_1$. This means that, according the initial definition, that x_2 is an *element of reality*. However, we might have chosen to measure, instead of x_1 the momentum p_1 of the first particle. This measurement would have allowed us to assess, without interfering in any way with the second particle, that its momentum $p_2 = p-p_1$ is an element of reality. This would have allowed to conclude that p_2 is an element of reality. Therefore, Einstein sums up, Quantum Mechanics is incomplete.

Bohr's answer stresses once more that one cannot speak of quantities existing independently of the actual procedure of measuring them: "From our point of view we now see that the wording of the above mentioned criterion of physical reality proposed by EPR contains an ambiguity as regards the meaning of the expression "without in any way disturbing a system". Of course there is, in a case like that just considered, no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system. Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete."

Einstein recognized that Bohr might be right, but remained attached to his own point of view (Bohr 1958b): ""To believe [that it should offer an exhaustive description of the individual phenomena] is logically possible without contradiction; - he admits - but it is so very contrary to my scientific instinct that I cannot forego the search for a more complete conception."

The question remained open for almost 50 years but was solved by two fundamental contributions. In 1964 John Bell (Bell 1964) showed that Einstein's hypothesis of the existence of hidden variables capable of describing reality in more detail than QM might lead to an experimental test. In order to sketch Bell's argument a reformulation of the original EPR proposal is necessary. Instead of chosing the relative distance and the total momentum as variables of the two-particle system with assigned initial value one assumes that they are two spin ½ particles in a state (singlet) of total angular momentum zero. In this state the components along three orthogonal directions are all zero, in spite of the fact that the three components of angular momentum are incompatible variable between themselves.

Bell's idea is the following. Rather than discussing the legitimacy of speaking of a physical variable without having measured it, he proposes of measuring the component of the spin of particle #1 in a direction **a** and the component ofthe spin of particle #2 in another direction **b**. After a series of measurements on a great number N of pairs the results are correlated by a function $C(\mathbf{a},\mathbf{b})=\sum a_i b_i$ (a_i and b_i may have values +/- 1) which depends on the angle θ between **a** et **b**. The point is, Bell shows, that Einstein's hypothesis of hidden variables leads to an inequality

$$|C(\mathbf{a},\mathbf{b}) - C(\mathbf{a},\mathbf{b}')| = (1/N)\,|\textstyle\sum_i a_i (b_i - b_i')| \le (1/N)\,|\sum(b_i - b_i')| =$$

$$= (1/N)\,|\textstyle\sum(1 - b_i b_i')| = 1 + C(\mathbf{b},\mathbf{b}') \qquad (1)$$

which is violated by the function $C(\mathbf{a},\mathbf{b}) = -\cos\theta$ of QM.

Bell's inequality shows that the difference between Einstein's and Bohr's views is not only a matter of interpretation, but that the formalism of QM contradicts the hypothesis that incompatible variables may have at the same time sharp, even if unknown, values. The debate between Bohr and Einstein has been settled in favour of Bohr by Alain Aspect and coworkers (Aspect 1982) who showed in a celebrated experiment that the inequality (3) is violated for (**ab**)= 22,5° et (**ab'**)= 67,5° by 5 standard déviations. Numerous other experiments have since then confirmed this result.

2. From quantons to objects

2.1 The existence of a classical world

We come back now to the ambiguous nature attributed by Bohr to the measuring apparatus. Does it belong to the classical or to the quantum world? In order to answer to this question we must preliminarly discuss the issue of the classical limit of Quantum Mechanics. We know that in the standard formulation of QM a system's state is represented by a wave function in the cohordinate's space (or a state vector in Hilbert space) which contains all the statistical properties of the system's variables. The wave function allows to calculate the probability of finding a given value of any variable of the system as a result of a measurement by means of a suitable instrument. More precisely, if the wave function is given by

$$\psi = c_1 \psi_1 + c_2 \psi_2 \tag{2}$$

(where ψ_1 (ψ_2) represents a state in which the variable G has with certainty the value g_1 (g_2)), the probability of finding g_1 (g_2) is $|c_1|^2$ ($|c_2|^2$). In Bohr's interpretation this means that the variable G does not have one of these values before its measurement but assumes one or the other value with the corresponding probability during the act of measurement. Now comes the question: is this interpretation always valid, even when g_1 and g_2 are macroscopically different?

The answer poses a serious problem. One can in effect prove that in the limit when Planck's constant *h* tends to zero the probability distribution of the quantum state represented by ψ tends to the probability distribution in phase space of the corresponding classical statistical ensemble labeled by the same values of the system's quantum variables. More precisely, in this limit $|c_1|^2$ and $|c_2|^2$ represent the probabilities of finding the values g_1 (g_2) of the classical variable corresponding to the quantum variable G. In this case, however, the interpretation of these probabilities is completely different. In classical statistical mechanics we assume that they express an incomplete knowlwdge of the values of G actually possessed by the different systems of the ensemble. We assume in fact that, if the ensemble is made of N systems, there are $N|c_1|^2$ systems wuth the value g_1 of G and $N|c_2|^2$ systems with the value g_2 of G to start with. Each system has a given value of G from the beginning, even if we don't know it.

We arrive therefore to a contradiction. The same mathematical expression represents on the one side (classical limit of QM) the probability that a given system of the ensemble acquires a given value of the variable G as a consequence of its interaction with a suitable measuring instrument, and on the other side (classical statistical mechanics) the probability that the system considered had that value of G before its measurement. Suppose, for exemple that y represents a quantum in a box with two communicating cpmpartments: Ψ_1 is different from zero in the left side compartment and Ψ_2 is different from zero in the right side one. The

corresponding probabilities of finding the quanton in one or the other one are respectively $|c_1|^2$ and $|c_2|^2$. Suppose now that the two compartments are separated by a shutter and displaced far away from each other. One of them is then opened: It may contain the quanton or may be empty. At this point it is undoubtably troubling to admit that, if one sticks strictly to Bohr's interpretation, the system in question instantly materializes in one or the other locality when the compartment is opened. Even more troubling is the fact that, if QM is the only true and universally valid theory of matter, the same conclusion must hold in principle also for macroscopic bodies.

2.2 Quantum and classical uncertainties

A way out of this dilemma, however, exists. We have shown, with Maurizio Serva (Cini M. Serva M. 1990, 1992), that, without changing the basic principlees and the predictions of Quantum Mechanics, one can save at the same time both Bohr's interpretation of the phenomena of the quantum domain, and Einstein's belief in the objective relity of the classical world in which we live. We have shown in fact that the uncertainty product between x and p can be written for any state of a quanton in the form

$$(\Delta x\ \Delta p)^2 = (\Delta x\ \Delta p)_{cl}^2 + (\Delta x\ \Delta p)_q^2 \tag{3}$$

where $(\Delta x\ \Delta p)_q$ is of the order of the minimum value $h/4\pi$ of the Heisenberg uncertainty relation and $(\Delta x\ \Delta p)_{cl}$ is the classical expression of the product of the indeterminations Δx and Δp predicted by the probability distribution of the classical statistical mechanics distribution corresponding to the quantum state when $h \to 0$. It is therefore reasonable to attribute to each of these two terms the meaning relevant to its physical domain.

In the typical quantum domain the clssical term vanishes and the indeterminacy is ontological, namely the variables x and p *do not have* a definite value before the system's interaction with a measuring instrument. When the accuracy of the act of measurement reduces the indeterminacy of one variable, the the indeterminacy of the other one increases. Their product cannot become smaller than $h/4\pi$.

As soon as the uncertainty product calculated from the state y acquires a classical term (which survives in the limit $h \to 0$) the total indeterminacy becomes epistemic, namely it represents an incomplete knowledge of the value that the measured variable really had before being measured. In this case it is possible to measure the variables x and p in such a way as to reduce at the same time both Δx et Δp without violating any quantum principle. These measurements reduce simply our ignorance. There is no instantaneous localization of the quanton in coordinate or momentum space as a consequence of the interaction between system and instrument, because position and momentum (within the intrinsic quantum uncertainty) were already localized.

This solution solves therefore the contradiction between the different interpretations of the total uncertainty product, and allows a reconciliation of the two alternative conceptions of physical reality proposed by Einstein and Bohr. It saves a realistic conception of the world as a whole by recognizing that macroscopic objects have objective properties independently of their being observed by any "observer", and, at the same time, that at the microscopic objects have properties dependent of the macroscopic objects with which they interact,

It allows also to clarify the ambiguity on the nature of the measurment apparatus mentioned above. One can in fact reformulate it in the following way. Assume that the microscopic

system S interacts with a part M_1 which at its turn interacts with a part M_2 and eventually other ones. We ask: at which point we pass the border between quantum domain and classical domain? The answer is not ambiguous. The border is where the values of the variable in one-to-one correspondence with the values of the quantum variable G, assume values which differ by each other by macroscopic quantities (e.g. charged or discharged counter). The part M_c when this happens is then the "pointer" of the instrument on whose unambiguous results all human observers agree.

This approch solves also a problem on which thousands of pages have been written, namely the problem of the "wave packet reduction" or "collapse" as a consequence of the act of measurement (Cini M, Levy Leblond J.M. 1991)(Wheeler J, Zurek W 1986). We recall that with this expression we mean that, after having measured G on a system S whose state is represented by ψ (eq.(1)) the wave function changes abruptly and instantaneously to ψ_1 or ψ_2 accordingly to the result g_1 or g_2 of the measurement. This change cannot be represented by a Schrödinger evolution, but must be postulated as a result of an instantaneous, irreversible and random evolution extraneous to QM. According to our findings (Cini M. et al 1979, Cini M. 1983) this additional and arbitrary mechanism is not necessary.

In fact, onsider the simplest case S+M, in which M is a counter which has two macroscopically different states (charged or discharged) represented by two state vectors Φ_1 and Φ_2. The wave funtion Ω of the total system may be written

$$\Omega = c_1 \psi_1 \Phi_1 + c_2 \psi_2 \Phi_2 \tag{4}$$

where we have assumed that the value g_1 (g_2) of the variable G of S is correlated with the charged (discharged) counter. The preceding discussion shows that, due to the macroscopic difference between Φ_1 and Φ_2, the total systen's state is, for all practical purposes, equivalent to a Gibbs classical ensemble made of $N|c_1|^2$ systems in which each counter is charged and S has the value g_1 of G and $N|c_2|^2$ systems in which each counter is discharged charged and S has the value g_2 of G. The wave packet reduction is therefore no longer needed as an additional postulate, and no additional misterious agent (even less the "observer's consciousness") is required to explain it. It simply turns out to be a well known consequence of classical statistical mechanics.

2.3 EPR and conservation laws

A similar "realistic" approach can be adopted to discuss the third counterintuitive quantum phenomenon, the famous EPR "paradox", whose solution, after the numerous experiments confirming the violation of Bell's inequalities, can only be expressed by saying that Einstein was wrong in concluding that quantum mechanics is an incomplete theory.

Usually people ask: how is it possible that when the first particle of a pair initially having zero total angular momentum acquires in interaction with its filter a sharp value of a given component of its angular momentum, the far away particle comes to "know" that its own angular momentum component should acquire the same and opposite value? I do not think that a realistic interpretation of this counterintuitive behaviour can be "explained" by minimizing the difference with its classical counterpart, because this difference has its roots, in my opinion, in the "ontological" (or irreducible) - not "epistemical" (or due to imperfect knowledge) - nature of the randomness of quantum events. If this is the case, one has in fact to accept that physical laws do not formulate detailed prescriptions, enforced by concrete physical entities, about what *must* happen in the world, but only provide constraints and

express prohibitions about what *may* happen. Random events just happen, provided they comply to these constraints and do not violate these prohibitions.
From this point of view, the angular momentum component of the far away particle *has* to be equal and opposite to the measured value of the first particle's component, because otherwise the law of conservation of angular momentum would be violated. In fact, the quantity "total angular momentum" *is* itself, by definition, a non-local quantity. Non locality therefore needs not to be enforced by a mysterious *action-at-a-distance*. The two filters are not two uncorrelated pieces of matter: they are two rigidly connected parts of one single piece of matter which "measures" this quantity. The non local constraint is therefore provided by the nature of the macroscopic "instrument". This entails that, once the quantum randomness has produced the first partial sharp result, there is no freedom left for the result of the final stage of the interaction: there is no source of angular momentum available to produce any other result except the equal and opposite sharp value needed to add up to zero for the total momentum.
We arrive to the conclusion that Bohr was right, but Einstein was not wrong in insisting that an uncritical acceptance of the current interpretation of QM would lead to absurd statements about the physical nature of the world we live in.

3. The randomness of quantum reality in phase space

3.1 The representation of the irreducible randomness of quantum world in phase space

After eighty years of Quantum Mechanics (QM) we have learned to live with wave functions without worrying about their physical nature. This attitude is certainly justified by the extraordinary success of the theory in predicting and explaining not only all the phenomena encountered in the domain of microphysics, but also some spectacular nonclassical macroscopic behaviours of matter. Nevertheless one cannot ignore that the *wave–particle duality* of quantum objects not only still raises conceptual problems among the members of the small community of physicists who are still interested in the foundations of our basic theory of matter, but also induces thousands and thousands of physics students all around the world to ask each year, at their first impact with Quantum Mechanics, embarassing questions to their teachers without receiving really convincing answers.
We have seen that typical examples of this insatisfaction are the nonseparable character of long distance correlated two-particle systems and the dubious meaning of the superposition of state vectors of measuring instruments, and in general of all macroscopic objects (Schrödinger 1935). In the former case experiments have definitely established that Einstein was wrong in claiming that QM has to be completed by introducing extra “hidden” variables, but have shed no light on the nature of the *entangled* two-particle state vector responsible for the peculiar quantum correlation between them, a correlation which exceeds the classical one expected from the constraints of conservation laws.
In the latter case, generations of theoretical physicists in neoplatonist mood have insisted in claiming that the realistic aspect of macroscopic objects is only an illusion valid For All Practical Purposes (in jargon FAPP). The common core of their views is the belief that the only entity existing behind any object, be it small or large, is its wave function, which rules the random occurrence of the object’s potential physical properties. The most extravagant and bold version of this approach is undoubtedly the one known as the Many Worlds Interpretation of QM Everett E.(1973), which goes a step further by eliminating the very

founding stone on which QM has been built, namely the essential randomness of quantum events. Chance disappears: the evolution of the whole Universe is written – a curious revival of Laplace - in the deterministic evolution of its wave function. "The Many-Worlds Interpretation (MWI) – in the words of Lev Vaidman, one of its most eminent supporters (Vaidman 2007) - is an approach to quantum mechanics according to which, in addition to the world we are aware of directly, there are many other similar worlds which exist in parallel at the same time and in the same space. The existence of the other worlds makes it possible to *remove randomness* and action at a distance from quantum theory and thus from all physics."

I believe that it is grossly misleading to attribute the epistemological status of "consistent physical theory" to this sort of science fiction, which postulates the existence of myriads and myriads of *physical objects* (indeed entire worlds!) which are *in principle undetectable*. My purpose is to show that these difficulties can only be faced by pursuing a line of research which goes in the opposite direction, namely which takes for granted the irreducible nature of randomness in the quantum world. This can be done by eliminating *from the beginning* the unphysical concept of wave function. I believe that this elimination is conceptually similar to the elimination of the aether, together with its paradoxical properties, from classical electrodynamics, accomplished by relativity theory. In our case the lesson sounds: No wave funtions, no problems about their physical nature.

Furthermore, the adoption of a statistical approach from the beginning for the description of the physical properties of quantum systems sounds methodologically better founded than the conventional *ad hoc* hybrid procedure of starting with the determination of a system's wave function of unspecified nature followed by a "hand made" construction of the probability distributions of its .physical variables. If randomness has an irreducible origin in the quantum world its fundamental laws should allow for the occurrence of *different* events under *equal* conditions. The language of probability, suitably adapted to take into account all the relevant constraints, seems therefore to be the only language capable of expressing this fundamental role of chance.

The proper framework in which a solution of the conceptual problems discussed above should be looked for is, after all, the birthplace of the quantum of action, namely phase space. It is of course clear that standard positive joint probabilities for both position and momentum having sharp given values cannot exist in phase space, because they would contradict the uncertainty principle. Wigner however, in order to represent Quantum Mechanics in phase space, introduced the functions called after his name (Wigner 1932) as pseudoprobabilities which may assume also negative values, and showed that by means of them one can compute any physically meaningful statistical property of quantum states.

A step further along this direction was made by Feynman (Feynman 1987), who has shown that, by dropping the assumption that the predictions of Quantum Mechanics can only be formulated by means of nonnegative probabilities, one can avoid the use of probability amplitudes, namely waves, in quantum mechanics. After all to the old questions about the physical meaning of probability amplitudes remains unanswered. Dirac said once "Nobody has ever seen quantum mechanical waves: only particles are detectable. Feynman is reported to have stated "It is safe to say that no one understands Quantum Mechanics". It is undeniable in fact that probability amplitudes are source of conceptual troubles (nonlocality of particle states, superposition of macroscopic objects' states).

The difficulty of introducing directly standard positive probability amplitudes in phase space in quantum mechanics arises, as is well known, from the impossibility of assigning

precise values to incompatible variables. No joint probability density of x and p exists in phase space. However, negative probabilities - argues Feynman - have a physical interpretation.

"The idea of negative numbers - he writes - is an exceedingly fruitful mathematical invention. Today a person who balks at making a calculation in this way is considered backward or ignorant, or to have some kind of mental block. It is the purpose of this paper to point out that we have a similar strong block against negative probabilities. By discussing a number of examples, I hope to show that they are entirely rational of course, and that their use simplifies calculations and thought in a number of calculations in physics."

"If a physical theory for calculating probabilities yields a negative probability for a given situation under certain assumed conditions, we need not conclude the theory is incorrect. Two other possibilities of interpretation exist. One is that the conditions (for example, initial conditions) may not be capable of being realized in the physical world. The other possibility is that the situation for which the probability appears to be negative is one that can not be verified directly. A combination of these two, limitation of verifiability and freedom in initial conditions, may also be a solution to the apparent difficulty."

Admittedly, as he recognizes, a "strong mental block" against this extention of the probability concept is widespread. Once this has been overcome, however, the road is open for a new reformulation of Quantum Mechanics, in which the concept of probability "waves" *is eliminated from the beginning*. After all, particles and waves *do not stand on the same footing* as far as their practical detection is concerned. We have already remarked that the position of a particle assumes a sharp value as a consequence of a single interaction with a suitable detector, but we need a beam of particles to infer the sharp value of their common momentum. This means that *we never detect waves*: we only infer their existence by detecting a large number of particles.

A striking exemple of the usefulness of this approach is that the troubles of entangled states disappear. In fact the Wigner pseutoprobability of the singlet state of the EPR paradox is the product of the Wigner pseudoprobabilities of the two spin ½ particles. This means no more questions about the "superluminal transmission" of information between them.

3.2 Classical ensembles with "Uncertainty Principle"

Feynman's program, however, is still based on the conventional formalism of QM: state vectors in Hilbert space or wave functions in coordinates' space. In fact, Wigner's function W(q,p) (pseudoprobability density for sharp values q, p of incompatible variables **q** and **p**) is defined by the expression

$$W(q,p) = \int dy \exp(-ipy)\, \psi(q+(1/2)y)\, \psi^*(q-(1/2)y) \qquad (5)$$

which contains explicitly the wave function of the state..In Feynman'sapproach waves are therefore still needed to start with, because pseudoprobabilities are first *expressed* in terms of wave functions, and then forgotten. We will show, however, that it is possible to express Quantum Mechanics from first principles in terms of pseudoprobabilities without ever introducing the concept of probability amplitudes. This program has been recently carried on [Cini 1999] by generalizing the formalism of classical statistical mechanics in phase space with the introduction of two postulates (*uncertainty* and *discreteness*), which impose mathematical constraints on the set of quantum variables in terms of which any physical quantity can be expressed. QM is therefore reformulated in terms of expectation values of

quantum variables as a generalization of the correspondent classical varibles of classical statistical mechanics, with the introduction of a single quantum postulate.
This goal will be attained in two steps. The first step is the formulation of a classical Uncertainty Principle. We consider all the classical *ensembles* of particles in phase space with coordinate **q** and momentum **p** in which a given variable **A**(**q**,**p**) has a well determined value α and its conjugate variable **B**(**q**,**p**) is completely undetermined[1]. *Only ensembles of this kind in fact are the classical limit of the quantum states.*
Following Moyal (1946), we will represent all the statistical properties of our ensembles, usually expressed by the joint probability distribution $P_\alpha(q,p)$, in terms of the expectation value $C_\alpha(k,x)$ (represented from now onwards by $<....>\alpha$) of the "characteristic variable" $\mathbf{C}(k,x) = e^{(-i/\hbar)(k\mathbf{q}+x\mathbf{p})}$ as follows

$$P_\alpha(q,p) = <\delta(q-\mathbf{q})\ \delta(p-\mathbf{p})>\alpha\ =$$

$$=(2\pi\hbar)^{-2}\iint dx\ dk\ \exp((-i/\hbar)(kq+xp))\ \ C_\alpha(k,x) \tag{6}$$

The requirement that all its systems have the value α of the variable **A**

$$<A^2>_\alpha\ \ = \alpha^2 \tag{7}$$

entails that $C_\alpha(k,x)$ must satisfy the equation

$$\iint dy\ dh\ a(h-k, y-x)\ C_\alpha(h, y)\ \ =\ \alpha\ C_\alpha(k,x) \tag{8}$$

where a(k,x) is the double Fourier transform of the function A(q,p).
Actually, eq. (8) is only apparently an integral equation, because it is easily reduced in terns of thr variables **A** and **B** to a simple algebraic functional equation with solution

$$P_\alpha(q,p) = \delta(\mathbf{A}(q, p) - \alpha) \tag{9}$$

In fact $P_\alpha(q,p)$ must be independent of **B** if this variable is indetermined in the ensemble. All this may seem trivial but actually it is not. Eq. (8) will be in fact one of our starting equations for the transition to QM.
We impose now that the result (9) should be invariant under the canonical transformations generated by any arbitrary function **L**

$$\mathbf{A'} = \mathbf{A} + \varepsilon\{\mathbf{A},\mathbf{L}\}_{PB} \tag{10}$$

Therefore the Poisson Bracket of **A** wiyh **L** must satisfy

$$<\{\mathbf{A},\mathbf{L}\}_{PB}>\alpha = 0 \tag{11}$$

from which it follows that the characteristic function must satisfiy, in addition to (9), also the equation

$$\iint dy\ dh\ a(h-k, y-x)\ (ky-hx)\ C_\alpha(h,y) = 0 \tag{12}$$

for all k,x.
Eqs. (8) (12) are the formal expression of a "classical uncertainty principle", representing the conditions to be fulfilled by classical ensembles having the property, invariant under

[1] In what follows the variables are written in boldface and their values are in ordinary typeset.

canonical trasformations, that a given variable **A** has the value α and its conjugate variable **B** is undetermined. Up to now we are still in the domain of classical statistical mechanics.

3.3 The quantum postulate

The second, essential, step is to introduce the quantum into this scheme. This is done by imposing the fulfilment of a second postulate, based on the assumption that the founding stone of quantum theory is the experimental fact that *physical quantities exist (the action of periodic motions, the angular momentum, the energy of bound systems..) whose possible values form a discrete set, invariant under canonical transformations, characteristic of each variable in question*. This means that we should request that α belongs to a discrete spectrum independent of the phase space variables.

This feature can only be ensured if eq. (8) for the classical characteristic function $C_\alpha(k,x)$, which yields a continuous spectrum α for the values of the classical variable **A**, is modified to become a true Fredholm homogeneous integral equation for the quantum characteristic function $C_i(k,y)$ with a nonseparable kernel g(ky-hx), allowing for the existence of a discrete set of eigenvalues α_i.

$$\iint dx\, dh\, a(h-k, y-x)\, g(ky-hx)\, C_i(h, y) = \alpha_i\, Ci(k,y) \tag{13}$$

Similarly, eq.(12) expressing the uncertainty principle between the classical variables **A** and B should be changed into

$$\iint dy\, dh\, a(h-k, y-x)\, f(ky-hx)\, C_i(h,y) = 0 \tag{14}$$

for the quantum characteristic function $\boldsymbol{C}_i(k,x)$ of the ensemble caracterized by one of the values α_i of the quantum variable ***A*** and by the complete indeterminacy of its quantum conjugate variable ***B***. The functions g( ) and f( ) should be determined by imposing new self consistent rules for the quantum variables involved.

The two eqs (13) (14), however, cannot be obtained from (7) and (11) as in the classical case by ordinary commuting numbers. In fact the only way to obtain (13) (14) is to replace the classical characteristic variables *C*(k,x) obeying the standard rule of multiplication of exponentials with quantum variables ***C***(k,x) having the property

$$(1/2)[\boldsymbol{C}(k,x)\, \boldsymbol{C}(h,y) + \boldsymbol{C}(h,y)\boldsymbol{C}(k,x)] =$$

$$g(ky-hx)\boldsymbol{C}(k+h,x+y) \tag{15}$$

and to replace their classical Poisson bracket with the Quantum Poisson Bracket

$$\{\boldsymbol{C}(k,x), \boldsymbol{C}(h,y)\}_{QPB} = f(ky-hx)\, \boldsymbol{C}[(k+h), (y+x)] \tag{16}$$

This means that, if we want to allow for the existence of discrete values of at least one variable **L** we are forced to represent all the variables **A** by means of noncommuting Dirac q-numbers.This means that the mathematical nature of the entities needed to represent the quantum variables is a consequence of the physical assumption of the discreteness of quantum variables and not viceversa, as the conventional view of reality underlying the conventional axiomatic formulation of Quantum Mecchanics assumes.

With (15) (16) the functions f( ) and g( ) turn out to have the expressions

$$g(ky-hx) = \cos[(ky-hx)/2\hbar] \quad ; \quad f(ky-hx) = (2/\hbar)\sin[(ky-hx)/2\hbar] \tag{17}$$

As expected, the quantum variables $\boldsymbol{C}(k,x)$ with the properties (15) (16) turn out to have the same exponential form of classical statistical mechanics where the classical variables **q** and **p** are replaced by quantum variables $\boldsymbol{q}$ and $\boldsymbol{p}$ satisfying the commutation relations

$$[\boldsymbol{q},\boldsymbol{p}]=i\hbar \tag{18}$$

of the standard variables of Quantum Mechanics
From the solution of equations (13) (14) one immediately obtains (by simple Fourier transform) the pseudoprobability $W_i(q,p)$ corresponding to the quantum caracteristic function $C_i(q,p)$ of the ensemble. This pseudoprobability coincides with the Wigner function obtained from the standard wave QM wave function of the state. It is important to mention that all pseudoprobabilities satisfy the condition

$$\iint dq\, dp\, W_i(q, p)\, Wi(q, p) = (2\pi\hbar)\text{-}1 \tag{19}$$

which expresses the uncertainty principle in the reformulation of quantum theory in phase space. It is remarkable that this principle is given by an equality , thus eliminating the ambiguity of the Heisenberg inequality due to the presence of the two physically different terms appearing in eq. (1)

3.4 Field quantization in phase space and wave/particle duality

These results however leaved some conceptual problems still open. First of all, once the Schrödinger waves have been eliminated from Quantum Mechanics, how does one generalize its principles to Quantum Field Theory? One should not forget that, historically, QED was invented by Dirac (Dirac 1927) by submitting "first quantized" Schrödinger amplitudes to the procedure of "second quantization". If no "first quantized" probability amplitudes exist any more how does one proceed? And, secondly, isn't one throwing away the baby with the dirty water by forgetting that after all a quantum field must still show some of the wavelike properties of its classical limit?
A second paper [Cini 2003] has been therefore devoted to answer to these questions, leading to the conclusion that: (a) one should not start from nonrelativistic quantum mechanics in order to formulate quantum field theory, but viceversa; (b) the wavelike behaviour of the quanta of a quantum field is, as already Pascual Jordan had understood in 1926 [Born, Heisernberg, Jordan 1926], a straightforward consequence of imposing the Einstein property of discreteness to the intensity of a classical field - clearly a nonlocal physical entity - which exists objectively in ordinary three dimensional space.
It is appropriate to recall that for Jordan, in fact, it is quantization which brings into existence particles, both photons and electrons. According to him, therefore, rather than trying to explain phenomena like diffraction and interference of single particles as properties of "probability waves" one should simply view them as primary properties of the field of which they represent the *quanta*. "These considerations show - we read in his paper "On waves and corpuscles in quantum mechanics" [Jordan 1927] - that the quantized field is equivalent, in all its physical properties and especially with respect to its inensity fluctuations, to a corpuscular system (with a symmetric eigenfunction)".
The derivation of Wigner functions from the principles of uncertainty and discreteness illustrated in the previous paragraph provides the formalism for deducing the kind of wave/particle duality suggested by Jordan (and forgotten by the physicist's community

since then) by simply imposing Einstein's quantization to the states of a classical field represented by means of statistical ensembles in the phase spaces of its normal modes.
Following the procedure sketched in the previous paragraph, we introduce a classical statistical ensemble for the r-th radiation oscillator of the field's normal modes defined by the constraint that the intensity $\mathbf{N}_r(q,p)$ has with certainty a given value ν_r. The equations (9) (11) remain valid, provided the variable **A** with its value α is replaced by the intensity **N** with its value ν and the conjugated variable **B** is replaced by the corresponding phase θ of each normal mode (we omit from now onwards the index r). Our procedure of field quantization will be based on the Einstein assumption of the existence of discrete field quanta. More precisely we *assume* that the spectrum of the quantum variable ***N*** of each field oscillator should be discrete. Eqs. (15) (16) remain unchanged and express now the result *that, the quantum variables should be represented by means of non commuting quantities (Dirac's q-numbers). Quantization is therefore now a consequence of the physical property of the existence of field quanta, and not viceversa.*
The field's states with a given number of quanta can now be represented by going from the quantum variables ***q***, ***p*** to the Dirac complex variables ***a***, ***a***** expressed in terms of each wave's intensity ***N*** and phase θ by means of their standard expressions

$$\boldsymbol{a}^* = \boldsymbol{N}^{1/2} \exp(-i\theta/\hbar) \qquad \boldsymbol{a} = \exp(i\theta/\hbar)\, \boldsymbol{N}^{1/2} \tag{20}$$

The eigenvalue equations (13) (14) can be rewritten for the characteristic functions $C_n(\beta, \beta^*)$ expressed in terms of the new variables β ,β* related to k,x and h,y by means of the same relations (20). These equations can be solved to give the eigenvalues ν_n of the quantum variable ***N*** and their characteristic functions $C_n(\beta, \beta^*)$ yielding

$$\nu_n = n+(1/2) \tag{21}$$

This result is expected, but remarkable, because it has been obtained by solving our new integral equations without any reference to Schrödinger wavefunctions. It is also easy with this formalism to treat the field's coherent states, as well as the processes of emission and absorption of photons from a source to reproduce the results obtained by Dirac in his seminal paper on the foundations of quantum electrodynamics. It turns out of course that the absorption rate is proportional to n_r and the emission rate to n_r+1 (Einstein's laws)

3.5 Conclusions

The main result of the reversal of the order of quantization from non relativistic quantum mechanics to quantum field theory gives a clear physical foundation to the mathematical nature of all quantum variables. The basic formal rules of quantum mechanics follow in this way from the Einstein postulate of the existence of field's quanta. The main conceptual result of this approach is therefore the clarification of the basic notion of wave/particle duality, which follows from this postulate, and simply reflects the dual nature of the quantum field as a unique physical entity objectively existing in ordinary three dimensional space (or ordinary four dimensional relativistic space, when is the case). From Jordan's point of view, in fact, the wavelike behaviour of any field's state with any number of discrete quanta simply reflects the property of a physical nonlocal entity which exists objectively in ordinary three dimensional space.
This goal has been achieved by imposing two requirements to the characteristic function (Moyal 1949) of the classical ensembles of thr field's normal modes. The first one is that the

probability distribution of the ensembles should be invariant under canonical transformations. The second requirement is quantization.

These two requirements are a reformulation of the principles introduced in the preceding nonrelativistic formulation of quantum mechnics where it was shown that the Wigner functions of the states of the one dimensional motion of a single particle can be directly derived without ever introducing Schrödinger wave functions. They lead to the two equations (18) and (21) whose solutions yield directly the quantum characteristic functions of the states of each mode, which turn out to be the double Fourier transforms of their Wigner functions. In the derivation of these equations one discovers that the field variables cannot be represented by ordinary numbers but should be represented by means of noncommuting mathematical objects.

With the direct construction of the Wigner functions of the states of quantum fields, the deBroglie-Schrödinger waves are thus eliminated from the formulation of quantum field theory. This means that, once that their nature of mathematical auxiliary tools has been recognized, the endless discussions about their queer physical properties, such as the nature of long distance EPR correlations between two or more particles or the meaning of the superposition of macroscopic states, become meanigless as those about the queer properties of the aether after its elimination declared by the theory of relativity.

Furthermore it supports the view that the most adequate representation of the random character of quantum phenomena ought to be based on Wigner-Feynman pseudoprobabilities in phase space, in which the constraints of the uncertainty principle are embodied, rather than insisting in representing them as events occurring in different spaces, (e.g. configuration *or* momentum) ruled by their correlated but separate classical probability laws. This view still meets a widespread resistance on the grounds that pseudoprobabilities are not positive definite, but is starting to acquire consensus in some domains of physics such as quantum optics (Leibfried et al 1988)leading even to a proposal for their experimental determination (Luttinger et al1997).

Finally, the direct deduction of Wigner functions from first principles solves a puzzling unanswered question which has been worrying all the beginners approaching the study of our fundamental theory of matter, all along its 75 years of life, namely "Why should one take the modulus squared of a wave amplitude in order to obtain the corresponding probability?" We can now say that there is no longer need of an answer, because there is no longer need to ask the question.

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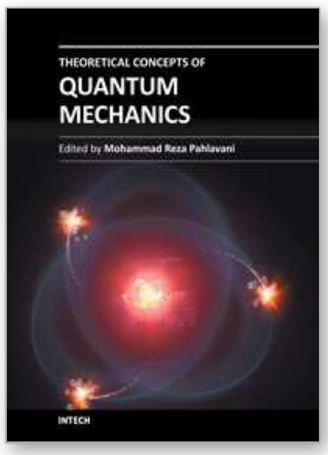
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Quantum theory as a scientific revolution profoundly influenced human thought about the universe and governed forces of nature. Perhaps the historical development of quantum mechanics mimics the history of human scientific struggles from their beginning. This book, which brought together an international community of invited authors, represents a rich account of foundation, scientific history of quantum mechanics, relativistic quantum mechanics and field theory, and different methods to solve the Schrodinger equation. We wish for this collected volume to become an important reference for students and researchers.

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