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Battle of the Sexes: A Quantum Games Theory Approach

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1. Introduction

Real life situations where there are strategies to be chosen in order to obtain a profit can be reproduced by games, so game theory is a way to describe the evolution of possible scenarios where players can select a scheme of play. Game theory takes importance in many areas, such as people decisions making, where their choices do affect others benefit (Davis, 1970; Myerson, 1991). It is to remark that the principles of game theory were initiated by trying to understand the behavior of economic strategies, however Von Neumann presented the concept of modern game theory in 1944 (Von Neumann & Morgenstern, 1947).

Quantum mechanics is a tool that creates another point of view for the traditional game theory due to multiple strategies offered for the players, whom possibilities are numerously expanded in contrast of classical ones (Eisert & Wilkens, 2000). There are also games where the player who uses quantum strategies, enhances his payoffs or even always wins against a player who only uses classical moves (Meyer, 1999). It is to remark that there are plenty of applications for quantum game theory such as quantum cryptography and computation, economics and biology (Piotrowski & Sladkowski, 2003; Hanauske et al.,2009).

The Battle of the Sexes game is a largely analyzed problem, based on two players: Alice and Bob and their choice about an activity for a Saturday night with each other. It is pretty important to remark that both want the best possible payoff in the decision, so the game can be developed normally, otherwise it would not be our case. Alice, really loves Opera, but wants to be with Bob; Bob likes Football but he wants to have Alice's company along the activity. This game has a lot of applications in real life scenarios such as the spread of some type of genes from a reproduction between two organisms (Dawkins, 2006); another interesting application is neuroeconomics (Montague & Berns, 2002), where brain studies have been done in order to incite neurons to choose either to "work" for a reward or to "shirk" (Glimcher, 2003).

2. Classical analysis

The global idea of the model used in this section is taken from Richard Dawkins' Battle of the Sexes Model, however the specific method implemented is taken from a worksheet made by Frank Wang (Wang, 2010), due to its facility to be developed in a computer algebra software such as Maple.

2.1 Method

M_A and M_B are payoff matrixes for Alice and Bob, respectively. Where M_{Aii} is the payoff matrix of a female player using strategy i against a male playing strategy j; and M_{Bij} is the payoff matrix for a male playing i against a female who plays j. x_i is the proportion of females playing i; and y_i is the proportion of males playing strategy i. As here we considerate two possible strategies whose proportions satisfy: $x_1 + x_2 = 1$; and the same applies for y_i. In order to reduce the number of variables, we describe x_2 and y_2 in terms of x_1 and y_1, respectively.

$$x_1(t) = x(t) \tag{1}$$

$$x_2(t) = 1 - x_1(t) \tag{2}$$

We substitute (1) in (2):

$$x_2(t) = 1 - x(t) \tag{3}$$

$$y_1(t) = y(t) \tag{4}$$

$$y_2(t) = 1 - y_1(t) \tag{5}$$

(4) in (5) to get a simpler equation for y:

$$y_2(t) = 1 - y(t) \tag{6}$$

The relevant fitness functions are defined as:

$$\begin{gathered} f(x,y) = x_1(t)\,(A_{11}\,y_1(t) + A_{12}\,y_2(t) \\ -\big(A_{11}\,x_1(t)\,y_1(t) + A_{12}\,x_1(t)\,y_2(t) + A_{21}\,x_2(t)\,y_1(t) + A_{22}\,x_2(t)\,y_2(t)\big)\big), \\ g(x,y) = y_1(t)\,(B_{11}\,x_1(t) + B_{12}\,x_2(t) \\ -\big(B_{11}\,x_1(t)\,y_1(t) + B_{12}\,y_1(t)\,x_2(t) + B_{21}\,y_2(t)\,x_1(t) + B_{22}\,y_2(t)\,x_2(t)\big)\big); \end{gathered} \tag{7}$$

The appropriate replicator equations in terms of the previously defined fitness functions are:

$$\begin{aligned} \frac{d\,x(t)}{dt} &= f(x,y), \\ \frac{d\,y(t)}{dt} &= g(x,y); \end{aligned} \tag{8}$$

In order to find the equilibrium values for x and y, we remove time-dependence by solving (9) when the derivatives are zero, hence $x(t) = x$ and $y(t) = y$. Then we solve the resulting system of two equations and two variables. In order to make the stability analysis, we search for a solution with the form:

$$\begin{aligned} x(t) &= x + \varepsilon(t), \\ y(t) &= y + \eta(t); \end{aligned} \tag{9}$$

where x and y are the equilibrium values previously found; $\varepsilon(t)$ and $\eta(t)$ are introduced as small perturbations respect to the equilibrium values. Now, we substitute (9) in equations (8). We can neglect the squares of η and ε and their product because their result is very small. So we can define the following simplifications in order to be replaced on the resulting replicator equations.

$$\begin{aligned} \eta(t)\,\varepsilon(t) &= 0, \\ \varepsilon(t)^2 &= 0, \\ \eta(t)^2 &= 0; \end{aligned} \tag{10}$$

The replicator equations are finally linear, so them can be analytically solved. From these solutions we can observe if the equilibrium values are stable.
An alternative method consists on the Jacobian's use and the computation of its eigenvalues, where the stability depends on the resulting sign of the real part of the eigenvalues. However, when the eigenvalues are pure imaginary numbers, the Jacobian method is not appropriate for the stability analysis. In this last case, we use the method of the vector field for the replicator equations; which is graphically implemented.

2.2 Standard case

The previously shown method will be illustrated using three possible examples of the traditional battle of the sexes game:

a. Dawkins example.
b. Wikipedia's example[1].
c. Wikipedia's example

2.2.1 Dawkins model by Frank Wang

This first example is taken from a Maple worksheet designed by Frank Wang, based on Richard Dawkins' Battle of the Sexes Model presented on Chapter 9 of the celebrated book titled "Selfish Gene". The classical payoff matrices for this scenario are:

$$\begin{aligned} A &= \begin{bmatrix} 2 & 0 \\ 5 & -5 \end{bmatrix}, \\ B &= \begin{bmatrix} 2 & 5 \\ 0 & 15 \end{bmatrix}; \end{aligned} \tag{11}$$

According with these payoff matrices, the corresponding fitness functions are:

$$\begin{aligned} f(x,y) &= x_1(t)\ (2\,y_1(t) - 2\,x_1(t)\,y_1(t) - 5\,x_2(t)\,y_1(t) + 5\,x_2(t)\,y_2(t)), \\ g(x,y) &= y_1(t)\ \left(2\,x_1(t) + 5\,x_2(t) - 2\,x_1(t)\,y_1(t) - 5\,x_2(t)\,y_1(t) - 15\,x_2(t)\,y_2(t)\right); \end{aligned} \tag{12}$$

which are reduced to:

$$\begin{aligned} f(x,y) &= x(t)\,(8\,y(t) - 5)\,(-1 + x(t)), \\ g(x,y) &= -2y(t)\ \left(-1 + y(t)\right)(6\,x(t) - 5). \end{aligned} \tag{13}$$

[1] Wikipedia – Battle of the Sexes (game theory)

With the previous found fitness, the replicator equations take the form:

$$\frac{d\,x(t)}{dt} = x(t)\,(8\,y(t) - 5)\,\big(-1 + x(t)\big),$$
$$\frac{d\,y(t)}{dt} = -2\,y(t)\,\big(-1 + y(t)\big)\,(6\,x(t) - 5). \tag{14}$$

In order to find the equilibrium values, we remove time dependence for x and y, making $x(t) = x$ and $y(t) = y$; and we obtain:

$$0 = -8\,y\,x + 5\,x + 8\,x^2 y - 5\,x^2$$
$$0 = 12\,y\,x - 12\,x\,y^2 - 10\,y + 10\,y^2 \tag{15}$$

Solving these last equations, we get:

$$\{x = 0, y = 0\}, \{x = 1, y = 0\}, \{x = 0, y = 1\},$$
$$\left\{x = \frac{5}{6}, y = \frac{5}{8}\right\}, \{x = 1, y = 1\}. \tag{16}$$

In order to make the stability analysis, we use the non-trivial equilibrium values and the following small perturbations around these values:

$$x(t) = \frac{5}{6} + \varepsilon(t)$$
$$y(t) = \frac{5}{8} + \eta(t) \tag{17}$$

The resulting replicator equations are:

$$\frac{d}{dt}\,\varepsilon(t) = \left(-\frac{10}{9}\right)\,\eta(t) + \left(\frac{16}{3}\right)\,\eta(t)\,\varepsilon(t) + 8\,\eta(t)\,\varepsilon(t)^2,$$
$$\frac{d}{dt}\,\eta(t) = \left(\frac{45}{16}\right)\,\varepsilon(t) - 3\,\eta(t)\,\varepsilon(t) - 12\,\varepsilon(t)\,\eta(t)^2; \tag{18}$$

and using (10), we obtain the following linear equations:

$$\frac{d}{dt}\,\varepsilon(t) = -\frac{10}{9}\,\eta(t),$$
$$\frac{d}{dt}\,\eta(t) = \frac{45}{16}\,\varepsilon(t); \tag{19}$$

The solution for the last system of equations is:

$$\left\{\varepsilon(t) = C_1 \sin\left(\frac{5}{4}\sqrt{2}\,t\right) + C_2 \cos\left(\frac{5}{4}\sqrt{2}\,t\right), \eta(t) = \frac{9}{8}\sqrt{2}\,\left(-C_1 \cos\left(\frac{5}{4}\sqrt{2}\,t\right) + C_2 \sin\left(\frac{5}{4}\sqrt{2}\,t\right)\right)\right\} \tag{20}$$

About the stability analysis, we observe that the small perturbations are oscillating functions without damping. It is to say that the perturbations never end, and therefore the equilibrium values are instable. It is worthwhile to note that in this case the Jacobian has eigenvalues that are pure imaginary numbers, so it is necessary to use the vector field method in order to see the results. The subsequent vector field plot is:

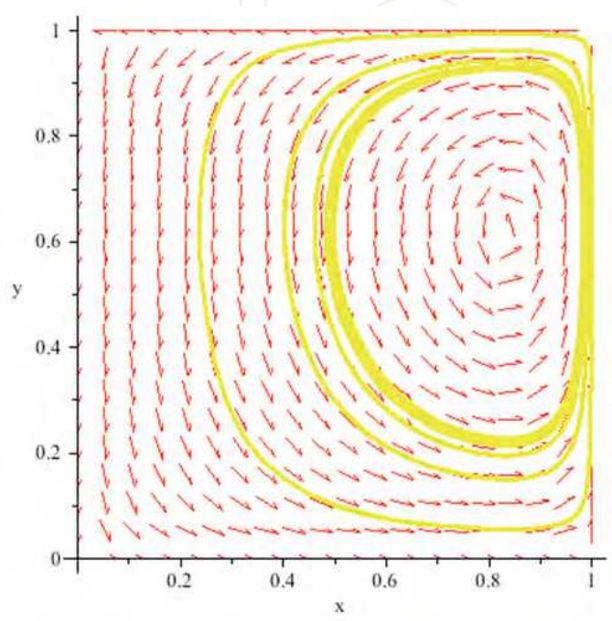

Fig. 1. Vector Field plot of the results.

We can observe that the trajectories in the phase plane are closed curves around the non-trivial equilibrium point, considered as a center. We can also make a numerical solution for the replicator equations, whose results are graphically shown:

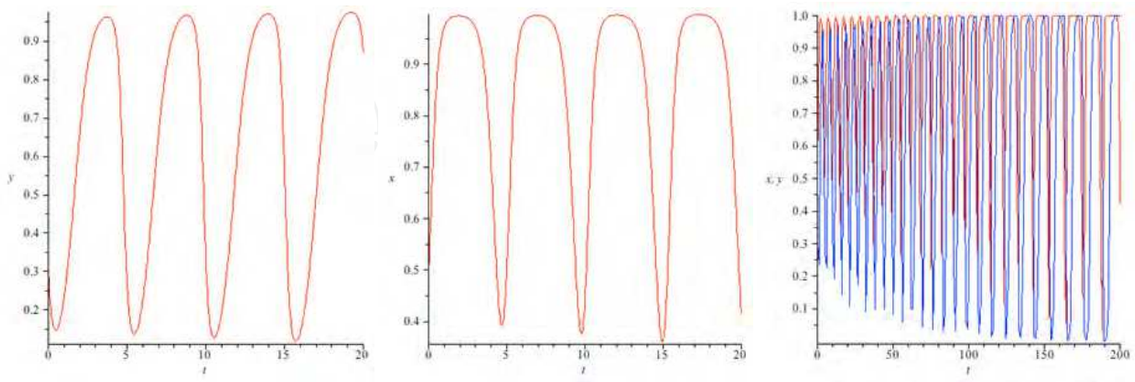

Fig. 2. a) Numerical solution for Alice's, b) Numerical solution for Bob's, c) Alice's and Bob's results plotted together with an extended time.

The three previous graphs are the time series for the player's proportions. The following graph is the phase plane for x and y:

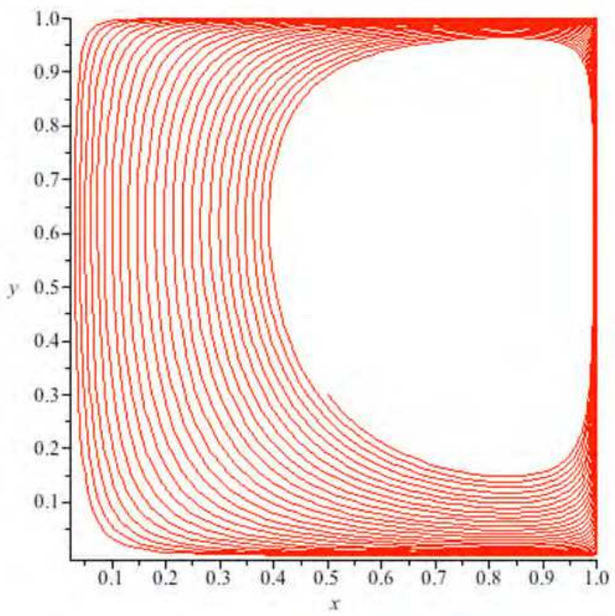

Fig. 3. Phase plane of the solutions.

2.2.2 Wikipedia case 1

The following example is taken from Wikipedia, where is referred as case 1. In this scenario, Alice and Bob receive pays only when both of them meet at the Opera, where Alice gets 3 and Bob 2; or at the football match, where Alice gets 2 and Bob 3. Also, both Alice and Bob receive no pay when they go to the wrong places either Opera or Football. It is to considerate the case where Alice receives a small payoff for going alone to the Opera and Bob for attending to the football match, but this will be analyzed in section 2.2.3. The previous scenario is shown on the following matrices, for Alice and Bob respectively:

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}; \tag{21}$$

Applying matrices at (21) in the method previously shown, the solutions obtained are:

$$\{x = 0, y = 0\}, \{x = 1, y = 0\}, \{x = 0, y = 1\}, \left\{x = \frac{3}{5}, y = \frac{2}{5}\right\}, \{x = 1, y = 1\}. \tag{22}$$

Following the steps presented in the method, the following linear equations are shown, which came from replicators equations:

$$\frac{d}{dt}\,\varepsilon(t) = \frac{6}{5}\,\eta(t), \qquad \frac{d}{dt}\,\eta(t) = \frac{6}{5}\,\varepsilon(t); \tag{23}$$

Finally we can find the solutions for the previous system (23):

$$\left\{\varepsilon(t) = C_1\, e^{-\frac{6}{5}t} + C_2\, e^{\frac{6}{5}t}, \eta(t) = -C_1\, e^{-\frac{6}{5}t} + C_2\, e^{\frac{6}{5}t}\right\} \tag{24}$$

From the previous solution we can deduce that both strategies meets at a specific point. A vector field plot is made:

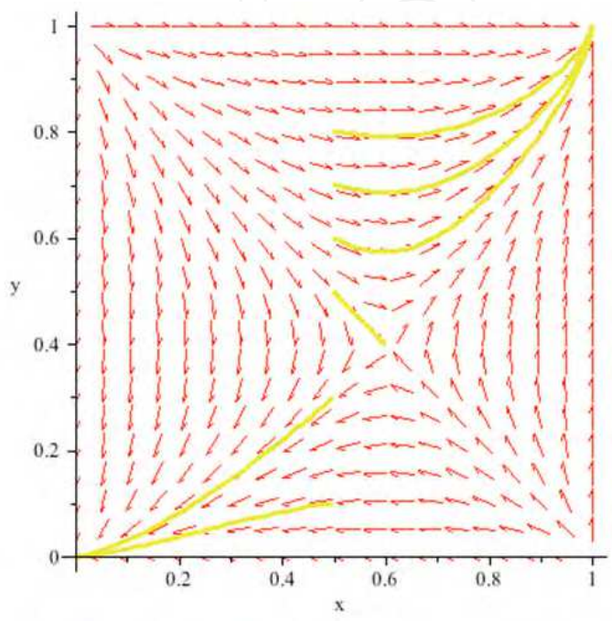

Fig. 4. Vector field plot.

Numerical solutions in the phase plane for both strategies are:

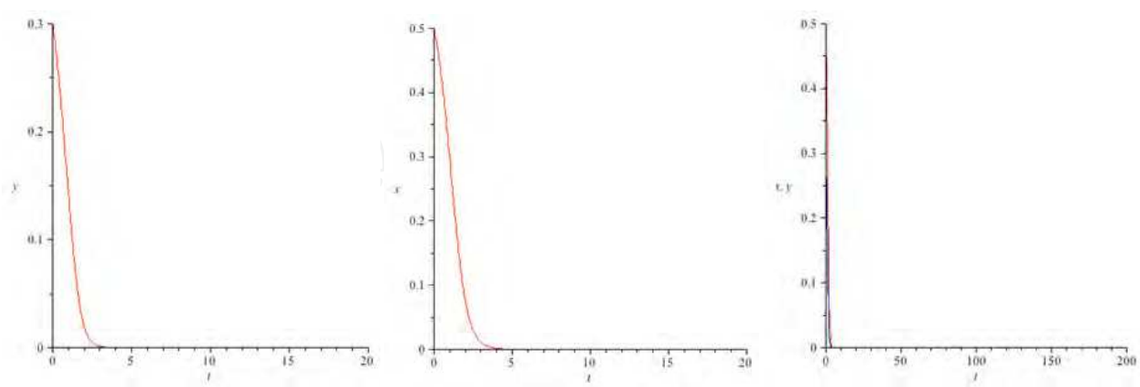

Fig. 5. a), b) Alice's and Bob's trajectories in the phase plane respectively, c) superposed graphs with a longer time period.

Figure 5 shows the evolution in time of the solutions obtained. The following graph is the phase plane for x and y:

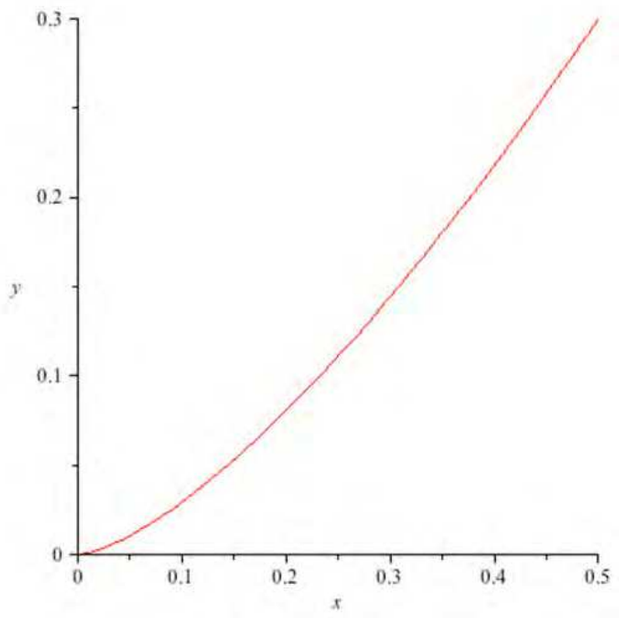

Fig. 6. Phase plane for x and y.

2.2.3 Wikipedia case 2

It's presented a possible modification for the previous case. Here, Alice receives a small pay for attending the Opera alone; likewise, Bob gets a positive payment for going to the football match without Alice. The amount received in both cases will be taken as 1; for the other possible situations, the payoffs keep being the same:

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}; \tag{25}$$

Using replicator's equation in the method presented, the equilibrium values obtained for the system are:

$$\{x = 0, y = 0\}, \{x = 1, y = 0\}, \{x = 0, y = 1\}, \left\{x = \frac{3}{4}, y = \frac{1}{4}\right\}, \{x = 1, y = 1\}. \tag{26}$$

Using equations (10) and the non-trivial equilibrium values, the following linear equations are found:

$$\frac{d}{dt}\,\varepsilon(t) = \frac{3}{4}\,\eta(t), \qquad \frac{d}{dt}\,\eta(t) = \frac{3}{4}\,\varepsilon(t); \tag{27}$$

which led to the solutions for $\varepsilon(t)$ and $\eta(t)$:

$$\left\{\varepsilon(t) = C_1 e^{\frac{3}{4}t} + C_2 e^{-\frac{3}{4}t}, \eta(t) = C_1 e^{\frac{3}{4}t} - C_2 e^{-\frac{3}{4}t}\right\} \tag{28}$$

In order to understand the stability analysis, a vector field plot of the solutions is made:

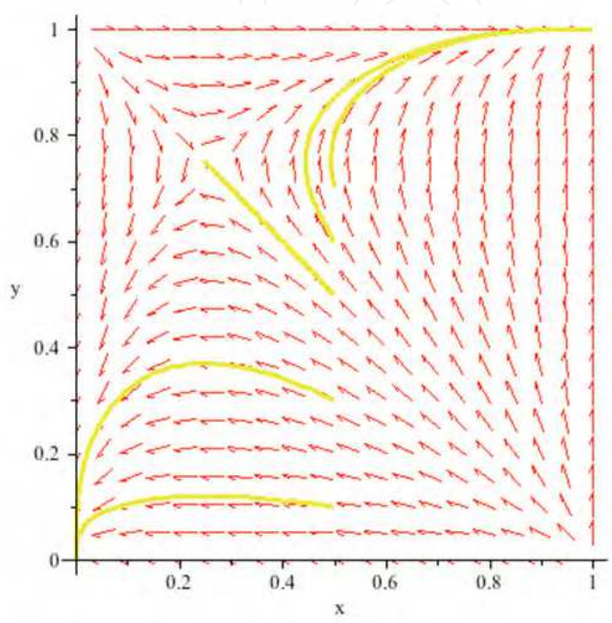

Fig. 7. Vector field plot.

Player's proportions on time's evolution are plotted in the next graphs:

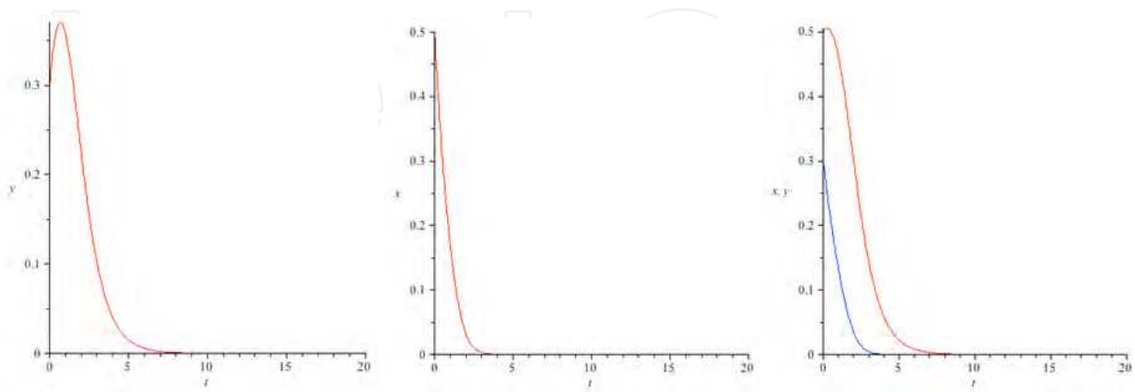

Fig. 8. a) Alice's proportion on time, b) Bob's strategy proportion, c) Alice's and Bob's time proportions.

From the previous three graphs is possible to observe the convergence. It is also presented a phase plane graph of the solutions:

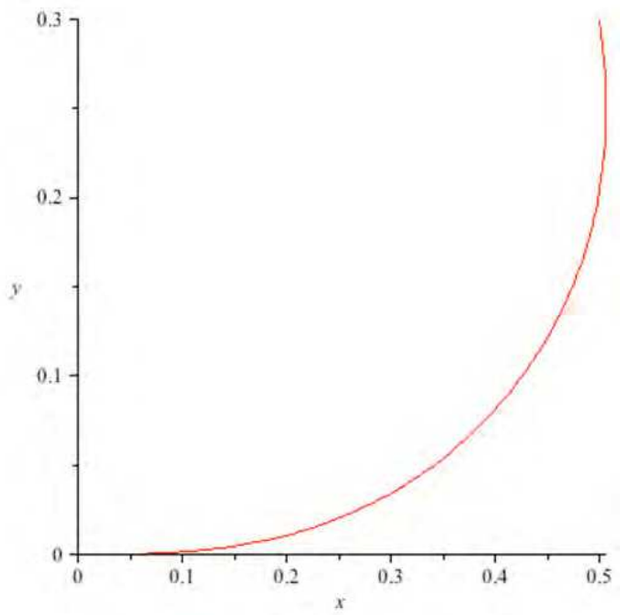

Fig. 9. Phase plane graph of the solution.

2.3 Burning money case

Allowing one of the players to "burn" money is really an interesting idea, remarking that both Alice and Bob are rational and pretend to have the best possible reward. In this work Alice will have the chance of burning money, while Bob plays only with selecting between opera or football match. The idea of burning money is presented as a game that helps understand weakly dominated strategies, modifying the normal game in order to let the players make a rational elimination by knowing that the other player eliminates strategies too. This case is presented in by Herbert Gintis (Gintis, 2009). The case proposed on section 2.2.2 can be taken as an un-burning money case.

2.3.1 Burning money case

The difference for this case is the possibility for one player (Alice) to "burn" money; that is to say, the player can have a negative payment by destroying some of her stuff, hence changing the strategies used by each player. In this case this option will be for Alice, who loses 2 points in each possible payment; while Bob keeps having the same payment:

$$A = \begin{bmatrix} 2 & -2 \\ -2 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}; \tag{29}$$

With the given matrices and the established method, equilibrium values for the system are found:

$$\{x = 0, y = 0\}, \{x = 1, y = 0\}, \{x = 0, y = 1\}, \left\{x = \frac{4}{5}, y = \frac{1}{5}\right\}, \{x = 1, y = 1\}. \quad (30)$$

Simplifying by (10) from the replicator's equation obtained in the method, we get:

$$\frac{d}{dt}\,\varepsilon(t) = \frac{4}{5}\,\eta(t), \qquad \frac{d}{dt}\,\eta(t) = \frac{4}{5}\,\varepsilon(t); \quad (31)$$

The solutions for this system are:

$$\left\{\varepsilon(t) = C_1\, e^{-\frac{4}{5}t} + C_2\, e^{\frac{4}{5}t}, \eta(t) = -C_1\, e^{-\frac{4}{5}t} + C_2\, e^{\frac{4}{5}t}\right\} \quad (32)$$

With the results obtained, it is possible to make a stability analysis and generate a vector field plot, which led us observe the specific point where the solution converges:

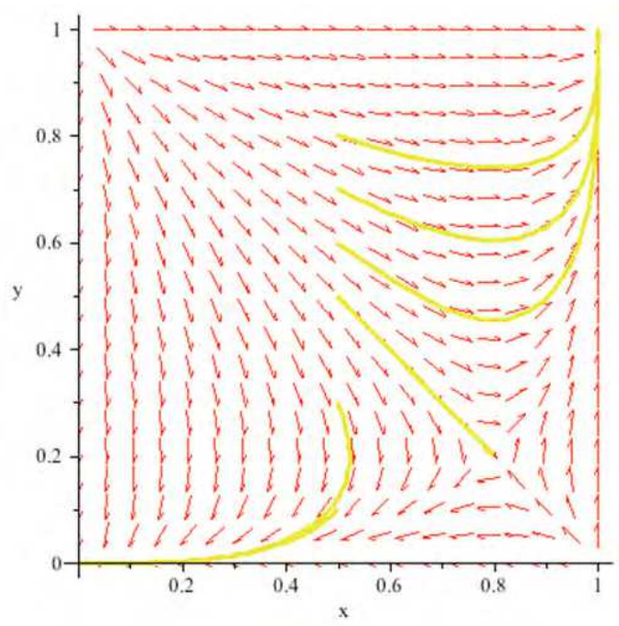

Fig. 10. Vector field plot

Plotting the proportions obtained over time, we can see that both Alice's and Bob's converge.

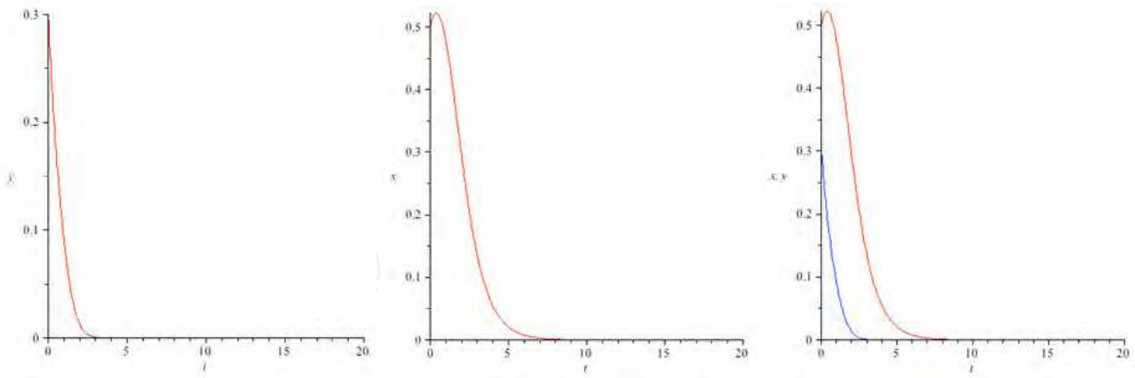

Fig. 11. a) Numerical solution for Alice's, b) Numerical solution for Bob's, c) Alice's and Bob's results plotted together with an extended time.

The three previous graphs are the time series for the player's proportions. The following graph is the phase plane for x and y:

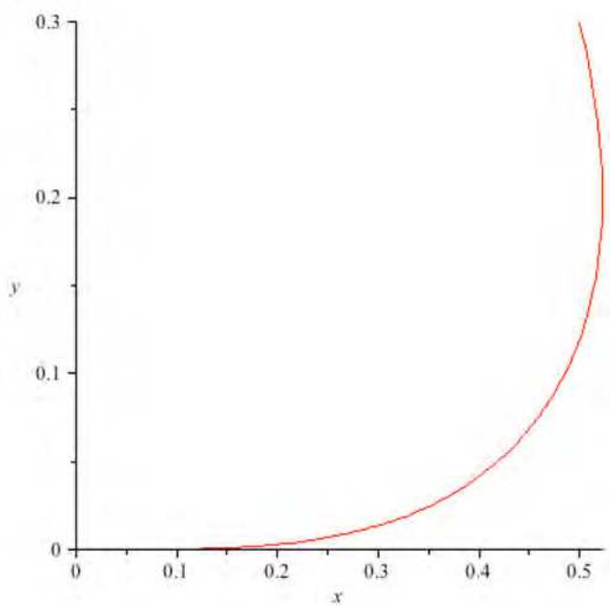

Fig. 12. Phase plane graph

3. Quantum game

Quantum mechanics has been applied to different games such as Prissioner Dilema (Eisert et al., 1999), hawk and dove game (López, 2010) and battle of the sexes game (Frackiewicz, 2009). Several versions for the battle of the sexes game have been proposed from quantum game theory (Neto, 2008; Du et al., 2001; Frackiewicz, 2009), however the conception of this work is to present the wider variety of strategies that a classical analysis. The method implemented is shown using quantum circuits, where operators can be understood as

quantum gates that transform the initial states in order to obtain the payoff matrices for both Alice and Bob. Three possible scenarios are presented, however only the first one is used in this work:

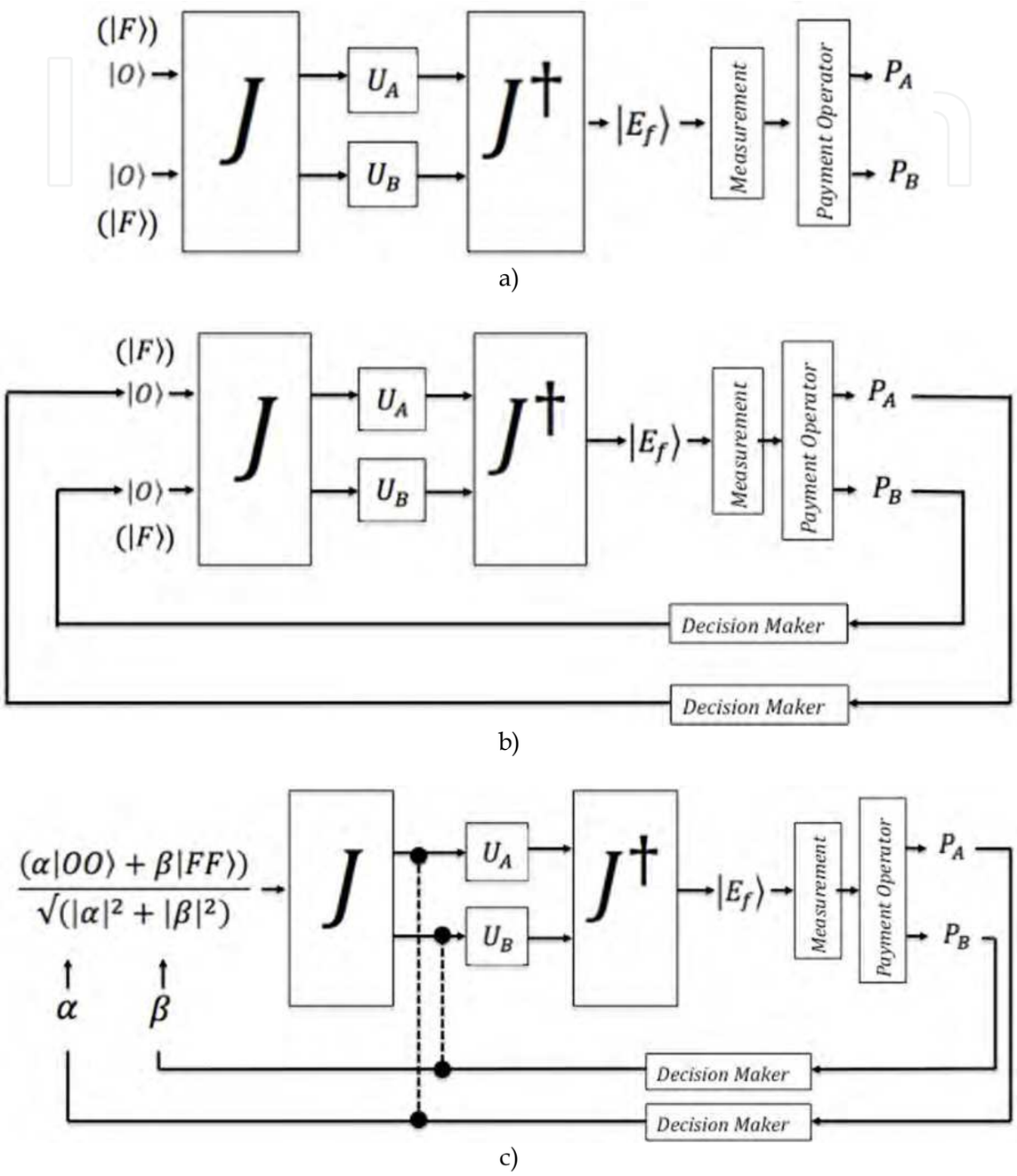

Fig. 13. Quantum circuit for quantum Battle of the Sexes Game: a) open loop case with non-entangled initial states, b) closed loop with non-entangled initial states, c) closed loop with entangled initial states.

3.1 Method

The method that will be used through this section is a formalization of the quantum circuit shown in Figure 13a), which corresponds to the open loop case with non-entangled initial states. The quantum gate named J in Figure 13a), has the form:

$$J = e^{\frac{i\Gamma}{2} C \otimes C}, \tag{33}$$

where

$$C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, C \otimes C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \frac{i\Gamma}{2} C \otimes C = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} I\,\Gamma \\ 0 & 0 & -\frac{1}{2} I\,\Gamma & 0 \\ 0 & -\frac{1}{2} I\,\Gamma & 0 & 0 \\ \frac{1}{2} I\,\Gamma & 0 & 0 & 0 \end{bmatrix}. \tag{34}$$

Then, J takes the form:

$$J = \begin{bmatrix} \cos\left(\frac{1}{2}\Gamma\right) & 0 & 0 & I \sin\left(\frac{1}{2}\Gamma\right) \\ 0 & \cos\left(\frac{1}{2}\Gamma\right) & -I \sin\left(\frac{1}{2}\Gamma\right) & 0 \\ 0 & -I \sin\left(\frac{1}{2}\Gamma\right) & \cos\left(\frac{1}{2}\Gamma\right) & 0 \\ I \sin\left(\frac{1}{2}\Gamma\right) & 0 & 0 & \cos\left(\frac{1}{2}\Gamma\right) \end{bmatrix} \tag{35}$$

The possible strategies are represented by the following Kets:

$$|FF\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, |FO\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, |OF\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, |OO\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{36}$$

where, F refers to Football and O to Opera.
The action of the quantum gate J over the Kets $|FF\rangle$ and $|OO\rangle$ is:

$$J \cdot |FF\rangle = \begin{bmatrix} \cos\left(\frac{1}{2}\Gamma\right) \\ 0 \\ 0 \\ I \sin\left(\frac{1}{2}\Gamma\right) \end{bmatrix}, J \cdot |OO\rangle = \begin{bmatrix} I \sin\left(\frac{1}{2}\Gamma\right) \\ 0 \\ 0 \\ cos\left(\frac{1}{2}\Gamma\right) \end{bmatrix} \tag{37}$$

The quantum gates associated with Alice and Bob are given respectively by:

$$U_A = \begin{bmatrix} a & b \\ -\bar{b} & \bar{a} \end{bmatrix} \quad , \quad U_B = \begin{bmatrix} c & d \\ -\bar{d} & \bar{c} \end{bmatrix} \tag{38}$$

and the tensor product between these, is:

$$U_A \otimes U_B = \begin{bmatrix} ac & ad & bc & bd \\ -a\bar{d} & a\bar{c} & -b\bar{d} & b\bar{c} \\ -\bar{b}c & -\bar{b}\bar{d} & \bar{a}c & \bar{a}d \\ \bar{b}\bar{d} & -\bar{b}\bar{c} & -\bar{a}\bar{d} & \bar{a}\bar{c} \end{bmatrix} \tag{39}$$

The action of combined quantum gate (39), over the Kets presented at (37) is:

$$(U_A \otimes U_B)\,(J \cdot |FF\rangle) = \begin{bmatrix} ac\cos\left(\frac{1}{2}\Gamma\right) + I\ bd\ \sin\left(\frac{1}{2}\Gamma\right) \\ -a\bar{d}\cos\left(\frac{1}{2}\Gamma\right) + I\ b\bar{c}\ \sin\left(\frac{1}{2}\Gamma\right) \\ -\bar{b}c\cos\left(\frac{1}{2}\Gamma\right) + I\ \bar{a}d\ \sin\left(\frac{1}{2}\Gamma\right) \\ \bar{b}\bar{d}\cos\left(\frac{1}{2}\Gamma\right) + I\ \bar{a}\bar{c}\ \sin\left(\frac{1}{2}\Gamma\right) \end{bmatrix} \tag{40}$$

$$(U_A \otimes U_B)\,(J \cdot |OO\rangle) = \begin{bmatrix} I\ ac\sin\left(\frac{1}{2}\Gamma\right) + \ bd\ \cos\left(\frac{1}{2}\Gamma\right) \\ -I\ a\bar{d}\sin\left(\frac{1}{2}\Gamma\right) + \ b\bar{c}\ \cos\left(\frac{1}{2}\Gamma\right) \\ -I\ \bar{b}c\sin\left(\frac{1}{2}\Gamma\right) + \ \bar{a}d\ \cos\left(\frac{1}{2}\Gamma\right) \\ I\ \bar{b}\bar{d}\sin\left(\frac{1}{2}\Gamma\right) + \ \bar{a}\bar{c}\ \cos\left(\frac{1}{2}\Gamma\right) \end{bmatrix} \tag{41}$$

According with the Figure 13a), the next step is to take the Hermitian transpose of the quantum gate J, and the result is the following:

$$J^{\dagger} = \begin{bmatrix} \cos\left(\frac{1}{2}\bar{\Gamma}\right) & 0 & 0 & -I\sin\left(\frac{1}{2}\bar{\Gamma}\right) \\ 0 & \cos\left(\frac{1}{2}\bar{\Gamma}\right) & I\sin\left(\frac{1}{2}\bar{\Gamma}\right) & 0 \\ 0 & I\sin\left(\frac{1}{2}\bar{\Gamma}\right) & \cos\left(\frac{1}{2}\bar{\Gamma}\right) & 0 \\ -I\sin\left(\frac{1}{2}\bar{\Gamma}\right) & 0 & 0 & \cos\left(\frac{1}{2}\bar{\Gamma}\right) \end{bmatrix} \tag{42}$$

The last step in the quantum circuit is the action of the quantum gate shown in (42) over the Kets given at (40) and (41):

$$J^{\dagger} \cdot ((U_A \otimes U_B)(J \cdot |FF\rangle))$$

$$= \begin{bmatrix} \cos\left(\frac{1}{2}\bar{\Gamma}\right)\left(ac\cos\left(\frac{1}{2}\Gamma\right) + I\ bd\ \sin\left(\frac{1}{2}\Gamma\right)\right) - I\sin\left(\frac{1}{2}\bar{\Gamma}\right)\left(\overline{bd}\cos\left(\frac{1}{2}\Gamma\right) + I\ \overline{ac}\ \sin\left(\frac{1}{2}\Gamma\right)\right) \\ \cos\left(\frac{1}{2}\bar{\Gamma}\right)(-a\bar{d}\cos\left(\frac{1}{2}\Gamma\right) + I\ b\bar{c}\ \sin\left(\frac{1}{2}\Gamma\right)) + I\sin\left(\frac{1}{2}\bar{\Gamma}\right)\left(-\bar{b}c\cos\left(\frac{1}{2}\Gamma\right) + I\ \bar{a}d\ \sin\left(\frac{1}{2}\Gamma\right)\right) \\ I\sin\left(\frac{1}{2}\bar{\Gamma}\right)(-a\bar{d}\cos\left(\frac{1}{2}\Gamma\right) + I\ b\bar{c}\ \sin\left(\frac{1}{2}\Gamma\right)) + \cos\left(\frac{1}{2}\bar{\Gamma}\right)\left(-\bar{b}c\cos\left(\frac{1}{2}\Gamma\right) + I\ \bar{a}d\ \sin\left(\frac{1}{2}\Gamma\right)\right) \\ -\mathrm{I}\sin\left(\frac{1}{2}\bar{\Gamma}\right)\left(ac\cos\left(\frac{1}{2}\Gamma\right) + I\ bd\ \sin\left(\frac{1}{2}\Gamma\right)\right) \cos\left(\frac{1}{2}\bar{\Gamma}\right)\left(\overline{bd}\cos\left(\frac{1}{2}\Gamma\right) + I\ \overline{ac}\ \sin\left(\frac{1}{2}\Gamma\right)\right) \end{bmatrix} \tag{43}$$

$$J^{\dagger}\cdot((U_A\otimes U_B)\,(J\cdot|OO\rangle))$$
$$=\begin{bmatrix} \cos\left(\frac{1}{2}\bar{\Gamma}\right)\left(I\,ac\sin\left(\frac{1}{2}\Gamma\right)+\ bd\ \cos\left(\frac{1}{2}\Gamma\right)\right) & -I\sin\left(\frac{1}{2}\bar{\Gamma}\right)\left(I\,\bar{b}\bar{d}\sin\left(\frac{1}{2}\Gamma\right)+\ \bar{a}\bar{c}\ \cos\left(\frac{1}{2}\Gamma\right)\right) \\ \cos\left(\frac{1}{2}\bar{\Gamma}\right)\left(-I\,a\bar{d}\sin\left(\frac{1}{2}\Gamma\right)+\ b\bar{c}\ \cos\left(\frac{1}{2}\Gamma\right)\right)+I\sin\left(\frac{1}{2}\bar{\Gamma}\right)\left(-I\,\bar{b}c\sin\left(\frac{1}{2}\Gamma\right)+\ \bar{a}d\ \cos\left(\frac{1}{2}\Gamma\right)\right) \\ I\sin\left(\frac{1}{2}\bar{\Gamma}\right)\left(-I\,a\bar{d}\sin\left(\frac{1}{2}\Gamma\right)+\ b\bar{c}\ \cos\left(\frac{1}{2}\Gamma\right)\right)+\cos\left(\frac{1}{2}\bar{\Gamma}\right)\left(-I\,\bar{b}c\sin\left(\frac{1}{2}\Gamma\right)+\ \bar{a}d\ \cos\left(\frac{1}{2}\Gamma\right)\right) \\ -\sin\left(\frac{1}{2}\bar{\Gamma}\right)\left(I\,ac\sin\left(\frac{1}{2}\Gamma\right)+\ bd\ \cos\left(\frac{1}{2}\Gamma\right)\right) & \cos\left(\frac{1}{2}\bar{\Gamma}\right)\left(I\,\bar{b}\bar{d}\sin\left(\frac{1}{2}\Gamma\right)+\ \bar{a}\bar{c}\ \cos\left(\frac{1}{2}\Gamma\right)\right) \end{bmatrix} \tag{44}$$

Finally a measurement is made over the final states (43) and (44), from which is possible to build the payment operators and the expected payments for Alice and Bob.
In figures 13b) and 13c), the quantum circuits are depicted as closed loops, where the feedback is implemented by special quantum gates that assume the role of "decision makers", it is to say, special quantum gates that transform the expected payments into new possible initial states and new quantum gates: U_A and U_B.

3.2 Simple cases

In this section some models presented by J.J. de Farias Neto in his work titled "Quantum Battle of the Sexes Revisited". These cases are a combination of different values for γ and different initial states, either non-entangled or entangled. In the first case, $\gamma = 0$ and the initial state is non-entangled. It will be initially made for $|OO\rangle$:

$$\left|E_{OO}\right. = (U_A \otimes\ U_B)\cdot\left|OO\right. \tag{45}$$

Replacing values from equation (44), we obtain:

$$\left|E_{OO}\right. = \begin{bmatrix} bd \\ b\bar{c} \\ \bar{a}d \\ \overline{ac} \end{bmatrix} \tag{46}$$

The resulting payoffs for Alice and Bob are:

$$\begin{aligned} P_A &= \left|b^2d^2\right| + 2\left|a^2c^2\right| \\ P_B &= 2\left|b^2d^2\right| + \left|a^2c^2\right| \end{aligned} \tag{47}$$

Likewise for $|FF\rangle$, the final state is given by:

$$\left|E_{FF}\right\rangle = (U_A \otimes\ U_B)\cdot\left|FF\right. \tag{48}$$

Substituting equation (43), we get:

$$|E_{FF}\rangle = \begin{bmatrix} ac \\ -a\bar{d} \\ -\bar{b}c \\ \bar{b}\bar{d} \end{bmatrix} \tag{49}$$

The outcomes in this case are:

$$\begin{aligned} P_A &= 2|b^2d^2| + |a^2c^2| \\ P_B &= |b^2d^2| + 2|a^2c^2| \end{aligned} \tag{50}$$

The second model maintains $\gamma = 0$, but the initial state is now entangled; hence the $|E_i\rangle$ has the form:

$$|E_i\rangle = \frac{1}{2}\sqrt{2}(\,|FF\rangle + |OO\rangle) \tag{51}$$

Replacing values for $|FF\rangle$ and $|OO\rangle$, we obtain the initial state:

$$|E_i\rangle = \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ 0 \\ 0 \\ \frac{1}{2}\sqrt{2} \end{bmatrix} \tag{52}$$

The final state is given by the following equation:

$$\left|E_f\right\rangle = (U_A \otimes U_B) \cdot |E_i\rangle \tag{53}$$

Substituting, the final state has the form:

$$\left|E_f\right\rangle = \begin{bmatrix} \frac{1}{2}\sqrt{2}ac + \frac{1}{2}\sqrt{2}bd \\ -\frac{1}{2}\sqrt{2}a\bar{d} + \frac{1}{2}\sqrt{2}b\bar{c} \\ -\frac{1}{2}\sqrt{2}\bar{b}c + \frac{1}{2}\sqrt{2}\bar{a}d \\ \frac{1}{2}\sqrt{2}\bar{b}\bar{d} + \frac{1}{2}\sqrt{2}\bar{a}\bar{c} \end{bmatrix} \tag{54}$$

Alice and Bob's payoffs are, respectively

$$\begin{aligned} P_A &= \frac{3}{2}(ac + bd)\left(\bar{a}\bar{c} + \bar{b}\bar{d}\right) \\ P_B &= \frac{3}{2}(ac + bd)\left(\bar{a}\bar{c} + \bar{b}\bar{d}\right) \end{aligned} \tag{55}$$

Another case is proposed by J.J. de Farias, where he defines a new initial state:

$$|E_i\rangle = \frac{1}{2}\sqrt{2}(\,I|FF\rangle + |OO\rangle) \tag{56}$$

By replacing values for the initial state previously given, we obtain:

$$|E_i\rangle = \frac{1}{2}\sqrt{2}\left(I\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right) \tag{57}$$

In (53), we replace the initial state, obtaining:

$$|E_f\rangle = \begin{bmatrix} I\frac{1}{2}\sqrt{2}ac + \frac{1}{2}\sqrt{2}bd \\ -I\frac{1}{2}\sqrt{2}a\bar{d} + \frac{1}{2}\sqrt{2}b\bar{c} \\ -I\frac{1}{2}\sqrt{2}\bar{b}c + \frac{1}{2}\sqrt{2}\bar{a}d \\ I\frac{1}{2}\sqrt{2}\bar{b}\bar{d} + \frac{1}{2}\sqrt{2}\bar{a}\bar{c} \end{bmatrix} \tag{58}$$

The outcome for Alice and Bob are:

$$P_A = -\frac{3}{2}(I\bar{b}\bar{d} + \bar{a}\bar{c})(-ac + Ibd)$$

$$P_B = -\frac{3}{2}(I\bar{b}\bar{d} + \bar{a}\bar{c})(-ac + Ibd) \tag{59}$$

The following case is a modification of the previous one. Here, the initial state is modified and has the form:

$$|E_i\rangle = \frac{1}{2}\sqrt{2}(\,|FF\rangle + I|OO\rangle) \tag{60}$$

By replacing the respective Kets, we get:

$$|E_i\rangle = \frac{1}{2}\sqrt{2}\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + I\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right) \tag{61}$$

The final state of the system is given by:

$$|E_f\rangle = \begin{bmatrix} \frac{1}{2}\sqrt{2}ac + I\frac{1}{2}\sqrt{2}bd \\ -\frac{1}{2}\sqrt{2}a\bar{d} + I\frac{1}{2}\sqrt{2}b\bar{c} \\ -\frac{1}{2}\sqrt{2}\bar{b}c + I\frac{1}{2}\sqrt{2}\bar{a}d \\ \frac{1}{2}\sqrt{2}\bar{b}\bar{d} + I\frac{1}{2}\sqrt{2}\bar{a}\bar{c} \end{bmatrix} \tag{62}$$

Alice and Bob's rewards are shown ahead:

$$P_A = -\frac{3}{2}\left(I\bar{b}\bar{d} + \bar{a}\bar{c}\right)(-ac + Ibd)$$
$$P_B = -\frac{3}{2}\left(I\bar{b}\bar{d} + \bar{a}\bar{c}\right)(-ac + Ibd) \tag{63}$$

3.2.1 Dawkins model

The same model for the classical analysis is presented, and so are the describing matrices:

$$M_A = \begin{bmatrix} 2 & 0 \\ 5 & -5 \end{bmatrix} \tag{64}$$

$$M_B = \begin{bmatrix} 2 & 5 \\ 0 & 15 \end{bmatrix} \tag{65}$$

Payments for Alice and Bob are:

$$\begin{aligned} P_A = {} & -5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a^2\,\bar{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 + 2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\bar{a}\,\bar{c} - 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{a}^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - 2\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}^2\,\bar{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 \\ & + 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{a}^2\,\bar{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - 5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\bar{b}\,\bar{d} + 4\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\bar{b}\,\bar{d}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) + 2\sin\left(\tfrac{1}{2}\right. \\ & \left.\bar{\Gamma}\right)\bar{a}^2\,\bar{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) - 10\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)a\,c\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\bar{a}\,\bar{c}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) - 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{b}^2\,\bar{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) \\ & + 10\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)a\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)d\,\bar{a}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) + 5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 c\,\bar{b} + 5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)b^2\,\bar{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) \\ & + 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{b}^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) + 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,\bar{c} + 5\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)a\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,\bar{c}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) + 5\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b \\ & \bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)a\,\bar{d} - 5\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}\,c\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)d\,\bar{a} - 5\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{a}\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2 c\,\bar{b}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) \\ & + 5\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)a\,c\cos\left(\tfrac{1}{2}\Gamma\right) - 5\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{a}\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\bar{b}\,\bar{d}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) - 5\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\bar{a}\,\bar{c} + 5\,\mathrm{I}\sin\left(\tfrac{1}{2}\right. \\ & \left.\bar{\Gamma}\right)a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,d - 8\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\bar{a}\,\bar{c} + 8\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)a\,c\sin\left(\tfrac{1}{2}\Gamma\right) - 8\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{a} \\ & \bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,d + 8\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\bar{b}\,\bar{d}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) - 2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 + 5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 + 2\cos\left(\tfrac{1}{2}\right. \\ & \left.\bar{\Gamma}\right)a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\sin\left(\tfrac{1}{2}\Gamma\right) + 2\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{a}\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,c - 5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)\cos\left(\tfrac{1}{2}\Gamma\right) - 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b} \\ & \bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,d - 2\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{b}\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2\bar{a}\,\bar{c}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) + 2\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right)b\,d\sin\left(\tfrac{1}{2}\Gamma\right) - 2\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{a}\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right) \\ & \bar{b}\,\bar{d} + 2\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)a\,c \end{aligned} \tag{66}$$

$$\begin{aligned} P_B = {} & 2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\bar{a}\,\bar{c} - 2\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}^2\,\bar{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 - 15\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{a}^2\,\bar{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 + 15\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\bar{b}\,\bar{d} + 4\cos\left(\tfrac{1}{2}\right. \\ & \left.\bar{\Gamma}\right)b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\bar{b}\,\bar{d}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) + 2\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{a}^2\,\bar{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) + 30\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)a\,c\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\bar{a}\,\bar{c}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) + 15\cos\left(\tfrac{1}{2}\right. \\ & \left.\bar{\Gamma}\right)\bar{b}^2\,\bar{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) - 5\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,\bar{d}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) + 5\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{b}\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2 d\,\bar{a}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) + 5\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 \\ & \bar{a}\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)c\,\bar{b} - 5\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)b\,\bar{c} - 15\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)a\,c\cos\left(\tfrac{1}{2}\Gamma\right) + 15\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{a}\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\bar{b} \\ & \bar{d}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) + 15\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\bar{a}\,\bar{c} - 15\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,d + 12\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\bar{a}\,\bar{c} \\ & - 12\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)a\,c\sin\left(\tfrac{1}{2}\Gamma\right) + 12\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{a}\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,d - 12\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\bar{b}\,\bar{d}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) + 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a \\ & \bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 d\,\bar{a} + 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)a^2\,\bar{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) + 5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{a}^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) + 5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{a}\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,\bar{d} \\ & - 2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - 15\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 + 2\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\sin\left(\tfrac{1}{2}\Gamma\right) + 2\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{a}\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,c \\ & + 15\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)\cos\left(\tfrac{1}{2}\Gamma\right) + 15\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}\,\bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,d + 10\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)c\,\bar{b}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right) - 2\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)\bar{b} \\ & \bar{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2\bar{a}\,\bar{c}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right) + 2\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right)b\,d\sin\left(\tfrac{1}{2}\Gamma\right) - 2\,\mathrm{I}\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{a}\,\bar{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\bar{b}\,\bar{d} + 2\,\mathrm{I}\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\sin\left(\tfrac{1}{2}\right. \\ & \left.\bar{\Gamma}\right)a\,c - 5\cos\left(\tfrac{1}{2}\bar{\Gamma}\right)^2 b^2\,\bar{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - 5\sin\left(\tfrac{1}{2}\bar{\Gamma}\right)^2\bar{b}^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 \end{aligned} \tag{67}$$

3.2.2 Wikipedia case 1

Like in section 2.2.2, the possible payments for both Alice and Bob has the form:

$$M_A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \tag{68}$$

$$M_B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \tag{69}$$

From the method established, the payoff matrices for Alice and Bob are presented in an extended form:

$$\begin{aligned} P_A = {} & 3\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2 \overline{a}\,\overline{c} - 3\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 \overline{b}^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 - 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 \overline{a}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 + 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2 \overline{b}\,\overline{d} \\ & + 6\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 3\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{a}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) + 4\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\sin\left(\tfrac{1}{2}\Gamma\right)\overline{a} \\ & \overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) + 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{b}^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right) + 21\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\overline{b} \\ & \overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 21\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\overline{a}\,\overline{c} - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d + 31\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right) \\ & - 31\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\overline{b}\,\overline{d} + 31\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c - 31\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2\overline{a}\,\overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) \\ & + 51\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\overline{a}\,\overline{c} - 51\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right) a\,c\sin\left(\tfrac{1}{2}\Gamma\right) + 51\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d - 51\cos\left(\tfrac{1}{2}\right. \\ & \left.\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) - 3\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 + 3\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\sin\left(\tfrac{1}{2}\Gamma\right) \\ & + 3\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,c + 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\cos\left(\tfrac{1}{2}\Gamma\right) + 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,d \end{aligned} \tag{70}$$

$$\begin{aligned} P_B = {} & 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\overline{a}\,\overline{c} - 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{b}^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 - 3\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{a}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 + 3\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\overline{b}\,\overline{d} \\ & + 4\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{a}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) + 6\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\overline{a} \\ & \overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) + 3\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{b}^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 51\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\overline{a}\,\overline{c} - 51\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{b} \\ & \overline{d}\cos\left(\tfrac{1}{2}\Gamma\right) a\,c\sin\left(\tfrac{1}{2}\Gamma\right) + 51\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d - 51\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) \\ & - 31\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right) + 31\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 31\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\overline{a}\,\overline{c} - 31\sin\left(\tfrac{1}{2}\right. \\ & \left.\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d - 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - 3\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 + 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\sin\left(\tfrac{1}{2}\Gamma\right) \\ & + 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,c + 3\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\cos\left(\tfrac{1}{2}\Gamma\right) + 3\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,d - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\overline{b} \\ & \overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2\overline{a}\,\overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) + 21\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right) - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\overline{b}\,\overline{d} + 21\cos\left(\tfrac{1}{2}\right. \\ & \left.\overline{\Gamma}\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c \end{aligned} \tag{71}$$

3.2.3 Wikipedia case 2

The values proposed in section 2.2.3 of the classical analysis are shown again for both Alice and Bob:

$$M_A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \tag{72}$$

$$M_B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \tag{73}$$

The expected payoff for Alice and Bob are:

$$P_A = 3\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\frac{1}{2}\Gamma\right)^2\overline{a}\,\overline{c} - 3\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}^2\,\overline{d}^2\cos\left(\frac{1}{2}\Gamma\right)^2 - 2\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}^2\,\overline{c}^2\sin\left(\frac{1}{2}\Gamma\right)^2 + 2\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\frac{1}{2}\Gamma\right)^2\overline{b}\,\overline{d}$$
$$+ 6\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\overline{b}\,\overline{d}\sin\left(\frac{1}{2}\overline{\Gamma}\right) + 3\sin\left(\frac{1}{2}\overline{\Gamma}\right)\overline{a}^2\,\overline{c}^2\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\overline{\Gamma}\right) + 4\sin\left(\frac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right)\overline{a}$$
$$\overline{c}\cos\left(\frac{1}{2}\overline{\Gamma}\right) + 2\cos\left(\frac{1}{2}\overline{\Gamma}\right)\overline{b}^2\,\overline{d}^2\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\overline{\Gamma}\right) - 2\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\frac{1}{2}\Gamma\right) a\,c\cos\left(\frac{1}{2}\Gamma\right) + 2\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right)\overline{a}\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)^2\overline{b}$$
$$\overline{d}\sin\left(\frac{1}{2}\overline{\Gamma}\right) + 2\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right)\overline{a}\,\overline{c} - 2\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\frac{1}{2}\Gamma\right)^2\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,d - 1\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 a\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right) b\,\overline{c}$$
$$- 1\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)^2 a\,\overline{d}\sin\left(\frac{1}{2}\overline{\Gamma}\right) + 1\sin\left(\frac{1}{2}\overline{\Gamma}\right)\overline{b}\,c\cos\left(\frac{1}{2}\Gamma\right)^2 d\,\overline{a}\cos\left(\frac{1}{2}\overline{\Gamma}\right) + 1\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,d\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right) c\,\overline{b} + \cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 a$$
$$\overline{d}\cos\left(\frac{1}{2}\Gamma\right)^2 d\,\overline{a} + \cos\left(\frac{1}{2}\overline{\Gamma}\right) a^2\,\overline{d}^2\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\overline{\Gamma}\right) + \sin\left(\frac{1}{2}\overline{\Gamma}\right)\overline{a}^2\,d^2\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\overline{\Gamma}\right) + \sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,d\sin\left(\frac{1}{2}\Gamma\right)^2 a\,\overline{d} \quad (74)$$
$$+ 3\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\frac{1}{2}\Gamma\right) b\,d\sin\left(\frac{1}{2}\Gamma\right) - 3\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\overline{b}\,\overline{d} + 3\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\frac{1}{2}\Gamma\right)^2\sin\left(\frac{1}{2}\overline{\Gamma}\right) a\,c - 3\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)$$
$$\overline{b}\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)^2\overline{a}\,\overline{c}\cos\left(\frac{1}{2}\overline{\Gamma}\right) - 3\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 b^2\,d^2\sin\left(\frac{1}{2}\Gamma\right)^2 - 2\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 a^2\,c^2\cos\left(\frac{1}{2}\Gamma\right)^2 + 3\cos\left(\frac{1}{2}\overline{\Gamma}\right) a^2\,c^2\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\overline{\Gamma}\right)\sin\left(\frac{1}{2}\Gamma\right)$$
$$+ 3\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)^2 a\,c + 2\sin\left(\frac{1}{2}\overline{\Gamma}\right)\overline{b}^2\,d^2\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\overline{\Gamma}\right)\cos\left(\frac{1}{2}\Gamma\right) + 2\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)^2 b\,d + 2\cos\left(\frac{1}{2}\overline{\Gamma}\right) b$$
$$\overline{c}\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right) c\,\overline{b}\sin\left(\frac{1}{2}\overline{\Gamma}\right) + 4\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\overline{a}\,\overline{c} - 4\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right) a\,c\sin\left(\frac{1}{2}\Gamma\right) + 4\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)\overline{a}$$
$$\overline{c}\sin\left(\frac{1}{2}\Gamma\right)^2\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,d - 4\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\frac{1}{2}\Gamma\right)^2\overline{b}\,\overline{d}\sin\left(\frac{1}{2}\overline{\Gamma}\right) - \cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 b^2\,\overline{c}^2\sin\left(\frac{1}{2}\Gamma\right)^2 - \sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}^2\,c^2\cos\left(\frac{1}{2}\Gamma\right)^2$$

$$P_B = -\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 a^2\,\overline{d}^2\cos\left(\frac{1}{2}\Gamma\right)^2 + 2\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\frac{1}{2}\Gamma\right)^2\overline{a}\,\overline{c} - \cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}^2\,d^2\sin\left(\frac{1}{2}\Gamma\right)^2 - 2\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}^2\,\overline{d}^2\cos\left(\frac{1}{2}\Gamma\right)^2$$
$$- 3\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}^2\,\overline{c}^2\sin\left(\frac{1}{2}\Gamma\right)^2 + 3\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\frac{1}{2}\Gamma\right)^2\overline{b}\,\overline{d} + 4\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\overline{b}\,\overline{d}\sin\left(\frac{1}{2}\overline{\Gamma}\right) + 2\sin\left(\frac{1}{2}\right.$$
$$\left.\overline{\Gamma}\right)\overline{a}^2\,\overline{c}^2\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\overline{\Gamma}\right) + 6\sin\left(\frac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right)\overline{a}\,\overline{c}\cos\left(\frac{1}{2}\overline{\Gamma}\right) + 3\cos\left(\frac{1}{2}\overline{\Gamma}\right)\overline{b}^2\,\overline{d}^2\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\overline{\Gamma}\right)$$
$$+ 2\sin\left(\frac{1}{2}\overline{\Gamma}\right) a\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right) d\,\overline{a}\cos\left(\frac{1}{2}\overline{\Gamma}\right) + \sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 b\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)^2 c\,\overline{b} + \sin\left(\frac{1}{2}\overline{\Gamma}\right) b^2\,\overline{c}^2\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\overline{\Gamma}\right) + \cos\left(\frac{1}{2}\right.$$
$$\left.\overline{\Gamma}\right)\overline{b}^2\,c^2\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\overline{\Gamma}\right) + \cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,c\cos\left(\frac{1}{2}\Gamma\right)^2 b\,\overline{c} - 3\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\frac{1}{2}\Gamma\right) a\,c\cos\left(\frac{1}{2}\Gamma\right) + 3\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right)\overline{a}\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)^2\overline{b}$$
$$\overline{d}\sin\left(\frac{1}{2}\overline{\Gamma}\right) + 3\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right)\overline{a}\,\overline{c} - 3\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\frac{1}{2}\Gamma\right)^2\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,d + 1\sin\left(\frac{1}{2}\overline{\Gamma}\right) a\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)^2 b\,\overline{c}\cos\left(\frac{1}{2}\overline{\Gamma}\right) \quad (75)$$
$$+ 1\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 b\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right) a\,\overline{d} - 1\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,c\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\Gamma\right) d\,\overline{a} - 1\cos\left(\frac{1}{2}\overline{\Gamma}\right)\overline{a}\,d\sin\left(\frac{1}{2}\Gamma\right)^2 c\,\overline{b}\sin\left(\frac{1}{2}\overline{\Gamma}\right)$$
$$- 2\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 b^2\,d^2\sin\left(\frac{1}{2}\Gamma\right)^2 - 3\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2 a^2\,c^2\cos\left(\frac{1}{2}\Gamma\right)^2 + 2\cos\left(\frac{1}{2}\overline{\Gamma}\right) a^2\,c^2\cos\left(\frac{1}{2}\Gamma\right)\sin\left(\frac{1}{2}\overline{\Gamma}\right)\sin\left(\frac{1}{2}\Gamma\right) + 2\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)^2 a\,c$$
$$+ 3\sin\left(\frac{1}{2}\overline{\Gamma}\right) b^2\,d^2\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\overline{\Gamma}\right)\cos\left(\frac{1}{2}\Gamma\right) + 3\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)^2 b\,d + 4\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\overline{a}\,\overline{c} - 4\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{b}$$
$$\overline{d}\cos\left(\frac{1}{2}\Gamma\right) a\,c\sin\left(\frac{1}{2}\Gamma\right) + 4\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)\overline{a}\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)^2\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,d - 4\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\frac{1}{2}\Gamma\right)^2\overline{b}\,\overline{d}\sin\left(\frac{1}{2}\overline{\Gamma}\right) - 2\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)\overline{b}\,\overline{d}\cos\left(\frac{1}{2}\Gamma\right)^2\overline{a}$$
$$\overline{c}\cos\left(\frac{1}{2}\overline{\Gamma}\right) + 2\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\frac{1}{2}\Gamma\right) b\,d\sin\left(\frac{1}{2}\Gamma\right) - 2\,1\sin\left(\frac{1}{2}\overline{\Gamma}\right)^2\overline{a}\,\overline{c}\sin\left(\frac{1}{2}\Gamma\right)\cos\left(\frac{1}{2}\Gamma\right)\overline{b}\,\overline{d} + 2\,1\cos\left(\frac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\frac{1}{2}\Gamma\right)^2\sin\left(\frac{1}{2}\overline{\Gamma}\right) a\,c$$

3.3 Burning money case

This section has the same assumptions made for the classical analysis. The burning money case is analyzed in order to find the payment matrices for Alice and Bob. The un-burning money case is presented in section 3.2.2, but with different values as reward.

3.3.1 Burning money

Here, Alice can have a negative reward due to the assumption of burning money as a possible strategy that modifies the outcomes of the game, while Bob plays using only two strategies to choose from. This is shown on the following matrices:

$$M_A = \begin{bmatrix} 2 & -2 \\ -2 & -1 \end{bmatrix}, M_B = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \quad (76,77)$$

The payoffs for Alice and Bob are:

$$
\begin{aligned}
P_A = {} & 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 + 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{a}\,\overline{c} + 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 \\
& + \cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - \sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{b}\,\overline{d} + 4\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\,\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 2\sin\left(\tfrac{1}{2}\right. \\
& \left.\overline{\Gamma}\right)\,\overline{a}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) - 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\,\overline{a}\,\overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) - \cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{b}^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) \\
& + 1\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right) + 1\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d - 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,\overline{a}\cos\left(\tfrac{1}{2}\Gamma\right)^2 d\,\overline{a} - 4\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a \\
& \overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right) d\,\overline{a}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) - 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 c\,\overline{b} - 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{b}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) - 2\cos\left(\tfrac{1}{2}\right. \\
& \left.\overline{\Gamma}\right)\,\overline{b}^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) - 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,\overline{c} - 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\,a^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) - 2\sin\left(\tfrac{1}{2}\right. \\
& \left.\overline{\Gamma}\right)\,\overline{a}^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) - 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,\overline{d} + 51\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\,\overline{a}\,\overline{c} - 51\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b} \\
& \overline{d}\cos\left(\tfrac{1}{2}\Gamma\right) a\,c\sin\left(\tfrac{1}{2}\Gamma\right) + 51\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d - 51\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) - 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 \\
& + \sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 + 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\sin\left(\tfrac{1}{2}\Gamma\right) + 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,c - \sin\left(\tfrac{1}{2}\overline{\Gamma}\right) b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\right. \\
& \left.\overline{\Gamma}\right)\cos\left(\tfrac{1}{2}\Gamma\right) - \cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,d - 4\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right) c\,\overline{b}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 21\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) \\
& - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{b}\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2 d\,\overline{a}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right) c\,\overline{b} + 21\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right) b\,\overline{c} \\
& + 21\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}\,c\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right) d\,\overline{a} + 21\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{a}\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2 c\,\overline{b}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,\overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) \\
& - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right) a\,\overline{d} - 1\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) - 1\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\,\overline{a}\,\overline{c} - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) \\
& \overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{a}\,\overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) + 21\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right) - 21\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\,\overline{b}\,\overline{d} + 21\cos\left(\tfrac{1}{2}\right. \\
& \left.\overline{\Gamma}\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c + 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 + 2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)^2
\end{aligned}
\tag{78}
$$

$$
\begin{aligned}
P_B = {} & \cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{a}\,\overline{c} - \sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 - 4\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 + 4\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{b}\,\overline{d} \\
& + 1\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a\,c\cos\left(\tfrac{1}{2}\Gamma\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right) + 2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\,\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 1\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d\sin\left(\tfrac{1}{2}\Gamma\right)^2\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c - 1\sin\left(\tfrac{1}{2}\right. \\
& \left.\overline{\Gamma}\right)\,\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{a}\,\overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) + \sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{a}^2\,\overline{c}^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) - 1\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\,\overline{b}\,\overline{d} + 8\sin\left(\tfrac{1}{2}\right. \\
& \left.\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\,\overline{a}\,\overline{c}\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) + 4\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{b}^2\,\overline{d}^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 51\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\Gamma\right)\,\overline{a}\,\overline{c} \\
& - 51\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right) a\,c\sin\left(\tfrac{1}{2}\Gamma\right) + 51\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d - 51\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) \\
& - \cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)^2 - 4\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)^2 - 41\sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2 b\,d\sin\left(\tfrac{1}{2}\Gamma\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right) + \cos\left(\tfrac{1}{2}\overline{\Gamma}\right) a^2\,c^2\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\right. \\
& \left.\overline{\Gamma}\right)\sin\left(\tfrac{1}{2}\Gamma\right) + \sin\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{a}\,\overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2 a\,c + 4\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) b^2\,d^2\sin\left(\tfrac{1}{2}\Gamma\right)\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\cos\left(\tfrac{1}{2}\Gamma\right) + 4\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)^2 b\,d + 41\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)\,\overline{a} \\
& \overline{c}\sin\left(\tfrac{1}{2}\Gamma\right)^2\,\overline{b}\,\overline{d}\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) + 41\cos\left(\tfrac{1}{2}\overline{\Gamma}\right)^2\,\overline{b}\,\overline{d}\cos\left(\tfrac{1}{2}\Gamma\right)\sin\left(\tfrac{1}{2}\Gamma\right)\,\overline{a}\,\overline{c} - 41\sin\left(\tfrac{1}{2}\overline{\Gamma}\right) a\,c\cos\left(\tfrac{1}{2}\Gamma\right)^2\cos\left(\tfrac{1}{2}\overline{\Gamma}\right) b\,d
\end{aligned}
\tag{79}
$$

4. Conclusion

In this work, we made a classical and quantum analysis of the Battle of the Sexes game. The classical analysis was based on the model proposed by Frank Wang, using ordinary differential equations and the corresponding stability analysis; while the quantum scheme was based on the method introduced by Neto (Neto, 2008) using quantum circuits with quantum gates for two qubits. Both quantum and classical analysis were implemented using computer algebra software, specifically Maple.

From the results obtained along this work, it is possible to observe that the quantum version has more versatility than the classical one, due to the effect of quantum superposition, the

quantum entanglement and the complexity of the payment operators from which resulted expected payments with more engineering possibilities. It is to remark that the burning money case could be analyzed using different reasoning methods, which may lead to specific solutions such as guaranteeing other player's final choice (Ginitis, 2009).
As a future investigation trend we propose the application of the Yang-Baxter operators (Zhang et al., 2005), which will play the role of the quantum gate J, hence generating completely different but more physically implementable quantum operators and expected payoffs.

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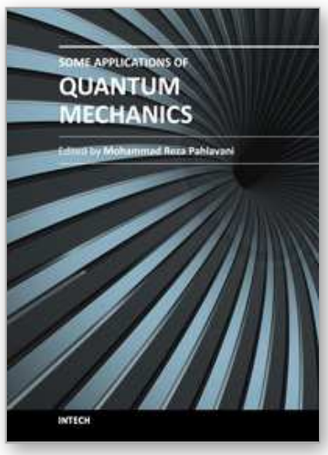

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Quantum mechanics, shortly after invention, obtained applications in different area of human knowledge. Perhaps, the most attractive feature of quantum mechanics is its applications in such diverse area as, astrophysics, nuclear physics, atomic and molecular spectroscopy, solid state physics and nanotechnology, crystallography, chemistry, biotechnology, information theory, electronic engineering... This book is the result of an international attempt written by invited authors from over the world to response daily growing needs in this area. We do not believe that this book can cover all area of application of quantum mechanics but wish to be a good reference for graduate students and researchers.

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