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# Quantum Statistics and Coherent Access Hypothesis

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## 1. Introduction

The properties of quantum systems at equilibrium are mainly displayed by the very nature of the particles involved. As for example, distinguishable particles behave in a quite different manner of indistinguishable bosons, which in turns behave very different from the also indistinguishable fermions. Among the standard methods of statistical mechanics to handle with quantum systems at equilibrium is included the way of counting the accessible states to a given particle (1). Thus, the thermodynamical probability  $\Omega$  comprising the set of possibilities for a system of  $N$  particles to populate the available states will depend on the type of particle being considered, as shown in Table 1. The method for calculating  $\Omega$  consists in supposing the existence of a number of energy levels, labeled by and index  $j$ , each level having energy  $\varepsilon_j$  and containing a total of  $n_j$  particles. Besides, it is assumed that each level contains  $g_j$  distinct sublevels, all of which having the same energy. The value of  $g_j$  associated with level  $j$  is called the "degeneracy" of that energy level. Any number of bosons can occupy the same sublevel, while the Pauli exclusion principle states that only one fermion can occupy any one of the sublevels. Classical behavior emerges when the density is relatively low, which is more common at high temperatures; in such cases, both fermions and bosons behave as if they were classical particles. After identifying what kind of particles we are taking into account, we use counting methods and combinatorial analysis to calculate the expected number  $n_j$  of those particles in level  $j$  with energy  $\varepsilon_j$ . This is done by first calculating the number  $\Omega_j$  of ways of distributing particles among the sublevels  $g_j$  of an energy level  $j$  and taking the product of the ways that each individual energy level can be populated to find the so called thermodynamical probability  $\Omega$ . Once we have  $\Omega$ , it is a standard task to find the statistical distribution for  $n_j$ .

In this chapter, the counting method is extended to handle with equilibrium as well as nonequilibrium quantum systems, in order to obtain the new thermodynamical probability emerging from these situations. Also, systems only slightly out of equilibrium are considered, such that a well defined temperature can be assigned to them, and the new results are pointed, which take into account eventual experimentally detectable changes in their statistical properties.

Another method to handle with equilibrium systems is to take advantage of the Boltzmann factor, which establishes that, given two levels  $a$  and  $b$  whose energies are  $E_a$  and  $E_b$ ,

| (1) | (2) | (3) |    | (1) | (2) | (3) |     |
|-----|-----|-----|----|-----|-----|-----|-----|
| ○   | ○   |     |    | ○   | ○   |     |     |
| ○   |     | ○   |    | ○   |     | ○   |     |
|     | ○   | ○   | a) |     | ○   | ○   | (b) |
| ○○  |     |     |    |     |     |     |     |
|     | ○○  |     |    |     |     |     |     |
|     |     | ○○  |    |     |     |     |     |

Table 1. Configurations of  $M = 3$  accessible states for a)  $N = 2$  indistinguishable bosons,  $\Omega = 6$  and b)  $N = 2$  indistinguishable fermions,  $\Omega = 3$ . (1) denotes the first available state, (2) and (3) denote the second and third available state, respectively.

respectively, the number  $n_a$  of particles in state  $a$  with energy  $E_a$  is given by

$$n_a = p_a \exp(-\beta E_a), \tag{1}$$

where  $p_a$  is a constant indicating the occurrence of probability for state  $a$ . The following condition is thus assumed to be satisfied:

$$n_a P_{MB}(E_{a \rightarrow b}) = n_b P_{MB}(E_{b \rightarrow a}), \tag{2}$$

where  $P_{MB}(E_{a \rightarrow b})$  stands for the Maxwell-Boltzmann (MB) distribution accounting for the transition rate from state  $a$  to another state  $b$ . This method will be heuristically used later to obtain a generalized statistics for both equilibrium and nonequilibrium situations concerning quantum systems.

1.1 Quons, anyons, and generalized statistics

The concept of particles with fractional statistics, called anyons, has been studied with increasing interest, now finding applications in, for example, quantum Hall effect (2) and superconductivity (3). Anyons are generally expected to be observed in two space dimensions for a system whose wave function behavior of multiparticle under the exchange of particles obeys  $\Psi(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4, \dots) = \exp(i\pi\alpha)\Psi(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4, \dots)$ , where  $\{\mathbf{q}\}$  represents generalized coordinates and  $\alpha$  is a real number defining the statistics. For bosons and fermions,  $\alpha = 0$  and 1, respectively, and  $\alpha$  is *any* real number for anyons. Since the concept of anyons was proposed (4; 5), there have been several approaches trying to understand its properties, including a q-deformed bosonic algebra (6) and certain ad hoc conjectures aiming to generalize the Bose-Einstein and Fermi-Dirac distributions (7). Without being exhaustive, it is worthwhile to mention at least a few attempts to achieve a generalized anyonic statistics. As for example, an interesting generalization of the Pauli exclusion principle can be advanced, without explicit reference to spatial dimension (8). This is remarkable, since anyons was generally accepted to exist only in two dimensions. The generalization as proposed in Ref.(8) consists on *defining* a statistical interaction  $g_{\alpha\beta}$  given by

$$\Delta d_\alpha = - \sum_\beta g_{\alpha\beta} \Delta N_\beta \tag{3}$$

where  $d_\alpha$  is the one-particle Hilbert space dimension, and  $\Delta N_\beta$  is an allowed change of the particle number  $N_\beta$  at fixed size and boundary conditions. Thus, for bosons it is required

$g_{\alpha\beta} = 0$ , while for fermions the exclusion principle requires  $g_{\alpha\beta} = \delta_{\alpha\beta}$ . Based on the thermodynamical limit, further requirements must be imposed, such as the independency of the number of particles and the rationality of  $g_{\alpha\beta}$ . According to the author, at a fixed particle numbers the counting state thus leads to the following size of the Hilbert space of many particle states:

$$\prod_a \frac{(d_a + N_a - 1)!}{N_a!(d_a - 1)!}, \quad (4)$$

with  $d_B = d_a$  and  $d_F = d_a + N_a - 1$  for the Fock space dimensions of bosons and Fermi, respectively.

We find another interesting proposal for generalizing Bose-Einstein and Fermi-Dirac statistics in (9), where the distribution function for the number of anyons in a given state  $j$  is postulated at once as

$$N_j = \frac{\gamma}{\exp[\beta(\epsilon_j - \mu)] - f(\alpha)}. \quad (5)$$

The parameter  $\mu$  characterizes the particle reservoir in the same way that  $\beta$  characterizes the thermal reservoir,  $\gamma = 2s + 1$  denotes the multiplicity of states and  $f(\alpha)$  generalizes the statistics, since  $f(\alpha) = 0, 1, -1$  recovers the Boltzmann, Bose-Einstein and Fermi-Dirac statistics and, for anyons, it is expected that  $f(\alpha)$  can assume any real number.

Although the interesting problem of what is exactly the distribution function for anyons in and out of equilibrium remains opened, the above studies provide some clues to this question. It is my purpose, in the remainder of this chapter, to show how to deep our understanding of anyons by looking for some reasons to justify their behavior in a manner that sounds plausible.

## 2. Generalized thermodynamical probability

It is possible to provide a formula taking into account more general behavior than that presented by fermions and bosons, as I show in the following, although, regarding to equilibrium situations, it is not clear at the present if nature allows for such *anyons*. The

general problem we have to solve is that of obtaining the number  $\Omega$  of ways that  $n_j$  balls can be distributed into  $g_j$  boxes, each box having capacity to accommodate  $p$  balls. This number is given by (10)

$$\Omega_p = \sum_{k=0}^{[n_j/(p+1)]} (-1)^k C_{g_j}^k \times C_{g_j+n_j-k(p+1)-1}^{n_j-k(p+1)}, \quad (6)$$

where  $C_n^m = \frac{n!}{m!(n-m)!}$  and the brackets  $[n_j/(p+1)]$  in the upper limit indicate the integer taken from  $n_j/(p+1)$ . As for example, if  $n_j = 2$  and  $p = 1$ , then  $[2/3] = 0$ ; if  $n_j = 5$  and  $p = 2$ , then  $[5/2] = 2$ , and so on. This relation can also be written as

$$\Omega_p = \frac{g_j!}{(g_j-1)!} \sum_{k=0}^{[n_j/(p+1)]} (-1)^k \frac{(g_j+n_j-1-k(p+1)!}{k!(g_j-k)![(n_j-k(p+1))!]} \quad (7)$$

From Eq.(6) it is easy to verify the number of configurations displayed in Tab.I: for the case of bosons,  $n_j = p = 2$ ,  $g_j = 3$ , and the summation is limited to  $k = 0$ , thus resulting  $\Omega =$

$C_4^2 = 6$ ; for the case of fermions,  $p = 1$ , such that in the summation  $k = 0, 1$ , thus resulting  $\Omega = \sum_{k=0}^1 (-1)^k C_3^k \times C_{4-2k}^{2-2k} = 3$ . Proceeding in a general manner, from Eq.(6) we can recover the statistics corresponding to the case  $p = 1$ , Fermi-Dirac (FD) distribution and  $p = n_j$ , Bose-Einstein (BE) distribution. For instance, taking  $p = n_j$  in Eq.(6), the upper limit  $[n_j / (n_j + 1)]$  implies  $k = 0$ , and it is straightforward to obtain the BE statistics:

$$\Omega_{BE}^j = C_{g_j}^0 \times C_{g_j+n_j-1}^{n_j} = \frac{(g_j + n_j - 1)!}{n_j! (g_j - 1)!}, \quad (8)$$

from which we obtain the thermodynamical probability for all the  $j$  levels:

$$\Omega_{BE} = \prod_j \Omega_{BE}^j = \prod_j \frac{(n_j + g_j - 1)!}{n_j! (g_j - 1)!}. \quad (9)$$

Following the standard route (11), since the maxima of  $\Omega_{BE}$  and  $\ln \Omega_{BE}$  occur at the same value, we choose to maximize this latter function, which is subject to the constraints  $\sum_j n_j = N$  and  $\sum_j n_j \epsilon_j = E$ . Using Lagrange multipliers  $\lambda$  and  $\beta$  to form the function

$$f(n_j) = \ln \Omega_{BE} + \lambda (\sum_j n_j - N) + \beta (\sum_j n_j \epsilon_j - E), \quad (10)$$

and the Stirling formula  $\ln n! \cong n \ln n - n$  for the factorials, we take the derivative with respect to  $n_j$ , setting the result to zero and solving for  $n_j$ , to find the Bose-Einstein distribution:

$$n_{jBE} = \frac{g_j}{\exp(\lambda + \beta E_j) - 1}. \quad (11)$$

From thermodynamics, we know that  $\beta = 1/kT$ ,  $k$  being the Boltzmann constant,  $T$  the temperature, and that  $\lambda$  is related to the chemical potential by  $\lambda = -\beta\mu$ , thus being zero for a photon gas. Similarly, when we take  $p = 1$  and use the closed relation

$$\sum_{k=0}^{[n_j/2]} (-1)^k \frac{(g_j + n_j - 1 - 2k)!}{k! (g_j - k)! (n_j - 2k)!} = \frac{(g_j - 1)!}{n_j! (g_j - n_j)!}, \quad (12)$$

it is straightforward to write

$$\Omega_{FD} = \prod_j \frac{g_j!}{n_j! (g_j - n_j)!}. \quad (13)$$

The same standard procedure now leads to

$$n_{jFD} = \frac{g_j}{\exp[\beta(E_j - \mu)] + 1}, \quad (14)$$

where the parameter of constraint  $\lambda = -\beta\mu$  was written explicitly. Note that Eq.(6) generalizes the concept of indistinguishable particles, and it is possibly a valid starting point to study the statistics of particles whose behavior at equilibrium at temperature  $T$  is neither that of bosons nor that of fermions. However, finding a closed relation like that of Eq.(12), for Eq.(7) giving a general capacity  $p$ , which would allow one to obtaining a general formula for

the thermodynamical probability  $\Omega_p$ , remains an opened problem. Of course, once we have such a closed relation, we can apply the standard methods to deduce a generalized occupation number  $n_p$ . I hope the readers can consider this problem interesting enough to pursue further. Another way to derive Eq.(11) and Eq.(14) for a gas of photons ( $\mu = 0$ ) is the following. Let  $E_a$  and  $E_b$  the energy of state  $a$  and  $b$ , respectively, for a two state system of distinguishable particles. As discussed in the introduction of this Chapter, the maxwell-Boltzmann distribution, Eq.(1), implies that, in equilibrium,

$$\frac{n_a}{n_b} = \frac{P_{MB}(E_a)}{P_{MB}(E_b)} = \exp[-\beta(E_a - E_b)], \quad (15)$$

where  $n_b$  and  $n_a$  are the number of distinguishable particles in state  $b$  and  $a$ , respectively. Now, if we consider indistinguishable particles, as for example bosons, the "Bose-Einstein" rate indicated by  $P_{BE}(E_{a \rightarrow b})$  for one particle to make a transition from one state ( $a$ ) to another ( $b$ ) and *vice-versa* must obey

$$P_{BE}(E_{a \rightarrow b}) = (1 + n_b)P_{MB}(E_{a \rightarrow b}), \quad (16)$$

$$P_{BE}(E_{b \rightarrow a}) = (1 + n_a)P_{MB}(E_{b \rightarrow a}), \quad (17)$$

with  $n_b, n_a = 0, 1, 2, \dots$ . We can assign the following meaning to Eq.(16) and Eq.(17): The transition rate governed by BE statistics, being proportional to the MB distribution, makes the probability to one bosonic particle goes from state  $a$  to state  $b$  (from state  $b$  to state  $a$ ) to increase with the occupation number of state  $b$  ( $a$ ). As the equilibrium is assumed, we can write

$$n_a P_{BE}(E_{a \rightarrow b}) = n_b P_{BE}(E_{b \rightarrow a}). \quad (18)$$

From Eqs(16)-(18), we find

$$\frac{n_a}{(1 + n_a)} \exp(-\beta E_a) = \frac{n_b}{(1 + n_b)} \exp(-\beta E_b). \quad (19)$$

If we now put  $\frac{n_b}{(1 + n_b)} \exp(-\beta E_b) = F(\beta)$  and solve to  $n_a$  we find, after changing the subindex  $a$  to  $BE$ ,

$$n_{BE} = \frac{1}{F(\beta) \exp(\beta E) - 1}, \quad (20)$$

which has the same form as Eq.(11).

Proceeding in the same way, it is straightforward to rederive Eq.(14), for fermions, by this method. We only need to change Eq.(16) and Eq.(17) by these ones:

$$P_{FD}(E_{a \rightarrow b}) = (1 - n_b)P_{MB}(E_{a \rightarrow b}), \quad (21)$$

$$P_{FD}(E_{b \rightarrow a}) = (1 - n_a)P_{MB}(E_{b \rightarrow a}), \quad (22)$$

with  $n_a, n_b = 0, 1$ . We must interpret Eq.(21) and Eq.(22) by saying that the probability for one fermion to populate the state  $b$  ( $a$ ) is either proportional to the Boltzmann factor if in state

$b$  ( $a$ ) there is no particle, or zero if in state  $b$  ( $a$ ) there exists another fermion. Now, writing  $n_a P_{FD}(E_{a \rightarrow b}) = n_b P_{FD}(E_{b \rightarrow a})$  and using Eqs.(21),(22) we obtain

$$n_{FD} = \frac{1}{F(\beta) \exp(\beta E) + 1}, \quad (23)$$

which is the same form as Eq.(14).

A heuristic argument can be developed, based on the proceedings which lead to the BE and FD statistics above, to deduce a generalized distribution function. Bearing in mind Eq.(7) for the generalized thermodynamical probability, we must note that it allows for some fraction  $p/N$  of the total number  $N$  of particles to populate a given available state. Thus, once the allowed fraction is attained for a certain level, the probability for any anyon making a transition to that level must be zero. With this reminds, consider the following generalization of Eqs.(16)-(17) and Eq.(21)-(22):

$$P_{ANY}(E_{a \rightarrow b}) = (\gamma - f n_b) P_{MB}(E_{a \rightarrow b}), \quad (24)$$

$$P_{ANY}(E_{b \rightarrow a}) = (\gamma - f n_a) P_{MB}(E_{b \rightarrow a}), \quad (25)$$

where  $n_a, n_b = 0, 1, \dots, p$ , and  $P_{ANY}(E_{b \rightarrow a})$  stands for the rate of transition from state  $b$  to state  $a$  governed by the anyonic statistics. Here  $\gamma$  and  $f$  are real parameters whose meaning will be cleared in the following. To interpret both Eqs.(24) and (25), we must say that the probability for one anyon to make a transition to state  $b$  is proportional to the MB factor, and this probability suddenly becomes zero if the fraction  $f n_b$  of particles attains the maximum  $\gamma$  value for a given state  $b$ . Eq.(25) can be interpreted in a similar way. If we now require, as it was done previously for the other transition rates, that in equilibrium we must have

$$n_a P_{ANY}(E_{a \rightarrow b}) = n_b P_{ANY}(E_{b \rightarrow a}), \quad (26)$$

we can combine Eq.(24)-(26) and use Eq.(15) to obtain, after proceeding in the very same way as we did previously:

$$n_{ANY} = \frac{\gamma}{g(\beta) \exp(\beta E) + f}, \quad (27)$$

Eq.(27) generalizes the FD statistics and becomes the FD distribution when, of course,  $f = 1$ , which means that transitions only occur for states with no particle.

Consider now the case when the transition rates are given by  $P_{ANY}(E_{a \rightarrow b}) = (\gamma + f n_b) P_{MB}(E_{a \rightarrow b})$ ,  $P_{ANY}(E_{b \rightarrow a}) = (\gamma + f n_a) P_{MB}(E_{b \rightarrow a})$  which generalize Eqs.(16) and (17). It is straightforward to obtain, now

$$n_{ANY} = \frac{\gamma}{g(\beta) \exp(\beta E) - f} \quad (28)$$

Note that the corresponding interpretation of these transition rates must be that the anyons now, being different from those governed by Eq.(27), have increasing probability to populate a given state when the fraction  $f$  becomes larger. We see, thus, that is possible to derive, on sound grounds, generalized statistics for anyons at equilibrium.



3. Nonequilibrium systems and coherent access hypothesis

Nonequilibrium situations require a little bit of complexification, since a given particle can now act *as if* it is populating more than one state at once. To convince that this is so it is enough to remind that any initially pure state can be described, using the completeness relation, as a superposition of each state physically accessible to the particle. I call this new situation as *coherent access hypothesis* (12).

3.1 Coherent access for bosons

As we saw from the preceding Section, Tab.1, to define a microstate in statistical mechanics, i.e., a given configuration, it is necessary to take into account the (in)distinguishability of the particles, which gives rise to different configurations. In nonequilibrium situation, to calculate all the possible configurations I now take into account, beside this characteristic, this another one: the possibility to the particle simultaneously access more than one state, or, to avoid eventual difficulties related to interpretations matter inherent to the quantum formalism, the possibility to the particle to coherently access the available states. This situation is shown in Tab. 2 for the case of  $p = g_j$  considering two identical particles having two accessible states ( $g_j = 2$ ). Note that if the particles were distinguishable, the corresponding configuration would be different.

| (1) | (2) | (12) |
|-----|-----|------|
| ○   | ○   |      |
| ○   |     | ○    |
|     | ○   | ○    |
| ○○  |     |      |
|     | ○○  |      |
|     |     | ○○   |

Table 2. A system out of equilibrium composed by two particles having two accessible states. (1) denotes the first available state, (2) denotes the second available state, and (12) denotes the coherent access to both states.

Comparing Tab.I and Tab.II, we see that, clearly, the nonequilibrium situation requires a new strategy for counting microstates. We can represent this new strategy to count, as shown in Tab.2, by the following sequences. The number between parentheses indicates the state to be occupied, while the letter following the parenthesis indicates the corresponding occupation by the particle  $a$ , which is identical to all the others:  $(1)a(2)a(12)$ ;  $(1)a(2)(12)a$ ;  $(1)(2)a(12)a$ ;  $(1)aa(2)(12)$ ;  $(1)(2)aa(12)$ ;  $(1)(2)(12)aa$ . The first sequence corresponds to one particle accessing the first available state and the other particle accessing the second available state, with no coherent access by both particles; the second sequence corresponds to one particle accessing the first state and the other one accessing coherently the states (1) and (2), and so on. As the sequence must initiate by a number, and existing three possible number of states, 1, 2, and 12, there will remain  $3 - 1$  numbers plus two letters  $a$  (particles) to be set in whatever order (permutation). Therefore, the number of unrepeated sequences is

$$\Omega^* = \frac{3 \times (3 - 1 + 2)!}{2!3!} = 6,$$

(29)



where I have put a superscript (\*) to remind that we are treating with nonequilibrium situation. Proceeding in a general manner, for  $g_j$  sublevels with  $N_j^*$  particles, the number  $\Omega_j^*$  of unrepeatd sequences is

$$\Omega_j^* = \frac{G_j(G_j + N_j^* - 1)!}{G_j!N_j^*!} = \frac{(G_j + N_j^* - 1)!}{(G_j - 1)!N_j^*!}, \quad (30)$$

where  $G_j = \sum_{k=1}^{g_j} C_{g_j}^k$  is the number of possible sequences formed from  $g_j$ , and  $C_n^m = n!/(n - m)!m!$ . Taking as example the configuration given by Tab.2, where  $g_j = 1$ ,  $N_j = 2$ ,  $G_j = \sum_{k=1}^2 C_2^k$ , thus  $G_j = C_2^1 + C_2^2 = 3$ ; then

$$\Omega_j^* = \frac{(3 + 2 - 1)!}{(3 - 1)!2!} = 6, \quad (31)$$

which is the number of sequences given in (1) $a$ (2) $a$ (12); (1) $a$ (2)(12) $a$ ; (1)(2) $a$ (12) $a$ ; (1) $aa$ (2)(12); (1)(2) $aa$ (12); (1)(2)(12) $aa$ , corresponding to Tab.2. Therefore, the nonequilibrium thermodynamical probability  $\Omega_j^*$  for a given macrostate  $j$  is

$$\Omega_j^* = \frac{(G_j + N_j^* - 1)!}{(G_j - 1)!N_j^*!}. \quad (32)$$

Also, as  $G_j = \sum_{k=1}^{g_j} C_{g_j}^k = C_{g_j}^1 + \sum_{k=2}^{g_j} C_{g_j}^k$ , and  $C_{g_j}^1 = g_j$ , letting  $L_{gj} = \sum_{k=2}^{g_j} C_{g_j}^k$ , then Eq.(32) can be rewritten and used to compose the thermodynamical probability  $\Omega^* = \prod_j \Omega_j^*$ :

$$\Omega^* = \prod_j \frac{(g_j + L_{gj} + N_j^* - 1)!}{(g_j + L_{gj} - 1)!N_j^*!}. \quad (33)$$

From Eq.(33) we can see that the only changing in the thermodynamical probability is the appearance of the factor  $L_{gj}$  modifying the degeneracy  $g_j$ , the number of macrostates  $\Omega_j^*$ , and also the thermodynamical probability and, consequently, the entropy  $S$  of the system, as should be, since  $S = k \ln \Omega^*$ . Remarkably, note the similarity between Eq.(33) and Eq.(4), provided that we identify  $d_j = g_j + L_{gj}$ .

We can easily verify that Eq.(32) gives rise to a BE-like statistics, with  $g_j$  replaced by  $G_j$ . That this is so can be checked proceeding by analogy with the equilibrium situation, as indicated previously (1): First, we take the  $\ln$  from both sides of Eq.(32). Second, we use the Stirling formula for factorials. Third, we differentiate with respect to  $N_j^*$  and use  $\partial \ln \Omega_j^* / \partial N_j^* = \epsilon_j^*$ , where  $\epsilon_j^*$  generalizes  $\epsilon_j = \beta E_j$ ,  $\beta = 1/kT$ , finding

$$N_j^* / G_j = \frac{1}{\exp(\epsilon_j^*) - 1}. \quad (34)$$

Therefore, we see that the Bose-Einstein statistics is corrected, since the equality  $G_j = g_j$  and  $\epsilon_j^* = \epsilon_j$  will be valid only when the complete equilibrium is reestablished.

Since we are treating with nonequilibrium systems, a fundamental point to be addressed is, then, what is the meaning of temperature here, and how temperature enters in these formulas. To apply these formulas, we can think that the system is brought into contact with a reservoir at a well defined temperature  $T$ . As will be discussed later, the reservoir, even at idealized zero temperature, will drive the system to a complete mixture of states at the end of the so-called decoherence time  $\tau_D$  (19). As a consequence, in a time scale shorter than that for occurring the thermalization between the system and the reservoir, the coherence of the system will be lost, implying its incapacity to access, coherently, every possible state, thus recovering the usual BE distribution. Although the temperature of the reservoir is well defined for all times, the temperature for the system is not. However, as I stressed before, we can think of taking the system only slightly out of equilibrium, such that we can correct its temperature using an expansion up to few orders in  $\epsilon_j^*$  and requiring that  $\epsilon_j^* \rightarrow \epsilon_j = \beta E_j$  when the equilibrium is restated.

Thus, by expanding  $\epsilon_j^*$  in power series of  $\epsilon_j$ , which formally can be written as

$$\epsilon_j^* = \epsilon_0^* + \left( \frac{\partial \epsilon_j^*}{\partial \epsilon_j} \right)_{\epsilon_0} \epsilon_j + \frac{1}{2!} \left( \frac{\partial^2 \epsilon_j^*}{\partial \epsilon_j^2} \right)_{\epsilon_0} \epsilon_j^2 + \frac{1}{3!} \left( \frac{\partial^3 \epsilon_j^*}{\partial \epsilon_j^3} \right)_{\epsilon_0} \epsilon_j^3 \dots, \quad (35)$$

and requiring that  $\epsilon_j^* \rightarrow \epsilon_j = \beta E_j$  when the equilibrium is restated, gives  $\epsilon^*(0) = \epsilon_0^* = 0$  and  $\left( \frac{\partial \epsilon_j^*}{\partial \epsilon_j} \right)_{\epsilon_{0j}} = 1$ , such that the first order correction to the Bose-Einstein distribution can be explicitly written as

$$N_j^*/G_j = \frac{1}{\exp \left[ \beta E + \alpha_1 (\beta E)^2 \right] - 1}, \quad (36)$$

where I have truncated the power series up to second order and put  $\frac{1}{2!} \left( \frac{\partial^2 \epsilon_j^*}{\partial \epsilon_j^2} \right)_{\epsilon_j=0} = \alpha_1$ . Note

that, from this approach, the net effect stemming from the nonequilibrium on a given system is the increasing in the degeneracy, which in turn increases the available states given by  $\Omega$ . Also note that for systems only slightly out of the equilibrium, the energy emitted should be slightly different from that corresponding to the system in equilibrium. I call attention to the fact that some experiments, see Ref.(16), seem to point for the importance of the BE-like statistics given by Eq.(36), which modifies the Boltzmann factor.

I now briefly discuss a point deserving further investigation, which regards to Fermi-Dirac statistics ( $p = 1$ ) for particles out of equilibrium. Take for example Table III, which shows the total of  $\Omega = 6$  possibilities of occupation for  $N = 2$  fermions having  $M = 3$  available states. The situation now is more complicated than that for bosons, since, for example, when the first and second state is being coherently accessed by one fermion, indicated by (12) in Table III, the other fermions cannot populate states labeled using either number 1 or 2; in this case, as shown in Tab. III, when one fermion is in (12) state, the other one only can exist in state (3). It remains a challenge to find the corresponding thermodynamical probability  $\Omega$  for  $g_j$  states and  $n_j$  fermions, and thus, the mean occupation number for fermions out of equilibrium, as was done in Eq.(36), for bosons.

| (1) | (2) | (3) | (12) | (13) | (23) | (123) |
|-----|-----|-----|------|------|------|-------|
| ○   | ○   |     |      |      |      |       |
| ○   |     | ○   |      |      |      |       |
| ○   |     |     |      |      | ○    |       |
|     | ○   | ○   |      |      |      |       |
|     | ○   |     |      | ○    |      |       |
|     |     | ○   | ○    |      |      |       |

Table 3. Configurations of  $M = 3$  accessible states for  $N = 2$  indistinguishable fermions. (1) denotes the first available state, (2) and (3) denote the second and third available state, respectively. (12) represents the first and second state being coherently populated, (123) represents the three available states being coherently populated, and so on.

3.2 Coherent access for fermions

Because the plain difficulty in obtaining  $\Omega$  for fermions out of equilibrium, as pointed above, another route for this purpose must be in order. Thus, let us focus our attention on the partition function, which, by definition, is defined as a sum in all microstates ( $ms$ ):

$$Z = \sum_{ms} \exp(-\beta E), \tag{37}$$

where  $\beta = 1/kT$ , and  $E$  is the energy of the system, which can also be written using the number of particles  $n_i$  in the state of  $i$ -th energy  $\epsilon(i)$  of the system as  $E = \sum_i n_i \epsilon(i)$ . In this

case, the total number of particles is simply  $N = \sum_i n_i$ .

For an out of equilibrium system consisting of fermions, I introduce the coherent access hypothesis to several states, which consists in maintaining the same form as that of Eq.(37), but replacing  $\sum_i n_i \epsilon(i)$  by  $\sum_{ij...} n_{ij...} \epsilon(i, j, ...)$ , which, for fermions, implies  $i \neq j \neq ...$ , and  $n_{kl}$  is either zero or one and must be interpreted as being the fermion coherently accessing the energy levels  $\epsilon(k)$  and  $\epsilon(l)$ . For example, as discussed in the beginning of this Section and represented in Table III,  $\epsilon(k, l)$  represents the coherent access related to the energy levels  $k$  and  $l$ , and  $\epsilon(1, 2)$  represents, for example, the states (1) and (2) being coherently populated by a single fermion. On the other hand, if we are taking into account bosons, there is no constraint on  $\sum_{ij...} n_{ij...} \epsilon(i, j, ...)$ , and of course  $n_{kl} = 0, 1, 2, ...$  is the number of bosons coherently accessing the energy levels  $\epsilon(k)$  and  $\epsilon(l)$ .

For what follows, I am assuming that the partition function preserves its form given by Eq.(37) even at the nonequilibrium situation. To demonstrate this, I make use of this following postulate, which is valid for equilibrium situation: that two systems, in contact with a third one, as for example a reservoir at temperature  $T$ , act independently of each other while both the systems exchange energy with the reservoir. Once this postulated is maintained, it is straightforward to proceed the demonstration (1), and, for sake of completeness, let us briefly outline the steps leading to this result. To this end, consider a system composed by two subsystems  $A$  and  $B$ . The probability for this composed system to be in the energy state  $E_{A+B}^*$  is  $P_{A+B}(E_{A+B}^*)$ , where the superscript (\*) is to remind us that the system is out of equilibrium. If, as usual, the interaction energy can be neglected, thus the energy of the composed system is  $E_{A+B}^* = E_A^* + E_B^*$ , and

$$P_{A+B}(E_{A+B}^*) = P_A(E_A^*) + P_B(E_B^*) \tag{38}$$

is the probability for the composed system to be in a particular state such that the subsystem  $A$  has an energy  $E_A^*$ , and, at the same time, the subsystem  $B$  has an energy  $E_B^*$ . Now, suppose that these two subsystems is put in contact with a third system, for example, a reservoir at temperature  $T$ . While persisting the nonequilibrium situation (and even after that, indeed), the two subsystems  $A$  and  $B$  act independently of each other, with both subsystems eventually exchanging energy with the reservoir. Besides, the energy exchanged with the reservoir by a given subsystem does not influence the energy that the other subsystem can exchange with this same reservoir. This assumption, valid for two systems in equilibrium with a reservoir, is here assumed to be valid also when the equilibrium was not reached. Therefore, as I am assuming these events as independent, it can be written

$$P(E_{A+B}^*) = P(E_A^*)P(E_B^*). \quad (39)$$

Differentiating Eq.(39) with respect to  $E_A^*$  and  $E_B^*$  and equating this result we obtain ( $dP/dE^* = P'$ )

$$P'_A(E_A^*)P_B(E_B^*) = P_A(E_A^*)P'_B(E_B^*). \quad (40)$$

Next, separating the variables and equating the result to a constant, we have

$$\frac{P'_A(E_A^*)}{P_A(E_A^*)} = \frac{P'_B(E_B^*)}{P_B(E_B^*)} = -\beta^* \quad (41)$$

where  $\beta^*$  is a constant independent from either  $E_A^*$  or  $E_B^*$ . Of course, in the equilibrium situation we must have  $\beta^* \rightarrow \beta = 1/kT$ . From Eq.(41) follows, therefore, our desired result

$$P(E^*) = \frac{\exp(-\beta^* E^*)}{Z^*}, \quad (42)$$

where the partition function for the nonequilibrium situation is  $Z^* = \sum_{ms} \exp(-\beta^* E^*)$  and the index were dropped given the validity of Eq.(42) for the two subsystems.

Thus, according to Eq.(42), if  $P(\epsilon_j^* = \beta^* E_j^*)$  is the probability for a given system out of the equilibrium is in a particular microstate whose configuration is described by  $\epsilon_j^* = \beta^* E_j^*$ , then

$$P(\epsilon_j^*) = \frac{\exp(-\epsilon_j^*)}{Z^*}. \quad (43)$$

Now, using Eq.(35) and requiring that  $\epsilon_j^* \rightarrow \beta E_j$  when the equilibrium is restated, Eq.(43) can now be written as

$$P(\epsilon_j^*) = \frac{1}{Z^*} \exp \left[ -\beta E_j - \alpha_1 (\beta E_j)^2 - \alpha_2 (\beta E_j)^3 + \alpha_3 (\beta E_j)^4 \dots \right], \quad (44)$$

where the other constants were renamed for convenience as  $\frac{1}{n!} \frac{\partial^n \epsilon_j^*}{\partial \epsilon_j^n} = \alpha_{n-1}$ . Such a state of affairs giving origin to an infinite number of free parameters was studied in Refs. (14; 15) in a different context. I will turn to this point in the next Section. Note that for systems only slightly out of the equilibrium this last equation can be written as

$$P(\epsilon^*) = \frac{1}{Z^*} \exp \left[ -\beta E - \alpha_1 (\beta E)^2 \right], \quad (45)$$

where I have dropped out the index  $i$ . As stressed before, some experiments seem to point for the importance of this last term, which modifies the Boltzmann factor (16). Eq.(45) applies as well as for bosons and fermions just eventually changing the parameter  $\alpha_1$  accounting for the difference in the occupation number for this two particles. It is to be noted that, in order to obtain an equation for the nonequilibrium fermions similar to that of Eq.(36), we must find the nonequilibrium partition function  $Z^*$  considering the fermionic nature of the particles, with the occupation number in Table Ib being replaced by the occupation number for out of equilibrium fermions as indicated in Table III. Unfortunately, this is also not an easy task, demanding additional efforts. Notwithstanding, Eq.(45) shows several applications, and exploring its experimental consequences would be a promising road to follow. Besides, following another route yet using Eq.(45), it is possible to find an occupation number for nonequilibrium fermions similar to Eq.(36), as desired. To this end, consider the following reasoning: since the equilibrium is slightly disturbed, it is reasonable to assume that Eq.(45), instead of Eq.(1), must be used to calculate the transition rates. Consider then a two state system of distinguishable particles. Let  $E_a$  and  $E_b$  the energy of state  $a$  and  $b$ , respectively. The MB-like distribution, Eq.(45), implies that

$$\frac{n_a^*}{n_b^*} = \frac{P_{MB}(\epsilon_a^*)}{P_{MB}(\epsilon_b^*)} = \exp \left\{ -\beta(E_a - E_b) - \alpha_1 \left[ (\beta E_a)^2 - (\beta E_b)^2 \right] \right\}, \quad (46)$$

where  $n_b^*$  and  $n_a^*$  are the number of distinguishable particles in state  $b$  and  $a$ , respectively. Eq.(46) thus corrects the usual rate of population of the two states. Now, if the equilibrium is only slightly disturbed, we can assume that the transition rate from  $a$  to  $b$  is nearly the transition rate from  $b$  to  $a$ , i.e.,

$$n_a^* P_{MB}(\epsilon_{a \rightarrow b}^*) \approx n_b^* P_{MB}(\epsilon_{b \rightarrow a}^*), \quad (47)$$

where I have indicated the single particle transition rate from state  $a$  to  $b$  as  $P_{MB}(\epsilon_{a \rightarrow b}^*)$ . By applying the same reasoning for two identical fermions as it was done previously - see Eq.(16)-(23), we can write to the rate  $P_{FD}(\epsilon_{a \rightarrow b}^*)$  for one particle to make a transition from one state ( $a$ ) to another ( $b$ ) and *vice-versa* as

$$P_{FD}(\epsilon_{a \rightarrow b}^*) = (1 - n_b^*) P_{MB}(\epsilon_{a \rightarrow b}^*), \quad (48)$$

$$P_{FD}(\epsilon_{b \rightarrow a}^*) = (1 - n_a^*) P_{MB}(\epsilon_{b \rightarrow a}^*), \quad (49)$$

where  $n_b^*$  and  $n_a^*$  is to be reminded as the nonequilibrium number of fermions which are either zero or one. By the same told, I am assuming that similar to Eq.(47) we can write

$$n_a^* P_{FD}(\epsilon_{a \rightarrow b}^*) \approx n_b^* P_{FD}(\epsilon_{b \rightarrow a}^*). \quad (50)$$

From Eqs(47)-(50), we find

$$\frac{n_a^*}{(1 - n_a^*)} \exp \left[ -\beta E_a - \alpha_1 (\beta E_a)^2 \right] \approx \frac{n_b^*}{(1 - n_b^*)} \frac{\exp \left[ -\beta E_b - \alpha_1 (\beta E_b)^2 \right]}{,}. \quad (51)$$

If we now put  $\frac{n_b^*}{(1-n_b^*)} \frac{\exp[-\beta E_b - \alpha_1 (\beta E_b)^2]}{F(\beta)} = F(\beta)$  and solve to  $n_a^*$  we find, after changing the subindex  $a$  to  $FD$ ,

$$n_{FD}^* \approx \frac{1}{F(\beta) \exp[\beta E + \alpha_1 (\beta E)^2] + 1}, \quad (52)$$

which is similar to Eq.(36), for bosons out of equilibrium. In fact, it is straightforward to rederive Eq.(36) by this method, provided we assume  $n_a^* P_{BE}(\epsilon_{a \rightarrow b}^*) \approx n_b^* P_{BE}(\epsilon_{b \rightarrow a}^*)$  and use, now,  $P_{FD}(\epsilon_{a \rightarrow b}^*) = (1 + n_b^*) P_{MB}(\epsilon_{a \rightarrow b}^*)$  and  $P_{FD}(\epsilon_{b \rightarrow a}^*) = (1 + n_a^*) P_{MB}(\epsilon_{b \rightarrow a}^*)$ , instead of Eqs.(48)-(49).

For a simple, yet illuminating application of Eq.(45), consider  $N$  two-level systems at equilibrium at temperature  $T_1$  with energies  $E_1 = \epsilon$  and  $E_2 = -\epsilon$ . Then, when this system is brought to a slightly different temperature  $T_2$ , we expect that Eq.(45) describes the route to thermalization. In view of Eq.(45) we can write

$$Z^* = \sum_{n=-1}^1 \exp[-\beta E_n - \alpha_1 (\beta E_n)^2] \quad (53)$$

$$= 2 \exp[-\alpha_1 (\beta \epsilon)^2] \cosh(\beta \epsilon), \quad (54)$$

such that the  $N$ -particle partition function is  $\mathcal{Z}^* = Z^{*N}$ :

$$\mathcal{Z}^* = 2^N \exp[-N \alpha_1 (\beta \epsilon)^2] \cosh^N(\beta \epsilon), \quad (55)$$

from which all the relevant experimental quantities can be deduced. As for example, the internal energy is

$$\begin{aligned} U^* &= -\frac{\partial \ln \mathcal{Z}^*}{\partial \beta} \\ &= -N \epsilon \tanh(\beta \epsilon) + 2N \alpha_1 \beta \epsilon^2, \end{aligned} \quad (56)$$

and the specific heat

$$\begin{aligned} C &= \frac{\partial U^*}{\partial T} \\ &= Nk(\beta \epsilon)^2 \cosh^{-2}(\beta \epsilon) - 2Nk\alpha_1 (\beta \epsilon)^2, \end{aligned} \quad (57)$$

where it is to remind that  $\alpha_1 (\beta \epsilon)^2 \ll 1$ , the correction  $2Nk\alpha_1 (\beta \epsilon)^2$  being very small.

#### 4. Connection with entropic forms

As discussed in Section II, since the thermodynamical probability  $\Omega$  was modified to  $\Omega^*$  to take into account the coherent access, a natural question emerging is what is the best entropic form related to  $\Omega^*$ . I note that, depending on the choice, we will face with different implications. Once there is a plenty of entropic forms at our disposal, let us take as examples two of them: the Boltzmann-Gibbs ( $S_{BG}$ ) and the Tsallis ( $S_q$ ) entropies. As is well known, while the first entropy is extensive, *i.e.*  $S_{BG}(A+B) = S_{BG}(A) + S_{BG}(B)$ , the second one in general is not, *i.e.*,  $S_q(A+B) \neq S_q(A) + S_q(B)$  if  $q \neq 1$ .



Let us begin by the Boltzmann-Gibbs entropy, assuming for now that the only effect of the nonequilibrium is to increase the degeneracy of the system, as seen in Section II. We shall see that is possible to relate Eq.(44), and thus Eq.(45), either with an extensive or nonextensive entropic form (15). To see how this can be accomplished, I again proceed in analogy with what was done previously, taking advantage of Lagrange multipliers. Thus, given the density operator  $\rho$  of the system and the Boltzmann constant  $k$ , we want to maximize the Boltzmann-Gibbs entropy

$$S_{BG} = -k \text{tr} \rho \ln \rho, \quad (58)$$

where  $\text{tr}$  stands for the trace operation, subjected to the constraints given by the moments

$$\langle (\Delta E)^n \rangle = \text{tr} \rho H^n, \quad (59)$$

where  $n = 0, 1, 2, 3, \dots$ . Note that for  $n = 0$  the constraint is just the normalization condition  $\text{tr} \rho = 1$ , while for  $n = 1$  the constraint becomes the mean energy  $\langle (\Delta E)^n \rangle = \text{tr} \rho H$ , which are, in general, the two constraints used in the maximization procedures of the entropy. We thus multiply each constraint by the undetermined Lagrange multiplier  $\beta_n$ , adding the result to Eq.(58). After varying  $\rho$ , we will obtain

$$\text{tr} \left( \mathbf{1} + \sum_{n=0}^{\infty} \beta_n H^n + \ln \rho \right) \delta \rho = 0. \quad (60)$$

Since all the variations are independent and  $\delta \rho$  is arbitrary, the extended (non-Maxwellian) distribution  $\ln \rho = -1 - \sum_{n=0}^{\infty} \beta_n H^n$  follows, or, equivalently,

$$\rho = Z^{-1} \exp \left( - \sum_{n=1}^{\infty} \beta_n H^n \right), \quad (61)$$

where the partition function is  $Z = \text{tr} \exp \left( - \sum_{n=1}^{\infty} \beta_n H^n \right)$ . In the energy representation where  $H |E\rangle = E |E\rangle$ , Eq.(61) reads

$$\begin{aligned} P(E) &= Z^{-1} \exp \left( - \sum_{n=1}^{\infty} \beta_n E^n \right) \\ &= Z^{-1} \exp \left( -\beta_1 E + \beta_2 E^2 + \beta_3 E^3 + \beta_4 E^4 \dots \right) \end{aligned} \quad (62)$$

where  $Z = \sum_E \exp \left( - \sum_{n=1}^{\infty} \beta_n E^n \right)$  and I have used  $P(E) = \text{tr} \rho |E\rangle \langle E|$ . The Lagrange multipliers  $\beta_k$  are formally obtained from  $\beta_k = -\frac{\partial \ln Z}{\partial E^k}$ , considering  $E^k = Y_k$  as independent variables. The equality between Eq.(62) and Eq.(44) is guaranteed, provided that  $\beta_1 = \beta$ ;  $\beta_n = \alpha_{n-1} \beta^n$ . Therefore, according to the present view, nonequilibrium systems remains extensive, although requiring a *posteriori* knowledge of the variance (second central moment), the coefficient of skewness (third central moment), the kurtosis (fourth central moment), and so on, thus giving rise to a virtually infinite number of free parameters. However, instead of using infinite parameters, we could just use a single one by redefining a new ensemble fully determined by this single parameter. An aesthetically appealing way to do so is to expand Eq.(44) in terms of



the Tsallis entropic index (13). To do this, consider the following expanded form of Eq.(44):

$$P(\epsilon_j^*) = \frac{1}{Z^*} \exp \left[ -\beta E_j - \frac{(1-q)}{2} (\beta E_j)^2 - \frac{(1-q)^2}{3} (\beta E_j)^3 + \dots \right], \quad (63)$$

where in general  $\alpha_n = \frac{(q-1)^{n-1}}{n}$ . This is equivalent to the statement that the old ensemble which depended of  $\beta, \{\alpha_n\}$  and  $E_j$  becomes now a function of only  $\beta, q$  and  $E_j$ . Eq.(63) can be rewritten as

$$P(\epsilon_j^*) = \frac{1}{Z^*} \exp \left\{ \frac{1}{1-q} \left[ (1-q) \beta E_j - \frac{(1-q)^2}{2} (\beta E_j)^2 - \frac{(1-q)^3}{3} (\beta E_j)^3 - \dots \right] \right\}, \quad (64)$$

where it is easily recognized the expanded form of the logarithm function  $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots$ ,  $x = (1-q)\beta E_j$ , such that Eq.(64) becomes

$$P(\epsilon_j^*) = \frac{1}{Z^*} \left[ 1 - (1-q) \beta E_j \right]^{\frac{1}{(1-q)}}, \quad (65)$$

which is the  $q$ -distribution stemming from the extremization of Tsallis entropy,

$$S_q = k \frac{1 - \sum_j p_j^q}{q-1}, \quad (66)$$

when considering a family of constraints determined by the  $q$ -expectation value of the energy

$$\langle E \rangle_q = \frac{\sum_j p_j^q E_j}{\sum_j p_j^q}, \quad (67)$$

besides the norm constraint  $\sum_j p_j = 1$ . Therefore, a complete formal agreement between Tsallis and Boltzmann-Gibbs entropies is possible. This formal equivalence gives rise to an important issue related to a possible pseudononextensivity of the entropy used to describe a given system, since this agreement suggest that nonextensivity can be removed by adding new constraints.

## 5. Coherent access and decoherence time

In Section II I mentioned that equilibrium destroys the possibility of the system to coherently access all the available states, thus preventing us to experimentally observe results diverging from BE and FD distributions - see Eqs.(36),(52) and (57). In this Section it is briefly discussed possible experiments to be done in order to measure deviation from the equilibrium distributions.

To understand how the coherent access capacity of the system is lost even before the reservoir to compel the system to the equilibrium, it will be instructive to analyze in some detail the decoherence time concept. As I mentioned before, there is a relevant time scale, much less

than that necessary to the system to acquire the thermal equilibrium, given by the decoherence time, which has been extensively studied (17–20). For sake of the clarity, I will briefly present the decoherence induced by the environment interpretation (19). In the so-called Caldeira-Leggett model (21), as proposed initially by Feynman and Vernon (22), a massive quantum system of interest and its environment are represented by a single Hamiltonian. The massive system is represented by a single quantum oscillator, while the environment is modeled as a thermal bath consisting of a set of  $N$  oscillators, the coupling occurring via position coordinates. The master equation for the density operator derived by Caldeira and Leggett from this model, in the position representation  $\rho(x, x', t)$  and in the high temperature limit, is

$$\dot{\rho} = -\frac{i}{\hbar} [H, \rho] - \gamma (x - x') \left( \frac{\partial \rho}{\partial x} - \frac{\partial \rho}{\partial x'} \right) - \frac{2m\gamma k_B T}{\hbar^2} (x - x')^2 \rho \quad (68)$$

where  $H$  is the Hamiltonian for the particle,  $\gamma$  is the relaxation rate,  $m$  is the mass of the system,  $k_B$  is the Boltzmann constant,  $T$  is the temperature of the heat bath and  $\Delta x = x - x'$  is the typical separation from two peaks of the density matrix  $\rho(x, x', t)$  in the phase state representation of a quantum superposition;  $x$  and  $x'$  standing for the diagonal and off-diagonal position of the peak, respectively. In the above equation, the first term corresponds to the unitary von Neumann evolution, while the second term is responsible for dissipation. The third term, having a classical counterpart related to Brownian motion, is the most important for our purposes, since it is responsible for eliminating the off-diagonal terms, thus destroying the quantum coherence, or in other words, the coherent access capacity of the system. For example, for a coherent superposition of two Gaussians, its evolution, according to Eq.(68), will initially present four peaks, two on the diagonal ( $x = x'$ ) and two off the diagonal ( $x \neq x'$ ). While dissipation is governed in a time scale given by the second term, decoherence is governed by the last term which, being proportional to the square of the difference between diagonal and off diagonal terms, it will be dominant for large  $\Delta x$ , eliminating the off-diagonal terms at the rate  $\dot{\rho} \cong -\tau_D \rho$ , which implies  $\rho \sim \exp(-t/\tau_D)$ , where  $\tau_D$  is the decoherence time

$$\tau_D \equiv \frac{\hbar^2}{2m\gamma k_B T (\Delta x)^2}. \quad (69)$$

Therefore, this model provides an indicative to the time scale for the coherent access to work and thus for the emergence of the Bose-Einstein and Fermi-Dirac statistics. Besides, it sheds light on the problem of nonexisting superpositions of macroscopic objects: as shown in Ref. (19), according to Eq.(69) it is enough for 1g of matter at room temperature  $T = 300$  K and separation of 1cm to the decoherence be destroyed in  $\tau_D \sim 10^{-21}$ s, even considering relaxations times of the order of the age of the Universe. However, as I am arguing, superposition is the core of the coherent access hypothesis. I shall, therefore, concentrate the attention on systems composed of relatively few particles. For instance, Bose-Einstein condensates (BEC) for  $N$  atoms at very low temperature indeed is a good candidate to observe deviations from Bose-Einstein statistics in accordance with Eq.(36). To be specific, considering, for example, a relaxation rate  $\gamma \sim 1\text{s}^{-1}$  taken from the average time of condensates of  $N$  atoms of rubidium-87 at  $10^{-7}$ K (23), then assuming  $\Delta x \sim 1\mu\text{m}$ , which is much greater than the atom size, the decoherence time will be scaled as  $\tau_D \sim N^{-1}\text{s}$ . For typical  $N$ , around  $10^2 - 10^6$ , we see that the decoherence time is high enough to be experimentally measured,

thus allowing the system to coherently populate the accessible states. Therefore, we can expect that BEC slightly out of equilibrium would emit radiation whose statistics, differing from that of Bose-Einstein, should agree with the predictions given by Eq.(36).

## 6. Conclusions

Generalized statistics as well as nonequilibrium thermodynamics, being an open subject under current investigation, provides us with several techniques to study the statistical properties of systems. In this Chapter I study possible ways to introduce generalized statistics in both equilibrium and nonequilibrium situations. Also, I developed a new approach to study a nonequilibrium system. This new approach consists in extending the counting methods taken from combinatorial analyses to include the possibility for a system out of equilibrium to act as if it is accessing more than one state at once, a hypothesis I named by coherent access. As a consequence of the coherent access hypothesis, the process of counting how the particles access the available states of a physical system is modified. According to this hypothesis, the statistics resulting from the spectrum of emission of systems out of equilibrium is different from the Bose-Einstein statistics, being this difference in principle experimentally detectable. I then pointed out that coherent access is inhibited at the time scale of the decoherence time, and I explored situations found in Bose-Einstein condensates (BEC) which can be suited to demonstrate the validity of the coherent access hypothesis. Finally, I note that, although BEC systems were mentioned, the above reasoning must be valid for whichever systems having a sufficiently high decoherence time for preserving the coherent access.

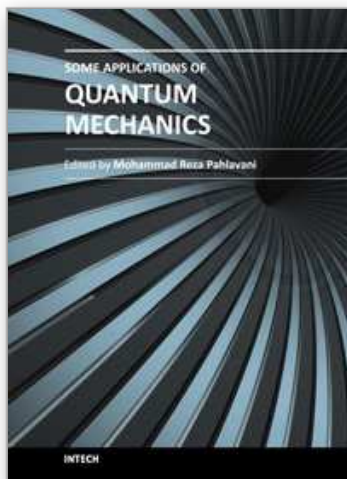
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Quantum mechanics, shortly after invention, obtained applications in different area of human knowledge. Perhaps, the most attractive feature of quantum mechanics is its applications in such diverse area as, astrophysics, nuclear physics, atomic and molecular spectroscopy, solid state physics and nanotechnology, crystallography, chemistry, biotechnology, information theory, electronic engineering... This book is the result of an international attempt written by invited authors from over the world to response daily growing needs in this area. We do not believe that this book can cover all area of application of quantum mechanics but wish to be a good reference for graduate students and researchers.

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