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Quantum Theory of Coherence
and Polarization of Light

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1. Introduction

When we open our eyes on earth for the first time, light generates the sensation of vision in our mind. If there was no light, our way of thinking would undoubtedly be quite different. Therefore, it is natural to ask the question, what is light? Our early inquisitiveness about light is documented in many stories and characters from almost all religions and cultures. Later, with the development of science, we tried to use more rigorous techniques to understand the nature of light. However, regardless of intense investigations for centuries on this subject, we are far from obtaining a convincing answer to the question "what is light?" Einstein seems to have put it best[1]:

"All the fifty years of conscious brooding have brought me no closer to answer the question, 'What are light quanta?' Of course today every rascal thinks he knows the answer, but he is deluding himself."

It is clear from the preceding remarks that this question represents one of the ever-lasting ones in physics. Although, at this stage one does not fully understand the nature of light, one can nevertheless study its properties. In this context, one may recall Feynman's illustration of the principle of scientific research (Feynman et al., 1985):

"...in trying to get some idea of what we're doing in trying to understand nature, is to imagine that the gods are playing some great game like chess, let's say, and you don't know the rules of the game, but you're allowed to look at the board, at least from time to time ..., and from those observations you try to figure out what the rules of the game are.

To understand the nature of light thus scientists have lead to investigate different properties of it. Coherence and polarization may be identified as the two important ones. In this chapter, we will give brief descriptions of them, and will show how the two apparently different phenomena can be described by analogous theoretical formulations, which involve incorporating the statistical fluctuations present in light.

Detailed descriptions of the history of the theories of coherence and polarization may be found in many scholarly articles [see, for example, (Born & Wolf, 1999; Brosseau, 2010; Mandel & Wolf, 1995). It may be said that the topic originated in Hooke's conjecture about the wave nature of light (Hooke, 1665), which was put in a sounder basis by Huygens (Huygens, 1690).

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[1] From Einstein's letter to Michael Besso, written in 1954.

Later Young discovered that light waves may produce interference fringes (Young, 1802). Young's discovery lead to many investigations concerning interference properties of light[2] and it turned out that light from different sources may differ in their abilities to interfere. Traditionally "coherence" of an optical field is understood as the ability of light to interfere. Early relevant investigations on interfering properties of light may be found in the works of Fresnel and Arago (Arago & Fresnel, 1819)[3]. An important contribution was made by Verdet (Verdet, 1865) when he determined the size of the region on the earth-surface, where "sunlight vibrations are in unison". Michelson established relationships between visibility of interference fringes and intensity distribution on the surface of an extended primary source (Michelson, 1890; 1891a;b; 1902; 1927). He also elucidated the connection which exists between the visibility of interference fringes and energy distribution in spectral lines. However, he did not interpret his results in terms of field correlations.

It has become customary in traditional optics to represent an optical field by a deterministic function. Although a deterministic model provides simple solutions to some problems, it often suffers from lack of self-consistency and leaves out many questions which can only be answered by taking the random nature of the field into account. The necessity of developing a statistical theory of light arises from the fact that all optical fields, whether found in nature or generated in a laboratory, have some random fluctuations associated with them. Even though these fluctuations are too rapid to be observed directly, their existence can be experienced by various experiments which involve effects of correlations among the fluctuating fields at a point, or at several points in space. In the quantum mechanical description, in addition to that, one also needs to consider the presence of a detector, which is an atom or a collection of atoms.

The first quantitative measure of the correlations of light vibrations was introduced by Laue (Laue, 1906; 1907). Later Berek used this concept of correlation in his work on image formation in microscopes (Berek, 1926a;b;c;d). A new era in this subject began when van Cittert determined the joint probability distribution for the light disturbances at any two points on a plane illuminated by an extended primary source (van Cittert, 1934) and also determined the probability distribution for light disturbances at one point, at two different instances of time (van Cittert, 1939).

A simpler and more profound approach for addressing such problems was developed by Zernike (Zernike, 1938), who also introduced the concept of the "degree of coherence" in terms of visibility of interference fringes, which is a measurable quantity. Although Zernike's work brought new light to this subject, it had some limitations, because it did not take into consideration the time-difference that may exist between interfering beams. Wolf formulated a more general theory of optical coherence by introducing more generalized correlation functions into the analysis (Wolf, 1955). Analyzing the propagation of such correlation functions is often quite complicated due to time-retardation factors, and in most of the cases it does not lead to any useful solution. Also, for the same reason, it is rather difficult in this formulation to analyze many practical problems. Such problems are treated much

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[2] For a detail discussion of the influence of Young's interference experiment on the development of coherence theory, see (Wolf, 2007a).

[3] Interpretation of the results obtained by Fresnel and Arago in terms of moderns coherence theory is given in (Mujat et al., 2004).

more conveniently by a different formulation, known as the space-frequency formulation of coherence theory (Wolf, 1981; 1982; 1986).

All the investigations mentioned above were carried out by the use of the scalar theory. To take into account another property of light, namely polarization, one needs to consider the vector nature of the fluctuating field. The credit of first notifying polarization properties of light is attributed to Erasmus Bartholinus, who studied the phenomenon of double refraction using calcite crystals. In 1672 Huygens (Huygens, 1690) provided an interpretation of the double refraction from the conception of spherical light waves. Use of double refraction became widely used in different fundamental and practical applications of optical sciences, which led to important investigations on this subject by several distinguished scientists. The foundation of the modern theory of polarization properties of light was laid down by Stokes (Stokes, 1852) as he formulated the theory of polarization in terms of certain parameters, now known as Stokes parameters [see also (Berry et al., 1977)]. Poincaré (Poincaré, 1892) provided a detailed mathematical treatment of the polarization properties of light, in which he introduced the concept of Poincaré sphere to specify any state of polarization of light. A matrix treatment of polarization was introduced by Wiener (Wiener, 1927; 1930; 1966), whose analysis related field correlations and polarization properties of light. Later, Wolf (Wolf, 1959) used a similar matrix formulation for systematic studies of polarization properties of statistically stationary light beams [see also, (Brosseau, 1995; Collett, 1993; Mandel & Wolf, 1995)]. A detailed description of the history of the theory of polarization can be found in a recently published article by Brosseau (Brosseau, 2010).

From the above discussion, it seems that coherence and polarization are two completely different phenomena with different histories and origins. However, according to classical theory, both of the can be interpreted as measures of correlations present between fluctuating electric field components. It is interesting to note that Verdet probably suspected, almost 140 years ago, that there is some analogy between the concepts of polarization and of coherence. The title of his paper (Verdet, 1865) states in loose translation: "Study of the nature of unpolarized and partially polarized light". Yet in spite of the stress on polarization, it is the very paper in which Verdet estimated the region of coherence of sunlight on the earth's surface[4]. This connection, became even more prominent in the works of Wiener and Wolf. However, for many years coherence and polarization have been considered as two independent branches of optics. The connection between them became evident with the introduction of a unified theory of coherence and polarization for stochastic electromagnetic beams (Wolf, 2003a;b).

All the investigations mentioned above are based on the classical theory of electromagnetic fields. In the early part of the last century, development of quantum mechanics began to provide deeper explanations of the intrinsic properties of light and its interaction with matter. Dirac's quantization of electromagnetic fields (Dirac, 1957) made it possible to analyze various properties of light by the use of quantum mechanical techniques. In 1963, Glauber introduced quantum mechanical formulation of coherence theory (Glauber, 1963), which has been followed by systematic investigations of the subject [see, for example, (Glauber, 2007; Mandel & Wolf, 1995)].

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[4] It was pointed out by Wolf in a conference talk (Wolf, 2009b), which was presented by the author.

This chapter intends to provide a quantum mechanical description of coherence and polarization properties of light with emphasis on some recently obtained results. We will begin with a brief discussion on the classical theories of coherence and polarization of light. We will emphasize how the two apparently different properties of light may be described by analogous theoretical techniques. We will stress the importance of describing the subject in terms of observable quantities. We will then recall some basic results of quantum theory of optical coherence in the space-time domain, followed by detailed description of coherence properties of optical fields in the space-frequency domain. After that, we will discuss a formulation of optical coherence in the space-frequency domain. We will also provide a quantum mechanical description of polarization properties of optical beams. In the end, we will point out an interesting observation on the connection of Bohr's complementarity principle with partial coherence, and with partial polarization.

2. Classical theory of stochastic fields

We already mentioned in the Introduction that a deterministic model of optical fields leads to many discrepancies in the analysis of properties of light. A systematic development of the theory of coherence and polarization properties of light requires to take account the random fluctuations present in the field. We will show how this theory may be used to elucidate coherence and polarization properties of light.

From the classic theory of Maxwell, it is well known that an optical field can be represented by an electric field $\mathbf{E}(\mathbf{r};t)$ and a magnetic field $\mathbf{B}(\mathbf{r};t)$, which obey the four Maxwell's equations [see, for example, (Jackson, 2004)]. For a stochastic (randomly fluctuating) three-dimensional optical field, each of them will be represented by three random components $E_i(\mathbf{r};t)$ and $B_i(\mathbf{r};t)$, $i = x, y, z$, where x, y, z are three arbitrary mutually orthogonal directions in space. In our discussion we will neglect the effects due to magnetic fields.

2.1 Optical coherence in the space-time domain

The word "coherence" refers to the ability of light to interfere. Coherence properties of light may, therefore, be understood by analyzing the interference fringes produced in an Young's interference experiment (Fig. 1). Many years ago, Zernike (Zernike, 1938) defined the "degree of coherence" of a wave field by the maximum value of visibility in the interference pattern produced by it "under the best circumstances"[5]. However, as already mentioned in the Introduction that Zernike did not take into account the time-delay between the fields arriving from different pinholes. Consequently, his theory could not address some interesting aspects of coherence, which was later generalized by Wolf (Wolf, 1955). We will now briefly discuss the main results of Wolf's theory in the space-time domain. For simplicity we begin with scalar description of fields, or in other words we assume that all the electric field components behave in the same way.

A randomly fluctuating generally complex optical scalar field[6], at a point $P(\mathbf{r})$, at a time t, may be represented by a statistical ensemble $\{V(\mathbf{r};t)\}$ of realizations. The second-order

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[5] By the term "best circumstances" Zernike meant that the intensities of the two interfering beams were equal and that only small path difference was introduced between them.

[6] The corresponding technical term is the "complex analytic signal" of a real electric field (Gabor, 1946).

cross-correlation function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; t_1, t_2)$ of the random fields at two different space-time points $(\mathbf{r}_1; t_1)$ and $(\mathbf{r}_2; t_2)$ is defined by the expression

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2; t_1, t_2) \equiv \langle V^*(\mathbf{r}_1; t_1) V(\mathbf{r}_2; t_2) \rangle, \tag{1}$$

where the asterisk denotes the complex conjugate and the angular brackets denote ensemble average. Now if the field is statistically stationary, at least in the wide sense (Mandel & Wolf, 1995; Wolf, 2007b), the expression on the right hand side of Eq. (1) then depends only on the time difference $\tau \equiv t_2 - t_1$. Therefore, for statistically stationary fields, the cross-correlation function takes the form

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) \equiv \langle V^*(\mathbf{r}_1; t) V(\mathbf{r}_2; t + \tau) \rangle. \tag{2}$$

The cross-correlation function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ is known as the *mutual coherence function*. It characterizes the second-order correlation properties of such fields in the space-time domain.

The average intensity $I(\mathbf{r})$ of light at a point $P(\mathbf{r})$, apart from a constant factor depending on the choice of units, is given by $\langle |V(\mathbf{r}; t)|^2 \rangle$. From Eq. (2) it follows that

$$I(\mathbf{r}) \equiv \langle |V(\mathbf{r}; t)|^2 \rangle = \Gamma(\mathbf{r}, \mathbf{r}; 0). \tag{3}$$

Evidently for statistically stationary light the average intensity *does not depend on time*.

Let us now consider a Young's two pinhole interference experiment (Fig. 1). Suppose that a light beam is incident from the half-space $z < 0$ onto an opaque screen A, placed in the plane $z = 0$ containing two pinholes $Q_1(\mathbf{r}_1)$ and $Q_2(\mathbf{r}_2)$. For the sake of simplicity, we assume that
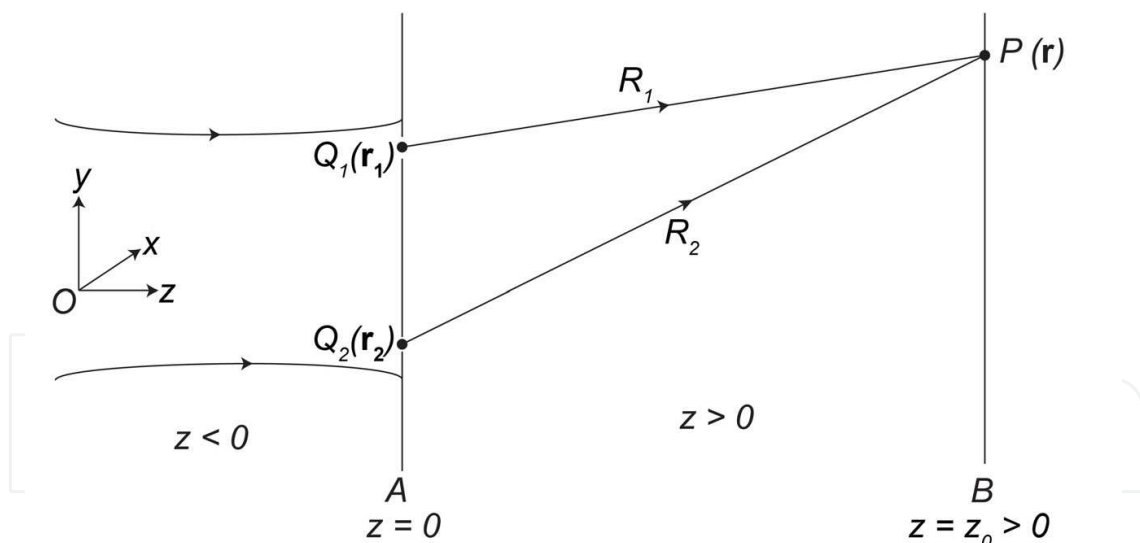

Fig. 1. Illustrating the notation relating to Young's interference experiment.

the beam is incident normally on the screen A. In general, interference fringes will be formed on a screen B, placed in a plane $z = z_0 > 0$, some distance behind the screen A (see Fig. 1). If we assume that the contributions to the total intensity from the two pinholes are equal to each other, i.e., that $I^{(1)}(\mathbf{r}) = I^{(2)}(\mathbf{r}) \equiv I^{(0)}(\mathbf{r})$ (say), then we find that

$$I(\mathbf{r}) = 2I^{(0)}(\mathbf{r}) \left\{ 1 + \left| \frac{\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)}{\sqrt{I(\mathbf{r}_1)} \sqrt{I(\mathbf{r}_2)}} \right| \cos \left[\alpha(\mathbf{r}_1, \mathbf{r}_2; \tau) \right] \right\}, \tag{4}$$

where

$$\tau \equiv t_2 - t_1 = \frac{R_2 - R_1}{c} \tag{5}$$

is the time delay between the fields arriving at $P(\mathbf{r})$ from pinholes $Q(\mathbf{r}_1)$ and $Q(\mathbf{r}_2)$, c being the speed of light in free space, and $\alpha(\mathbf{r}_1, \mathbf{r}_2; \tau) = \arg\{\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)\}$. The visibility $\mathcal{V}$ of the fringes is defined by the famous formula due to Michelson, viz.,

$$\mathcal{V} \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}, \qquad 0 \le \mathcal{V} \le 1. \tag{6}$$

One can readily show from Eq. (4) and (6) that the visibility of the fringes at the point $P(\mathbf{r})$ is given by ((Wolf, 2007b), Sec. 3.1, Eq. (19))

$$\mathcal{V} = \left| \frac{\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)}{\sqrt{I(\mathbf{r}_1)}\sqrt{I(\mathbf{r}_2)}} \right|. \tag{7}$$

The normalized cross-correlation function in this expression is defined as the *degree of coherence* $\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$, i.e.,

$$\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) \equiv \frac{\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)}{\sqrt{I(\mathbf{r}_1)}\sqrt{I(\mathbf{r}_2)}}, \tag{8}$$

Since the visibility is always bounded by zero and by unity, so is the modulus of degree of coherence. It can also be proved explicitly by use of the Cauchy-Schwarz inequality that

$$0 \le |\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)| \le 1. \tag{9}$$

When $|\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)| = 1$, sharpest possible fringes are obtained and the field is said to be *completely coherent*, for the time delay τ, at the pair of points $Q(\mathbf{r}_1)$ and $Q(\mathbf{r}_2)$. In the other extreme case, when $|\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)| = 0$, no fringe is obtained and the field is said to be *incoherent*, for the time delay τ, at the two points. In the intermediate case $0 < |\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)| < 1$, the field is said to be *partially coherent*. It is to be noted that the degree of coherence is, in general, a complex quantity. Its phase is also a meaningful physical quantity and can be determined from measurements of positions of maximum and minimum in the fringe pattern [see, for example, (Mandel & Wolf, 1995), p-167]. It must be noted that the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ obeys certain propagation laws which make it possible to determine changes in correlation properties of light on propagation. These propagation laws are often called Wolf's equations (see, for example, (Wolf, 2007b), Sec. 3.5).

The theory can be immediately generalized to vector fields. If we restrict ourselves to a stationary stochastic light beam propagating along positive z direction, then the coherence properties can be described by a 2×2 matrix $\overleftrightarrow{\Gamma}(\mathbf{r}_1, \mathbf{r}_2; \tau)$, which is defined by the formula

$$\overleftrightarrow{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, \tau) \equiv \left[\Gamma_{ij}(\mathbf{r}_1, \mathbf{r}_2; \tau)\right] \equiv \left[\langle E_i^*(\mathbf{r}_1; t) E_j(\mathbf{r}_2; t + \tau)\rangle\right], \quad (i = x, y; \; j = x, y), \tag{10}$$

where E_i is a component of the electric field vector. In this case, the average intensity at a point $P(\mathbf{r})$ is given by

$$I(\mathbf{r}) \equiv \mathrm{Tr} \, \overleftrightarrow{\Gamma}(\mathbf{r}, \mathbf{r}; 0), \tag{11}$$

where Tr represents trace of a matrix. The degree of coherence can be shown to be given by the formula (Karczewski, 1963)

$$\gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) \equiv \frac{\text{Tr} \overleftrightarrow{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, \tau)}{\sqrt{I(\mathbf{r}_1)I(\mathbf{r}_2)}}. \tag{12}$$

2.2 Optical coherence in the space-frequency domain

The space-time formulation of the coherence theory, discussed in the previous section, is quite natural, intuitive and often useful. However, as mentioned in the Introduction, there are some problems in statistical optics which turn out to be almost impossible to solve by the use of this formulation. For example, attempts to solve problems involving change in coherence properties of light on propagation through various media, or the problems involving scattering of partially coherent light, presents considerable difficulties in this formulation. These types of problems can be much more conveniently addressed by the use of a somewhat different formulation, known as the *space-frequency formulation* of coherence theory (Wolf, 1981; 1982; 1986). This space-frequency formulation has led to discoveries and understanding of some new physical phenomena, such as correlation-induced spectral changes (Wolf, 1987) and changes in polarization properties of light on propagation (James, 1994). Recent studies have also revealed a great usefulness of this theory in connection with determining the structure of objects by inverse scattering technique [see, for example, Refs. (Lahiri et al., 2009; Wolf, 2009a; 2010a; 2011)]. In this section we will briefly present some basic results in the theory of optical coherence in the space-frequency domain for scalar fields which are statistically stationary, at least in the wide sense.

A stationary random function $V(\mathbf{r}; t)$ is not square integrable and, consequently, its Fourier transform does not exist. However, for most statistically stationary optical fields, it is reasonable to assume that the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ exists and is a square integrable function of τ. One can then define a function $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$ which together with $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ form a Fourier transform pair, i.e.,

$$W(\mathbf{r}_1, \mathbf{r}_2; \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) e^{i\omega\tau} \, d\tau, \tag{13a}$$

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \int_0^{\infty} W(\mathbf{r}_1, \mathbf{r}_2; \omega) e^{-i\omega\tau} \, d\omega, \tag{13b}$$

where ω denotes the frequency. The quantity $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$ is called the cross-spectral density function (to be abbreviated by CSDF) of the field. It can be shown that CSDF is also a correlation function, i.e., that it can be represented in the form

$$W(\mathbf{r}_1, \mathbf{r}_2; \omega) = \langle U^*(\mathbf{r}_1; \omega) U(\mathbf{r}_2; \omega) \rangle_\omega, \tag{14}$$

where $U(\mathbf{r}; \omega)$ is a typical member of a suitably constructed ensemble of monochromatic realizations[7], all of frequency ω ((Wolf, 2007b), Sec. 4.1). In the special case, when the two points $\mathbf{r}_1$ and $\mathbf{r}_2$ coincide, it follows from generalized Wiener-Khinchin theorem (see, for example, (Mandel & Wolf, 1995), sec. 2.4.4) that the CSDF represents the spectral density

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[7] It is important to note that $U(\mathbf{r}; \omega)$ is not the Fourier transform of the field $V(\mathbf{r}; t)$.

$S(\mathbf{r}, \omega)$ of the field, i.e., that

$$S(\mathbf{r}, \omega) = W(\mathbf{r}, \mathbf{r}; \omega). \tag{15}$$

The spectral density $S(\mathbf{r}, \omega)$ is a physically meaningful quantity which represents the average intensity at a particular frequency.

The CSDF is the key quantity in the second-order coherence theory in space-frequency domain. As Eq. (15) shows, one can obtain the spectral density directly from it by taking the two spatial arguments to be equal. The spectral coherence properties of scalar fields are also completely described by the CSDF. To see that let us consider, once again, an Young's interference experiment (Fig. 1), but now we consider the fringe pattern produced by each frequency component present in the spectrum of the light. This situation can be realized by imagining that the incident light is filtered around the frequency ω before reaching the pinholes. The distribution of the spectral density $S(\mathbf{r}; \omega)$ on the screen B is given by the expression [(Wolf, 2007b), Sec. 4.2]

$$S(\mathbf{r}; \omega) = S^{(1)}(\mathbf{r}; \omega) \left\{ 1 + |\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)| \cos \left[\beta(\mathbf{r}_1, \mathbf{r}_2; \omega) - \delta \right] \right\}. \tag{16}$$

Here $S^{(1)}(\mathbf{r}; \omega)$ is the contribution of light reaching at $P(\mathbf{r})$ from either of the two pinholes, $\delta = \omega(R_2 - R_1)/c$, and $\beta(\mathbf{r}_1, \mathbf{r}_2; \omega)$ is the phase of the so-called *spectral degree of coherence* $\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)$ which is given by the expression ((Wolf, 2007b), Sec. 4.2)

$$\mu(\mathbf{r}_1, \mathbf{r}_2; \omega) \equiv \frac{W(\mathbf{r}_1, \mathbf{r}_2; \omega)}{\sqrt{S(\mathbf{r}_1, \omega)} \sqrt{S(\mathbf{r}_2, \omega)}}. \tag{17}$$

The formula (16) is known as the *spectral intensity law*. By analogy with the space-time formulation, one can readily show that $|\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)|$ is equal to the fringe visibility associated with the frequency component ω, in the experiment sketched out in Fig. 1. It should be noted that in this case, unlike in the case of the space-time formulation, the fringe visibility is constant over the screen B. It can be shown that [(Mandel & Wolf, 1995), Sec. 4.3.2]

$$0 \le |\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)| \le 1. \tag{18}$$

When $|\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)| = 1$, the field at the two points $Q_1(\mathbf{r}_1)$ and $Q_2(\mathbf{r}_2)$ is said to be *spectrally completely coherent* at the frequency ω. If $\mu(\mathbf{r}_1, \mathbf{r}_2; \omega) = 0$, the field is said to be *spectrally completely incoherent* at the two points, at that frequency. In the intermediate case, it is said to be spectrally partially coherent at frequency ω. Like the mutual coherence function, the cross-spectral density function also obey certain propagation laws [see, for example, (Mandel & Wolf, 1995), Sec. 4.4.1].

The theory can also be generalized to the vector fields. For an optical beam propagating along positive direction of z axis, one can define a 2×2 matrix, known as the cross-spectral density matrix (CSDM), which is the Fourier transform of mutual coherence matrix [(Wolf, 2007b), Chapter 9, Eqs. (1) and (2)]:

$$\overleftrightarrow{W}(\mathbf{r}_1, \mathbf{r}_2; \omega) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} \overleftrightarrow{\Gamma}(\mathbf{r}_1, \mathbf{r}_2; \tau) \exp[i\omega\tau] \, d\tau. \tag{19}$$

As was in the scalar case, it can be shown that each element of the CSDM is a correlation function [see, (Wolf, 2007b), Chapter 9]. In this case, the spectral density $S(\mathbf{r}, \omega)$ is given by

the expression [(Wolf, 2007b), Sec. 9.2, Eq. (2)]

$$S(\mathbf{r}, \omega) = \text{Tr} \overleftrightarrow{W}(\mathbf{r}, \mathbf{r}; \omega), \tag{20}$$

and the spectral degree of coherence is given by the expression [(Wolf, 2007b), Sec. 9.2, Eq. (8)]

$$\mu(\mathbf{r}_1, \mathbf{r}_2; \omega) = \frac{\text{Tr} \overleftrightarrow{W}(\mathbf{r}_1, \mathbf{r}_2; \omega)}{\sqrt{S(\mathbf{r}_1; \omega)} \sqrt{S(\mathbf{r}_2; \omega)}}. \tag{21}$$

The relationship between coherence properties of light in the space-time and in the space-frequency domains have been subject of interest [for details see, for example, (Friberg & Wolf, 1995; Lahiri & Wolf, 2010a;d; Wolf, 1983)].

2.3 Polarization properties of stochastic beams in the space-time domain

In this section, we will briefly discuss some basic results in the matrix theory of polarization of electromagnetic beams, following the work of Wolf. For detailed discussions on this topic see any standard textbook, for example, (Born & Wolf, 1999; Brosseau, 1995; Collett, 1993; Mandel & Wolf, 1995). Let us consider a statistically stationary light beam characterized by a randomly fluctuating electric field vector $\mathbf{E}(\mathbf{r}, t)$. Without any loss of generality, we assume that the beam propagation direction is along positive z axis. Therefore, $\mathbf{E}(\mathbf{r}, t)$ may be represented by the two mutually orthogonal random components $E_x(\mathbf{r}, t)$ and $E_y(\mathbf{r}, t)$. Suppose that these components are represented by the ensembles of realizations $\{E_x(\mathbf{r}, t)\}$ and $\{E_y(\mathbf{r}, t)\}$, respectively. One can construct a 2×2 correlation matrix, known as coherency matrix (Wiener, 1927), which is given by [see, for example, (Mandel & Wolf, 1995), Sec. 6.2, Eq. 6.2-6]

$$\overleftrightarrow{J}(\mathbf{r}) \equiv \overleftrightarrow{\Gamma}(\mathbf{r}, \mathbf{r}; 0) \equiv \left[\left\langle E_i^*(\mathbf{r}; t) E_j(\mathbf{r}, t) \right\rangle \right], \qquad i = x, y, \quad j = x, y. \tag{22}$$

The elements of this matrix are *equal-time* correlation functions; consequently for statistically stationary fields they are time independent. This matrix contains all information about polarization properties of a stationary stochastic light beam at a point. Each element of this matrix can be determined from a canonical experiment, which involves passing the beam thought a compensator plate and polarizer, and then measuring the intensities for different values of the polarizer angle and of different values time delays introduced by the compensator plate among the components of electric field (Born & Wolf, 1999). A similar experiment will be discussed in detail in section 7. The Stokes parameters can be expressed in terms of the elements of a coherency matrix [see, for example, (Born & Wolf, 1999), section 10.9.3].

2.3.1 Unpolarized light beam

It can be shown that if a light beam is unpolarized at a point $P(\mathbf{r})$, then at that point the coherency matrix is proportional to a unit matrix, i.e., it has the form ((Mandel & Wolf, 1995), Sec. 6.3.1)

$$\overleftrightarrow{J}^{(\text{u})}(\mathbf{r}) = \mathscr{A}(\mathbf{r}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{23}$$

It implies that in this case x and y components of the electric field are completely uncorrelated $[J_{xy}(\mathbf{r}) = J_{yx}(\mathbf{r}) = 0]$, and intensities of x and y components of the field are equal $[J_{xx}(\mathbf{r}) = J_{yy}(\mathbf{r}) \equiv \mathscr{A}(\mathbf{r})]$. It immediately follows that in this case the normalized correlation function

$$j_{xy}(\mathbf{r}) \equiv J_{xy}(\mathbf{r}) / \sqrt{J_{xx}(\mathbf{r}) J_{yy}(\mathbf{r})} = 0 \qquad (24)$$

2.3.2 Polarized light beam

In the other extreme case when $|j_{xy}(\mathbf{r})| = 1$, one can readily show that the elements of the coherency matrix factorize in the form

$$J_{ij}^{(\mathrm{p})}(\mathbf{r}) = \mathcal{E}_i^*(\mathbf{r})\mathcal{E}_j(\mathbf{r}), \quad (i = x, y; \; j = x, y), \qquad (25)$$

where $\mathcal{E}_i(\mathbf{r})$ is a time-independent deterministic function of position. This coherency matrix is identical with that of a monochromatic field which is given by the expression

$$\mathbf{E}(\mathbf{r}; t) = \mathcal{E}_i(\mathbf{r}) e^{-i\omega t}. \qquad (26)$$

In analogy with monochromatic beams, completely polarized beams are traditionally defined by the the coherency matrices which can be expressed in the form (25). However, a polarized light beam must not be confused with a monochromatic light beam. Recently, the distinction between the two has been clearly pointed out (Lahiri & Wolf, 2009; 2010a).

2.3.3 Partially polarized light beam

Any optical beam, which is neither unpolarized, nor polarized, is said to be partially polarized. Evidently, any coherency matrix which cannot be expressed in the form (23), or in the form (25) represents a partially polarized beam. However, it is remarkable that any such coherency matrix, can always be uniquely decomposed into the sum of two matrices representing a polarized beam and an unpolarized beam (Wolf, 1959) (see also, (Mandel & Wolf, 1995), Sec. 6.3.3), i.e., that

$$\overleftrightarrow{J}(\mathbf{r}) = \overleftrightarrow{J}^{(\mathrm{u})}(\mathbf{r}) + \overleftrightarrow{J}^{(\mathrm{p})}(\mathbf{r}). \qquad (27)$$

consequently, the average intensity $[I(\mathbf{r}) \equiv \mathrm{Tr}\,\overleftrightarrow{J}(\mathbf{r})]$ of any light beam, at a point, has contributions from a completely polarized and a completely unpolarized beam. The degree of polarization at a point $P(\mathbf{r})$ is defined as the ratio of the average intensity of the polarized part to the total average intensity at that point (Wolf, 1959). One can show that it is given by the expression (Wolf, 1959) (see also, Ref. (Born & Wolf, 1999), Sec. 10.9.2, Eq. (52))

$$P(\mathbf{r}) \equiv \frac{I^{(\mathrm{p})}(\mathbf{r})}{I(\mathbf{r})} = \sqrt{1 - \frac{4\,\mathrm{Det}\,\overleftrightarrow{J}(\mathbf{r})}{[\mathrm{Tr}\,\overleftrightarrow{J}(\mathbf{r})]^2}}, \qquad (28)$$

where Det denotes the determinant and Tr denotes the trace. One can show that the degree of polarization is always bounded between zero and unity:

$$0 \leq P(\mathbf{r}) \leq 1. \qquad (29)$$

If the degree of polarization is unity at a point **r**, then the beam is said to be *completely polarized* at that point and if it has zero value, the light is said to be *completely unpolarized* at that point.

2.4 Polarization properties of stochastic beams in the space-frequency domain

As was in the case of coherence, polarization properties of light can also be analyzed in the space-frequency domain. Such a theory has certain advantages, because it makes it easier to study the change in polarization properties of light on propagation and scattering. The theory involves analyzing polarization properties of light at each frequency present in its spectrum. It is similar to the theory in the space-time domain, except that the coherency matrix $\overleftrightarrow{J}(\mathbf{r})$ is replaced by equal-point CSDM $\overleftrightarrow{W}(\mathbf{r}, \mathbf{r}; \omega)$. In this case the spectral degree of polarization is given by the expression ((Wolf, 2007b), Sec. 9.2, Eq. (14))

$$\mathcal{P}(\mathbf{r}; \omega) \equiv \sqrt{1 - \frac{4 \operatorname{Det} \overleftrightarrow{W}(\mathbf{r}, \mathbf{r}; \omega)}{[\operatorname{Tr} \overleftrightarrow{W}(\mathbf{r}, \mathbf{r}; \omega)]^2}}. \tag{30}$$

The relationship between space-time and space-frequency description of polarization properties of light are being investigated recently [for details see, for example, (Lahiri, 2009; Lahiri & Wolf, 2010a; Setälä et al., 2009)].

2.5 Unified theory of coherence and polarization

For many years coherence and polarization properties of light have been considered as independent subjects. However, the matrix formulation, especially in the space-frequency domain, shows that they are intimately related. This fact was firmly established by the introduction of unified theory of coherence and polarization (Wolf, 2003b). According to the unified theory, both the coherence and polarization properties of a stochastic electromagnetic beam, can be described in terms of the 2×2 cross-spectral density matrix (CSDM) $\overleftrightarrow{W}(\mathbf{r}_1, \mathbf{r}_2; \omega)$. The spectral density, spectral degree of coherence, and the spectral degree of polarization are described by Eqs. (20), (21), and (30) respectively.

Elements of the CSDM obey definite propagation laws [see, for example, (Wolf, 2007b), Sec. 9.4.1, Eq. (3)]. If the CSDM is specified at all pairs of points at any cross-sectional plane of a beam, then it is possible to determine the CSDM at all pairs of points on any other cross-section of that beam, both in free space and in a medium. Therefore, it is possible to study the changes in spectral, coherence and polarization properties of a beam on propagation.

3. Optics in terms of observable quantities

At this point, we would like to emphasize that every optical phenomenon, which will be addressed in this chapter, will be illustrated and interpreted in terms of *observable quantities*. Formulating optical physics in terms of observable quantities is due to valuable work of Emil Wolf (Wolf, 1954). His effort was highly appreciated by Danis Gabor in a lecture on "Light and Information" (Gabor, 1955), as he mentioned "perhaps the most satisfactory feature of the theory is that it operates entirely with quantities which are in principle observable, in line with the valuable efforts of E. Wolf to rid optics of its metaphysical residues."

The analysis of coherence and polarization properties of light are based on the theory of electromagnetic fields. However, one must note that even today, one is not able to "directly" detect such a field at an optical frequency, or at a higher frequency. Any optical phenomenon that one observes in a laboratory, or in nature, is the result of generation of electric currents in detectors. Such currents originate from the so-called *destruction of photons* by light-matter interactions. An example of commonly available sophisticated optical detectors is a human eye. Another example of an optical detector is an EM-CCD camera, which is often used in today's laboratories. From the measurement of current in a detector one may predict the so-called photon detection rate, or intensity of light.

In section 2, we interpreted spatial coherence properties of light in terms of correlation between classical electric field at a pair of points. However, since electric field in an optical frequency is not observable, one must not be too much carried away with such an interpretation. It must be kept in mind that coherence is the ability of light to interfere and a physical measure of coherence is the visibility of fringes in an interference experiment, not a correlation function. On the other hand, polarization properties of light beams were interpreted as correlation between electric field components at a particular point. However, the physical phenomenon which leads to such a mathematical formulation is the modulation of intensity of the beam, as it is passed through polarization controlling devices, such as polarizers, compensator plates etc.

4. Quantum theory of optical fields

We begin by recalling some basic properties of quantized electromagnetic fields (Dirac, 1957). A quantized electric field may be represented by a Hermitian operator (Mandel & Wolf, 1995)

$$\widehat{\mathbf{E}}(\mathbf{r},t) = i \sum_{\mathbf{k}} \sum_{s} \left(\frac{1}{2}\hbar\omega \right)^{\frac{1}{2}} \left[\widehat{a}_{\mathbf{k},s} C_{\mathbf{k},s} \boldsymbol{\epsilon}_{\mathbf{k},s} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} - \widehat{a}^{\dagger}_{\mathbf{k},s} C^{*}_{\mathbf{k},s} \boldsymbol{\epsilon}^{*}_{\mathbf{k},s} e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \right], \qquad (31)$$

where the wave vectors $\mathbf{k}$ labels plane wave modes, $|\mathbf{k}| = k = \omega/c$, c is the speed of light in free space, $C_{\mathbf{k},s}$ is a constant, and $\boldsymbol{\epsilon}_{\mathbf{k},s}$ ($s = 1,2$), are mutually orthonormal base vectors, which obey the conditions[8]

$$\mathbf{k} \cdot \boldsymbol{\epsilon}_{\mathbf{k},s} = 0, \qquad \boldsymbol{\epsilon}^{*}_{\mathbf{k},s} \cdot \boldsymbol{\epsilon}_{\mathbf{k},s'} = \delta_{ss'}, \qquad \boldsymbol{\epsilon}_{\mathbf{k},1} \times \boldsymbol{\epsilon}_{\mathbf{k},2} = \mathbf{k}/k. \qquad (32)$$

In the expansion (31), $\widehat{a}_{\mathbf{k},s}$ and $\widehat{a}^{\dagger}_{\mathbf{k},s}$ are the photon annihilation and the photon creation operators respectively, for the mode labeled by $(\mathbf{k},s)$. These operators obey the well known commutation relations [see, for example, (Mandel & Wolf, 1995), Sec. 10.3]

$$\begin{aligned} \left[\widehat{a}_{\mathbf{k},s}, \widehat{a}^{\dagger}_{\mathbf{k}',s'} \right] &= \delta^{3}_{\mathbf{k}\mathbf{k}'} \delta_{ss'}, \\ \left[\widehat{a}_{\mathbf{k},s}, \widehat{a}_{\mathbf{k}',s'} \right] &= 0, \\ \left[\widehat{a}^{\dagger}_{\mathbf{k},s}, \widehat{a}^{\dagger}_{\mathbf{k}',s'} \right] &= 0, \end{aligned} \qquad (33)$$

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[8] The unit base vectors $\boldsymbol{\epsilon}_{k1}$, $\boldsymbol{\epsilon}_{k2}$ may be chosen to be complex for general expansion of the field into two orthogonal polarization components, for example, in connection with elliptic polarization.

where δ_{ij} is the Kronecker symbol. As evident from Eq. (31), the electric field operators consists of a positive frequency part[9]

$$\widehat{\mathbf{E}}^{(+)}(\mathbf{r}, t) = i \sum_{\mathbf{k}} \sum_{s} \left(\frac{1}{2} \hbar \omega \right)^{\frac{1}{2}} \widehat{a}_{\mathbf{k},s} C_{\mathbf{k},s} \boldsymbol{\epsilon}_{\mathbf{k},s} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}, \tag{34}$$

and a negative frequency part $\widehat{\mathbf{E}}^{(-)}(\mathbf{r}, t)$, which is the Hermitian adjoint of the positive frequency part. The expansion (31) represents the field in discrete modes. Such a representation is appropriate, for example, when treating an electric field in a cavity. In more general situations, a continuous mode representation may be more appropriate [for discussion on such representation see (Mandel & Wolf, 1995), Sec. 10.10].

The state of the field is described by a state vector $|i\rangle$ in the Fock space, or, more generally, by a density operator $\widehat{\rho} = \{\langle i| \, |i\rangle\}_{\text{average}}$, where the average is taken over an appropriate ensemble. The expectation value $\left\langle \widehat{O} \right\rangle$ of any operator $\widehat{O}$ is given by the well known expression

$$\left\langle \widehat{O} \right\rangle = \text{Tr} \left\{ \widehat{\rho} \, \widehat{O} \right\}, \tag{35}$$

where Tr denotes the trace. An informative description of how the measurable quantities may be interpreted in terms of quantized field and density operators, was given by Glauber (Glauber, 1963). In the following section, we will briefly go over some basic concepts from the Glauber's interpretation of quantum theory of optical coherence.

5. Summary of some basic results of the quantum theory of optical coherence in the space-time domain

In the quantum-mechanical interpretation, a photon can be detected only by destroying it. The photo-detector is assumed to be ideal in the sense that it is of negligible size and has a frequency-independent photo absorbtion probability. Let us now consider absorption (detection) of a photon by an ideal detector at a space-time point $(\mathbf{r}; t)$. Suppose that due to this absorption the field goes from the initial state $|i\rangle$ to a final state $|f\rangle$. The probability of the detector for absorbing a photon in final state $|f\rangle$ is $\left| \langle f| \, \widehat{\mathbf{E}}^{(+)}(\mathbf{r}; t) \, |i\rangle \right|^2$. The counting rate in the detector is obtained by summing over all the final states which can be reached from $|i\rangle$, by absorption of a photon. One can extend the summation over a complete set of final states, since the states which cannot be reached in this process will not contribute to the result [for details see (Glauber, 2007)]. The counting rate of the detector then becomes proportional to

$$\sum_{f} \left| \langle f| \, \widehat{\mathbf{E}}^{(+)}(\mathbf{r}; t) \, |i\rangle \right|^2 = \langle i| \, \widehat{\mathbf{E}}^{(-)}(\mathbf{r}; t) \cdot \widehat{\mathbf{E}}^{(+)}(\mathbf{r}; t) \, |i\rangle . \tag{36}$$

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[9] The classical analogue of the positive frequency part of electric field operator is the so-called complex analytic signal of a real electric field, introduced by Gabor (Gabor, 1946). For a discussion of the physical interpretation of the positive and negative frequency parts of the quantized electric field operator see (Glauber, 1963).

If one considers the random fluctuations associated with light, one leads to a more general expression involving the density operator. The average counting rate of a photo-detector placed at a position **r** at time t then becomes proportional to [c.f (Glauber, 1963), Eq. (3.3)]

$$\mathscr{R}(\mathbf{r}, t) \equiv \text{Tr}\left\{\widehat{\rho}\, \widehat{\mathbf{E}}^{(-)}(\mathbf{r}, t) \cdot \widehat{\mathbf{E}}^{(+)}(\mathbf{r}, t)\right\}. \tag{37}$$

Here the dot $(\cdot)$ denotes scalar product.

Using similar arguments, one can obtain quantum-mechanical analogues of all correlation functions (such as mutual coherence function etc.) used in the classical theory. The first-order correlation properties (corresponds to second-order properties in the classical theory) of the field may be specified by a 3×3 correlation matrix (Glauber, 1963)

$$\begin{aligned}
\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}', t') &\equiv \left[G^{(1)}_{\mu\nu}(\mathbf{r}, t; \mathbf{r}', t')\right] \\
&\equiv \left[\text{Tr}\left\{\widehat{\rho}\, \widehat{E}^{(-)}_{\mu}(\mathbf{r}, t)\widehat{E}^{(+)}_{\nu}(\mathbf{r}', t')\right\}\right],
\end{aligned} \tag{38}$$

where μ, ν label, mutually orthogonal components of the electric field operator. For the sake of simplicity, let us neglect the polarization properties of the light, restricting our analysis to scalar fields. Hence, with a suitable choice of axes, *only one* element $G^{(1)}_{\mu\mu}(\mathbf{r}, t; \mathbf{r}', t')$ [no summation over repeated indices] of $\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}', t')$ will completely characterize all first-order correlation properties of the field in the space-time domain. We omit the suffix μ and write

$$G^{(1)}(\mathbf{r}, t; \mathbf{r}', t') = \text{Tr}\left\{\widehat{\rho}\, \widehat{E}^{(-)}(\mathbf{r}; t)\widehat{E}^{(+)}(\mathbf{r}'; t')\right\}. \tag{39}$$

The simplest coherence properties of light, in the space-time domain, are characterized by the first-order correlation function $\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}', t')$. In terms of it one can define the first-order degree of coherence by the formula [(Glauber, 1963), Eq. (4)]

$$g^{(1)}(\mathbf{r}, t; \mathbf{r}', t') \equiv \frac{G^{(1)}(\mathbf{r}, t; \mathbf{r}', t')}{\sqrt{G^{(1)}(\mathbf{r}; t; \mathbf{r}; t)}\sqrt{G^{(1)}(\mathbf{r}'; t'; \mathbf{r}'; t')}}. \tag{40}$$

The modulus of $g^{(1)}(\mathbf{r}, t; \mathbf{r}', t')$ may be shown to be bounded by zero and unity [(Glauber, 1963), Eq. (4.2)], i.e.,

$$0 \leq |g^{(1)}(\mathbf{r}, t; \mathbf{r}', t')| \leq 1. \tag{41}$$

It can be shown that this quantity is related to fringe visibility in interference experiments [see, for example, (Glauber, 2007), Sec. 2.7.2]. Complete first-order coherence (corresponding to second-order coherence in classical theory) is characterized by the condition $|g^{(1)}(\mathbf{r}, t; \mathbf{r}', t')| = 1$, and complete first-order incoherence by the other extreme, $g^{(1)}(\mathbf{r}, t; \mathbf{r}', t') = 0$. Equations (8), and (40) may look similar, but one must appreciate the fact that the quantum mechanical interpretation is much more effective as one goes to a low-intensity domain, where absorption or emission of one, or, few numbers of photons may affect the experimental observations.

Detailed descriptions on this topic have been discussed in many scholarly articles [see, for example, (Glauber, 1963; 2007; Mandel & Wolf, 1995)].

6. Quantum theory of optical coherence in space-frequency domain

In this section we present a quantum-mechanical theory of first-order optical coherence for statistically non-stationary light in the space-frequency domain. We discuss some relevant correlation functions, associated with the quantized field and the density operator, which can be introduced in the space-frequency representation. We also show, by use of the technique of linear filtering, that these new correlation functions may be related to the photo-counting rate and to well known coherence-functions of space-time domain. We consider non-stationary light[10], because the assumption of statistical stationarity is not always appropriate, especially in many situations encountered in quantum optics where light emission may take place over a finite time-interval. A detailed description of the main material presented in this section has been recently published (Lahiri & Wolf, 2010c).

6.1 Correlation functions in the space-frequency domain

Let us first note some properties of the operator $\widehat{\mathbf{e}}(\mathbf{r}, \omega)$, which is the Fourier transform of $\widehat{\mathbf{E}}(\mathbf{r}; t)$, i.e., which is given by

$$\widehat{\mathbf{e}}(\mathbf{r}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{\mathbf{E}}(\mathbf{r}; t) e^{i\omega t} \, dt. \tag{42}$$

Using the fact that $\widehat{\mathbf{E}}(\mathbf{r}; t) = \widehat{\mathbf{E}}^{(+)}(\mathbf{r}; t) + \widehat{\mathbf{E}}^{(-)}(\mathbf{r}; t)$, one may express $\widehat{\mathbf{e}}(\mathbf{r}, \omega)$ in the form

$$\widehat{\mathbf{e}}(\mathbf{r}, \omega) = \widehat{\mathbf{e}}^{(+)}(\mathbf{r}, \omega) + \widehat{\mathbf{e}}^{(-)}(\mathbf{r}, \omega), \tag{43}$$

where

$$\widehat{\mathbf{e}}^{(+)}(\mathbf{r}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{\mathbf{E}}^{(+)}(\mathbf{r}; t) e^{i\omega t} \, dt, \tag{44a}$$

$$\widehat{\mathbf{e}}^{(-)}(\mathbf{r}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{\mathbf{E}}^{(-)}(\mathbf{r}; t) e^{i\omega t} \, dt. \tag{44b}$$

Using the property $\left\{ \widehat{\mathbf{E}}^{(+)}(\mathbf{r}; t) \right\}^{\dagger} = \widehat{\mathbf{E}}^{(-)}(\mathbf{r}; t)$, it follows from Eqs. (44) that $\left\{ \widehat{\mathbf{e}}^{(+)}(\mathbf{r}, \omega) \right\}^{\dagger} = \widehat{\mathbf{e}}^{(-)}(\mathbf{r}, -\omega)$.

Let us now consider the following 3×3 correlation matrix

$$\overleftrightarrow{\mathscr{W}}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') \equiv \left[\mathscr{W}_{\mu\nu}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') \right] \equiv \mathrm{Tr} \left\{ \widehat{\rho} \, \widehat{e}_{\mu}^{(-)}(\mathbf{r}, -\omega) \widehat{e}_{\nu}^{(+)}(\mathbf{r}', \omega') \right\}. \tag{45}$$

On using Eqs. (44) and (45), one can readily show that the elements of the correlation matrices $\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}', t')$ and $\overleftrightarrow{\mathscr{W}}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')$ are Fourier transforms of each other, i.e., that

$$\mathscr{W}_{\mu\nu}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') = \left(\frac{1}{2\pi} \right)^2 \iint_{-\infty}^{\infty} G_{\mu\nu}^{(1)}(\mathbf{r}, t; \mathbf{r}', t') e^{i(-\omega t + \omega' t')} \, dt \, dt', \tag{46a}$$

$$G_{\mu\nu}^{(1)}(\mathbf{r}, t; \mathbf{r}', t') = \iint_{0}^{\infty} \mathscr{W}_{\mu\nu}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') e^{i(\omega t - \omega' t')} \, d\omega \, d\omega'. \tag{46b}$$

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[10] Attempts to formulate coherence theory for classical non-stationary fields have been made (Bertolotti et al., 1995; Sereda et al., 1998).

We will restrict our analysis to scalar fields. Therefore, the 3×3 correlation matrix $\overleftrightarrow{\mathscr{W}}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')$ may now be replaced by the correlation function

$$\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') = \text{Tr}\left\{\widehat{\rho}\,\widehat{e}^{(-)}(\mathbf{r}, -\omega)\widehat{e}^{(+)}(\mathbf{r}', \omega')\right\}, \tag{47}$$

which, in analogy with the relations (46), is the Fourier transform of $G^{(1)}(\mathbf{r}, t; \mathbf{r}', t')$; viz,

$$\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') = \left(\frac{1}{2\pi}\right)^2 \iint\limits_{-\infty}^{\infty} G^{(1)}(\mathbf{r}, t; \mathbf{r}', t')e^{i(-\omega t + \omega' t')}\, dt\, dt', \tag{48a}$$

$$G^{(1)}(\mathbf{r}, t; \mathbf{r}', t') = \iint\limits_{0}^{\infty} \mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')e^{i(\omega t - \omega' t')}\, d\omega\, d\omega'. \tag{48b}$$

We will refer to $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')$ as the *two-frequency cross-spectral density function* (to be abbreviated by two-frequency CSDF) and the single-frequency correlation function $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega)$ as the *cross-spectral density function* (CSDF) of the field, in analogy with terminology used in the classical theory.

6.2 Physical interpretation of correlation functions in the space-frequency domain

Let us now assume that a light beam is transmitted by a linear filter which allows only a narrow frequency band to pass through it. Suppose that the light, emerging from the filter, has mean frequency $\bar{\omega}$ and effective bandwidth $\Delta\omega \ll \bar{\omega}$. The field operators representing this filtered narrow-band light in the space-frequency domain, may then be represented by the formulas

$$\widehat{e}_{(\bar{\omega})}^{(+)}(\mathbf{r}, \omega) = T_{(\bar{\omega})}(\omega)\widehat{e}^{(+)}(\mathbf{r}, \omega), \tag{49a}$$

$$\widehat{e}_{(\bar{\omega})}^{(-)}(\mathbf{r}, -\omega) = T_{(\bar{\omega})}^{*}(\omega)\widehat{e}^{(-)}(\mathbf{r}, -\omega), \tag{49b}$$

where $T_{(\bar{\omega})}(\omega)$ is the transmission function of the filter, whose modulus is negligible outside the pass-band $\bar{\omega} - \Delta\omega/2 \leq \omega \leq \bar{\omega} + \Delta\omega/2$ of the filter. Using Eqs. (47) and (49), it follows that for the filtered light,

$$\mathscr{W}_{(\bar{\omega})}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') = T_{(\bar{\omega})}^{*}(\omega)T_{(\bar{\omega})}(\omega')\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega'). \tag{50}$$

Here, the function $\mathscr{W}_{(\bar{\omega})}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')$ is the two-frequency CSDF of the filtered light, of mean frequency $\bar{\omega}$, and $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')$ on the right hand side is the two-frequency CSDF of the unfiltered light incident on the filter.

From Eqs. (48b) and (50), one readily finds that the space-time correlation function $G_{(\bar{\omega})}^{(1)}(\mathbf{r}; t; \mathbf{r}'; t')$ of the filtered narrow-band light is given by the expression

$$G_{(\bar{\omega})}^{(1)}(\mathbf{r}; t; \mathbf{r}'; t') = \iint\limits_{\bar{\omega}-\Delta\omega/2}^{\bar{\omega}+\Delta\omega/2} T_{(\bar{\omega})}^{*}(\omega)T_{(\bar{\omega})}(\omega')\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')e^{i(\omega t - \omega' t')}\, d\omega\, d\omega'. \tag{51}$$

Because of the assumption that $\Delta\omega/\bar{\omega} \ll 1$, the function $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')$ in Eq. (51) does not change appreciably as function of ω and ω' over the ranges $\bar{\omega} - \Delta\omega/2 \leq \omega \leq \bar{\omega} + \Delta\omega/2$, $\bar{\omega} - \Delta\omega/2 \leq \omega' \leq \bar{\omega} + \Delta\omega/2$ and is approximately equal to $\mathscr{W}^{(1)}(\mathbf{r}, \bar{\omega}; \mathbf{r}', \bar{\omega})$. From Eq. (51) it then follows that

$$G^{(1)}_{(\bar{\omega})}(\mathbf{r}; t; \mathbf{r}'; t') \approx \mathscr{W}^{(1)}(\mathbf{r}, \bar{\omega}; \mathbf{r}', \bar{\omega}) \iint_{\bar{\omega}-\Delta\omega/2}^{\bar{\omega}+\Delta\omega/2} T^*_{(\bar{\omega})}(\omega) T_{(\bar{\omega})}(\omega') e^{i(\omega t - \omega' t')} \, d\omega \, d\omega'. \tag{52}$$

6.3 Physical significance of $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$

Let us consider light generated by some physical process which begins at time $t = 0$ and ceases at time $t = \mathcal{T}$, say; for example light emitted by a collection of exited atoms. Clearly, such light is not statistically stationary. Suppose now that this light is filtered and is then incident on a photo-detector. It follows from Eq. (52) that the average counting rate $\mathscr{R}_{(\bar{\omega})}(\mathbf{r}; t) \equiv G^{(1)}_{(\bar{\omega})}(\mathbf{r}; t; \mathbf{r}; t)$ of the detector placed at a point $\mathbf{r}$, at time t, is given by the expression

$$\mathscr{R}_{(\bar{\omega})}(\mathbf{r}; t) \approx \mathscr{W}^{(1)}(\mathbf{r}, \bar{\omega}; \mathbf{r}, \bar{\omega}) \iint_{\bar{\omega}-\Delta\omega/2}^{\bar{\omega}+\Delta\omega/2} T^*_{(\bar{\omega})}(\omega) T_{(\bar{\omega})}(\omega') e^{i(\omega - \omega')t} \, d\omega \, d\omega'. \tag{53}$$

Since, $\mathscr{R}_{(\bar{\omega})}(\mathbf{r}; t) = 0$, unless $0 < t < \mathcal{T}$, the total energy $\mathscr{E}(\mathbf{r}; \bar{\omega})$ detected (total counts) by the photo-detector will be proportional to

$$\mathscr{E}(\mathbf{r}; \bar{\omega}) \equiv \int_0^{\mathcal{T}} \mathscr{R}_{(\bar{\omega})}(\mathbf{r}; t) \, dt = \int_{-\infty}^{\infty} \mathscr{R}_{(\bar{\omega})}(\mathbf{r}; t) \, dt$$

$$\approx \mathscr{W}^{(1)}(\mathbf{r}, \bar{\omega}; \mathbf{r}, \bar{\omega}) \left\{ 2\pi \int_{\bar{\omega}-\Delta\omega/2}^{\bar{\omega}+\Delta\omega/2} \left| T_{(\bar{\omega})}(\omega) \right|^2 \, d\omega \right\} \tag{54}$$

Thus, we may conclude that the total energy $\mathscr{E}(\mathbf{r}; \omega)$ detected by the photo-detector, placed at a point $\mathbf{r}$ in the path of a narrow-band light beam of mean frequency ω, is proportional to $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$, i.e., that

$$\mathscr{E}(\mathbf{r}; \omega) \equiv \int_0^{\mathcal{T}} \mathscr{R}_{(\bar{\omega})}(\mathbf{r}; t) \, dt \propto \mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega), \tag{55}$$

the proportionality constant being dependent on the filter and on the choice of units. In other words, the correlation function $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$ provides a measure of the energy density, at a point $\mathbf{r}$, associated with the frequency ω of light. It is evident that $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega) \geq 0$.

$\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$ must not be confused with the well-known Wiener's spectral density (Wiener, 1930) of statistically stationary light. In the quantum mechanical interpretation, Wiener's spectral density is equivalent to the *counting rate* of the photo-detector, associated with a frequency. On the other hand, the quantity $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$ represents the *total counts* in the photo-detector associated with the frequency ω. Therefore, $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$ has a different dimension than Wiener's spectral density. Even in the stationary limit, $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$ does not reduce to the Wiener's spectral density. In fact, in such a case, the energy density $\mathscr{E}(\mathbf{r}; \omega)$ will be infinitely large and hence, in that limit, the quantity $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$ will not be useful.

For a light that is not statistically stationary, defining spectral density is a nontrivial problem which is still a subject of an active discussion. Many publications have been dedicated to it and several different definitions have been proposed [see, for example, (Eberly & Wódkievicz, 1977; Lampard, 1954; Mark, 1970; Page, 1952; Ponomarenko et al., 2004; Silverman, 1957)]. Nevertheless, for statistically non-stationary processes, $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)$ is measurable and a physically meaningful quantity. In the following sections we show that it has an important role in defining the spectral-degree of coherence of non-stationary light.

6.4 First-order coherence

We will now consider the first-order coherence properties of non-stationary light, again restricting ourselves to scalar fields. As in the previous Section, we will consider a filtered narrow-band light. On using Eqs. (40) and (52), one then has

$$g_{(\bar{\omega})}^{(1)}(\mathbf{r}, t; \mathbf{r}', t') \approx \frac{\mathscr{W}^{(1)}(\mathbf{r}, \bar{\omega}; \mathbf{r}', \bar{\omega})}{\sqrt{\mathscr{W}^{(1)}(\mathbf{r}, \bar{\omega}; \mathbf{r}, \bar{\omega})}\sqrt{\mathscr{W}^{(1)}(\mathbf{r}', \bar{\omega}; \mathbf{r}', \bar{\omega})}} \Theta(t, t'), \tag{56}$$

where

$$\Theta(t, t') = \frac{\iint_{\bar{\omega}-\Delta\omega/2}^{\bar{\omega}+\Delta\omega/2} T_{(\bar{\omega})}^{*}(\omega) T_{(\bar{\omega})}(\omega') e^{i(\omega t - \omega' t')}\, d\omega\, d\omega'}{\left|\int_{\bar{\omega}-\Delta\omega/2}^{\bar{\omega}+\Delta\omega/2} T_{(\bar{\omega})}(\omega) e^{-i\omega t}\, d\omega\right| \left|\int_{\bar{\omega}-\Delta\omega/2}^{\bar{\omega}+\Delta\omega/2} T_{(\bar{\omega})}(\omega) e^{-i\omega t'}\, d\omega\right|}. \tag{57}$$

Since, the numerator on the right hand side of Eq. (57) factorizes into a product of the two integrals which appear in the denominator, it is evident that

$$|\Theta(t, t')| = 1, \tag{58}$$

for all values of t and t'. From Eqs. (56) and (58), one readily finds that the modulus of the first-order "space-time" degree of coherence of the filtered narrow-band light of mean frequency $\bar{\omega}$, is given by the formula

$$\left|g_{(\bar{\omega})}^{(1)}(\mathbf{r}, t; \mathbf{r}', t')\right| \approx \left|\frac{\mathscr{W}^{(1)}(\mathbf{r}, \bar{\omega}; \mathbf{r}', \bar{\omega})}{\sqrt{\mathscr{W}^{(1)}(\mathbf{r}, \bar{\omega}; \mathbf{r}, \bar{\omega})}\sqrt{\mathscr{W}^{(1)}(\mathbf{r}', \bar{\omega}; \mathbf{r}', \bar{\omega})}}\right|. \tag{59}$$

The expression within the modulus signs on the right-hand side is the normalized CSDF at the frequency $\bar{\omega}$.

It may be concluded from Eq. (59) that this normalized CSDF provides a measure of first-order coherence of the filtered light of mean frequency $\bar{\omega}$. Consequently, one may define the *first-order spectral degree of coherence* at frequency ω by the formula

$$\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega) \equiv \frac{\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega)}{\sqrt{\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)}\sqrt{\mathscr{W}^{(1)}(\mathbf{r}', \omega; \mathbf{r}', \omega)}}. \tag{60}$$

It can be immediately shown that (Lahiri & Wolf, 2010c)

$$0 \leq |\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega)| \leq 1. \tag{61}$$

When $|\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega)| = 1$, the light may be said to be completely coherent at the frequency ω, and when $\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega) = 0$, it may be said to be completely incoherent at that frequency. We stress, once again, that $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega)$ and $\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega)$ being *single-frequency* quantities, provide measures of the ability of a particular frequency-component of the light to interfere.

We will now return to the two-frequency correlation function $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')$ given by formula (47). It characterizes correlations between different frequency components of light, i.e., the ability of two different frequency components to interfere. Interference of fields of different frequencies has generally not been considered in the literature, probably because it is not a commonly observed phenomenon. According to the classical theory, different frequency components of *statistically stationary* light do not interfere (see, for example, (Wolf, 2007b), Sec. 2.5). For the sake of completeness, we will now show that this fact is also true in the non-classical domain. The proof is similar to that for classical fields. If one considers light whose fluctuations are statistically stationary in the wide sense, i.e., if $G^{(1)}(\mathbf{r}, t; \mathbf{r}', t') = G^{(1)}(\mathbf{r}, \mathbf{r}'; t' - t)$ then, ignoring some mathematical subtleties, one finds from Eq. (48a) that the two-frequency CSDF $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')$ has the form

$$\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') = f[\mathbf{r}, \mathbf{r}'; (\omega + \omega')/2]\delta(\omega - \omega'), \tag{62}$$

where f is, in general, a complex function of its arguments and δ denotes the Dirac delta function. This formula shows that when $\omega \neq \omega'$, the two-frequency CSDF $\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') = 0$, implying that different frequency components of statistically stationary light do not interfere. However, for non-stationary random processes, this correlation function may have a non-zero value and, consequently, interference among different frequency components may take place. One may define a normalized two-frequency correlation function

$$\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') = \frac{\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')}{\sqrt{\mathscr{W}^{(1)}(\mathbf{r}, \omega; \mathbf{r}, \omega)\mathscr{W}^{(1)}(\mathbf{r}', \omega'; \mathbf{r}', \omega')}} \tag{63}$$

as a generalized first-order spectral degree of coherence for a pair of frequencies ω and ω'. It can also be shown that (Lahiri & Wolf, 2010c)

$$0 \leq |\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')| \leq 1. \tag{64}$$

The extreme value $|\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega')| = 1$, which corresponds to maximum possible fringe visibility observed in interference experiments, represents complete coherence in the space-frequency domain. The other extreme value, $\eta^{(1)}(\mathbf{r}, \omega; \mathbf{r}', \omega') = 0$, implies that no interference fringes will be present, i.e that there is complete incoherence between different frequency components.

7. Polarization properties of optical beams

In this section we will discuss the polarization properties of light. As is clear from previous discussions that a scalar treatment will no more be sufficient for his purpose, and we have to consider the vector nature of the field. However, since, we will restrict our analysis to beam-like optical fields, we will encounter 2×2 correlation matrices.

We consider the canonical experiment depicted in Fig. 2. Suppose that a light beam, propagating along the positive *z* axis, passes through a compensator, followed by a polarizer (see Fig. 2). Light emerging from the polarizer is linearly polarized along some direction,
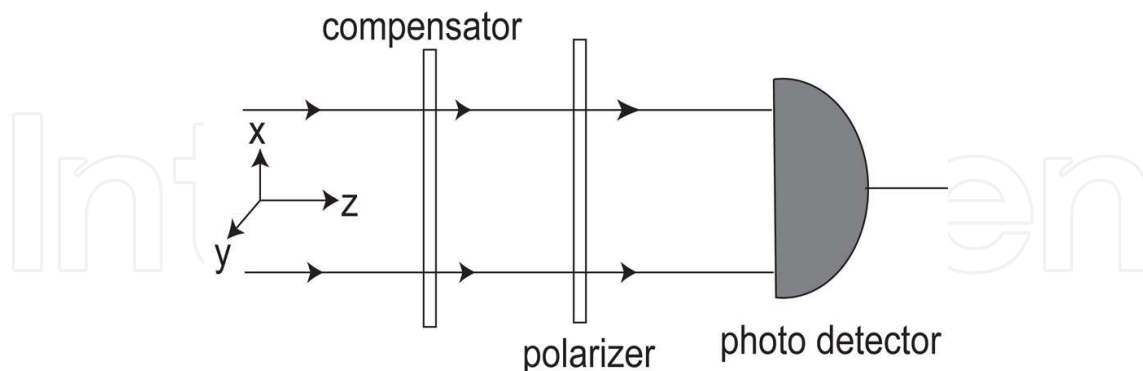

Fig. 2. Illustrating notation relating to the canonical polarization experiment.

which makes an angle θ, say, with the positive direction of a chosen *x* axis. We call θ the polarizer angle. Effects of the compensator may be described by introducing phase delays α_x and α_y in the *x* and the *y* components of the field operator $\widehat{\mathbf{E}}^{(+)}$ respectively. Suppose now that a photodetector is placed behind the polarizer (see Fig. 2), which detects photons that emerge from the polarizer. From Eq. (37), it follows that the counting rate $\mathscr{R}_{\theta,\alpha}(\mathbf{r};t)$ in the detector will be given by the formula

$$\mathscr{R}_{\theta,\alpha}(\mathbf{r};t) = \mathrm{Tr}\left\{\widehat{\rho}\,\widehat{E}_{\theta,\alpha}^{(-)}(\mathbf{r};t)\widehat{E}_{\theta,\alpha}^{(+)}(\mathbf{r};t)\right\}, \tag{65}$$

where

$$\widehat{E}_{\theta,\alpha}^{(\pm)} = \widehat{E}_x^{(\pm)}e^{i\alpha_x}\cos\theta + \widehat{E}_y^{(\pm)}e^{i\alpha_y}\sin\theta. \tag{66}$$

Using Eqs. (65) and (66), one readily finds that the average counting rate of the photo-detector is given by the expression

$$\mathscr{R}_{\theta,\alpha}(\mathbf{r};t) = G_{xx}^{(1)}(\mathbf{r},t;\mathbf{r},t)\cos^2\theta + G_{yy}^{(1)}(\mathbf{r},t;\mathbf{r},t)\sin^2\theta$$
$$+ 2\sqrt{G_{xx}^{(1)}(\mathbf{r},t;\mathbf{r},t)}\sqrt{G_{yy}^{(1)}(\mathbf{r},t;\mathbf{r},t)}\sin\theta\cos\theta|g_{xy}^{(1)}(\mathbf{r};t)|\cos\left[\beta_{xy}(\mathbf{r};t) - \alpha\right], \tag{67}$$

where $\alpha = \alpha_y - \alpha_x$ and

$$g_{xy}^{(1)}(\mathbf{r};t) \equiv \frac{G_{xy}^{(1)}(\mathbf{r},t;\mathbf{r},t)}{\sqrt{G_{xx}^{(1)}(\mathbf{r},t;\mathbf{r},t)}\sqrt{G_{yy}^{(1)}(\mathbf{r},t;\mathbf{r},t)}} \equiv |g_{xy}^{(1)}(\mathbf{r};t)|e^{i\beta_{xy}(\mathbf{r};t)}. \tag{68}$$

It is important to note that only the "equal-point" ($\mathbf{r}_1 = \mathbf{r}_2 \equiv \mathbf{r}$) and "equal-time" ($t_1 = t_2 \equiv t$) correlation matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)$ contributes to the photon counting rate. We will refer to this matrix as the *quantum polarization matrix*. Equation (67) makes it possible to determine the elements of the quantum polarization matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)$ in a similar way as is done for the elements of the analogous correlation matrix for a classical field, introduced in Ref. (Wolf,

1959). One can readily show that

$$G_{xx}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) = \mathscr{R}_{0,0}(\mathbf{r}; t), \tag{69a}$$

$$G_{yy}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) = \mathscr{R}_{\pi/2,0}(\mathbf{r}; t), \tag{69b}$$

$$G_{xy}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) = \frac{1}{2} \left[\mathscr{R}_{\pi/4,0}(\mathbf{r}; t) - \mathscr{R}_{3\pi/4,0}(\mathbf{r}; t) \right] + \frac{i}{2} \left[\mathscr{R}_{\pi/4,\pi/2}(\mathbf{r}; t) - \mathscr{R}_{3\pi/4,\pi/2}(\mathbf{r}; t) \right], \tag{69c}$$

$$G_{yx}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) = \frac{1}{2} \left[\mathscr{R}_{\pi/4,0}(\mathbf{r}; t) - \mathscr{R}_{3\pi/4,0}(\mathbf{r}; t) \right] - \frac{i}{2} \left[\mathscr{R}_{\pi/4,\pi/2}(\mathbf{r}; t) - \mathscr{R}_{3\pi/4,\pi/2}(\mathbf{r}; t) \right]. \tag{69d}$$

Let us now briefly examine some properties of the quantum polarization matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}, t)$. Each element of this matrix, being a correlation function, has the properties of a scalar product[11]. In particular, the elements of any such matrix satisfy the constraints

$$G_{\mu\mu}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) \geq 0, \tag{70a}$$

$$G_{\mu\nu}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) = \left\{ G_{\nu\mu}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) \right\}^*, \tag{70b}$$

$$\mathrm{Det}\, \overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) \geq 0, \tag{70c}$$

where $\mu = x, y; \nu = x, y$ and Det denotes the determinant. Conditions (70a) and (70c) imply that a quantum polarization matrix is always *non-negative definite*. Formula (70c) follows from the Cauchy-Schwarz inequality. From Eqs. (68) and (70c), it follows that

$$0 \leq |g_{xy}^{(1)}(\mathbf{r}; t)| \leq 1. \tag{71}$$

7.1 Unpolarized light beam

We will now assume that an *unpolarized* photon-beam[12] is used in the experiment depicted in Fig. 2. For such a beam, the photon detection rate $\mathscr{R}_{\theta,\alpha}(\mathbf{r}; t)$, has to be independent of θ and α and consequently, one has from Eq. (67) that

$$g_{xy}^{(1)}(\mathbf{r}; t) = 0, \tag{72a}$$

$$\text{and} \quad G_{xx}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) = G_{yy}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) \equiv A(\mathbf{r}; t), \quad \text{say,} \tag{72b}$$

where $A(\mathbf{r}; t)$ is a real function of space and time and $A(\mathbf{r}; t) \geq 0$. From Eqs. (72) it is evident that for an unpolarized beam, the quantum polarization matrix $\overleftrightarrow{G}_{(u)}^{(1)}(\mathbf{r}; t; \mathbf{r}, t)$ is proportional to a unit matrix, i.e., that

$$\overleftrightarrow{G}_{(u)}^{(1)}(\mathbf{r}; t; \mathbf{r}, t) = A(\mathbf{r}; t) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{73}$$

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[11] The proof is similar to that given for scalar field operators (Titulaer & Glauber, 1965).

[12] For discussion on unpolarized radiation see, for example, Refs. (Agarwal, 1971; Prakash et al., 1971)

7.2 Polarized light beam

Let us now consider the other extreme case, namely when the beam is fully polarized[13], i.e., when $|g_{xy}^{(1)}(\mathbf{r};t)| = 1$. In this case, it follows from Eq. (68) that the elements of $\overleftrightarrow{G}_{(p)}^{(1)}(\mathbf{r};t;\mathbf{r};t)$ can be expressed in the factorized form

$$\left\{ \overleftrightarrow{G}_{(p)}^{(1)}(\mathbf{r};t;\mathbf{r};t) \right\}_{\mu\nu} = \mathcal{E}_\mu^*(\mathbf{r};t)\mathcal{E}_\nu(\mathbf{r};t), \quad (\mu = x,y; \ \nu = x,y), \tag{74}$$

where

$$\mathcal{E}_x(\mathbf{r};t) = \sqrt{G_{xx}^{(1)}(\mathbf{r},t;\mathbf{r},t)} \exp\left[i\phi_{xx}(\mathbf{r};t)\right], \tag{75a}$$

$$\mathcal{E}_y(\mathbf{r};t) = \sqrt{G_{yy}^{(1)}(\mathbf{r},t;\mathbf{r},t)} \exp\left[i\phi_{yy}(\mathbf{r};t)\right]. \tag{75b}$$

Here $\phi_{yy}(\mathbf{r};t) - \phi_{xx}(\mathbf{r};t) = \beta_{xy}(\mathbf{r};t)$, where $\beta_{xy}(\mathbf{r};t)$ is the phase of $g_{xy}^{(1)}(\mathbf{r};t)$, defined in Eq. (68). The fact that the elements of $\overleftrightarrow{G}_{(p)}^{(1)}(\mathbf{r};t;\mathbf{r};t)$ can be factorized, was also noted in Ref. (Glauber, 1963). The condition (74) may readily shown to imply that for a completely polarized light beam

$$\mathrm{Det}\, \overleftrightarrow{G}_{(p)}^{(1)}(\mathbf{r};t;\mathbf{r};t) = 0. \tag{76}$$

The converse of this statement is also true, i.e., if condition (76) holds at a point $\mathbf{r}$, at time t, one can readily show by use of properties (70a) and (70b) that $|g_{xy}^{(1)}(\mathbf{r};t)| = 1$; hence the light is then completely polarized at that point at time t.

We will next establish a necessary and sufficient condition of complete polarization[14]. We express it in the form of the following theorem:

Theorem 7.1. *In order that a beam is completely polarized, it is necessary and sufficient that the quantized field components and the density operator satisfy the condition*

$$\widehat{E}_x^{(+)}(\mathbf{r};t)\widehat{\rho} = A(\mathbf{r};t)\widehat{E}_y^{(+)}(\mathbf{r};t)\widehat{\rho}, \tag{77a}$$

$$\text{where} \quad A(\mathbf{r};t) = \frac{G_{yx}^{(1)}(\mathbf{r},t;\mathbf{r},t)}{G_{yy}^{(1)}(\mathbf{r},t;\mathbf{r},t)}, \tag{77b}$$

Proof. To prove that Eq. (77) is a necessary condition, let us introduce the operator

$$\widehat{M}(\mathbf{r};t) = \widehat{E}_x^{(+)}(\mathbf{r};t) - A(\mathbf{r};t)\widehat{E}_y^{(+)}(\mathbf{r};t), \tag{78}$$

where $A(\mathbf{r};t)$ is given by Eq. (77b). On using Eqs. (77b) and (78), one readily finds that

$$\mathrm{Tr}\left\{ \widehat{\rho}\, \widehat{M}^\dagger \widehat{M} \right\} = \frac{\mathrm{Det}\, \overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)}{G_{yy}^{(1)}(\mathbf{r},t;\mathbf{r},t)}. \tag{79}$$

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[13] For discussion on polarized radiation see Ref. (Mehta et al., 1974)

[14] An analogous condition hold for complete first-order coherence in the space-time domain (Titulaer & Glauber, 1965). In the classical limit, these conditions resembles the recently introduced concept of statistical similarity (Roychowdhury & Wolf, 2005; Wolf, 2010b) for statistically stationary beams.

If the field is completely polarized, Eqs. (76) and (79) imply that

$$\text{Tr}\left\{\hat{\rho}\,\hat{M}^{\dagger}\hat{M}\right\} = 0,\tag{80}$$

and, consequently,

$$\hat{M}\hat{\rho} = \hat{\rho}\hat{M}^{\dagger} = 0.\tag{81}$$

Substituting for $\hat{M}$ from Eq. (78) into Eq. (81), one readily find that condition (77) is satisfied. Thus we have proved that Eq. (77) *is a necessary condition for complete polarization* of a light beam at a point $\mathbf{r}$, at time t.

On the other hand, if one imposes condition (77) on the components of the field operator and evaluates $\text{Det}\,\overleftrightarrow{G}^{(1)}_{(p)}(\mathbf{r};t;\mathbf{r};t)$, one readily obtains Eq. (76), i.e., one finds that if condition (77) holds, the beam is completely polarized. Hence this condition is also a sufficiency condition. $\qquad\qquad\square$

7.3 Partially polarized light beam

We have discussed the properties of completely polarized and completely unpolarized beams. Next we consider beams which are partially polarized, i.e., beams for which $0 < |g^{(1)}_{xy}(\mathbf{r};t)| < 1$.

We will first establish the following result: *If a beam of partially polarized photons is incident on a photo-detector, the average counting rate of the detector, at any time t, can always be decomposed into two parts, one which represents the counting rate for a polarized beam and the other the counting rate for an unpolarized beam*. It can be proved that any quantum polarization matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)$ can be uniquely decomposed in the form (Lahiri & Wolf, 2010b)

$$\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t) = \overleftrightarrow{G}^{(1)}_{(p)}(\mathbf{r};t;\mathbf{r};t) + \overleftrightarrow{G}^{(1)}_{(u)}(\mathbf{r};t;\mathbf{r};t),\tag{82}$$

where the elements of $\overleftrightarrow{G}^{(1)}_{(p)}(\mathbf{r};t;\mathbf{r};t)$ and $\overleftrightarrow{G}^{(1)}_{(u)}(\mathbf{r};t;\mathbf{r};t)$ can be uniquely expressed in terms of the elements of $\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)$. From Eqs. (37) and (82), one can at once deduce that the counting rate of the photo-detector has contribution from an unpolarized part and from a polarized part; i.e., that

$$\mathscr{R}(\mathbf{r},t) = \text{Tr}\,\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t) = \mathscr{R}_{(p)}(\mathbf{r},t) + \mathscr{R}_{(u)}(\mathbf{r},t).\tag{83}$$

By analogy with the classical theory of stochastic electromagnetic beams, we define the degree of polarization $\mathscr{P}$, at a point $P(\mathbf{r})$, at time t, as the ratio of the photon counting rate for the polarized part to the total counting rate:

$$\mathscr{P}(\mathbf{r};t) \equiv \frac{\mathscr{R}_{(p)}(\mathbf{r},t)}{\mathscr{R}(\mathbf{r},t)} = \frac{\text{Tr}\,\overleftrightarrow{G}^{(1)}_{(p)}(\mathbf{r};t;\mathbf{r};t)}{\text{Tr}\,\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)}.\tag{84}$$

On expressing the elements of $\overleftrightarrow{G}^{(1)}_{(p)}(\mathbf{r};t;\mathbf{r};t)$ in terms of the elements of $\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)$, one can show that (Lahiri & Wolf, 2010b)

$$\mathscr{P}(\mathbf{r};t) = \sqrt{1 - \frac{4\,\mathrm{Det}\,\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)}{\left\{\mathrm{Tr}\,\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)\right\}^2}}. \tag{85}$$

From Eqs. (73) and (85), one can readily show that for a beam of unpolarized photons $\mathscr{P} = 0$; and from Eqs. (82) and (85) it follows, at once, that for a completely polarized beam of photons $\mathscr{P}(\mathbf{r};t) = 1$.

Although, formulas (28), and (85) look similar in form, one must bear in mind that the definition used in classical theory is not appropriate for light of low intensity, nor has it been shown that it is valid for fields that are not necessarily statistically stationary. On the other hand, the expression (85) for the degree of polarization applies also to low intensity light and for light whose statistical properties are characterized by non-stationary ensembles, such as, for example, fields associated with a non-stationary ensemble of pulses.

8. Wave-particle duality, partial coherence and partial polarization of light

Quantum systems (quantons[15]) possess properties of both particles and waves. Bohr's complementarity principle (Bohr, 1928) suggests that these two properties are mutually exclusive. In other words, depending on the experimental situation, a quanton will behave as a particle or as a wave. In the third volume of his famous lecture series (Feynman et al., 1966), Feynman emphasized that this *wave-particle duality* may be understood from Young's two-pinhole interference experiments (Young, 1802). In such an experiment, a quanton may arrive at the detector along two different paths. If one can determine which path the quanton traveled, then no interference fringe will be found (i.e., the quanton will show complete particle behavior). On the other hand, if one *cannot* obtain any information about the quanton's path, then interference fringes, with *unit visibility*, will be obtained (i.e., the quanton will show complete wave behavior), assuming that the intensities at the two pinholes are the same. In the intermediate case when one has partial "which-path information" (WPI), fringes with visibility smaller than unity are obtained, even if the intensities at the two pinholes are equal. For the sake of brevity, we will use the term "best circumstances" to refer to the situation when in an Young's interference experiment, the intensities at the two pinholes are equal, or to equivalent situations in other interferometric setups.

The relation between fringe visibility (degree of coherence) and WPI has been investigated by researchers [see, for example, (Englert, 1996; Jaeger et al., 1995; Mandel, 1991)]. It has been established that a quantitative measure of WPI and fringe-visibility obey a certain inequality. In this section, we will first recollect the results obtained by Mandel and then will show that it is not only the fringe-visibility, but also the polarization properties of the superposed light, which may depend on WPI in an interference experiment.

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[15] This abbreviation is due to M. Bunge [see, for example, J.-M. Lévy-Leblond, Physica B **151**, 314 (1988)].

Suppose, that $|\psi_1\rangle$ and $|\psi_2\rangle$ represent two normalized single-photon states (eigenstates of the number operator), so that

$$\langle\psi_1|\psi_2\rangle = \langle\psi_2|\psi_1\rangle = 0, \tag{86a}$$

$$\langle\psi_1|\psi_1\rangle = \langle\psi_2|\psi_2\rangle = 1. \tag{86b}$$

We will consider the superposition of the two states in some interferometric arrangement, where a photon may travel along two different paths. Suppose that $|\psi_{\rm ID}\rangle$ represents a state of light, which is formed by coherent superposition of the two states $|\psi_1\rangle$ and $|\psi_2\rangle$, i.e., that

$$|\psi_{\rm ID}\rangle = \alpha_1 |\psi_1\rangle + \alpha_2 |\psi_1\rangle, \quad |\alpha_1|^2 + |\alpha_2|^2 = 1, \tag{87}$$

where α_1 and α_2 are, in general, two complex numbers. In this case, a photon may be in the state $|\psi_1\rangle$ with probability $|\alpha_1|^2$, or in the state $|\psi_2\rangle$ with probability $|\alpha_2|^2$, but the two possibilities are intrinsically *indistinguishable*. The density operator $\widehat{\rho}_{\rm ID}$ will then have the form

$$\widehat{\rho}_{\rm ID} = |\alpha_1|^2 |\psi_1\rangle\langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle\langle\psi_2| + \alpha_1^*\alpha_2 |\psi_2\rangle\langle\psi_1| + \alpha_2^*\alpha_1 |\psi_1\rangle\langle\psi_2|. \tag{88}$$

In the other extreme case, when the state of light is due to incoherent superposition of the two states, the density operator $\widehat{\rho}_{\rm D}$ will be given by the expression

$$\widehat{\rho}_{\rm D} = |\alpha_1|^2 |\psi_1\rangle\langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle\langle\psi_2|. \tag{89}$$

Here $|\alpha_1|^2$ and $|\alpha_2|^2$ again represent the probabilities that the photon will be in state $|\psi_1\rangle$ or in state $|\psi_2\rangle$, but now the two possibilities are intrinsically *distinguishable*. Mandel (Mandel, 1991) showed that in any intermediate case, the density operator

$$\widehat{\rho} = \rho_{11} |\psi_1\rangle\langle\psi_1| + \rho_{12} |\psi_1\rangle\langle\psi_2| + \rho_{21} |\psi_2\rangle\langle\psi_1| + \rho_{22} |\psi_2\rangle\langle\psi_2| \tag{90}$$

can be uniquely expressed in the form

$$\widehat{\rho} = \mathscr{I}\widehat{\rho}_{\rm ID} + (1-\mathscr{I})\widehat{\rho}_{\rm D}, \qquad 0 \le \mathscr{I} \le 1. \tag{91}$$

Mandel defined $\mathscr{I}$ as the *degree of indistinguishability*. If $\mathscr{I} = 0$, the two paths are completely distinguishable, i.e., one has complete WPI; and if $\mathscr{I} = 1$, they are completely indistinguishable, i.e., one has no WPI. In the intermediate case $0 < \mathscr{I} < 1$, the two possibilities may be said to be partially distinguishable. Clearly, $\mathscr{I}$ may be considered as a measure of WPI. According to Eqs. (90) and (91), one can always express $\widehat{\rho}$ in the form

$$\widehat{\rho} = |\alpha_1|^2 |\psi_1\rangle\langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle\langle\psi_2| + \mathscr{I}(\alpha_1^*\alpha_2 |\psi_2\rangle\langle\psi_1| + \alpha_2^*\alpha_1 |\psi_1\rangle\langle\psi_2|). \tag{92}$$

Clearly, the condition of "best circumstances" requires that $|\alpha_1| = |\alpha_2|$.

8.1 WPI and partial coherence

Mandel considered a Young's interference experiment, in which the two pinholes (secondary sources) were labeled by 1 and 2. He assumed $|n\rangle_j$ to be a state representing n photons originated from pinhole j ($n = 0, 1$; $j = 1, 2$). Clearly, in this case

$$|\psi_1\rangle = |1\rangle_1 |0\rangle_2, \qquad |\psi_2\rangle = |0\rangle_1 |1\rangle_2. \tag{93}$$

Calculations show that [for details see (Mandel, 1991)] "visibility $\leq \mathscr{I}$", and the equality holds in the special case when $|\alpha_1| = |\alpha_2|$. Since fringe-visibility is a measure of coherence properties of light (modulus of degree of coherence is equal to the fringe visibility), Mandel's result displays an intimate relationship between wave-particle duality and partial coherence.

8.2 WPI and partial polarization

Let us now assume that $|\psi_1\rangle$ and $|\psi_2\rangle$ are of the form

$$|\psi_1\rangle = |1\rangle_x |0\rangle_y, \tag{94a}$$

$$|\psi_2\rangle = |0\rangle_x |1\rangle_y, \tag{94b}$$

where the two states are labeled by the same (vector) mode $\mathbf{k}$, and x, y are two mutually orthogonal directions, both perpendicular to the direction of $\mathbf{k}$. For the sake of brevity, $\mathbf{k}$ is not displayed in Eqs. (94). Clearly $|\psi_1\rangle$ represents the state of a photon polarized along the x direction, and $|\psi_2\rangle$ represents that along the y direction. In the present case, one may express $\widehat{E}_i^{(+)}(\mathbf{r};t)$ in the form

$$\widehat{E}_i^{(+)}(\mathbf{r};t) = Ce^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\widehat{a}_i, \qquad (i = x, y), \tag{95}$$

where the operator $\widehat{a}_i$ represents annihilation of a photon in mode $\mathbf{k}$, polarized along the $i-$axis, and C is a constant. From Eqs. (38), (92), and (95), one readily finds that the quantum polarization matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}, t)$ has the form

$$\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) = |C|^2 \begin{pmatrix} |\alpha_1|^2 & \mathscr{I}\alpha_1^*\alpha_2 \\ \mathscr{I}\alpha_1\alpha_2^* & |\alpha_2|^2 \end{pmatrix}. \tag{96}$$

From Eqs. (84) and (96) and using the fact $|\alpha_1|^2 + |\alpha_2|^2 = 1$, one finds that, in this case, the degree of polarization is given by the expression

$$\mathscr{P} = \sqrt{(|\alpha_1|^2 - |\alpha_2|^2)^2 + 4|\alpha_1|^2|\alpha_2|^2\mathscr{I}^2}. \tag{97}$$

It follows from Eq. (97) by simple calculations that

$$\mathscr{P}^2 - \mathscr{I}^2 = (1 - \mathscr{I}^2)(2|\alpha_1|^2 - 1)^2. \tag{98}$$

Using the fact that $0 \leq \mathscr{I} \leq 1$, one readily finds that

$$\mathscr{P} \geq \mathscr{I}. \tag{99}$$

Thus, the degree of polarization of the out-put light in a single-photon interference experiment is always greater or equal to the degree of indistinguishability ($\mathscr{I}$) which a measure of "which-path information".

Let us now assume that the condition of "best circumstances" holds, i.e., one has $|\alpha_1| = |\alpha_2|$. It then readily follows from Eq. (97) that

$$\mathscr{P} = \mathscr{I}. \tag{100}$$

This formula shows that under the "best circumstances", the degree of indistinguishability (a measure of WPI) and the degree of polarization are equal. The physical interpretation of this result may be understood from the following consideration: If one has complete "which-path information" (i.e., $\mathscr{I} = 0$), then it follows from Eq. (100) that the degree of polarization of the light emerging from the interferometer is equal to zero. Complete "which-path information" in an single-photon interference experiment implies that a photon shows complete particle nature, and our analysis suggests that in such a case light is completely unpolarized. In the other extreme case, when one has no "which-path information", i.e., when a photon does *not* display any particle behavior, the output light will be completely polarized. Any intermediate case will produce partially polarized light. For details analysis see (Lahiri, 2011).

9. Conclusions

We conclude this chapter by saying that we have given a description of first-order coherence and polarization properties of light. The main aim of this chapter was to emphasize the fact that although, coherence and polarization seem to be two different optical phenomena, both of them can be described by analogous theoretical techniques. Our discussion also emphasizes some newly obtained results in quantum theory of optical coherence in the space-frequency domain, and in quantum theory of polarization of light beams.

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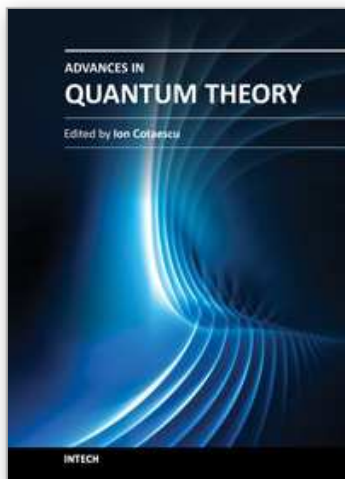
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