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Quantum Gravity in Cantorian Space-Time

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1. Introduction

Einstein's contribution to relativity was initially an intuitive approach based on a basic elimination of simultaneousity and a mathematical reformulation using the Lorenz transformation. In this respect Einstein just added some more physics to what Poincaré and Lorenz have done much earlier. However, it was Minkowski who introduced the geometrical ideas and the use of a four-dimensional space with time as the fourth dimension. Einstein took over Minkowski's idea and initiated what we may call the program of geometrizing physics, starting with gravity. Later on Einstein and Hilbert attempted the unification of electro-magnetism and gravity while Kaluza and Klein tried the same using an extra fifth dimension. This may have been the beginning of the higher dimensional space-time theories culminating in super strings, super gravity and the Cantorian space-time theory [1].

In special relativity there is no absolute time. We have a space and each slice has its own time. Thus each point in the Minkowski's space is specified by four coordinates, three spatial and one temporal in a four-dimensional space-time rather than the 3+1 space plus time coordinates of the classical mechanics [1].

A fundamental role in this new geometry is played by the constancy of the velocity of light that cannot be exceeded without violating the causal structure as well documented experimental facts show. A change of things began by adding quantum mechanics to special relativity.

By replacing Euclidean geometry by curved Riemannian one, Einstein was the first to give gravity a geometrical interpretation as a curvature of space-time due to matter. Einstein never fixed the topology of his theory nor did he use or was aware of the existence of non-classical geometry which was in any case in its infancy [1-5]. The possibly only encounter of Einstein with M. S. El Naschie's Cantorian like transfinite geometry was when K. Menger presented a paper in a conference held in his honour [1, 6-17].

2. Transfinite sets and quantum mechanics

Let us examine the basic concept of a line or more generally a curve. Classical geometry used in classical mechanics and general relativity the fact that a line is a one-dimensional object, while a point is zero-dimensional. Furthermore, it would seem at first sight that a line consists of infinite number of points and that it is simply the path drawn by a zero-

dimensional point moving in the two or three-dimensional space. Classical geometry similar to classical mechanics has made various tacit simplifications and ignored several subtle topological facts [2].

If a line is one-dimensional and if it is made of infinite number of points then the sum of infinitely many zeros should be equal to one. That is of course not true. On the other hand we know that there is a curve called the Peano-Hilbert curve which is area filling and two-dimensional [1, 7-20]. By contrast, we can construct a three dimensional cube known as the Menger sponge which has a fractal dimension more than two and less than three, namely $D = \frac{\log 20}{\log 3}$ as explained for instance in the classical book of Mandelbrot [2].

The existence of all these non-conventional forms described in modern parlance, following Mandelbrot, as fractals, may be traced back to the archetypal transfinite set known as Cantor triadic set [6].

A Cantor set is a set of disjoint points which possesses the same cardinality as the continuum. It may be this coincidence that makes it an ideal compromise between the discrete and the continuum. It is transfinite discrete. Our Cantorian space-time which we will use to "topologize" physics is based on these transfinite sets. The main idea behind the Cantorian space-time approach is to replace the formal analysis of quantum mechanics and the Riemannian space-time geometry of general relativity by a transfinite fractal Cantorian space-time manifold [1, 8, 11, 13].

3. A short historical overview of ideas leading to fractal space-time

The idea of a hierarchy and fractal-like self-similarity in science started presumably first in cosmology before moving to the realm of quantum and particle physics [1]. It is possible that the English clergyman T. Right was the first to entertain such ideas (Fig. 1). Later on the idea reappeared in the work of the Swedish scientist Emanuel Swedenborg (1688-1772) and then much later and in a more mathematical fashion in the work of another Swedish astrophysicist C. Charlier (1862-1934) (Fig. 2).

In 1983, the English-Canadian physicist Garnet Ord wrote a seminal paper [3] and coined the phrase Fractal Space-time. Ord set on to take the mystery out of analytical continuation. We should recall that analytical continuation is what converts an ordinary diffusion equation into a Schrödinger equation and a telegraph equation into a Dirac equation. Analytical continuation is thus a short cut quantization. However what really happened is totally inexplicable. Ord showed using his own (invented) quantum calculus, that analytical continuation which consist of replacing ordinary time t by imaginary time it where $i = \sqrt{-1}$ is not needed if we work in a fractal-like setting, i.e. a fractal space-time. Although rather belated Ord's work has gained wider acceptance in the mean time and was published for instance, in Physics Review [4]. Therefore one is hopeful that his message has found wider understanding. It is the transfinite geometry and not quantization which produces the equations of quantum mechanics. Quantization is just a very convenient way to reach the same result fast, but understanding suffers in the process of a formal analytical continuation.

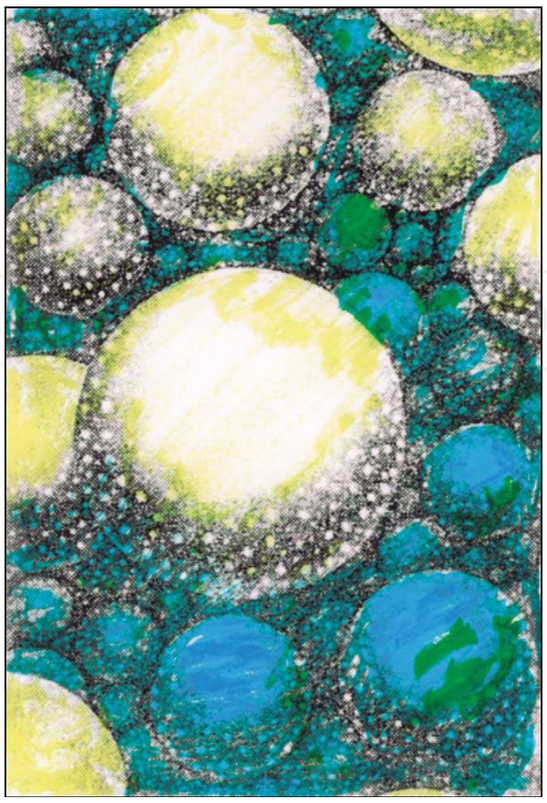


Fig. 1. A vision of T. Right's cosmos as a form of sphere packing, on all scales [1].

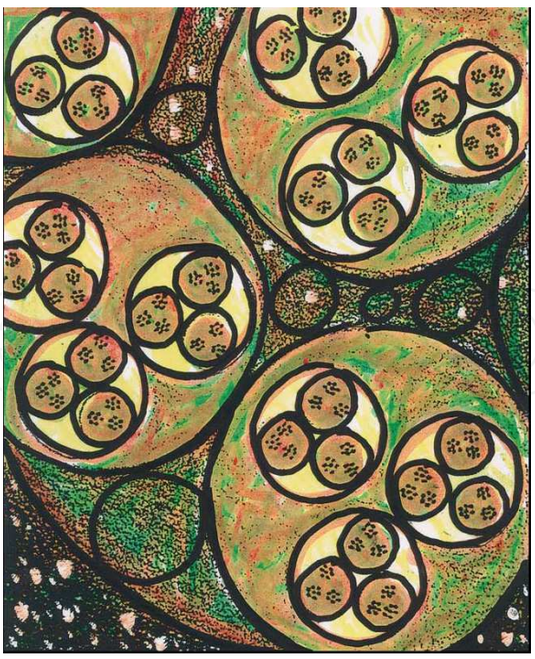


Fig. 2. A vision of a fractal-like universe, with clusters of clusters ad infinitum as envisaged by the Swedish astronomer C. Charlier who lived between 1862 and 1934. This work was clearly influenced by the work of the Swedish astrophysicist A. Swedenborg (1688–1772) [1].

Similar work, but not identical, was carried out by the French cosmologist Laurent Nottale, fifteen years ago. Nottale connected scaling and Einstein's relativity to what is now called scale relativity theory [5]. Around 1990, M. S. El Naschie began to work on his Cantorian version of fractal space-time [6]. In M. S. El Naschie's work on high energy physics and electromagnetic weak interactions the golden mean plays a very important role.

In the year 1995 Nobel laureate Prof. Ilya Prigogine, Otto Rössler and M. S. El Naschie edited an important book [7] in which the basic principles of fractal space-time were spelled out. Sometime later El Naschie using the work of Prigogine on irreversibility showed that the arrow of time may be explained in a fractal space-time. Recently El Naschie gave for the first time a geometrical explanation of quantum entanglement and calculated a probability of the golden mean to the power of five ϕ^5 for the entanglement of two quantum particles [8, 9].

4. Fractals

In this section, we give a very brief account of Cantor sets and fractals which are fundamental to the Cantorian space-time theory.

4.1 Triadic Cantor set and the random Cantor set

The archetypal fractal is what is known as Cantor triadic set. We start by describing the fundamental construction. Consider a unit interval. Let us delete the middle third but leave the end points. We repeat the procedure with the two segments left and so on as shown in Fig. 3 infinitely many times. At the end we obtain an uncountable set of points of measure zero. This means adding all these points together and we obtain a zero length. However, from the point of view of transfinite set theory something very profound is left, namely a transfinite points set with a finite dimension, the so-called Hausdorff-Besicovitch dimension [2]

$$d_C = \frac{\ln 2}{\ln 3} \cong 0.63$$

Mauldin and Williams replaced the orderly triadic construction by a random construction. In their original paper [10] they said they used a uniform probabilistic distribution. The Mauldin-Williams theorem which states that with the probability equal to one, a one dimensional randomly constructed Cantor set will have the Hausdorff-Besicovitch dimension

$$d_C^{(0)} = \frac{\sqrt{5}-1}{2}$$

The Menger-Urysohn dimension of all Cantor sets is zero, while the empty set has the dimension minus one [11].

4.2 The Sierpinski triangle, Menger sponge and their random analogues

The generalization of the one-dimensional triadic Cantor set to two-dimensions is called the Sierpinski triangle. It is constructed as shown in Fig. 3 and the Hausdorff-Besicovitch dimension is given by the inverse of the triadic Cantor set [2]

$$d_S = \frac{\ln 3}{\ln 2} \cong 1.5849$$

It is important to note that the Sierpinski triangle is a curve and its dimension lies between the classical line and the classical area.

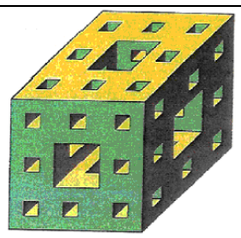
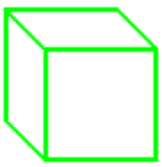
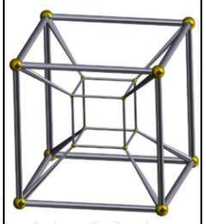
Type of fractal	Geometrical shape	Menger–Urysohn dimension	Hausdorff dimension	Corresponding random Hausdorff dimension	Embedding dimension	Corresponding Euclidean shape
Cantor Set		0	d_C=*ln2/ln3* *0.630929*	$\phi = 0.618033$	1	*Line*
Sierpinski gasket		2	d_S=*ln3/ln2* *1.5849625*	$\frac{1}{\phi} = 1 + \phi =$ 1.618033	2	 *Square*
Menger sponge		3	d_M= *ln20/ln3* *2.7268*	$2 + \phi =$ 2.618033	3	 *Cube*
The 4 dimension random Cantor set		4	$d_C^{(4)} =$ *4.236068*	$4 + \phi^3 =$ 4.236067	5	 *Hyper cube*

Fig. 3. In this figure we draw analogy between smooth spaces as a line, a square, a cube, a higher-dimensional cube and the Cantor set, the Sierpinski triangle, the Menger sponge and the Cantorian space-time which is difficult to draw. The calculation of the Hausdorff-Besicovitch dimension of classical fractals and their random version is presented [1, 13].

It was shown in the Cantorian space-time theory [12] that the generalization of the formula connecting the triadic Cantor set with the Sierpinski triangle is possible for n dimension and is given by the so-called bijection formula

$$d_C^{(n)} = \left(\frac{1}{d_C^{(0)}} \right)^{n-1}$$

For $d_C^{(0)} = \phi$, the random contra part of the Sierpinski triangle will have the Hausdorff-Besicovitch dimension equal to [13]

$$d_C^{(2)} = \left(\frac{1}{\phi}\right)^{2-1} = \frac{1}{\phi} \cong 1.61803$$

A most remarkable 3D fractal is the Menger sponge which is shown in Fig. 3. The Hausdorff-Besicovitch dimension of this fractal is given by [2]

$$d_M = \frac{\ln 20}{\ln 3} \cong 2.7268$$

The volume of the Menger sponge is zero. The random version of the Menger sponge has a Hausdorff- Besicovitch dimension equal to [1, 13]

$$d_C^{(3)} = \left(\frac{1}{\phi}\right)^{3-1} = \left(\frac{1}{\phi}\right)^{2} = 2 + \phi$$

Using the bijection formula we can calculate any higher dimensional fractals [8, 11].

One of the most far reaching and fundamental discoveries using the zero measure Cantor sets is undoubtedly that of El Naschie probability of quantum entanglement. His result for two entangled particles is a generic and universal value of the golden mean to the power of five. This is exactly equal to the famous result of Lucien Hardy [8, 9]. Quantum entanglement is thus explained as a consequence of zero measure gravity. Similarly one could explain any velocity larger than the speed of light [8, 9].

5. Construction of a random Cantor set and the Cantorian space-time

The main idea of the Cantorian space-time theory is in fact a sweeping generalization of what Einstein did in his general relativity, namely introducing a new geometry of space-time which differs considerably from the space-time of our sensual experience. This space-time is taken to be Euclidean. By contrast, general relativity persuaded us that the Euclidean 3+1 dimensional space-time is only an approximation and that the true geometry of the universe in the large is in reality a four-dimensional curved manifold [1, 11].

In the Cantorian space-time theory we take a similar step and allege that space-time at quantum scales is far from being the smooth, flat and passive space which we use in the classical physics. On extremely small scales, at very high observational resolution equivalent to a very high energy, space-time resembles a vacuum fluctuation and in turn modeling this fluctuation using the mathematical tools of non-linear dynamics, complexity theory and chaos. In particular, the geometry of chaotic dynamics, namely the fractal geometry is reduced to its quintessence, i.e. Cantor sets. A Cantor set has no ordinary real physical existence, because its Lebegue measure is zero and nonetheless it exists indirectly because it does have a well defined non-zero quantity, namely its Hausdorff-Besicovitch dimension. The triadic Cantor set possesses a Hausdorff-Besicovitch dimension equal to $D = \frac{\log 2}{\log 3} \cong 0.63.$

For a randomly constructed Cantor set on the other hand, the Hausdorff-Besicovitch dimension is found to take the surprising value of the inverse of the golden mean $D = \frac{\sqrt{5}-1}{2} \cong 0.61803$ by virtue of the Mauldin-Williams theorem [10].

In 1986 R. Mauldin and S. Williams proved a remarkable theorem which confirmed the main conclusion of the Hausdorff-Besicovitch dimension of the Cantorian space-time. To explain the Mauldin-Williams theorem let us construct a Cantor set of the interval *[0, 1]* via a random algorithm as follows. First we chose at random an *x* according to the uniform distribution on *[0, 1]*, then between *x* and *1* we chose *y* at random according to the uniform distribution on *[x, 1]*. That way we obtain two intervals *[0, X]* and *[Y, 1]*. Next we repeat the same procedure on *[0, X]* and *[Y, 1]* independently and so on. Continuation of this procedure leads then to a random Cantor dust and the Hausdorff-Besicovitch dimension of this set will be with a probability one equal to $\phi = \frac{\sqrt{5}-1}{2} \cong 0.61803$ [10].

Cantorian space-time is made of an infinite number of intersections and unions of the randomly constructed Cantor sets. Let us denote the Hausdorff-Besicovitch dimension of these Cantor sets by $d_C^{(0)}$. Next we use $(d_C^{(0)})^n$ as a statistical weight for the topological dimension *n=1* to ∞ and determine the average dimension $\langle n \rangle$, i.e. the expectation value of *n*. This value is easy to find following the centre of gravity theorem of probability theory to be [1, 8, 11]

$$\langle n \rangle = \frac{\sum_{n=0}^{\infty} n^2 (d_C^{(0)})^n}{\sum_{n=0}^{\infty} n (d_C^{(0)})^n}$$

Since

$$\sum_{n=0}^{\infty} n (d_C^{(0)})^n = \frac{d_C^{(0)}}{(1-d_C^{(0)})^2}$$

and

$$\sum_{n=0}^{\infty} n^2 (d_C^{(0)})^n = \frac{d_C^{(0)}(1+d_C^{(0)})}{(1-d_C^{(0)})^3}$$

one finds that

$$\langle n \rangle = \frac{1+d_C^{(0)}}{1-d_C^{(0)}}.$$

Next let us calculate the average Hausdorff-Besicovitch dimension $\langle d_C \rangle$. We sum together all the Hausdorff-Besicovitch dimensions $(d_C^{(0)})^{(0)}$, $(d_C^{(0)})^{(1)}$, $(d_C^{(0)})^{(2)}$ ....., following the

formula for the infinite convergent geometric sequence, $(d_C^{(0)})^{(0)}$, $(d_C^{(0)})^{(1)}$, $(d_C^{(0)})^{(2)}$,....., where $0\langle d_C^{(0)}\langle 1$, we obtain [1, 11]

$$\sum_{n=0}^{\infty}(d_C^{(0)})^n = \frac{1}{1-d_C^{(0)}}.$$

The average Hausdorff-Besicovitch dimension is thus

$$\langle d_C \rangle = \frac{\sum_{n=0}^{\infty}(d_C^{(0)})^n}{d_C^{(0)}} = \frac{1}{d_C^{(0)}(1-d_C^{(0)})}.$$

If the Cantorian space-time is to be without gapes and overlapping [1, 14] then we must set $\langle n \rangle$ equal to $\langle d_C \rangle$. Proceeding that way one finds from [1, 14] the following Peano-Hilbert space filling condition $\langle n \rangle = \langle d_C \rangle$ that

$$\frac{1+d_C^{(0)}}{1-d_C^{(0)}} = \frac{1}{d_C^{(0)}(1-d_C^{(0)})}.$$

Thus we have

$$(1+ d_C^{(0)})\, d_C^{(0)} = 1$$

or

$$(d_C^{(0)})^{(2)} + d_C^{(0)} - 1 = 0.$$

This is a quadratic equation with two solutions

$$d_{C,1}^{(0)} = \frac{\sqrt{5}-1}{2} = \phi$$

$$d_{C,2}^{(0)} = -\frac{1}{\phi}.$$

Inserting back in $\langle n \rangle$ and $\langle d_C \rangle$ the solution $d_{C,1}^{(0)} = \phi$, one finds that

$$\langle n \rangle = \frac{1+\varphi}{1-\varphi} = \frac{1}{\varphi^3} = 4+ \phi^3$$

and

$$\langle d_C \rangle = \frac{1}{\phi(1-\phi)} = \frac{1}{\phi^3} = 4 + \phi^3$$

where $\phi + \phi^2 = 1,\ \phi = \frac{\sqrt{5}-1}{2}$.

Our next aim is to solve the problem of lifting the random Cantor set $d_C^{(0)}$ to higher dimensions *n* and find $d_C^{(n)}$ for a given $d_C^{(0)}$.

The solution of this problem comes from the fact that the generalisation of the triadic set to two dimensions is the Sierpinski gasket. The Hausdorff-Besicovitch dimension of the gasket is the inverse value of the Hausdorff-Besicovitch dimension of the triadic set $\frac{\log 2}{\log 3}$. Therefore one could write [1, 11]

$$d_C^{(2)} = \frac{1}{d_C^{(0)}} = \left(\frac{1}{d_C^{(0)}}\right)^{2-1}.$$

The generalisation by analogy and induction can thus be written as [12]

$$d_C^{(n)} = \left(\frac{1}{d_C^{(0)}}\right)^{n-1}.$$

Now let us examine the case for space filling, i.e. $d_C^{(0)} = \phi$ and four-dimensionality.

This way we obtain the Hausdorff-Besicovitch dimension of the Cantorian space-time

$$d_C^{(4)} = \left(\frac{1}{\phi}\right)^3 = 4 + \phi^3 \cong 4.236067.$$

This is a remarkable result which means that the formally infinite dimensional but hierarchical Cantorian space-time looks from a distance as if it were four-dimensional with the Hausdorff-Besicovitch dimension equal to $4 + \phi^3 \cong 4.236067$.

The preceding derivation could be regarded as a proof for the essential four-dimensionality of our physical space-time. We perceive space-time to be four-dimensional because this is the expectation value of our infinite dimensional Cantorian space-time.

6. Summing over paths and summing over all dimensions in the Cantorian space-time

We recall that Feynman gave an alternative formulation of quantum mechanics in which one calculates amplitudes by summing over all possible trajectories of a system weighted by $e^{\frac{is}{\hbar}}$, where *s* is the classical action, $i = \sqrt{-1}$ and $\hbar$ the Planck quantum. For one particle the path integral is thus [15]

$$Z = \int e^{\frac{is}{\hbar}} [dx]$$

where [dx] means that we are summing over all possible paths of the concerned particle. What is important here is to realize that from all of infinitely many paths which a quantum particle can take some are more probable than others. The probability of the actual path, that is to say the amplitude of an event is the sum over the amplitude corresponding to all paths. Thus we have a weight assigned to each path in the Feynman formulation of quantum mechanics.

In the Cantorian space-time theory we proceed in an analogous way. However, instead of summing over all paths, we sum over all dimensions of infinite dimensional hierarchical Cantorian space-time. El Naschie has recently demonstrated that E-Infinity is a Suslin operation and the so-called Suslin A operation [9]. In this theory Suslin scaling replace the classical Lagrangian and the classical calculus using descriptive set theory [16, 17].

7. Cantorian space-time and Newton's non-dimensional gravity constant

Quantum non-dimensional gravity constants can be derived from descriptive set theory [16]. In descriptive set theory and theory of polish spaces it is shown that [16, 17]:

Definition 1:

When a space A^N is viewed as the product of infinitely many copies of A with discrete topology and is completely metrizable and if A is countable then the space is said to be polish.

Two cases are of considerable importance.

Definition 2:

When a space is polish and when $A = 2 = [0,1]$, then we call $C = 2^N$ the Cantor space.

Definition 3:

When a space is polish and when $A = N$ then we call $B = N^N$ the Baire space.

Now we can proceed to explain the relationship between the Cantor space and Cantorian space-time. The relationship comes from the solution of the cardinality problem of a Borel set in polish spaces. Thus, we call a subset of a topological space a Cantor set if it is homeomorphic to a Cantor space [16, 17].

Theorem 1:

Let X be polish and $Y \subseteq X$ be a Borel set. Then either Y is countable or else it contains a Cantor set. In particular every uncountable standard Borel space has cardinality 2.

A Cantor space is homeomorphic to a triadic Cantor set and also to the random Cantor set [17]. The relation between the triadic Cantor set and the Cantor space establishes the relationship between the Cantor space and the Cantorian space-time, since the Cantorian space-time is a hierarchical infinite dimensional Cantor set with the expectation Hausdorff-Besicovitch dimension $4 + \phi^3 \cong 4.236067$.

In particular, it has been shown [17] that when interpreting $\frac{1}{d_C^{(0)}}$ in the bijection formula as the average $\left\langle \frac{1}{d_C^{(0)}} \right\rangle = 2$ of the fundamental Wisse-Abbot theorem and taking N = 128,

($\overline{\alpha}_{ew} = 128$ is the inverse coupling constant measured at the electroweak scale) then the bijection formula

$$d_C^{(n)} = \left(\frac{1}{d_C^{(0)}} \right)^{n-1}$$

gives for $N = n = \overline{\alpha}_{ew} = 128$ the following

$$d_C^{(128)} = \left(\left\langle \frac{1}{d_C^{(0)}} \right\rangle \right)^{128-1} = 2^{127} \cong (1.70141)(10)^{38}$$

where $C = 2^N = d_C^{(128)}$. The value $(1.70141)(10)^{38}$ is the non-dimensional gravity constant $\overline{\alpha_G}$ which is defined as

$$\overline{\alpha_G} = \frac{\hbar c}{G m_p^2} \cong (1.7)(10)^{38}$$

It is of interest to mention that a similar result was found empirically by F. Parker Rhodes which was the subject of extensive discussions by Noyes [18]

$$\overline{\alpha_G} = 2^{127} + 137 = (1.7)(10)^{38}$$

Here $\hbar$ is the Planck quantum, c the speed of light, G the Newton's gravity constant, m_p the Planck mass.

8. Cantorian space-time and the connectivity dimension

Next we show the logarithmic scaling which will connect the non-dimensional gravity constant to the most fundamental equation namely the bijection formula. We start by taking the logarithm of both sides of the equation

$$\ln d_C^{(n)} = \ln \left(\frac{1}{d_C^{(0)}} \right)^{n-1}.$$

That means

$$\ln d_C^{(n)} = (n-1) \ln \left(\frac{1}{d_C^{(0)}} \right)$$

solving for n one finds that

$$n = \frac{\ln d_C^{(n)}}{\ln \left(\frac{1}{d_C^{(0)}} \right)} + 1.$$

Setting $\left(\frac{1}{d_C^{(0)}}\right)=2$ and $d_C^{(n)}$ =Z, where Z is the partition function, one finds

$$n=\frac{\ln Z}{\ln 2}+1.$$

The above formula is very well-known in the combinatorial topology [14, 19, 20] and is called the connectivity dimension. Now if we conceive of $\overline{\alpha_G}$ as being the expectation value of the partition function of the observable universe then the connectivity dimension would be

$$D=\frac{\ln\overline{\alpha_G}}{\ln 2}+1\cong 128=\overline{\alpha_{ew}}$$

This is the inverse of the Sommerfeld electromagnetic fine structure constant measured at the electroweak scale [1, 13].

9. Fundamental constants of Cantorian space-time

The fine-structure constant usually denoted with *a*, is a fundamental physical constant, namely the coupling constant characterizing the strength of the electromagnetic interaction. It is a dimensionless quantity and is defined as

$$\alpha=\frac{e^2}{(4\pi\varepsilon_0)\hbar c}=\frac{1}{137.035999074}$$

or as the inverse fine-structure constant

$$\overline{\alpha}=\frac{1}{\alpha}=137.03599907$$

where e is the unit electron, $\hbar = h/2\pi$ is the Planck constant, c is the speed of light, ε_0 permittivity of free space.

In the Cantorian space-time theory the inverse fine-structure constant $\overline{\alpha_0}$ can be written in a remarkable short form based upon the multiplication and addition theorems of probability theory [1]. This is done by interpreting $d_C^{(0)}=\phi$ as a topological probability of a Cantor set formed by the ratio of the Hausdorff- Besicovitch dimension $d_C^{(0)}=\phi$ and the embedding topological dimension $d_C^{(1)}=1$.

That way one finds

$$\overline{\alpha_0}=(2)(10)(\frac{1}{d_C^{(0)}})^4$$

or

$$\overline{\alpha_0}=(2)(10)(\frac{1}{\phi})^4=137.082039.$$

The value 137.082039 is in excellent agreement with the measured experimental value.

From the inverse fine-structure constant $\overline{\alpha_0}$ we can derive the inverse coupling constant of the non-super symmetric unification of all forces $\overline{\alpha_g}$ and the inverse coupling constant of the super symmetric unification of all forces $\overline{\alpha_{gs}}$ using the scaling arguments in the Cantorian space-time [1] . The scaling factor in the Cantorian space-time is ϕ. To derive the inverse coupling constant of the non-super symmetric unification of all forces $\overline{\alpha_g}$ we start with the Cooper pair. That means we multiply $\frac{\overline{\alpha_0}}{2}$ with ϕ and obtain the following result [20]

$$\frac{\overline{\alpha_0}}{2}\phi = 42.360679 = \overline{\alpha_g}.$$

Proceeding in this way one finds the inverse coupling constant of the super symmetric unification of all forces $\overline{\alpha_{gs}}$. We multiply $\frac{\overline{\alpha_0}}{2}$ with ϕ^2 and obtain

$$\frac{\overline{\alpha_0}}{2}\phi^2 = 26.18033989 = \overline{\alpha_{gs}}.$$

Both inverse coupling constants are in full agreement with the experimental values [1, 13, 20].

10. Conclusion

In the present review article we gave a short overview of ideas leading to the fractal space-time and the Cantorian space-time theory. The triadic set, the Sierpinski gasket, the Menger sponge and their random analogous are introduced. The Cantorian space-time is determined by three dimensions, the formal $n_f = \infty$, the topological n_T = 4 and the Hausdorff-Besicovitch dimension equal to $4 + \phi^3 \cong 4.236067$.

Feynman introduced a procedure which consists of summing over all possible paths of the concerned particle. In the Cantorian space-time theory the procedure is analogous, but instead of summing over all paths we sum over all dimensions of the infinite dimensional but hierarchical Cantorian space-time.

We establish a conceptual and quantitative connection between classical gravity and the electro-weak field using the Cantorian space-time theory and the descriptive set theory. This led El Naschie to a fundamental discovery for quantum entanglement [8, 9, 21, 22].

With the use of the golden mean scaling operator we derive an expectation value of the inverse electromagnetic fine structure constant $\overline{\alpha_0}$, the inverse coupling constant of the non-super symmetric unification of all forces $\overline{\alpha_g}$ and the inverse coupling constant of the super symmetric unification of all forces $\overline{\alpha_{gs}}$.

11. Acknowledgement

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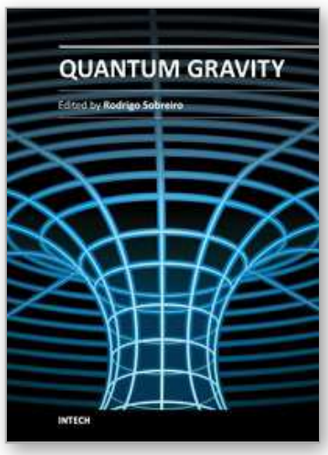
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The unification between gravity and quantum field theory is one of the major problems in contemporary fundamental Physics. It exists for almost one century, but a final answer is yet to be found. Although string theory and loop quantum gravity have brought many answers to the quantum gravity problem, they also came with a large set of extra questions. In addition to these last two techniques, many other alternative theories have emerged along the decades. This book presents a series of selected chapters written by renowned authors. Each chapter treats gravity and its quantization through known and alternative techniques, aiming a deeper understanding on the quantum nature of gravity. Quantum Gravity is a book where the reader will find a fine collection of physical and mathematical concepts, an up to date research, about the challenging puzzle of quantum gravity.

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