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Optical Solitons in a Nonlinear Fiber Medium with Higher-Order Effects

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1. Introduction

Nowadays we can see many interesting applications of solitons in different areas of physical sciences such as plasma physics (1), nonlinear optics (2; 3), Bose-Einstein condensate (4; 5), fluid mechanics (6), and so on. Solitons are so robust particles that they are unlikely to breakdown under small perturbations. The most interesting factor about the soliton, however, is that their interactions with the medium through which it propagates is elastic. Recent researches on nonlinear optics have shown that dispersion-managed pulse can be more useful if the pulse is in the form of a power series of a stable localized pulse which is called soliton.
Optical solitons have been the objects of extensive theoretical and experimental studies during the last four decades, because of their potential applications in long distance communication. In 1973, the pioneering results of Hasegawa and Tappert (7) proved that the major constraint in the optical fiber, namely, the group velocity dispersion (GVD) could be exactly counterbalanced by the self-phase modulation (SPM). SPM is the dominant nonlinear effect in silica fibers due to the Kerr effect. The theoretical results of Hasegawa and Tappert were greatly supported by the experimental demonstration of optical solitons by Mollenauer et al. (8) in 1980. Since then many theoretical and experimental works have been done to achieve a communication system based on optical solitons.
The solitons, localized-in-time optical pulses, evolve from a nonlinear change in the refractive index of the material, known as Kerr effect, induced by the light intensity distribution. When the combined effects of the intensity-dependent refractive index nonlinearity and the frequency-dependent pulse dispersion exactly compensate for one another, the pulse propagates without any change in its shape, being self-trapped by the waveguide nonlinearity. The propagation of optical solitons in a nonlinear dispersive optical fiber is governed by the well-known completely integrable nonlinear Schrödinger (NLS) equation

$$i\frac{\partial q}{\partial z}+\epsilon\frac{\partial^2 q}{\partial \tau^2}+|q|^2 q=0, \qquad \epsilon=\pm 1, \tag{1}$$

where q is the complex amplitude of the pulse envelope, τ and z represent the spatial and temporal coordinates, and the $+$ and $-$ sign of ϵ before the dispersive term denote the anomalous and normal dispersive regimes, respectively. In the anomalous dispersive regime, this equation possesses a bright soliton solution, and in the normal dispersive regime it possesses dark solitons. The bright soliton and dark soliton solutions can be derived by

the inverse-scattering transform method with vanishing (9; 11) and nonvanishing boundary conditions (10).
However, if optical pulses are shorter, the standard NLS equation becomes inadequate. Therefore, some higher-order effects such as third-order dispersion, self-steepening, and stimulated Raman scattering, will play important roles in the propagation of optical pulses. In such a case, the governing equation is the one known widely as the higher-order NLS equation, first derived by Kodama and Hasegawa (12). The effect of these effects in uncoupled and coupled systems for bright solitons is well explained (13; 14). Inelastic Raman scattering is due to the delayed response of the medium, which forces the pulse to undergo a frequency shift which is known as a self-frequency shift. The effect of self-steepening is due to the intensity-dependent group velocity of the optical pulse, which gives the pulse a very narrow width in the course of propagation. Because of this, the peak of the pulse will travel more slowly than the wings.
In practice, the refractive index or the core diameter of the optical fiber are fucntions of the axial coordinate, which means that the fiber is actually axially inhomogeneous. In this case, the parameters which characterize the dispersive and nonlinear properties of the fiber exhibit variations and the corresponding nonlinear wave equations are NLS equations with variable coefficients. Moreover, the problem of ultrashort pulse propagation in nonlinear and axially inhomogeneous optical fibers near the zero dispersion point is more complicated because the high order effects have to be taken into account as well. In order to understand such phenomena, we consider the higher-order NLS (HNLS) equation with variable coefficients

$$\frac{\partial u}{\partial z} = i(d_1\frac{\partial^2 u}{\partial \tau^2} + d_2|u|^2u) + d_3\frac{\partial^3 u}{\partial \tau^3} + d_4\frac{\partial(u|u|^2)}{\partial \tau} + d_5u\frac{\partial|u|^2}{\partial \tau} + d_6u, \tag{2}$$

where u is the slowly varying envelope of the pulse, d_1, d_2, d_3, d_4, d_5 and d_6 are the z-dependent real parameters related to GVD, SPM, third-order dispersion (TOD), self-steepening, and stimulated Raman scattering (SRS), and the heat-insulating amplification or loss, respectively. Though Eq. (2) was first derived in the year 1980s, only for the past few years, it has attracted much attention among the researchers from both theoretical and experimental points of view. For example, Porsezian and Nakkeeran (13) derive all parametric conditions for soliton-type pulse propagation in HNLS equation using the Painlevé analysis, and generalize the Ablowitz-Kaup-Newell-Segur method to the 3×3 eigenvalue problem to construct the Lax pair for the integrable case. Papaioannou et al. (15) give an analytical treatment of the effect of axial inhomogeneity on femtosecond solitary waves near the zero dispersion point which governed by the variable-coefficient HNLS equation. The exact bright and dark soliton wave solutions of this variable-coefficient equation are derived and their behaviors in the presence of the inhomogeneity are analyzed. Mahalingam and Porsezian (16) analyze the propagation of dark solitons with higher-order effects in optical fibers by Painlevé analysis and Hirota bilinear method. Xu et al. (17) investigate the modulation instability and solitons on a cw background in an optical fiber with higher-order effects. In addition, there have recently been several papers giving W-shaped solitary wave solution in the HNLS equation.
However, in recent years the studies of Eq. (2) have not been widespread. In this chapter, we consider equation (2) again and derive some exact soliton solutions in explicit form for specified soliton management conditions. We first change the variable-coefficient HNLS equation into the well-known constant-coefficient HNLS equation through similarity transformation. Then the Lax pairs for two integrable cases of the constant-coefficient HNLS equation are constructed explicitly by prolongation technique, and the novel exact

bright N-soliton solutions for the bright soliton version of HNLS equation are obtained by Riemann-Hilbert formulation. Finally, we examine the dynamics and present the features of the optical solitons. It is seen that the bright two-soliton solution of the HNLS equation behaves in an elastic manner characteristic of all soliton solutions. These results are useful in the design of transmission lines with spatial parameter variations and soliton management to future research.

2. Similarity transformation

A direct and efficient method for investigating the variable-coefficient nonlinear wave equation is to transform them into their constant-coefficient counterparts by similarity transformation. To do so, we firstly take the similarity transformation (18; 19)

$$u = \rho\, q\,(T, X)\, e^{i(a_1\tau + a_2)}, \tag{3}$$

to reduce Eq. (2) to the constant-coefficient HNLS equation

$$\frac{\partial q}{\partial T} = i(\alpha_1 \frac{\partial^2 q}{\partial X^2} + \alpha_2 |q|^2 q) + \epsilon(\alpha_3 \frac{\partial^3 q}{\partial X^3} + \alpha_4 \frac{\partial (q|q|^2)}{\partial X} + \alpha_5 q \frac{\partial |q|^2}{\partial X}), \tag{4}$$

where $q = q(T, X)$ is the complex amplitude of the pulse envelope, the parameter ϵ ($0 < \epsilon < 1$) denotes the relative width of the spectrum that arises due to the quasi-monochromocity, $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are the real constant parameters. In Eq. (3), ρ, T, a_1 and a_2 are functions of z, and X is a function of τ and z.

Substituting Eq. (3) into Eq. (2) and asking $q\,(T, X)$ to satisfy the constant-coefficient HNLS equation (4), we have a set of partial differential equations (PDEs)

$$d_1 X_{\tau\tau} + 3\, d_3 X_{\tau\tau} a_1 = 0,\ \ d_3 {X_\tau}^3 = \alpha_3 T_z,\ \ \rho_z = d_6 \rho,$$

$$2\, d_1 X_\tau a_1 + X_z + 3\, d_3 X_\tau {a_1}^2 = d_3 X_{\tau\tau\tau},\ \ \rho^2 d_4 a_1 + \rho^2 d_2 = \alpha_2 T_z,$$
$$2\,\rho^2 X_\tau d_4 + \rho^2 X_\tau d_5 = 2\,\alpha_4 T_z + \alpha_5 T_z,\ \ \rho^2 X_\tau d_4 + \rho^2 X_\tau d_5 = \alpha_4 T_z + \alpha_5 T_z,$$
$$d_3\, {a_1}^3 + a_{1z}\tau + a_{2z} + d_1\, {a_1}^2 = 0,\ \ 3\, d_3 {X_\tau}^2 a_1 + d_1 {X_\tau}^2 = \alpha_1 T_z,\ \ X_{\tau,\tau} = 0,$$

where the subscript denotes the derivative with respect to z and τ. Solving this set of PDEs, we have $X = k\tau + f$ and

$$a_1 = c,\ \ d_1 = \frac{T_z\,(k\alpha_1 - 3\,\alpha_3 c)}{k^3},\ \ d_2 = \frac{T_z\,(\alpha_2 k - \alpha_4 c)}{\rho^2 k},\ \ d_3 = \frac{\alpha_3 T_z}{k^3},$$

$$d_4 = \frac{\alpha_4 T_z}{\rho^2 k},\ \ d_5 = \frac{\alpha_5 T_z}{\rho^2 k},\ \ f = \frac{c\,(3\,\alpha_3 c - 2\,k\alpha_1)\,T}{k^2},\ \ a_2 = \frac{(2\,\alpha_3 c - k\alpha_1)\,c^2 T}{k^3},$$

where $\rho = \rho_0\, e^{\int d_6 dz}$, k, ρ_0 and c are constants, and T and d_6 are arbitrary functions of z. So the similarity transformation (3) becomes

$$u = \rho_0\, e^{\int d_6 dz} q\left(T, \frac{k^3\tau - 2\,ck\alpha_1 T + 3\,c^2\alpha_3 T}{k^2}\right) e^{ic(k^3\tau + 2\,c^2\alpha_3 T - ck\alpha_1 T)/k^3}. \tag{5}$$

Therefore, if we can get the exact soliton solutions of the constant-coefficient HNLS equation (4) we can obtain the exact soliton solutions for HNLS equation (2) through Eq. (5). In the next section, we will investigate the integrable condition of equation (4) by prolongation technique.

3. Prolongation structures of the constant-coefficient HNLS equation

In this section, we investigate the prolongation structures of the constant-coefficient HNLS equation (4) by means of the prolongation technique (20–22). Firstly, the complex conjugate of the dependent variable q in Eq. (4) is denoted as $q^* = u$. Then, Eq. (4) and its conjugate become

$$i\alpha_1 q_{XX} + i\alpha_2 q^2 u + \epsilon[\alpha_3 q_{XXX} + (\alpha_4+\alpha_5)q^2 u_X + (2\alpha_4+\alpha_5)quq_X] - q_T = 0, \tag{6a}$$

$$-i\alpha_1 u_{XX} - i\alpha_2 u^2 q + \epsilon[\alpha_3 u_{XXX} + (\alpha_4+\alpha_5)u^2 q_X + (2\alpha_4+\alpha_5)quu_X] - u_T = 0. \tag{6b}$$

Next we introduce four new variables p, r, v and w by

$$q_X = p, \;\; p_X = r, \;\; u_X = v, \;\; v_X = w, \tag{7}$$

and define a set of differential 2-form $I = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\}$ on solution manifold $M = \{T, X, u, v, w, p, q, r\}$, where

$$\theta_1 = dq \wedge dT + pdT \wedge dX, \;\; \theta_2 = dp \wedge dT + rdT \wedge dX,$$

$$\theta_3 = du \wedge dT + vdT \wedge dX, \;\; \theta_4 = dv \wedge dT + wdT \wedge dX,$$

$$\theta_5 = dq \wedge dX + \alpha_3 dr \wedge dT + \rho_1 dX \wedge dT, \;\; \theta_6 = du \wedge dX + \alpha_3 dw \wedge dT + \rho_2 dX \wedge dT,$$

with

$$\rho_1 = i\alpha_1 r + i\alpha_2 q^2 u + \epsilon[(\alpha_4+\alpha_5)q^2 v + (2\alpha_4+\alpha_5)qup],$$

$$\rho_2 = -i\alpha_1 w - i\alpha_2 u^2 q + \epsilon[(\alpha_4+\alpha_5)u^2 p + (2\alpha_4+\alpha_5)quv].$$

When these differential 2-forms restricted on the solution manifold M become zero, we recover the original constant-coefficient HNLS equation (4). It is easy to verify that I is a differential closed idea, i.e. $dI \subset I$.

We further introduce n differential 1-forms

$$\Omega^i = d\zeta^i - \tilde{F}^i dX - \tilde{G}^i dT, \tag{8}$$

where $i = 1, 2, \cdots, n$, $\tilde{F}^i$ and $\tilde{G}^i$ are functions of $u, v, w, p, q, r, \zeta^i$ and are assumed to be both linearly dependent on ζ^i, namely $\tilde{F}^i = F^i\zeta^i$, $\tilde{G}^i = G^i\zeta^i$. For the sake of simplification, we drop the indices by rewriting ζ^i as ζ, F^i as F and G^i as G. When restricting on solution manifold, the differential 1-forms Ω^i are null, i.e. $\Omega^i = 0$ which is just the linear spectral problem $\zeta_X = F\zeta$ and $\zeta_T = G\zeta$.

Following the well-known prolongation technique, the extended set of differential form $\tilde{I} = I \bigcup \left\{\Omega^i\right\}$ must be a closed ideal under exterior differentiation, i.e. $d\tilde{I} \subset \tilde{I}$. Because $dI \subset I \subset \tilde{I}$, we only need to let $d\left\{\Omega^i\right\} \subset \tilde{I}$, which denotes that

$$d\Omega^i = \sum_{j=1}^{6} f_j^i \theta^j + \eta^i \wedge \Omega^i, \qquad i = 1, 2, \cdots, n, \tag{9}$$

where f_j^i $(j = 1, 2, 3, 4, 5, 6)$ are functions of (T, X), and $\eta^i = g^i(T, X)dX + h^i(T, X)dT$ are differential 1-forms.

When Eq. (9) is written out in detail, after dropping the indices we have the following PDEs about F and G as

$$G_r = \epsilon\alpha_3 F_q, \quad G_w = \epsilon\alpha_3 F_u,$$

$$G_q p + G_p r + G_u v + G_v w - F_q\left[i\alpha_1 r + i\alpha_2 q^2 u + \epsilon\left(\alpha_4+\alpha_5\right)q^2 v + \epsilon\left(2\,\alpha_4+\alpha_5\right)qup\right] \tag{10}$$

$$-F_u\left[-i\alpha_1 w - i\alpha_2 u^2 q + \epsilon\left(\alpha_4+\alpha_5\right)u^2 p + \epsilon\left(2\,\alpha_4+\alpha_5\right)quv\right] - [F,G] = 0,$$

with $[F,G] = FG - GF$.
Solving Eq. (10), we have the expressions of F and G as

$$F = x_0 + x_1 q + x_2 u, \tag{11}$$

$$G = \epsilon\,\alpha_3 x_1 r + \epsilon\,\alpha_3 x_2 w + vq\epsilon\,\alpha_3 x_5 + v\epsilon\,\alpha_3 x_4 - pu\epsilon\,\alpha_3 x_5 + p\epsilon\,\alpha_3 x_3 - iqu\alpha_1 x_5$$
$$+ipx_1\alpha_1 + qx_2\epsilon\,u^2\alpha_4 + \frac{2}{3}\,qx_2\epsilon\,u^2\alpha_5 + \frac{1}{2}\,\epsilon\,\alpha_3 u^2 x_{13} + q^2\epsilon\,x_1 u\alpha_4 + \frac{2}{3}\,q^2\epsilon\,x_1 u\alpha_5$$
$$+qu\epsilon\,\alpha_3 x_8 + qu\epsilon\,\alpha_3 x_{10} + \epsilon\,\alpha_3 x_7 u + \frac{1}{2}\,q^2\epsilon\,\alpha_3 x_9 + q\epsilon\,\alpha_3 x_6 + iq\alpha_1 x_3 - i\alpha_1 x_4 u - ivx_2\alpha_1 + x_{15},$$

where $L = \{x_0, x_1, x_2, \cdots, x_{15}\}$ is an incomplete Lie algebra which is called prolongation algebra and it satisfies the following commutation relations

$$[x_2,x_5] = x_{14},\ \ x_2\alpha_5 = 3\,\alpha_3 x_{14}, 2\,x_8 + x_{10} = x_{12}, x_1\alpha_5 + 3\,\alpha_3 x_{11} = 0,$$

$$[x_0,x_1] = x_3,\ \ [x_0,x_2] = x_4,\ \ [x_0,x_3] = x_6,\ \ [x_0,x_4] = x_7,\ \ [x_0,x_5] = x_8,$$
$$[x_1,x_2] = x_5,\ \ [x_1,x_3] = x_9,\ \ [x_1,x_4] = x_{10},\ \ [x_1,x_5] = x_{11},\ \ [x_2,x_3] = x_{12},$$
$$[x_2,x_4] = x_{13},\ \ \epsilon\,\alpha_3[x_0,x_9] + 2\,i\alpha_1 x_9 + 2\,\epsilon\,\alpha_3[x_1,x_6] = 0,\ \ [x_1,x_{15}] + i\alpha_1 x_6 + \epsilon\,\alpha_3[x_0,x_6],$$
$$\alpha_3[x_1,x_9] = 0,\ \ \alpha_3[x_2,x_{13}] = 0,\ \ [x_0,x_{15}] = 0,\ \ 2\,\epsilon\,\alpha_3[x_2,x_7] + \epsilon\,\alpha_3[x_0,x_{13}] = 2\,i\alpha_1 x_{13},$$
$$[x_2,x_{15}] + \epsilon\,\alpha_3[x_0,x_7] = i\alpha_1 x_7,\ \ 6\,\epsilon\,\alpha_3([x_0,x_{10}] + [x_1,x_7] + [x_2,x_6] + [x_0,x_8]) + 6\,i\alpha_1 x_8 = 0,$$
$$\left(6\,\epsilon\,[x_1,x_8] + 6\,\epsilon\,[x_1,x_{10}] + 3\,\epsilon\,[x_2,x_9]\right)\alpha_3{}^2 + \left(4\,\epsilon\,\alpha_5 x_3 + 6\,\epsilon\,\alpha_4 x_3 + 6\,ix_1\alpha_2\right)\alpha_3 + 2\,i\alpha_1 x_1\alpha_5 = 0,$$
$$\left(6\,\epsilon\,[x_2,x_{10}] + 3\,\epsilon\,[x_1,x_{13}] + 6\,\epsilon\,[x_2,x_8]\right)\alpha_3{}^2 + \left(4\,\epsilon\,\alpha_5 x_4 + 6\,\epsilon\,\alpha_4 x_4 - 6\,ix_2\alpha_2\right)\alpha_3 - 2\,i\alpha_1 x_2\alpha_5 = 0.$$

It is known that nontrivial matrix representations of prolongation algebra L correspond to nontrivial prolongation structures. To find the matrix representation of L, following the procedure of Fordy (23), we try to embed it into Lie algebra $sl(n,C)$. Starting from the case of $n = 2$, we found that $sl(2,C)$ is the whole algebra for some special coefficients $\alpha_j (j = 1,2,3,4,5)$. For the case of $n = 3$, we can also find that $sl(3,C)$ will be the whole algebra for some other special coefficients $\alpha_j (j = 1,2,3,4,5)$. In this paper, we only examine the case of $sl(2,C)$ algebra.
From the above commutation relations, we have the special relations among elements x_1, x_2 and x_5 as

$$[x_2,x_5] = \frac{\alpha_5}{3\alpha_3}x_2,\ \ [x_1,x_5] = -\frac{\alpha_5}{3\alpha_3}x_1,\ \ [x_1,x_2] = x_5, \tag{12}$$

from which we know that x_1 and x_2 are nilpotent elements and x_5 is a neutral element. So we have $\alpha_5 = \pm 6\,\delta^2\alpha_3$ and

$$x_1 = \begin{pmatrix} 0 & \delta \\ 0 & 0 \end{pmatrix},\ \ x_2 = \begin{pmatrix} 0 & 0 \\ \pm\delta & 0 \end{pmatrix},\ \ x_5 = \begin{pmatrix} \pm\delta^2 & 0 \\ 0 & \mp\delta^2 \end{pmatrix}, \tag{13}$$

with δ a nonzero constant. Substituting (13) into the commutation relations of prolongation algebra L, we finally get the 2×2 matrix representations of F and G. Therefore, we obtain two integrable HNLS equations with 2×2 spectral problems.
When $\alpha_2 = 2\,\delta^2\alpha_1, \alpha_4 = 6\,\delta^2\alpha_3$ and $\alpha_5 = -6\,\delta^2\alpha_3$, Eq. (4) becomes the bright soliton version of Hirota equation

$$q_T = i\alpha_1 q_{XX} + 2\,i\alpha_1\delta^2|q|^2q + \epsilon\,\alpha_3 q_{XXX} + 6\,\epsilon\delta^2\alpha_3\,|q|^2q_X, \tag{14}$$

with linear spectral problem

$$\zeta_X = F\zeta, \quad \zeta_T = G\zeta, \tag{15}$$

and

$$F = \begin{pmatrix} -i\lambda & \delta q \\ -\delta q^* & i\lambda \end{pmatrix}, \tag{16}$$

$$\begin{aligned} G = {} & 4\,i\alpha_3\epsilon\lambda^3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - 2\,\lambda^2\begin{pmatrix} i\alpha_1 & 2\epsilon\,\alpha_3\delta\,q \\ -2\,\epsilon\,\alpha_3\delta q^* & -i\alpha_1 \end{pmatrix} \\ & + 2\,\lambda\begin{pmatrix} -i\epsilon\,\alpha_3\delta^2|q|^2 & \alpha_1\delta q - i\epsilon\,\alpha_3\delta q_X \\ -\alpha_1\delta q^* - i\epsilon\,\alpha_3\delta q_X^* & i\epsilon\,\alpha_3\delta^2|q|^2 \end{pmatrix} \\ & + \begin{pmatrix} \epsilon\,\alpha_3\delta^2 q_X q^* - \epsilon\,\alpha_3\delta^2 q_X^* q + i\alpha_1\delta^2|q|^2 & \epsilon\,\alpha_3\delta q_{XX} + i\alpha_1\delta q_X + 2\,\epsilon\delta^3\alpha_3\,|q|^2q \\ i\alpha_1\delta q_X^* - \epsilon\,\alpha_3\delta q_{XX}^* - 2\,\epsilon\delta^3\alpha_3\,|q|^2q^* & \epsilon\,\alpha_3\delta^2 q_X^* q - \epsilon\,\alpha_3\delta^2 q_X q^* - i\alpha_1\delta^2|q|^2 \end{pmatrix}, \end{aligned} \tag{17}$$

where λ is a spectral parameter and $\zeta(T, X, \lambda)$ is a vector or matrix function.
When $\alpha_2 = -2\,\delta^2\alpha_1, \alpha_4 = -6\,\delta^2\alpha_3$ and $\alpha_5 = 6\,\delta^2\alpha_3$, Eq. (4) becomes the dark soliton version of Hirota equation

$$q_T = i\alpha_1 q_{XX} - 2\,i\delta^2\alpha_1|q|^2q + \epsilon\,\alpha_3 q_{XXX} - 6\,\epsilon\,\delta^2\alpha_3|q|^2q_X, \tag{18}$$

with linear spectral problem Eq. (15) and

$$F = \begin{pmatrix} -i\lambda & \delta q \\ \delta q^* & i\lambda \end{pmatrix}, \tag{19}$$

$$\begin{aligned} G = {} & 4\,i\epsilon\,\lambda^3\alpha_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - 2\lambda^2\begin{pmatrix} i\alpha_1 & 2\epsilon\,\alpha_3\delta\,q \\ 2\,\epsilon\,\alpha_3\delta q^* & -i\alpha_1 \end{pmatrix} \\ & + 2\lambda\begin{pmatrix} i\epsilon\,\alpha_3\delta^2|q|^2 & -i\epsilon\,\alpha_3\delta q_X + \alpha_1\delta q \\ \alpha_1\delta q^* + i\epsilon\,\alpha_3\delta q_X^* & -i\epsilon\,\alpha_3\delta^2|q|^2 \end{pmatrix} \\ & + \begin{pmatrix} \epsilon\,\alpha_3\delta^2 q_X^* q - \epsilon\,\alpha_3\delta^2 q_X q^* - i\alpha_1\delta^2|q|^2 & \epsilon\,\alpha_3\delta q_{XX} + i\alpha_1\delta q_X - 2\,\epsilon\,\delta^3\alpha_3|q|^2q \\ \epsilon\,\alpha_3\delta q_{XX}^* - i\alpha_1\delta q_X^* - 2\,\epsilon\,\delta^3\alpha_3|q|^2q^* & \epsilon\,\alpha_3\delta^2 q_X q^* - \epsilon\,\alpha_3\delta^2 q_X^* q + i\alpha_1\delta^2|q|^2 \end{pmatrix}. \end{aligned} \tag{20}$$

4. The bright soliton solutions for Eq. (14)

In this section, we propose the N-bright soliton solutions of Eq. (14) using the Riemann-Hilbert formulation (24–28). Let us consider Eq. (14) for localized solutions, i.e. assuming that potential function q decay to zero sufficiently fast as $X, T \to \pm\infty$. In the Riemann-Hilbert formulation, we treat ζ as a fundamental matrix of the two linear equations in (15). From (15) we note that when $X, T \to \pm\infty$, one has $\zeta = e^{-i\lambda\Lambda X+(4i\alpha_3\epsilon\lambda^3-2i\lambda^2\alpha_1)\Lambda T}$ with $\Lambda = \mathrm{diag}(1,-1)$. This motivates us to introduce the variable transformation

$$\zeta = Je^{-i\lambda\Lambda X+(4i\alpha_3\epsilon\lambda^3-2i\lambda^2\alpha_1)\Lambda T}, \tag{21}$$

where J is (X,T)-independent at infinity. Inserting (21) into (15) with (16)-(17), we get

$$J_X = -i\lambda[\Lambda, J] + \delta QJ, \tag{22a}$$

$$J_T = -(2i\,\alpha_1\lambda^2 - 4\,i\alpha_3\epsilon\lambda^3)[\Lambda, J] + VJ, \tag{22b}$$

with

$$Q = \begin{pmatrix} 0 & q \\ -q^* & 0 \end{pmatrix}, \quad V = (2\,\lambda\alpha_1\delta - 4\,\lambda^2\epsilon\,\alpha_3\delta)Q + 2\,\lambda \begin{pmatrix} -i\epsilon\,\alpha_3\delta^2|q|^2 & -i\epsilon\,\alpha_3\delta q_X \\ -i\epsilon\,\alpha_3\delta q_X^* & i\epsilon\,\alpha_3\delta^2|q|^2 \end{pmatrix}$$
$$+ \begin{pmatrix} \epsilon\,\alpha_3\delta^2 q_X q^* - \epsilon\,\alpha_3\delta^2 q_X^* q + i\alpha_1\delta^2|q|^2 & \epsilon\,\alpha_3\delta q_{XX} + i\alpha_1\delta q_X + 2\,\epsilon\delta^3\alpha_3\,|q|^2 q \\ i\alpha_1\delta q_X^* - \epsilon\,\alpha_3\delta q_{XX}^* - 2\,\epsilon\delta^3\alpha_3\,|q|^2 q^* & \epsilon\,\alpha_3\delta^2 q_X^* q - \epsilon\,\alpha_3\delta^2 q_X q^* - i\alpha_1\delta^2|q|^2 \end{pmatrix}.$$

Here $[\Lambda, J] = \Lambda J - J\Lambda$ is the commutator, $\mathrm{tr}(Q) = \mathrm{tr}(V) = 0$ and

$$Q^\dagger = -Q, \quad V^\dagger = -V, \tag{23}$$

where † represents the Hermitian of a matrix.
In what folows, we consider the scattering problem of the Eq. (22a). By doing so, the variable T is fixed and is a dummy variable. We first introduce the matrix Jost solutions $J_\pm(X,\lambda)$ of (22a) with the asymptotic condition

$$J_\pm \to I, \quad \text{when} \;\; X \to \pm\infty, \tag{24}$$

where I is a 2×2 unit matrix. Here the subscripts in $J_\pm$ refer to which end of the X-axis the boundary conditions are set. Then due to $\mathrm{tr}(Q) = 0$ and Abel's formula we have $\det(J_\pm) = 1$ for all X. Next we denote $E = e^{-i\lambda\Lambda X}$. Since $\Psi \equiv J_+E$ and $\Phi \equiv J_-E$ are both solutions of the first equation in (15), they must be linearly related, i.e.

$$J_-E = J_+ES(\lambda), \qquad \lambda \in \mathbb{R} \tag{25}$$

where

$$S(\lambda) = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}, \quad \lambda \in \mathbb{R}$$

is the scattering matrix, and $\mathbb{R}$ is the set of real numbers. Notice that $\det(S(\lambda)) = 1$ since $\det(J_\pm) = 1$. If we denote (Φ, Ψ) as a collection of columns,

$$\Phi = [\phi_1, \phi_2], \quad \Psi = [\psi_1, \psi_2], \tag{26}$$

by using the same formulation as (24; 25; 27), we have the Jost solution

$$P^+ = [\phi_1, \psi_2]e^{i\lambda\Lambda X} = J_- H_1 + J_+ H_2, \tag{27}$$

is analytic in $\lambda \in C_+$, and Jost solution

$$P^- = e^{-i\lambda\Lambda X}\begin{bmatrix}\hat{\phi}_1\\ \hat{\psi}_2\end{bmatrix} = H_1 J_-^{-1} + H_2 J_+^{-1}, \tag{28}$$

is analytic in $\lambda \in C_-$, with

$$\Phi^{-1} = \begin{bmatrix}\hat{\phi}_1\\ \hat{\phi}_2\end{bmatrix}, \quad \Psi^{-1} = \begin{bmatrix}\hat{\psi}_1\\ \hat{\psi}_2\end{bmatrix},$$

and

$$H_1 = \mathrm{diag}(1,0), \qquad H_2 = \mathrm{diag}(0,1).$$

In addition, it is easy to see that

$$P^+(X,\lambda) \to I, \quad \text{as } \lambda \in \mathbb{C}_+ \to \infty, \tag{29}$$

and

$$P^-(X,\lambda) \to I, \quad \text{as } \lambda \in \mathbb{C}_- \to \infty. \tag{30}$$

In addition, if we express S^{-1} as

$$S^{-1} = \begin{pmatrix}\hat{s}_{11} & \hat{s}_{12}\\ \hat{s}_{21} & \hat{s}_{22}\end{pmatrix}, \quad \lambda \in \mathbb{R},$$

from $\det(S(\lambda)) = 1$ we have

$$\hat{s}_{11} = s_{22}, \ \ \hat{s}_{22} = s_{11}, \ \ \hat{s}_{12} = -s_{12}, \ \ \hat{s}_{21} = -s_{21}. \tag{31}$$

Hence we have constructed two matrix functions P^+ and P^- which are analytic in C_+ and C_-, respectively. On the real line, using Eqs. (25), (27) and (28), it is easily to see that

$$P^-(X,\lambda)P^+(X,\lambda) = G(X,\lambda), \qquad \lambda \in \mathbb{R}, \tag{32}$$

with

$$G = E(H_1 + H_2 S)(H_1 + S^{-1}H_2)E^{-1} = E\begin{pmatrix}1 & \hat{s}_{12}\\ s_{21} & 1\end{pmatrix}E^{-1}.$$

This determines a matrix Riemann-Hilbert problem with asymptotics

$$P^{\pm}(X,\lambda) \to I, \quad \text{as } \lambda \to \infty, \tag{33}$$

which provide the canonical normalization condition for this Riemann-Hilbert problem. If this problem can be solved, one can readily reconstruct the potential $q(X,T)$ as follows. Notice that P^+ is the solution of the spectral problem (22a). Thus if we expand P^+ at large λ as

$$P^+(X,\lambda) = I + \frac{1}{\lambda}P_1^+(X) + O(\lambda^{-2}), \quad \lambda \to \infty, \tag{34}$$

and inserting this expansion into (22a), then comparing $O(1)$ terms in (34), we find that

$$\delta Q = i[\Lambda, P_1^+] = \begin{pmatrix}0 & 2iP_{12}\\ -2iP_{21} & 0\end{pmatrix}. \tag{35}$$

Thus, recalling the definition of Q the potentials q is reconstructed immediately as

$$q = 2iP_{12}/\delta, \tag{36}$$

where $P_1^+ = (P_{ij})$. In addition, from the definitions of P^+, P^- and Eq. (25) we have

$$\det P^+ = \hat{s}_{22} = s_{11}, \quad \det P^- = s_{22} = \hat{s}_{11}. \tag{37}$$

The symmetry properties of the potential Q and V in (23) give rise to symmetry properties in the scattering matrix as well as in the Jost functions. In fact, after some computation we have $J_\pm$ satisfies the involution property

$$J_\pm^\dagger(X, \lambda^*) = J_\pm^{-1}(X, \lambda), \tag{38}$$

analytic solutions $P^\pm$ satisfy the involution property

$$(P^+)^\dagger(\lambda^*) = P^-(\lambda), \tag{39}$$

and S satisfies the involution property

$$S^\dagger(\lambda^*) = S^{-1}(\lambda). \tag{40}$$

Let λ_k and $\bar{\lambda}_k$ are zero points of $\det P^+$ and $\det P^-$, respectively. We see from (37) that $(\lambda_k, \bar{\lambda}_k)$ are zeros of the scattering coefficients $\hat{s}_{22}(\lambda)$ and $s_{22}(\lambda)$. Due to the above involution property, we have the symmetry relation

$$\bar{\lambda}_k = \lambda_k^*. \tag{41}$$

For simplicity, we assume that all zeros $\{(\lambda_k, \bar{\lambda}_k), k = 1, 2, \cdot, N\}$ are simple zeros of $\hat{s}_{22}(\lambda)$ and $s_{22}(\lambda)$, then each kernal of $P^+(\lambda_k)$ and $P^-(\bar{\lambda}_k)$ contains only a single column vector v_k and row vector $\bar{v}_k$,

$$P^+(\lambda_k)v_k = 0, \quad \bar{v}_k P^-(\bar{\lambda}_k) = 0.$$

Taking the Hermitian of the above equations and using the involution properties, we have

$$\bar{v}_k = v_k^\dagger. \tag{42}$$

To obtain the soliton solutions, we set $G = I$ in (32). In this case, the solutions to this special Riemann-Hilbert problem have been derived in (25; 26) as

$$P_1^+(T, X, \lambda) = \sum_{j,k=1}^{N} v_j \left(M^{-1}\right)_{jk} \bar{v}_k, \tag{43}$$

where

$$M_{jk} = \frac{\bar{v}_j v_k}{\bar{\lambda}_j - \lambda_k}. \tag{44}$$

The zeros λ_k and $\bar{\lambda}_k$ are T-independent. To find the spatial and temporal evolutions for vectors $v_k(T, X)$, we take the X-derivative to equation $P^+ v_k = 0$. By using (22a), one gets

$$P^+(X, \lambda_k)\left(\frac{\partial v_k}{\partial X} + i\lambda_k \Lambda v_k\right) = 0, \tag{45}$$

thus we have

$$\frac{dv_k}{dX} + i\lambda_k \Lambda v_k = 0. \tag{46}$$

Similarly, taking T-derivative to equation $P^+ v_k = 0$ and using (22b), one has

$$P^+(T, X, \lambda_k)(\frac{\partial v_k}{\partial T} + (2i\,\alpha_1 {\lambda_k}^2 - 4\,i\alpha_3 \epsilon {\lambda_k}^3) v_k) = 0, \tag{47}$$

thus we have

$$\frac{\partial v_k}{\partial T} + (2i\,\alpha_1 {\lambda_k}^2 - 4\,i\alpha_3 \epsilon {\lambda_k}^3) v_k = 0. \tag{48}$$

Solving (46) and (48) we get

$$v_k(T, X) = e^{-i\lambda_k \Lambda X + (4\,i\alpha_3 \epsilon {\lambda_k}^3 - 2i\,{\lambda_k}^2 \alpha_1)\Lambda T} v_{k0}, \tag{49a}$$

$$\bar{v}_k(T, X) = \bar{v}_{k0} e^{i\lambda_k \Lambda X + (-4\,i\alpha_3 \epsilon {\lambda_k}^3 + 2i\,{\lambda_k}^2 \alpha_1)}, \tag{49b}$$

where $(v_{k0}, \bar{v}_{k0})$ are constant vectors.

In summary, the N-bright soliton solutions to Eq. (14) are obtained from the analytical functions P_1^+ in (43) together with the potential reconstruction formula (36) as

$$q(T, X) = 2iP_{12}/\delta = 2i \left(\sum_{j,k=1}^{N} v_j \left(M^{-1} \right)_{jk} \bar{v}_k \right)_{12} / \delta, \tag{50}$$

where the vectors v_j are given by (49). Without loss of generality, we take $v_{k0} = [b_k, 1]'$ with b_k constants. And if we denote

$$\xi_k = -i\lambda_k X + (4\,i\alpha_3 \epsilon {\lambda_k}^3 - 2i\,{\lambda_k}^2 \alpha_1) T, \tag{51}$$

the general N-soliton solution to Eq. (14) can be written out explicitly as

$$q(T, X) = \frac{2i}{\delta} \sum_{j,k=1}^{N} b_j e^{\xi_j - \xi_k^*} (M^{-1})_{jk}, \tag{52}$$

with

$$M_{jk} = \frac{1}{\lambda_j^* - \lambda_k} \left(b_j^* c_k e^{\xi_k + \xi_j^*} - e^{-\xi_k - \xi_j^*} \right). \tag{53}$$

In what follows, we investigate the dynamics of the one-soliton and two-soliton solutions in Eqs. (14) in detail.

4.1 Examples of single and two bright solitons in Eq. (14)

To get the single bright soliton solution for Eq. (14), we set $N = 1$ in (52) to have

$$q(T, X) = \frac{2i(\lambda_1^* - \lambda_1)}{\delta} \frac{b_1 e^{\xi_1 - \xi_1^*}}{e^{-\xi_1 - \xi_1^*} + |b_1|^2 e^{\xi_1 + \xi_1^*}}. \tag{54}$$

If setting $\lambda_1 = \zeta_1 + i\eta_1$, $b_1 = e^{-2\eta_1 X_0 + i\omega_0}$, the single soliton solution (54) can be rewritten as

$$q(T, X) = \frac{2\eta_1}{\delta} \mathrm{sech}[2\,\eta_1 (X + \left(4\,\alpha_3 \epsilon\, {\eta_1}^2 + 4\,\alpha_1 \zeta_1 - 12\,\alpha_3 \epsilon\, {\zeta_1}^2 \right) T - X_0)] \exp^{i\theta}, \tag{55}$$

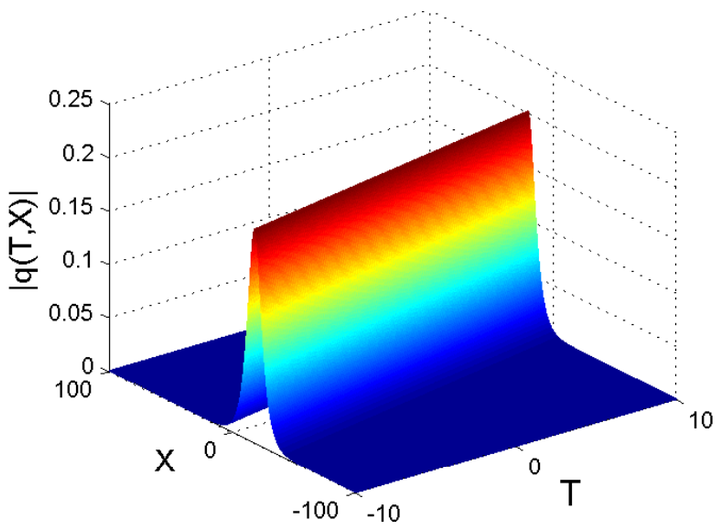


Fig. 1. (color online). Evolution of single soliton $|q(T,X)|$ in (55) with parameters (56). It is similar to single soliton in standard NLS equation.

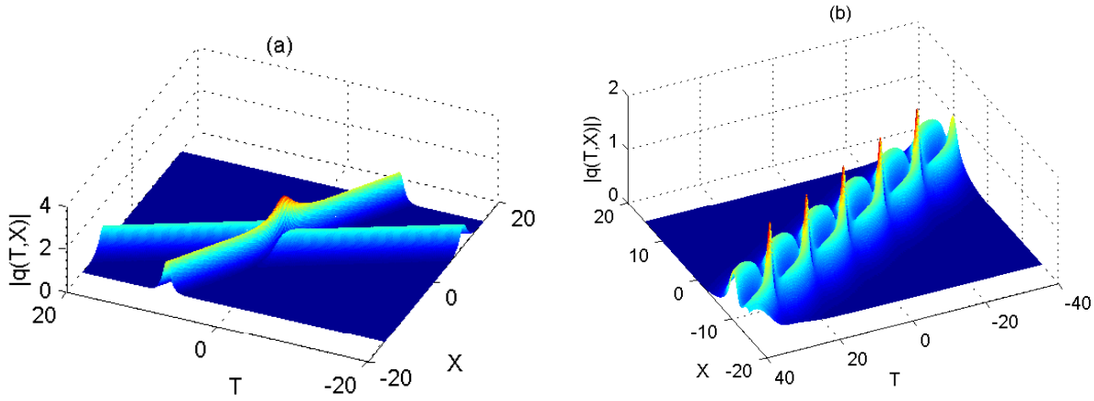


Fig. 2. (color online). The shapes of two-soliton solutions $|q(T,X)|$ in (52) with (53). (a) soliton collision with parameters (57); (b) bound state with parameters (58).

with $\theta = -2\zeta_1 X + \left(-4\alpha_1\zeta_1{}^2 + 4\alpha_1\eta_1{}^2 + 8\alpha_3\epsilon\zeta_1{}^3 - 24\alpha_3\epsilon\zeta_1\eta_1{}^2\right)T + \omega_0$, and X_0, ω_0 are constants. This solution is similar to the solitary wave solution in the standard NLS equation (1). Its amplitude function has the shape of a hyperbolic secant with peak amplitude $2\eta_1/\delta$, and its velocity depends on several parameters, which is $12\alpha_3\epsilon\zeta_1{}^2 - 4\alpha_3\epsilon\eta_1{}^2 - 4\alpha_1\zeta_1$. The phase θ of this solution depends linearly both on space X and time T. We show this single soliton solution in Fig. 1 with parameters

$$\zeta_1 = 0.5, \eta_1 = 0.1, X_0 = 1.5, \omega_0 = 2, \delta = 1, \alpha_1 = 0.5, \alpha_3 = 1, \epsilon = 1. \tag{56}$$

The two-soliton solution in Eq. (14) corresponds to $N = 2$ in the general N-soliton solution (52) with (53). This solution can also be written out explicitly, however, we prefer to showing

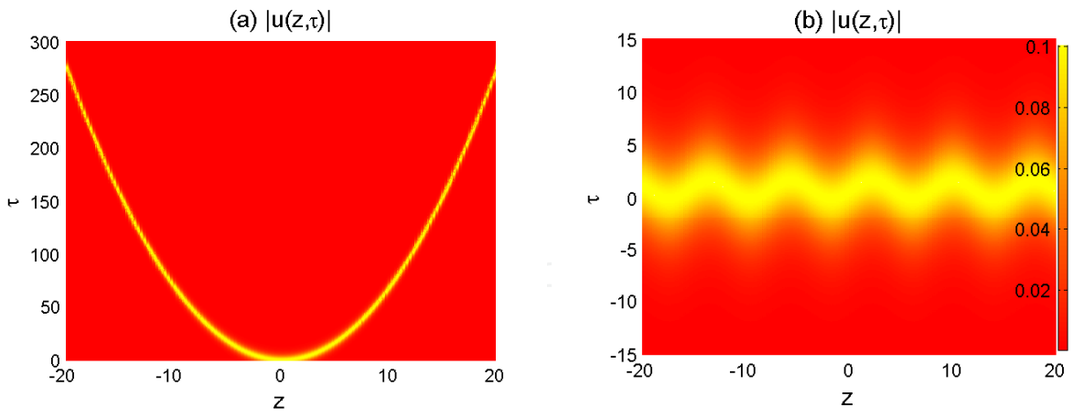


Fig. 3. (color online). Evolution of single soliton solutions $|u(z,\tau)|$ in HNLS equation (2) with controlable coefficients (59) and (62), respectively. (a) Soliton solution (61) with parameter (56) and $\rho_0 = 0.5, c = 1, k = 2$. (b) Soliton solution (64) with parameter (59) and $\rho_0 = 0.5, c = 1, k = 2$.

their behaviors by figures, see Fig. 2(a)-(b). Below we take $\lambda_1 = \zeta_1 + i\eta_1$ and $\lambda_2 = \zeta_2 + i\eta_2$ and examine this solution with various velocity parameters: one is $12\,\alpha_3\epsilon\,{\zeta_1}^2 - 4\,\alpha_3\epsilon\,{\eta_1}^2 - 4\,\alpha_1\zeta_1 = 12\,\alpha_3\epsilon\,{\zeta_2}^2 - 4\,\alpha_3\epsilon\,{\eta_2}^2 - 4\,\alpha_1\zeta_2$, i.e. the collision between two solitons, and the other is $12\,\alpha_3\epsilon\,{\zeta_1}^2 - 4\,\alpha_3\epsilon\,{\eta_1}^2 - 4\,\alpha_1\zeta_1 \neq 12\,\alpha_3\epsilon\,{\zeta_2}^2 - 4\,\alpha_3\epsilon\,{\eta_2}^2 - 4\,\alpha_1\zeta_2$, i.e. bound state. In Fig. 2(a), the two soliton parameters in Eq. (52) with (53) are

$$\alpha_1 = 0.5,\ \alpha_3 = 0.8,\ \epsilon = 1,\ \delta = 1,\ \lambda_1 = 0.2 + 0.7i,\ \lambda_2 = -0.1 + 0.5i,\ b_1 = 1,\ b_2 = 1. \tag{57}$$

Under these parameters, the velocity of the two solitons are different. It is observed that interactions between two soliton don't change the shape and velocity of the solitons, and there is no energy radiation emitted to the far field. Thus the interaction of these solitons is elastic, which is a remarkable property which signals that the HNLS equation (14) is integrable.
Fig. 2(b) displays a bound state in Eq. (14), and the soliton parameters here are

$$\alpha_1 = 0.5,\ \alpha_3 = 0.8,\ \epsilon = 1,\ \delta = 1,\ \lambda_1 = 0.3i,\ \lambda_2 = -0.1 + 0.4272i,\ b_1 = 1,\ b_2 = 1. \tag{58}$$

Under these parameters, the two constituent solitons have equal velocities, thus they will stay together to form a bound state which moves at the common speed. It can be seen that the width of this solution changes periodically with time, thus this solution is called breather soliton.

5. Dynamics of solitons in HNLS equation (2)

In what follows, we investigate the dynamic behavior of solitons in the variable-coefficients HNLS equation (2) with special soliton management parameters $d_j (j = 1, 2, 3, 4, 5, 6)$.

5.1 Single soliton solutions

We choose two cases of soliton management parameters $d_j (j = 1, 2, 3, 4, 5, 6)$ to study the dynamics of the single solitons in HNLS equation (2). Firstly, if we take the soliton management parameters to satisfy

$$d_1 = 1.6\ (k\alpha_1 - 3\,\alpha_3 c)\, z/k^3,\ d_2 = 1.6\ (\alpha_2 k - \alpha_4 c)\, z/{\rho_0}^2 k,$$

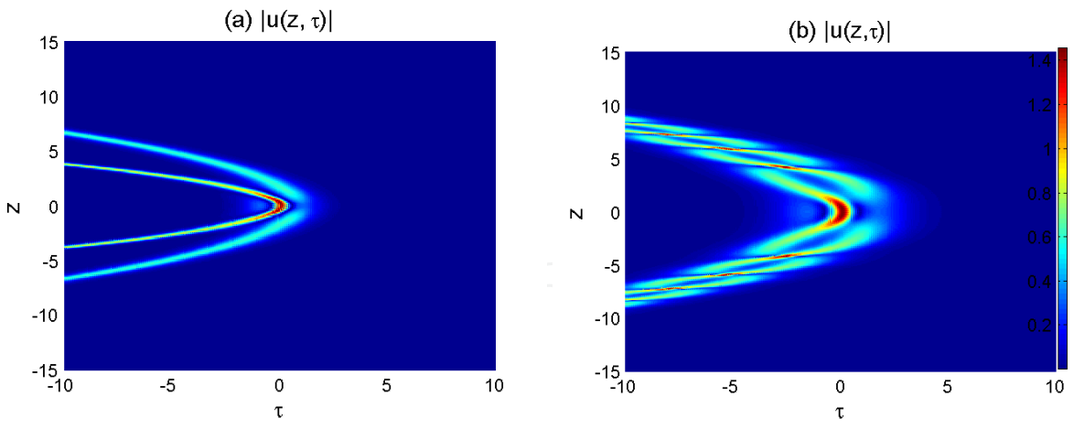


Fig. 4. (color online). The two-soliton solutions $|u(z,\tau)|$ in HNLS equation (2) with coefficients (59). (a) soliton collision with parameter (57) and $\rho_0 = 0.5, c = 1, k = 2$; (b) bound state with parameter (58) and $\rho_0 = 0.5, c = 1, k = 2$.

$$d_3 = 1.6\,\alpha_3 z/k^3,\ d_4 = 1.6\,\alpha_4 z/{\rho_0}^2 k,\ d_5 = 1.6\,\alpha_5 z/{\rho_0}^2 k,\ d_6 = 0, \tag{59}$$

the variables ρ, T and X in similarity transformation (3) are

$$\rho = \rho_0,\ \ T = 0.8\,z^2,\ \ X = k\tau + (2.4\,c^2\alpha_3 - 1.6\,ck\alpha_1)z^2/k^2. \tag{60}$$

So the single soliton solution in HNLS equation (2) with coefficients (59) is

$$u(z,\tau) = \rho_0 q\,(T,X)\,e^{ic(k^3\tau + 1.6\,c^2\alpha_3\,z^2 - 0.8\,ck\alpha_1 z^2)/k^3}, \tag{61}$$

where $q\,(T,X)$ satisfies Eq. (55) and T, X satisfy Eq. (60).
Secondly, if we take the soliton management parameters to satisfy

$$d_1 = 0.8\,\cos\,(0.8\,z)\,(k\alpha_1 - 3\,\alpha_3 c)\,/k^3, d_2 = 0.8\,\cos\,(0.8\,z)\,(\alpha_2 k - \alpha_4 c)\,/{\rho_0}^2 k, d_6 = 0,$$

$$d_3 = 0.8\,\alpha_3\cos\,(0.8\,z)\,/k^3,\ d_4 = 0.8\,\alpha_4\cos\,(0.8\,z)\,/{\rho_0}^2 k,\ d_5 = 0.8\,\alpha_5\cos\,(0.8\,z)\,/{\rho_0}^2 k, \tag{62}$$

the variables ρ, T and X in similarity transformation (3) are

$$\rho = \rho_0,\ \ T = \sin\,(0.8\,z)\,,\ \ X = k\tau + (3\,c^2\alpha_3 - 2\,ck\alpha_1)\,\sin\,(0.8\,z)\,/k^2. \tag{63}$$

In this case the single soliton solution in HNLS equation (2) with coefficients (62) is

$$u(z,\tau) = \rho_0 q\,(T,X)\,e^{ic(k^3\tau + 2\,c^2\alpha_3\,\sin(0.8\,z) - ck\alpha_1\,\sin(0.8\,z))/k^3}, \tag{64}$$

where $q\,(T,X)$ satisfies Eq. (55) and T, X satisfy Eq. (63).
In Fig. 3, we show the single soliton solutions (61) and (64) in HNLS equation (2) with coefficients (59) and (62), respectively. Here the solution parameters are given in (56) and $\rho_0 = 0.5, c = 1, k = 2$. It is observered that when the soliton management parameters $d_j (j = 1,2,3,4,5)$ are linearly dependent on variable z and $d_6 = 0$ (see Eq. (59)), the trajectory of the optical soliton is a localized parabolic curve, as shown in Fig. 3(a). When the soliton management parameters $d_j (j = 1,2,3,4,5)$ are periodically dependent on variable z and $d_6 = 0$ (see Eq. (62)), the trajectory of the optical soliton is a periodical localized nonlinear wave, as shown in Fig. 3(b).

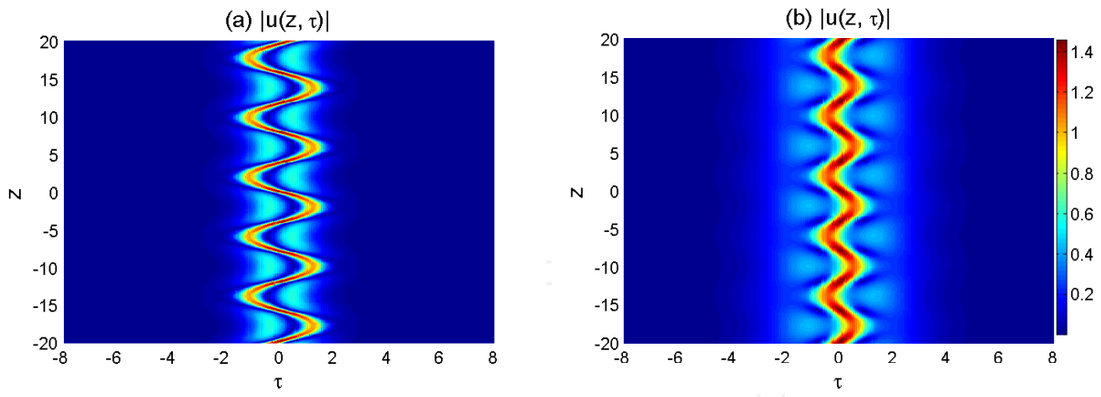


Fig. 5. (color online). The two-soliton solutions $|u(z,\tau)|$ in HNLS equation (2) with coefficients (62). (a) soliton collision with parameter (57) and $\rho_0 = 0.5, c = 1, k = 2$; (b) bound state with parameter (58) and $\rho_0 = 0.5, c = 1, k = 2$.

5.2 Collisions of the two-solitons

We now demonstrate various collision scenarios in HNLS equation (2) with coefficients (59) and (62), respectively. As in Section 4.1, we consider the two-soliton collisions and bound states in equation (2).

When the coefficients of equation (2) satisfies (59), its two-soliton solution is

$$u(z,\tau) = \rho_0 q\left(T, X\right) e^{ic\left(k^3\tau + 1.6\, c^2\alpha_3\, z^2 - 0.8\, ck\alpha_1 z^2\right)/k^3}, \tag{65}$$

where T, X satisfy Eq. (60), and $q\left(T, X\right)$ satisfies Eq. (52) with (53) and $N = 2$.

When the coefficients of equation (2) satisfies (62), its two-soliton solution is

$$u(z,\tau) = \rho_0 q\left(T, X\right) e^{ic\left(k^3\tau + 2\, c^2\alpha_3 \sin(0.8\, z) - ck\alpha_1 \sin(0.8\, z)\right)/k^3}, \tag{66}$$

where T, X satisfy Eq. (63), and $q\left(T, X\right)$ satisfies Eq. (52) with (53) and $N = 2$.

In Fig. 4, we display the evolutions of the two-soliton solutions (65) in HNLS equation (2) with coefficients (59). Fig. 4(a) shows the soliton collision with parameter (57) and $\rho_0 = 0.5, c = 1, k = 2$, and Fig. 4(b) shows the bound state with parameter (58) and $\rho_0 = 0.5, c = 1, k = 2$. In Fig. 5, we display the evolutions of the two-soliton solutions (66) in HNLS equation (2) with coefficients (62). Fig. 5(a) shows the soliton collision with parameter (57) and $\rho_0 = 0.5, c = 1, k = 2$, and Fig. 5(b) shows the bound state with parameter (58) and $\rho_0 = 0.5, c = 1, k = 2$.

6. Conclusions

In summary, we have studied the variable-coefficient higher order nonlinear Schrödinger equation which describes the wave propagation in a nonlinear fiber medium with higher-order effects such as third order dispersion, self-steepening and stimulated Raman scattering. By means of similarity transformation, we first change this variable-coefficient equation into the constant-coefficient HNLS equation. Then we investigate the integrability of the constant-coefficient HNLS equation by prolongation technique and find two Lax integrable HNLS equations. The exact bright N-soliton solutions for the bright soliton version of HNLS equation are obtained using Riemann-Hilbert formulation. Finally, the dynamics of the optical solitons in both constant-coefficient and variable-coefficient HNLS equations is examined and the effects of higher-order effects on the velocity and shape of the optical soliton

are observed. In addition, it is seen that the bright two-soliton solution of the HNLS equation behaves in an elastic manner characteristic of all soliton solutions.

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8. References

[1] K. E. Lonngren, Soliton experiments in plasmas, Plas. Phys. 25, 943 (1983).
[2] G.P. Agrawal, Nonlinear Fiber Optics, Academic Press, San Diego (1989).
[3] A. Hasegawa and Y. Kodama, Solitons in Optical Communications, Clarendon, Oxford, 1995.
[4] J. Denschlag, J. E. Simsarian, D. L. Feder, Charles W. Clark, L. A. Collins, J. Cubizolles1, L. Deng, E. W. Hagley, K. Helmerson, W. P. Reinhardt1, S. L. Rolston, B. I. Schneider and W. D. Phillips, Generating Solitons by Phase Engineering of a Bose-Einstein Condensate, Science, 287, 97-101 (2000).
[5] B. P. Anderson, P. C. Haljan, C. A. Regal, D. L. Feder, L. A. Collins, C. W. Clark, and E. A. Cornell, Watching Dark Solitons Decay into Vortex Rings in a Bose-Einstein Condensate, Phys. Rev. Lett. 86, 2926-2929 (2001).
[6] M. A. Helal, Soliton solution of some nonlinear partial differential equations and its applications in fluid mechanics, Chaos, Solitons and Fractals, 13, 1917-1929 (2002).
[7] A. Hasegawa and F. Tappert, Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion, Appl. Phys. Lett. 23, 142-144 (1973); Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. II. Normal dispersion, Appl. Phys. Lett. 23, 171-172 (1973).
[8] L. F. Mollenauer, R. H. Stolen, and J. P. Gordon, Experimental observation of picosecond pulse narrowing and solitons in optical fibers, Phys. Rev. Lett. 45, 1095-1098 (1980).
[9] V. E. Zakharov and A. B. Shabat, Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of manes in nonlinear media, Zh. Eksp. Teor. Fiz. 61, 118 (1971) (Sov. Phys. JETP 34, 62, 1972).
[10] V. E. Zakharov and A. B. Shabat, On interactions between solitons in a stable medium, Zh. Eksp. Teor. Fiz. 64, 1627 (1973) (Sov. Phys. JETP 37, 823, 1974).
[11] J. Yang, Nonlinear waves in integrable and non-integrable systems, Society for Industrial and Applied Mathematics, U.S. (2010).
[12] Y. Kodama and A. Hasegawa, Nonlinear pulse propagation in a monomode dielectric guide, IEEE J. Quantum Electron. 23, 510 (1987).
[13] K. Porsezian and K. Nakkeeran, Optical Solitons in Presence of Kerr Dispersion and Self-Frequency Shift, Phys. Rev. Lett. 76, 3955 (1996).
[14] K. Nakkeeran, K. Porsezian, P. Shanmugha Sundaram, and A. Mahalingam, Optical solitons in N-coupled higher order nonlinear Schrodinger equations, Phys. Rev. Lett. 80, 1425 (1998).
[15] E. Papaioannou, D. J. Frantzeskakis, K. Hizanidis, An analytical treatment of the effect of axial inhomogeneity on femtosecond solitary waves near the zero dispersion point, 32, 145-154 (1996).
[16] A. Mahalingam and K. Porsezian, Propagation of dark solitons with higher-order effects in optical fibers, Phys. Rev. E 64, 046608 (2001).

[17] Z. Y. Xu, L. Li, Z. H. Li, and G. S. Zhou, Modulation instability and solitons on a cw background in an optical fiber with higher-order effects, Phys. Rev. E 67, 026603 (2003).
[18] D. S. Wang, X. H. Hu, J. P. Hu, and W. M. Liu, Quantized quasi-two-dimensional Bose-Einstein condensates with spatially modulated nonlinearity, Phys. Rev. A 81, 025604 (2010).
[19] D. S. Wang, X. H. Hu, and W. M. Liu, Localized nonlinear matter waves in two-component Bose-Einstein condensates with time- and space-modulated nonlinearities, Phys. Rev. A 82, 023612 (2010).
[20] H.D. Wahlquist and F.B. Estabrook, Prolongation structures of nonlinear evolution equations. J. Math. Phys. 16, 1-7 (1975).
[21] F.B. Estabrook and H.D. Wahlquist, Prolongation structures of nonlinear evolution equations II, J.Math.Phys.17, 1293-1297 (1976).
[22] D. S. Wang, Integrability of the coupled KdV equations derived from two-layer fluids: Prolongation structures and Miura transformations, Nonlinear Analysis, 73, 270-281 (2010). Deng-Shan Wang
[23] R. Dodd and A. P. Fordy, The prolongation structure of quasi-polynomial flows Proc. R. Soc. London A 385, 389-429 (1983).
[24] V.S. Gerdjikov, Basic aspects of soliton theory, Sixth International Conference on Geometry, Integrability and Quantization June 3-10, 2004, Varna, Bulgaria Ivailo M. Mladenov and Allen C. Hirshfeld, Editors, Sofia 1-48 (2005).
[25] V. E. Zakharov, S. V. Manakov, S. P. Novikov, and L. P. Pitaevskii, *The Theory of Solitons: The Inverse Scattering Method*, Consultants Bureau, New York, 1984.
[26] V.S. Shchesnovich and J. Yang, General soliton matrices in the Riemann-Hilbert problem for integrable nonlinear equations. J. Math. Phys. 44, 4604 (2003).
[27] E.V. Doktorov, J. Wang, and J. Yang, Perturbation theory for bright spinor Bose-Einstein condensate solitons, Phys. Rev. A. 77, 043617 (2008).
[28] D. S. Wang, D. J. Zhang and J. Yang, Integrable properties of the general coupled nonlinear Schrodinger equations, J. Math. Phys. 51, 023510 (2010).

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This book presents a comprehensive account of the recent progress in optical fiber research. It consists of four sections with 20 chapters covering the topics of nonlinear and polarisation effects in optical fibers, photonic crystal fibers and new applications for optical fibers. Section 1 reviews nonlinear effects in optical fibers in terms of theoretical analysis, experiments and applications. Section 2 presents polarization mode dispersion, chromatic dispersion and polarization dependent losses in optical fibers, fiber birefringence effects and spun fibers. Section 3 and 4 cover the topics of photonic crystal fibers and a new trend of optical fiber applications. Edited by three scientists with wide knowledge and experience in the field of fiber optics and photonics, the book brings together leading academics and practitioners in a comprehensive and incisive treatment of the subject. This is an essential point of reference for researchers working and teaching in optical fiber technologies, and for industrial users who need to be aware of current developments in optical fiber research areas.

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