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Polygonal Representation of Digital Curves

Dilip K. Prasad[1] and Maylor K. H. Leung[2]
[1]*Nanyang Technological University*
[2]*Universiti Tunku Abdul Rahman (Kampar)*
[1]*Singapore*
[2]*Malaysia*

1. Introduction

Approximating digital curves using polygonal approximations is required in many image processing applications [Kolesnikov & Fränti, 2003, 2005; Lavallee & Szeliski, 1995; Leung, 1990; Mokhtarian & Mackworth, 1986; Prasad, *et al.*, 2011; Prasad & Leung, 2010a, 2010b; Prasad & Leung, 2010; Prasad & Leung, 2012; Prasad, *et al.*, 2011a]. Such representation is used for representing noisy digital curves in a more robust manner, reducing the computational resources required for processing and storing them, and for computing various geometrical properties of digital curves. Specifically, properties like curvature estimation, tangent estimation, detecting inflexion points, perimeter of the curves, etc., which are very sensitive to the digitization noise. Polygonal approximation is also useful in topological representation, segmentation and contour feature extraction in the applications of object detection, face detection, etc.

Most contemporary methods require some form of control parameter for selecting the most representative points (referred to as the dominant points) in the digital curve to be used as the vertices of the polygonal approximation [Arcelli & Ramella, 1993; Bhowmick & Bhattacharya, 2007; Carmona-Poyato, *et al.*, 2005; Carmona-Poyato, *et al.*, 2010; Carmona-Poyato, *et al.*, 2011; Chung, *et al.*, 2008; Chung, *et al.*, 1994; Davis, 1999; Debled-Rennesson, *et al.*, 2005; Douglas & Peucker, 1973; Gritzali & Papakonstantinou, 1983; Kanungo, *et al.*, 1995; Kolesnikov, 2008; Kolesnikov & Fränti, 2003, 2005, 2007; Latecki, *et al.*, 2009; Lavallee & Szeliski, 1995; Leung, 1990; Lowe, 1987; Marji & Siy, 2004; Mokhtarian & Mackworth, 1986; Pavlidis, 1976; Perez & Vidal, 1994; Phillips & Rosenfeld, 1987; Prasad & Leung, 2010c; Ramer, 1972; Ray & Ray, 1992; Rosin, 1997, 2002; Salotti, 2002; Sankar & Sharma, 1978; Sarkar, 1993; Sato, 1992; Sklansky & Gonzalez, 1980; Tomek, 1975; Wall & Danielsson, 1984; Wang, *et al.*, 2008]. The value of control parameter in all the known algorithms is chosen heuristically. In reality, choosing such control parameter can be very challenging because a suitable value of such control parameters depends upon the nature of the digital curve and one value may not be suitable for all the curves in an image and definitely not suitable for all images in a dataset or an application. In section 2, we first propose a parameter independent method for polygonal approximation of the digital curves.

In addition to the heuristics, another issue is the problem of measuring the quality of the polygon fitted by an algorithm. It was shown in [Carmona-Poyato, *et al.*, 2011; Rosin, 1997] that most contemporary metrics to compare and benchmark such algorithms are ineffective for different types of digital curves. The reason for this is that the polygonal approximation has conflicting requirements in terms of the local and global quality of fit. In section 3, we show explicitly that these requirements are conflicting. Quality metrics for local and global characteristics are presented in section 3.5. The presented metrics can be used to measure the quality of not only one edge of the approximated polygons, but also for the complete polygon for a digital curve and for all curves in an image.

A few contemporary methods are discussed qualitatively in section 4 and numerical comparisons are provided in section 5. The conclusions are presented in section 6.

2. Parameter independent polygonal approximation method

The proposed method uses the framework of the method proposed by Lowe [Lowe, 1987] and Ramer-Douglas-Peucker [Douglas & Peucker, 1973; Ramer, 1972] (referred to as L-RDP method for convenience). The L-RDP method of fitting a series of line segment over a digital curve is described here. For a digital curve $e = \{P_1 \quad P_2 \quad \dots \quad P_N\}$, where P_i is the i th edge pixel in the digital curve e. The line passing through a pair of pixels $P_a(x_a, y_a)$ and $P_b(x_b, y_b)$ is given by:

$$x(y_a - y_b) + y(x_b - x_a) + y_b x_a - y_a x_b = 0 . \tag{1}$$

Then the deviation d_i of a pixel $P_i(x_i, y_i) \in e$ from the line passing through the pair $\{P_1, P_N\}$ is given as:

$$d_i = \left| x_i(y_1 - y_N) + y_i(x_N - x_1) + y_N x_1 - y_1 x_N \right| . \tag{2}$$

Accordingly, the pixel with maximum deviation can be found. Let it be denoted as $P_{\max}$. Then considering the pairs $\{P_1, P_{\max}\}$ and $\{P_{\max}, P_N\}$, we find two new pixels from e using the concept in the equations (1) and (2). It is evident that the maximum deviation goes on decreasing as we choose newer pixels of maximum deviation between a pair. This process can be repeated till a certain condition (depending upon the method) is satisfied by all the line segments. This condition shall be referred to as the optimization goal for the ease of reference.

The condition used by L-RDP [Douglas & Peucker, 1973; Lowe, 1987; Ramer, 1972] is that for each line segment, the maximum deviation of the pixels contained in its corresponding edge segment is less than a certain tolerance value:

$$\max(d_i) < d_{\text{tol}} . \tag{3}$$

where d_{tol} is the chosen threshold.

In general, the value of d_{tol} is chosen heuristically to be a few pixels and d_{tol} functions as the control parameter. Now, we present the method to choose the value of d_{tol} automatically using the characteristics of the line, such that the user does not need to specify

the value of d_{tol} [Prasad, *et al.*, 2011a]. First we show that if a continuous line segment is digitized then the maximum distance between that digital line segment and the continuous line segment is bounded and can be computed analytically. Then, this bound can be used to choose the value of d_{tol} adaptively.

We consider the effect of digitization on the slope of a line connecting two points (which may or may not be pixels) [Prasad, *et al.*, 2011a]. Due to digitization in the case of images, a general point $P(x,y)$ is approximated by a pixel $P'(x',y')$ as follows:

$$x' = \text{round}(x); \quad y' = \text{round}(y); \quad \Rightarrow x', y' \in \mathbb{Z} \tag{4}$$

where $\text{round}(x)$ denotes the rounding of the value of real number x to its nearest integer. $P'(x',y')$ satisfy the following:

$$x' = x + \Delta x; \quad y' = y + \Delta y; \quad -0.5 \le \Delta x \le 0.5, \quad -0.5 \le \Delta y \le 0.5 \tag{5}$$

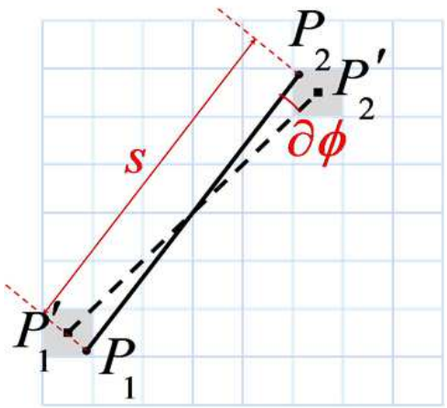


Fig. 1. Representation of the line P_1P_2 and the digitized line $P_1'P_2'$.

Let the slope of the line P_1P_2 (actual line) be denoted as m and the slope of the line $P_1'P_2'$ (digital line) be denoted as m', where P_1' and P_2' are obtained by digitization of P_1 and P_2 using (4). See Fig. 1 for the illustration. Then m and m' are given as:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{6}$$

$$m' = \frac{y_2' - y_1'}{x_2' - x_1'} = \left(m + \frac{\Delta y_2 - \Delta y_1}{x_2 - x_1} \right) \Bigg/ \left(1 + \frac{\Delta x_2 - \Delta x_1}{x_2 - x_1} \right) \tag{7}$$

The angular difference between the numeric tangent and the digital tangent is used as the estimate of the error. This angular difference is given as:

$$\begin{aligned}\partial\phi &= \left|\tan^{-1}(m)-\tan^{-1}(m')\right| = \left|\tan^{-1}\left(\frac{m-m'}{1+mm'}\right)\right| \\ &= \left|\tan^{-1}\left(\frac{m(\Delta x_2-\Delta x_1)-(\Delta y_2-\Delta y_1)}{(1+m^2)(x_2-x_1)+(\Delta x_2-\Delta x_1)+m(\Delta y_2-\Delta y_1)}\right)\right| \\ &= \left|\tan^{-1}\left(\left(\frac{x_2-x_1}{s^2}\right)(1+t)^{-1}\left(m(\Delta x_2-\Delta x_1)-(\Delta y_2-\Delta y_1)\right)\right)\right|\end{aligned} \tag{8}$$

where, $s=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ and $t=\frac{(\Delta x_2-\Delta x_1)(x_2-x_1)}{s^2}+\frac{(\Delta y_2-\Delta y_1)(y_2-y_1)}{s^2}$. Due to (5), the maximum value of $|\Delta x_2-\Delta x_1|$ and $|\Delta y_2-\Delta y_1|$ is 1. Further, $|(x_2-x_1)/s|$ and $|(y_2-y_1)/s|$ are both less than 1. Thus, $|t|<1$ if $s>\sqrt{2}$, which is true for any line made of more than 2 pixels (i.e. 3 pixels or more). Thus, infinite geometric series expansion can be used in (8) and $\partial\phi$ can be approximated as:

$$\begin{aligned}\partial\phi &\approx \left|\tan^{-1}\left(\left(\frac{x_2-x_1}{s^2}\right)(m(\Delta x_2-\Delta x_1)-(\Delta y_2-\Delta y_1))(1-t+t^2)\right)\right| \\ &\approx \left|\left(\frac{x_2-x_1}{s^2}\right)(m(\Delta x_2-\Delta x_1)-(\Delta y_2-\Delta y_1))(1-t+t^2)\right|\end{aligned} \tag{9}$$

Further we note that, $\partial\phi$ has a maximum value when $|\Delta x_2-\Delta x_1|=|\Delta y_2-\Delta y_1|=1$:

$$\partial\phi_{\max}=\max\left(\frac{1}{s^3}(|\sin\phi\pm\cos\phi|)\left(\left|s^2-s(\pm\cos\phi\pm\sin\phi)+(\pm\cos\phi\pm\sin\phi)^2\right|\right)\right) \tag{10}$$

where, $\phi=\tan^{-1}(m)$. Then, the maximum deviation is given by:

$$d_{\max}=s\partial\phi_{\max}. \tag{11}$$

Based on the above analysis, in L-RDP, the suggested value of d_{tol} at every iteration is $d_{\max}=s\partial\phi_{\max}$. At each step in the recursion, if the length of the line segment most fit on the curve (or sub-curve) is s and the slope of the line segment is m, then using (10), we compute $d_{\text{tol}}=s\partial\phi_{\max}$ and use it in (3).

3. Global vs. local characteristics of line fit

It is expected that while fitting a polygon on a digital curve, which is effectively fitting a series of line segments on the digital curve, either we have to take very small local area in order to achieve high precision or we have to take a larger area in order to have a reliable and practically usable fit. This was formally stated and explained by Strauss in the context of Hough transform [Strauss, 1996; Strauss, 1999], "This duality could be set out as follows: as the shape detection precision increases, the reliability of the detection decreases. This seems to be due to the binary aspect of the vote in the classical Hough transform."

While Strauss is right in pointing out the duality between the precision (quality of local fit) and reliability (quality of global fit), he is incorrect in attributing it to the nature of Hough transform. It can be shown using simple metrics, precision and reliability measures, that there is a perennial conflict in the quality of fit in the local scale (precision at the level of few pixels) and global scale (reliability at the level of complete curve) [Prasad & Leung, 2010c]. It is due to this reason, most absolute measures fail in quantifying the quality of fit properly [Carmona-Poyato, *et al.*, 2011; Rosin, 1997].

Assuming that we are not bound by the limitation that the points used for representing the digital curve should be a subset of the digital curve, we just use least squares method to get the best line(s) fit for a digital curve and show that the precision and reliability measures are at conflict with each other. Suppose, for a digital curve with the sequence of pixels $S = \{P_i(x_i, y_i)\}$, $i = 1$ to N, we intend to fit a line $ax + by = 1$. Then, the coefficients of the line, a and b, can be determined by casting the problem of fitting into the following matrix equation [Acton, 1984]:

$$\mathbf{X}\bar{\mathbf{A}} = \bar{\mathbf{J}}, \tag{12}$$

where $\mathbf{X} = \begin{bmatrix} [x_1 \quad x_2 \quad \cdots \quad x_M]^{\mathrm{T}} & [y_1 \quad y_2 \quad \cdots \quad y_M]^{\mathrm{T}} \end{bmatrix}$, $\bar{\mathbf{A}} = [a \quad b]^{\mathrm{T}}$, the superscript T denotes the transpose operation, and $\bar{\mathbf{J}}$ is a column matrix containing M rows, whose every element is 1.

3.1 Precision

The precision of fitting can be modeled using the residue of the least squares method:

$$\varepsilon_p = \left\| \mathbf{X}\bar{\mathbf{A}} - \bar{\mathbf{J}} \right\| = \left\| \mathbf{B}\bar{\mathbf{J}} - \bar{\mathbf{J}} \right\|, \tag{13}$$

where $\|\bullet\|$ represents the Euclidean norm, $\mathbf{B} = \mathbf{X}\left(\mathbf{X}^{\mathrm{T}}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}$ is obtained by substituting $\mathbf{A}$ obtained using (12). The subscript p in ε_p represents precision, and we shall refer to ε_p as the precision parameter for the ease of reference. The lower the value of ε_p, the greater the precision. Noting that $\mathbf{B} = \mathbf{B}^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{B}$, (13) can simplified as $\varepsilon_p = \sqrt{\left(\mathbf{B}\bar{\mathbf{J}} - \bar{\mathbf{J}}\right)^{\mathrm{T}}\left(\mathbf{B}\bar{\mathbf{J}} - \bar{\mathbf{J}}\right)} = \sqrt{\left(\left\|\bar{\mathbf{J}}\right\|\left\|\bar{\mathbf{J}} - \mathbf{B}\bar{\mathbf{J}}\right\|\cos\theta\right)} = \sqrt[4]{M}\sqrt{\left(\left\|\bar{\mathbf{J}} - \mathbf{B}\bar{\mathbf{J}}\right\|\cos\theta\right)}$, where $\left\|\bar{\mathbf{J}}\right\| = \sqrt{N}$ (since $\bar{\mathbf{J}}$ contains N elements, each equals to 1) and θ is the angle between $\bar{\mathbf{J}}$ and $\left(\bar{\mathbf{J}} - \mathbf{B}\bar{\mathbf{J}}\right)$. However, since $\varepsilon_p = \left\|\bar{\mathbf{J}} - \mathbf{B}\bar{\mathbf{J}}\right\|$, ε_p can be written as:

$$\varepsilon_p = \sqrt{N}\cos\theta. \tag{14}$$

It is evident that by choosing lesser number of pixels, i.e., reducing N, ε_p can be reduced and hence, the precision can be increased. It should be noted that with the decrease in the number of pixels, $\mathbf{X}$ and consequently $\mathbf{B}$ change, and thus the contribution from $\cos\theta$ may vary. However, if continuous pixels are considered, the overall variance in $\mathbf{X}$ is reduced, and hence the impact of $\cos\theta$ is also reduced. In effect, this means that fitting the line in a smaller local region is more precise than a large region.

3.2 Reliability

In this sub-section, we first present a quantitative measure of reliability that can be understood and compared with respect to the precision measure. Generally, for the reliability of a fit, the fit is expected to satisfy at least two conditions. First, the fit should be valid for a sufficiently large region (or in this case a long edge) and second, it should not be sensitive to occasional spurious large deviations in the edge. A combination of both these properties can be sought by defining a reliability parameter as follows:

$$\varepsilon_r = \sum_i \left| \mathbf{X}_i \bar{\mathbf{A}} - 1 \right| \Big/ s_{\max} \tag{15}$$

where $\mathbf{X}_i = \begin{bmatrix} x_i & y_i \end{bmatrix}$, $|\bullet|$ represents the magnitude and $s_{\max}$ is the maximum Euclidean distance between any two pair of pixels. Subscript r in ε_r denotes reliability. As before, lower the value of ε_r, higher the reliability.

3.3 Duality

As evident from (14) and (15), and the discussion between them, there is always a contradiction between precision and reliability. In order to increase the precision, we need to consider smaller regions for fitting, whereas for increasing the reliability, we need to consider larger regions for fitting (largest region being the region spanned by the connected edge pixels under consideration). Indeed the contradiction does not occur in ideal lines as shown in Fig. 2(a)-(d). It is also not an issue if the lines are in general smooth, so that the precision within a large region is already very high, such that reliability and precision are already sufficiently high and there is no practical need to increase the precision or reliability. Some such examples are presented in Fig. 2(e)-(g). This is illustrated in Fig. 3 and Table 1. However, if indeed an application calls for still higher precision, the reliability will have to be compromised and the duality comes into picture. Examples of more practical cases are shown in Fig. 2(h)-(p). In such cases, the duality comes into picture strongly and a balance has to be achieved in order to obtain a fit that is sufficiently reliable as well as precise.

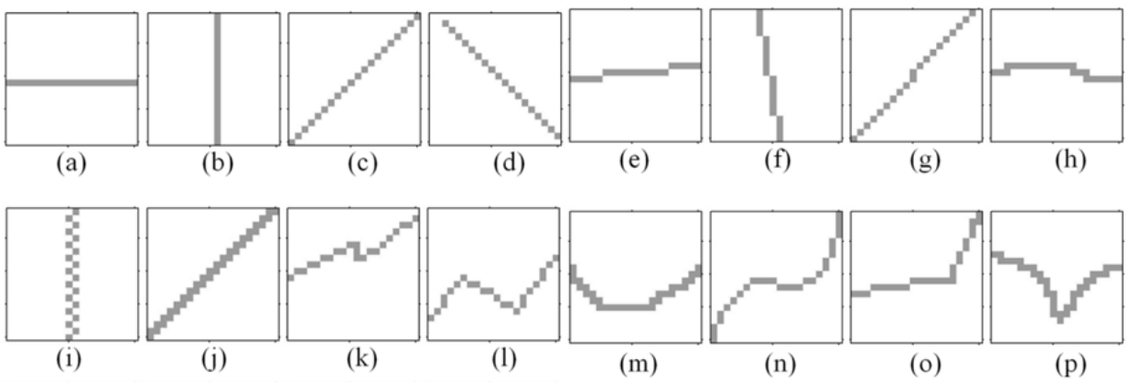


Fig. 2. Example of small images. Each image is of size 20×20 pixels. The grey pixels are the edge pixels, on which line has to be fit.

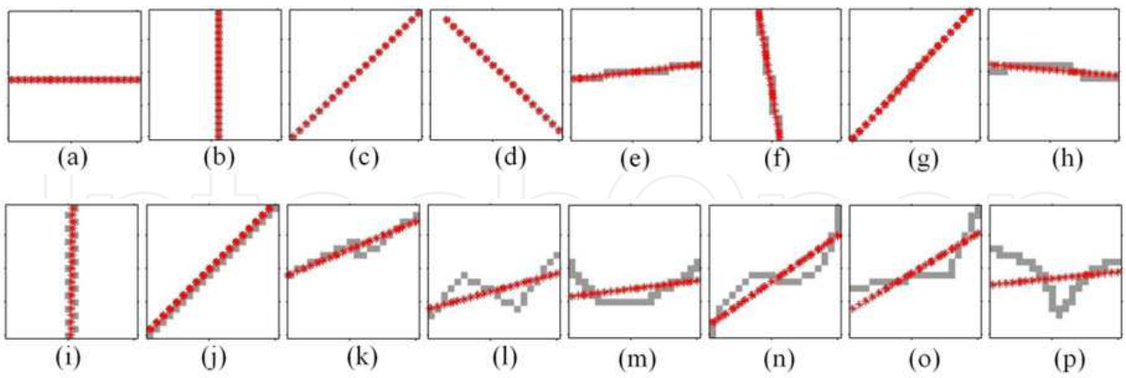


Fig. 3. Lines that have been fit on the images using least squares approach. The lines are shown in red asterisk.

Image	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
ε_p'	0.000	0.000	0.000	0.000	0.024	0.035	0.013	0.071
ε_r	0.000	0.000	0.000	0.000	0.022	0.031	0.008	0.066
Image	(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
ε_p'	0.047	0.015	0.072	0.121	0.150	0.111	0.121	0.233
ε_r	0.048	0.007	0.057	0.114	0.135	0.107	0.115	0.248

Table 1. Value of parameters ε_p' and ε_r corresponding to Fig. 3. Values are shown till 3 decimal points.

3.4 Performance measures in the context of precision and reliability

We use various performance measures for comparing various algorithms.

1. Maximum deviation of any pixel on the edge from the fitted polygon ($\varepsilon_{\max}$):

In the context of a general line given by (12), the maximum deviation $\varepsilon_{\max}$ is specified by $\varepsilon_{\max} = \left\|\mathbf{X}\bar{\mathbf{A}} - \bar{\mathbf{J}}\right\|_\infty \Big/ \left\|\bar{\mathbf{A}}\right\|$, where $\|\bullet\|_\infty$ denotes the infinity norm (i.e. maximum norm). Since $\|\bullet\|_\infty \le \|\bullet\|$, $\varepsilon_{\max} \le \varepsilon_p \Big/ \left\|\bar{\mathbf{A}}\right\|$. Thus, it can be concluded that $\varepsilon_{\max}$ is a form of precision measure.

2. Integral square error (ISE):

This is the sum of squares of deviation of each pixel from the approximated polygon. It is given by $\text{ISE} = \left(\varepsilon_p \Big/ \left\|\bar{\mathbf{A}}\right\|\right)^2 = \varepsilon_p^2 \Big/ \left(\bar{\mathbf{A}}^{\mathrm{T}}\bar{\mathbf{A}}\right)$. Thus, effectively, ISE is also a precision measure.

3. Dimensionality reduction (DR) ratio or compression ratio (CR):

The compression ratio is the ratio of number of pixels in the digital curve (N) to the number of vertices of the polygonal approximation (M), $CR = N/M$. Though this measure is not related to either precision or reliability, it is an important performance metric in

practice. In addition to other metrics representing precision and/or reliability, a larger value of this is beneficial for reduction of data and computational resources. Instead of compression ratio, its reciprocal dimensionality reduction ratio $DR = CR^{-1} = M/N$ can be used as a minimization metric (i.e. the lesser, the better).

4. Figure of merit (FOM)

Figure of merit is given by $\mathrm{FOM}=\mathrm{CR}/\mathrm{ISE}$. This is a maximization metric, i.e., larger value of FOM is preferred over a lower value. However, it is well known that FOM is biased towards ISE [Carmona-Poyato, *et al.*, 2010]. For example, if the break points of a digital curve [Masood, 2008] are considered as the dominant points, the ISE is zero and inconsequent of the CR, FOM is infinity. If we intend to use a minimization metric, we may consider $\mathrm{WE}^{1}=1/\mathrm{FOM}$ [Marji & Siy, 2004]. It suffers with the same deficiency as FOM.

5. Fidelity, Efficiency and Merit

Researchers tried relative measures like fidelity, efficiency, merit to quantify the quality of fit [Carmona-Poyato, *et al.*, 2011; Rosin, 1997]. In relative measures, a so-called optimal algorithm is considered as the reference for comparing the performance of the algorithm being tested. The method proposed by Perez and Vidal [Perez & Vidal, 1994] based on dynamic programming is generally used by the researchers as the reference algorithm. This is because it targets $\min-\varepsilon$ and $\min-\#$ such that the fitting error is minimized for a certain number of points ($\min-\varepsilon$) or the number of points for fitting is minimized for a given value of fitting error ($\min-\#$). It is logical that there is no way of determining an optimal value for the fixed number of points ($\min-\varepsilon$) or the fixed value of fitting error ($\min-\#$), because such a value depends upon the nature of the digital curve for which polygonal approximation is sought.

3.5 Proposed performance measures

As seen in section 3.4, none of the existing methods cater for the global nature of the fit. Thus, the reliability measure is very important addition to the performance metrics of the polygonal approximation method. For the line segments (edges of the polygon), the precision and reliability measures are computed:

$$\varepsilon'_p = \frac{\bar{\mathbf{J}} \cdot \left(\bar{\mathbf{J}} - \mathbf{X}\bar{\mathbf{A}}\right)}{\left|\bar{\mathbf{A}}\right|} \tag{16}$$

$$\varepsilon_r = \sum_i \left|\mathbf{X}_i \bar{\mathbf{A}} - 1\right| \Big/ s_{\max} \tag{17}$$

where ε'_p and ε_r are the precision and reliability measures, $\mathbf{X}_i = \begin{bmatrix} x_i & y_i \end{bmatrix}$, and $s_{\max}$ is the maximum Euclidean distance between any two pair of pixels [Prasad & Leung, 2010c]. Notation $|\bullet|$ represents the magnitude in the scalar case and the Euclidean norm in the case of vectors.

1. Precision measure for an edge

Suppose J line segments are fitted upon a digital curve. Then we define the net precision measure for the digital curve as follows:

$$\left(\varepsilon'_p\right)_{\text{Curve}} = \text{mean}\left(\varepsilon'^{j}_p ; j = 1 \text{ to } J\right), \tag{18}$$

where ε'^{j}_p is the precision measure of the j th line segment, defined using (16).

2. Reliability measure for an edge

The net reliability measure of the digital curve is defined as follows:

$$\left(\varepsilon_r\right)_{\text{Curve}} = \frac{\sum_{j=1}^{J}\sum_{i}\left|\mathbf{X}_i^j \bar{\mathbf{A}}^j - 1\right|}{\sum_{j=1}^{J} s_{\max}^j}, \tag{19}$$

where $\mathbf{X}_i^j$, $\bar{\mathbf{A}}^j$, and $s_{\max}^j$ correspond to $\mathbf{X}_i$, $\bar{\mathbf{A}}$, and $s_{\max}$ defined after (17) for the j th line segment.

3. Precision measure for a dataset of images

Suppose a dataset contains L number of images, the number of edges in the l th image is K_l, then, the precision measure for the dataset is:

$$\begin{aligned}\left(\varepsilon'_p\right)_{\text{Dataset}} &= \text{mean}\left(\left(\varepsilon'_p\right)^{l}_{\text{Image}} ; l = 1 \text{ to } L\right)\\ \left(\varepsilon'_p\right)_{\text{Image}} &= \max\left(\left(\varepsilon'_p\right)^{k}_{\text{Curve}} ; k = 1 \text{ to } K\right)\end{aligned}, \tag{20}$$

4. Reliability measure for a dataset of images

In a manner similar to (20), the reliability measure for a dataset is:

$$\begin{aligned}\left(\varepsilon_r\right)_{\text{Dataset}} &= \text{mean}\left(\left(\varepsilon_r\right)^{l}_{\text{Image}} ; l = 1 \text{ to } L\right)\\ \left(\varepsilon_r\right)_{\text{Image}} &= \max\left(\left(\varepsilon_r\right)^{k}_{\text{Curve}} ; k = 1 \text{ to } K\right)\end{aligned}, \tag{21}$$

4. Contemporary polygonal approximation method in the perspective of duality and the upper bound

4.1 Optimal polygonal representation of Perez and Vidal [Perez & Vidal, 1994]

The algorithm proposed by Perez and Vidal (PV) [Perez & Vidal, 1994] is by far the most popular algorithm used as a benchmark for comparing the performance of polygonal fitting algorithms. The reason for its popularity is twofold. For a given number of points $N' \le N$, where N is the number of pixels in the digital curve, it computes the optimal choice of N' points from the digital curve such that some error metric is minimized. Since the error metric can be flexibly defined by a user, it is versatile in its use. Further, for the purpose of benchmarking, the designers of other algorithms can first perform the polygonal fitting using their own algorithms, obtain a value of N' as obtained by their own algorithms, use this value of N' in the algorithm by PV and simply compare the points obtained by their method against the optimal points obtained by PV.

Since PV can use any error metric to be minimized, it is interesting to note that we can either use the precision score or the reliability score as the error to be minimized. If precision score is used as the error function, PV attempts to fit segments such that all the line segments are of approximately the same length. If reliability score is used as the error function, PV attempts to fit segments that are combination of two types: first type are the small segment with small value of $d_{\max}$ but with very small (close to zero) value of $\sum_i \left|\mathbf{X}_i\bar{\mathbf{A}} - 1\right|$; the second type are long segments with comparatively larger values of $\sum_i \left|\mathbf{X}_i\bar{\mathbf{A}} - 1\right|$ but significantly larger value of $d_{\max}$ such that the reliability score is also small valued.

On the other hand, PV do not guarantee that the maximum deviation of the pixels in curve is within the upper limit of the error due to digitization. If the value of N' is very large, it is likely that PV will fit the segments such that the maximum deviation is lesser than the upper bound. This means that the polygonal approximation will over fit and be sensitive to the error due to digitization. On the other hand, if the value of N' is small, the maximum deviation of the fitted segments is larger than the upper bound, thus indicating under-fitting. In essence, this means that using a fixed value of N' or solving min-ε problem is not suitable for optimal polygonal approximation of the digital curves.

4.2 Lowe [Lowe, 1987] and Ramer-Douglas-Peucker [Douglas & Peucker, 1973; Ramer, 1972] (L-RDP method)

Lowe [Lowe, 1987] and Ramer-Douglas-Peucker [Douglas & Peucker, 1973; Ramer, 1972] is basically a splitting method in which the point of maximum deviation is found recursively till the maximum deviation of any edge pixel from the nearest line segment is less than a fixed value. Since this is a splitting algorithm, it begins with a very high value of $d_{\max}$, which reduces as the edge is split further. The algorithm stops at a point where the maximum deviation satisfies a minimum criterion. Thus, this algorithm focuses more on reliability and attempts to barely satisfy a precision requirement.

In the sense of the upper bound, this algorithm gives a mixed performance. For a few segments, the chosen threshold may be below the upper bound and the result is an over-fitting for this segment. On the other hand, the chosen threshold may be above the upper bound for certain line segments, thus resulting in under-fitting for such segments.

4.3 Precision and reliability based optimization (PRO)

In this method, though the method of optimization is the same as the L-RDP method, the optimization goal is different than (3). Instead of (3), the optimization goal is:

$$\max\left(\varepsilon_p', \varepsilon_r\right) < \varepsilon_0, \tag{22}$$

where, ε_0 is the chosen heuristic parameter. Since this method explicitly uses the precision and reliability measures as the optimization functions, this method is expected to perform well for both precision and reliability measures.

However, this method does not take into account the upper bound of the error due to digitization.

4.4 Break point supression method of Masood [Masood, 2008]

Masood begins with the sequence of the break points, i.e., the smallest set of line segments such that each pixel of the curve lies exactly on the line segments, which is considered as the initial set of dominant points. Then, he proceeds with recursively deleting one break point at a time such that removing it has a minimum impact in its immediate neighborhood and optimizing the locations of the dominant points for minimum precision score. Although the aim of optimization is to improve the global fit and thus indirectly improve the reliability, evidently, Masood's method is tailored for optimizing the precision and performs poorly in terms of reliability.

Since Masood begins with largest possible set of dominant points and removes the dominant points till a certain termination criterion is satisfied, if the termination criterion is not very relaxed, the maximum deviation is in general lesser than the upper bound. Thus, in essence, Masood's method is sensitive to the digitization effects and gives an unnecessarily close fit to the digital curve.

4.5 Dominent point detection method of Carmona-Poyato [Carmona-Poyato, *et al.*, 2010]

Like Masood [Masood, 2008], Carmona also begins with the sequence of break points and the initial set of dominant points. However, unlike Masood, Carmona recursively deletes the dominant points with minimum impact on the global fit of the line segments. Thus, inherently Carmona-Poyato focuses more on reliability than on precision. It is evident in the results reported in [Carmona-Poyato, *et al.*, 2010] that this method has a tendency to be lenient in the maximum allowable deviation in the favor of general shape representation for the whole curve.

5. Numerical examples

We consider the following methods for comparison:

1. L-RDP_max (from section 2)
2. L-RDP0.5, L-RDP1.0, L-RDP1.5, and L-RDP2.0 (from sections 2 and 4.2) correspond to the values of d_{tol} as 0.5, 1.0, 1.5, and 2.0 pixels respectively.
3. PRO0.2, PRO0.4, PRO0.6, PRO0.8, and PRO1.0 (from section 4.3) correspond to the values of ε_0 as 0.2, 0.4, 0.6, 0.8, and 1.0, respectively.
4. Masood (section 4.4, [Masood, 2008]) using the termination criterion specified in [Masood, 2008], i.e., $\varepsilon_{\max} < 0.9$.
5. Carmona-Poyato (section 4.5, [Carmona-Poyato, *et al.*, 2010]), using the termination condition specified in [Carmona-Poyato, *et al.*, 2010], i.e., $r_i < 0.4$.

5.1 Images of Fig. 2

First, we consider L-RDP_max. The results are in the second row of Fig. 4. We see that L-RDP_max is able to avoid the fluctuations due to digitization and noise (in Fig. 4, see columns (g-p), row 2). In the meanwhile, it is able to retain good fit for snippets with important curvature changes, see columns (h,m,n,p), row 2 of Fig. 4. Conclusively, due to the consideration of the upper bound of digitization error, L-RDP_max considers the general features of the digital curve rather than concentrating on every single small scale feature of the curve.

The first observation is that L-RDP algorithms are very sensitive to the tolerance values. L-RDP0.5 algorithm gives a performance comparable to PRO0.2 and PRO0.4, both qualitatively (specifically note the columns (h,i,m,n) of Fig. 4) and quantitatively (see Table 2). A slight increase in tolerance from 0.5 to 1 changes the quality and performance parameters of the line fitting significantly, as evident in Fig. 4 and Table 2. The performance of L-RDP1 is closer to PRO0.6 and the performance of L-RDP1.5 is closer to L-RDP_max and PRO0.8. As the tolerance is increased further in L-RDP algorithms, the fitted line segments start losing information about the major curvature changes and represent the digital curves only crudely. Thus, though L-RDP2.0 provides significant dimensionality reduction (see DR in Table 2), it performs poorly for all the remaining performance parameters.

Next, we consider the results of PRO algorithms. It can be seen in the row PRO0.2 of Fig. 4 that it follows the digital curves very closely. As a consequence it is very sensitive to digitization and generates numerous small line segments to represent the curve, strongly evident in columns (e-i,k-p) of Fig. 4. Though definitely very reliable and precise, as evident from $\left(\varepsilon_p'\right)_{\text{Dataset}}$ and $\left(\varepsilon_r\right)_{\text{Dataset}}$ in Table 1, due to the tendency to fit the curves very closely, it performs poorly in dimensionality reduction (see DR values in Table 1). In the next set: PRO0.4-0.8, we see that these algorithms tend to follow the curvature of the digital curve, better than PRO0.2. We highlight the results in column (m) of Fig. 4. While PRO0.2 generated many line segment for the right side of the curve, PRO0.4-0.8 are more selective in fitting the line segments and fit the line segments focusing at the location of changes in curvature. Further, as the value of ε_0 increases from 0.4 to 0.8, the tendency to concentrate further on the general characteristics of the curve (rather than following every small scale feature of the curve) increases. This is significantly evident in the results in columns (h), (n), and (p) of Fig. 4. In the last PRO algorithm, PRO1.0, we see that rather than focusing on small features of the digital curve, these algorithms tend to follow the general characteristics of the digital curve on a relatively larger scale. Due to this reason, the results of PRO0.8-1.0 are closer to L-RDP_max. As a consequence of this characteristic, PRO0.8-1.0 and L-RDP_max have significantly better dimensionality reduction as compared to other PRO algorithms (see DR in Table 2).

With lower value of $\left(\varepsilon_p'\right)_{\text{Dataset}}$ than $\left(\varepsilon_r\right)_{\text{Dataset}}$ in Table 2, we see that Masood targets improving the precision (reducing) rather than the reliability. Thus, as noted in columns (k-m) of Fig. 4, Masood fails in representing the nature of the curves effectively. On the other hand, for Carmona-Poyato, the value of $\left(\varepsilon_r\right)_{\text{Dataset}}$ is lower than $\left(\varepsilon_p'\right)_{\text{Dataset}}$ in Table 2. However, due to the small length of the digital curves here and the fact that Carmona fits the polygonal approximation depending upon the length of the curve, in these figures with very small digital curves, it tends to fit the curves very closely (see columns (e,f,i,n,o) of Fig. 4), thus demonstrating better precision as well as reliability as compared to Masood. On the other hand for these images, L-RDP_max gives a performance in between Masood and Carmona-Poyato. This indicates that L-RDP_max avoids both under-fitting and over fitting.

5.2 Example of large closed curve

In this section, we consider an example of digital closed curve which is significantly large and contains 458 pixels. The digital curve is derived by scanning the image of dog from Figure 14 of [Masood, 2008] at 300 dpi, followed with blurring using Adobe Photoshop with

a brush size of 2 pixels. Then the polygonal approximation obtained using various methods are presented in Fig. 5. As in section 5.1, the performance of L-RDP_max, L-RDP1.5, and PRO0.8 are similar. For this curve, the performance of Carmano-Poyato is also similar to L-RDP_max. Not only the number of vertices in the polygonal approximation for these cases are similar, the location of the vertices are also similar. L-RDP0.5 and PRO0.2 over fit the curve with numerous points. The quantitative performances are listed in Table 2.

5.3 Large datasets used in real applications

We consider 7 datasets used in object detection algorithms for the purpose of training. These datasets, namely afright [McCarter & Storkey, 2003], Caltech101 [Fei-Fei, *et al.*, 2007], Caltech 256 [Griffin, *et al.*], Pascal 2007 [Everingham, *et al.*, 2007], Pascal, 2008 [Everingham, *et al.*, 2008], Pascal 2009 [Everingham, *et al.*, 2009], and Pascal 2010[Everingham, *et al.*, 2010], contain a total of 97178 images, with the smallest image being only 80 pixels wide and the largest image being 748 pixels wide. The values of $\left(\varepsilon_p'\right)_{\text{Dataset}}$, $\left(\varepsilon_r\right)_{\text{Dataset}}$ and DR for all the datasets and algorithms are plotted in Fig. 6, Fig. 7, and Fig. 8 respectively. Even over such wide range of images, L-RDP and PRO algorithms give consistent performances, as seen in Fig. 6, Fig. 7, and Fig. 8. Further, all L-RDP and PRO algorithms give better performance in terms of precision and reliability, as seen in Fig. 6 and Fig. 7. As a final note, L-RDP_max gives better DR than both Masood and Carmona-Poyato, as seen in Fig. 8.

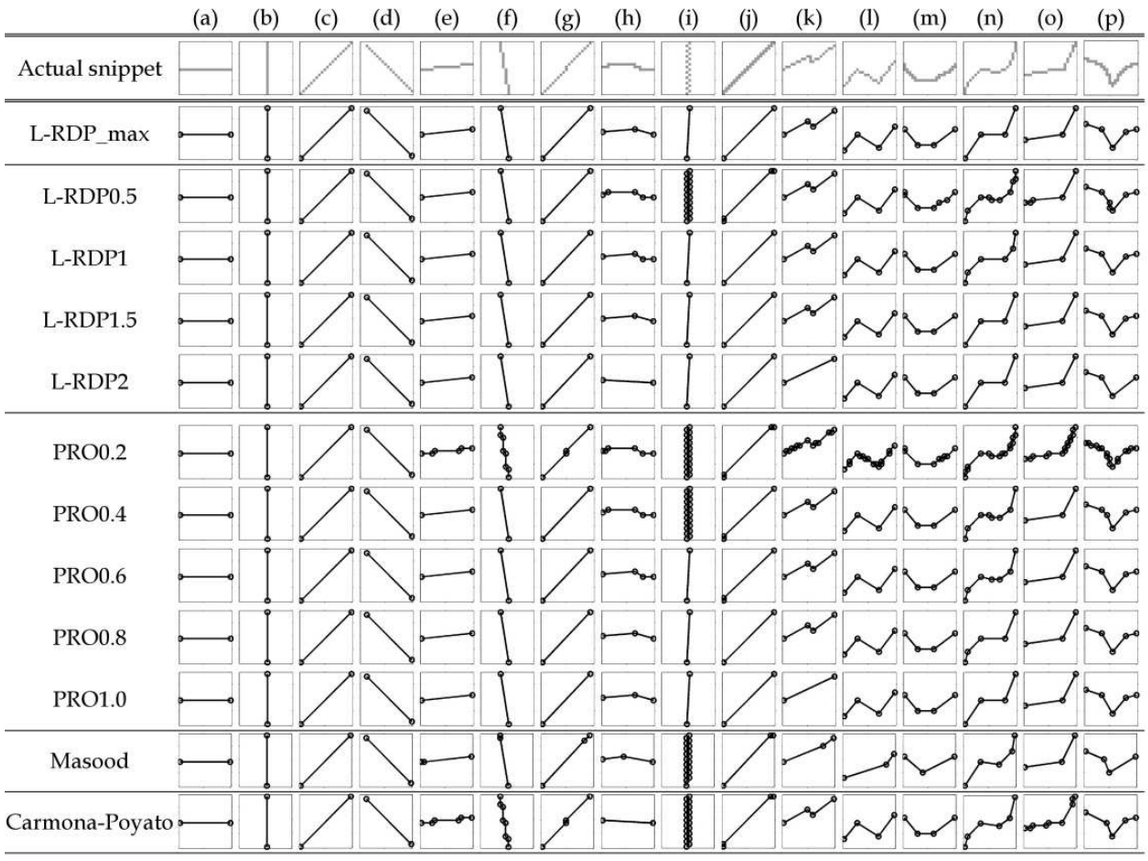
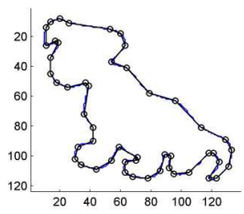


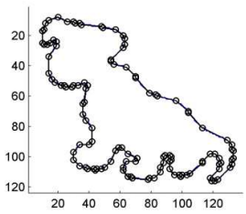
Fig. 4. The polygonal approximations obtained using various methods for images in Fig. 2.

	Dataset of the 16 images in Fig. 2			Digital curve in section 5.2			
	$\left(\varepsilon'_p\right)_{\text{Dataset}}$	$\left(\varepsilon_r\right)_{\text{Dataset}}$	DR	$\left(\varepsilon'_p\right)_{\text{Dataset}}$	$\left(\varepsilon_r\right)_{\text{Dataset}}$	DR	Time (seconds)
L-RDP_max	0.2751	0.2198	0.1286	0.3954	0.3189	0.1114	0.3059
L-RDP0.5	0.1108	0.0931	0.2233	0.0897	0.0964	0.2642	0.8533
L-RDP1	0.2295	0.1884	0.1356	0.2829	0.2397	0.1332	0.3498
L-RDP1.5	0.2751	0.2198	0.1286	0.4133	0.3508	0.1048	0.2571
L-RDP2	0.3650	0.2899	0.1174	0.5298	0.4341	0.0917	0.2225
PRO0.2	0.0032	0.0030	0.3933	0.0055	0.0062	0.4672	1.6157
PRO0.4	0.1486	0.1227	0.2008	0.1629	0.1607	0.1769	0.5004
PRO0.6	0.1974	0.1700	0.1405	0.2748	0.2467	0.1288	0.3330
PRO0.8	0.2563	0.2086	0.1307	0.4066	0.3483	0.1048	0.2613
PRO1.0	0.3185	0.2471	0.1228	0.5626	0.4635	0.0873	0.2100
Masood	0.2970	0.3144	0.1845	0.1203	0.1155	0.2249	26.1754
Carmona-Poyato	0.1306	0.1110	0.2436	0.3349	0.3152	0.1157	0.6324

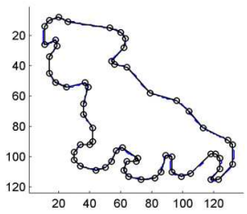
Table 2. Performance metrics for the dataset of the 16 images in Fig. 2 and the digital curve in section 5.2.



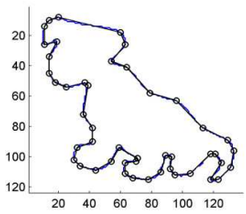
(a) L-RDP-max (M=51)



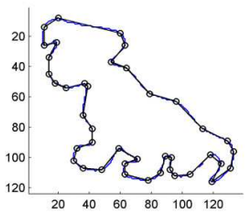
(b) L-RDP0.5 (M=121)



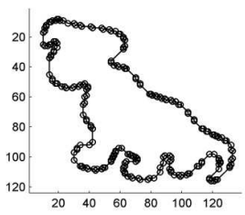
(c) L-RDP1 (M=61)



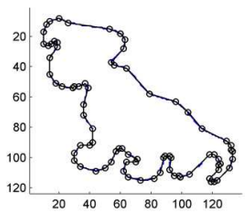
(d) L-RDP1.5 (M=48)



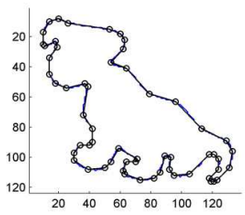
(e) L-RDP2 (M=42)



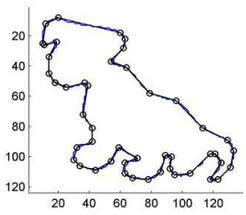
(f) PRO0.2 (M=214)



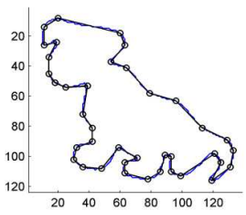
(g) PRO0.4 (M=81)



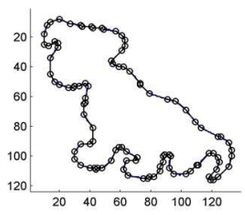
(h) PRO0.6 (M=59)



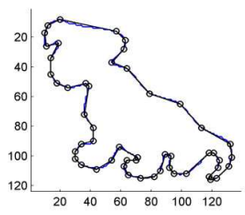
(i) PRO0.8 (M=48)



(j) PRO1.0 (M=40)



(k) Masood (M=103)



l) Carmona-Poyato (M=53)

Fig. 5. Example of a large curve, shape of a dog. The length of the digital curve is N=458.

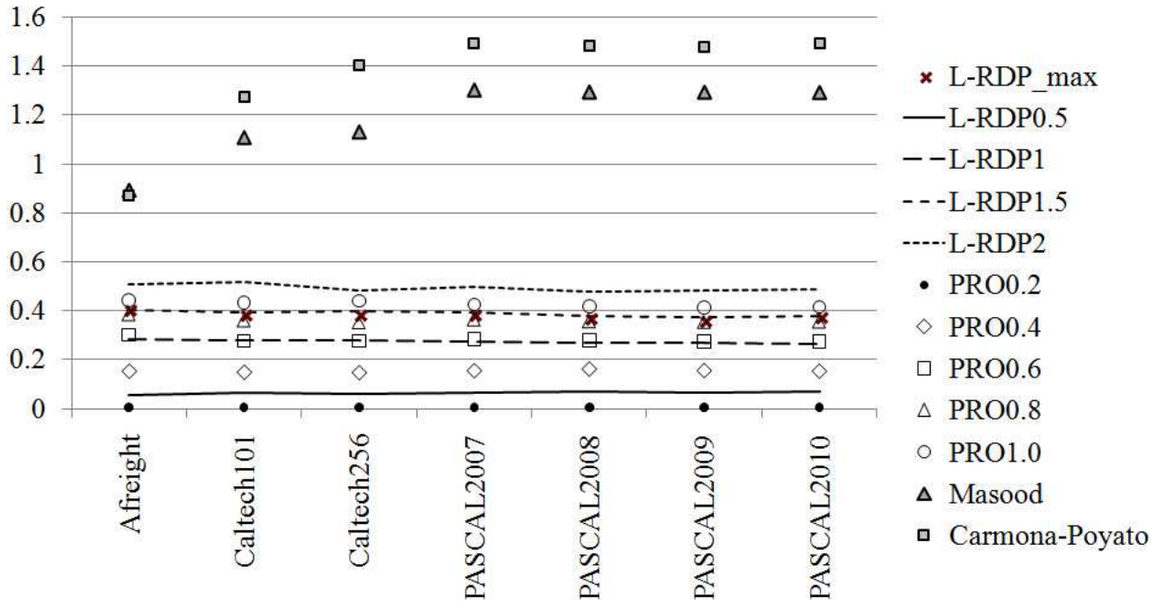


Fig. 6. Precision measure $\left(\varepsilon'_p\right)_{\text{Dataset}}$ for various datasets obtained by different algorithms.

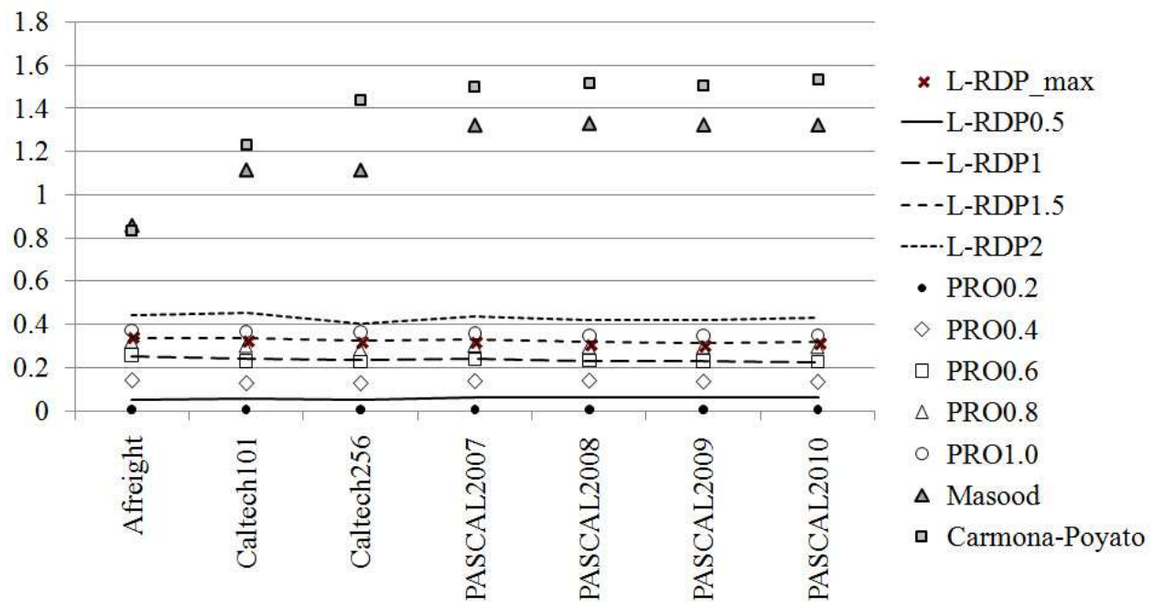


Fig. 7. Reliability measure $\left(\varepsilon_r\right)_{\text{Dataset}}$ for various datasets obtained by different algorithms.

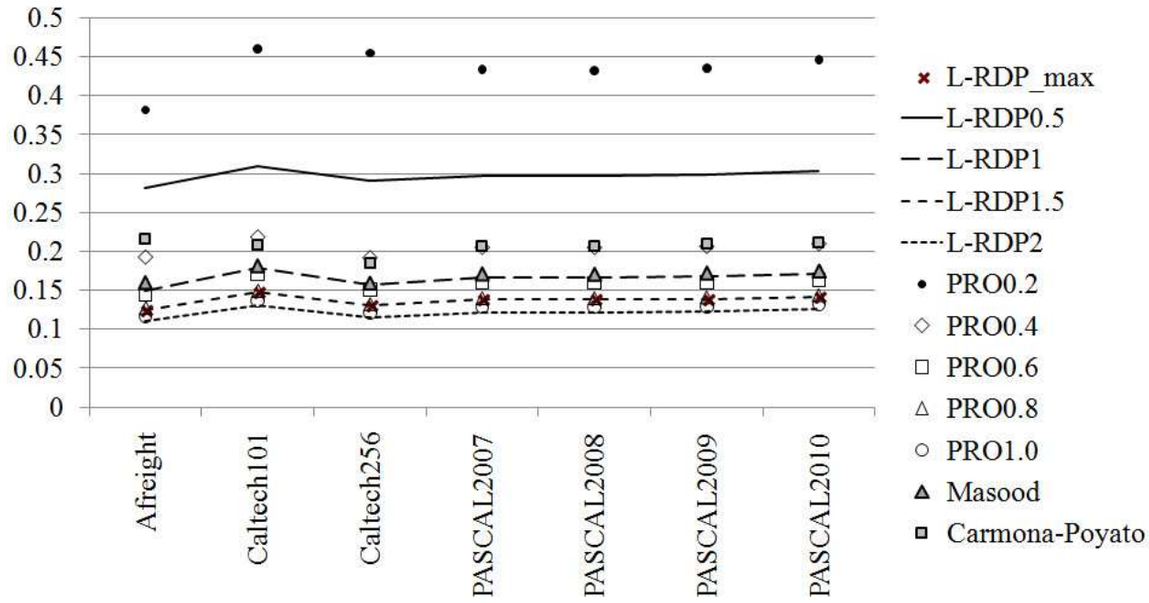


Fig. 8. Average dimensionality reduction for various datasets obtained by different algorithms.

6. Conclusion

Polygonal approximation of digital curves is an important step in many image processing applications. It is important that the fitted polygons are significantly smaller than the original curves, less sensitive to the digitization effect in the digital curves, and good representations of the curvature related properties of the digital curves. Thus, we need methods to deal with the digitization and consider both local and global properties of the fit.

First, we show that the maximum deviation of a digital curve obtained from a line segment has a definite upper bound. We show that this definite upper bound can be incorporated in a polygonal approximation method like L-RDP for making it parameter independent. Various results are shown to demonstrate the effectiveness of the parameter independent L-RDP method against digitization, dimensionality reduction, and retaining good global and local properties of the digital curve. In the future, we are hopeful that this error bound shall be incorporated in various recent and more sophisticated polygonal approximation and give a good performance boost to them, while making them free from heuristic choice of control parameters.

Second, we show that global and local properties of the fit are in contradiction with each other in general. We propose precision measure for measuring the local quality of fit and reliability measure for measuring the global quality of fit. Using them we show that better local fits are achieved by considering small edges of the polygons, while better global fit is

achieved by making the edges of the polygon as long as possible. Further, we show that most contemporary measure of quality of fit are either directly related to precision or correspond to the local nature of fit. Since these measures are used in most contemporary algorithms, most of them concentrate on improving the local quality of the fit only. However, as demonstrated by the upper bound of the maximum deviation due to digitization, it may not be worth to reduce the precision below a certain level, since it is difficult to predict if the actual deviation is below the error bound due to digitization, some form of noise, or due to the nature of the curve. In our knowledge, only Carmona-Poyato includes reliability (though indirectly) in its algorithm [Carmona-Poyato*, et al.*, 2010].

We also propose line fitting algorithm that specifically optimize the curves for increasing both precision and reliability simultaneously. We hope that these measures are paid attention to by the research community and better algorithms for polygonal fitting are developed, which provide good local as well as global fit. In the future, it shall be useful to further improve the design of the precision and reliability measures such that they are more representative of the quality of fit. Such improvements in design will also influence the quality of polygonal approximation achieved by the polygonal approximation methods.

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Everingham, M., Gool, L. V., Williams, C. K. I., Winn, J., & Zisserman, A. (2008). The PASCAL Visual Object Classes Challenge 2008 (VOC2008). http://www.pascal-network.org/challenges/VOC/voc2008/workshop/index.html

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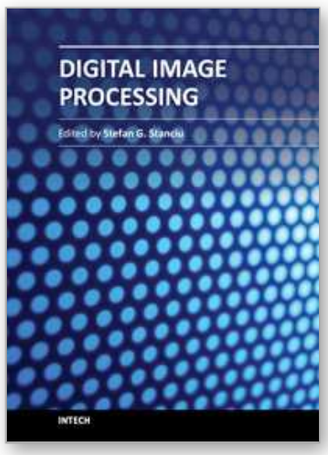
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This book presents several recent advances that are related or fall under the umbrella of 'digital image processing', with the purpose of providing an insight into the possibilities offered by digital image processing algorithms in various fields. The presented mathematical algorithms are accompanied by graphical representations and illustrative examples for an enhanced readability. The chapters are written in a manner that allows even a reader with basic experience and knowledge in the digital image processing field to properly understand the presented algorithms. Concurrently, the structure of the information in this book is such that fellow scientists will be able to use it to push the development of the presented subjects even further.

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InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

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