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Vibration Energy Harvesting: Linear and Nonlinear Oscillator Approaches

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1. Introduction

An important question that must be addressed by any energy harvesting technology is related to the type of energy available (Paradiso et al., 2005; Roundy et al., 2003). Among the renewable energy sources, kinetic energy is undoubtedly the most widely studied for applications to the micro-energy generation[1]. Kinetic energy harvesting requires a transduction mechanism to generate electrical energy from motion. This can happen via a mechanical coupling between the moving body and a physical device that is capable of generating electricity in the form of an electric current or of a voltage difference. In other words a kinetic energy harvester consists of a mechanical moving device that converts displacement into electric charge separation.

The design of the mechanical device is accomplished with the aim of maximising the coupling between the kinetic energy source and the transduction mechanism.

In general the transduction mechanism can generate electricity by exploiting the motion induced by the vibration source into the mechanical system coupled to it. This motion induces displacement of mechanical components and it is customary to exploit relative displacements, velocities or strains in these components.

Relative displacements are usually exploited when electrostatic transduction is considered. In this case two or more electrically charged components move performing work against the electrical forces. This work can be harvested in the form of a varying potential at the terminals of a capacitor.

Velocities are better exploited when electromagnetic induction is the transduction mechanism under consideration. In this case the variation of the magnetic flux through a coil due to the motion of a permanent magnet nearby is exploited in the form of an electric current through the coil itself.

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[1] Clearly kinetic energy is not the only form of energy available at micro and nanoscale. As an example light is a potentially interesting source of energy and nanowires have been studied also in this respect (see e.g. Tian, B. Z. et al. *Coaxial silicon nanowires as solar cells and nanoelectronic power sources*. Nature 449, 885–890, 2007), however in this chapter we will focus on kinetic energy only.

Finally, strains are considered when the transduction mechanism is based on piezoelectric effects. Here electric polarization proportional to the strain appears at the boundaries of a strained piezo material. Such a polarization can be exploited in the form of a voltage at the terminals of an electric load.

In this chapter we will be mainly dealing with transduction mechanisms based on the exploitation of strains although most of the conclusions obtained conserve their validity if applied also to the other two mechanisms.

Before focusing on the characters of the available vibrations it is worth considering the energy balance we are facing in the energy harvesting problem. In the following figure we summarize the balance of the energies involved.
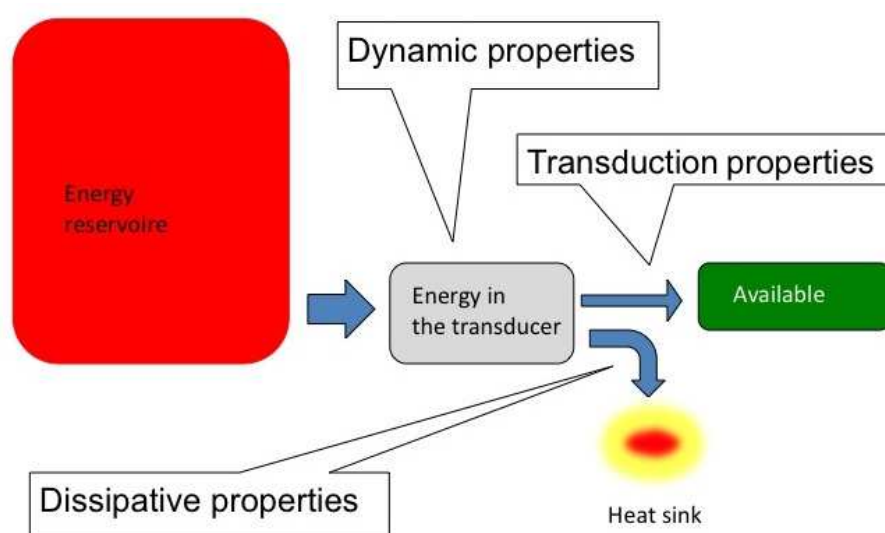

Fig. 1. Energy balance for the harvesting problem.

The kinetic energy available in the environment (red tank) inputs the transducer under the form of work done by the vibrational force to displace the mechanical components of the harvester. At the equilibrium, part of this energy is stored in the device as elastic and/or kinetic energy, part is dissipated in the form of heat and finally part of it is transduced into electric energy available for further use. Different fractions of the incoming energy have different destinations. The relative amount of the different parts depends on the dynamic properties of the transducer, its dissipative and transduction properties, each of them playing a specific role in the energy transformation process. We will come back to the energy balance problem below, when we will deal with a specific transduction mechanism.

2. The character of available energies

At micro and nanoscale kinetic energy is usually available as random vibrations or displacement noise. It is well known that vibrations potentially suitable for energy harvesting can be found in numerous aspects of human experience, including natural events (seismic motion, wind, water tides), common household goods (fridges, fans, washing machines, microwave ovens etc), industrial plant equipment, moving structures such as automobiles and aeroplanes and structures such as buildings and bridges. Also human and

animal bodies are considered interesting sites for vibration harvesting. As an example in Fig. 2 we present three different spectra computed from vibrations taken from a car hood in motion, an operating microwave oven and a running train floor.
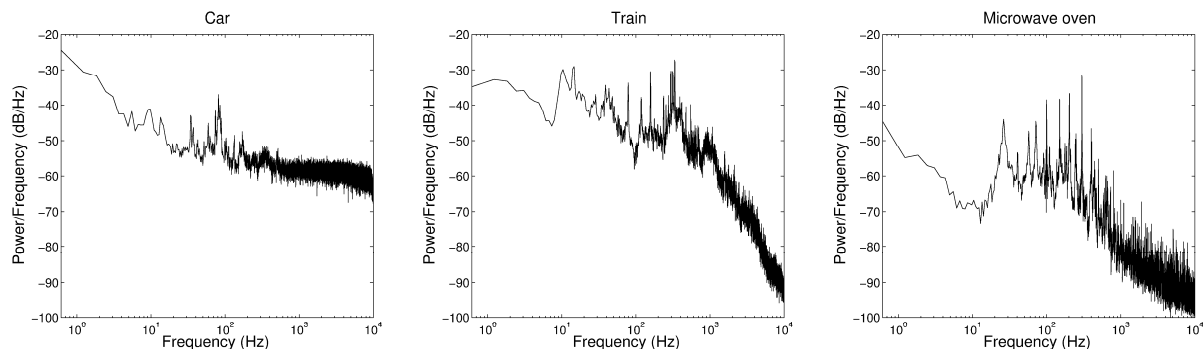

Fig. 2. Vibration power spectra. Figure shows acceleration magnitudes (in db/Hz) vs frequency for three different environments.

All these different sources produce vibrations that vary largely in amplitude and spectral characteristics. Generally speaking the human motion is classified among the high-amplitude/low-frequency sources. These very distinct behaviours in the vibration energy sources available in the environment reflect the difficulty of providing a general viable solution to the problem of vibration energy harvesting.

Indeed one of the main difficulties that faces the layman that wants to build a working vibration harvester is the choice of a suitable vibration to be used as a test bench for testing his/hers own device. In the literature is very common to consider a very special vibration signal represented by a sinusoidal signal of a given frequency and amplitude. As in (Roundy et al. 2004) where a vibration source of 2.5m s−2 at 120Hz has been chosen as a baseline with which to compare generators of differing designs and technologies. Although this is a well known signal that being deterministic in its character (can provide an easy approach both for generation and also for mathematical treatment), the results obtained with this signal are very seldom useful when applied to operative conditions where the vibration signal comes in the form of a random vibration with broad and often non stationary spectra. As we will see more extensively below, the specific features of the vibration spectrum can play a major role in determining the effectiveness of the harvester. In fact, most of the harvesters presently available in the market are based on resonant oscillators whose oscillating amplitude can be significantly enhanced due to vibrations present at the oscillator resonance frequency. On the other hand this kind of harvester results to be almost insensitive to vibrations that fall outside the usually narrow band of its resonance. For this reason it is highly recommended that the oscillator is built with specific care at tuning the resonance frequency in a region where the vibration spectrum is especially rich in energy. As a consequence it is clear that a detailed knowledge of the spectral properties of the hunted vibrations is of paramount importance for the success of the harvester. Unfortunately there is only a limited amount of public knowledge available of the spectral properties of vibrations widely available. In order to fill this gap, a database that collects time series from a wide variety of vibrating bodies was created (Neri et al. 2011). The database, still in the "accumulation phase" has to be remotely accessible without dedicated software. For this reason a data presentation via a web interface is implemented.

3. Micro energies for micro devices and below

An interesting limiting case of kinetic energy present in the form of random vibration is represented by the thermal fluctuations at the nanoscale. This very special environment represents also an important link between the two most promising sources of energy at the nanoscale: thermal gradients and thermal non-equilibrium fluctuations (Casati 2008).

Energy management issues at nanoscales require a careful approach. At the nanoscale, in fact, thermal fluctuations dominates the dynamics and concepts like "energy efficiency" and work-heat relations imply new assumptions and new interpretations. In recent years, assisted by new research tools (Ritort 2005), scientists have begun to study nanoscale interactions in detail. Non-equilibrium work relations, mainly in the form of "fluctuation theorems", have shown to provide valuable information on the role of non-equilibrium fluctuations. This new branch of the fluctuation theory was formalized in the chaotic hypothesis by Gallavotti and Cohen (Gallavotti 1995). Independently, Jarzynski and, then, Crooks derived interesting equalities (Jarzynski 1997), which hold for both closed and open classical statistical systems: such equalities relate the difference of two equilibrium free energies to the expectation of an ad hoc stylized non-equilibrium work functional.

In order to explore viable solutions to the harvesting of energies down to the nanoscales a number of different routes are currently explored by researchers worldwide. An interesting approach has been recently proposed within the framework of the race "Toward Zero-Power ICT"[2]. Within this perspective three classes of devices have been recently proposed (Gammaitoni et al., 2010):

- Phonon rectifiers
- Quantum harvesters
- Nanomechanical nonlinear vibration oscillators

The first device class (**Phonon rectifiers**) deals with the exploitation of thermal gradients (here interpreted in terms of phonon dynamics) via spatial or time asymmetries. The possibility of extracting useful work out of unbiased random fluctuations (often called *noise rectification*) by means of a device where all applied forces and temperature gradients average out to zero, can be considered an educated guess, for a rigorous proof can hardly be given. P. Curie postulated that if such a venue is not explicitly ruled out by the dynamical symmetries of the underlying microscopic processes, then it will generically occur.

The most obvious asymmetry one can try to advocate is spatial asymmetry (say, under mirror reflection, or chiral). Yet, despite the broken spatial symmetry, equilibrium fluctuations alone cannot power a device in a preferential direction of motion, lest it operates as a Maxwell demon, or *perpetuum mobile* of the second kind, in apparent conflict with the Second Law of thermodynamics. This objection, however, can be reconciled with Curie's criterion: indeed, a necessary (but not sufficient!) condition for a system to be at thermal equilibrium can also be expressed in the form of a dynamical symmetry, namely reversibility, or time inversion symmetry (detailed balance). Time asymmetry is thus a second crucial ingredient one advocates in the quest for noise rectification. Note, however,

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[2] "Toward Zero-Power ICT" is an initiative of Future and Emerging Technologies (FET) program within the ICT theme of the Seventh Framework Program for Research of the European Commission.

that detailed balance is a subtle probabilistic concept, which, in certain situations, is at odds with one's intuition. For instance, as reversibility is not a sufficient equilibrium condition, rectification may be suppressed also in the presence of time asymmetry. On the other hand, a device surely operates under non-equilibrium conditions when stationary external perturbations act directly on it or on its environment.

In the concept device studied by the NANOPOWER project the asymmetry is related to the discreteness of phonon modes in cavities. By playing on the mismatch between the energy levels between a small cavity and a bigger one enabling the continuum to be reached, one could find that the transmission are not equal from left to right and vice versa. Phonon rectification occurs as the heat flow between the cavities becomes imbalanced due to non-matching phonon energy levels in it.

Quantum harvesters is a novel class of devices based on mesoscopic systems where unconventional quantum effects dominate the device dynamics. A significant example of this new device class is the so-called Buttiker-Landauer motor (Benjamin 2008) based on a working principle proposed by M. Buttiker (Buttiker 1987) and dealing with a Brownian particle moving in a sinusoidal potential and subject to non-equilibrium noise and a periodic potential. The motion of an underdamped classical particle subject to such a periodic environmental temperature modulation was investigated by Blanter and Buttiker (Blanter 1998). Recently this phenomenology has been experimentally investigated in a system of electrons moving in a semiconductor system with periodic grating and subjected terahertz radiation (Olbrich et al. 2009). The grating is shaped in such a way that it provides both the spatial variation for electron motion as well as a means to absorb radiation of much longer wavelength than the period of the grating.

The third device class is represented by **nano-mechanical nonlinear vibration oscillators**. Nanoscale oscillators have been considered a promising solution for the harvesting of small random vibrations of the kind described above since few years. A significant contribution to this area has been given by Zhong Lin Wang and colleagues at the Georgia Institute of Technology (Wang et al. 2006). In a recent work (Xu et al. 2010) they grew vertical lead zirconate titanate (PZT) nanowires and, exploiting piezoelectric properties of layered arrays of these structures, showed that can convert mechanical strain into electrical energy capable of powering a commercial diode intermittently with operation power up to 6 mW. The typical diameter of the nanowires is 30 to 100 nm, and they measure 1 to 3 μm in length.

A different nano-mechanical generator has been realized by Xi Chen and co-workers (Chen Xi et al. 2010), based on PZT nanofibers, with a diameter and length of approximately 60 nm and >500 μm, aligned on Platinum interdigitated electrodes and packaged in a soft polymer on a silicon substrate. The PZT nanofibers employed in this generator have been prepared by electrospinning process and exhibit an extremely high piezoelectric voltage constant (g33: 0.079 Vm/N) with high bending flexibility and high mechanical strength (unlike bulk, thin films or microfibers). Also Zinc-Oxide (ZnO) material received significant attention in the attempt to realize reliable nano-generators. Min-Yeol Choi and co-workers (Min-Yeol Choi et al. 2009) have recently proposed a transparent and flexible piezoelectric generator based on ZnO nanorods. The nanorods are vertically sandwiched between two flat surfaces producing a thin mattress-like structure. When the structure is bended the nanorods get compressed and a voltage appear at their ends.

At difference with these existing approaches, in the NANOPOWER project attention is focussed mainly on the dynamics of nanoscale structures and for a reason that will be discussed below, it concentrates in geometries that allowed a clear nonlinear dynamical behaviour, like bistable membranes. Recently (Cottone et al. 2009, Gammaitoni et al. 2009) a general class of bistable/multistable nonlinear oscillators have been demonstrated to have noise-activated switching with an increased energy conversion efficiency. In order to reach multi-stable operation condition, in NANOPOWER a clamped membrane is realised under a small compressive strain, forcing it to either of the two positions. The membrane vibrates between the two potential minima and has also intra-minima modes. The kinetic energy of the nonlinear vibration is converted into electric energy by piezo membrane sandwiched between the electrodes.

4. Fundaments of vibration harvesting

As we have anticipated above, kinetic energy harvesting requires a transduction mechanism to generate electrical energy from motion. This is typically achieved by means of a transduction mechanism consisting in a massive mechanical component attached to an inertial frame that acts as the fixed reference. The inertial frame transmits the vibrations to a suspended mass, producing a relative displacement between them.

4.1 A simple scheme for vibration harvesting

The scheme reproduced in Fig. 3 shows the inertial mass m that is acted on by the vibrations transmitted by the vibrating body to the reference frame.
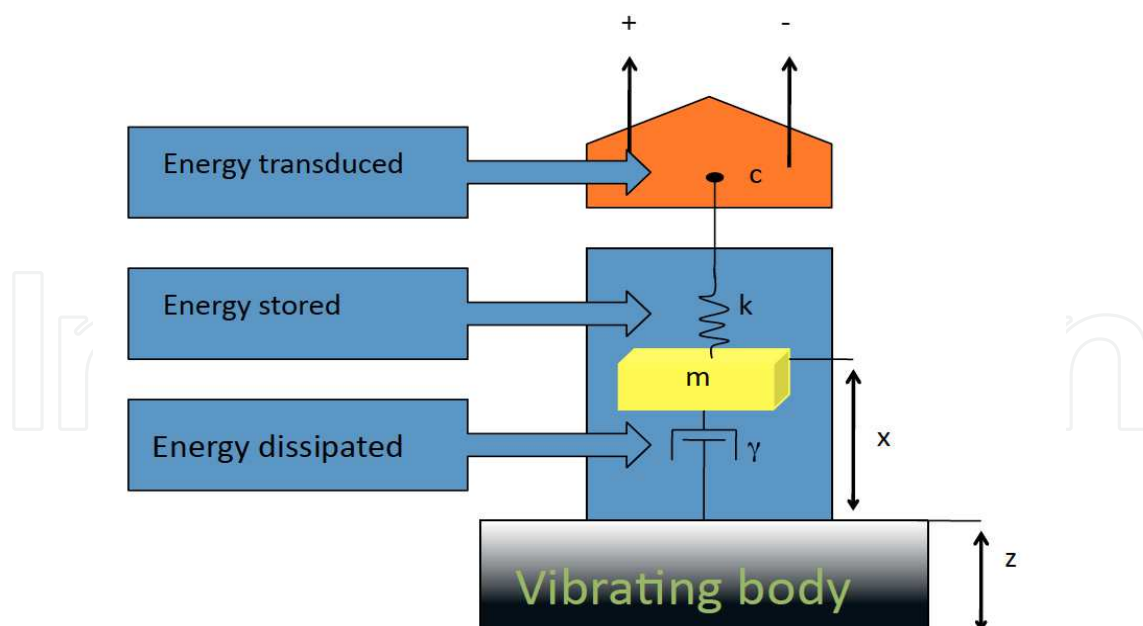

Fig. 3. Vibration-to-electricity dynamic conversion scheme. Energy balance: the kinetic energy input into the system from the contact with the vibrating body is partially stored into the system dynamics (potential energy of the spring), partially dissipated through the dashpot and partially transduced into electric energy available for powering electronic devices.

In terms of energy balance, the input energy, represented by the kinetic energy of the vibrating body, is transmitted to the harvester. This input energy is divided into three main components:

1. Part of the energy is **stored** into the dynamics of the mass and is usually expressed as the sum of its kinetic and potential energy: when the spring is completely extended (or compressed), the mass is at rest and all the dynamic energy is represented by the potential (elastic) energy of the spring.
2. Part of the energy is **dissipated** during the dynamics meaning with this that it is converted from kinetic energy of a macroscopic degree of freedom into heat, i.e. the kinetic energy of many microscopic degrees of freedom. This is represented in Fig. 3 by the dashpot. There are different kinds of dissipative effects that can be relevant for a vibration harvester. One simple example is the internal friction of the material undergoing flexure. Another common case is *viscous damping* a source of dissipation due to the fact that the mass is moving within a gas and the gas opposes some resistance.
3. Finally, some of the energy is **transduced** into electric energy. The transducer is represented in Fig. 3 by the block with the two terminals + and -, thus indicating the existence of a voltage difference V.

4.2 A mathematical model for our scheme

The functioning of the vibration harvester, within this scheme, can usually be quantitatively described in terms of a simple mathematical model that addresses the dynamics of the two relevant quantities: the mass displacement x and the voltage difference V. Both quantities are function of time and obey proper equations of motion.

For the displacement x the dynamics is described by the standard Newton equation, i.e. a second order ordinary differential equation:

$$m\ddot{x} = -\frac{d}{dx}U(x) - \gamma\dot{x} - c(x,V) + \xi_z \qquad (1)$$

where

$U(x)$	Represents the energy stored
$\gamma\dot{x}$	Represents the dissipative force
$c(x,V)$	Represents the reaction force due to the transduction mechanism
ξ_z	Represents the vibration force

The quantity ξ_z represents here the vibration force that acts on the oscillator. In general this is a stochastic quantity due to the random character of practically available vibrations. For this reason the equation of motion cannot be considered an ordinary differential equation but it is more suitably defined a stochastic differential equation, also know as *Langevin equation* by the name of the French physicist who introduced it in 1908 in order to describe the Brownian motion (Langevin, 1908).

All the components of the energy budget that we mentioned above are in this equation represented in term of forces. In particular the quantity γ is the damping constant and

multiplies the time derivative of the displacement, i.e. the velocity. Thus this term represents a dissipative force that opposes the motion with an intensity proportional to the velocity: a condition typical of viscous damping (the faster the motion the greater the force that opposes it).

The quantity $c(x,V)$ is a general function that represents the reaction force due to the motion-to-electricity conversion mechanism. It has the same sign of the dissipative force and thus opposes the motion. In physical terms this arises from the energy fraction that is taken from the kinetic energy and transduced into electric energy.

The dynamics of the voltage V is described by:

$$\dot{V} = F(\dot{x}, V) \tag{2}$$

This is a first order differential equation that connects the velocity of the displacement with the electric voltage generated. In order to reach a full description of the motion-to-electric-energy conversion we need to specify the form of the two connecting functions

$$F(\dot{x}, V) , \quad c(x, V)$$

These two functions are determined once we specify the physical mechanism employed to transform kinetic energy into electric energy.

4.3 The piezoelectric transduction case

As we pointed out earlier there are three main physical mechanisms that are usually considered at this aim: piezoelectric conversion (dynamical strain of piezo material is converted into voltage difference), electromagnetic induction (motion of magnets induces electric current in coils) and capacitive coupling (geometrical variations of capacitors induce voltage difference).

For a number of practical reasons (Roundy et al. 2003) mainly related to the possibility to miniaturize the generator maintaining an efficient energy conversion process, piezoelectricity is generally considered the most interesting mechanism.

For the case of piezoelectric conversion the two connecting functions assume a simple expression:

$$c(x, V) = K_V V$$
$$F(\dot{x}, V) = K_c \dot{x} - \frac{1}{\tau_p} V$$

The dynamical equations thus become:

$$m\ddot{x} = -\frac{d}{dx} U(x) - \gamma \dot{x} - K_V V + \xi_z$$
$$\dot{V} = K_c \dot{x} - \frac{1}{\tau_p} V \tag{3}$$

where K_V and K_c are piezoelectric parameters that depend on the physical properties of the piezo material employed and τ_p is a time constant that can be expressed in terms of the parameters of the electric circuit connected to the generator as:

$$\tau_p = R_L C \tag{4}$$

where C is the electrical capacitance of the piezo component and R_L is the load resistance across which the voltage V is exploited. In this scheme the power extracted from the harvester is given by

$$W = \frac{V^2}{R_L} \tag{5}$$

5. The linear oscillator approach: Performances and limitations

In order to proceed further in our analysis of the vibration harvester we need to focus our attention on the quantity $U(x)$ that following the schematic in Fig. 3, represents the potential energy. The mathematical form of this function is a consequence of the geometry and of the dynamics of the vibration harvester that we want to address.

One of the most common models of harvester is the so-called *cantilever configuration*. A typical cantilever is reproduced in Fig. 4.
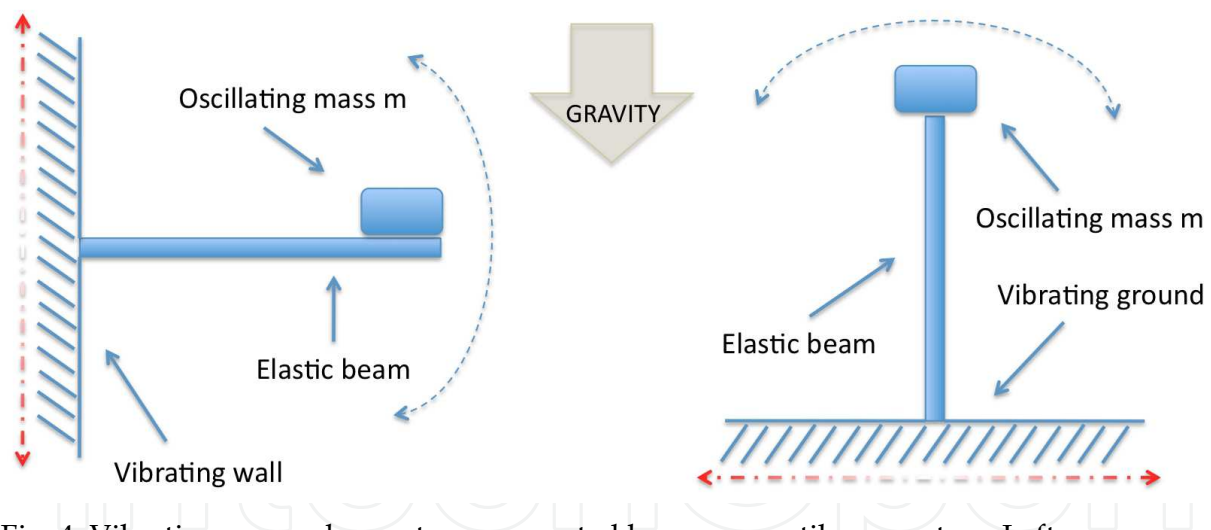

Fig. 4. Vibration energy harvester represented here as a cantilever system. Left: configuration for harvesting vertical vibrations. Right: configuration for harvesting horizontal vibrations.

According to our schematic in Fig. 3, the "spring like" behaviour of the harvester is represented here by the bending of the beam composing the cantilever. When the beam is completely bent (corresponding to the case where the spring is completely extended or compressed), as we have seen above, the mass is at rest and all the dynamic energy is represented by the (potential) elastic energy.

A common assumption is that the potential energy grows with the square of the bending. This is based on the idea that the force acted by the beam is proportional to the bending.

Thus is $F = kx$ than $U(x) = -1/2\, k\, x^2$. The idea that the force is proportional to the bending is quite reasonable and has been verified in a number of different cases. An historically relevant example is the Galileo's pendulum. In this case the "bending" is represented by the displacement of the mass from the vertical position. Due to the action of the gravity, the restoring force that acts on the pendulum mass is $F = -mg\, sin(x/l)$ where g is the gravity acceleration and l is the pendulum length. When the displacement angle x/l is small the function $sin(x/l)$ is approximately equal to x/l (first term of the Taylor expansion around $x/l = 0$) and thus $F = -mg/l\, x$ or $F = -k\, x$. This condition is usually known as "small oscillation approximation" and can be applied any time we have a *small*[3] oscillation condition around an equilibrium point.

5.1 The linear vibration harvester

Within the small oscillation approximation we can treat most of the vibration energy harvesters by introducing a potential energy function like the following:

$$U(x) = \frac{1}{2}kx^2 \tag{6}$$

This form is also known as *harmonic potential*. By substituting (6) in (3) and taking the derivative, the equations of motion now become:

$$m\ddot{x} = -kx - \gamma\dot{x} - K_V V + \xi_z$$
$$\dot{V} = K_c\dot{x} - \frac{1}{\tau_p}V \tag{7}$$

This often called the linear oscillator approximation, due to the fact that, as we have seen, in the dynamical equation of the displacement x the force is linearly proportional to the displacement itself.

A linear oscillator is a very well known case of Newton dynamics and its solution is usually studied in first year course in Physics. A remarkable feature of a linear oscillator is the existence of the *resonance frequency*. When the system is driven by a periodic external force with frequency equal to the resonance frequency then the system response reaches the maximum amplitude.

This occurrence is well described by the so-called system transfer function $H(w)$ whose study is part of the *linear response theory* addressed in physics and engineering course in dynamical systems.

A detailed treatment of the linear response theory is well beyond the scope of this chapter. For our purposes it is sufficient to observe that a linear system represents a good approximation of a number of real oscillators (in the small oscillation approximation) and that their behaviour is characterized by the existence of a resonance frequency that maximizes the oscillator amplitude. This condition has led vibration energy harvester designer to try to build cantilevers (Williams CB et al., 1996, Mitcheson et al., 2004, Stephen

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[3] The oscillation angle is considered small when the terms following the first one in the Taylor expansion of the sine are negligible compare to the leading one

N.G., 2006, Renno J.M. et al., 2009) that operates in the linear oscillation regime and present a resonant frequency that can be tuned to match the characteristic frequency of the application environment. Various attempts to design tuneable harvesters have been proposed (Challa V.R. et al., 2008, Morris D. et al., 2008). Thus most of the present working solutions for vibration-to-electricity conversion are based on linear, i.e. resonant, mechanical oscillators that convert kinetic energy by tuning their resonant frequency in the spectral region where most of the energy is available. However, as we have observed above, in the vast majority of cases the ambient vibrations have their energy distributed over a wide frequency spectrum, with significant predominance of low frequency components and frequency tuning is not always possible due to geometrical/dynamical constraints.
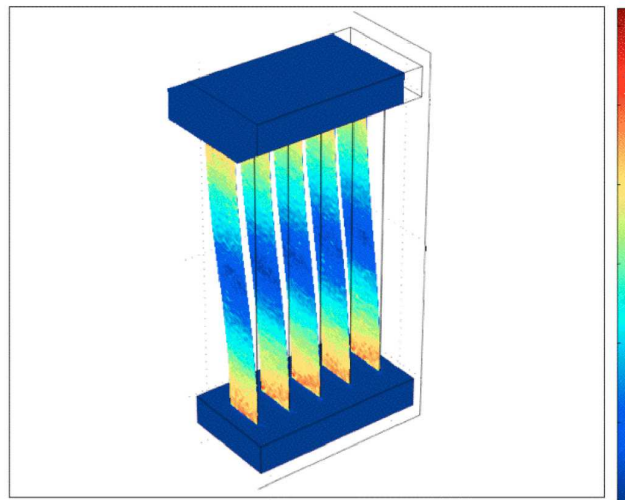
Fig. 5. Composite model of a microscale linear vibration energy harvester. The five beams represented here are 1 mm high and 25 μm thick. The resonant frequency (fundamental mode) for this system is as high as 10 KHz. Colours represents the map of stresses during the dynamics.

5.2 Main limitations in the linear vibration harvester

In general, for a generic linear system, even more complicated that the simple cantilever in Fig. 4, the transfer function presents one or more peaks corresponding to the resonance frequencies and thus it is effective mainly when the incoming energy is abundant in that frequency regions.

Unfortunately this is a serious limitation when it is required to build a vibration energy harvester of small dimension, for at least two main reasons:

a. as we have discussed above, the frequency spectrum of ambient available vibrations instead of being sharply peaked at some frequency is usually very broad.
b. The frequency spectrum of available vibrations is particularly rich in energy in the low frequency range, and it is very difficult, if not impossible, to build small, low-frequency resonant systems.

As an example we can consider the composite system sketched in Fig. 5. In this case the resonant frequency (fundamental mode) is of the order of 10 KHz, a frequency region where in most practical cases the presence of ambient vibrations is almost negligible. As a

consequence we are forced to build vibration harvesters that have geometrical dimensions compatible with low resonance frequencies. Also in this case, however there is a serious limitation arising from the need to tune the resonance frequency according to the specific application. As an example we can consider the following situation: we have designed a vibration harvester that is supposed to operate at 100 Hz (a typical value for commercial harvesters of few centimetres) because for the specific application that we are considering there is enough vibration amplitude at that very frequency. If the ambient condition changes, e.g. the frequency peak of the ambient vibration moves from 100 Hz to 80 Hz our harvester needs a tuning operation in order to lower its resonant frequency. Such an operation can be made, not without difficulties, by changing some physical parameters, like the length or the stiffness of the cantilever beam or the mass attached to its tip. These operations are clearly not easy to perform and thus a linear vibration harvester in general does not allow for wide *tunability*.

5.3 In search of the ideal vibration harvester

Based on the considerations developed so far it is clear that vibration harvesters inspired by cantilever-like configurations present a number of drawbacks that limit seriously their field of application. If we try to summarize what we have learned so far we see that the search for the ideal vibration harvester have to cover at least the following aspects:

- Capability of harvesting energy on a broadband of frequencies and not just at the resonance frequency. This seems to exclude resonant oscillators. In fact a resonant oscillator is capable of harvesting energy only in a very narrow band, i.e. around its resonant frequency. Moreover for a linear oscillator the narrower is the resonance the better is the efficiency in harvesting energy at that frequency. Thus, if we want to keep the requirement 1) we should look for non-resonant oscillators.
- Capability of harvesting energy at low frequency (below few hundred Hz) because this is where most of the ambient available energy is. Also in this case, due to the conflicting requirements of small dimensions and low operating frequencies, we should abandon linear, i.e. resonant oscillators for some other dynamical system that shows a significant response also at low frequencies.
- No need for frequency tuning after the initial set-up of the harvester. Frequency tuning is a feature of resonant systems, thus if we move to non-resonant systems this requirement will be automatically satisfied.

As we have seen before the search for the best solution in terms of non-resonant systems should start from the potential energy function *U(x)*. In fact *U(x)* plays the role of dynamical energy storage facility (before transduction) for our mechanical oscillator and thus it is here that we should focus our attention.

6. The nonlinear oscillator approach: Performances and limitations

The natural candidates to replace linear oscillators seem clearly to be the so called "non-linear" oscillators. Unfortunately this is not a well-defined category and one is left with the sole condition

$$U(x) \neq \frac{1}{2}kx^2 \tag{8}$$

meaning with this expression oscillators whose potential energy is not quadratically dependent on the relevant displacement variable. In recent years few possible candidates have been explored (Cottone et al., 2009; Gammaitoni et al., 2009, 2010, Ferrari M. et al, 2009, Arrieta A.F. et al., 2010, Ando B. et al., 2010, Barton D.A.W. et al., 2010, Stanton S.C. et al., 2010) running from

$$U(x) = ax^{2n} \tag{9}$$

to other more complicated expressions.

For nonlinear oscillators it is not possible to define a transfer function (i.e. a response function independent from the external force acting on the system) and thus a properly defined resonant frequency even if the power spectrum density of the system can show one or more well defined peaks for specific values of the frequencies.

In this section we will briefly address one of these nonlinear potential cases, with the specific purpose of describing and testing the power generated in common environments.

Specifically we will show that, if we consider bistable or monostable oscillators under proper operating conditions, they can provide better performances compared to a linear oscillator in terms of energy extracted from a generic wide spectrum vibration.

6.1 The bistable cantilever

An interesting option for a nonlinear oscillator is to look for a potential energy that is multi-stable, instead of mono-stable (like the linear case, i.e. the harmonic potential). A particularly simple and instructive example on how to move from the linear (mono-stable) to a possible nonlinear (bi-stable) dynamics is represented by a slightly modified version of our vibration harvester cantilever (see Fig. 6).

This is a common cantilever operated vertically (in a configuration sometimes called *inverted pendulum configuration*) with a bending piezoelectric beam. On top of the beam mass a small magnet (tip magnet) has been added. Under the action of the vibrating ground the pendulum oscillates alternatively bending the piezoelectric beam and thus generating a measurable voltage signal V. The dynamics of the inverted pendulum tip can be controlled with the introduction of an external magnet conveniently placed at a certain distance D and with polarities opposed to those of the tip magnet. The external magnet introduces a force dependent from the distance that opposes the elastic restoring force of the bended beam. As a result, the inverted pendulum dynamics can show three different types of behaviours as a function of the distance D:

- when the two magnets are separated by a large distance ($D \gg D_0$) the inverted pendulum behaves like a linear oscillator whose dynamics is resonant with a resonance frequency determined by the system parameters. This situation accounts well for the usual operating condition of traditional piezoelectric vibration-to-electric energy converters.
- On the other hand, when D is small ($D \ll D_0$) the pendulum is forced to oscillates at the left or at the right of the vertical. In the limit of small oscillations, this can still be described in terms of a linear, i.e. resonant, oscillator with a resonant frequency higher that in the previous case.

- Finally, in between the two previous cases it exists an intermediate condition ($D=D_0$) where the pendulum swings in a more complex way with small oscillations around each of the two equilibrium positions (left and right of the vertical) and large excursions from one to the other.
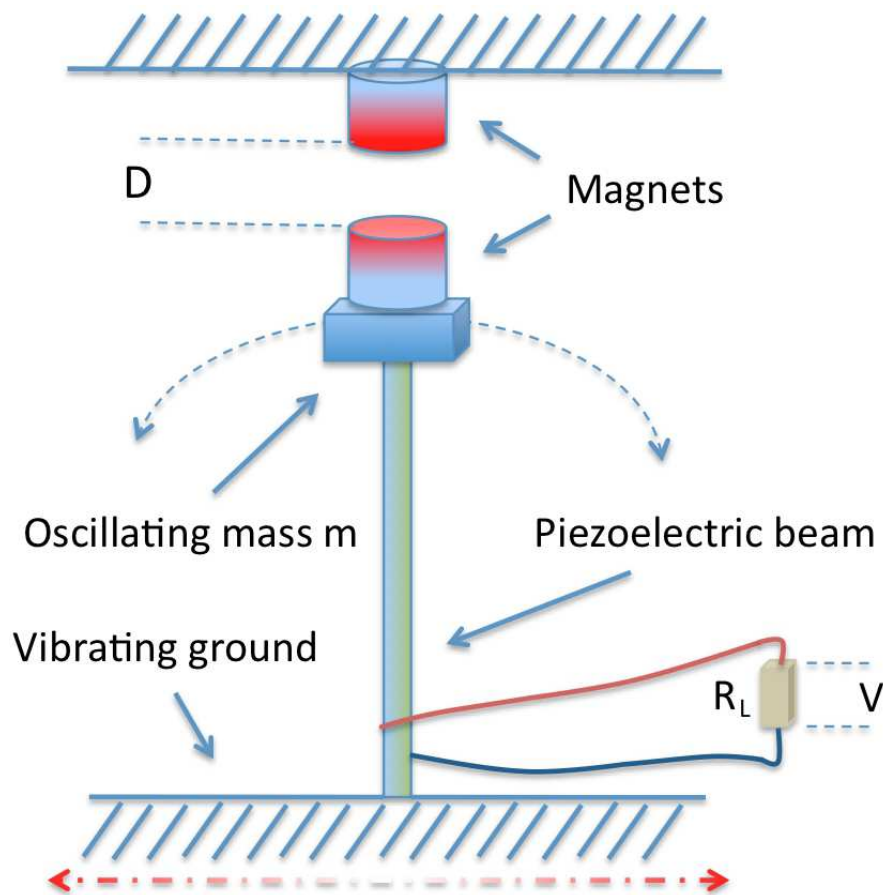

Fig. 6. Piezoelectric inverted pendulum showing bistable dynamics.

In Fig. 7, 8 and 9 we present an analysis of the simulated dynamics with a modified potential that takes into account the presence of the repulsion force due to the magnets. This new potential can be written as:

$$U(x) = \frac{1}{2}k_e x^2 + \left(Ax^2 + BD^2\right)^{-\frac{3}{2}} \tag{10}$$

with k_e, A and B representing constants related to the physical parameters of the pendulum (Lefeuvre et al., 2006, Shu et al., 2006) like the permeability constant and the effective magnetic moment of the magnets. Clearly when the distance D between the magnets grows very large the second term in (10) becomes negligible and the potential tends to the harmonic potential of the linear case, typical of the cantilever harvester.

This is the condition represented in Fig. 7 (top) where we plot the potential $U(x)$ *vs x*. Under the picture of the potential we plot the displacement *x* time series. This is measured in meters and it is referred to an inverted pendulum of few centimetres long. For details please

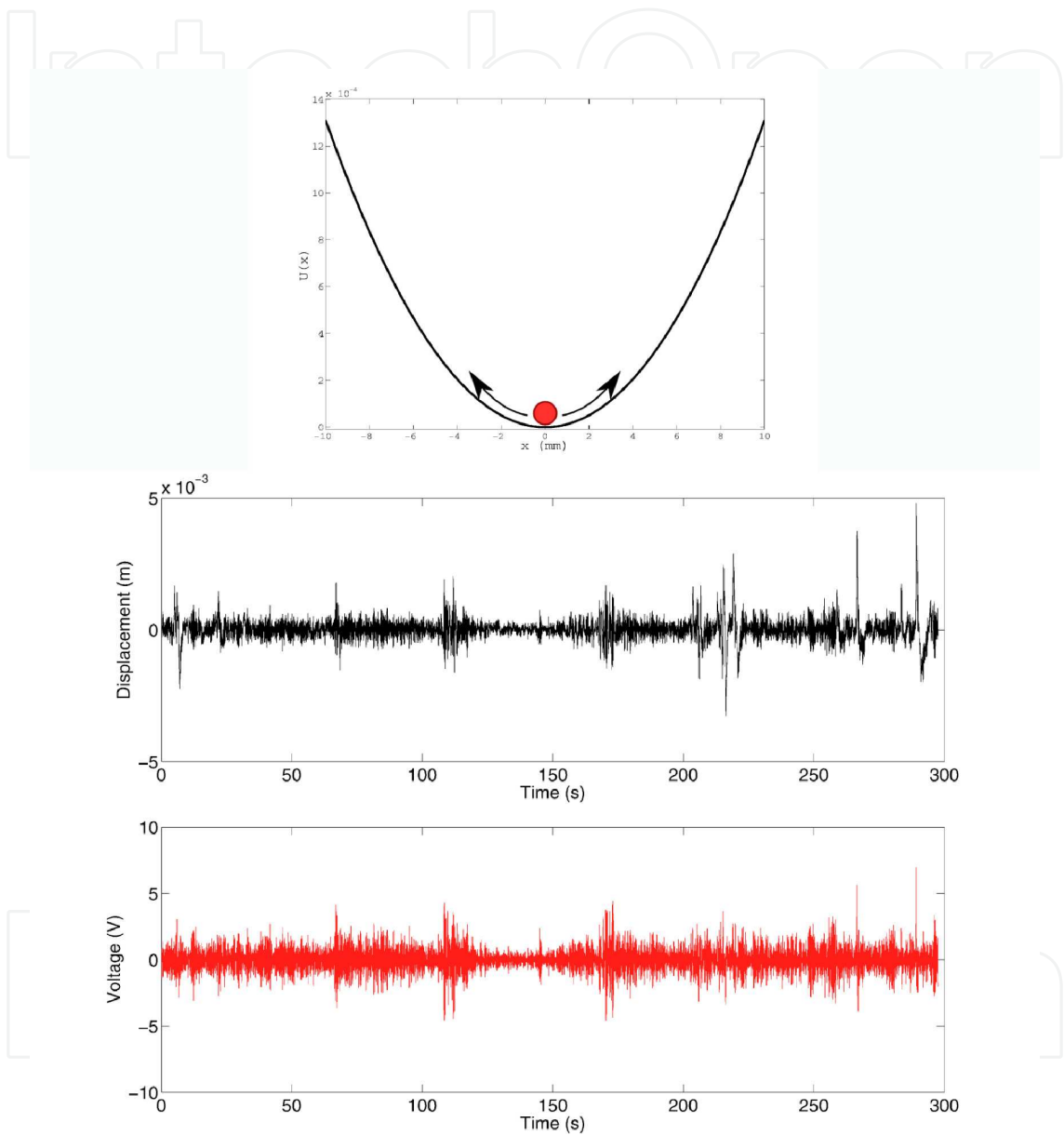see (Cottone et al. 2009). Further below there is the time series for the voltage V across the load resistor R_L.
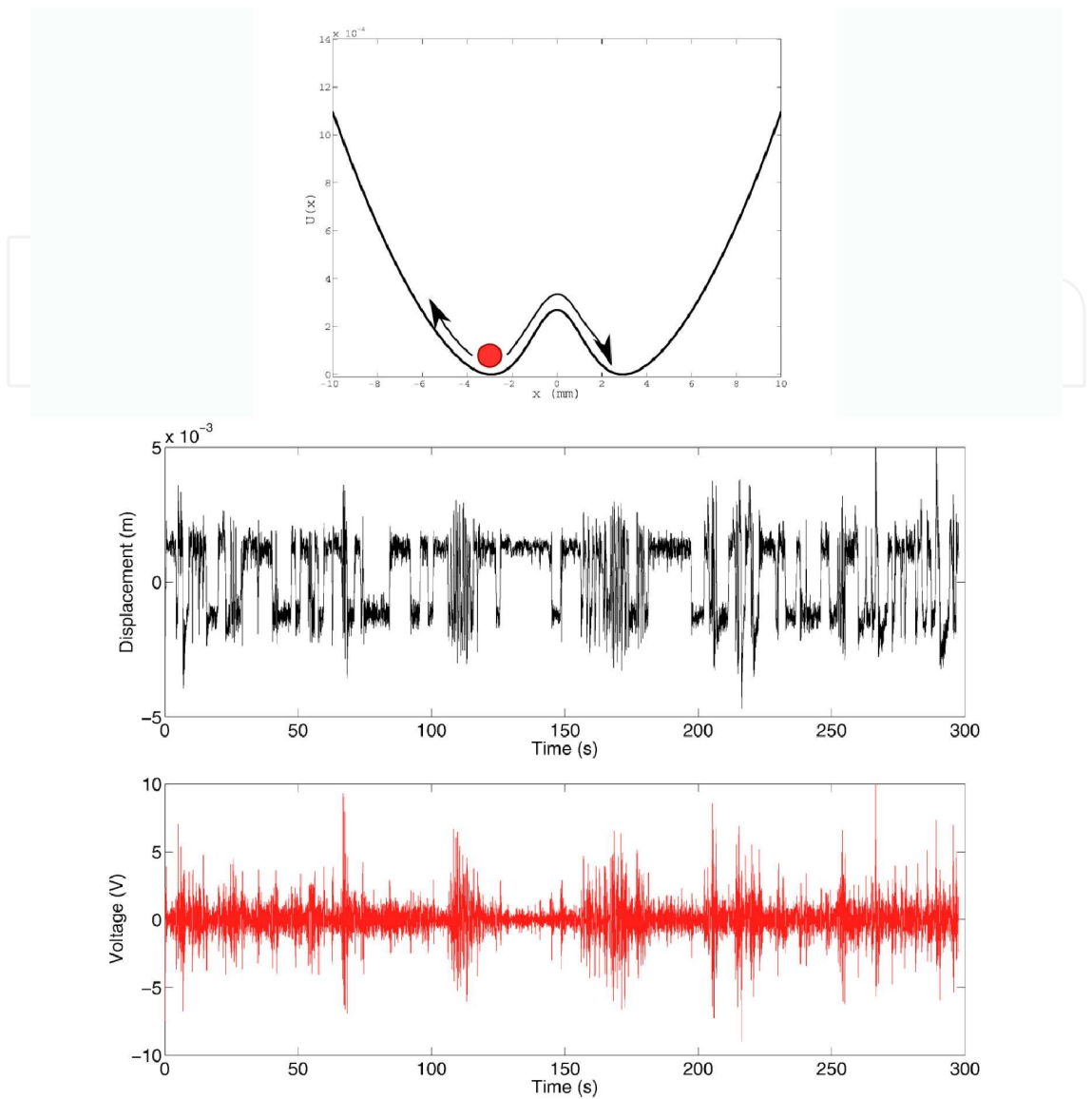

Fig. 7. Upper panel: potential energy $U(x)$ in (10) in arbitrary units when (D>>D$_0$).
Middle panel: displacement x time series and Lower panel: voltage V time series.
Both quantities have been obtained via a numerical solution of the stochastic differential equation (3) with potential (10). The stochastic force is an exponentially correlated noise with fixed standard deviation and correlation time 0.1 s.
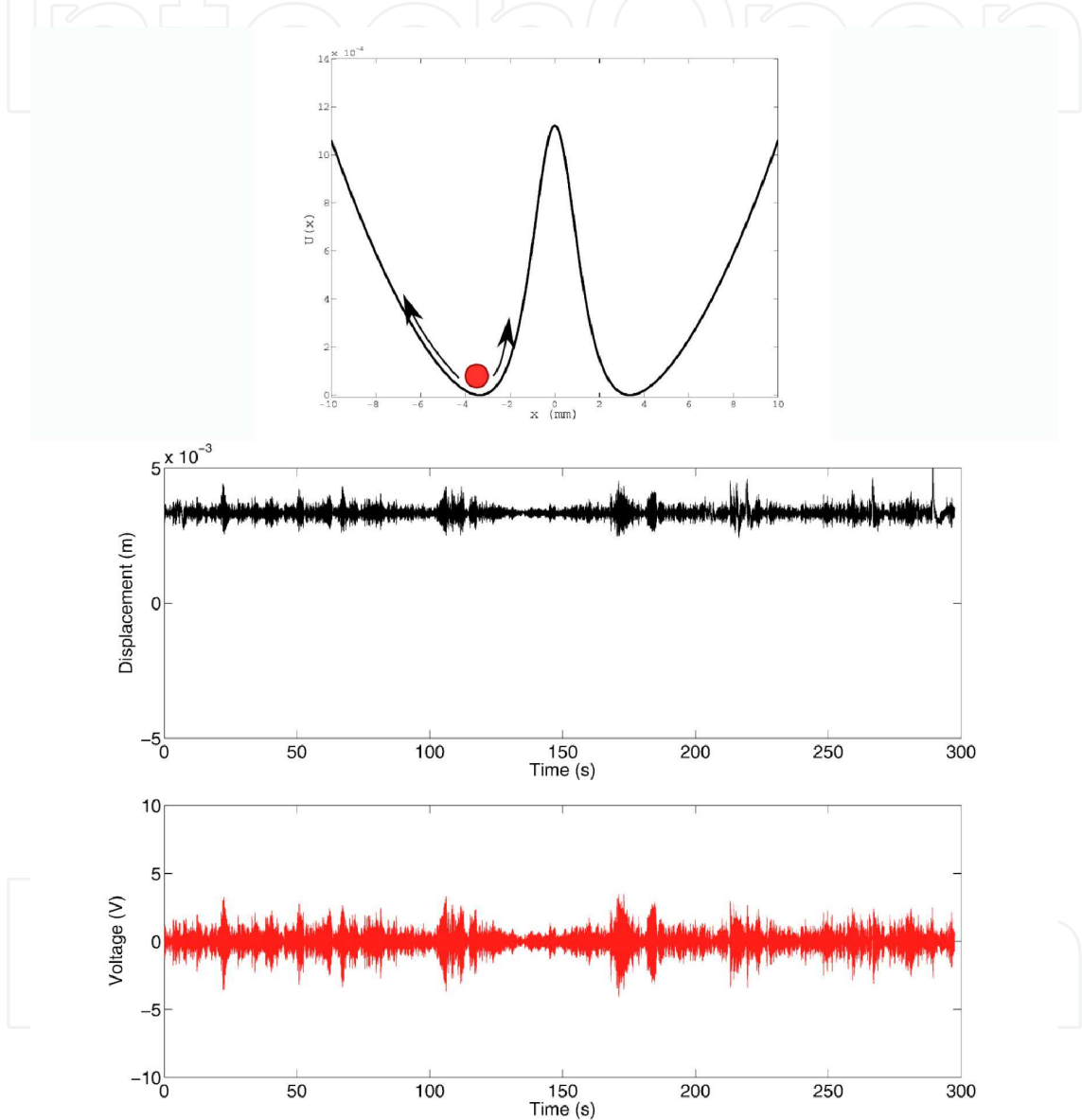
Fig. 8. Upper panel: potential energy $U(x)$ in (10) in arbitrary units when ($D=D_0$). Middle panel: displacement x time series and Lower panel: voltage V time series. Both quantities have been obtained via a numerical solution of the stochastic differential equation (3) with potential (10). The stochastic force is an exponentially correlated noise with fixed standard deviation and correlation time 0.1 s.

It is worth notice that the voltage V, obtained from the solution of the coupled differential stochastic equations (3), follows quite closely the displacement x time series. This is due to the special form of the voltage dynamics equation:

$$\dot{V} = K_c \dot{x} - \frac{1}{\tau_p} V \qquad (11)$$

In fact such an equation represent an high-pass filter whose input signal is the displacement derivative (the velocity) and where the cut-on frequency is represented by $1/\tau_p$ with τ_p given by (4). By Physical point of view this means that the piezoelectric material introduces

a characteristic time constant determined among other things by the physical properties of the piezoelectricity mechanism, i.e. the rearrangement dynamics of the internal electric dipole moments, once the strain is applied. In other words if we produce a bending of the beam too slowly the charges accumulated are easily dispersed and little or no global voltage difference V appears at the ends of the sample. On the other hand, if the time constant τ_p is very large compared to the strain dynamics, then the second term in (11) becomes negligible and *V* is simply proportional to *x*.

Fig. 9. Upper panel: potential energy *U(x)* in (10) in arbitrary units when (D<<D_0). Middle panel: displacement x time series and Lower panel: voltage V time series. Both quantities have been obtained via a numerical solution of the stochastic differential equation (3) with potential (10). The stochastic force is an exponentially correlated noise with fixed standard deviation and correlation time 0.1 s.

In Fig. 8 we present the same quantities of Fig. 7, with the only difference that now the two magnets are not far away from each other but at a certain distance D=D_0 . In this case the

potential energy shows clearly two distinct equilibrium points separated by an energy barrier. The displacement dynamics (below) shows an evident bistable character with the displacement x that switches frequently between the two corresponding potential minima positions. As a consequence the voltage V (further below) shows a random behavior with large excursions in correspondence with the switches.

Finally, in Fig. 9, on further decreasing the distance of the two magnets ($D << D_0$) the potential energy becomes even more bistable and the barrier grows up to a point in which the jumps between the two minima become fewer and less probable. The displacement dynamics gets confined in one well and it shows lower amplitude that reflects into a smaller voltage amplitude V.

This overall qualitative behaviour is somehow summarized in a more quantitative way in Fig. 10, where the average power (average V^2/R_L) extracted from this vibration harvester is presented as a function of the distance parameter D. As it is well evident there is an optimal distance D_0 where the power peaks to a maximum. Most importantly such a maximum condition is reached in a full nonlinear regime (bistable condition of the potential) and results to be quite larger (at least a factor 4) than the value in the linear operation condition (far right in Fig. 10).
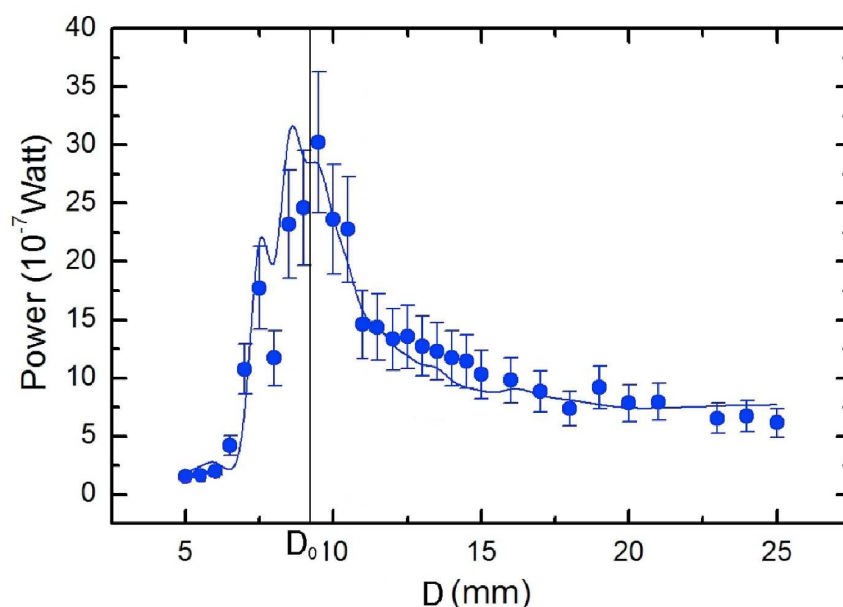

Fig. 10. Piezoelectric nonlinear vibration harvester mean electric power as a function of the distance D between the two magnets. The symbols correspond to experimental values measured from a prototype apparatus (see Cottone et al., 2009). The continuous curve has been obtained from the numerical solution of the stochastic differential equation (3) with potential (10). Both in the experiment and in the numerical solution, the stochastic force has the same statistical properties: an exponentially correlated signal with correlation time 0.1 s. Every data point is obtained from averaging the rms values of ten time series sampled at a frequency of 1 kHz for 200 s. The rms is computed after zero averaging the time series. The expected relative error in the numerical solution is within 10%. For further details on the numerical quantities please see (Cottone et al. 2009).

6.2 More general potentials

The results presented here show that, following our previous reasoning, there might be nonlinear potentials that allow better performances in terms of power generated by a vibration energy harvester, compared to the standard linear cantilever. Although the shape of potential in (10) is quite peculiar, there are more general potentials that allow good performances as well. We have explored few simpler forms, both in the bistable and in the monostable configuration. Among the bistable ones, a popular form is represented by the well know "Duffing oscillator" described by:

$$U(x) = \frac{1}{2}ax^2 + \frac{1}{4}bx^4 \tag{12}$$

This potential $U(x)$ is a good approximation to a number of practically available nonlinear oscillators, like pre-bended membranes or beams.

Among the monostable (nonlinear) oscillators, as previously anticipated, we have studied the simple class of potentials in (9) with $a > 0$ and $n = 1,2,..$ The $n = 1$ case corresponds to the harmonic potential. In this case the dynamics is the standard linear oscillator dynamics. For $n > 1$ we still have a monostable potential but the dynamics is not linear anymore. In Fig. 11 we show the displacement (x_{rms}) as a function of a and n. It is evident that if a is larger than a certain threshold (a_{th}) then x_{rms} increases with n and a nonlinear potential can easily outperform a linear one, thus extending the main finding of the bistable potential (10) also to the monostable case. On the other hand if $a < a_{th}$ then x_{rms} decreases with n and the linear case performs better than the nonlinear one.
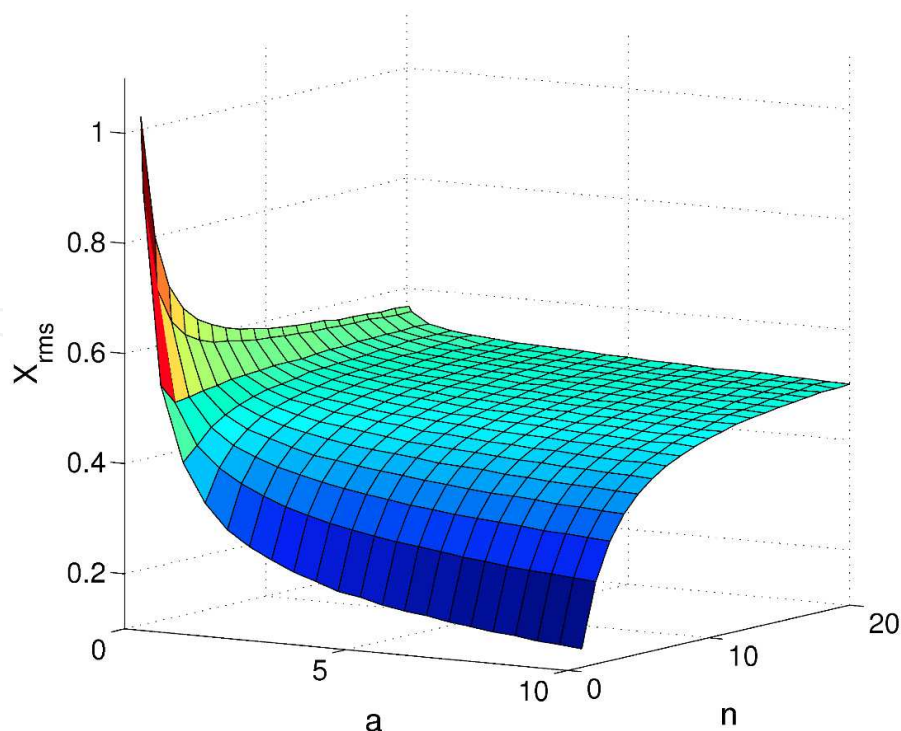

Fig. 11. 3D plot of the displacement (x_{rms}) versus a and n for the potential case in equation (9). For further details on numerical parameters please see (Gammaitoni et al. 2009, 2010).

It has been shown in (Gammaitoni et al. 2009, 2010) that the value a_{th} is linearly dependent on the ambient noise intensity. This result is relevant in the design of an efficient vibration harvesting device. In fact, in real-world applications, the value of the vibration intensity is set by the ambient and cannot be arbitrarily set. The value of the parameter a is usually fixed by the dynamical constraints or by the material properties (stiffness, inertia, etc.) while the value of n can sometimes be selected by a proper design of the geometry of the device itself. Thus, once the noise intensity and a are fixed the choice of a linear ($n = 1$) or nonlinear potential ($n > 1$) can be made in order to maximize the x_{rms} and consequently the power obtained at the device output.

7. Conclusion

In this chapter we have discussed problem of vibration energy harvesting with specific reference to the role of linearity and nonlinearity in the potential energy characterizing the mechanical transduction system. We have shown that linear cantilevers are clearly limited in their practical applications and that more complex nonlinear oscillators can outperform them in a number of realistic energy harvesting scenarios.

8. Acknowledgment

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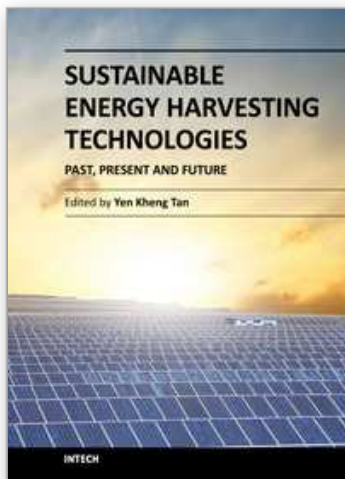
Sustainable Energy Harvesting Technologies - Past, Present and Future
Edited by Dr. Yen Kheng Tan

In the early 21st century, research and development of sustainable energy harvesting (EH) technologies have started. Since then, many EH technologies have evolved, advanced and even been successfully developed into hardware prototypes for sustaining the operational lifetime of low?power electronic devices like mobile gadgets, smart wireless sensor networks, etc. Energy harvesting is a technology that harvests freely available renewable energy from the ambient environment to recharge or put used energy back into the energy storage devices without the hassle of disrupting or even discontinuing the normal operation of the specific application. With the prior knowledge and experience developed over a decade ago, progress of sustainable EH technologies research is still intact and ongoing. EH technologies are starting to mature and strong synergies are formulating with dedicate application areas. To move forward, now would be a good time to setup a review and brainstorm session to evaluate the past, investigate and think through the present and understand and plan for the future sustainable energy harvesting technologies.

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