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Can a Lorentz Invariant Equation Describe Thermal Energy Propagation Problems?

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1. Introduction

In the new technologies the development towards the small scales initiates and encourages the reformulation of those well-known transport equations, like heat and electric conduction, that were applied for bulk materials. The reason of it is that there are several physical evidences for the changes of the behavior of the signal propagation as the sample size is decreasing (Anderson & Tamma, 2006; Cahill et al., 2003; Chen, 2001; Liu & Asheghi, 2004; Schwab et al., 2000; Vázquez et al., 2009). The constructed different mathematical models clearly belong to the phenomena of the considered systems. However, presently, there is no a well-trodden way how to establish the required formulations in general. A great challenge is to establish and exploit the Lagrangian and Lorentz invariant formulation of the thermal energy propagation, since, on the one hand, the connection with other field theories including the interactions of fields can be done on this level, on the other hand, these provide the finite physical action and signal propagation. The results of the presented theory ensures a deeper insight into the phenomena, thus hopefully it will contribute to the technical progress in the near future.
It is an old and toughish question how to introduce the finite speed propagation of action in such physical processes like the thermal energy propagation (Eckart, 1940; Joseph & Preziosi, 1989; Jou et al., 2010; Márkus & Gambár, 2005; Sandoval-Villalbazo & García-Colín, 2000; Sieniutycz, 1994; Sieniutycz & Berry, 2002). There is no doubt that the solution must exist somehow and the suitable description should be Lorentz invariant. Moreover, this Lorentz invariant formulation needs to involve anyway the Fourier heat conduction as the classical limit. The elaborated theory ensures that in the case of Lorentz invariant formulation both the speed of the signal and the action propagation is finite. Furthermore, for the Fourier heat conduction the temperature propagation is finite, however, the speed of action is infinite.
This chapter treats the consequent mathematical formulation of a suitable relativistic invariant description of the above problem and its consequences, connections with other topics are also treated. As the author hopes it will be noticeable step-by-step that this synthesis theory may have a prominent role in the phenomena of nature. The construction of the Lorentz invariant thermal energy propagation, the Klein-Gordon equation with negative "mass term" providing the expected propagation modes, the limit to the classical heat conduction and the related dynamic phase transition between the dissipative – non-dissipative dynamic phase transitions are discussed in a coherent frame within Sec. 2. Two mechanical analogies are shown in Sec. 3 for the two kinds of Klein-Gordon type equations to see the distinct behavior due to

the opposite sign of the mass term. On the one hand, it will be convincing to see how the negative "mass term" can govern the above mentioned change in the dynamics, and, on the other hand, it clarifies the physical role of the similar term in the Lorentz invariant propagation studied in Sec. 2. It is assumable that the efficiency of the relativistic invariant theory can be demonstrated via other physical phenomena. The spectacular description of the inflationary cosmology with the inflaton-thermal field coupling, the resulted time evolution of the inflaton field and the dynamic temperature show this fact clearly in Sec. 4. Finally, to achieve a deeper insight into the soul of this new theory and to be sure that the causality principle is completed, for this reason the Wheeler propagator is calculated in Sec. 5 as well. The main ideas, results of the chapter and some concluding remarks are summarized in Sec. 6. Finally, Sec. 7 is for the acknowledgment.

2. Lorentz invariant thermal energy propagation

The mathematical description is based on the least action principle (Hamilton's principle)

$$S = \int L \, d^3x dt = extremum, \tag{1}$$

i.e., there exists a Lagrange density function L by which the calculated action S is extremal for the real physical processes. The Hamiltonian formulation can be also achieved for certain differential equations involving non-selfadjoint operators like the first time derivative in the classical Fourier heat conduction. Then such potential functions are required to introduce by which the Lagrange functions can be expressed and the whole Hamiltionian theory can be constructed (Gambár & Márkus, 1994; Gambár, 2005; Márkus, 2005). The long scientific experience on this topic showed that the theories are comparable and connectable on this — Lagrangian-Hamiltonian — level, thus in the further development of the theory it is useful to apply this idea and scheme. In order to generate a dynamic temperature and the related covariant Klein-Gordon type field equation, to describe the heat propagation with finite speed — less than the speed of light — of action an abstract scalar potential field has been introduced (Gambár & Márkus, 2007). In this case the thermal energy propagation has wave-like modes. It is important to emphasize that, on the other hand, this scalar field can be connected to the usual (local equilibrium) temperature and the Fourier's heat conduction in the classical limit. This treating is an attempt to point out that the dynamic phase transition (Ma, 1982) between the two kinds of propagation, between a wave and a non-wave, or with another context it is better to say — between a non-dissipative and a dissipative thermal process — has a more general role and manifestation in the processes.

As a starting point the Lagrange functions are given for both the Lorentz invariant heat propagation (Márkus & Gambár, 2005) and for the classical heat conduction (Fourier's heat conduction) (Gambár & Márkus, 1994). The first description is based on a Klein-Gordon type equation formulated by a negative "mass term". It will be shown that this pertains to a repulsive potential, which repulsive interaction produces a tachyon solution leading to the so-called spinodal instability which effect is often applied in modern field theories (Borsányi et al., 2000; 2002; 2003). Now, the Hamiltonian descriptions are written side by side — to prepare the later comparison — showing how the Lorentz invariant solution provides the classical solution in the limit of speed of light. The relevant Lagrangians, L_w for the wave-like solution (Márkus & Gambár, 2005) and L_c for the classical heat conduction (Gambár & Márkus, 1994) restricting our examination for the one dimensional case, are

$$L_w = \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2 + \frac{1}{2c^4}\left(\frac{\partial^2\varphi}{\partial t^2}\right)^2 - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{1}{2}\frac{c^4c_v^4}{16\lambda^4}\varphi^2, \tag{2a}$$

$$L_c = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 + \frac{1}{2}\frac{\lambda^2}{c_v^2}\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2, \tag{2b}$$

where φ is a four times differentiable and Lorentz invariant scalar field that generates the measurable thermal field, and c denotes the speed of light, λ is the heat conductivity, c_v is the specific heat. Applying the calculus of variation the corresponding Euler-Lagrange equations as equations of motion for the field φ can be obtained

$$0 = \frac{1}{c^4}\frac{\partial^4\varphi}{\partial t^4} + \frac{\partial^4\varphi}{\partial x^4} - \frac{2}{c^2}\frac{\partial^4\varphi}{\partial t^2\partial x^2} - \frac{c^4c_v^4}{16\lambda^4}\varphi, \tag{3a}$$

$$0 = -\frac{\partial^2\varphi}{\partial t^2} + \frac{\lambda^2}{c_v^2}\frac{\partial^4\varphi}{\partial x^4}. \tag{3b}$$

It is expected that the above scalar field is able to define the measurable physical quantities, namely, in the present case, the temperature. Let the temperature T be a Lorentz invariant temperature, which is defined from a dynamical point of view, thus it can be considered as the dynamic temperature. Furthermore, temperature $\mathcal{T}$ denotes the usual local equilibrium temperature

$$T = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} + \frac{c^2c_v^2}{4\lambda^2}\varphi, \tag{4a}$$

$$\mathcal{T} = -\frac{\partial\varphi}{\partial t} - \frac{\lambda}{c_v}\frac{\partial^2\varphi}{\partial x^2}. \tag{4b}$$

Eliminating the potentials in Equations (3a) and (3b) by the help of the corresponding Equations (4a) and (4b), for the relevant case, a differential equation for the time evolution of the temperature can be obtained

$$\frac{1}{c^2}\frac{\partial^2T}{\partial t^2} - \frac{\partial^2T}{\partial x^2} - \frac{c^2c_v^2}{4\lambda^2}T = 0, \tag{5a}$$

$$\frac{\partial\mathcal{T}}{\partial t} - \frac{\lambda}{c_v}\frac{\partial^2\mathcal{T}}{\partial x^2} = 0. \tag{5b}$$

Here, Equation (5a) — the hyperbolic one — is a Klein-Gordon type equation with a negative "mass term" $-(c^2c_v^2/4\lambda^2)T$ which means a kind of repulsive interaction. This term is responsible for the tachyon solution leading to a spinodal instability as it will be also seen in Sec. 3 in the case of classical Klein-Gordon equation of the mechanical analogy. On the other hand, Equation (5b) — the parabolic one — pertains to the Fourier's heat equation. The signal propagation mechanism can be examined by the calculation of the dispersion relations for both cases

$$\omega(k) = \sqrt{c^2k^2 - \frac{c^4c_v^2}{4\lambda^2}} = ck\sqrt{1 - \frac{c^2}{4D^2k^2}}, \tag{6a}$$

$$\omega(k) = -i\frac{\lambda}{c_v}k^2 = -iDk^2. \tag{6b}$$

Here, the diffusivity parameter $D = \lambda / c_v$ is introduced to simplify the forms. The dispersion relation in Equation (6a) pertains to the Klein-Gordon wave equation in Equation (5a) from which we obtain the phase velocity w_f

$$w_f(k) = \frac{\omega}{k} = c\sqrt{1 - \frac{c^2}{4D^2k^2}}. \tag{7}$$

The dispersion relation in Equation (6b) belongs to the classical (non-wave) Fourier's heat conduction. The models can be compared by the calculation of the group velocities since these pertain to the signal propagations. Thus, from Equation (6a) the group velocity $v_g = d\omega/dk$ of the wave-like propagation can be directly calculated. Then, tending to the infinity with the speed of light, the group velocity $v_\mathcal{T}$ of the classical heat conduction can be obtained, as it is expected

$$v_g = \frac{d\omega}{dk} = \frac{c}{\sqrt{1 - \frac{c^2}{4D^2k^2}}} \quad \longrightarrow \quad \left.\frac{d\omega}{dk}\right|_{c\to\infty} = -i2Dk; \quad v_\mathcal{T} = 2Dk \ll c. \tag{8}$$

This limit shows clearly that the Lorentz invariant description covers both cases, and the wave-like and the non-wave heat propagation can be discussed in the same frame.

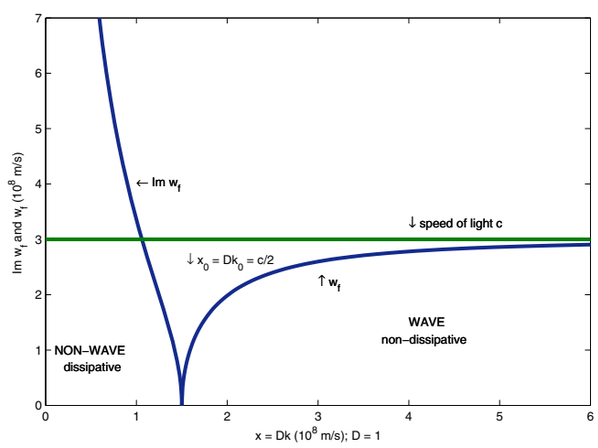


Fig. 1. Phase transition between the non-wave (dissipative) [left] and the wave (non-dissipative) solution [right]. The critical transition point is at $x_0 = Dk_0 = c/2$. The value of diffusity is taken $D = 1$. The phase velocity w_f of the wave-like propagation is always smaller than the speed of light.

It can be recognized that there is a value of the wave number k when the discriminant changes its sign in Equation (7) at the value $k_0 = c/2D$. Now, the solutions can be split into two parts. On the one hand, we can consider the case $k > k_0$, when the solution is real and wave-like (non-dissipative), and on the other hand, we take the case $k < k_0$, when the solution is imaginary and non-wave (dissipative). The real and the imaginary part of the phase velocity w_f can be written for both cases

$$w_f = \frac{\omega}{k} = c\sqrt{1 - \frac{c^2}{4D^2k^2}} < c \quad k > k_0, \tag{9a}$$

$$Im\ w_f = \frac{\omega}{k} = c\sqrt{\frac{c^2}{4D^2k^2} - 1} \quad k < k_0. \tag{9b}$$

The above physical discussion can be easily followed in Fig. 1.
In order to couple the thermal field given in Equation (2a) with other fields (like the inflaton field in the cosmology shown in Sec. 4) it is worthy to reformulate it for this later use. It has been shown in the literature (Márkus & Gambár, 2005) that the quantization of the thermal field generates quasi particles and these particles may have a mass

$$M_0 = \frac{\hbar}{2D}, \tag{10}$$

where $\hbar$ is the Planck constant. Moreover, the Planck units are applied for the present case ($c = 1; \hbar = 1$). Then the 3D Lagrangian given by Eq. (2a) should be rewritten

$$L_w = \frac{1}{2}(\Delta\varphi)^2 + \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial t^2}\right)^2 - \frac{\partial^2\varphi}{\partial t^2}\Delta\varphi - \frac{1}{2}M_0^4\varphi^2, \tag{11}$$

where Δ is the Laplace operator.

3. Mechanical analogies for the two kinds of Klein-Gordon equations

It is instructive to study the set-up of the classical model of the Klein-Gordon equation (Morse & Feshbach, 1953) to make comparisons and conclusions on the physical meaning of the relevant terms that may appear similarly in a more general and abstract theory. The mechanical model is a stretched string with little vertically oriented springs along the string which pull back the spring to the equilibrium position as it is shown in Fig. 2(a). The equation of motion of the string can be formulated applying the Lagrangian formalism. To achieve this, the kinetic and potential energy terms are needed to calculate. The string has a kinetic energy from its movement

$$T = \frac{1}{2}\varrho A\int\left(\frac{\partial\Psi}{\partial t}\right)^2 dx, \tag{12}$$

where Ψ is the displacement from the equilibrium position, ϱ is the density, A is the cross section of the string. The mass element is $dm = \varrho A dx$. The either of the potential energy terms comes from the small deformation (elongation) of the stretching which is

$$V = F\int\left[\sqrt{1 + \left(\frac{\partial\Psi}{\partial x}\right)^2} - 1\right]dx \sim \frac{1}{2}F\int\left(\frac{\partial\Psi}{\partial x}\right)^2 dx, \tag{13}$$

F is the stretching force. The other attractive potential energy term pertains to the little springs which is

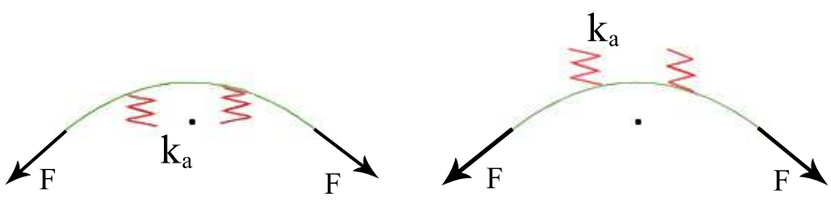
$$V_s = \frac{1}{2}k_a\int\Psi^2 dx. \tag{14}$$

Here, k_a is the spring direction coefficient density along the string as is shown in Fig 2(a). The Lagrangian of the system can be formulated with the usual construction $L = T - V - V_s$, by which the Euler-Lagrange equation as equation of motion — a Klein-Gordon equation with positive "mass term" —

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{F}{\varrho A}\frac{\partial^2 \Psi}{\partial x^2} + \frac{k_a}{\varrho A}\Psi = 0. \tag{15}$$

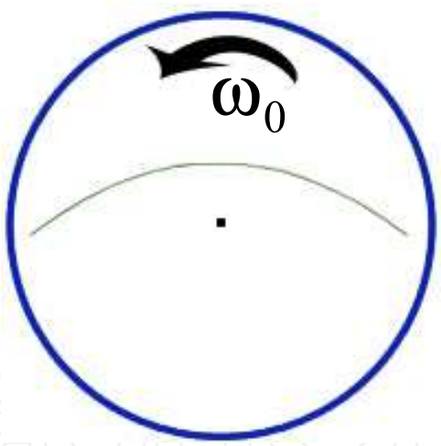
can be deduced. Now, if a "repulsive" potential is imagined at the places of the springs shown in Fig. 2(b) then a Klein-Gordon type equation with negative "mass term" (Gambár & Márkus, 2008) is obtained

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{F}{\varrho A}\frac{\partial^2 \Psi}{\partial x^2} - \frac{k_a}{\varrho A}\Psi = 0. \tag{16}$$



(a) A stretched string (green line) with an additional attractive interaction by the springs k_a

(b) A stretched string (green line) with an additional "repulsive" interaction by the springs k_a



(c) A stretched string (green line) on a rotating disc; ω_0 is the angular velocity

Fig. 2. The three physical situations of the stretched string; the acting force is F for each cases. The equations of motion due to the attractive or "repulsive" interactions pertain to the different figures: Equation (15) for Fig. (a); Equation (16) for Fig. (b); Equation (18) for Fig. (c).

The structure of this equation is exactly the same as in the case of Lorentz invariant thermal energy propagation in Equation (5a). Since, it is clear from this mechanical example that the negative sign of the third term in Equation (16) pertains to a repulsive interaction, thus, this is the reson why the negative "mass term" may relate to a repulsive interaction in the relativistic

case in Equation (5a), in general. Maybe, it is complicated to prepare a device to ensure the repulsive interaction from little springs. However, if the stretched string is placed on the diameter of a rotating disk — shown in Fig. 2(c) that moves with the angular velocity ω_0, then the centrifugal force can produce the similar repulsive interaction.
The centrifugal potential of a point-like mass m moving on a circle with a radius r

$$-\frac{1}{2}mr^2\omega_0^2$$

can be generalized to the present case. This gives the potential V_{rot} pertaining to the rotational motion of the string

$$V_{rot} = -\frac{1}{2}\varrho A\omega_0^2 \int \Psi^2 dx. \tag{17}$$

The relevant Lagrangian is $L = T - V - V_{rot}$, by which the calculated equation of motion can be obtained

$$\frac{\partial^2\Psi}{\partial t^2} - \frac{F}{\varrho A}\frac{\partial^2\Psi}{\partial x^2} - \omega_0^2\Psi = 0. \tag{18}$$

The same mathematical structure can be immediately recognized comparing this equation with the Equations (5a) and (16). This means that these three equations must involve the similar physical behavior: the spinodal instability and the dynamic phase transition (Gambár, 2010). All together these examples clearly prove the physical reality of the Klein-Gordon equation with negative "mass term" in nature.
Finally, for the completeness the dispersion relation for Equation (18) can be also calculated

$$\Omega(k, \omega_0) = \sqrt{\frac{F}{\varrho A}k^2 - \omega_0^2}. \tag{19}$$

This formula shows again the same physical behavior clearly as it has been found in Equation (6a). The phase velocity is

$$w_{ph} = \frac{\Omega}{k} = \sqrt{\frac{F}{\varrho A} - \left(\frac{\omega_0}{k}\right)^2}. \tag{20}$$

It is easy to recognize that for small angular velocity ω_0 while

$$\sqrt{\frac{F}{\varrho A}} > \frac{\omega_0}{k} \tag{21}$$

is completed, then wave modes exist. The opposite case is when

$$\sqrt{\frac{F}{\varrho A}} < \frac{\omega_0}{k}, \tag{22}$$

there are no wave modes. The physical meaning is that, above a certain value of ω_0, the centrifugal force elongates the string to infinity, the string cannot have vibrating modes. The change in the propagation modes is an angular velocity controlled dynamic phase transition that divides the dissipative – non-dissipative transition like in Equations (7), (9a) and (9b) for the thermal case.

4. Inflationary cosmology with the dynamic temperature

It is a great challenge to experience and understand how the Lorentz invariant propagating thermal energy field φ can interact with other physical fields. In this way new physical relations, considerations and explanations may be expected for the relevant phenomena. As an advanced example, to point out the strength of the formulation, the thermal and cosmological inflaton fields are coupled within the Lagrangian framework (Márkus et al., 2009).

4.1 Linde's model of the inflaton field

In the present model the cosmological model is based on the Einstein's equation in the Friedman-Robertson-Walker metric. Now, the action S can be expressed as

$$S = \int \sqrt{-\tilde{g}} L_{FRW} \, d^4x, \tag{23}$$

where the expression $\sqrt{-\tilde{g}} = a^3$ is the Friedman-Robertson-Walker metric. Here, the $a(t) = R(t)/R_0$ is taken as the 'radius' of the universe. The Lagrange density function L_{FRW} of the inflaton field ϕ

$$L_{FRW} = \left(\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2a^2} (\nabla \phi)^2 - V(\phi) \right) \tag{24}$$

is the starting point in the description; ∇ is the gradient operator. Then, the equation of motion for the inflaton can be calculated

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{1}{a^2} \Delta \phi + 3H \frac{\partial \phi}{\partial t} = - \frac{\delta V(\phi)}{\delta \phi}, \tag{25}$$

where $\delta V(\phi)/\delta \phi$ means a functional derivative. The Hubble parameter $H(t)$ is defined by

$$H = \frac{\dot{a}}{a}. \tag{26}$$

The fate of the universe depends on the potential $V(\phi)$. The hybrid inflation model suggested by Linde (Felder et al., 1999; 2001; Linde, 1982; 1994) introduces an additional scalar field σ (in fact the Higgs field) into the effective potential

$$V(\sigma, \phi) = - \frac{1}{2a^2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g^2 \phi^2 \sigma^2 + \frac{1}{4\lambda} (M^2 - \lambda \sigma^2)^2. \tag{27}$$

Here, the first term on the right hand side pertains to the second term — the space derivate term — on the left hand side in Equation (25). The second term generates the inflation process, the third one couples the inflaton field to the introduced additional field σ and the last one produces mass generation through the spontaneous symmetry breaking. The canonical momentum of the inflaton field can be calculated

$$\Pi_\phi = \frac{\partial L_{FRW}}{\partial \dot{\phi}} = \dot{\phi}. \tag{28}$$

Then the Hamiltonian $\tilde{H}$ of the field which is the energy density can be obtained

$$\tilde{H} = \Pi_\phi \dot{\phi} - L_{FRW} = \left(\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2a^2} (\nabla \phi)^2 + V(\phi) \right). \tag{29}$$

It is often used different notations for $\tilde{H}$

$$\tilde{H} = \varrho_\phi = T_{00}, \tag{30}$$

where T_{00} is called as the time-time component of the energy-momentum tensor. Furthermore, the Einstein's equation can be expressed in the FRW metric as

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \varrho, \tag{31}$$

where G is the gravitational constant and ϱ is the mass density. Substituting the energy density ϱ_ϕ and the Planck mass

$$M_{pl} = \sqrt{\frac{\hbar c}{8\pi G}} \tag{32}$$

into Equation (31) and applying Planck units, the Friedman's equation can be written in the following form

$$H^2 = \frac{1}{3M_{pl}^2} \varrho_\phi, \tag{33}$$

which corresponds to a flat universe. If it is assumed that the universe is growing homogeneously in the space we can neglect those terms where the spatial derivates (∇ and Δ) appear in Equation (25), then an ordinary differential equation can be obtained

$$\frac{d^2 \phi_0}{dt^2} + 3H \frac{d\phi_0}{dt} = -\frac{\delta V(\phi_0)}{\delta \phi_0}, \tag{34}$$

the 'field variable' ϕ_0 depends on the time parameter only. In this case the energy density ϱ_ϕ has a simplified form

$$\varrho_\phi = \left(\frac{1}{2} \left(\frac{d\phi_0}{dt} \right)^2 + V(\phi) \right), \tag{35}$$

by which the equation $H^2 = (1/3M_{pl}^2)\varrho_\phi$ naturally also remains valid, i.e.,

$$H^2 = \frac{1}{3M_{pl}^2} \left(\frac{1}{2} \left(\frac{d\phi_0}{dt} \right)^2 + V(\phi) \right). \tag{36}$$

Soon it will be seen that the above equations, (35) and (36), with the modifying effect of the thermal field φ_0 will become those equations which are going to be considered as the time-evolution equations of the inflaton field.

4.2 The coupling of the fields

The introduction of the dynamic temperature and the laws of thermodynamics into the theory of cosmology requires the same mathematical frame of the description. Now, the tool is ready

to make this willing. The interaction of the thermal potential field φ [see Equation (11)] and the inflaton field ϕ [see Equation (24)] can be constructed by adding the Lagrangians of the different fields

$$L_{int} = \left(\frac{1}{2a^4}(\Delta\varphi)^2 + \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial t^2}\right)^2 - \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}\Delta\varphi - \frac{1}{2}M_0^4\varphi^2 \right) + \left(\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2a^2}(\nabla\phi)^2 - V(\phi,\varphi) \right). \tag{37}$$

This Lagrangian L_{int} of the coupled inflaton-thermal field by the following interaction potential can also realize the spontaneous symmetry breaking

$$V(\phi,\varphi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g_0^2\phi^2\varphi^2, \tag{38}$$

where m denotes the mass of the inflaton, and g_0 is the coupling constant, moreover, this description can involve the temperature of the inflaton field (Márkus et al., 2009). This fact is very interesting, since at this stage, there is no need for the Higgs field and the mass generation.

After all, applying the calculus of variation, two Euler-Lagrange equations as equations of motion are arisen from the variation with respect to the variables ϕ and φ

$$\frac{\partial^2\phi}{\partial t^2} - \frac{1}{a^2}\Delta\phi + 3\frac{\dot{a}}{a}\frac{\partial\phi}{\partial t} = -\frac{\delta V(\phi,\varphi)}{\delta\phi}, \tag{39}$$

and

$$\frac{1}{a^4}\Delta\Delta\varphi + \frac{\partial^4\varphi}{\partial t^4} + 6\frac{\dot{a}}{a}\frac{\partial^3\varphi}{\partial t^3} + \frac{1}{a^3}\frac{\partial^2(a^3)}{\partial t^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{2}{a^2}\Delta\frac{\partial^2\varphi}{\partial t^2} - \frac{\ddot{a}}{a^3}\Delta\varphi - 2\frac{\dot{a}}{a^3}\Delta\frac{\partial\varphi}{\partial t} - M_0^4\varphi = \frac{\delta V(\phi,\varphi)}{\delta\varphi}. \tag{40}$$

An important remark is needed here. Since, for the cases when the Lagrangian contains second order time derivatives the Hamiltonian $\tilde{H}$ must be expressed as follows (Gambár & Márkus, 1994; Márkus & Gambár, 1991),

$$\tilde{H} = \frac{\partial\varphi}{\partial t}\frac{\partial L}{\partial\dot{\varphi}} - \frac{\partial\varphi}{\partial t}\frac{\partial}{\partial t}\frac{\partial L}{\partial\ddot{\varphi}} + \frac{\partial^2\varphi}{\partial t^2}\frac{\partial L}{\partial\ddot{\varphi}} - L. \tag{41}$$

By substituting the Lagrangian L_{int} from Equation (37), the Hamiltonian — energy density regarding the whole space with all interactions — can be calculated

$$\varrho_{\phi,\varphi} = \tilde{H} = -\frac{\partial\varphi}{\partial t}\frac{\partial^3\varphi}{\partial t^3} + \frac{\partial\varphi}{\partial t}\frac{\partial}{\partial t}\left(\frac{1}{a^2}\right)\Delta\varphi + \frac{1}{a^2}\frac{\partial\varphi}{\partial t}\frac{\partial}{\partial t}\Delta\varphi + \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial t^2}\right)^2 - \frac{1}{2a^4}(\Delta\varphi)^2 + \frac{1}{2}M_0^4\varphi^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi,\varphi). \tag{42}$$

In the case of a rapidly growing universe in a homogeneous space, the terms containing the operators ∇ and Δ can be omitted, thus the obtained field equations are simplified to the following coupled nonlinear ordinary differential equations:

$$\frac{d^2\phi_0}{dt^2} + 3H\frac{d\phi_0}{dt} = -\left(m^2 + g_0^2\varphi_0^2\right)\phi_0, \tag{43}$$

$$\frac{d^4\varphi_0}{dt^4} + 6H\frac{d^3\varphi_0}{dt^3} = M_0^4\varphi_0 + g_0^2\phi_0^2\varphi_0 \tag{44}$$

and

$$H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2}\left(\frac{d^2\varphi_0}{dt^2}\right)^2 - \frac{d\varphi_0}{dt}\frac{d^3\varphi_0}{dt^3} + \frac{1}{2}\left(\frac{d\phi_0}{dt}\right)^2 + \frac{1}{2}M_0^4\varphi_0^2 + \frac{1}{2}m^2\phi_0^2 + \frac{1}{2}g_0^2\phi_0^2\varphi_0^2\right]. \tag{45}$$

Here, the field ϕ_0 and φ_0 depend on time only. The three coupled nonlinear ordinary differential equations, Equations (43), (44) and (45), can be considered as the equations of motion of the inflationary model. It is easy to recognize that Equation (45) can be considered as the modified version of Friedman's equation given in Equation (33). The temperature generated by the thermal field φ_0 can then be expressed as [see Equation (4a) and taking into account Equation (10) with Planck units]

$$T = \frac{d^2\varphi_0}{dt^2} + M_0^2\varphi_0. \tag{46}$$

4.3 On the time evolution of the fields

The mathematical and numerical examinations show that the solution of these coupled differential equations describes fairly well the time evolution of the inflationary universe including its thermodynamical behavior. Due to the complicated nonlinear Equations (43-45) the solutions can be achieved by numerical calculations for the time-dependence of the scalar fields and the dynamic temperature T. These equations are needed to solve simultaneously for the scalar field ϕ_0 and the thermal potential φ_0 first. After then the time evolution equation for the (thermo)dynamic temperature can be obtained.

In the present model there are two adjustable parameters, namely, the mass M_0 of the thermal field and the coupling constant g_0. The time scales of the temperature and the scalar inflaton field can be synchronized by the change of values for these two parameters. The mass of the scalar field m is chosen in the same order of magnitude as it is proposed by Linde Linde (1994), namely, $m = 80GeV$. The two fitted parameters are $M_0 = 52.2GeV$ and $g_0 = 0.12GeV$. It is important to set relevant initial conditions to find reasonable numerical solutions for Equations (43) – (45). Thus, a big acceleration is assumed at the beginning of the expansion and the thermal field has a given initial value. This results an initial value for the temperature $T_0 \sim 2.5 \times 10^6 GeV \sim 10^{19}K$. (Presently, the exact magnitude of the temperature has not too much importance, since another value can be obtained by rescaling, i.e., it does not touch the shape of the temperature function. However, it is sure, that this value is rather far from the theoretically possible $\sim 1.4 \times 10^{32}K$ value (Lima & Trodden, 1996; Márkus & Gambár, 2004).) In order to ensure the thermal and the inflaton field decay the first time derivatives of them are needed to be negative.

After finding a set of the numerical solutions, two main stages can be distinguished for the time evolution of the inflaton field ϕ_0. The first short period is when it decreases rapidly.

This follows the second rather long time interval in which the inflaton field oscillates with decreasing amplitude. Both of these processes can be recognized well in Fig. 3.

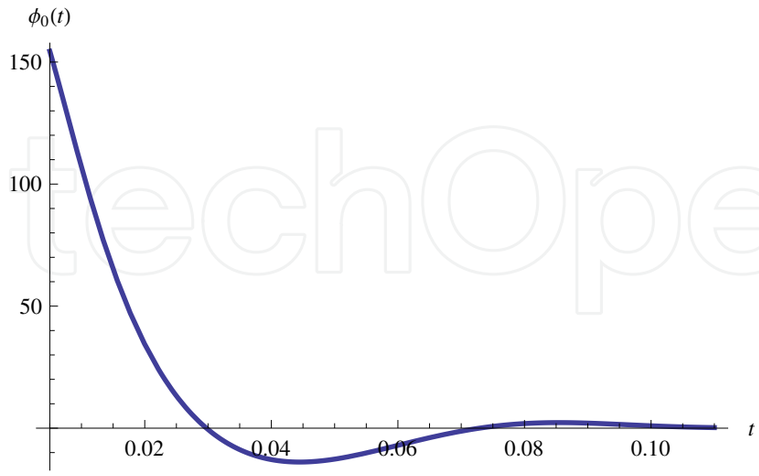


Fig. 3. The time evolution of the inflaton field $\phi_0(t)$ is shown. The short decreasing (deacying) period is followed by a rather long damped oscillating process. Time is in arbitrary units.

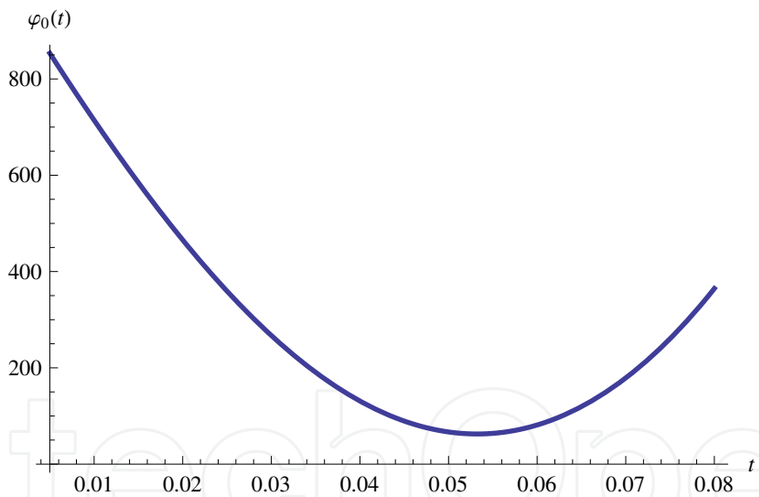


Fig. 4. The time evolution of the thermal field $\varphi_0(t)$. The field decays in the first period and reaches its minimal value. It begins to increase monotonically when the inflaton field $\phi_0(t)$ starts to oscillate. Time is in arbitrary units.

It is noticable that the above described behavior of the inflaton field is in line with Linde's cosmology model (Felder et al., 2002; Linde, 1982; 1990; 1994) based on a potential energy expression given by $V(\phi_0) = (m^2/2)\phi_0^2 + V_0$ with $V_0 > 0$ which is similar to Equation (38), here. The physically coupled thermal field φ_0 produces a completely different behavior. During inflation era, the field φ_0 decreases. Probably, the reason of this effect is strongly the radius and the volume increase of the universe. Once it reaches a minimum which happens about the same time when field ϕ_0 starts to oscillate. After then, the thermal field increases

monotonically since the decaying inflaton field ϕ_0 with a time delay pumps up it as plotted in Fig. 4.
The temperature field T is coupled to the thermal field φ_0 by Equation (46), thus mathematically this can be obtained directly. The time evolution of the temperature can be followed in Fig. 5. In the first era of the inflation process the temperature decreases. After reaching its minimal value, which is at the same instantaneous of the minimum of the thermal field, it increases quite rapidly. This period of the cosmology is known as the reheating process of the universe. The present elaboration of the model can describe and reproduce to this stage of the life of the early universe.

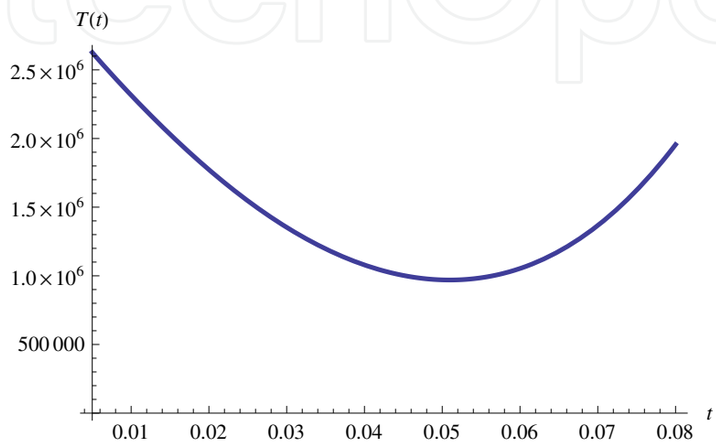


Fig. 5. The time evolution of the temperature field $T(t)$. The temperature follows the change of the thermal field φ_0. It decreases in the first period of the expansion while its reaches a minimal value. The, due to the pumping of the inflaton field ϕ_0 into the thermal field φ_0, the temperature starts increasing. This growing temperature period can be identified as the reheating process in Linde's cosmology model. Time is in arbitrary units.

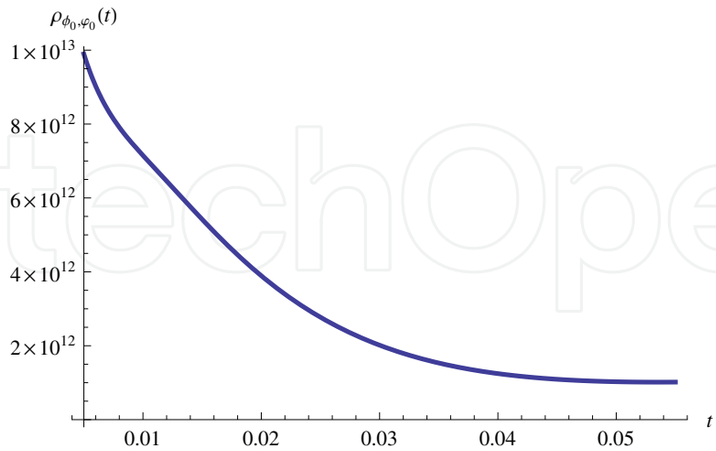


Fig. 6. The time evolution of the energy density $\rho_{\phi_0\varphi_0}(t)$. As it is expected the energy density decreases monotonically during the expansion. Time is in arbitrary units.

Since the whole energy of the universe is conserved during the expansion, the energy density is needed to decrease. This tendency can be seen in Fig. 6. Finally, the radius $a(t)$ of the universe is plotted in Fig. 7.

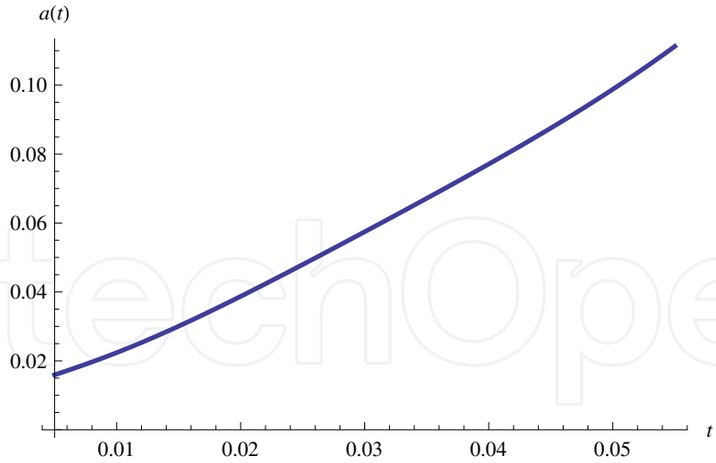


Fig. 7. The time evolution of the radius $a(t)$ of the universe. As it is expected the radius increases monotonically during the expansion. Time is in arbitrary units.

The presented model of the inflationary period is not complete in that sense that e.g., the Higgs mechanism is dropped by the elimination of the fourth term of the effective potential in Equation (27) comparing with the applied potential in Equation (38). However, hopefully, the strength of the theory can be read out from the most spectacular results: the thermal field can generate not only the spontaneous symmetry breaking involving the correct time evolution of the inflaton field, but it ensures a really dynamic Lorentz invariant thermodynamic temperature. The further development of this cosmological model would be to add the particle generator Higgs mechanism again.

5. Wheeler propagator of the Lorentz invariant thermal energy propagation

As it has been shown previously that the Lorentz invariant description involves different physically realistic propagation modes. However, the development of the theory is needed to learn more about propagation, the transition amplitude and the completeness of causality, i.e., the field equation in Equation (5a) does not violate the causality principle.

5.1 The Green function

A common way to examine these questions is based on the Green function method. Mathematically, the solution of the equation

$$\frac{1}{c^2}\frac{\partial^2 G}{\partial t^2} - \frac{\partial^2 G}{\partial x^2} - \frac{c^2 c_v^2}{4\lambda^2} G = -\delta^n(x - x') \tag{47}$$

for the Green function G is needed to find. The n-dimensional source function is $\delta^n(x - x') = \delta^{n-1}(\mathbf{r} - \mathbf{r}')\delta(t - t')$ which can be expressed by the delta function

$$\delta^n(x - x') = \frac{1}{(2\pi)^n} \int d^n k e^{ik(x-x')}. \tag{48}$$

Here, the vector $k = (\mathbf{k}, \omega_0)$ is n-dimensional; the $n - 1$ dimensional $\mathbf{k}$ pertains to the space and the 1-dimensional ω_0 is to time. Moreover, the d'Alembert operator is

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta. \tag{49}$$

To shorten the formulations the following abbreviation is also introduced

$$m^2 = \frac{c^2 c_v^2}{4\lambda^2}. \tag{50}$$

Now, Equation (47) has a simpler form

$$(\Box - m^2)G = \delta^n(x - x'). \tag{51}$$

Since, the equality holds

$$(\Box - m^2)^{-1} e^{ik(x-x')} = -\frac{e^{ik(x-x')}}{k^2 - m^2}, \tag{52}$$

then we obtain

$$(\Box - m^2)^{-1}\delta^n(x - x') = -\frac{1}{(2\pi)^n}\int d^n k \frac{e^{ik(x-x')}}{k^2 - m^2}. \tag{53}$$

After all, the Green function can be formally expressed as

$$G(x, x') = \frac{1}{(2\pi)^n}\int d^n k \frac{e^{ik(x-x')}}{k^2 - m^2}. \tag{54}$$

To calculate this integral the zerus points of the denominator $k^2 - m^2 = \mathbf{p}^2 - p_0^2 - m^2 = 0$ are needed, from which

$$p_0 = \pm\sqrt{\mathbf{p}^2 - m^2}. \tag{55}$$

can be obtained. After then, the propagator should be expressed in proper way taking Equation (54)

$$G(p) = \frac{1}{\mathbf{p}^2 - p_0^2 - m^2}. \tag{56}$$

In the sense of the theory the retarded $G_{ret}(p) = 1/(\mathbf{p}^2 - p_0^2 - m^2)_{ret}$ and the advanced $G_{adv}(p) = 1/(\mathbf{p}^2 - p_0^2 - m^2)_{adv}$ propagators are needed to be expressed for the tachyons due to the presence of the imaginary poles. Now, the construction of the Wheeler propagator (Wheeler, 1945; 1949) can be expounded as a half sum of the above propagators

$$G(p) = \frac{1}{2}G_{adv}(p) + \frac{1}{2}G_{ret}(p). \tag{57}$$

5.2 The Bochner's theorem

The calculation of propagators is based on the Bochner's theorem (Bochner, 1959; Bollini & Giambiagi, 1996; Bollini & Rocca, 1998; 2004; Jerri, 1998). It states that if the function $f(x_1, x_2, ..., x_n)$ depends on the variable set $(x_1, x_2, ..., x_n)$ then its Fourier transformed is — without the factor $1/(2\pi)^{n/2}$ —

$$g(y_1, y_2, ..., y_n) = \int d^n x f(x_1, x_2, ..., x_n) e^{ix_i y_i} \quad (i = 1, ..., n). \tag{58}$$

However, it is useful to introduce the variables $x = (x_1^2 + x_2^2 + ... + x_n^2)^{1/2}$ and $y = (y_1^2 + y_2^2 + ... + y_n^2)^{1/2}$ instead of the original sets. Now, the examinations are restricted to the spherically symmetric functions $f(x)$ and $g(y)$. In these cases the above Fourier transform given by Equation (58) can be calculated by applying the Hankel (Bessel) transformation by which we obtain

$$g(y, n) = \frac{(2\pi)^{n/2}}{y^{n/2-1}} \int_0^\infty f(x) x^{n/2} J_{n/2-1}(xy) dx. \tag{59}$$

Here, J_α is a first kind α order Bessel function. Later it will be very useful to calculate the function f with causal functions depending on the momentum space p thus we write

$$f(x, n) = \frac{(2\pi)^{n/2}}{x^{n/2-1}} \int_0^\infty g(p) p^{n/2} J_{n/2-1}(xp) dp. \tag{60}$$

It can be seen that the singularity at the origin depends on n analytically.

5.3 Calculation of the Wheeler propagator

To obtain the Wheeler propagator, first, e.g., the integral in Equation (54) for the advanced propagator can be calculated

$$G_{adv}(x) = \frac{1}{(2\pi)^n} \int d^{n-1} p e^{i\mathbf{pr}} \int_{adv} dp_0 \frac{e^{-ip_0 x_0}}{\mathbf{p}^2 - p_0^2 - m^2}. \tag{61}$$

The path of integration runs parallel to the real axis and below both the poles for the advanced propagator. (For the retarded propagator the path runs above the poles.) Thus, considering the propagator $G_{adv}(p)$ for $x_0 > 0$ the path is closed on the lower half plane giving null result. In the opposite case, when $x_0 < 0$, there is a non-zero finite contribution of the residues at the poles

$$p_0 = \pm\omega = \sqrt{\mathbf{p}^2 - m^2} \quad if \quad \mathbf{p}^2 \geq m^2 \tag{62}$$

and

$$p_0 = \pm i\omega' = \sqrt{\mathbf{p}^2 - m^2} \quad if \quad \mathbf{p}^2 \leq m^2. \tag{63}$$

After applying the Cauchy's residue theorem for the integration with respect to p_0 we obtain an $n-1$ order integral

$$G_{adv}(x) = -\frac{H(-x_0)}{(2\pi)^{n-1}} \int d^{n-1} p e^{i\mathbf{pr}} \frac{sin[(\mathbf{p}^2 - m^2 + i0)^{\frac{1}{2}} x_0]}{(\mathbf{p}^2 - m^2 + i0)^{\frac{1}{2}}}, \tag{64}$$

where H is the Heaviside's function. The retarded propagator can be similarly obtained

$$G_{ret}(x) = \frac{H(x_0)}{(2\pi)^{n-1}} \int d^{n-1} p e^{i\mathbf{pr}} \frac{sin[(\mathbf{p}^2 - m^2 + i0)^{\frac{1}{2}} x_0]}{(\mathbf{p}^2 - m^2 + i0)^{\frac{1}{2}}}. \tag{65}$$

Considering the form of the propagator in Equation (57) and taking the propagators in Equations (64) and (65) we obtain the Wheeler-propagator

$$G(x) = \frac{Sgn(x_0)}{2(2\pi)^{n-1}} \int d^{n-1} p e^{i\mathbf{pr}} \frac{sin[(\mathbf{p}^2 - m^2 + i0)^{\frac{1}{2}} x_0]}{(\mathbf{p}^2 - m^2 + i0)^{\frac{1}{2}}}. \tag{66}$$

To evaluate the above propagators the integrals can be rewritten by the Hankel transformation based on Bochner's theorem [Equation (59)]

$$\frac{1}{(2\pi)^{n-1}} \int d^{n-1} p e^{i\mathbf{pr}} \frac{sin[(\mathbf{p}^2 - m^2 + i0)^{\frac{1}{2}} x_0]}{(\mathbf{p}^2 - m^2 + i0)^{\frac{1}{2}}} =$$

$$\frac{1}{(2\pi)^{\frac{n-1}{2}}} \frac{1}{x^{\frac{n-1}{2}-1}} \int_0^\infty p^{\frac{n-1}{2}} \frac{\sin(\mathbf{p}^2 - m^2)^{\frac{1}{2}} x_0}{(\mathbf{p}^2 - m^2)^{\frac{1}{2}}} J_{\frac{n-1}{2}-1}(xp)\, dp, \tag{67}$$

where $p = \sqrt{p_1^2 + p_2^2 + ... + p_{n-1}^2}$ and $r = \sqrt{x_1^2 + x_2^2 + ... + x_{n-1}^2}$. The following integrals (Gradshteyn & Ryzhik, 1994) are applied for the above calculations such as

$$\int_0^\infty dy\, y^{\gamma+1} \frac{\sin\left(a\sqrt{b^2+y^2}\right)}{\sqrt{b^2+y^2}} J_\gamma(cy) = \sqrt{\frac{\pi}{2}} b^{\frac{1}{2}+\gamma} c^\gamma (a^2 - c^2)^{-\frac{1}{4}-\frac{1}{2}\gamma} J_{-\gamma-\frac{1}{2}}(b\sqrt{a^2-c^2}), \tag{68}$$

if $0 < c < a, \quad Re\, b > 0, \quad -1 < Re\, \gamma < 1/2$, and

$$\int_0^\infty dy\, y^{\gamma+1} \frac{\sin\left(a\sqrt{b^2+y^2}\right)}{\sqrt{b^2+y^2}} J_\gamma(cy) = 0, \tag{69}$$

if $0 < a < c, \quad Re\, b > 0, \quad -1 < Re\, \gamma < \frac{1}{2}$. The parameters of the model can be fitted by

$$a = x_0, \quad b = im = i\frac{cc_v}{2\lambda}, \quad c = r, \quad \gamma = \frac{n}{2} - \frac{3}{2}. \tag{70}$$

and we consider the relation between the Bessel functions

$$J_\alpha(ix) = i^\alpha I_\alpha(x), \tag{71}$$

where $I_\alpha(x)$ is the modified Bessel function. Now, we can express the advanced Wheeler propagator Equation (64) of the tachyonic thermal energy propagation

$$W_{adv}(x) = H(-x_0) \frac{\pi}{(2\pi)^{n/2}} \left(\frac{cc_v}{2\lambda}\right)^{\frac{n}{2}-1} (x_0^2 - r^2)_+^{\frac{1}{2}(1-\frac{n}{2})} I_{1-\frac{n}{2}}\left(\frac{cc_v}{2\lambda}(x_0^2 - r^2)_+^{\frac{1}{2}}\right). \tag{72}$$

The calculation for the retarded propagator can be similarly elaborated by Equations (65) and (67)

$$W_{ret}(x) = H(x_0) \frac{\pi}{(2\pi)^{n/2}} \left(\frac{cc_v}{2\lambda}\right)^{\frac{n}{2}-1} (x_0^2 - r^2)_+^{\frac{1}{2}(1-\frac{n}{2})} I_{1-\frac{n}{2}}\left(\frac{cc_v}{2\lambda}(x_0^2 - r^2)_+^{\frac{1}{2}}\right). \tag{73}$$

Comparing the results of Equations (72) and (73) it can be seen that we can write one common formula easily to express the complete propagator. Thus the Wheeler-propagator in the n dimensional space-time — remembering the construction in Equation (57) — is

$$W^{(n)}(x) = \frac{\pi}{2(2\pi)^{n/2}} \left(\frac{cc_v}{2\lambda}\right)^{\frac{n}{2}-1} (x_0^2 - r^2)_+^{\frac{1}{2}(1-\frac{n}{2})} I_{1-\frac{n}{2}} \left(\frac{cc_v}{2\lambda}(x_0^2 - r^2)_+^{\frac{1}{2}}\right). \tag{74}$$

The calculated Wheeler propagator in the $3+1$ dimensional space-time can be expressed for the thermal energy propagation

$$W^{(4)}(r, x_0) = \frac{1}{8\pi} \left(\frac{cc_v}{2\lambda}\right) (x_0^2 - r^2)^{-\frac{1}{2}} I_{-1} \left(\frac{cc_v}{2\lambda}(x_0^2 - r^2)^{\frac{1}{2}}\right). \tag{75}$$

The expected causality can be immediately recognized from the plot of the propagator in Fig. 8, since it differs to zero just within the light cone.

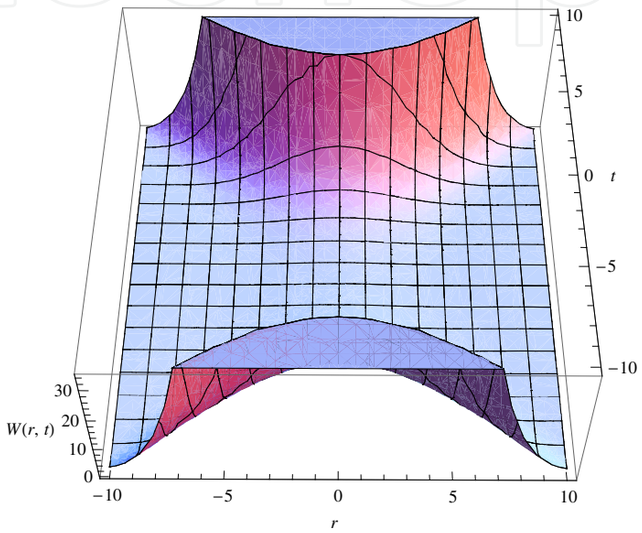


Fig. 8. The causal Wheeler propagator in the space-time — in arbitrary units — which is zero out of the light cone.

Finally, it is important to mention and emphasize that the participating particles of the above treated thermal energy propagation cannot be observable directly as Bollini's and Rocca's detailed studies (Bollini & Rocca, 1997a;b; Bollini et al., 1999) show. This is a consequence of the fact that the tachyons do not move as free particles, thus they can be considered as the mediators of the dynamic phase transition (Gambár & Márkus, 2007; Márkus & Gambár, 2010).

6. Summary and concluding remarks

This chapter of the book is dealing with the hundred years old open question of how it could be formulated and exploited the Lorentz invariant description of the thermal energy propagation. The relevant field equation as the leading equation of the theory providing the finite speed of action is a Klein-Gordon type equation with negative "mass term". It has been shown via the dispersion relations that the classical Fourier heat conduction equation is also involved, naturally. The tachyon solution of this kind of Klein-Gordon equation ensures that both wave-like (non-dissipative, oscillating) and the non-wave-like (dissipative, diffusive) signal propagations are present. The two propagation modes are divided by a spinodal instability pertaining to a dynamic phase transition. It is important to emphasize that in this

way, finally, the concept of the dynamic temperature has been introduced.
Then, a mechanical system is discussed to point out clearly that Klein-Gordon equations with the same mathematical structure and similar physical meaning can be found in the other disciplines of physics, too. The model involves a stretched string put on the diameter of a rotating disc. Collecting the kinetic and potential energy terms and formulating the Lagrange function of the problem, it has been shown that the equation of motion as Euler-Lagrange equation is exactly the above mentioned Klein-Gordon equation. The calculated dispersion relation points out unambiguously that the dynamics is similar to the case of Lorentz invariant heat conduction. The motion is vibrating (oscillating) below a system parameter dependent angular velocity, or diffusive (decaying) above this value.
The great challenge is to embed the concept of dynamic temperature into the general framework of physics. One of the aims via this step is to introduce the second law of thermodynamics by which the most basic law of nature may appear in the physical theories. Thus, such categories like dissipation, irreversibility, direction of processes can be handled directly within a description. This was the motivation to elaborate the coupling of the inflaton and the thermal field. As it can be concluded from the results, the introduced thermal field can generate the spontaneous symmetry breaking in the theory — without the Higgs mechanism — due to its property including the spinodal instability and the dynamic phase transition. The inflation decays into the thermal field by which the reheating process can start during the expansion of the universe. The time evolution of the inflation field is reproduced so well as it is known from the relevant cosmological models. It is important to emphasize that the thermal field generates a really dynamic temperature. A further progress could be achieved by the adding again the Higgs mechanism to generate massive particles in the space. This elaboration of the model remains for a future work.
Finally, it is an important step to justify that the above theory of thermal propagation completes the requirement of the causality. This question comes up due to the tachyon solutions. The arisen doubts can be eliminated in the knowledge of the propagator of the process. The relevant causal Wheeler propagator can be deduced by a longer, direct, analytic mathematical calculation applying the Bochner's theorem. The results clearly shows that the causality is completed since the propagator is within the light cone, i.e., the theory is consistent.
The presented theory of this chapter is put into the general framework of the physics coherently. These results mean a good base how to couple the thermodynamic field with the other fields of physics. Hopefully, it opens new perspectives towards in the understanding of irreversibility and dissipation in the field theoretical processes.

7. Acknowledgment

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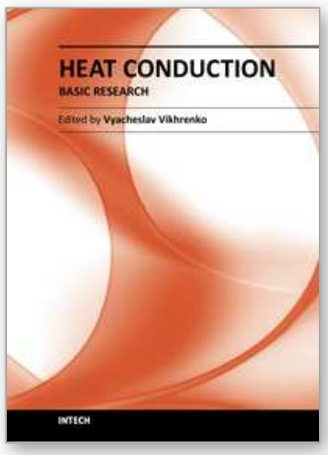
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