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Exact Travelling Wave Solutions for Generalized Forms of the Nonlinear Heat Conduction Equation

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1. Introduction

"*The most incomprehensible thing about the world is that it is at all comprehensible*" (Albert Einstein), but the question is how do we fully understand incomprehensible things? Nonlinear science provides some clues in this regard (He, 2009).

The world around us is inherently nonlinear. For instance, nonlinear evolution equations (NLEEs) are widely used as models to describe complex physical phenomena in various fields of sciences, especially in fluid mechanics, solid-state physics, plasma physics, plasma waves, and biology. One of the basic physical problems for these models is to obtain their travelling wave solutions. In particular, various methods have been utilized to explore different kinds of solutions of physical models described by nonlinear partial differential equations (PDEs). For instance, in the numerical methods, stability and convergence should be considered, so as to avoid divergent or inappropriate results. However, in recent years, a variety of effective analytical and semi-analytical methods have been developed to be used for solving nonlinear PDEs, such as the variational iteration method (VIM) (He, 1998; He et al., 2010), the homotopy perturbation method (HPM) (He, 2000, 2006), the homotopy analysis method (HAM) (Abbasbandy, 2010), the tanh-method (Fan, 2002; Wazwaz, 2005, 2006), the sine-cosine method (Wazwaz, 2004), and others. Likewise, He and Wu (2006) proposed a straightforward and concise method called the Exp-function method to obtain the exact solutions of NLEEs. The method, with the aid of Maple or Matlab, has been successfully applied to many kinds of NLEE (He & Zhang, 2008; Kabir & Khajeh, 2009; Borhanifar & Kabir, 2009, 2010; Borhanifar et al., 2009; Kabir et al., 2011). Lately, the (G'/G)-expansion method, first introduced by Wang et al. (2008), has become widely used to search for various exact solutions of NLEEs (Bekir & Cevikel, 2009; Zhang et al., 2009; Zedan, 2010; Kabir et al., 2011). The results reveal that the two recent methods are powerful techniques for solving nonlinear partial differential equations (NPDEs) in terms of accuracy and efficiency. This is important, since systems of NPDEs have many applications in engineering.

The generalized forms of the nonlinear heat conduction equation can be given as

$$u_t - a(u^n)_{xx} - u + u^n = 0, \qquad a > 0\ , n > 1 \tag{1.1}$$

and in (2 + 1)-dimensional space

$$u_t - a(u^n)_{xx} - a(u^n)_{yy} - u + u^n = 0. \tag{1.2}$$

The heat equation is an important partial differential equation which describes the distribution of heat (or variation in temperature) in a given region over time. The heat equation is a consequence of Fourier's law of cooling. In this chapter, we consider the heat equation with a nonlinear power-law source term. The equations (1.1) and (1.2) describe one-dimensional and two-dimensional unsteady thermal processes in quiescent media or solids with the nonlinear temperature dependence of heat conductivity. In the above equations, u= u(x,y,t) is temperature as a function of space and time; u_t is the rate of change of temperature at a point over time; u_{xx} and u_{yy} are the second spatial derivatives (thermal conductions) of temperature in the x and y directions, respectively; also u_x and u_y are the temperature gradient.

Many authors have studied some types of solutions of these equations. Wazwaz (2005) used the tanh-method to find solitary solutions of these equations and a standard form of the nonlinear heat conduction equation (when $n = 3$ in Eq. (1.1)). Also, Fan (2002) applied the solutions of Riccati equation in the tanh-method to obtain the travelling wave solution when $n = 2$ in Eq. (1.1). More recently, Kabir et al. (2009) implemented the Exp-function method to find exact solutions of Eq. (1.1), and obtained more general solutions in comparison with Wazwaz's results.

Considering all the indispensably significant issues mentioned above, the objective of this paper is to investigate the travelling wave solutions of Eqs. (1.1) and (1.2) systematically, by applying the (G'/G)-expansion and the Exp-function methods. Some previously known solutions are recovered as well, and, simultaneously, some new ones are also proposed.

2. Description of the two methods

2.1 The (G'/G)-expansion method

Suppose that a nonlinear PDE, say in two independent variables x and t, is given by

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \cdots) = 0, \tag{2.1}$$

or in three independent variables x, y and t, is given by

$$P(u, u_t, u_x, u_y, u_{xx}, u_{yy}, u_{tt}, u_{tx}, u_{ty}, \cdots) = 0, \tag{2.2}$$

where P is a polynomial in its arguments, which include nonlinear terms and the highest order derivatives.

Introducing a complex variable η defined as

$$u(x,t) = U(\eta) \quad , \quad \eta = k(x - ct) \tag{2.3}$$

or

$$u(x,y,t) = U(\eta) \quad , \quad \eta = k(x + y - ct) \tag{2.4}$$

Eq. (2.1) and (2.2) reduce to the ordinary differential equations (ODEs)

$$P(U, -kcU', kU', k^2U'', k^2c^2U'', -k^2cU'', \cdots) = 0, \tag{2.5}$$

and

$$P(U, -kcU', kU', kU', k^2U'', k^2U'', k^2c^2U'', -k^2cU'', -k^2cU'', \cdots) = 0, \tag{2.6}$$

respectively, where k and c are constants to be determined later. According to the (G'/G)-expansion method, it is assumed that the travelling wave solution of Eq. (2.5) or (2.6) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$U(\eta) = \sum_{i=1}^{m} \alpha_i \left(\frac{G'}{G}\right)^i + \alpha_0 \ , \qquad \alpha_m \neq 0 \tag{2.7}$$

where α_0, and α_i, for $i = 1, 2, \ldots, m$, are constants to be determined later, and $G(\eta)$ satisfies a second-order linear ordinary differential equation (LODE):

$$\frac{d^2G(\eta)}{d\eta^2} + \lambda \frac{dG(\eta)}{d\eta} + \mu\, G(\eta) = 0 \tag{2.8}$$

where λ and μ are arbitrary constants. Using the general solutions of Eq. (2.8), we have

$$\frac{G'(\eta)}{G(\eta)} = \begin{cases} \dfrac{\sqrt{\lambda^2 - 4\mu}}{2} \left(\dfrac{C_1 \sinh\left(\dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right) + C_2 \cosh\left(\dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right)}{C_1 \cosh\left(\dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right) + C_2 \sinh\left(\dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right)} \right) - \dfrac{\lambda}{2}, & \lambda^2 - 4\mu > 0, \\ \dfrac{\sqrt{4\mu - \lambda^2}}{2} \left(\dfrac{-C_1 \sin\left(\dfrac{\sqrt{4\mu - \lambda^2}}{2}\eta\right) + C_2 \cos\left(\dfrac{\sqrt{4\mu - \lambda^2}}{2}\eta\right)}{C_1 \cos\left(\dfrac{\sqrt{4\mu - \lambda^2}}{2}\eta\right) + C_2 \sin\left(\dfrac{\sqrt{4\mu - \lambda^2}}{2}\eta\right)} \right) - \dfrac{\lambda}{2}, & \lambda^2 - 4\mu < 0, \end{cases} \tag{2.9}$$

and it follows from (2.7) and (2.8), that

$$U' = -\sum_{i=1}^{m} i\alpha_i \left[\left(\frac{G'}{G}\right)^{i+1} + \lambda \left(\frac{G'}{G}\right)^{i} + \mu \left(\frac{G'}{G}\right)^{i-1} \right],$$

$$U'' = \sum_{i=1}^{m} i\alpha_i \left[\begin{array}{l} (i+1)\left(\frac{G'}{G}\right)^{i+2} + \lambda\,(2i+1)\left(\frac{G'}{G}\right)^{i+1} + i\,(\lambda^2+2\mu)\left(\frac{G'}{G}\right)^{i} \\ +\mu\lambda\,(2i-1)\left(\frac{G'}{G}\right)^{i-1} + \mu^2(i-1)\left(\frac{G'}{G}\right)^{i-2} \end{array} \right] \tag{2.10}$$

and so on. Here, the prime denotes the derivative with respective to η.

To determine u explicitly, we take the following four steps:

Step 1. Determine the integer m by substituting Eq. (2.7) along with Eq. (2.8) into Eq. (2.5) or (2.6), and balancing the highest-order nonlinear term(s) and the highest-order partial derivative.

Step 2. Substitute Eq. (2.7) with the value of m determined in *Step 1*, along with Eq. (2.8) into Eq. (2.5) or (2.6) and collect all terms with the same order of $\left(\frac{G'}{G}\right)$ together; the left-hand side of Eq. (2.5) or (2.6) is converted into a polynomial in $\left(\frac{G'}{G}\right)$. Then set each coefficient of this polynomial to zero to derive a set of algebraic equations for k, c, α_0 and α_i, for $i = 1, 2, \ldots, m$.

Step 3. Solve the system of algebraic equations obtained in *Step 2*, for k, c, α_0 and α_i, for $i = 1, 2, \ldots, m$, by use of Maple.

Step 4. Use the results obtained in the above steps to derive a series of fundamental solutions $u(\eta)$ of Eq. (2.5) or (2.6) depending on $\left(\frac{G'}{G}\right)$; since the solutions of Eq. (2.8) have been well known for us, we can obtain exact solutions of Eqs. (2.1) and (2.2).

2.2 The Exp-function method

According to the classic Exp-function method, it is assumed that the solution of ODEs (2.5) or (2.6) can be written as

$$u(\eta) = \frac{\sum_{n=-f}^{g} a_n \exp(n\eta)}{\sum_{m=-p}^{q} b_m \exp(m\eta)} = \frac{a_f \exp(f\eta) + \cdots + a_{-g} \exp(-g\eta)}{b_p \exp(p\eta) + \cdots + b_{-q} \exp(-q\eta)}, \tag{2.11}$$

where f, g, p and q are positive integers which are unknown, to be further determined, and a_n and b_m are unknown constants.

3. A generalized form of the nonlinear heat conduction equation

3.1 Application of the (G'/G)-expansion method

Introducing a complex variable η defined as Eq. (2.3), Eq. (1.1) becomes an ordinary differential equation, which can be written as

$$-kcU' - ak^2(U^n)'' - U + U^n = 0, \qquad a > 0 \tag{3.1}$$

or, equivalently,

$$-kcU' - ak^2 n(n-1)U^{n-2}U'^2 - ak^2 nU^{n-1}U'' - U + U^n = 0, \tag{3.2}$$

To get a closed-form analytic solution, we use the transformation (Kabir & Khajeh, 2009; Wazwaz, 2005)

$$U(\eta) = V^{\frac{-1}{n-1}}(\eta), \tag{3.3}$$

which will convert Eq. (3.2) into

$$kc(n-1)V'V^2 + ak^2 n(1-2n)V'^2 + ak^2 n(n-1)VV'' - (n-1)^2 V^3 + (n-1)^2 V^2 = 0, \tag{3.4}$$

According to Step 1, considering the homogeneous balance between VV'' and $V'V^2$ in Eq. (3.4) gives

$$2m + 2 = 3m + 1, \tag{3.5}$$

so that

$$m = 1. \tag{3.6}$$

Suppose that the solutions of (3.4) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$V(\eta) = \alpha_0 + \alpha_1\left(\frac{G'}{G}\right), \qquad \alpha_1 \neq 0. \tag{3.7}$$

where α_0 and α_1, are constants which are unknown, to be determined later.

Substituting Eq. (3.7) along with Eq. (2.8) into Eq. (3.4) and collecting all terms with the same power of $\left(\frac{G'}{G}\right)$ together, the left-hand side of Eq. (3.4) is converted into a polynomial in $\left(\frac{G'}{G}\right)$. Equating each coefficient of this polynomial to zero yields a set of simultaneous algebraic equations for $\alpha_0, \alpha_1, k, c, \lambda$ and μ. Solving the system of algebraic equations with the aid of Maple 12, we obtain the following.

Case A: When $\lambda^2 - 4\mu > 0$

Case A-1.

$$\alpha_0 = \frac{1}{2} + \frac{\lambda}{2\sqrt{\lambda^2 - 4\mu}},\ \alpha_1 = \frac{1}{\sqrt{\lambda^2 - 4\mu}},\ k = \pm\frac{n-1}{n\sqrt{a}}.\frac{1}{\sqrt{\lambda^2 - 4\mu}},\ c = \mp\sqrt{a} \tag{3.8}$$

where λ and μ are arbitrary constants.
By using Eq. (3.8), expression (3.7) can be written as

$$V(\eta) = \frac{1}{2} + \frac{\lambda}{2\sqrt{\lambda^2 - 4\mu}} + \frac{1}{\sqrt{\lambda^2 - 4\mu}}\left(\frac{G'}{G}\right), \tag{3.9}$$

Substituting the general solution of (2.9) into Eq. (3.9), we get the generalized travelling wave solution as follows:

$$V(\eta) = \frac{1}{2}\left(1 + \left[\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right)}\right]\right), \tag{3.10}$$

where

$$\eta = \pm\frac{n-1}{n\sqrt{a}}.\frac{1}{\sqrt{\lambda^2 - 4\mu}}\left(x \pm \sqrt{a}t\right).$$

inserting Eq. (3.10) into Eq. (3.3), it yields the following exact solution of Eq. (1.1):

$$u(x,t) = \left[\frac{1}{2}\left(1 + \left[\frac{C_1 \sinh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \pm \sqrt{a}t\right)\right) + C_2 \cosh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \pm \sqrt{a}t\right)\right)}{C_1 \cosh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \pm \sqrt{a}t\right)\right) + C_2 \sinh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \pm \sqrt{a}t\right)\right)}\right]\right)\right]^{\frac{-1}{n-1}} \tag{3.11}$$

in which C_1 and C_2 are arbitrary parameters that can be determined by the related initial and boundary conditions.

Now, to obtain some special cases of the above general solution, we set $C_2 = 0$; then (3.11) leads to the formal solitary wave solution to (1.1) as follows:

$$u(x,t) = \left[\frac{1}{2}\left(1 + \tanh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \pm \sqrt{a}t\right)\right)\right)\right]^{\frac{-1}{n-1}}, \tag{3.12}$$

and, when $C_1 = 0$, the general solution (3.11) reduces to

$$u(x,t) = \left[\frac{1}{2}\left(1 + \coth\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \pm \sqrt{a}t\right)\right)\right)\right]^{\frac{-1}{n-1}}, \tag{3.13}$$

Comparing the particular cases of our general solution, Eqs. (3.12) and (3.13), with Wazwaz's results (2005), Eqs. (73) and (74), it can be seen that the results are exactly the same.

Case A-2.

$$\alpha_0 = \frac{1}{2} - \frac{\lambda}{2\sqrt{\lambda^2 - 4\mu}},\ \alpha_1 = \frac{-1}{\sqrt{\lambda^2 - 4\mu}},\ k = \pm\frac{n-1}{n\sqrt{a}}.\frac{1}{\sqrt{\lambda^2 - 4\mu}},\ c = \pm\sqrt{a} \tag{3.14}$$

Inserting Eq. (3.14) into (3.7) yields

$$V(\eta) = \frac{1}{2} - \frac{\lambda}{2\sqrt{\lambda^2 - 4\mu}} - \frac{1}{\sqrt{\lambda^2 - 4\mu}}\left(\frac{G'}{G}\right), \tag{3.15}$$

Substituting the general solution of (2.9) into Eq. (3.15), we obtain

$$V(\eta) = \frac{1}{2}\left(1 - \left[\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta\right)}\right]\right), \tag{3.16}$$

where $\eta = \pm\frac{n-1}{n\sqrt{a}}.\frac{1}{\sqrt{\lambda^2 - 4\mu}}\left(x \mp \sqrt{a}t\right)$.

Substituting Eq. (3.16) into the transformation (3.3) leads to the generalized solitary wave solution of Eq. (1.1) as follows:

$$u(x,t) = \left[\frac{1}{2}\left(1 - \left[\frac{C_1 \sinh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \mp \sqrt{a}t\right)\right) + C_2 \cosh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \mp \sqrt{a}t\right)\right)}{C_1 \cosh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \mp \sqrt{a}t\right)\right) + C_2 \sinh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \mp \sqrt{a}t\right)\right)}\right]\right)\right]^{\frac{-1}{n-1}} \tag{3.17}$$

Similarly, to derive some special cases of the above general solution (3.17), we choose $C_2 = 0$; then (3.17) leads to

$$u(x,t) = \left[\frac{1}{2}\left(1 - \tanh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \mp \sqrt{a}t\right)\right)\right)\right]^{\frac{-1}{n-1}}, \tag{3.18}$$

and, when $C_1 = 0$, the general solution (3.17) reduces to

$$u(x,t) = \left[\frac{1}{2}\left(1 - \coth\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x \mp \sqrt{a}t\right)\right)\right)\right]^{\frac{-1}{n-1}}, \tag{3.19}$$

Validating our results, Eqs. (3.18) and (3.19), with Wazwaz's solutions (2005), Eqs. (71) and (72), we can conclude that the results are exactly the same.

Case B: When $\lambda^2 - 4\mu < 0$

Case B-1.

$$\alpha_0 = \frac{1}{2} + \frac{\lambda i}{2\sqrt{4\mu - \lambda^2}},\ \alpha_1 = \frac{i}{\sqrt{4\mu - \lambda^2}},\ k = \pm\frac{n-1}{n\sqrt{a}}.\frac{i}{\sqrt{4\mu - \lambda^2}},\ c = \mp\sqrt{a} \tag{3.20}$$

Inserting Eq. (3.20) into (3.7) results

$$V(\eta) = \frac{1}{2} + \frac{\lambda i}{2\sqrt{4\mu - \lambda^2}} + \frac{i}{\sqrt{4\mu - \lambda^2}}\left(\frac{G'}{G}\right), \tag{3.21}$$

Substituting the general solution of (2.9) for $\lambda^2 - 4\mu < 0$ into Eq. (3.21), we get

$$V(\eta) = \frac{1}{2}\left(1 + i\left[\frac{-C_1 \sin\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\eta\right) + C_2 \cos\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\eta\right)}{C_1 \cos\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\eta\right) + C_2 \sin\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\eta\right)}\right]\right), \tag{3.22}$$

where

$$\eta = \pm\frac{n-1}{n\sqrt{a}}.\frac{i}{\sqrt{4\mu - \lambda^2}}\left(x \pm \sqrt{a}t\right).$$

Using the following transformation,

$$\begin{aligned} &\eta = i\zeta, \\ &\sinh\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\zeta\right) = -i\sin\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\eta\right), \\ &\cosh\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\zeta\right) = \cos\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\eta\right). \end{aligned} \tag{3.23}$$

in Eq. (3.22) and substituting the result into (3.3), we obtain the following exact solution of Eq. (1.1):

$$u(x,t)=\left[\frac{1}{2}\left(1+\left[\frac{C_1\sinh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x\pm\sqrt{a}t\right)\right)+C_2 i\cosh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x\pm\sqrt{a}t\right)\right)}{C_1\cosh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x\pm\sqrt{a}t\right)\right)+C_2 i\sinh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x\pm\sqrt{a}t\right)\right)}\right]\right)\right]^{\frac{-1}{n-1}} \tag{3.24}$$

We note that if we set $C_2=0$ and $C_1=0$ in the general solution (3.24), we can recover the solutions (3.12) and (3.13), respectively.

Case B-2.

$$\alpha_0=\frac{1}{2}-\frac{\lambda i}{2\sqrt{4\mu-\lambda^2}},\ \alpha_1=\frac{-i}{\sqrt{4\mu-\lambda^2}},\ k=\pm\frac{n-1}{n\sqrt{a}}.\frac{i}{\sqrt{4\mu-\lambda^2}},\ c=\pm\sqrt{a} \tag{3.25}$$

Inserting Eq. (3.25) into (3.7) leads to

$$V(\eta)=\frac{1}{2}-\frac{\lambda i}{2\sqrt{4\mu-\lambda^2}}-\frac{i}{\sqrt{4\mu-\lambda^2}}\left(\frac{G'}{G}\right), \tag{3.26}$$

Substituting the general solution of (2.9) for $\lambda^2-4\mu<0$ into Eq. (3.26), we have

$$V(\eta)=\frac{1}{2}\left(1-i\left[\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}\right]\right), \tag{3.27}$$

in which $\eta=\pm\frac{n-1}{n\sqrt{a}}.\frac{i}{\sqrt{4\mu-\lambda^2}}\left(x\mp\sqrt{a}t\right)$.

Using the transformation (3.23) into Eq. (3.27), and substituting the result into (3.3) yields the following exact solution:

$$u(x,t)=\left[\frac{1}{2}\left(1-\left[\frac{C_1\sinh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x\mp\sqrt{a}t\right)\right)+C_2 i\cosh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x\mp\sqrt{a}t\right)\right)}{C_1\cosh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x\mp\sqrt{a}t\right)\right)+C_2 i\sinh\left(\pm\frac{n-1}{2n\sqrt{a}}\left(x\mp\sqrt{a}t\right)\right)}\right]\right)\right]^{\frac{-1}{n-1}} \tag{3.28}$$

Similarly, if we set $C_2=0$ and $C_1=0$ in the general solution (3.28), we arrive at the same solutions (3.18) and (3.19), respectively.

3.2 Application of the Exp-function method

In order to determine values of f and p, we balance the term v^3 with vv'' in Eq. (3.4); we have

$$v^3=\frac{c_1\exp(3f\eta)+\cdots}{c_2\exp(3p\eta)+\cdots}, \tag{3.29}$$

$$vv'' = \frac{c_3 \exp([2f+3p]\eta)+\cdots}{c_4 \exp(5p\eta)+\cdots}, \tag{3.30}$$

where c_i are determined coefficients only for simplicity. Balancing the highest order of the Exp-function in Eqs. (3.29) and (3.30), we have

$$3f+2p = 2f+3p, \tag{3.31}$$

which leads to the result

$$p = f, \tag{3.32}$$

Similarly, to determine values of g and q, we have

$$v^3 = \frac{\cdots + d_1 \exp(-3g\eta)}{\cdots + d_2 \exp(-3q\eta)}, \tag{3.33}$$

$$vv'' = \frac{\cdots + d_3 \exp(-[2g+3q]\eta)}{\cdots + d_4 \exp(-5p\eta)}, \tag{3.34}$$

where d_i are determined coefficients for simplicity. Balancing the lowest order of the Exp-function in Eqs. (3.33) and (3.34), we have

$$3g+2q = 2g+3q, \tag{3.35}$$

which leads to the result

$$q = g. \tag{3.36}$$

Case A: $p = f = 1, q = g = 1$

We can freely choose the values of p and q. For simplicity, we set $p = f = 1$ and $q = g = 1$, so Eq. (2.11) reduces to

$$v(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{\exp(\eta) + b_0 + b_{-1} \exp(-\eta)}, \tag{3.37}$$

Substituting Eq. (3.37) into Eq. (3.4), and making use of Maple, we arrive at

$$\begin{aligned} &\frac{1}{A}[c_4 \exp(4\eta) + c_3 \exp(3\eta) + c_2 \exp(2\eta) + c_1 \exp(\eta) + c_0 + c_{-1} \exp(-\eta) \\ &\quad + c_{-2} \exp(-2\eta) + c_{-3} \exp(-3\eta) + c_{-4} \exp(-4\eta)] = 0, \end{aligned} \tag{3.38}$$

in which

$$A = [\exp(\eta) + b_0 + b_{-1} \exp(-\eta)]^4, \tag{3.39}$$

And the c_n are coefficients of $\exp(n\eta)$. Equating to zero the coefficients of all powers of $\exp(n\eta)$ yields a set of algebraic equations for $a_0, b_0, a_1, a_{-1}, b_{-1}, k$, and c. Solving the system of algebraic equations with the aid of Maple 12, we obtain:

Case 1.

$$a_0 = 0,\ b_0 = 0,\ a_1 = 0,\ a_{-1} = b_{-1},\ b_{-1} = b_{-1},\ k = \pm\frac{n-1}{2n\sqrt{a}},\ c = \pm\sqrt{a} \tag{3.40}$$

Substituting Eq. (3.40) into (3.37) and inserting the result into the transformation (3.3), we get the generalized solitary wave solution of Eq. (1.1) as follows:

$$u(x,t) = \left[\frac{b_{-1}\exp(-\eta)}{\exp(\eta) + b_{-1}\exp(-\eta)}\right]^{\frac{-1}{n-1}}, \tag{3.41}$$

where $\eta = \pm\frac{n-1}{2n\sqrt{a}}\left(x \mp \sqrt{a}t\right)$ and b_{-1} is an arbitrary parameter which can be determined by the initial and boundary conditions.

If we set $b_{-1} = 1$ and $b_{-1} = -1$ in (3.41), the solutions (3.18) and (3.19) can be recovered, respectively.

Case 2.

$$a_0 = 0,\ b_0 = 0,\ a_1 = 1,\ a_{-1} = 0, b_{-1} = b_{-1},\ k = \pm\frac{n-1}{2n\sqrt{a}},\ c = \mp\sqrt{a} \tag{3.42}$$

By the same procedure as illustrated above, we obtain

$$u(x,t) = \left[\frac{\exp(\eta)}{\exp(\eta) + b_{-1}\exp(-\eta)}\right]^{\frac{-1}{n-1}}, \tag{3.43}$$

in which $\eta = \pm\frac{n-1}{2n\sqrt{a}}\left(x \pm \sqrt{a}t\right)$ and b_{-1} is a free parameter.

If we set $b_{-1} = 1$ and $b_{-1} = -1$ in (3.43), then it can be easily converted to the same solutions (3.12) and (3.13), respectively.

Case 3.

$$a_1 = 0,\ b_{-1} = 0,\ a_0 = a_0,\ b_0 = b_0,\ a_{-1} = a_0 b_0,\ k = \pm\frac{n-1}{n\sqrt{a}},\ c = \mp n\sqrt{a} \tag{3.44}$$

and consequently we get

$$u(x,t)=\left[\frac{a_0+a_0b_0\exp(-\eta)}{\exp(\eta)+b_0}\right]^{\frac{-1}{n-1}}, \tag{3.45}$$

where $\eta=\pm\frac{n-1}{n\sqrt{a}}\left(x\pm n\sqrt{a}t\right)$ and a_0, b_0, are arbitrary parameters; for example, if we put $b_0=0$, solution (3.45) reduces to

$$u(x,t)=\left[a_0\left(\cosh\eta-\sinh\eta\right)\right]^{\frac{-1}{n-1}}, \tag{3.46}$$

Case 4.

$$\begin{aligned}&a_1=0,\ a_0=a_0,\ b_0=b_0,\ b_{-1}=-a_0(a_0-b_0),\ a_{-1}=-a_0(a_0-b_0),\\&k=\pm\frac{n-1}{n\sqrt{a}},\ c=\pm\sqrt{a}\end{aligned} \tag{3.47}$$

and

$$u(x,t)=\left[\frac{a_0-a_0(a_0-b_0)\exp(-\eta)}{\exp(\eta)+b_0-a_0(a_0-b_0)\exp(-\eta)}\right]^{\frac{-1}{n-1}}, \tag{3.48}$$

where $\eta=\pm\frac{n-1}{n\sqrt{a}}\left(x\mp\sqrt{a}t\right)$ and a_0, b_0 are free parameters; for example, if we set $a_0=1,\ b_0=0$ in Eq. (3.48), it can be easily converted to

$$u(x,t)=\left[\frac{1}{2}(1-\coth\eta+\operatorname{csch}\eta)\right]^{\frac{-1}{n-1}}, \tag{3.49}$$

Case 5.

$$a_1=1,\ a_0=0,\ b_0=b_0,\ b_{-1}=0,\ a_{-1}=0,\ k=\pm\frac{n-1}{n\sqrt{a}},\ c=\mp\sqrt{a} \tag{3.50}$$

and finally we obtain

$$u(x,t)=\left[\frac{\exp(\eta)}{\exp(\eta)+b_0}\right]^{\frac{-1}{n-1}}. \tag{3.51}$$

in which $\eta=\pm\frac{n-1}{n\sqrt{a}}\left(x\pm\sqrt{a}t\right)$ and b_0 is a free parameter.

Case B: $p=f=2,\ q=g=1$

Since the values of g and f can be freely chosen, we can put $p = f = 2$ and $q = g = 1$, the trial function, Eq. (2.11) becomes

$$v(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{\exp(2\eta) + b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}, \tag{3.52}$$

By the same manipulation as illustrated above, we have the following sets of solutions:

Case 1.

$$a_1 = 0,\ a_0 = a_0,\ b_0 = 0,\ b_{-1} = 0,\ a_{-1} = 0,\ a_2 = 0,\ b_1 = 0,\ k = \pm\frac{n-1}{2n\sqrt{a}},\ c = \mp n\sqrt{a} \tag{3.53}$$

Substituting Eq. (3.53) into (3.52), we have

$$v(\eta) = a_0 \exp(-2\eta), \tag{3.54}$$

Substituting Eq. (3.54) into Eq. (3.3), we get the generalized solitary wave solution of Eq. (1.1) as

$$u(x,t) = [a_0 \exp(-2\eta)]^{\frac{-1}{n-1}}, \tag{3.55}$$

where $\eta = \pm\frac{n-1}{2n\sqrt{a}}(x \pm n\sqrt{a}t)$ and a_0 is an arbitrary parameter. Using the transformation $\begin{cases}\exp(\eta) = \cosh\eta + \sinh\eta \\ \exp(-\eta) = \cosh\eta - \sinh\eta\end{cases}$, Eq. (3.55) yields the same solution (3.46).

Case 2.

$$a_1 = 0,\ a_0 = b_0,\ b_0 = b_0,\ b_{-1} = 0,\ a_{-1} = 0,\ a_2 = 0,\ b_1 = 0,\ k = \pm\frac{n-1}{2n\sqrt{a}},\ c = \pm\sqrt{a} \tag{3.56}$$

Substituting Eq. (3.56) into (3.52), we have

$$v(\eta) = \frac{b_0}{\exp(2\eta) + b_0}, \tag{3.57}$$

Inserting Eq. (3.57) into (3.3), it admits to the generalized solitary wave solution of Eq. (1.1) as follows:

$$u(x,t) = \left[\frac{b_0}{\exp(2\eta) + b_0}\right]^{\frac{-1}{n-1}}, \tag{3.58}$$

where $\eta = \pm\frac{n-1}{2n\sqrt{a}}(x \mp \sqrt{a}t)$ and b_0 is a free parameter.

We note that if we set $a_0 = b_0$ in Eq. (3.48), we can recover the solution (3.58).

Case 3.

$$a_1 = 0,\ a_0 = 0,\ b_0 = 0,\ b_{-1} = b_{-1},\ a_{-1} = b_{-1},\ a_2 = 0,\ b_1 = 0,\ k = \pm\frac{n-1}{3n\sqrt{a}},\ c = \pm\sqrt{a} \tag{3.59}$$

Substituting Eq. (3.59) into (3.52) we obtain

$$v(\eta) = \frac{b_{-1}\exp(-\eta)}{\exp(2\eta) + b_{-1}\exp(-\eta)}, \tag{3.60}$$

and by inserting Eq. (3.60) into (3.3), we get the generalized solitary wave solution of (1.1) as

$$u(x,t) = \left[\frac{b_{-1}\exp(-\eta)}{\exp(2\eta) + b_{-1}\exp(-\eta)}\right]^{\frac{-1}{n-1}}, \tag{3.61}$$

in which $\eta = \pm\frac{n-1}{3n\sqrt{a}}(x \mp \sqrt{a}t)$ and b_{-1} is a free parameter that can be determined by the initial and boundary conditions.

4. The generalized nonlinear heat conduction equation in two dimensions

4.1 Application of the (G'/G)-expansion method

Using the wave variable (2.4) transforms Eq. (1.2) to the ODE

$$-kcU' - 2ak^2(U^n)'' - U + U^n = 0, \qquad a > 0 \tag{4.1}$$

or, equivalently,

$$-kcU' - 2ak^2 n(n-1)U^{n-2}U'^2 - 2ak^2 nU^{n-1}U'' - U + U^n = 0, \tag{4.2}$$

Then we use the transformation (3.3), which will convert Eq. (4.2) into

$$kc(n-1)V'V^2 + 2ak^2 n(1-2n)V'^2 + 2ak^2 n(n-1)VV'' - (n-1)^2V^3 + (n-1)^2V^2 = 0, \tag{4.3}$$

By the same manipulation as illustrated in Section 3.1, we obtain the following sets of solutions.

Case A: When $\lambda^2 - 4\mu > 0$

Case A-1.

$$\alpha_0 = \frac{1}{2} + \frac{\lambda}{2\sqrt{\lambda^2 - 4\mu}},\ \alpha_1 = \frac{1}{\sqrt{\lambda^2 - 4\mu}},\ k = \pm\frac{n-1}{n\sqrt{2a}}.\frac{1}{\sqrt{\lambda^2 - 4\mu}},\ c = \mp\sqrt{2a} \tag{4.4}$$

By the same procedure as illustrated in Case A-1 of Section 3.1, Eqs. (3.9) and (3.10), we can finally find the generalized solitary wave solution of Eq. (1.2) as

$$u(x,y,t)=\left[\frac{1}{2}\left(1+\left[\frac{C_1\sinh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\pm\sqrt{2a}t\right)\right)+C_2\cosh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\pm\sqrt{2a}t\right)\right)}{C_1\cosh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\pm\sqrt{2a}t\right)\right)+C_2\sinh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\pm\sqrt{2a}t\right)\right)}\right]\right)\right]^{\frac{-1}{n-1}} \tag{4.5}$$

in which C_1 and C_2 are arbitrary parameters that can be determined by the related initial and boundary conditions.

Now, to obtain some special cases of the above general solution, we set $C_2=0$; then (4.5) leads to

$$u(x,y,t)=\left[\frac{1}{2}\left(1+\tanh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\pm\sqrt{2a}t\right)\right)\right)\right]^{\frac{-1}{n-1}}, \tag{4.6}$$

and, when $C_1=0$, the exact solution (4.5) reduces to

$$u(x,y,t)=\left[\frac{1}{2}\left(1+\coth\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\pm\sqrt{2a}t\right)\right)\right)\right]^{\frac{-1}{n-1}}, \tag{4.7}$$

Comparing the particular cases of our general solution, Eqs. (4.6) and (4.7), with Wazwaz's results (2005), Eqs. (87) and (88), it can be seen that the results are exactly the same.

Case A-2.

$$\alpha_0=\frac{1}{2}-\frac{\lambda}{2\sqrt{\lambda^2-4\mu}},\ \alpha_1=\frac{-1}{\sqrt{\lambda^2-4\mu}},\ k=\pm\frac{n-1}{n\sqrt{2a}}.\frac{1}{\sqrt{\lambda^2-4\mu}},\ c=\pm\sqrt{2a} \tag{4.8}$$

By the similar process as illustrated in Case A-2 of Section 3.1, Eqs. (3.15) and (3.16), we can easily gain the following exact solution of Eq. (1.2):

$$u(x,y,t)=\left[\frac{1}{2}\left(1-\left[\frac{C_1\sinh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\mp\sqrt{2a}t\right)\right)+C_2\cosh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\mp\sqrt{2a}t\right)\right)}{C_1\cosh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\mp\sqrt{2a}t\right)\right)+C_2\sinh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\mp\sqrt{2a}t\right)\right)}\right]\right)\right]^{\frac{-1}{n-1}} \tag{4.9}$$

Similarly, to derive some special cases of the above general solution, we choose $C_2=0$; then (4.9) leads to the formal solitary wave solution as follows:

$$u(x,y,t)=\left[\frac{1}{2}\left(1-\tanh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x+y\mp\sqrt{2a}t\right)\right)\right)\right]^{\frac{-1}{n-1}}, \tag{4.10}$$

and, when $C_1 = 0$, the general solution (4.9) reduces to

$$u(x,y,t) = \left[\frac{1}{2}\left(1 - \coth\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \mp \sqrt{2a}t\right)\right)\right)\right]^{\frac{-1}{n-1}}, \tag{4.11}$$

Validating our results, Eqs. (4.10) and (4.11), with Wazwaz's solutions (2005), Eqs. (85) and (86), it can be seen that the results are exactly the same.

Case B: When $\lambda^2 - 4\mu < 0$

Case B-1.

$$\alpha_0 = \frac{1}{2} + \frac{\lambda i}{2\sqrt{4\mu - \lambda^2}},\ \alpha_1 = \frac{i}{\sqrt{4\mu - \lambda^2}},\ k = \pm\frac{n-1}{n\sqrt{2a}}.\frac{i}{\sqrt{4\mu - \lambda^2}},\ c = \mp\sqrt{2a} \tag{4.12}$$

By the same manipulation as illustrated in Case B-1 of Section 3.1, Eqs. (3.21)-(3.23), we can finally obtain the following exact solution:

$$u(x,y,t) = \left[\frac{1}{2}\left(1 + \left[\frac{C_1 \sinh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \pm \sqrt{2a}t\right)\right) + C_2 i \cosh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \pm \sqrt{2a}t\right)\right)}{C_1 \cosh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \pm \sqrt{2a}t\right)\right) + C_2 i \sinh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \pm \sqrt{2a}t\right)\right)}\right]\right)\right]^{\frac{-1}{n-1}} \tag{4.13}$$

We note that, if we set $C_2 = 0$ and $C_1 = 0$ in the general solution (4.13), we can recover the solutions (4.6) and (4.7), respectively.

Case B-2.

$$\alpha_0 = \frac{1}{2} - \frac{\lambda i}{2\sqrt{4\mu - \lambda^2}},\ \alpha_1 = \frac{-i}{\sqrt{4\mu - \lambda^2}},\ k = \pm\frac{n-1}{n\sqrt{2a}}.\frac{i}{\sqrt{4\mu - \lambda^2}},\ c = \pm\sqrt{2a} \tag{4.14}$$

Similar to Case B-2 of Section 3.1, we can find the following result:

$$u(x,y,t) = \left[\frac{1}{2}\left(1 - \left[\frac{C_1 \sinh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \mp \sqrt{2a}t\right)\right) + C_2 i \cosh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \mp \sqrt{2a}t\right)\right)}{C_1 \cosh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \mp \sqrt{2a}t\right)\right) + C_2 i \sinh\left(\pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \mp \sqrt{2a}t\right)\right)}\right]\right)\right]^{\frac{-1}{n-1}} \tag{4.15}$$

In particular, if we take $C_2 = 0$ and $C_1 = 0$ in the general solution (4.15), we arrive at the same solutions (4.10) and (4.11), respectively.

4.2 Application of the Exp-function method

By the same manipulation as illustrated in Section 3.2, we obtain the following sets of solutions.

Case 1.

$$a_{-1} = a_{-1},\ a_1 = 0,\ a_0 = 0,\ b_0 = 0,\ b_{-1} = a_{-1},\ k = \pm\frac{n-1}{2n\sqrt{2a}},\ c = \pm\sqrt{2a} \tag{4.16}$$

Substituting Eq. (4.16) into (3.37) and inserting the result into the transformation (3.3), we get the generalized solitary wave solution of Eq. (1.2) as follows:

$$u(x,y,t) = \left[\frac{a_{-1}\exp(-\eta)}{\exp(\eta) + a_{-1}\exp(-\eta)}\right]^{\frac{-1}{n-1}}, \tag{4.17}$$

where $\eta = \pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \mp \sqrt{2a}t\right)$ and a_{-1} is an arbitrary parameter which can be determined by the initial and boundary conditions.

If we set $a_{-1} = 1$ and $a_{-1} = -1$ in (4.17), the solutions (4.10) and (4.11) can be recovered, respectively.

Case 2.

$$a_0 = 0,\ b_0 = 0,\ a_1 = 1,\ a_{-1} = 0,\ b_{-1} = b_{-1},\ k = \pm\frac{n-1}{2n\sqrt{2a}},\ c = \mp\sqrt{2a} \tag{4.18}$$

By the same process as illustrated in the previous case, we obtain

$$u(x,y,t) = \left[\frac{\exp(\eta)}{\exp(\eta) + b_{-1}\exp(-\eta)}\right]^{\frac{-1}{n-1}}, \tag{4.19}$$

in which $\eta = \pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \pm \sqrt{2a}t\right)$ and b_{-1} is a free parameter.

If we set $b_{-1} = 1$ and $b_{-1} = -1$ in (4.19), then it can be easily converted to the same solutions (4.6) and (4.7), respectively.

Case 3.

$$a_{-1} = a_{-1},\ a_1 = 0,\ a_0 = 0,\ b_{-1} = 0,\ b_0 = 0,\ k = \pm\frac{n-1}{2n\sqrt{2a}},\ c = \mp n\sqrt{2a} \tag{4.20}$$

and consequently we get

$$u(x,y,t) = \left[a_{-1}\exp(-2\eta)\right]^{\frac{-1}{n-1}} = \left[a_{-1}\left(\cosh 2\eta - \sinh 2\eta\right)\right]^{\frac{-1}{n-1}}, \tag{4.21}$$

where $\eta = \pm\frac{n-1}{2n\sqrt{2a}}\left(x + y \pm n\sqrt{2a}t\right)$ and a_{-1} is an arbitrary parameter.

Case 4.

$$a_1 = 1,\ a_0 = a_0,\ a_{-1} = 0,\ b_{-1} = b_{-1},\ b_0 = \frac{b_{-1} + {a_0}^2}{a_0},\ k = \pm\frac{n-1}{n\sqrt{2a}},\ c = \mp\sqrt{2a} \tag{4.22}$$

and

$$u(x,y,t) = \left[\frac{\exp(\eta) + a_0}{\exp(\eta) + \frac{b_{-1} + {a_0}^2}{a_0} + b_{-1}\exp(-\eta)}\right]^{\frac{-1}{n-1}}, \tag{4.23}$$

where $\eta = \pm\frac{n-1}{n\sqrt{2a}}\left(x + y \pm \sqrt{2a}t\right)$ and a_0, b_{-1} are free parameters.

Case 5.

$$a_1 = 0,\ a_{-1} = a_{-1},\ a_0 = a_0,\ b_{-1} = a_{-1},\ b_0 = \frac{a_{-1} + a_0^2}{a_0},\ k = \pm\frac{n-1}{n\sqrt{2a}},\ c = \pm\sqrt{2a} \tag{4.24}$$

and finally we obtain

$$u(x,y,t) = \left[\frac{a_0 + a_{-1}\exp(-\eta)}{\exp(\eta) + \frac{a_{-1} + a_0^2}{a_0} + a_{-1}\exp(-\eta)}\right]^{\frac{-1}{n-1}} \tag{4.25}$$

in which $\eta = \pm\frac{n-1}{n\sqrt{2a}}(x + y \mp \sqrt{2a}t)$ and a_0, a_{-1} are free parameters.

Remark 1. We have verified all the obtained solutions by putting them back into the original equations (1.1) and (1.2) with the aid of Maple 12.

Remark 2. The solutions (3.12), (3.13), (3.18), (3.19), (4.6), (4.7), (4.10), (4.11) have been obtained by the tanh method (Wazwaz, 2005); the other solutions are new and more general solutions for the generalized forms of the nonlinear heat conduction equation.

5. Conclusions

To sum up, the purpose of the study is to show that exact solutions of two generalized forms of the nonlinear heat conduction equation can be obtained by the (G'/G)-expansion and the Exp-function methods. The final results from the proposed methods have been compared and verified with those obtained by the tanh method. New exact solutions, not obtained by the previously available methods, are also found. It can be seen that the Exp-function method yields more general solutions in comparison with the other method. Overall, the results reveal that the (G'/G)-expansion and the Exp-function methods are powerful mathematical tools to solve the nonlinear partial differential equations (NPDEs) in the terms

of accuracy and efficiency. This is important, since systems of NPDEs have many applications in engineering.

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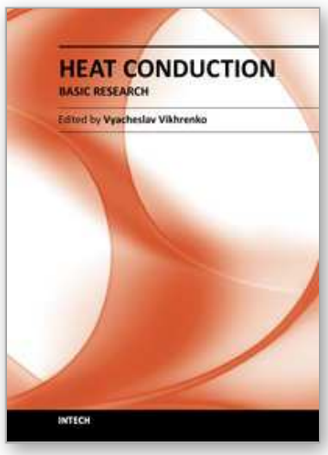

Heat Conduction - Basic Research

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The content of this book covers several up-to-date approaches in the heat conduction theory such as inverse heat conduction problems, non-linear and non-classic heat conduction equations, coupled thermal and electromagnetic or mechanical effects and numerical methods for solving heat conduction equations as well. The book is comprised of 14 chapters divided into four sections. In the first section inverse heat conduction problems are discuss. The first two chapters of the second section are devoted to construction of analytical solutions of nonlinear heat conduction problems. In the last two chapters of this section wavelike solutions are attained.The third section is devoted to combined effects of heat conduction and electromagnetic interactions in plasmas or in pyroelectric material elastic deformations and hydrodynamics. Two chapters in the last section are dedicated to numerical methods for solving heat conduction problems.

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