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Positioning Control of One Link Arm with Parametric Uncertainty Using the QFT Method

Takayuki Kuwashima, Jun Imai and Masami Konishi
Okayama University
Japan

1. Introduction

Many manufacturing robots are currently operated in various factories, with the aim of saving labor and cost. In particular, automatic sorting robots for products in assortment lines are used to improve productivity. In case of having the robots operate in such ways, the manipulators are demanded to have positioning performance of high precision to the target position. But in condition that products of various weights are handled, control performance such as positioning precision and settling time would be deteriorated if the controller is designed based on a fixed mathematical model; and they include parameters such as weights of grabbing product with their robot hands. So the controller needs to have robust control performance against parameter uncertainty in plant dynamics. It means that a control system should maintain its performance even if there exists uncertainty. In decades, control problems under plant uncertainty have been much studied because conventional control theory is based on the assumption that the dynamics of plant is fully well-known and certain, and that the mathematical model accurately reflects behavior of controlled plant in the real world. Nowadays, robust control theory is developed in order to cope with such problems. H_∞ control theory, among robust control theories, has been applied to many control systems and can handle unstructured uncertainty of plant, but it can not directly deal with structured uncertainty such as parameter variation.

In this chapter, a robust controller design using Quantitative Feedback Theory(QFT) (Houpis et al., 2006) is presented for one link arm with parametric uncertainty. QFT is a robust control theory developed based mainly on classical control. QFT can cope with parametric uncertainty in a plant(Khodabakhshian & Golbon, 2005; Barve & Nataraj, 1998; Zolotas & Halikias, 1999; Ryoo et al., 2002). In design procedure, the region where the plant can exists in accordance with parameter variation is illustrated on Nichols chart for each frequency, which is called template. By using templates, the controller is designed in order to satisfy performance specification for all possible plants.

2. Model of one link arm

Let us consider the one link arm as shown in Fig.1. It simulates the sorting robot in manufacturing facility. It consists of a Direct-Drive (DD) motor as an actuator, a rigid arm and a payload mass at free end of the arm, simulating a carried product. The arm is driven by the DD motor and rotates in the horizontal plane. The DD motor is actuated by input torque τ[Nm]. The specification of the motor is as shown in Table. 1 and the arm is made of stainless

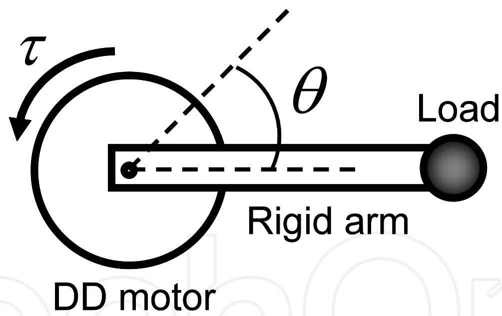

Fig. 1. One link arm

Rated voltage[V]	200
Rated torque[N·m]	25
Rated current[A]	2.7
Rated velocity[min^{-1}]	150
Inertia moment J_0[kg·m^2]	0.0909

Table 1. Specification of the DD motor

steel with length of L=0.3m, line density ρ=0.79 Kg/m and cross-section 1cm^2. Now, let $\theta(t)$ be the angle of rotation from the initial position, an equation of motion describing this system is

$$J\ddot{\theta} + D\dot{\theta} = \tau. \tag{1}$$

The Laplace transform of Eq.(1) gives the transfer function from τ to θ as

$$P(s) = \frac{1}{Js^2 + Ds}. \tag{2}$$

Here, J donates rotating inertia of the whole arm system, and is represented as

$$J = J_0 + \frac{\rho L^3}{3} + ML^3. \tag{3}$$

D is the friction coefficient around the motor rotary axis. From Eq.(3), it is found that value of J varies depending on weight of product. This time, we considered 6 samples of products, and their weights are from 0.5 through 3.0 at 0.5kg intervals. On the other hand, D varies with time. So observation experiment of D using step response method has been performed for 200 times. The variation ranges for J and D are as

$$J \in (0.098,\ 0.368)\,,\ D \in (0.475,\ 0.814)\,. \tag{4}$$

Then, we consider 7 samples of J and 10 samples of D as parameters of $P(s)$, and let $\mathcal{P}$ donates the set of the plant transfer functions, that is

$$\mathcal{P} = \{P_l\} \quad \text{where } l = 1, 2, ..., 70. \tag{5}$$

3. Effect of parameter variation

In this section, the effect of plat parameter variation on control performance of the one link arm system is studied. As described in Section 2, values of J and D vary in a certain range. We employ average values

$$\bar{J} = 0.233\text{kg} \cdot \text{m}^2, \ \ \bar{D} = 0.644\text{N} \cdot \text{m} \cdot \text{s} \tag{6}$$

for each parameter as the nominal parameter. Let $\bar{P}(s)$ donates the nominal plant. A PD controller has been designed for $\bar{P}(s)$ in order to satisfy following performance specifications:

$$\text{Overshoot}: \ O_s < 10\% \tag{7}$$

$$\text{Peak time}: \ t_p < 1.0\text{sec}. \tag{8}$$

Designed controller is

$$C(s) = 3.0 + \frac{0.5}{1 + 0.001s}. \tag{9}$$

Numerical experiment has been done on the control system shown in Fig. 2. The target rotary angle $\theta_r(t)$ is aimed at 1 rad and computational interval is 0.005s. Simulation result to control $\bar{P}(s)$ is shown in Fig. 3. On the other hand, J and D can take various values. Control results for $\mathcal{P}$ are shown in Fig. 4, and it is found that the responses of output $\theta(t)$ are scattered and some responses violate performance specifications. From these results, it can be seen that the controller designed for $\bar{P}(s)$ does not always ensure the desired performance with variation in parameters of $\bar{P}(s)$.

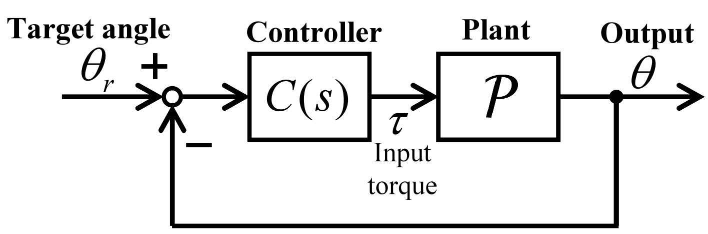

Fig. 2. Feedback Control System

4. Control system design using QFT

In control systems, it is desirable for controlled output of plant $\theta(t)$ to satisfy performance specification in spite of parameter uncertainty in a plant. In QFT design procedure, designer can determine the performance specifications that response of $\theta(t)$ should lie in. It has certain width as tolerance in time and frequency domains. Controller is designed on Nichols Chart (NC) so that all $\theta(t)$ lie between these specifications. These specifications are described on NC as boundary.

Consider feedback control system shown in Fig.5. Control objective is to control angular position of arm. We design $C(s)$ and $F(s)$ in order to satisfy performance specifications.

QFT design procedure is as follows:

STEP1. Determine performance specification in time and frequency domains.

STEP2. Gain the area where varying plant can exist on NC, called template.

STEP3. Construct performance specification on NC as bounds.

STEP4. Form the open loop transfer function $C(s)P(s)$ to satisfy bounds.

STEP5. Design the filter $F(s)$.

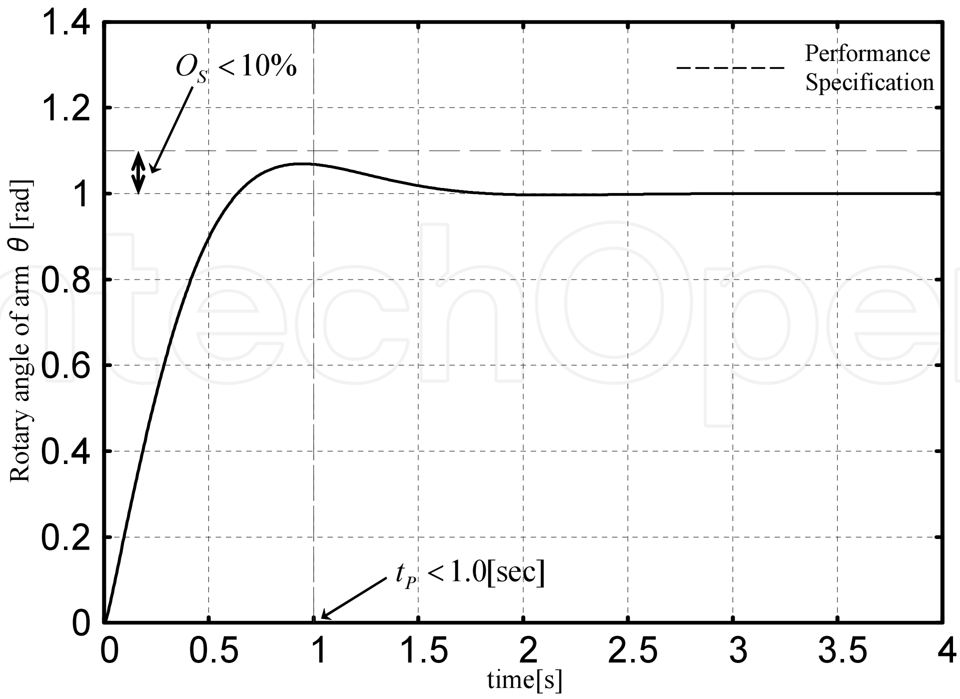

Fig. 3. Simulation result for $\bar{P}(s)$

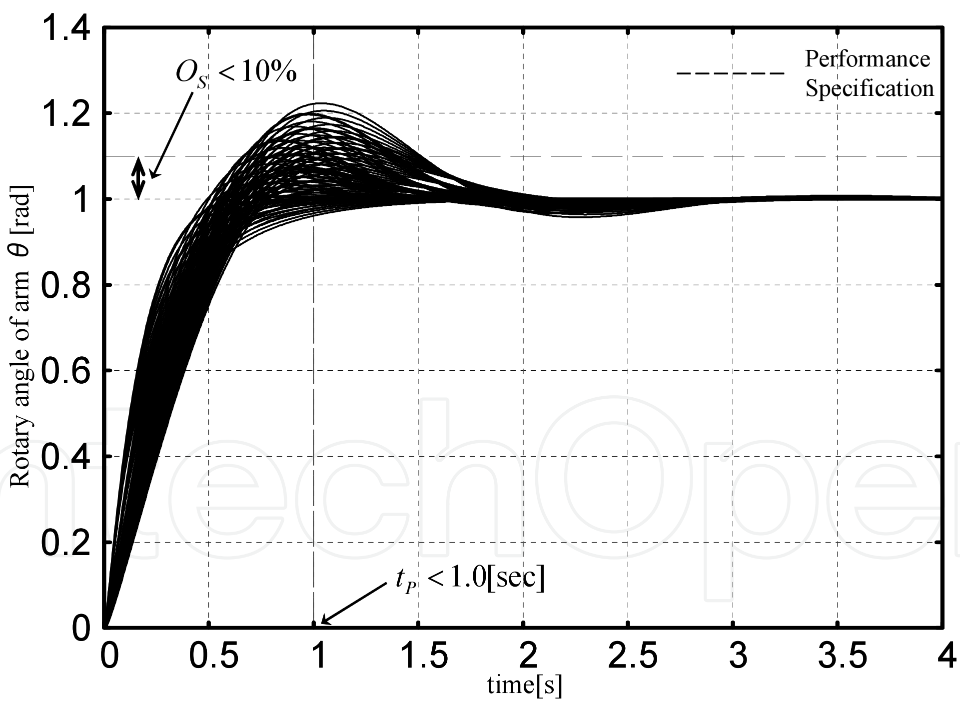

Fig. 4. Simulation result for $\mathcal{P}(s)$

4.1 Performance specification

Here, performance specifications in time domain is determined as

$$\text{Overshoot}: \; O_s = 0 \sim 10\%, \tag{10}$$

$$\text{Settling time}: \; t_s < 2.0\text{sec}. \tag{11}$$

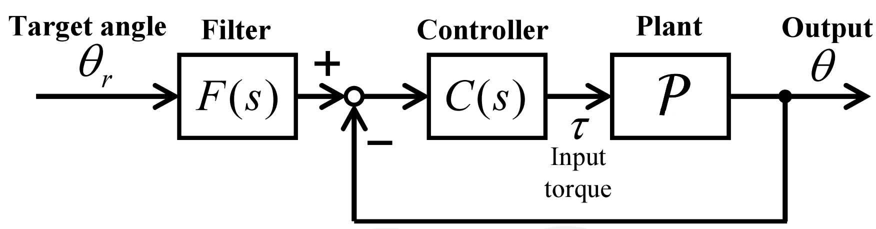

Fig. 5. QFT Control System

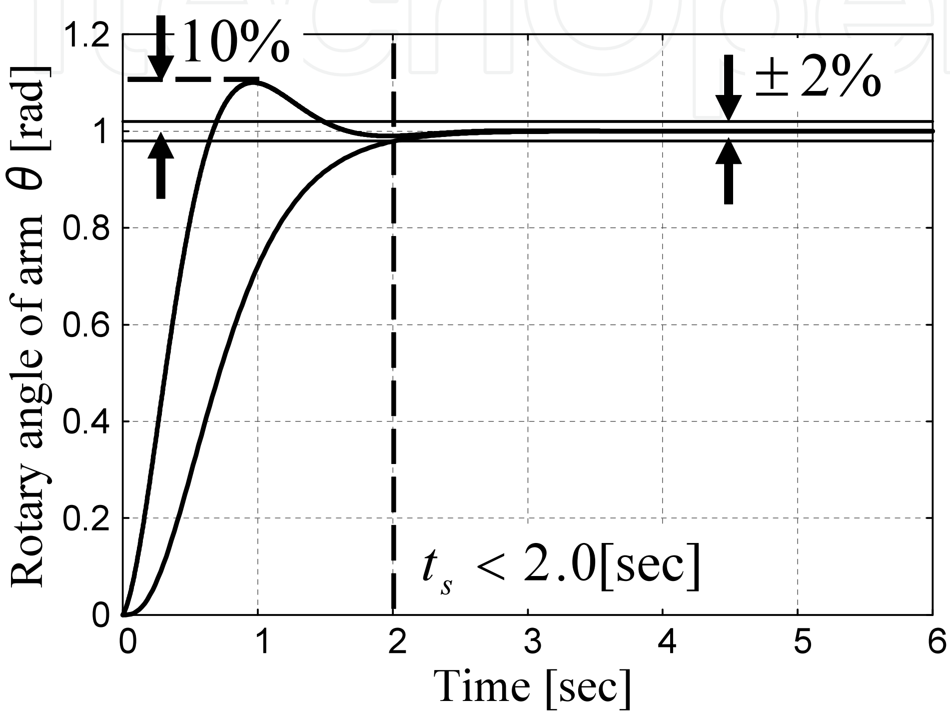

Fig. 6. Time domain performance specification

In QFT, more concrete specification is constructed so as to have certain width by determining upper and lower bounds according to Eqs.(10) and (11). Transfer function T_{R_U} and T_{R_L} that represents upper and lower bounds respectively, is designed as follows:

$$T_{R_U}(s) = \frac{0.5(s+30)}{s^2+4.6148s+15.2}, \tag{12}$$

$$T_{R_L}(s) = \frac{52.8}{(s+3.3)(s+4)^2}. \tag{13}$$

The time and frequency responses of these functions are shown in Figs.6 and 7.

4.2 Template

One of the features of QFT is to describe the plant as a contour. On NC, a plant is expressed as a point at certain frequency. If parameters of the plant model vary, the plant is expressed on NC as contour for each frequency. It is referred to as a *template*. Therefore the template represents the region where the plant can exist. So the designer can recognize uncertainty in plant visually from templates. Fig.8 shows templates of $P(s)$ with variations of J and D at each frequency.

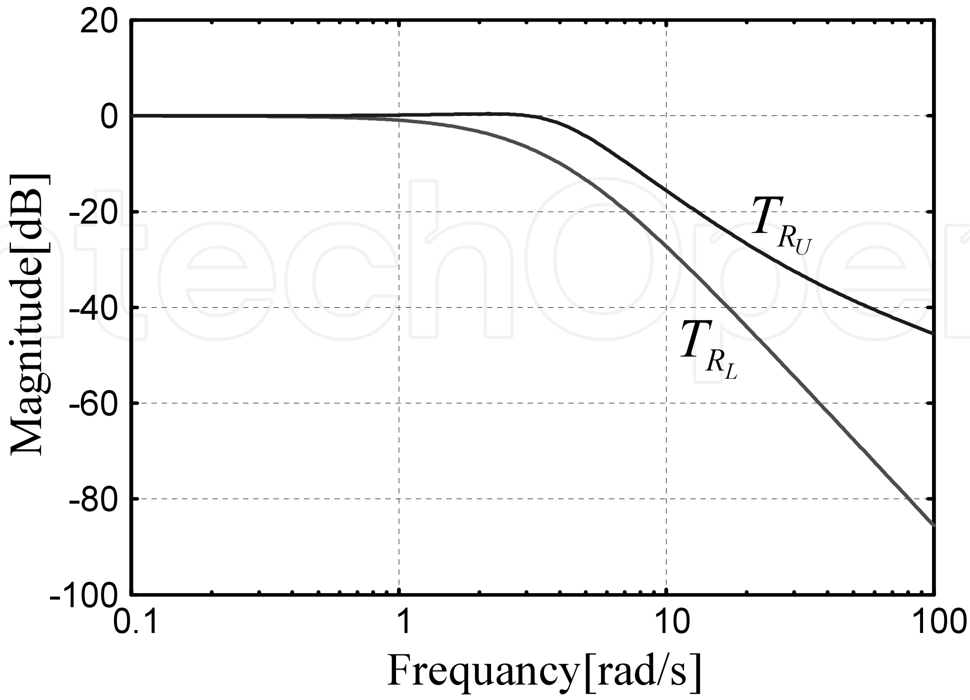

Fig. 7. Frequency domain performance specification

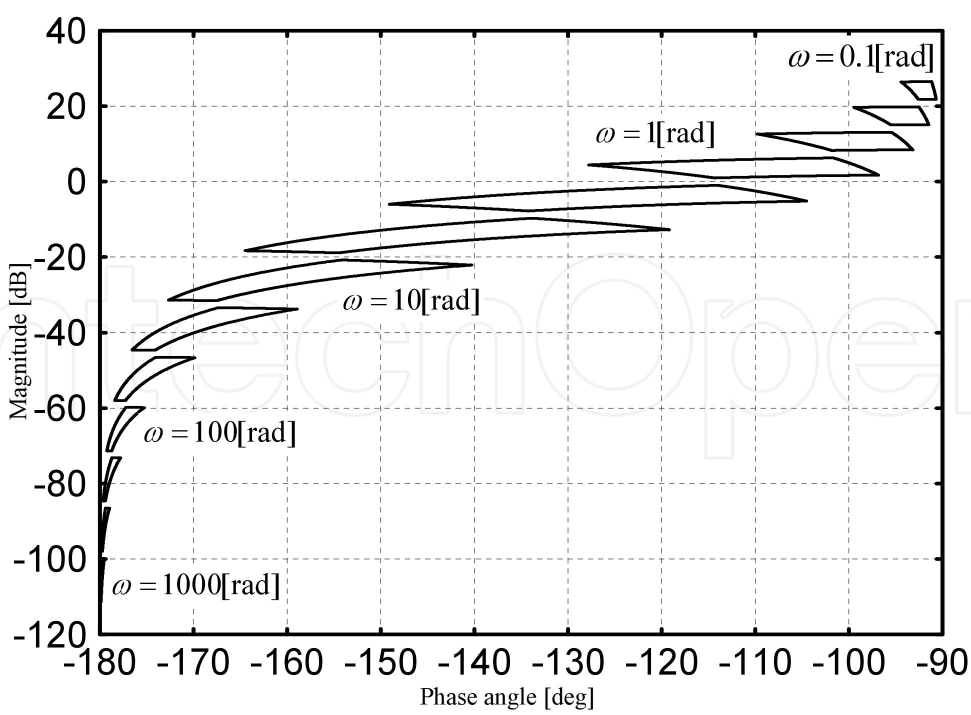

Fig. 8. Plant's template on NC

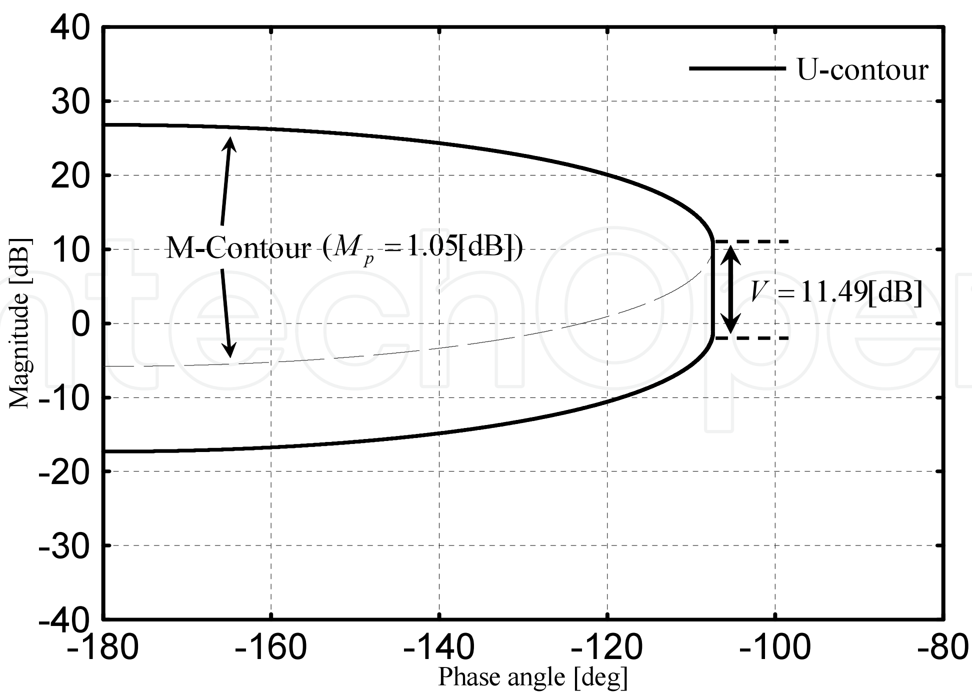

Fig. 9. U-contour on NC

4.3 Bound on NC

Let us determine performance specification on NC. As one of the features of NC, we can obtain a peak gain of a closed loop system by plotting gain-phase curve of open loop transfer function on NC. This peak gain is given by constant M curve to which gain-phase curve is tangent, called M-contour. Therefore, if peak gain M_p(dB) of performance specification in frequency domain is given, M-contour representing M_p can be obtained on NC, which indicate performance specification about the peak gain. So this specification can be satisfied unless gain-phase curve of open loop transfer function $C(s)P(s)$ does not enter this M-contour. However, gain-phase curve of $C(s)P(s)$ is represented as certain region if plant model $P(s)$ has parametric uncertainty, as stated in former section. So consider that we take down lower half of M-contour by V(dB), which is maximum variation of templates in bandwidth frequency, to make gain margin. The region constructed by M-contour and gain margin is called *U-contour*. If gain-phase curve of $C(s)P(s)$ does not enter U-contour, it means that performance specification about the peak gain is satisfied for all varying plant $\mathcal{P}$.

This time, peak gain of frequency domain performance specification, M_p, and maximum variation of templates, V(dB) take the values

$$M_p = 1.05\text{dB}, V = 11.49\text{dB}. \tag{14}$$

Then, U-contour has form as shown in Fig.9.

4.4 Loop shaping

From upper specification in Eq.(12), the main object of controller design is written as

$$\left| \frac{CP}{1+CP} \right| < M_p = 1.05\text{dB}. \tag{15}$$

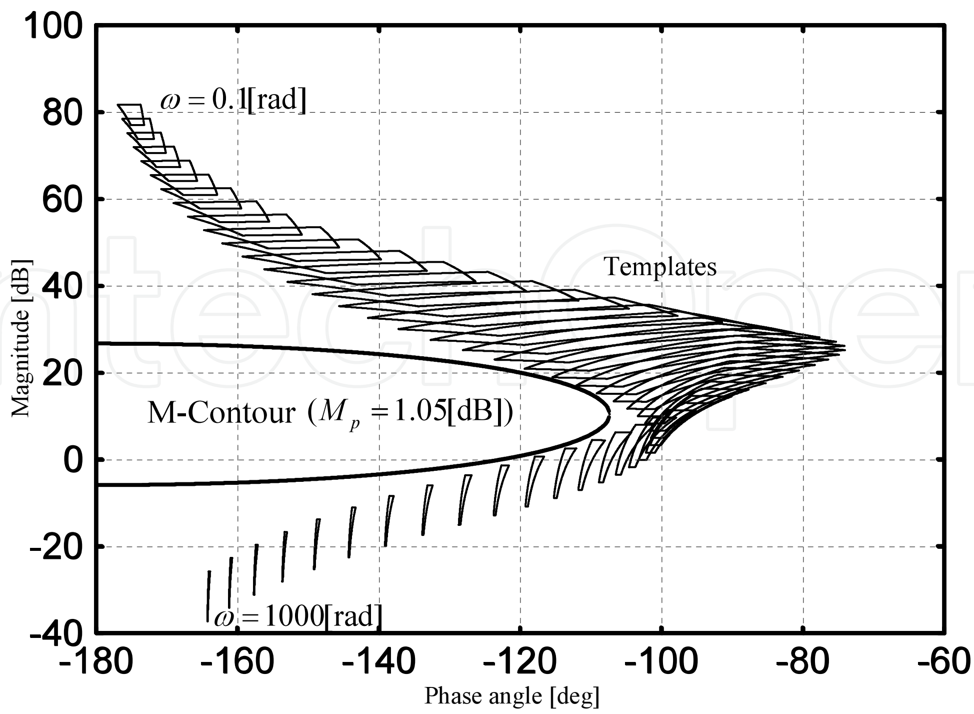

Fig. 10. M-contour and templates on NC

M_p is a peak magnitude of closed loop transfer function. This specification is described on NC as bound, called as U-contour (See Fig.9). As stated in Sec.4.3, this bound means the region that the gain-phase curve of open loop transfer function $C(s)P(s)$ must not enter to satisfy Eq.(15) for all possible plant. $C(s)$ is designed by trial and error in order not to penetrate U-contour. This time, $C(s)$ is consist of integrator and phase-delay compensator. Then, $C(s)$ has been decided as

$$C(s) = 5.2 \cdot 10^3 \frac{(s+1)(s+3.2)}{s(s+285)}. \tag{16}$$

Fig.10 shows templates of $C(s)\mathcal{P}(s)$ and M-contour representing M_p. From this figure, it is found that all templates do not enter the M-contour. It means that the system including $\mathcal{P}(s)$ satisfies Eq.(15).

4.5 Design of $F(s)$

In the final step, filter $F(s)$ is designed so that transfer function from input to output

$$\frac{F(s)C(s)\mathcal{P}(s)}{1 + C(s)\mathcal{P}(s)}$$

lies between performance specifications T_{R_U} and T_{R_L}. Then, $F(s)$ was decided as

$$F(s) = \frac{(s/25+1)^2}{(s/3+1)(s/4+1)}. \tag{17}$$

5. Simulation

In this section, based on numerical simulation, control performance of QFT control system designed in former section are compared with conventional PID control system. We consider 70 samples of set of J and D in plant dynamics. As a conventional controller to be compared, the following PID controller

$$C_P(s) = 1.3 + \frac{0.001}{s} + \frac{0.8}{1 + 0.001s} \tag{18}$$

is adopted. The target rotary angle $\theta_r(t)$ is 1 rad and computational time step is 0.005s. The simulation results for two control system are shown in Figs.11 and 12. We found that scattering of the output $\theta(t)$ due to the parametric uncertainty are suppressed, and all $\theta(t)$ lies between upper and lower performance specifications, while the PID controller can not cope with parametric uncertainty. Now let us consider settling time t_s[s] and overshoot O_s[rad] as performance indices and use standard deviation to assess the scattering of these indices. The results are shown in Table 2. From this table, it is found that standard deviation of t_s[s] is reduced to 1/80 and that of O_s[s] is decreased to 1/7, compared with PID.

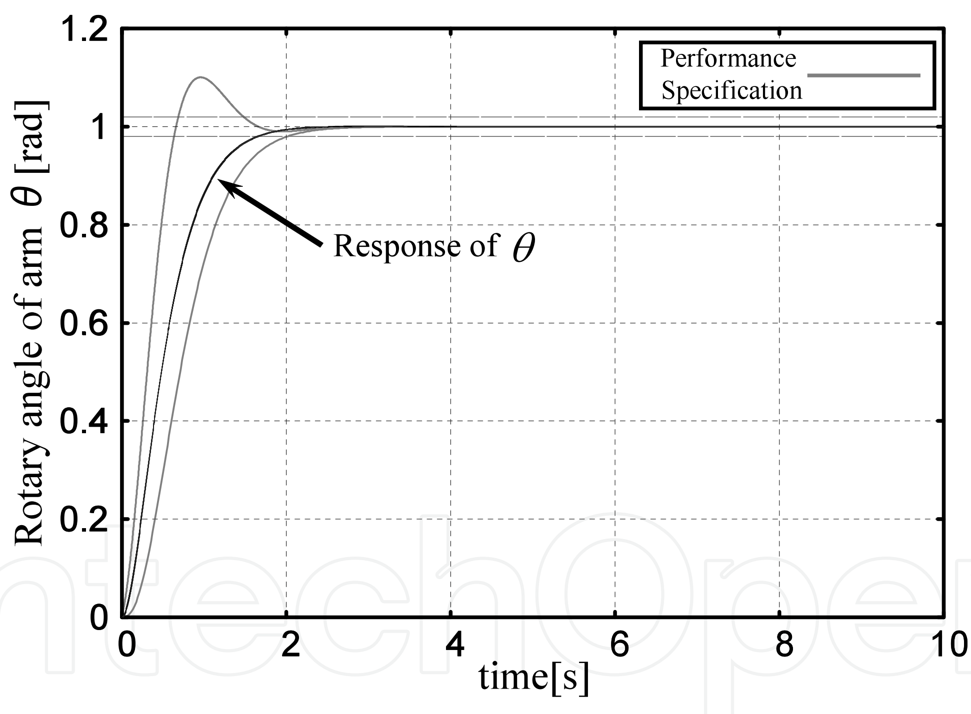

Fig. 11. Simulation result with QFT

	PID	QFT
t_s[s]	64.0×10^{-2}	0.875×10^{-2}
O_s[rad]	1.95×10^{-3}	0.278×10^{-3}

Table 2. Standard deviation of t_s and O_s

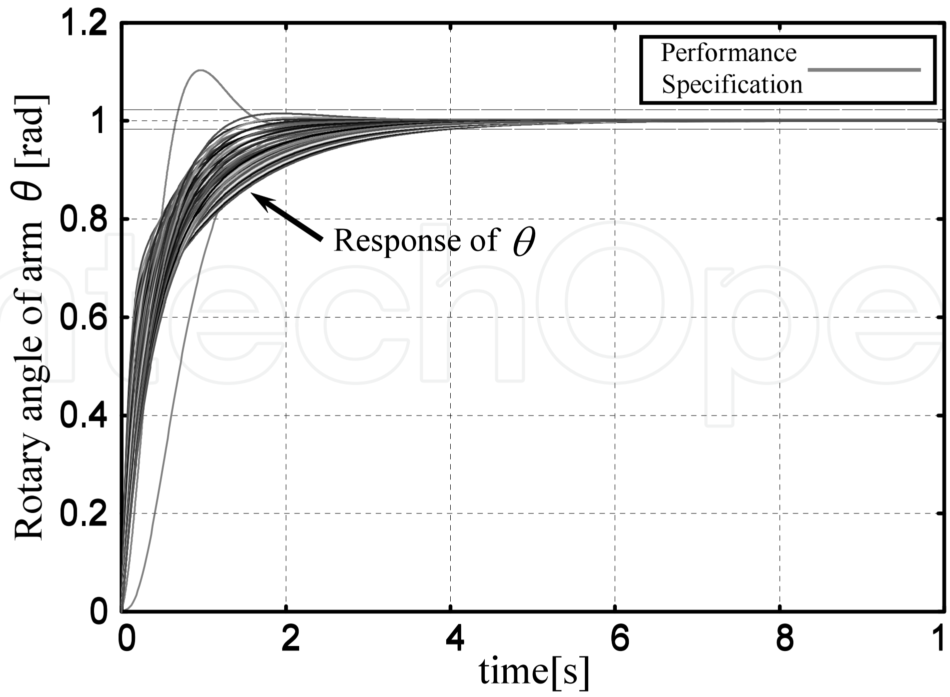

Fig. 12. Simulation result with PID

6. Conclusion

We presented a controller design for a one link arm with parametric uncertainty. In condition that there is uncertainty in plant's dynamics, the desirable control performance may not be obtained because the controller is designed based on only nominal model of plant. In this chapter, robust controller based on QFT is designed for plant with parametric uncertainty. By numerical experiments, systems designed by using QFT and conventional method are compared. It is found that the control system designed by QFT shows robust performance and can suppress the undesirable output due to parametric uncertainty.

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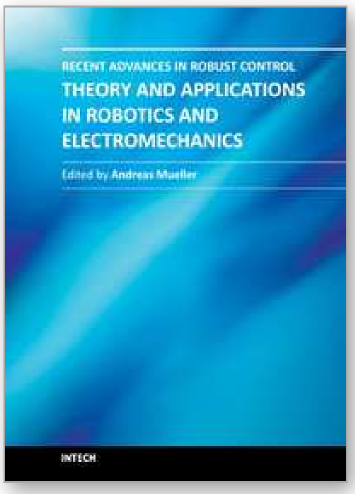

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