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Optimizing the Tracking Performance in Robust Control Systems

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1. Introduction

A typical control engineering problem deals with the design of a control system subject to closed-loop stability and certain performance requirements. The requirements may include the figures of merit such as gain/phase margin, bandwidth, and tracking error to a reference command. The control system is required to achieve the design objectives against unknown or unmeasurable disturbances. The difficulty arises since the plant is often poorly modeled and the set of performance requirements is typically stringent. The robust control theory attempts to address the question of stability and performance of multivariable systems in the face of modeling errors and unknown disturbances (Zhou et al., 1996).

In robust control theory, the question concerning the achievable performance limits is generally posed as an optimization problem in an appropriate mathematical setting. A major benefit of this approach is that it provides a means to optimize the system performance by trading off various stringent, and often conflicting, specifications against each other. In the last three decades, H_∞ control theory has evolved as the primary multivariable optimization and synthesis tool that can effectively deal with the modeling errors and unknown disturbances (Skogesttad & Postlethwaite, 2007).

In a tracking problem, the reference command is usually specified as a step or ramp signal. Accordingly, the tracking error is also specified in terms of such signals. This class of signals, however, does not model all command signals of interest. For example, a servo control system may be required to track a periodic signal of a fixed period. For this class of applications, the tracking performance must instead be specified in terms of a periodic command signal. Since every periodic signal can be represented by its Fourier series for all time, the steady state tracking performance of a linear feedback system with a periodic command signal can be studied in terms of the steady state tracking performance of each of its sinusoidal components. Design of the control systems that can track periodic reference signals falls in the category of repetitive control (Hara et al., 1998; Lee & Smith, 1998; Sugimoto & Washida, 1997). This has been an active area of research in the last three decades where many successful applications have been reported in the literature. However, applications of the results to certain high performance positioning systems have proved to be more challenging. For example, in (Broberg & Molyet, 1994) a robust repetitive control system is designed to improve the turn-around sinusoidal tracking performance of the imaging mirror system of

a weather satellite in face of stringent tracking error specifications. A similar situation has been investigated recently by (Aphale et al., 2008; Salapaka et. al, 2002) who considered a robust control design for a high bandwidth nano-positioning system.

An important step in studying the tracking performance of a control system to a sinusoidal reference signal is to investigate the inherent limitations of a feedback system. These limitations provide a deeper understanding of the problem and help a designer to evaluate his/her design against the best attainable tracking error obtained over all possible controller design. The topic been investigated thoroughly in (Su et al., 2003; 2005). The results show that the best achievable performance can be characterized in terms of the inherent properties, mainly the nonminimum phase zeros of the plant and the frequency of the reference signal.

After gaining the necessary insight into the fundamental limitations on the best achievable tracking performance, the next step is to pose the problem as an H_∞ robust performance problem. Among the various approaches reported in the literature, the mixed-sensitivity H_∞ control (Kwakernaak H., 2002), signal-based H_∞ control (Skogesttad & Postlethwaite, 2007), and H_∞ loop-shaping design (Balas et al., 998) have perhaps gained more popularity with designers. The mixed-sensitivity H_∞ design is particularly attractive as it gives the designer the ability to directly shape the sensitivity and complementary sensitivity functions. This, in turn, greatly facilitates the trade-off study among several competing performance objectives. The mixed sensitivity design is a conceptually attractive method, but how easily does it lend itself in practical applications? To apply the design, the designer starts by selecting certain weights such that the H_∞ optimal controller can provide a good trade-off between conflicting objectives in various frequency ranges. After several iterations, the designer is in a position to assess the design to see if all objectives have been met by the controller. If not, the next logical step is to go back and change the weights and repeat the process until a satisfactory result is obtained. Evidently, this is a tedious and often a long process, especially when the system dimension is high. To shorten the design cycle, it is of great interest to have a set of guidelines that can help the designer in selecting the appropriate weights in the optimization process.

The selection of optimal weights for the H_∞ control has received attention only very recently (Chiang & Hadaegh, 1994; Lanzon, 2001). In (Lanzon, 2000), the problem is formulated in such a way that the controller and the weights are obtained simultaneously and in an iterative manner. However, the question of the suitability of the weights and the complexity of the algorithm employed are yet to be judged. As an alternative, a new set of simple guidelines have been developed recently that can greatly facilitate the selection of appropriate weights (Oloomi & Shafai, 2003). These guidelines are derived using elementary arguments based on phasors and straight-line approximation of the magnitude response, and in the same spirit as what is usually done in the classical control theory. These results are simple to interpret and provide insights into the interplay among various design parameters including the peaks of the sensitivity and complementary sensitivity functions and the system bandwidth.

The chapter is outlines as follows. In Section 2, we briefly discuss the general guidelines used for the selection of the weighting functions in the mixed S/T sensitivity design. In Section 3, we study the problem of the weights selection for tracking sinusoidal reference signals and obtain certain expressions which relate the parameters of the weights to the steady state tracking error specifications. We then outline a procedure for the selection of the parameters of the weighting functions using the derived expressions. The approximate formulae obtained in this chapter are derived using elementary arguments from phasors and straight-line approximation of the magnitude response, in the same spirit as what is

usually done in the classical control theory. The results obtained are simple to interpret and provide insights into the interplay among various design parameters including the peaks of the sensitivity and complementary sensitivity functions and the system bandwidth. In Section 4, we briefly demonstrate how these results can be used to obtain the weights in a robust control mixed sensitivity design of a high bandwidth nano-positioning system. We conclude the chapter in Section 4.

2. Weights selection in general mixed sensitivity design

We initiate the discussion by considering the feedback system shown in Figure 1. Let $S(s) = 1 + G(s)K(s)$ and $T(s) = 1 - S(s)$ be the sensitivity and complementary sensitivity transfer functions, respectively. In the S/T mixed sensitivity design, the objective is to minimize the infinity norm

$$\left\| \begin{matrix} W_P S \\ W_T T \end{matrix} \right\|_\infty$$

where $W_P(s)$ and $W_T(s)$ are the performance and the stability weights, respectively (Skogesttad and Postlethwaite, 2000; Zhou et $al.$, 1996). These weights are often taken to be

$$W_P(s) = \left(\frac{s/\sqrt[m]{M_S} + \omega_B^\star}{s + \omega_B^\star \sqrt[m]{A_S}} \right)^m, \qquad W_T(s) = \left(\frac{s/\omega_{BT}^\star + 1/\sqrt[n]{M_T}}{\sqrt[n]{A_T}s/\omega_{BT}^\star + 1} \right)^n.$$

The amplitude responses of these weights and their inverses are shown in Figure 2.
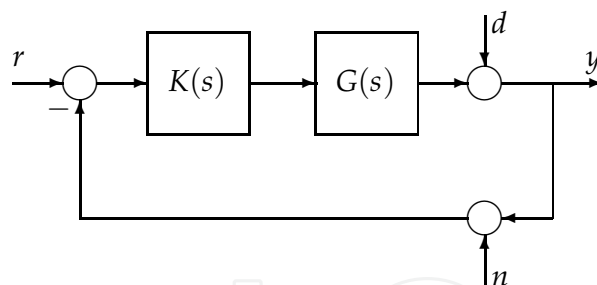

Fig. 1. One degree of freedom feedback control system.

Typically M_S and M_T are chosen to be in the interval 1.5 to 2 so that sufficient gain margin, GM, and sufficient phase margin, PM, are attained according to the inequalities

$$GM \geq \frac{M_S}{M_S - 1} \quad PM \geq 2\arcsin\left(\frac{1}{2M_S}\right)$$
$$GM \geq 1 + \frac{1}{M_T} \quad PM \geq 2\arcsin\left(\frac{1}{2M_T}\right).$$

However, larger values of M_S and M_T are unavoidable for nonminimum phase systems. Ideally, $A_S = A_T = 0$ so that $1/|W_P|$ and $1/|W_T|$ have the desirable Butterworth highpass and Butterworth lowpass characteristics. This ensures that the frequency responses of $1/|W_P|$ and $1/|W_T|$ are maximally flat in the high and low frequency ranges respectively, where they take the general shapes of the sensitivity and the complementary sensitivity functions. Although, due to the numerical difficulties (Balas et $al.$, 1998), one is often forced to set the parameters

A_S and A_T to some small non-zero values, the forgoing observations still hold true in the frequency ranges of interest. Keeping this into consideration, A_S and A_T are chosen to be sufficiently small so that poles of $1/W_P(s)$ are at least two decades above the zeros of $1/W_P(s)$, and zeros of $1/W_T(s)$ are at least two decades above the poles of $1/W_T(s)$. In general, it is required to have $A_S \ll M_S$ and $A_T \ll M_T$. Assuming that M_S, A_S, M_T, and A_T are chosen based on these observations, we now concentrate on selecting the remaining parameters of the weighting functions, namely, m, $\omega_{BT}^\star$, n, and $\omega_{BT}^\star$. General guidelines for selecting these parameters are given below (Skogesttad and Postlethwaite, 2000).
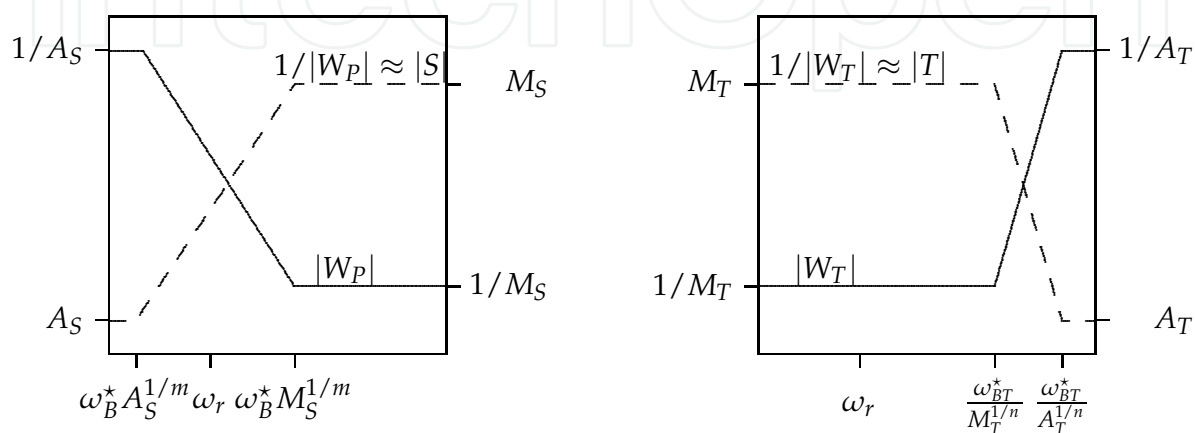

Fig. 2. Stability and performance weighting functions and their inverses.

2.1 General guidelines

1. For systems with $PM \leq 90°$, it is well known that $\omega_B \leq \omega_c \leq \omega_{BT}$ where ω_B, ω_{BT}, and ω_c are the closed loop bandwidth measured on the basis of S, the closed loop bandwidth measured on the basis of T, and the gain crossover frequency, respectively. Therefore, it is required that $\omega_B^\star \leq \omega_{BT}^\star$. It should be noted that the presence of nonminimum phase zeros places restriction on the achievable bandwidth. Moreover, for high performance tracking applications with noticeable measurement noise it often becomes necessary to make a compromise and instead choose $\omega_{BT}^\star < \omega_B^\star$.

2. When disturbance attenuation is the control objective, the general rule is to increase $\omega_B^\star$ as much as possible. However, increasing $\omega_B^\star$ more than necessary causes the appearance of a peak in the sensitivity curve. This implies that the system will have less stability margins which manifests itself in an increased overshoot in the step response.

3. When the control objective is to reduce the effect of the measurement noise, the general rule is to decrease $\omega_{BT}^\star$ as much as possible. However, decreasing $\omega_{BT}^\star$ more than necessary causes a reduction in the system bandwidth and this manifests itself in a poor tracking performance.

4. Increasing m and n can improve the disturbance rejection and measurement noise attenuation, respectively. However, m and n should be kept as low as possible since large values of these parameters adversely affect the stability margins, and the controller order becomes unnecessarily high. (Controller order is $N + n + m$ where N is the order of the plant.)

3. Weights selection for sinusoidal tracking performance

In this section, we study the tracking performance of the feedback system in Figure 1 to a sinusoidal command signal. Tracking of other periodic command waveforms can be reduced to this case since every periodic signal can be represented by its Fourier series and ω_r can be chosen to represent the highest frequency component of $r(t)$ beyond which all other components are negligible. For example, when tracking a triangular waveform, ω_r can represent the frequency of the third harmonic of $r(t)$ since higher frequency harmonics have negligible amplitudes for this signal. Thus, let us assume that $d = n = 0$ in Figure 1 and consider the sinusoidal reference command

$$r(t) = A_r \cos \omega_r t, \quad \omega_r \ll \omega_B^\star.$$

Then the sinusoidal steady state output is

$$
\begin{aligned}
y_{ss}(t) &= A_r |T(j\omega_r)| \cos(\omega_r t + \angle T(j\omega_r)) \\
&= A_r |T(j\omega_r)| \cos\left[\omega_r\left(t + \frac{\angle T(j\omega_r)}{\omega_r}\right)\right] \\
&= A_r |T(j\omega_r)| \cos\left[\omega_r(t - \tau_e)\right],
\end{aligned}
$$

where the tracking delay is given by

$$\tau_e = -\frac{\angle T(j\omega_r)}{\omega_r}.$$

This delay is an increasing function of the tracking frequency.

In tracking applications, the complementary sensitivity function is shaped so that at least up to the tracking frequency the system behaves as an all-pass filter with negligible phase shift, that is $|T(j\omega_r)| \approx 1$ and $\angle T(j\omega_r) \approx 0$. This ensures that the peak steady state error and delay are small so that $y_{ss}(t) \approx r(t)$. However, as was mentioned earlier, for high performance applications even small deviation of $y_{ss}(t)$ from the reference signal $r(t)$ may exceed the performance requirements. Thus, our objective in this chapter is to address this issue by outlining a procedure for selecting the parameters m, $\omega_B^\star$, n, and $\omega_{BT}^\star$. To this end, we first define what we mean by the steady state tracking errors.

Using basic results from trigonometry, it is readily seen that the steady state error signal

$$e_{ss}(t) = A_r \cos \omega_r t - A_r |T(j\omega_r)| \cos\left[\omega_r(t - \tau_e)\right]$$

can be written in the compact form

$$e_{ss}(t) = R_e \cos(\omega_r t + \phi_e)$$

where

$$R_e = A_r \sqrt{1 + |T(j\omega_r)|^2 - 2|T(j\omega_r)| \cos \omega_r \tau_e}, \tag{1}$$

$$\phi_e = \arctan\left(\frac{|T(j\omega_r)| \sin \omega_r \tau_e}{1 - |T(j\omega_r)| \cos \omega_r \tau_e}\right). \tag{2}$$

The result is depicted in Figure 3 where the sinusoidal components of the steady state error signal are represented as phasors in the quadrature plane with the reference axis taken as $\cos \omega_r t$. It is seen that the steady state error phasor is rotated by an angle of ϕ_e in the counter-clockwise direction due to the presence of the tracking delay τ_e, and that the peak amplitude of the steady state tracking error, namely R_e, is influenced by this rotation as well as the gain of the closed loop system at the tracking frequency ω_r. It should be noted that when $|T(j\omega_r)| \cos \omega_r \tau_e \leq 1$, this phasor resides in the first quadrant so that $\tan \phi_e > 0$. However, when $|T(j\omega_r)| \cos \omega_r \tau_e > 1$, the steady state error phasor moves to the second quadrant for which $\tan \phi_e < 0$. Therefore, in obtaining ϕ_e from $\tan \phi_e$ in the latter case, we must interpret ϕ_e as being in the second quadrant ant not in the fourth. Typical sinusoidal tracking waveforms with small peak steady state error and small delay are also shown in Figure 4 where the lead property of the steady state error signal is cleanly seen.
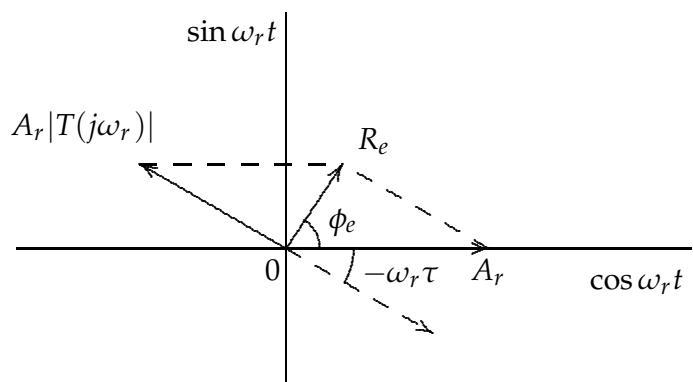

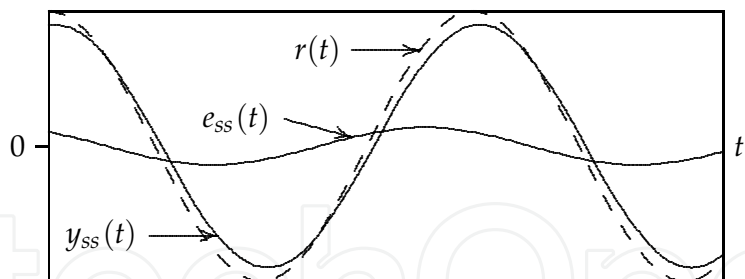Fig. 3. Phasor diagram for the steady state sinusoidal tracking error.

Fig. 4. Steady state sinusoidal tracking error signal.

We now derive expressions for the parameters of the weighting functions in terms of the tracking error parameters R_e and τ_e. To this end, recall from (Skogesttad and Postlethwaite, 2000) that in the mixed sensitivity design the weighting functions W_P and W_T are used to scale the closed loop transfer functions S and T, respectively in order to satisfy the performance and stability requirements, and that the inverse of these weighting functions are upper bounds, up to constant scaling factors, on the transfer functions they are used to scale. These constant factors can be absorbed in the weighting functions themselves so that the approximations $W_P S \approx 1$ and $W_T T \approx 1$ are reasonable for appropriate weights. However, the discrepancies can become noticeable if the controller is not designed properly or when the nonminimum

phase zeros are located near the origin for which large peaks appear in the sensitivity and complementary sensitivity response curves.

We first derive an expression for the tracking delay. To simplify notation, let

$$\alpha := \frac{\omega_r}{\omega_B^{\star}}, \qquad \beta := \frac{\omega_r}{\omega_{BT}^{\star}} \tag{3}$$

and note that $0 < \alpha, \beta \ll 1$. Using the approximation $W_T T \approx 1$, we have

$$|T(j\omega_r)| \approx \frac{1}{|W_T(j\omega_r)|}, \qquad \angle T(j\omega_r) \approx -\angle W_T(j\omega_r).$$

Therefore, using the straight line approximation

$$
\begin{aligned}
|T(j\omega_r)| &\approx \frac{1}{|W_T(j\omega_r)|} \\
&= M_T \left| \frac{1 + j\beta \sqrt[n]{A_T}}{1 + j\beta \sqrt[n]{M_T}} \right|^n \\
&\approx M_T, \qquad \text{for } \beta \ll \frac{1}{\sqrt[n]{M_T}},
\end{aligned} \tag{4}
$$

and

$$
\begin{aligned}
\tau_e &\approx \frac{\angle W_T(j\omega_r)}{\omega_r} \\
&\approx \frac{n}{\omega_r} \left[\arctan\left(\beta \sqrt[n]{M_T}\right) - \arctan\left(\beta \sqrt[n]{A_T}\right) \right].
\end{aligned} \tag{5}
$$

Next, we derive an expression for the peak steady state error. Since $W_P S \approx 1$, we have

$$|S(j\omega_r)| \approx \frac{1}{|W_P(j\omega_r)|}, \qquad \angle S(j\omega_r) \approx -\angle W_P(j\omega_r).$$

Therefore,

$$
\begin{aligned}
|S(j\omega_r)| &\approx \frac{1}{|W_P(j\omega_r)|} \\
&= A_S \left| \frac{1 + j\frac{\alpha}{\sqrt[m]{A_S}}}{1 + j\frac{\alpha}{\sqrt[m]{M_S}}} \right|^m \\
&\approx \alpha^m, \qquad \text{for } \sqrt[m]{A_S} \ll \alpha \ll \sqrt[m]{M_S}.
\end{aligned}
$$

On the other hand, $E(s) = S(s)R(s)$ so that at the steady state we also have

$$R_e \approx A_r \alpha^m. \tag{6}$$

Therefore, by equating (1) and (6) and using (4) we obtain

$$\alpha^m \approx \sqrt{1 + M_T^2 - 2M_T \cos \omega_r \tau_e}. \tag{7}$$

An expression relating (5) to (7) can now be derived noting that

$$\angle W_P(j\omega_r) \approx m \left[\arctan \left(\frac{\alpha}{\sqrt[m]{M_S}} \right) - \arctan \left(\frac{\alpha}{\sqrt[m]{A_S}} \right) \right]. \tag{8}$$

Since

$$\begin{aligned} \angle E(j\omega_r) &= \angle S(j\omega_r) + \angle R(j\omega_r) \\ &= \angle S(j\omega_r) \\ &= -\angle W_P(j\omega_r), \end{aligned} \tag{9}$$

from (2), (4), (8), and (9) we obtain

$$\arctan \left(\frac{M_T \sin \omega_r \tau_e}{1 - M_T \cos \omega_r \tau_e} \right) \approx m \left[\arctan \left(\frac{\alpha}{\sqrt[m]{A_S}} \right) - \arctan \left(\frac{\alpha}{\sqrt[m]{M_S}} \right) \right]. \tag{10}$$

Expressions (5), (6), (7), and (10) are the basic expressions to be used in the selection of the weighting functions. In order to gain insight into the relationships among various parameters involved in these equations, we make further simplifications by noting that A_S, A_T, α and β are small positive numbers. Thus, by neglecting appropriate terms, these equations reduce to

$$\omega_B^\star \approx \omega_r \left(\frac{A_r}{R_e} \right)^{\frac{1}{m}}, \tag{11}$$

$$M_T \approx \cos \omega_r \tau_e + \sqrt{ \left(\frac{\omega_B^\star}{\omega_r} \right)^{2m} - \sin^2 \omega_r \tau_e }, \tag{12}$$

$$\omega_{BT}^\star \approx \frac{\omega_r \sqrt[n]{M_T}}{\tan \left(\frac{\omega_r \tau_e}{n} \right)}, \tag{13}$$

$$M_S \approx \left[\frac{\omega_r}{\omega_B^\star \left| \tan \left(\frac{m\pi}{2} - \gamma \right) \right|} \right]^m, \qquad (\gamma \neq m\pi/2) \tag{14}$$

where

$$\gamma = \arctan \left(\frac{M_T \sin \omega_r \tau_e}{1 - M_T \cos \omega_r \tau_e} \right). \tag{15}$$

Note that $\omega_B^\star \gg \omega_r$ so that (12) is well defined. For (13), we have used the trigonometric identity $\tan(x - y) = (\tan x - \tan y)/(1 + \tan x \tan y)$ to obtain the quadratic equation

$$\sqrt[n]{M_T A_T} \tan \left(\frac{\omega_r \tau_e}{n} \right) \beta^2 + \left(\sqrt[n]{A_T} - \sqrt[n]{M_T} \right) \beta + \tan \left(\frac{\omega_r \tau_e}{n} \right) = 0,$$

and then have set $A_T \approx 0$. Derivation of the remaining equations is straightforward. When $m = 1$, (14) and (15) can be combined using the trigonometric identity $\tan(x - y) = (\tan x - \tan y)/(1 + \tan x \tan y)$ resulting in

$$M_S \approx \frac{\omega_r M_T \sin \omega_r \tau_e}{\omega_B^\star |1 - M_T \cos \omega_r \tau_e|}, \qquad (M_T \cos \omega_r \tau_e \neq 1). \tag{16}$$

3.1 Guidelines for sinusoidal tracking performance

Assume that A_r, ω_r, and the upper bounds on the tracking errors R_e and τ_e are specified. Further, assume that the parameters A_S and A_T are chosen to be some small positive numbers on the basis of our earlier guidelines. A procedure for selecting the remaining parameters of the weighting functions $W_P(s)$ and $W_T(s)$ are given below assuming that R_e and τ_e are the only specifications to be dealt with.

1. Initially, let $m = 1$ and calculate $\omega_B^\star$ from (11). If this value is too large, increase m and re-calculate $\omega_B^\star$.

2. Calculate M_T from (12) using the values of m and $\omega_B^\star$ obtained in Step 1.

3. Let $n = 1$ and calculate $\omega_{BT}^\star$ from (13) with the values of m, $\omega_B^\star$, and M_T calculated in Steps 1 and 2. If $\omega_{BT}^\star$ is not large enough, increase n and recalculate $\omega_{BT}^\star$ from (13) till a satisfactory result is obtained.

4. Finally, calculate M_S from (14) and (15), or from (16) if $m = 1$, using the values of $\omega_B^\star$, m, and M_T calculated in Steps 1 and 2.

4. Application

The importance of nanotechnology has been brought to full attention by the scanning probe microscopy and is the result of new techniques used to explore the properties of near atomic-scale structure (Aphale et al., 2008; Barrett & Quate, 1991; Teoh et al., 2008). However, most schemes of nanotechnology impose severe specifications on positioning systems, making the control system design more challenging. For example, micro/nano positioning systems are essential in auto focus systems, fast mirror scanners, image steering devices in optics; disk spin stands and vibration cancelation in disk drives; wafer and mask positioning in microelectronics; micropumps, needle valve actuation, linear drives, and piezo hammers in precision mechanics; and cell penetration and microdispensing devices in medicine and biology (Daniele et al., 1999; Salapaka et. al, 2002; Tamer & Dahleh, 1994).

In (Salapaka et. al, 2002), a mixed sensitivity robust control has been successfully applied to a noano-poistioning device, suited to biological samples as part of an atomic force microscope, where it is shown that substantial improvement in the positioning and precision is attainable over the conventional PI control. The improvement reported in this chapter is judged on the basis of the system ability to track a "high frequency" triangular reference waveform with a small peak error (in order of micro-meter) and a small delay (in order of milli-seconds). However, it is notable that the success of the design reported in (Salapaka et. al, 2002), as well as other mixed sensitivity designs, depends largely on the appropriate selection of the weights used in the optimization process. While for typical applications appropriate weights are often easily chosen after several trials and errors, the stringent performance requirements imposed for the ultra-high performance applications makes the selection of appropriate weights difficult, or at least time-consuming.

In the last section, we derived certain approximate expressions in terms of the tracking performance specifications and provided a guideline for the selection of the weights in the mixed sensitivity design. These expressions should prove valuable to the designer as they expedite the weights selection process in the simulation/design cycle. In order to demonstrate the usefulness of the guideline, consider the mixed sensitivity robust control design for a high

bandwidth nano-positioning system as discussed in (Salapaka et. al, 2002). A model of the device obtained experimentally is a fourth order nonminimum phase transfer function

$$G(s) = \frac{9.7 \times 10^4 (s - (7.2 \pm 7.4j) \times 10^3)}{(s + (1.9 \pm 4.5j) \times 10^3)(s + (1.2 \pm 15.2j) \times 10^2)}.$$

The design considered is a mixed S/T/KS design where the weight on the controller transfer function KS is chosen to be $W_u = 0.1$ in order to restrict the magnitude of the input signal within the saturation limit. The other weights chosen are

$$W_P(s) = \frac{0.1667s + 2827}{s + 2.827}, \qquad W_T(s) = \frac{s + 235.6}{0.01s + 1414}.$$

A simulation result presented in this chapter which shows a sinusoidal tracking response with $R_e \approx 1(\mu m)$ and $\tau_e \approx 2$ [msec] when system is subjected to a 100 [Hz] command signal with peak value of 5 [μm]. From the selected weights, it is seen that

$$m = 1, \ \omega_B^\star = 2827, \ M_S \approx 36, \ A_S = 10^{-6},$$
$$n = 1, \ \omega_{BT}^\star = 1414, \ M_T \approx 36, \ A_T = 10^{-4}.$$

We like to demonstrate how the initial weights can be obtained using the expressions derived earlier. Starting with $m = 1$, $A_r/R_e = 5$, and $\omega_r \approx 628$ [rad/sec], we obtain $\omega_B^\star \approx 3142$ [rad/sec] which is not too far from the given value of 2827 [rad/sec]. Since $\omega_r \tau_e \approx 70.4$ and $\omega_B^\star/\omega_r \approx 5$, equation (12) gives $M_T \approx 5.245$ which is better than the one chosen in (Salapaka et. al, 2002). With the calculated values and from (13) we next obtain $\omega_{BT}^\star \approx 1176$ [rad/sec] which is again not too far from the given value in (Salapaka et. al, 2002). Finally, from (16) we obtain $M_S \approx 1.5$ which is lower than what is considered in that chapter. Therefore, we see that while $\omega_B^\star$ and $\omega_{BT}^\star$ are fairly close in the first try, the values of M_S and M_T are considerably lower. This is however expected since large values of M_S and M_T are unavoidable here due to the presence of a complex pair of RHP zeros (Su et al., 2003; 2005).

In conclusion, we see that using the expressions derived in this chapter, a designer can start off with a fairly reasonable set of parameters and further adjust these parameters for the desired performance. Additionally, if larger values of M_T and M_S are to be allowed, the derived expressions can be used to see how these changes affect the remaining parameters like $\omega_B^\star$ and $\omega_{BT}^\star$. For example, it is seen from (12) that a larger M_T is obtained at the expense of a larger value for $\omega_B^\star$. From (13), this in turn implies a larger value for $\omega_{BT}^\star$ as well, and the same can be said for M_S form (14) and (15). In summary, the values obtain from the derived expressions in this chapter can form the basis of the first try in the simulation and as such should prove valuable to the designers.

5. Conclusion

In this chapter, the mixed sensitivity robust tracking problem of a feedback system with sinusoidal command waveforms is studied. Approximate expressions relating the tracking errors specifications to various parameters of the weighting functions used in the mixed S/T sensitivity design are derived. The derivation presented in this chapter uses simple arguments using phasors and straight line approximation of magnitude response. We have outlined guidelines for the selection of the weighting functions parameters using the derived

expressions. Application of the results in minimizing the tracking errors of a nano-positioning system is demonstrated.

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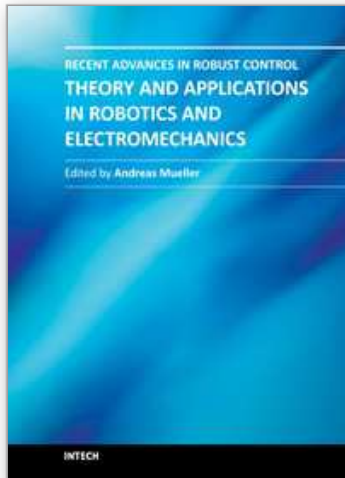
Recent Advances in Robust Control - Theory and Applications in Robotics and Electromechanics

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Robust control has been a topic of active research in the last three decades culminating in H_2/H_\infty and \mu design methods followed by research on parametric robustness, initially motivated by Kharitonov's theorem, the extension to non-linear time delay systems, and other more recent methods. The two volumes of Recent Advances in Robust Control give a selective overview of recent theoretical developments and present selected application examples. The volumes comprise 39 contributions covering various theoretical aspects as well as different application areas. The first volume covers selected problems in the theory of robust control and its application to robotic and electromechanical systems. The second volume is dedicated to special topics in robust control and problem specific solutions. Recent Advances in Robust Control will be a valuable reference for those interested in the recent theoretical advances and for researchers working in the broad field of robotics and mechatronics.

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