We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900
Open access books available

186,000
International authors and editors

200M
Downloads

Our authors are among the

154
Countries delivered to

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

CLARIVATE ANALYTICS
BOOK CITATION INDEX
INDEXED

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Design of Robust Policies for Uncertain Natural Resource Systems: Application to the Classic Gordon-Schaefer Fishery Model

Armando A. Rodriguez[1], Jeffrey J. Dickeson[2],
John M. Anderies[3] and Oguzhan Cifdaloz[4]
Arizona State University
USA

1. Introduction

Introduction. A critical challenge faced by sustainability science is to develop robust strategies to cope with highly uncertain social and ecological dynamics. The increasing intensity with which human societies utilize (limited) natural resources is fueling the global debate and urging the development of resource management methodologies/policies to effectively deal with very demanding socio-bio-economical issues. Unfortunately, despite concerted efforts by governments, many natural resources continue to be poorly managed. The collapse of many fisheries worldwide is the most notable example (Clark, 2006; Clark et al., 2006; Holland, Gudmundsson; Myers, Worm 2003; Sethi et al., 2005) but other examples include forests (Moran, Ostrom), groundwater basins (Shah, 2000), and soils (ISRIC, 1990). The suggested causes are varied but (Clark, 2006) highlights two: (1) lack of consideration of economic incentives actually faced by economic agents and (2) uncertainty associated with the dynamics of biological populations. In the case of fisheries, Clark notes that "complexity and uncertainty will always limit the extent to which the effects of fishing can be understood or predicted" (Clark, 2006, p. 98). This suggests that we need policies capable of effectively managing natural resource systems despite the fact that we understand them poorly at best.

Real-World Management Issues. Real-world resource management must address three components: goal setting, practical (robust) implementation, and learning. Clark and others (Clark, 2007; 2006; Clark et al., 2006) have recently noted that practical implementation issues are frequently at the root of fishery management failures. For most fisheries, the necessary institutional contexts exist (Wilen, Homans) and we know what to do, yet management efforts fail. This suggests a need to focus on the actual *process* of resource management. For example, how can managers make decisions with incomplete information concerning how the resource and the resource users will respond to management actions?

---

[1]Electrical Engineering, Ira A. Fulton School of Engineering
[2]Electrical Engineering, Ira A. Fulton School of Engineering
[3]School of Human Evolution and Social Change, School of Sustainability
[4]ASELSAN, Inc. Microelectronics, Guidance and Electro-Optics Division, Turkey

When managers can't learn fast enough, yet still must make decisions, how should they proceed?

Stochastic Optimization. A common approach to such policy[1] problems is stochastic optimization. Examples include studies of the performance of management instruments in the face of a single source of specific uncertainty such as in the size of the resource stock (Clark, Kirkwood; Koenig, 1984), the number of new recruits (Ludwig, Walters; Weitzman, 2002), or price (Andersen, 1982). Unfortunately, because they require assigning probabilities to possible outcomes, the insights from stochastic optimization techniques can be somewhat restricted. As Weitzman puts it, "The most we can hope to accomplish with such an approach is to develop a better intuition about the direction of the pure effect of the single extra feature being added...when the rest of the model is isolated away from all other forms of fisheries uncertainty" (Weitzman, 2002, p. 330). Such models generate interesting insights regarding how uncertain resources *should* be managed, but they contribute little to improving actual resource management practice. In our presentation, we attempt to provide some guidance through the development and application of a set of tools for practical (robust) policy implementation decisions in situations with multiple sources of uncertainty. While our approach is fundamentally deterministic, we show how probabilistic information can be accommodated within our framework.

Literature Survey. Several different threads concerning practical policy implementation challenges have emerged in the literature. Adaptive management (Walters, 1986) and resilience-based management (Holling, Gunderson; 1986; 1973; Ludwig et al., 1997) are examples from ecology. In parallel, robust control ideas from engineering (Zhou, Doyle) have begun to permeate macroeconomics (Hansen, Sargent; Kendrick, 2005) and there is recent work on resource management problems in the engineering literature (Belmiloudi, 2006; 2005; Dercole et al., 2003). A concept of robust optimization has also been developed in the operations research and management science literature (Ben-Tal, Nemirovski; Ben-Tal et al., 2000; Ben-Tal, Nemirovski) with some specific applications of these ideas to environmental problems (Babonneu et al., 2010; Lempert et al., 2006; 2000). The overarching theme of robust optimization is to select the best solution from those "immunized" against data uncertainty, i.e. solutions that remain feasible for all realizations of the data (Ben-Tal, Nemirovski).

Our Approach: Exploiting Concepts from Robust Control. This chapter presents a sensitivity-based robustness-vulnerability framework for the study of policy implementation in highly uncertain natural resource systems in which uncertainty is characterized by parameter bounds (not probability distributions). This approach is motivated by the fact that probability distributions are often difficult to obtain. Despite this, it is shown how one might exploit distributions for uncertain model parameters within the presented framework. The framework is applied to parametric uncertainty in the classic Gordon-Schaefer fishery model to illustrate how performance (income) can be sacrificed (traded-off) for reduced sensitivity, and hence increased robustness, with respect to model parameter uncertainty. Our robustness-vulnerability approach provides tools to systematically compare policy uncertainty-performance properties so that policy options can be systematically discussed.

More specifically, within this chapter, we exploit concepts from robust control in order to analyze the classic Gordon-Schaefer fishery model (Clark, 1990). Classic maximization of net present revenue is shown to result in an optimal control law that exhibits limit

---

[1] We use the terms "policies" and "control laws" interchangeably in this presentation.

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

417

cycle behavior (nonlinear oscillations) when parametric uncertainty is present. As such, it cannot be implemented in practice (because of prohibitively expensive switching costs). This motivates the need for robust policies that (1) do not exhibit limit cycle behavior and (2) offer performance (returns) as close to the optimal perfect information policy as model parameter (and derived fishery biomass target) uncertainty permits. Given the state of most world fisheries, our presentation focuses on a fishery that is nominally (i.e. believed to be) biologically over exploited (BOE); i.e. the optimal equilibrium biomass lies below the maximum sustainable yield biomass (Clark, 2006; Clark et al., 2006; Clark, 1990; Holland, Gudmundsson; Myers, Worm 2003; Sethi et al., 2005). By so doing, we directly address a globally critical renewable resource management problem. As in our prior work (Anderies et al., 2007), (Rodriguez et al., 2010), we do not seek "a best policy." Instead, we seek families of policies that are robust with respect to uncertainties that are likely to occur. Such families can, in principle, be used by a fishery manager to navigate the many tradeoffs (biological, ecological, social, economic, political) that must be confronted. More specifically, our effort to seek robust performance focuses on reducing the worst case downside performance; i.e. maximizing returns when we have the worst case combination of parameters. Such worst case (conservative) planning is critical to avoid/minimize the possibility of major regional/societal economical shortfalls; case in point, the recent "Great Recession." It is important to note that the simplicity of our model (vis-a-vis our performance objective of maximizing the net present value of returns) permits us to readily determine the worst case combination of model parameters (i.e. growth rate, carrying capacity, catchability, discount rate, price, cost of harvesting). Given this, we seek control laws that do not exhibit limit cycle behavior and whose returns are close (modulo limitations imposed by uncertainty) to that of the worst case perfect information optimal control policy - the best we could do in terms of return if we knew the parameters perfectly. Other design strategies are also examined; e.g. designing for the best case set of parameters. "Blended strategies" that attempt to do well for the worst case downside perturbation (i.e. minimize the economic downside) as well as the best case upside perturbation (i.e. maximize the economic upside) are also discussed. Such strategies seek to flatten the return-uncertainty characteristics over a broad range of likely parameters. The above optimal control (derived) policies are used as performance benchmarks/targets for the development of robust control policies. While our focus is on bounded deterministic parametric uncertainty, we also show how probability distributions for uncertain model parameters can be exploited to help in the selection of benchmark (optimal) policies. After targeting a suitable optimal (benchmark) policy, we show how robust policies can be used to approximate the benchmark (as closely as the uncertainty will permit) in order to achieve desired performance-robustness-vulnerability tradeoffs; e.g. have a return that is robust to worst case parameter perturbations.

While the presentation is intended to provide an introduction into how concepts from optimal and robust control may be used to address critical issues associated with renewable resource management, the presentation also attempts to shed light on challenges for the controls community. Although the presentation builds on the prior work presented in (Anderies et al., 2007), (Rodriguez et al., 2010), the focus here is more on defining the problem, describing the many issues, and sufficiently narrowing the scope to permit the presentation of a design methodology (framework) for robust control policies.

Finally, it must be noted that the robust policies that we present are not intended to be viewed as final policies to be implemented. Rather, they should be viewed as policy targets -

providing guidance to resource managers for the development of final implementable policies (based on taxes, quotas, etc. (Clark, 1990, Chapter 8)) that will (in some sense) approximate our robust policies. While our focus has been on parametric uncertainty, it must be noted that robustness to unmodeled dynamics (e.g. lags, time delays) is also important. While some discussion on this is provided, this will be examined in future work.

Contributions of Work. The main contributions of this chapter are as follows:

- *Benefits of Robust Control in Renewable Resource Management.* The chapter shows how robust control laws can be used to eliminate the limit cycle behavior of the optimal control law while increasing robustness to parametric uncertainty and achieving a return that is close (modulo limitations imposed by uncertainty) to the perfect information optimal control law. Special attention is paid to minimizing worst case economic downside. As such, the policies presented shed light on fundamental performance limitations in the presence of (parametric) uncertainty. The policies presented are intended to serve as targets/guidelines that fishery managers may try to approximate using available tools (e.g. taxes, quotas, etc. (Clark, 1990, Chapter 8).

- *Tutorial/Introductory Value.* The chapter serves as an introduction for the controls community to a very important resource management problem in the area of global sustainability. As such, the chapter offers a myriad of challenging problems for the controls community to address in future work.

Organization of Chapter. The remainder of the chapter is organized as follows.

- Section 2 describes the classic Gordon-Schaefer nonlinear fishery model (Clark, 1990) to be used.

- Section 3 describes the optimal control law and its properties. The latter motivates the need for robust control laws for fishery management - laws that try to achieve robust near optimal performance in some sense.

- Section 4 describes a class of robust control laws to be examined.

- Section 5 contains the main results of the work - comparing the properties of the optimal policy to those of the robust policies being considered.

- Finally, Section 6 summarizes the chapter and presents directions for future research.

2. Nonlinear bioeconomic model

In this section, we describe the nonlinear bioeconomic model to be used for control design. The model is then analyzed.

2.1 Description of bioeconomic model

The nonlinear Gordon-Schaefer bioeconomic model (Clark, 1990; Gordon, 1954; Schaefer, 1957) is now described.

Nonlinear Gordon-Schaefer Bioeconomic Model.

The nonlinear model to be used is as follows:

$$\dot{x} = F(x) - qxu_p \qquad x(0) = x_o, \tag{1}$$

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

419

where

$$F(x) = rx \left(1 - \frac{x}{k}\right) \tag{2}$$

represents the natural regeneration rate of the resource and x, x_o, and u_p represent resource biomass, initial resource biomass, and harvesting effort, respectively. The parameters r, k, and q, retain their traditional definitions of intrinsic growth rate, carrying capacity, and catchability, respectively. Table 1 in Section 2.5 summarizes model parameter definitions, units, nominal values, and ranges. Model uncertainty will be addressed in Section 2.6.

Saturating Nonlinearity. Typically, effort is bounded above by some maximum and below by zero, i.e. $u_p \in [0, u_{\max}]$. Typically, this physical constraint is implicitly taken into account when the optimal control problem is solved. However, a more general family of controls may generate control signals outside the allowable range, and it is important to be explicit about how these signals are "clipped" by physical constraints. We thus define the saturation function

$$\text{sat}(x; x_{\min}, x_{\max}) \stackrel{\text{def}}{=} \begin{cases} x_{\min} & -\infty < x < x_{\min} \\ x & x_{\min} \leq x \leq x_{\max} \\ x_{max} & x_{\max} < x < \infty. \end{cases} \tag{3}$$

The feasibility condition can then be written in terms of (3), i.e.

$$u_p \in [0, u_{\max}] \Leftrightarrow u_p = \text{sat}(u; 0, u_{max}) \tag{4}$$

where u is the control signal. When there is no risk of confusion, we will write $u_p = \text{sat}(u)$.

Performance Measure. The fishery performance measure to be used, denoted J, is the net present value of future returns:

$$J(u_p) \stackrel{\text{def}}{=} \int_0^T e^{-\delta\tau}(pqx - c)u_p \, d\tau \tag{5}$$

where price p, cost per unit effort c, discount rate δ, and planning horizon T are assumed constant. (We will use $T = \infty$ to develop the optimal control law.)

2.2 Equilibrium analysis of bioeconomic model

One of the desired control objectives will be for the fishery to operate at specific equilibrium (set) points. Given this, the set of equilibria for the nonlinear model are as follows:

$$x_e = o \qquad\qquad u_e = 0 \tag{6}$$

$$when \quad u_e \in (0,1] \qquad\qquad x_e = k\left(1 - \frac{q}{r}u_e\right). \tag{7}$$

Observe that as the equilibrium effort increases, the equilibrium biomass decreases (as expected).

2.3 LTI small signal model

To further understand the local characteristics of the above nonlinear model, we can linearize it about equilibria. Doing so yields the following small signal linear time invariant (LTI)

model:

$$\delta\dot{x} \;=\; a\,\delta x \;+\; b\,\delta u \tag{8}$$

$$f(x) = rx\left(1 - \frac{x}{k}\right) - qxu \tag{9}$$

$$a = \left[\frac{\partial f}{\partial x}\right]_{(x_e, u_e)} = r - \frac{2rx_e}{k} - qu_e = -\left(\frac{r}{k}\right)x_e \qquad b = \left[\frac{\partial f}{\partial u}\right]_{(x_e, u_e)} = -qx_e \tag{10}$$

$$\delta x(t) = x(t) - x_e \qquad \delta x(0) = x(0) - x_e = x_o - x_e \qquad \delta u(t) = u(t) - u_e \tag{11}$$

The associated transfer function from δu to δx is given by:

$$P(s) = \frac{b}{s - a} \tag{12}$$

Since $a = -\left(\frac{r}{k}\right)x_e < 0$, it follows that the equilibrium point (x_e, u_e) is asymptotically stable with the rate of convergence (pole) being proportional to the equilibrium biomass x_e, the fishery growth rate r, and inversely proportional to the fishery's carrying capacity k. The dc gain associated with P is $P(0) = -\frac{kq}{r}$; the minus sign implying that fishing reduces the equilibrium biomass.

Utility of LTI Small Signal Model. The above LTI model can be used to approximate the response x of the nonlinear model. If the response of the LTI model is denoted

$$\hat{x} = x_e + \delta x \tag{13}$$

then $\hat{x} \approx x$ when $u \approx u_e$ (i.e. $\delta u(t) \approx 0$) and $x_o \approx x_e$ (i.e. $\delta x(0) = x_o - x_e \approx 0$).

2.4 Control objectives

The control objectives for the fishery may be summarized (roughly) as follows:

1. Maximize the net present value of future returns

$$\text{maximize } J \stackrel{\text{def}}{=} \int_0^\infty e^{-\delta t}(pqx - c)u_p \, dt \tag{14}$$

   Note: We would be willing to give up some return for increased robustness.

2. Closed loop stability

   (a) Limit cycle behavior is not acceptable because it can have an prohibitively expensive implementation cost. While this is not captured in J, it could be addressed by introducing an additional $\dot{u}_p$ term within J.

   (b) Closed loop responses should be "relatively smooth" (continuous) when we have nearly continuous sampling of the biomass x. It is understood that sampling is inevitable in practice; i.e. continuous sampling is prohibitively expensive and hence impossible. As such, closed loop responses should be robust with respect to some discrete sampling.

3. Follow (achievable) step biomass commands issued by the fishery manager in the steady state

4. Reject additive step input and output disturbances in the steady state

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

421

5. Attenuate high frequency sensor noise so that it does not significantly impact control action

6. Ensure that the fishery biomass overshoot to step reference biomass commands is suitably bounded

7. Robustness with respect to model parametric uncertainty

2.5 Nominal model parameters

Nominal parameter values to be used are given below in Table 1.

Symbol θ	Description	Unit	Nominal θ_o	Range
Biological Parameters				
x_o	Initial resource biomass	*Kilotons, KT*	varies	$[0.5x_o, 1.5x_o]$
u_{min}	Minimum harvesting effort	*fleet · power · year/year*	0	-
u_{max}	Maximum harvesting effort	*fleet · power · year/year*	1	-
r	Intrinsic growth rate	*1/year*	0.3	[0.15,0.45]
q	Catchability	*1/fleet · power · year*	0.3	[0.15,0.45]
k	Carrying capacity	*KT*	100	[50,150]
Economic Parameters				
p	Resource market price	*M\$ per kiloton*	10	[5,15]
c	Cost of harvesting per effort	*M\$ per year*	13.24	$[6.62, 19.86]$
δ	Annual discount rate	*1/year*	0.1	[0.05,0.15]
T	Planning horizon	*years*	50	N/A

Table 1. Nominal Parameter Values Used

A planning horizon of $T = 50$ years was selected because the nominal discount rate is $\delta = 0.1$ and in roughly $T = \frac{5}{\delta} = 50$ years, the integrand within J is negligible.

Focus of Work: Biologically Exploited (BOE) Fishery. The focus of our presentation will be on a fishery that biologically overly expoilted (BOE) as opposed to biologically under exploited (BUE). This is because most of the world's critical fisheries are overly exploited (Clark, 1990).

- *BOE with the 'low cost' c = 13.24.* BOE occurs when the cost is sufficiently small. For the parameters indicated, it can be shown that:

$$x_e^* = 0.75 \cdot x_{MSY} = 37.5 < x_{MSY} = \frac{k}{2} = 50$$

i.e. the optimal equilibrium biomass is below the maximum sustainable yield biomass.

2.6 Model uncertainty and scope of presentation

Within this presentation, we focus on uncertainty associated with the nominal model parameters: r, k, q, p, c, δ. The following uncertainty will not be addressed in this presentation but it is duly noted:

1. The structure of F may be different than considered above. For example, if F has the form $F(x) geq 0$ for $x \in [k_c, k]$ and $F(x) < 0$ for $x \in (0, k_c)$ where $F(0) = 0$ and $F(k) = 0$, then we say that the fishery exhibits *critical depensation* (Clark, 1990, p. 17). In short, this implies that if x ever drops below the critical depensation parameter $k_c > 0$, then x will decrease toward zero regardless of u; i.e. the fishery will be lost.

2. All plant parameters are uncertain. They may even change with time. Moreover, the plant contains additional dynamics; e.g. it takes time for the fishery workers to mobilize. This can contribute additional lags, time delays, and rate limiters within the plant. One can use a decentralized or distributed model in order to capture the decision making made by individual fisher people (Clark, 1990, Ch. 8 & 9).

3. Input and output disturbances are uncertain.

4. Measurement noise is uncertain.

5. The biomass is not known; it must be estimated

6. The output (biomass) is sampled at some rate; if this rate is not sufficiently high, it could cause aliasing (Ogata, 1995); the sampling rate should be (as a rule-of-thumb)greater than ten times the control system bandwidth.

In contrast to many control applications where the "controller" is implemented with great fidelity, fishery controllers are implemented by an organization. As such, there can be considerable implementation issues/uncertainty. This will be discussed further below.

3. Optimal control law and properties

Within this section, we present the optimal control problem, the associated solution (optimal control law), and the properties of the optimal control law.

3.1 Optimal control law

We begin with a brief derivation of the classical optimal control policy stated in a way that will facilitate comparison to the class of LTI policies described later in this section.
The solution of the traditional optimal control problem:

$$\text{maximize } J \stackrel{\text{def}}{=} \int_0^\infty e^{-\delta t}(pqx(t) - c)u_p(t) \, dt \tag{15}$$

$$\text{s.t. } \dot{x}(t) = F(x(t)) - qx(t)u_p(t) \qquad x(0) = x_o \tag{16}$$

$$u_{min} = 0 \leq u_p(t) \leq u_{max} \tag{17}$$

is obtained by forming the Hamiltonian:

$$\mathcal{H}(x,u,\lambda) \stackrel{\text{def}}{=} e^{-\delta t}(pqx - c)u + \lambda\left[F(x) - qxu\right] = G(x,t)u - \lambda F(x) \tag{18}$$

where $G(x,t) \stackrel{\text{def}}{=} e^{-\delta t}(pqx - c) - \lambda qx$ and λ is the co-state variable. *Pontryagin's Maximum Principle* then implies that an optimal control policy will satisfy:

$$u(t) = \begin{cases} -\infty \text{ when } & G(x,t) < 0 \\ \infty \quad \text{ when } & G(x,t) > 0. \end{cases} \tag{19}$$

Because the objective functional is linear, the Maximum Principle says nothing about the case when $G(x,t) = 0$. However, using the co-state variable relationship $\dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial x}$, the well-known implicit formula for the singular control path can be determined (Clark, 1990):

$$F'(x) + \frac{cF(x)}{x(pqx - c)} = \delta \tag{20}$$

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model
423

Optimal Steady State Equilibrium Biomass. When $F(x) = rx(1 - x/k)$, the above equation can be used to determine the optimal (steady state) equilibrium biomass x_e^*:

$$x_e^* = \frac{\left[\frac{x_\infty}{2} - x_{MSY}\left(\frac{\delta}{r}\right) + x_{MSY}\right] + \sqrt{\left[\frac{x_\infty}{2} - x_{MSY}\left(\frac{\delta}{r}\right) + x_{MSY}\right]^2 + 4x_{MSY}x_\infty\left(\frac{\delta}{r}\right)}}{2}. \quad (21)$$

where

$$x_{MSY} = \frac{k}{2} \quad (22)$$

is the maximum sustainable yield biomass and

$$x_\infty = \frac{c}{pq} \quad (23)$$

is the optimal equilibrium when $\delta = \infty$; i.e. open-access equilibrium (Clark, 1990). The above shows that the optimal biomass x_e^* depends on the three independent parameters x_∞, x_{MSY}, and $\frac{\delta}{r}$. It can be shown that

$$x_\infty \quad \leq \quad x_e^* \quad \leq \quad x_{MSY} \quad + \quad \frac{x_\infty}{2} \quad (24)$$

for all $\delta \in [0, \infty]$ where the quantity $x_{MSY} + \frac{x_\infty}{2}$ is the optimal x_e^* for $\delta = 0$. The associated optimal (steady state) equilibrium control is given by:

$$u_e^* \overset{\text{def}}{=} \frac{r}{q}\left(1 - \frac{x_e^*}{k}\right). \quad (25)$$

Optimal Control Policy. Define the *tracking error* as the difference between the desired (reference) state and the actual state, i.e.

$$e \overset{\text{def}}{=} x_{ref} - x. \quad (26)$$

Setting $x_{ref} = x_e^*$ and combining (19) with (25) yields following expression for the control law:

$$u(t) = \begin{cases} -\infty & \text{when} \quad e > 0 \\ u_e^* & \text{when} \quad e = 0 \\ \infty & \text{when} \quad e < 0. \end{cases} \quad (27)$$

The saturation function is then applied to this control signal to capture the physical constraints on the system, i.e. $u_p(t) = \text{sat}(u(t))$. This control law implies the following:

- If $e > 0$, set $u_p(t) = u_{min} = 0$, allow $x(t)$ to increase until $x(t) = x_e^*$, then set $u_p(t) = \text{sat}(u_e^*)$.

- If $e < 0$, set $u_p(t) = u_{max}$ until $x(t)$ decreases to x_e^*, then set $u_p(t) = \text{sat}(u_e^*)$.

- If $e = 0$, set $u_p(t) = \text{sat}(u_e^*)$.

Below, we show that this policy (in general) exhibits limit cycle behavior in the presence of parameter uncertainty (see Figure 6).

3.2 Nominal optimal control policy numerics

The numerics for the nominal optimal perfect information control law are summarized in Table 2.

Case	x_e^*	u_e^*	x_∞	u_∞	Optimal Control Law	Optimal Return, J_o^*	
						small IC	large IC
BOE	37.5	0.625	4.41	0.987	$u(t) = \begin{cases} 0 & x(t) < 37.5 \\ u_e^* \stackrel{\text{def}}{=} \frac{F(x_e^*)}{qx_e^*} = 0.625 & x(t) = 37.5 \\ u_{max} = 1 & x(t) > 37.5 \end{cases}$	451	782

Table 2. Summary of Optimal Control Policy Numerics. BOE corresponds to $c = 13.24$ which yields $x_e^* = 0.75 \cdot x_{\text{MSY}}$. Small IC corresponds to $x_o = 0.5 \cdot x_e^* < x_e^*$. Large IC corresponds to $x_o = 1.5 \cdot x_e^* > x_e^*$. $x_\infty = \frac{c}{pq}$ and $u_\infty = \frac{r}{q}(1 - \frac{x_\infty}{k})$ correspond to infinite discounting (open-access); i.e. $\delta = \infty$.

3.3 Properties of the optimal control law

In this section, we describe the properties of the optimal control law assuming perfect information (i.e. model parameters are known with no error) and imperfect information (i.e. model parameters are not perfectly known). Understanding the properties of the optimal policy is very important for several reasons. We wish to understand (1) the fundamental robustness properties (e.g. economic inefficiency) of an optimal policy (e.g. one based on nominal, worst case, or best case parameters); (2) implementation issues associated with the optimal control policy; (3) how the robustness properties for our robust policies compare to those of a particular optimal control policy; (4) how x_e^* depends on parameter perturbations. The latter is important because we will using x_e^* as the reference command x_{ref} for our robust control law policies. This is an issue because the optimal x_e^* (in general) is uncertain; i.e. x_e^* is only known for specific value selections (e.g. nominal, worst case, best case). As such, we will have to address this uncertainty to clearly understand what our robust control policies (with built-in command following) will be driving the state of the fishery to.

In short, we show below that: (1) Since x_e^* is, in general, uncertain, if x_e^* is the desired (reference) state, then we have a major issue in that we will be driving the fishery to the incorrect state. This can have severe economic as well as biological implications (e.g. driving x below the critical depensation parameter k_c, will destroy the fishery). (2) The optimal policy exhibits limit cycle behavior when x_e^* is uncertain. Moreover, it is very sensitive to any discrete sampling. As such, the (imperfect information) optimal policy is prohibitively expensive to implement (see Figure 6).

Optimal Perfect Information Control Law Sensitivity: Single Parameter Results ($x_o = x_e^*$).
The following shows how the the performance of the optimal perfect information control law changes with parameter perturbations. Results for our BOE fishery when $x_o = x_e^*$ are as follows:

1. (J_e^*, x_e^*, u_e^*) increase with increasing k or increasing r.

2. J_e^* increases while (x_e^*, u_e^*) decrease with increasing q.

3. J_e^* decreases while x_e^* increases and u_e^* decreases with increasing δ.

4. J_e^* increases while x_e^* decreases and u_e^* increases with increasing p.

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

425

5. (J_e^*, x_e^*) decrease while u_e^* increases with increasing c.

Robustness with Respect to Parametric Uncertainty: Imperfect Versus Perfect Information.
Figure 1 shows how the optimal control law performs in the presence of parametric uncertainty. $x_o = x_e^*$ for the perfect information policy. x_o is at the unperturbed/nominal x_e^* for the imperfect information policy. The plots compare the performance of the optimal control law with imperfect parameter knowledge to that with perfect parameter knowledge. The perfect information optimal control law (by definition) results in the maximum achievable return. While it represents a suitable benchmark to compare with, it must be emphasized that x_e^* is always uncertain. This is particularly crucial when x_e^* is being used as the target biomass (reference command) for a robust control law (see Sections 4, 5) because an incorrect reference command x_{ref} will fundamentally limit the achievable performance. Moreover, no (inner loop) robust policy can address this. To properly address this, one needs some combination of parameter estimation, system identification, and learning coupled with some adaptive outer loop policy that adjusts the target based on collected information. While this is challenging and exciting to pursue, it is beyond the scope of our presentation.

Figure 1 specifically shows the maximum theoretical (perfect information) return on the left in blue. The return associated with the imperfect information optimal policy (designed for nominal parameter values) is shown on the left in red. On the right in blue, we see how much the imperfect information optimal control law under performs the perfect information optimal control law. When k is perturbed by -30%, the imperfect law under performs the perfect information law by nearly 10%. Figure 1 shows that for a similar perturbation in r, the imperfect policy under performs by nearly 2%. It can be shown (figures not provided) that for a similar perturbation in δ, the imperfect policy under performs by less than 1%. It can be shown (figures not provided) that for similar perturbations in p, c, or q the imperfect policy under performs by a very small percentage. Why is it that the biological parameters k and r matter more in closing the perfect-imperfect information performance gap than δ, p, c, or q? This is because x_e^* is more sensitive to uncertainty in k and r for the BOE case under consideration. In short, the plots show that we should be concerned primarily with uncertainty in k. More generally, we seek (robust) policies that perform closer to the perfect information optimal policy for likely parametric modeling errors. Imperfect information obviously limits how close we can get. This and associated issues will be addressed below.

Impact of Extremal Parameter Uncertainty on Perfect Information (J_e^*, x_e^*, u_e^*) - At Optimal Equilibrium. In what follows, x_e^* will be used as a reference command x_{ref} to a robust control law with good command following properties. Since x_e^* is uncertain, it is important to understand how commanding an incorrect target will limit achievable performance. Given this, suppose that $x_o = x_e^*$.

We now ask, what is the worst case combination of perturbations for the model parameters (r, k, q, p, c, δ)? While an analytical proof is difficult, it can be shown (numerically) that

J_e^*, in general, decreases when (k, r, p, q) are decreased and/or (c, δ) are increased.

This result is independent of the initial condition x_o for the BOE case under consideration. Given uncertainty bounds for each of the model parameters, this observation permits us to readily determine the worst case set of parameter perturbation - something that, in general, is very difficult to do.
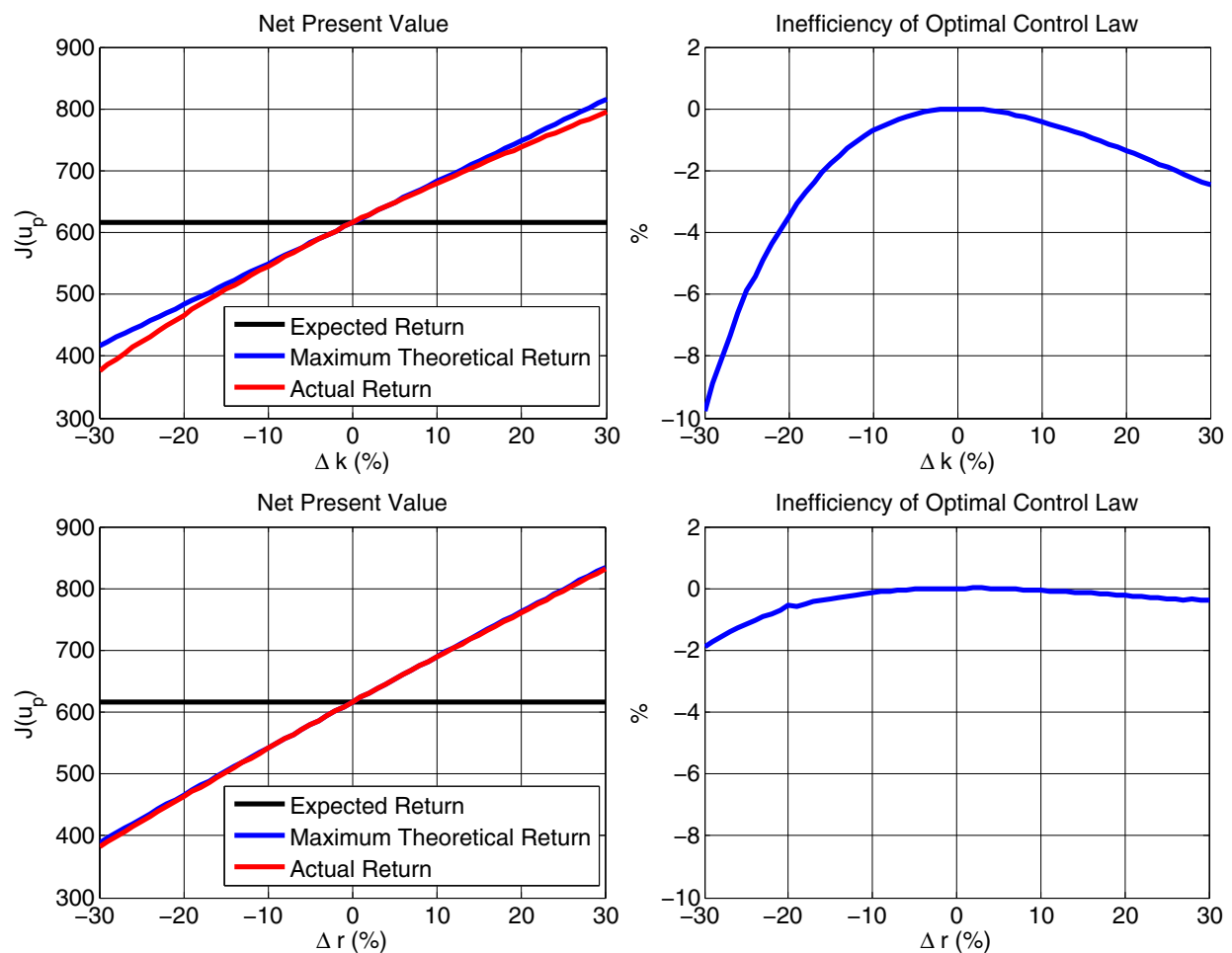
Fig. 1. Economic Inefficiency for Imperfect Information (Nominal) Optimal Control Law: Capacity & Growth Rate Uncertainty

Consider figures 2-3 for (J_e^*, x_e^*, u_e^*), respectively. Within these figures, $x_o = x_e^*$ and perfect information is assumed. The figures show the dependence of the perfect information optimal control law on worst case and best case (extremal) parameter perturbations as defined below.

- *Worst Case Extremal Parameter Perturbations.* Within figures 2-3, negative (worst case extremal) parameter perturbations correspond to

$$\frac{\Delta r}{r_o} = \frac{\Delta k}{k_o} = \frac{\Delta q}{q_o} = \frac{\Delta p}{p_o} < 0 \quad \text{and} \quad \frac{\Delta c}{c_o} = \frac{\Delta \delta}{\delta_o} > 0 \tag{28}$$

i.e. equal parametric perturbations that result in a smaller return. Here, $\Delta \theta \stackrel{\text{def}}{=} \theta - \theta_o$ represents a perturbation in the parameter θ with respect to the nominal parameter θ_o.

- *Best Case Extremal Parameter Perturbations.* Within figures 2-3, positive (best case extremal) parameter perturbations correspond to

$$\frac{\Delta r}{r_o} = \frac{\Delta k}{k_o} = \frac{\Delta q}{q_o} = \frac{\Delta p}{p_o} > 0 \quad \text{and} \quad \frac{\Delta c}{c_o} = \frac{\Delta \delta}{\delta_o} < 0 \tag{29}$$

i.e. equal parametric perturbations that result in a larger return.

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

427

The green curves within figures 2-3 represent actual optimal perfect information values. The blue curves give the percent deviation with respect to the nominal value.
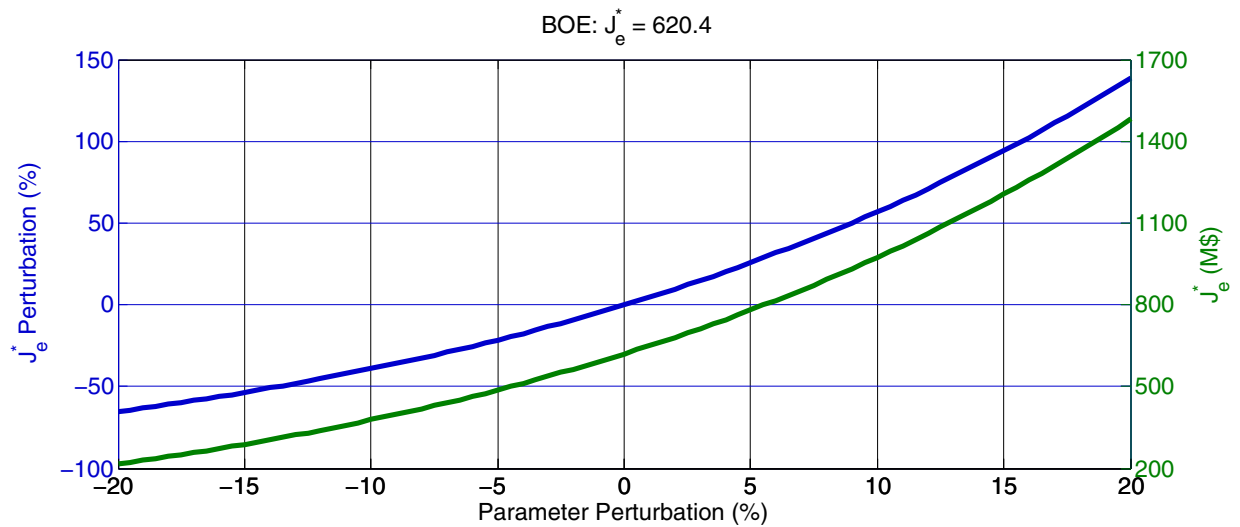

Fig. 2. Perfect Information Optimal Control Law Returns: Extremal Percent Parameter Perturbations, $x_o = x_e^*$

Assuming $\pm 20\%$ uncertainty for each nominal parameter value, figure 2 shows that the worst case perfect information optimal return is \$215.6 M (65.25% below the nominal of $J_e^* = 620.4M$). In contrast, the best case perfect information optimal return is \$1482M (138.95% above the nominal of $J_e^* = 620.4M$) - a 687% improvement with respect to the worst case perfect information optimal return. Also note from figure 3 that the worst case parameter combination results in a 20% reduction in x_e^* with respect to the nominal. From figure 3, we see that u_e^* is increased by less than 1%.
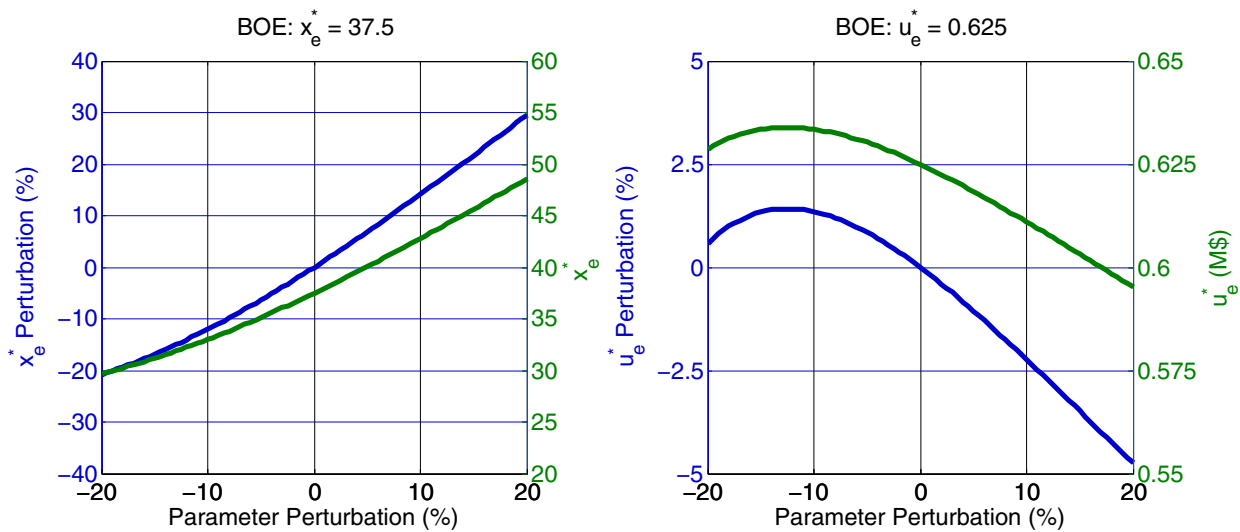
Fig. 3. Perfect Information Optimal Control Law (x_e^*, u_e^*): Extremal Percent Parameter Perturbations, $x_o = x_e^*$

Dealing with Uncertain x_e^* and J_e^*. Let x_{ref} denote the reference biomass at which the fishery manager wishes to operate the fishery [2]. How does a manager choose the target fishery biomass x_{ref}? A biologically conservative manager may wish to keep the fishery at the maximum sustainable yield $x_{ref} = x_{MSY} = \frac{k}{2}$. A financially aggressive manager may chose to operate the fishery at the infinite discount ($\delta = \infty$)) optimal value $x_{ref} = x_\infty = \frac{c}{pq}$. More generally, a manager could use the optimal value $x_{ref} = x_e^*$ as the point at which to operate. Given that x_e^* is known to within a percentage $\frac{\Delta x_e^*}{x_e^*}$, it follows that a fishery manager might try to operate at (1) $x_e^* - \Delta x_e^*$ if economic aggression is desired, or at (2) $x_e^* + \Delta x_e^*$ if biological conservatism is desired. The x_e^* concept gives the fishery manager a way to systematically think about fishery biomass targets.

Uncertainty In (x_o, x_e^*, x_{ref}): 6 Cases. In general, x_o and x_e^* are uncertain. How does one choose the target x_{ref}. We'd ideally like $x_{ref} = x_e^*$, but x_e^* is uncertain. What can a manager do? The table below contains the six possible relations that can exist amongst the three scalars (x_o, x_e^*, x_{ref}) - from smallest to biggest. In general, we would (ideally) like the state to move from x_o toward $x_{ref} = x_e^*$. Since x_e^* is uncertain, it follows that x_{ref} (in general) will differ from x_e^*. As such, it follows that we may issue reference commands x_{ref} that move the state x in an incorrect direction. Since the state moves from x_o toward x_{ref}, it follows from the table below that in two cases the state moves in the incorrect direction. In the four other cases, the state moves in the correct direction.

Smallest	$\rightarrow$	Biggest	Direction	
x_{ref}	x_o	x_e^*	Incorrect	Way too much fishing (Way Too Aggressive)
x_{ref}	x_e^*	x_o	Correct	Too much fishing (Very Aggressive)
x_o	x_{ref}	x_e^*	Correct	Too much fishing (Moderately Aggressive)
x_o	x_e^*	x_{ref}	Correct	Too little fishing (Moderately Conservative)
x_e^*	x_{ref}	x_o	Correct	Too little fishing (Very Conservative)
x_e^*	x_o	x_{ref}	Incorrect	Way too little fishing (Way Too Conservative)

Table 3. Six Possible Inequality Relations for (x_o, x_e^*, x_{ref})

To select x_{ref}, we offer the following approaches.

1. **Best-Worst Case Approach.** Assume that we have good bounds on parametric uncertainty (not necessarily tight, but encompassing) for the 6 model parameters under consideration: k, r, q, p, c, δ. Suppose that we design for the best worst case scenario; i.e. try to approach the return of the perfect information optimal policy when the worst case parameter perturbations occur; i.e. $\frac{\Delta k}{k_o} = \frac{\Delta r}{r_o} = \frac{\Delta q}{q_o} = \frac{\Delta p}{p_o} < 0$ and $\frac{\Delta c}{c_o} = \frac{\Delta \delta}{\delta_o} > 0$. We assume worst case maximal parameter perturbations. (For simplicity, we assume that all parameters are perturbed by their maximum worst case percentage and that this percentage is the same for all of the parameters.)

   One could, for example, pick a worst case percentage which bounds all of the parameters. Doing so can be conservative. Parameter estimation can be used to narrow tighten this worst case percentage. If we have fixed percentage bounds for each of the parameters, our approach remains the same. (Recall: Determining the worst case perturbation in our problem is easy. This is not true in most practical scenarios.)

---

[2] It is understood that x_{ref} can change with time. For now, we assume x_{ref} is fixed.

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model
429

Now choose x_{ref} equal to the associated worst case x_e^*; i.e. the x_e^* that results from choosing the worst case parameters. By so doing, the actual x_e^* will be greater than x_{ref}. As such, only cases 1-3 can occur; i.e. cases 4-6 cannot occur. The only way, cases 4-6 can occur is if our uncertainty bounds were not truly encompassing.

2. **Best-Best Case Approach.** Assume that we have good bounds on parametric uncertainty (not necessarily tight, but encompassing)for the 6 model parameters under consideration: k, r, q, p, c, δ. Suppose that we design for the best best case scenario; i.e. try to approach the return of the perfect information optimal policy when the best case parameter perturbations occur; i.e. $\frac{\Delta k}{k_o} = \frac{\Delta r}{r_o} = \frac{\Delta q}{q_o} = \frac{\Delta p}{p_o} > 0$ and $\frac{\Delta c}{c_o} = \frac{\Delta \delta}{\delta_o} < 0$. We assume best case maximal parameter perturbations. (For simplicity, we assume that all parameters are perturbed by their maximum best case percentage and that this percentage is the same for all of the parameters.)

Now choose x_{ref} equal to the associated best case x_e^*; i.e. the x_e^* that results from choosing the best case parameters. By so doing, the actual x_e^* will be smaller than x_{ref}. As such, only cases 4-6 can occur; i.e. cases 1-3 cannot occur. The only way, cases 1-3 can occur is if our uncertainty bounds were not truly encompassing.

3. **Blended Best-Worst-Best-Best Approach.** One can also try to offer a blended approach that attempts to offer decent returns when either worse case or best case parameter perturbations occur. We shall illustrate this below.

4. **Probabilistic Approach.** If a probability density function for the parameter percentage θ is available, it can be used to determine where to operate. Let f_θ denote a density function for θ. This can be used to derive the density function f_J for J. Given this, the expected value for J is given by $E[J] = \int_J J \, f_J(J) \, dJ = \int_\theta J(\theta) \, f_J(J(\theta)) \, J'(\theta) \, d\theta$. The density function for θ can be used to reflect what parameter perturbations are most likely to occur. The above expectation can then be used to choose x_{ref} to maximize the expectation.

To illustrate the above ideas, consider figures 4-5 for small and large initial conditions, respectively under extremal parameter perturbations. The figures show results for the perfect information designs (black), best worst case design (blue), best best case design (red), and the nominal design (green).

To summarize, the following specific optimal control laws were implemented:

1. Perfect Information Optimal Designs: $\qquad x_{ref} = x_e^*, u_{ref} = u_e^*$
2. A Best-Worst Case Design: $\qquad x_{ref} = 29.7, u_{ref} = 0.629$
3. A Nominal Design Based on the Nominal Parameters: $\qquad x_{ref} = 37.5, u_{ref} = 0.625$
4. A Best-Best Case Design: $\qquad x_{ref} = 48.6, u_{ref} = 0.595$

The performance of the perfect information designs are always best (by definition). The performance of the best-worst case design (blue) duplicates that of the perfect information design for 20% worst case perturbations since it is based on the worst case parameter model and x_e^*. The performance of the best-best case design (blue) duplicates that of the perfect information design for 20% best case perturbations since it is based on the best case parameter model and x_e^*. The following key observations are in order within figure 4 (small IC case):

1. The best-worst case design does better than the best-best case design when its parameter assumptions are maximally incorrect; falling by less than 20% (with respect to perfect information optimal return) while the best best falls by more than 40% (with respect

to perfect information optimal return) when its parameter assumptions are maximally incorrect.

2. The nominal design can be viewed as a nice compromise or blend between the two prior policies. It is based on the nominal parameter model and x_e^*. Its returns deteriorates by a little more than 10% for worst case parameter uncertainty and by a little more than 5% for best case parameter uncertainty. In short, the returns associated with this nominal (blended) policy offers flatter returns over a wider range of extremal parameter perturbations.

Each of the above three approaches offer a specific design model (to base the control design upon) and a specific x_e^* to use as a target. Control laws are always evaluated with the true (nonlinear) plant. In what follows, we will use the above as benchmarks whose performance we shall target via robust control laws. Similar patterns are observed for the large IC case in figure 5.

Sensitivity Analysis: Extremal Perturbations, Small Initial Condition. The expected value for each of the design cases considered are as follows:
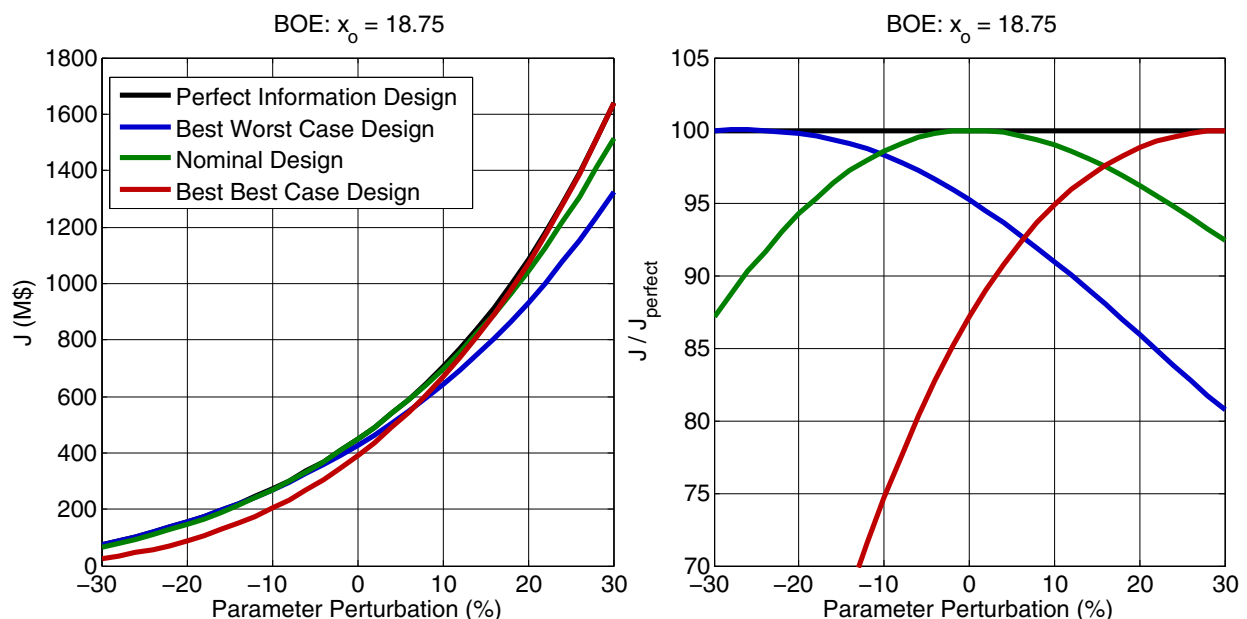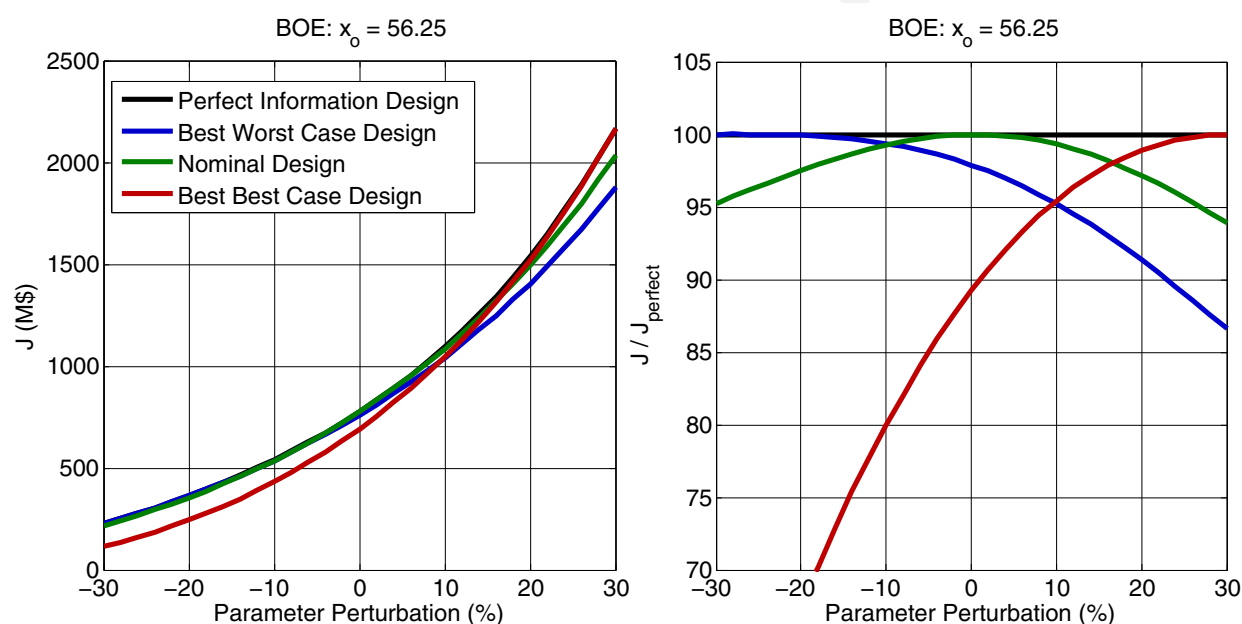

Fig. 4. Economic Inefficiencies for Various Optimal Control Laws: Extremal Perturbations (Small IC)

- $E[J] = 867.8$ for the Perfect Information Optimal Designs
- $E[J] = 772.0$ for the Best-Worst Case Design
- $E[J] = 838.0$ for the Nominal Design
- $E[J] = 800.4$ for the Best-Best Case Design

A uniform distribution has been assumed for the parameter uncertainty. The optimal perfect information control law is included for comparison purposes. Its performance can only be approximated over a range of parameter perturbations. This is because the (1) design plant parameters differ from those of the true plant and the desired target x_{ref} differs from the perfect information target x_e^*.

The following additional points are in order:

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

431

- Although the best-best case design appears worse in terms of percentages at off design conditions, it has a higher expected return across all cases versus the best-worst case design.

- A manager could readily design a policy that limited the worst case downside return to a certain percentage of the maximum possible. For example, if the manager wanted a worst case downside return no worse than 5% of the maximum possible, a policy should be designed around roughly a -2 % parameter perturbation.

- A manager may also be interested in implementing the following policy: $\max_\theta E[J(\theta)]$.

Sensitivity Analysis: Extremal Perturbation, Large Initial Condition. The expected value for each of the design cases considered are as follows:

Fig. 5. Economic Inefficiencies for Various Optimal Control Laws: Extremal Perturbations (Large IC)

- $E[J] = 1368.2$ for the Perfect Information Optimal Designs
- $E[J] = 1284.5$ for the Best-Worst Case Design
- $E[J] = 1336.8$ for the Nominal Design
- $E[J] = 1259.3$ for the Best-Best Case Design

Finally, it should be noted that in contrast to the low initial condition study conducted, the Best-Best Case Design performs worse both in terms of the percentage possible and the expected return when compared to the Best-Worst Case Design.

Limit Cycles In the Presence of Uncertainty. Finally, consider figure 6. The optimal control law is based on the nominal BOE parameters. The initial condition is above the uncertain x_e^*. The simulation is conducted with a truth plant possessing a 10% reduction in k - hence the limit cycle behavior. The figure shows that: (1) The optimal control policy (in general) will exhibit limit cycle behavior when we have imperfect information; i.e. model parameters are not known exactly. (2) The optimal control policy (in general) will be very sensitive to finer

sampling (ΔT smaller) under imperfect information; i.e. more oscillations (switching) will be exhibited as our x time samples are spaced closer together. The figure also shows that low pass filtering the optimal with a lag can be used to smooth oscillations a bit. To significantly reduce the oscillations, however, there is no easy fix. We either need a penalized $\dot{u}_p$ term within J to penalize switching or we need policies that are inherently more robust (like the ones we will describe subsequently). As such, this implies that, in practice, the optimal control policy is prohibitively expensive to implement in the presence of parametric uncertainty because of the inherent limit cycle behavior and the associated switching costs.
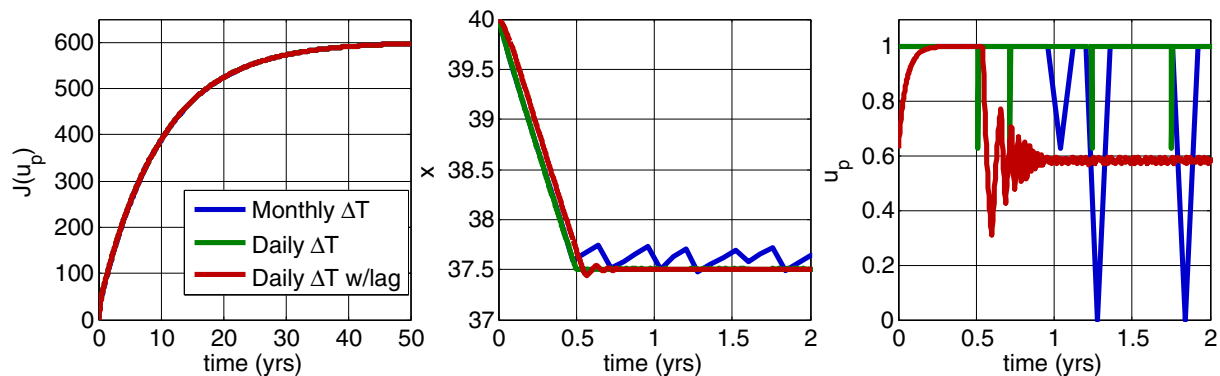
Fig. 6. Optimal Control Law Robustness: Limit Cycles In Presence of (-10% Capacity) Uncertainty

Motivation for Robust Control Laws. The above motivates the need for more robust control laws; i.e. control laws that (1) exhibit an acceptable return (i.e. return robustness) in the presence of anticipated (likely) parametric uncertainty; (2) do not exhibit limit cycle behavior in the presence of anticipated (likely) parametric uncertainty. As such, the above motivates the robust control laws to be considered in our presentation.

Control Law Implementation Issues. Unlike many control applications where controllers are implemented with great fidelity (within state-of-the-art digital computing units), controllers within a resource management system are implemented by an organization by setting rules for the fishery worker community (e.g. quotas, taxes (Clark, 1990, Chapter 8). As such, many types of uncertainties can be introduced by the organization. These could include any of the following: (1) parameter uncertainty, (2) additional uncertain actuation/sensing dynamics (e.g. lags, time delays, rate limiters, etc.), (3) nonlinearities (e.g. rate limiters, saturations, quantization, dead zones), (4) actuation/incentive errors (e.g. quota/tax miscalculations), (5) sensing, measurement, and estimation errors (e.g. sensor dynamics, biomass sampling/aliasing/quntization errors, noise, disturbances).

4. Robust control laws

The model under consideration is very simple. Many tools from the controls literature may be applied (e.g. classical control (Rodriguez, 2003), H-infinity (Rodriguez, 2004), feedback linearization, SDRE's, etc.). Given the introductory/tutorial nature of the paper, the simplicity of the model being used, as well as the fact that this text covers advanced control methodologies, we shall focus on simple control strategies from classical control theory. We will show that such control laws can be used to avoid limit cycles, increase robustness with

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

433

respect to parametric uncertainty, and achieve returns that are close to those of the perfect information optimal control law.

Control System Structure. The structure of the control system may be visualized as shown in Figure 7.
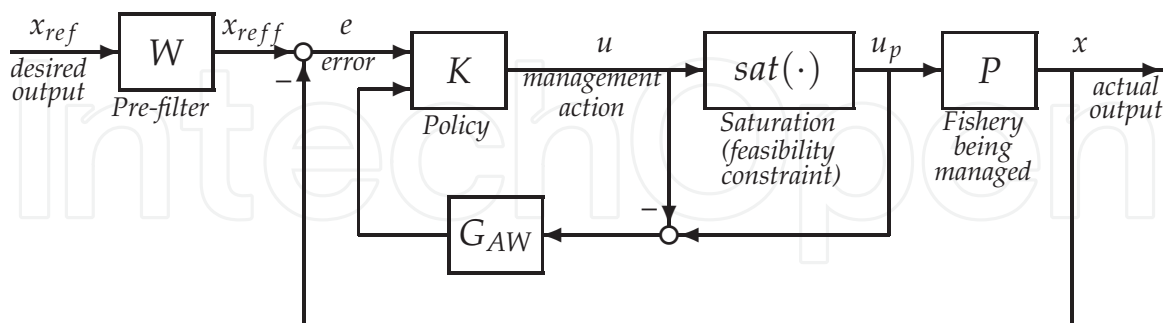

Fig. 7. Renewable Resource Management Problem Represented as a Standard Negative Feedback System with a Pre-Filter and Anti-Windup Logic

1. *Plant.* Here, P represents the plant under control. We shall use an LTI small signal model to approximate our nonlinear plant.

2. *Reference State or Command.* x_{ref} is the desired reference biomass state. Ideally, we would like to use $x_{ref} = x_e^*$. Parameter uncertainty prevents us from commanding the desired state. As such, we are forced to choose x_{ref} more judiciously. Given this, we will give special attention to maximizing our return under the worst case parameter uncertainty.

3. *PI Controller.* K is a proportional-plus-integral (PI) controller possessing the structure:

$$K(s) = \frac{g(s+z)}{s} \left[\frac{p_{ro}}{s + p_{ro}} \right] \tag{30}$$

where $g > 0$, $z > 0$, and $p_{ro} > 0$. The integrator within the controller will ensure that step biomass commands are followed in the steady state while step input/output disturbances are rejected in the steady state. The $(s + z)$ term will ensure that the LTI plant-integrator pair will be stabilized. The term $\left[\frac{p_{ro}}{s+p_{ro}} \right]$ provides high frequency roll-off to ensure that high frequency sensor noise n is suitably attenuated.

4. *Command Pre-Filter.* W is a reference command pre-filter possessing the structure:

$$W(s) = \left[\frac{z}{s + z} \right] \tag{31}$$

This pre-filter can be used to ensure that the overshoot to step reference commands is suitably bounded.

5. *Observer-Based Integrator Anti-Windup Logic.* Anti-windup logic is included so that the integrator in the PI controller does not windup. That is, the integrator is turned off so that it does not integrate constant errors which occur when the input to the plant is saturated (Aström, Hägglund). The structure of the anti-windup logic is as follows $\dot{x}_k = A_k x_k + B_k e + L(sat(u) - u)$ where L is an observer gain matrix. The PI controller with the anti-windup logic may be described by the following equations:

$$\dot{x}_1 = e + G_{AW}(sat(u) - u) \qquad \dot{x}_2 = gzx_1 - p_{ro}x_2 + ge \qquad u = p_{ro}x_2 \qquad (32)$$

where G_{AW} is the anti-windup gain.

Nominal Design Methodology. The nominal design methodology can be described as follows (Rodriguez, 2003):

1. *Plant Approximant.* The following small signal LTI model $P \approx P_d \overset{\text{def}}{=} \frac{b}{s-a}$ will be used to approximate our nonlinear plant. Here, P_d is referred to as the *design plant*; i.e. the plant upon which we will base our control law design. While any design we obtain can be evaluated using plant approximants such as P_d, control designs must be evaluated with the actual nonlinear plant model.

2. *Controller Approximant.* Use the controller approximant $K \approx \frac{g(s+z)}{s}$ where $g > 0$ and $z > 0$.

3. *Nominal Open Loop Approximant.* Form the open loop transfer function approximant

$$L = P_d K \approx \frac{bg(s+z)}{s(s-a)} = \frac{n(s)}{d(s)}. \qquad (33)$$

4. *Nominal Closed Loop Characteristic Equation.* Form the nominal closed loop characteristic equation

$$\Phi_{cl}(s) = d(s) + n(s) = s^2 + (bg - a)s + bgz = 0 \qquad (34)$$

This polynomial has the "standard second order form"

$$\Phi_{cl}(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 \qquad (35)$$

where $\zeta = \frac{bg-a}{2\sqrt{bgz}}$ is the damping factor and $\omega_n = \sqrt{gz}$ is the undamped natural frequency. For stable nominal complex closed loop poles, we require $0 < \zeta < 1$.

5. *Closed Loop Poles.* Determine the nominal closed loop poles (assumed complex for rapid transient response):

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} \qquad (36)$$

Given this, we will have nominal (local) closed loop exponential stability with an associated time constant $\tau = \frac{1}{\zeta\omega_n}$. The associated (approximate 1%) settling time is $t_s = 5\tau$.

6. *Standard Second Order Closed Loop Transfer Function and Percent Overshoot.* With the command pre-filter W, the associated closed loop transfer function takes the standard second order form:

$$T_{x_{ref}x} = \frac{WPK}{1 + PK} \approx \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (37)$$

As such, the associated percent overshoot to a step reference command is given by

$$M_p = e^{-\zeta\omega_n t_p} = e^{-\left(\frac{\zeta\pi}{1-\zeta^2}\right)} \qquad (38)$$

where $t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ is the time at which the peak overshoot occurs.

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

435

7. *Damping Factor from Percent Overshoot Specification.* Determine ζ from the overshoot specification:

$$\zeta = \frac{|lnM_p|}{\sqrt{\pi^2 + |lnM_p|^2}} \tag{39}$$

8. *Undamped Natural Frequency from Settling Time Specification.* Determine ω_n from the settling time specification:

$$\omega_n = \frac{5}{\zeta t_s} \tag{40}$$

9. *PI Controller Parameters.* Determine the PI controller gain g and zero z from:

$$g = \frac{2\zeta\omega_n + a}{b} \qquad\qquad z = \frac{\omega_n^2}{bg} \tag{41}$$

10. *Controller Roll-Of Parameter.* Choose the roll-off parameter p_{ro} as follows:

$$p_{ro} = 10\omega_n \tag{42}$$

so that the added high frequency roll-off does not significantly degrade the nominal phase margin within the loop. It could also be selected in order to satisfy a specific sinusoidal steady state noise attenuation specification.

11. *Anti-Windup Gain.* Choose the anti-windup gain $G_{AW} > 0$ to be sufficiently large so that the integrator suitably shuts down in order to "maximally recapture" the dominant second order response characteristics described above. A family of gains is examined below.

5. Control law comparisons

In this section, we compare the properties of the nominal optimal control law with those for the robust policies based upon the nominal LTI plant model $P_d = \frac{b}{s-a}$.

5.1 Sample control law time responses

Within this section, sample time responses are provided for families of robust control laws (based upon the nominal LTI plant model) - families that approximate the performance of the nominal optimal control law. (Note: There will be an approximation gap when uncertainty is considered.)

Reference Biomass Tracking: Anti-Windup Gain Study. Figure 8 shows closed loop biomass tracking time responses for a family of robust control law designs where $\zeta = 1$, $t_s = 1$. The anti-windup gain G_{AW} is varied to control how well the responses approximate that of the optimal with no limit cycle behavior. As the anti-windup gain G_{AW} is increased, the responses come closer to the (nominal) optimal control law (with no limit cycle behavior). The limit cycle behavior of the (nominal) optimal has been cleaned up in order to improve the readability of the figure (see Figure 6).

A Note On Robustness with Respect to High Frequency Unmodeled Dynamics. It should be noted that as the speed of a policy is increased, the significance of unmodeled high frequency dynamics within the fishery or within the policy implementing organization/evironment (e.g. lags, time delays, rate limiters) becomes an issue to consider in final policy evaluation.
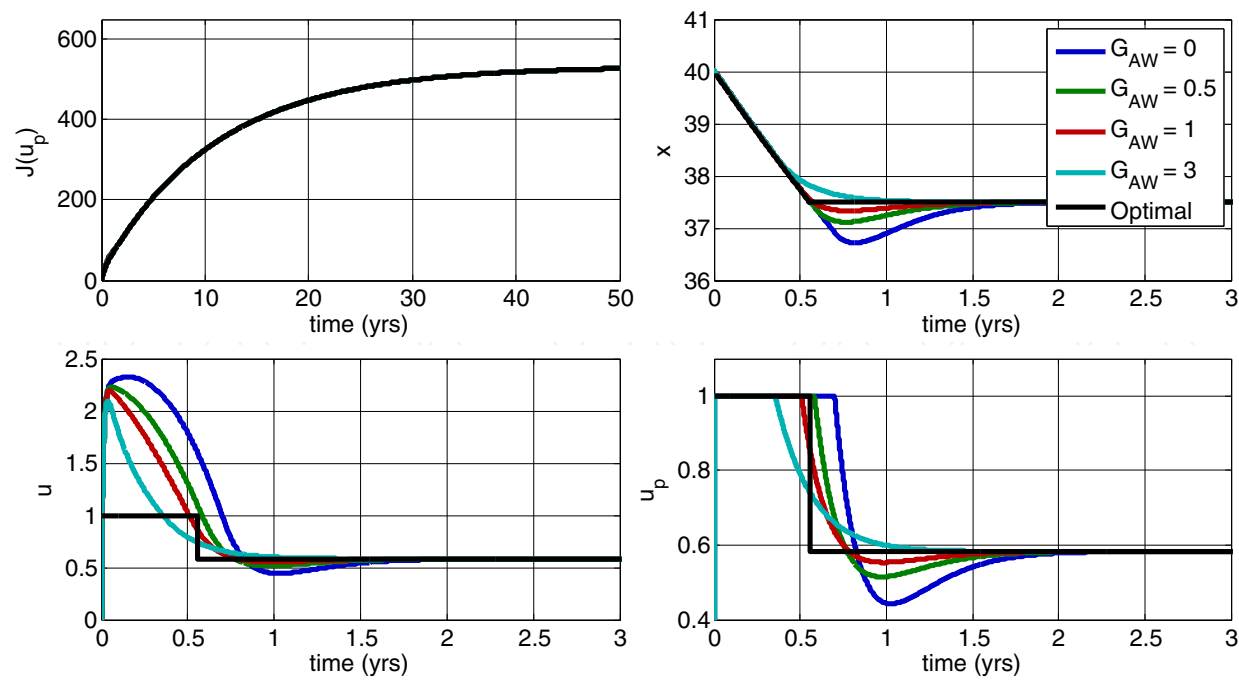
Fig. 8. Reference Biomass Tracking: Anti-Windup Gain Study ($\zeta = 1$, $t_s = 1$)

It is well known from fundamental robustness theory (Rodriguez, 2004; 2003) that fast control laws can result in closed loop oscillatory responses or instability when high frequency unmodeled dynamics are "significantly excited." This issue will be examined in future work. **Reference Biomass Tracking: Damping Factor Study.** Figure 9 shows shows closed loop biomass tracking time responses for a family of robust control law designs where $t_s = 1$, $G_{AW} = 3$. The damping factor ζ is varied in order to control the speed of the response as
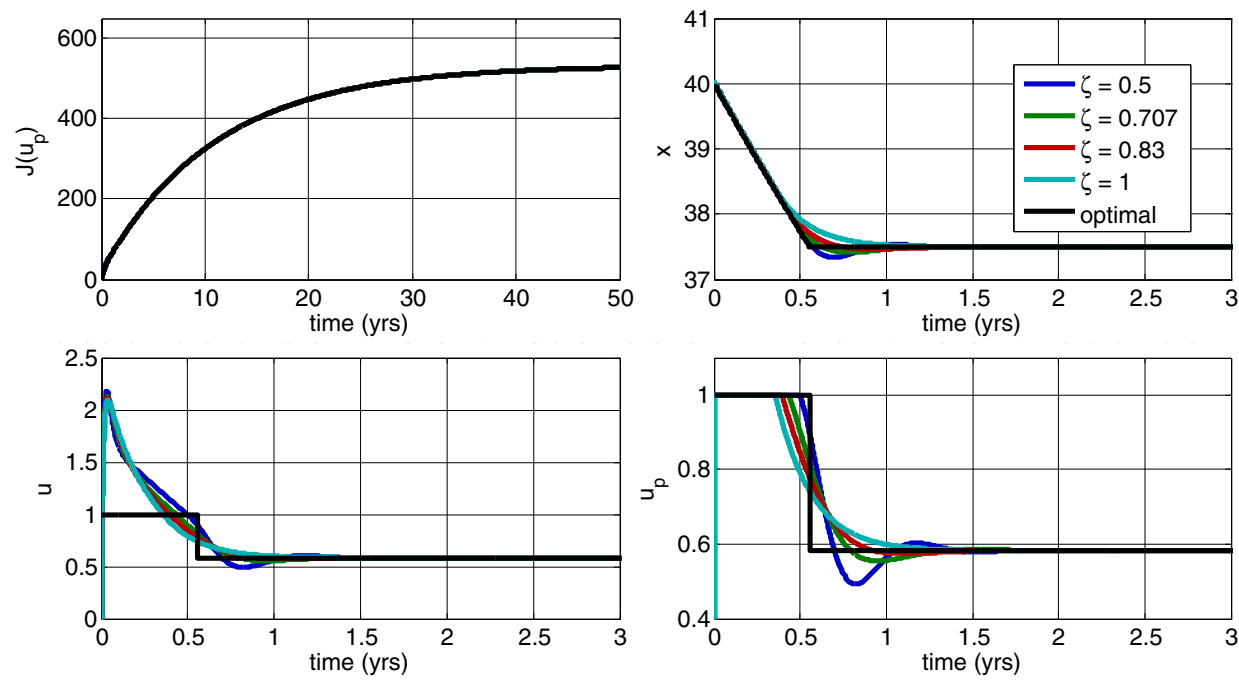
Fig. 9. Reference Biomass Tracking: Damping Factor Study ($G_{AW} = 3$, $t_s = 1$)

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

437

well as the undershoot. As the damping factor ζ is reduced, the response speeds up (getting closer to that of the (nominal) optimal with no limit cycle behavior), although the observed undershoot increases. The limit cycle behavior of the (nominal) optimal has been cleaned up in order to improve the readability of the figure (see Figure 6).
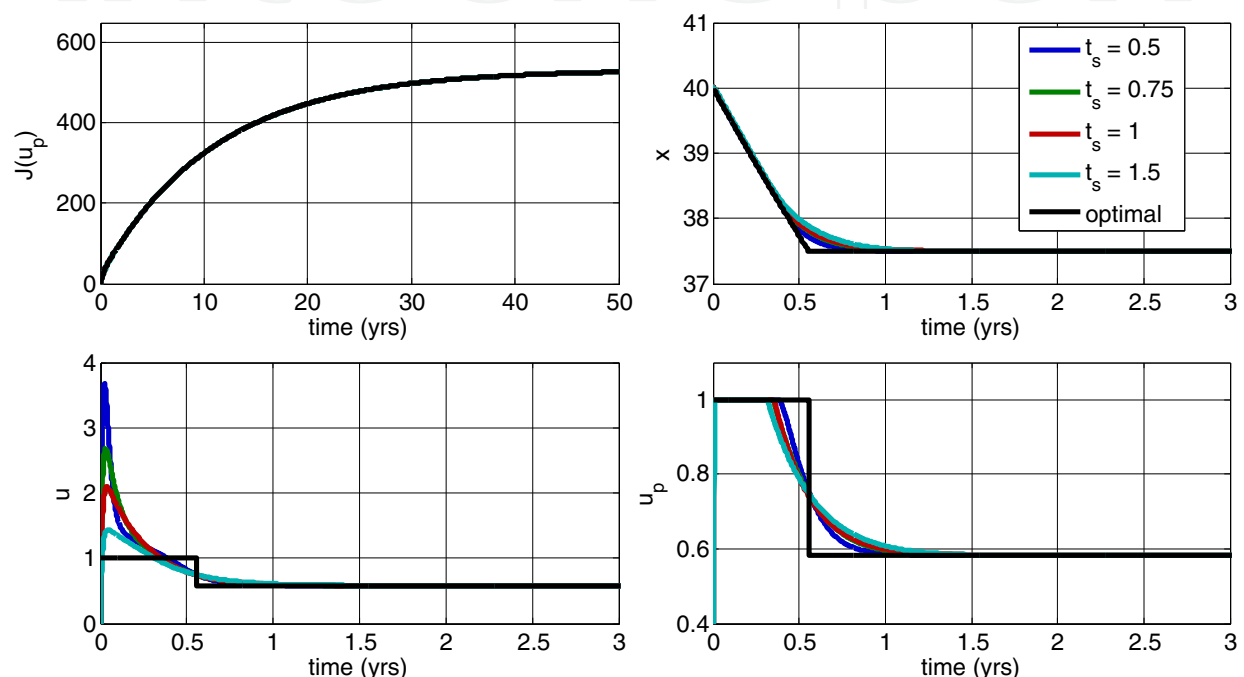
Reference Biomass Tracking: Settling Time Study. Figure 10 shows closed loop biomass tracking time responses for a family of robust control law designs where $\zeta = 1$ (critically damped, $M_p = 0$) and $G_{AW} = 3$. As the settling time t_s of the closed loop system is reduced, the responses come closer to the (nominal) optimal (with no limit cycle behavior). The limit cycle behavior of the (nominal) optimal has been cleaned up in order to improve the readability of the figure.

Fig. 10. Reference Biomass Tracking: Settling Time Study ($G_{AW} = 3$, $\zeta = 1$)

5.2 Utility of linear design methodology
In this section, we try to shed light on the utility of our linear time invariant (LTI) based robust control system design methodology and how linear simulation can be used to approximate/predict the behavior of the nonlinear simulations. All designs are based upon nominal parameter values.

Linear vs Nonlinear Biomass Tracking: x_o Near x_{ref}. Figure 11 compares linear and nonlinear closed loop biomass tracking simulations where the the initial condition (IC) is near the desired set point (target biomass). Four responses are shown for x and u_p: (1) purely linear; i.e. linear plant model, linear controller, and no saturation, (2) linear with plant saturation; i.e. linear plant model, linear controller, and plant saturation, (3) linear with anti-windup logic; i.e. linear plant model, linear controller, plant saturation, and anti-windup logic, (4) nonlinear; i.e. nonlinear plant model, linear controller, plant saturation, and anti-windup logic. Here, the reference command is very small ($x_{ref} = 0.375$), the control does not saturate, and all of the responses match one another. This shows that the "pure linear theory" suffices under small signal conditions (as expected).
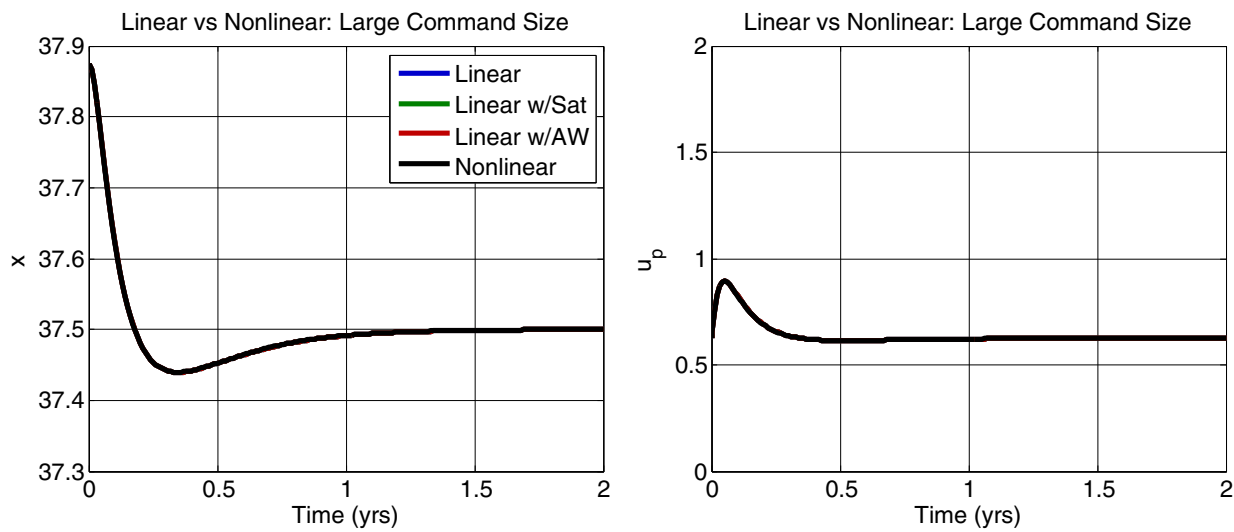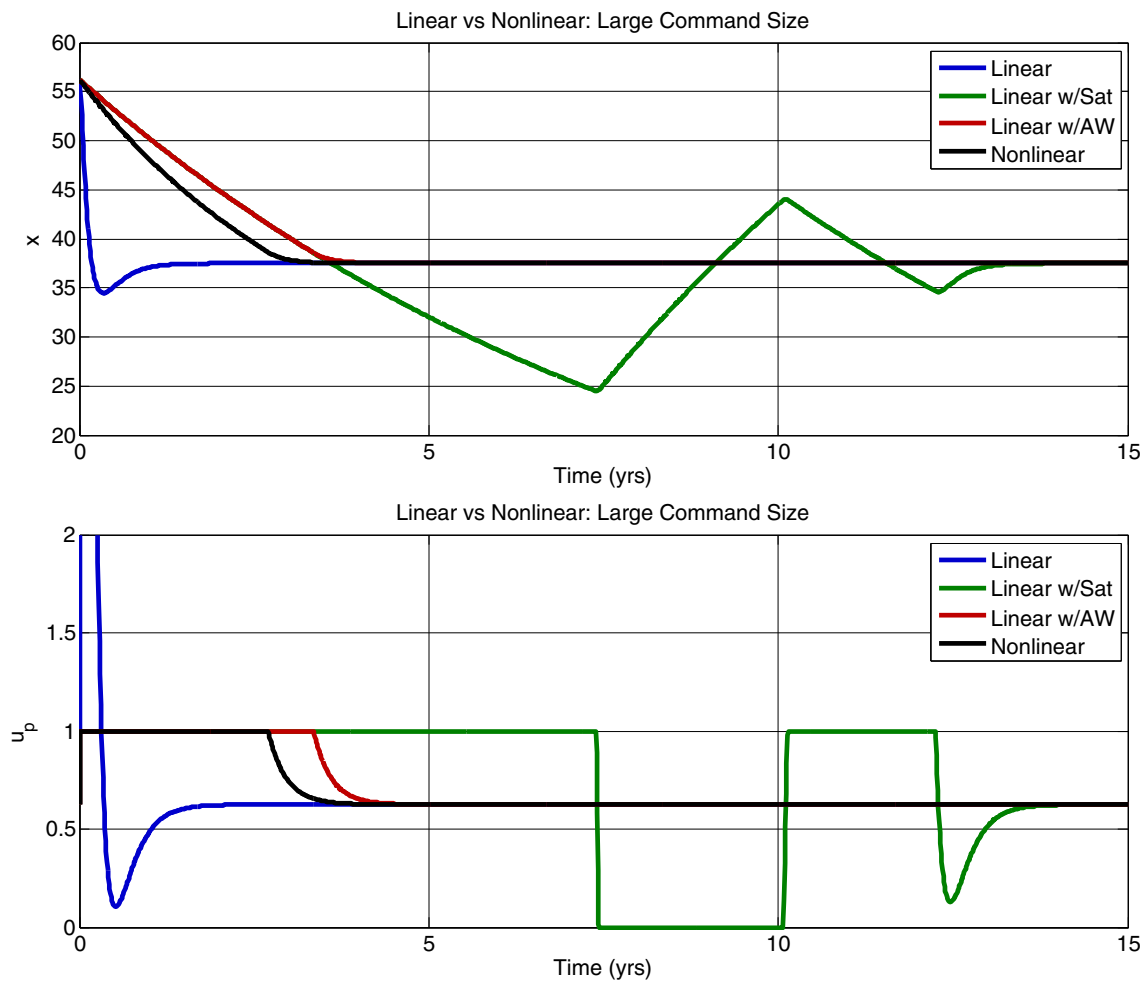
Fig. 11. Linear vs Nonlinear Biomass Tracking: x_o Near x_{ref}

Linear vs Nonlinear Biomass Tracking: x_o **Far From** x_{ref}. Figure 12 compares linear and nonlinear closed loop biomass tracking simulations where the the initial condition (IC) is

Fig. 12. Linear vs Nonlinear Biomass Tracking: x_o Far From x_{ref}

Design of Robust Policies for Uncertain Natural Resource Systems:
Application to the Classic Gordon-Schaefer Fishery Model

439

far from the desired set point (target biomass). Here, the reference command is large ($x_{ref} = 18.75$), the controls saturate, windup is exhibited in the linear w/Sat case, and we observe relatively good agreement between the linear (particularly linear w/AW) and nonlinear responses.

Biomass Tracking Robustness In Presence Of Capacity Uncertainty: Anti-Windup Gain Study. Figure 13 shows how our robust control laws can be adjusted to achieve the "flatter" economic inefficiency of the nominal optimal control law (see Figures 4-5). We observe the following:
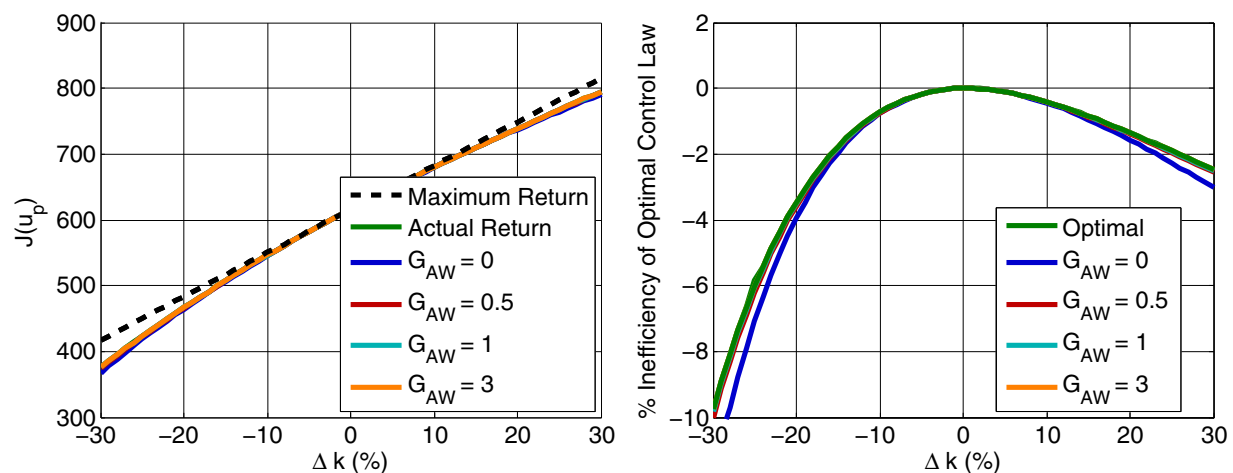
Fig. 13. PI Biomass Tracking Robustness (Capacity Uncertainty): Anti-Windup Gain Study ($\zeta = 1$, $t_s = 1$)

- With an anti-windup implementation, a PI control law can come arbitrarily close to matching the performance of the nominal optimal control law with imperfect information.

- Improving upon the nominal optimal control law with imperfect information requires some outer loop control logic as well as system identification to more appropriately select the reference/target biomass.

The observed performance gap (or inefficiency) is fundamentally because the target x_{ref} differs from the perfect information x_e^*; not because the nominal design plant differs from the truth plant. Closing the observed performance gap further requires an outer loop controller and/or parameter estimation techniques in order to get a more accurate target x_{ref} that is closer to the perfect information target x_e^*.

6. Summary and future directions

Summary. This chapter has shown how ideas from robust control may be applied to a fishery. It has been specifically shown how some small amount of income may be sacrificed for increased robustness with respect to uncertain fishery parameters.

Directions for Future Research. Future work will examine more complex models (e.g. decentralized, distributed), pros/cons associated with parameter estimation schemes, more complex robust control laws (e.g. use of receding horizon control for long-term management), robustness with respect to plant and controller uncertainty (parametric and dynamic).

7. References

Anderies, J.M., Rodriguez, A., Janssen, M. & Cifdaloz, O. (2007). Panaceas, uncertainty, and the robust control framework in sustainability science. *Proceedings of the National Academy of Sciences USA 104*, 15,194–15,199

Andersen, P. (1982). Commercial fisheries under price uncertainty. *Journal of Environmental Economics and Management 9*(1), 11–28

Aström, K. & Hägglund, T. (1995). PID Controllers: Theory, Design, and Tuning. *Int. Society for Measurement and Control*, (2nd Ed.), Research Triangle Park, N.C.

Babonneau, F., Vial, J.P. & Apparigliato, R.: Robust optimization for environmental and energy planning. *In: J.A. Filar, A. Haurie (eds.) Uncertainty and Environmental Decision Making, International Series in Operations Research & Management Science*, vol. 138, chap. 3, pp. 79–126. Springer (2010)

Belmiloudi, A. (2006). Minimax control problems of periodic competing parabolic systems with logistic growth terms. *International Journal of Control 79*(2), 150–161

Belmiloudi, A. (2005). Nonlinear optimal control problems of degenerate parabolic equations with logistic time-varying delays of convolution type. *Nonlinear Analysis-theory Methods & Applications 63*(8), 1126–1152

Ben-Tal, A., Nemirovski, A. (2008). Selected topics in robust convex optimization. *Mathematical Programming 112*(1), 125–158

Ben-Tal, A., Nemirovski, A. (2002). Robust optimization—methodology and applications. *Mathematical Programming 92*(3, Ser. B), 453–480. ISMP 2000, Part 2 (Atlanta, GA)

Ben-Tal, A., El-Ghaoui L. & Nemirovski, A. (2000) Robust semidefinite programming. *In: R. Saigal, L. Vandenberghe, H. Wolkowicz (eds.) Semidefinite Programming and Applications. Kluwer Dordrecht*

Ben-Tal, A., Nemirovski, A.(1998). Robust convex optimization. *Mathematics of Operations Research 23*(4), 769–805

Clark, C. (2007) The Worldwide Crisis in Fisheries: Economic Models and Human Behavior. *Cambridge University Press, Cambridge*, UK

Clark, C. (2006). Fisheries bioeconomics: why is it so misunderstood?, In: *Population Ecology 48*, 95-98

Clark, C., Munro, G. & U.R., S. (2006). Subsidies, buybacks, and sustainable fisheries. *Journal of Environmental Economics and Management 50*, 47–58

Clark, C.W. (1990). Mathematical Bioeconomics: The Optimal Management of Renewable Resources, 2 edn. *J. Wiley*, N.Y., U.S.A.

Clark, C. & Kirkwood, G.: On uncertain renewable resource stocks - optimal harvest policies and the value of stock surveys. *Journal of Environmental Economics and Management 13*(3), 235–244 (1986)

Dercole, F., Gragnani, A., Kuznetsov, Y.A. & Rinaldi, S. (2003) Numerical sliding bifurcation analysis: an application to a relay control system. *Ieee Transactions on Circuits and Systems I-Fundamental Theory and Applications 50*(8), 1058–1063

Gordon, H. (1954) The economic theory of a common property resource: the fishery. *Journal of Political Economy 62*, 124–142

Hansen, L.P. & Sargent, T.J. (2007) Robustness. *Princeton University Press*, Princeton, NJ

Holland, D. & Gudmundsson, E., J., G. (1999) Do fishing vessel buyback programs work? a survey of the evidence. *Marine Policy 23*, 47–69

Holling, C.S., Gunderson, L.H. (2002). Resilience and adaptive cycles. *In: L.H. Gunderson, C.S. Holling (eds.) Panarchy: Understanding Transformations in Systems of Humans and Nature*, chap. 2. Island Press

Holling, C.S. (1986) The resilience of terrestrial ecosystems, local surprise and global change. *In: C.W. C., R.E. Munn (eds.) Sustainable development of the biosphere*, pp. 292–317. Cambridge University Press, Cambridge

Holling, C.S.(1973). Resilience and stability of ecological systems. *Annual Review of Ecology and Systematics 4*, 1–23

ISRIC. (1990). Global assessment of the status of human induced soil degradation (glasdod). *dataset, International Soil Reference and Information Centre*, Wageningen, The Netherlands

Kendrick, D.A. (2005). Stochastic control for economic models: past, present and the paths ahead. *Journal of Economic Dynamics and Control 29*, 3–30

Koenig, E.F. (1984). Controlling stock externalities in a common property fishery subject to uncertainty. *Journal of Environmental Economics and Management 11*(2), 124–138

Lempert, R.J., Groves, D.G., Popper, S.W. & Bankes, S.C. (2006) A general, analytic method for generating robust strategies and narrative scenarios. *Management Science 52*(4), 514–528

Lempert, R.J., Schlesinger, M.E. (2000). Robust strategies for abating climate change - an editorial essay. *Climatic Change 45*(3-4), 387–401

Ludwig, D., Walker, B. & Holling, C.S. (1997). Sustainability, stability, and resilience. *Conservation Ecology 1*(1), (online) URL: http://www.consecol.org/vol1/iss1/art7/

Ludwig, D. & Walters, C. (1982). Optimal harvesting with imprecise parameter estimates. *Ecological Modelling 14*, 273–292

Moran, E.F., Ostrom, E. (eds.) (2006). Seeing the Forest and the Trees: Human-Environment Interactions in Forest Ecosystems. *MIT Press*, Cambridge, MA

Myers, R. & Worm, B. (2003). Rapid worldwide depletion of predatory fish communities. *Nature 423*, 280–283

Ogata, K. (1995). Discrete-Time Control Systems. *Prentice Hall*, (2nd Ed.), N.J.

Rodriguez, A.A., Cifdaloz, O., Anderies, J.M., Janssen, J. & Dickeson, J., (2010). Confronting Management Challenges in Highly Uncertain Natural REsource Systems: a Robustness-Vulnerability Trade-off Approach," *Environmental Modeling & Assessment*, Volume 16, Issue 1 (2011), Page 15.

Rodriguez, A.A. (2004). Analysis and Design of Multivariable Feedback Control Systems. *Control3D, LLC.*, Tempe, AZ

Rodriguez, A.A. (2003). Analysis and Design of Feedback Control Systems. *Control3D, LLC.*, Tempe, AZ

Schaefer, M. (1957). Some considerations of population dynamics and economics in relation to the management of the commercial marine fisheries. *Journal of the Fisheries Research Board of Canada 14*(5), 669–681

Sethi, G., Costello, C., Fisher, A., Hanemann, M. & Karp, L. (2005). Fishery management under multiple uncertainty. *Journal of Environmental Economics and Management 50*(2), 300–318

Shah, T., Molden, D., Sakthivadivel, R., Seckler. (2000). The global groundwater situation: Overview of opportunities and challenges. *Tech. rep., International Water Management Institute, Colombo, Sri Lanka*

Walters, C. (1986). Adaptive Management of Renewable Resources. *MacMillan Publishing Co.*

Weitzman, M. (2002). Landing fees vs harvest quotas with uncertain fish stocks. *Journal of environmental economics and management 43*(2), 325–338

Wilen, J., Homans, F. (1998). What do regulators do? Dynamic behavior of resource managers in the North Pacific Halibut Fishery 1935-1978. *Ecological Economics 24*(2-3), 289–298

Zhou, K., Doyle, J. (1998). Essentials of Robust Control. *Prentice Hall*
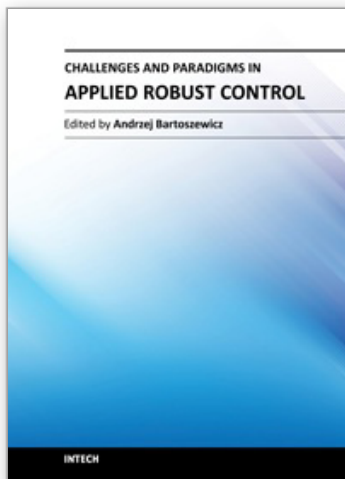
Challenges and Paradigms in Applied Robust Control

Edited by Prof. Andrzej Bartoszewicz

The main objective of this book is to present important challenges and paradigms in the field of applied robust control design and implementation. Book contains a broad range of well worked out, recent application studies which include but are not limited to H-infinity, sliding mode, robust PID and fault tolerant based control systems. The contributions enrich the current state of the art, and encourage new applications of robust control techniques in various engineering and non-engineering systems.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Armando A. Rodriguez, Jeffrey J. Dickeson, John M. Anderies and Oguzhan Cifdaloz (2011). Design of Robust Policies for Uncertain Natural Resource Systems: Application to the Classic Gordon-Schaefer Fishery Model, Challenges and Paradigms in Applied Robust Control, Prof. Andrzej Bartoszewicz (Ed.), ISBN: 978-953-307-338-5, InTech, Available from: http://www.intechopen.com/books/challenges-and-paradigms-in-applied-robust-control/design-of-robust-policies-for-uncertain-natural-resource-systems-application-to-the-classic-gordon-s

INTECH
open science | open minds