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Robust Control of Mechanical Systems

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1. Introduction

Control of mechanical systems has been an important problem since several years ago. For free-motion systems, the dynamics is often modeled by ordinary differential equations arising from classical mechanics. Controllers based on feedback linearization, adaptive, and robust techniques have been proposed to control this class of systems (Brogliato et al., 1997; Slotine & Li, 1988; Spong & Vidyasagar, 1989).

Many control algorithms proposed for these systems are based on models where practical situations like parameter uncertainty, external disturbances, or friction force terms are not taken into account. In addition, a complete availability of the state variables is commonly assumed (Paden & Panja, 1988; Takegaki & Arimoto, 1981; Wen & Bayard, 1988). In practice, however, the position is usually the only available measurement. In consequence, the velocity, which may play an important role in the control strategy, must be calculated indirectly, often yielding an inaccurate estimation.

In (Makkar et al., 2007), a tracking controller that includes a new differentiable friction model with uncertain nonlinear terms is developed for Euler-Lagrange systems. The technique is based on a model and the availability of the full state. In (Patre et al., 2008), a similar idea is presented for systems perturbed by external disturbances. Moreover, some robust controllers have been proposed to cope with parameter uncertainty and external disturbances. H_∞ control has been a particularly important approach. In this technique, the control objective is expressed as a mathematical optimization problem where a ratio between some norms of output and perturbation signals is minimized (Isidori & Astolfi, 1992). It is used to synthesize controllers achieving robust performance of linear and nonlinear systems.

In general, the control techniques mentioned before yield good control performance. However, the mathematical operations needed to calculate the control signal are rather complex, possibly due to the compensation of gravitational, centrifugal, or Coriolis terms, or the need to solve a Hamilton-Jacobi-Isaacs equation. In addition, if an observer is included in the control system, the overall controller may become rather complex.

Another method exhibiting good robustness properties is the sliding mode technique (Perruquetti & Barbot, 2002; Utkin, 1992). In this method, a surface in the state space is made attractive and invariant using discontinuous terms in the control signal, forcing the system to converge to the desired equilibrium point placed on this surface, and making the controlled dynamics independent from the system parameters. These controllers display good performance for regulation and tracking objectives (Utkin et al., 1999; Weibing & Hung, 1993;

Yuzhuo & Flashner, 1998). Unfortunately, they often exhibit the chattering phenomenon, displaying high-frequency oscillations due to delays and hysteresis always present in practice. The high-frequency oscillations produce negative effects that may harm the control devices (Utkin et al., 1999). Nevertheless, possibly due to the good robust performance of sliding mode controllers, several solutions to alleviate or eliminate chattering have been developed for some classes of systems (Bartolini et al., 1998; Curk & Jezernik, 2001; Erbatur & Calli, 2007; Erbatur et al., 1999; Pushkin, 1999; Sellami et al., 2007; Xin et al., 2004; Wang & Yang, 2007).

In the previous works, it is also assumed that the full state vector is available. However, in practice it is common to deal with systems where only some states are measured due to technological or economical limitations, among other reasons. This problem can be solved using observers, which are models that, based on input-output measurements, estimate the state vector.

To solve the observation problem of uncertain systems, several approaches have been developed (Davila et al., 2006; Rosas et al., 2006; Yaz & Azemi, 1994), including sliding mode techniques (Aguilar & Maya, 2005; Utkin et al., 1999; Veluvolu et al., 2007). The sliding mode observers open the possibility to use the equivalent output injection to identify disturbances (Davila et al., 2006; Orlov, 2000; Rosas et al., 2006).

In this chapter, we describe a control structure designed for mechanical systems to solve regulation and tracking objectives (Rosas et al., 2010). The control technique used in this structure is combined with a discontinuous observer. It exhibits good performance with respect to parameter uncertainties and external disturbances. Because of the included observer, the structure needs only the generalized position and guarantees a good convergence to the reference with a very small error and a control signal that reduces significantly the chattering phenomenon. The observer estimates not only the state vector but, using the equivalent output injection method, it estimates also the plant perturbations produced by parameter uncertainties, non-modeled dynamics, and other external torques. This estimated perturbation is included in the controller to compensate the actual disturbances affecting the plant, improving the performance of the overall control system.

The robust control structure is designed in a modular way and can be easily programed. Moreover, it can be implemented, if needed, with analog devices from a basic electronic circuit having the same structure for a wide class of mechanical systems, making its analog implementation also very easy (Alvarez et al., 2009). Some numerical and experimental results are included, describing the application of the control structure to several mechanical systems.

2. Control objective

Let us consider a mechanical system with $n-$degree of freedom (DOF), modeled by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \Phi(q,\dot{q},\ddot{q})\theta + \gamma(t) = u = \tau_0 + \Delta_\tau. \tag{1}$$

$q \in \mathbb{R}^n$, $\dot{q} = dq/dt$, $\ddot{q} = d^2q/dt^2$ denote the position, velocity, and acceleration, respectively; M and C are the inertia and Coriolis and centrifugal force matrices, G is the gravitational force, $\Phi\theta$ includes all the parameter uncertainties, and γ, which we suppose bounded by a constant σ, that is, $||\gamma(t)|| < \sigma$, denotes a external disturbance. τ_0 and Δ_τ are control inputs. Note that, under this formulation, the terms M, C, and G are well known. If not, it is known that they can be put in a form linear with respect to parameters and can be included in $\Phi\theta$ (Sciavicco & Siciliano, 2000).

We suppose that τ_0, which may depend on the whole state $(q, \dot{q})$, denotes a feedback controller designed to make the state $(q, \dot{q})$ follow a reference signal $(q_r, \dot{q}_r)$, with an error depending on the magnitude of the external disturbance γ and the uncertainty term $\Phi\theta$, but keeping the tracking error bounded. We denote this control as the "nominal control". We propose also to add the term Δ_τ, and design it such that it confers the following properties to the closed-loop system.

1. The overall control $u = \tau_0 + \Delta_\tau$ greatly reduces the steady-state error, provided by τ_0 only, under the presence of the uncertainty θ and the disturbance γ.
2. The controller uses only the position measurement.

Note that, for the nominal control, the steady state error is normally different to zero, usually large enough to be of practical value, and the performance of the closed-loop system may be poor. The role of the additional control term Δ_τ is precisely to improve the performance of the system driven by the nominal control.
The nominal control can be anyone that guarantees a bounded behavior of system (1). In this chapter we use a particular controller and show that, under some conditions, it preserves the boundedness of the state. In particular, suppose the control aim is to make the position q track a smooth signal q_r, and define the plant state as

$$e_1 = q - q_r, \quad e_2 = \dot{q} - \dot{q}_r. \tag{2}$$

Suppose also that the nominal control law is given by

$$\tau_0 = -M(\cdot)\left[K_p e_1 + K_v e_2 - \ddot{q}_r(t)\right] + C(\cdot)(e_2 + \dot{q}_r) + G(\cdot), \tag{3}$$

where K_p and K_v are $n \times n$-positive definite matrices. However, because the velocity is not measured, we need to use an approximation for the velocity error, which we denote as $\hat{e}_2 = \dot{\hat{q}} - \dot{q}_r$. This will be calculated by an observer, whose design is discussed in the next section. Suppose that the exact velocity error and the estimated one are related by $e_2 = \hat{e}_2 + \epsilon_2$. Then, if we use the estimated velocity error, the practical nominal control will be given by

$$\hat{\tau}_0 = -M(\cdot)(K_p e_1 + K_v \hat{e}_2 - \ddot{q}_r) + \hat{C}(\cdot)(\hat{e}_2 + \dot{q}_r) + G(\cdot). \tag{4}$$

Moreover, the approximated Coriolis matrix $\hat{C}$ can be given the form

$$\hat{C}(\cdot) = C(q, \dot{\hat{q}}) = C(\cdot, \hat{e}_2 + \dot{q}_r) = C(\cdot, e_2 + \dot{q}_r) - \Delta C(\cdot),$$

where $\Delta C = \mathcal{O}(\|\epsilon_2\|)$. Then the state space representation of system (1), with the control law (4), is given by

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= -K_p e_1 - K_v e_2 + \xi(e, t) + \Delta u, \end{aligned} \tag{5}$$

where

$$\xi(\cdot) = -M^{-1}\left[(\hat{C} - MK_v)\epsilon_2 + \Delta C(e_2 + \dot{q}_r) + \Phi\theta + \gamma\right], \tag{6}$$

and $\Delta u = M^{-1}(\cdot)\Delta_\tau$ is a control adjustment to robustify the closed-loop system. When $\Delta u = 0$, a well established result is that, if

$$||\xi(e,t)|| < \rho_1||e|| + \rho_0, \ \ \rho_i > 0, \tag{7}$$

then there exist matrices K_p and K_v such that the state e of system (5) is bounded (Khalil, 2002). In fact, the bound on the state e can be made arbitrarily small by increasing the norm of matrices K_p and K_v.

The control objective can now be established as design a control input Δu that, depending only on the position, improves the performance of the control $\hat{\tau}_0$ by attenuating the effect of parameter uncertainty and disturbances, concentrated in ξ.

Note that disturbances acting on system (5) satisfy the matching condition (Khalil, 2002). Hence, it is theoretically possible to design a compensation term Δu to decouple the state e_1 from the disturbance ξ. The problem analyzed here is more complicated, however, because the velocity is not available.

In the next Section we solve the problem of velocity estimation using two observers that guarantee convergence to the states (e_1, e_2). Moreover, an additional property of these observers will allow us to have an estimation of the disturbance term ξ. This estimated perturbation will be used in the control Δu to compensate the actual disturbances affecting the plant.

3. Observation of the plant state

In this section we describe two techniques to estimate the plant state, yielding exponentially convergent observers.

3.1 A discontinuous observer

Discontinuous techniques for designing observers and controllers have been intensively developed recently, due to their robustness properties and, in some cases, finite-time convergence. In this subsection we describe a simple technique, just to show the observer performance.

The observer has been proposed in (Rosas et al., 2006). It guarantees exponential convergence to the plant state, even under the presence of some kind of uncertainties and disturbances.

Let us consider the system (5). The observer is described by

$$\begin{bmatrix} \dot{\hat{e}}_1 \\ \dot{\hat{e}}_2 \end{bmatrix} = \begin{bmatrix} \hat{e}_2 + C_2\epsilon_1 \\ -K_p e_1 - K_v \hat{e}_2 + \Delta u + C_1\epsilon_1 + C_0 \mathrm{sign}(\epsilon_1) \end{bmatrix}, \tag{8}$$

where $\hat{e}_1 \in \mathbb{R}^n$ and $\hat{e}_2 \in \mathbb{R}^n$ are the states of the observer, $\epsilon_1 = e_1 - \hat{e}_1$. C_0, C_1, and C_2 are diagonal, positive-definite matrices defined by

$$C_i = \mathrm{diag}\{c_{i1}, c_{i2}, \ldots, c_{in}\} \qquad \text{for } i = 0, 1, 2.$$

The signum vector function $\mathrm{sign}(\cdot)$ is defined as

$$\mathrm{sign}(v) = [\mathrm{sign}(v_1), \mathrm{sign}(v_2), \ldots, \mathrm{sign}(v_n)]^T.$$

Then, the dynamics of the observation error $\epsilon = (\epsilon_1, \epsilon_2) = (e_1 - \hat{e}_1, e_2 - \hat{e}_2)$, are described by

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \end{bmatrix} = \begin{bmatrix} \epsilon_2 - C_2\epsilon_1 \\ -C_1\epsilon_1 - K_v\epsilon_2 - C_0\text{sign}(\epsilon_1) + \xi(e,t) \end{bmatrix}. \tag{9}$$

An important result is provided by (Rosas et al., 2006) for the case where $\rho_1 = 0$ (see equation (7)). Under this situation we can establish the conditions to have a convergence of the estimated state to the plant state.

Theorem 1. *(Rosas et al., 2006) If (7) is satisfied with $\rho_1 = 0$, then there exist matrices C_0, C_1, and C_2, such that system (9) has the origin as an exponentially stable equilibrium point. Therefore, $\lim_{t\to\infty} \hat{e}(t) = e(t)$.*

The proof of this theorem can be found in (Rosas et al., 2006). In fact, a change of variables given by $v_1 = \epsilon_1$, $v_2 = \epsilon_2 - C_2\epsilon_1$, allows us to express the dynamics of system (9) by

$$\begin{aligned} \dot{v}_1 &= v_2, \\ \dot{v}_2 &= -(C_1 + K_vC_2)v_1 - (C_2 + K_v)v_2 - C_0\text{sign}(v_1) + \xi(e,t), \end{aligned} \tag{10}$$

where v_1 and v_2 are vectors with the form

$$v_i = (v_{i1}, v_{i2}, \ldots, v_{in})^T; \;\; i = 1, 2.$$

Then system (10) can be expressed as a set of second-order systems given by

$$\begin{aligned} \dot{v}_{1i} &= v_{2i}, \\ \dot{v}_{2i} &= -\tilde{c}_{1i}v_{1i} - \tilde{c}_{2i}v_{2i} - c_{0i}\text{sign}(v_{1i}) + \xi_i(\cdot), \end{aligned} \tag{11}$$

where $\tilde{c}_{1i} = c_{1i} + k_{vi}c_{2i}$, $\tilde{c}_{2i} = c_{2i} + k_{vi}$, for $i = 1, \ldots, n$, and $|\xi_i| \leq \beta_i$, for some positive constants β_i. The conditions to have stability of the origin are given by

$$\tilde{c}_{1i} > 0, \tag{12}$$

$$\tilde{c}_{2i} > 0, \tag{13}$$

$$c_{0i} > 2\lambda_{\max}(P_i)\sqrt{\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_i)}}\left(\frac{\tilde{c}_{1i}\beta_i}{\theta}\right), \tag{14}$$

for some $0 < \theta < 1$, where P_i is a 2×2 matrix that is the solution of the Lyapunov equation $A_i^T P_i + P_i A_i = -I$, and the matrix A_i is defined by

$$A_i = \begin{bmatrix} 0 & 1 \\ -\tilde{c}_{1i} & -\tilde{c}_{2i} \end{bmatrix}.$$

System (10) displays a second-order sliding mode (Perruquetti & Barbot, 2002; Rosas et al., 2010) determined by $v_1 = \dot{v}_1 = \ddot{v}_1 = 0$. To determine the behavior of the system on the sliding surface, the equivalent output injection method can be used (Utkin, 1992), hence

$$\ddot{v}_1 = -u_{eq} + \xi(e,t) = 0, \tag{15}$$

where u_{eq} is related to the discontinuous term $C_0\text{sign}(v_1)$ of equation (10). The equivalent output injection u_{eq} is then given by (Rosas et al., 2010; Utkin, 1992)

$$u_{eq} = \xi(e,t). \tag{16}$$

This means that the equivalent output injection corresponds to the perturbation term, which can be recovered by a filter process (Utkin, 1992). In fact, in this reference it is shown that the equivalent output injection coincides with the slow component of the discontinuous term in (10) when the state is in the discontinuity surface. Hence, it can be recovered using a low pass filter with a time constant small enough as compared with the slow component response, yet sufficiently large to filter out the high rate components.
For example, we can use a set of n second-order, low-pass Butterworth filter to estimate the term u_{eq}. These filters are described by the following normalized transfer function,

$$F_i(s) = \frac{\omega_{c_i}^2}{s^2 + 1.4142\omega_{c_i}s + \omega_{c_i}^2}, \qquad i = 1,\ldots,n, \tag{17}$$

where ω_{c_i} is the cut-off frequency of each filter. Here, the filter input is the discontinuous term of the observer, $c_{0_i}\text{sign}(v_{1i})$. By denoting the output of the filter set of as $x_f \in \mathbb{R}^n$, and choosing a set of constants ω_{c_i} that minimizes the phase-delay, it is possible to assume

$$\lim_{t\to\infty} x_f = \tilde{\xi}(\cdot) \approx \xi(\cdot), \tag{18}$$

where $\|\tilde{\xi}(\cdot) - \xi(\cdot)\| \leq \tilde{\rho}$ for $\tilde{\rho} \ll \rho_0$.

3.2 An augmented, discontinuous observer

A way to circumvent the introduction of a filter is to use an augmented observer. To simplify the exposition, consider a 1-DOF whose tracking error equations have the form of system (5). An augmented observer is proposed to be

$$\begin{aligned}\dot{\hat{e}}_1 &= w_1 + c_{21}(e_1 - \hat{e}_1),\\ \dot{w}_1 &= c_{11}(e_1 - \hat{e}_1) + c_{01}\text{sgn}(e_1 - \hat{e}_1),\\ \dot{\hat{e}}_2 &= w_2 + c_{22}(w_1 - \hat{e}_2) - K_p e_1 - K_v\hat{e}_2 + \Delta u,\\ \dot{w}_2 &= c_{12}(w_1 - \hat{e}_2) + c_{02}\text{sgn}(w_1 - \hat{e}_2).\end{aligned} \tag{19}$$

If we denote the observation error as $\epsilon_1 = e_1 - \hat{e}_1$, $\epsilon_2 = e_2 - \hat{e}_2$, we arrive at

$$\begin{aligned}\dot{\epsilon}_1 &= -c_{21}\epsilon_1 - w_1 + e_2,\\ \dot{w}_1 &= c_{11}\epsilon_1 + c_{01}\text{sgn}(\epsilon_1),\\ \dot{\epsilon}_2 &= -(K_v + c_{22})\epsilon_2 - w_2 - c_{22}(w_1 - e_2) + \xi,\\ \dot{w}_2 &= c_{12}(w_1 - e_2 + \epsilon_2) + c_{02}\text{sgn}(w_1 - e_2 + \epsilon_2).\end{aligned} \tag{20}$$

A change of variables given by

$$\begin{aligned}v_{11} &= \epsilon_1,\\ v_{12} &= -c_{21}\epsilon_1 - w_1 + e_2,\end{aligned}$$

$$
\begin{aligned}
v_{21} &= w_1 - e_2 + \epsilon_2, \\
v_{22} &= \dot{v}_{21} = -c_{22}v_{21} - K_v\epsilon_2 + \dot{w}_1 - \dot{e}_2 - w_2 + \xi
\end{aligned}
$$

converts the system to

$$
\begin{aligned}
\dot{v}_{11} &= v_{12}, \\
\dot{v}_{12} &= -c_{11}v_{11} - c_{21}v_{12} - c_{01}\mathrm{sgn}(v_{11}) + \dot{e}_2, \\
\dot{v}_{21} &= v_{22}, \\
\dot{v}_{22} &= -\tilde{c}_{12}v_{21} - c_{22}v_{22} - c_{02}\mathrm{sgn}(v_{21}) + \tilde{\dot{\xi}},
\end{aligned} \tag{21}
$$

where $\tilde{c}_{12} = c_{12} - K_v c_{22}$ and $\tilde{\dot{\xi}}$ is a disturbance term that we suppose bounded. Under some similar conditions discussed in the previous section, particularly the boundedness of $\dot{e}_2$ and $\tilde{\dot{\xi}}$, we can assure the existence of positive constants c_{ij} such that v_{ij} converges to zero, so $\hat{e}_1$ converges to e_1, w_1 and $\hat{e}_2$ to e_2, and w_2 converges to the disturbance ξ. This observer Hence we propose to use the redesigned control Δu, or Δ_τ, as (see equation (5))

$$
\Delta u = -w_2 \ \rightarrow -\xi, \quad \Delta_\tau = -M(\cdot)w_2
$$

to attenuate the effect of disturbance ξ in system (5) or in system (1), respectively.

4. The controller

As we mentioned previously, we propose to use the nominal controller (4) because the velocity is not available from a measurement. We can use any of the observers previously described, and replace the velocity e_2 by its estimation, $\hat{e}_2$. The total control is then given by

$$
\tau = \tau_0 + \Delta_\tau = -M(\cdot)\left[\nu + K_p e_1 + K_v \hat{e}_2 - \ddot{q}_r(t)\right] + C(\cdot)(\hat{e}_2 + \dot{q}_r) + G(\cdot), \tag{22}
$$

where ν is the redesigned control. This control adjustment is proposed to be $\nu = x_f$, where x_f is the output of filter (17), if the first observer is used (system (8)), or $\nu = w_2$, where w_2 is the last state of system (19), if the second observer is chosen.

The overall structure is shown in figure 1 when the first observer is used.

A similar structure is used for the second observer. An important remark is that the nominal control law (a PD-controller with compensation of nonlinearities in this case) can be chosen independently; the analysis can be performed in a similar way. However, this nominal controller must provide an adequate performance such that the state trajectories remain bounded.

5. Control of mechanical systems

To illustrate the performance of the proposed control structure we describe in this section its application to control some mechanical systems, a Mass-Spring-Damper (MSD), an industrial robot, and two coupled mechanical systems which we want them to work synchronized.

5.1 An MSD system

This example illustrates the application of the first observer (equation (8), Section 3.1).

Consider the MSD system shown in figure 2. Its dynamical model is given by equation (1),

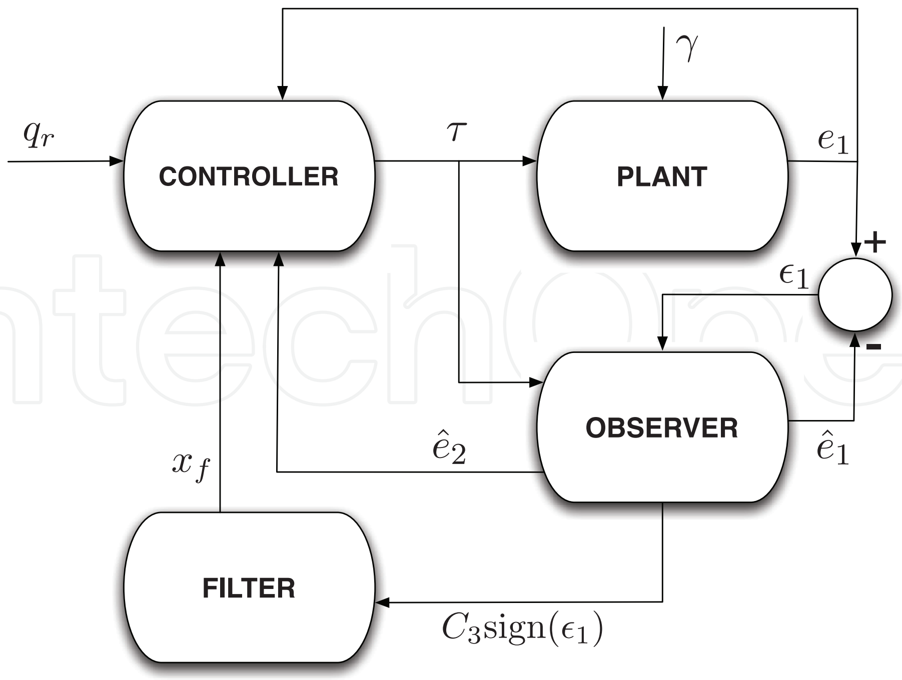

Fig. 1. The robust control structure.

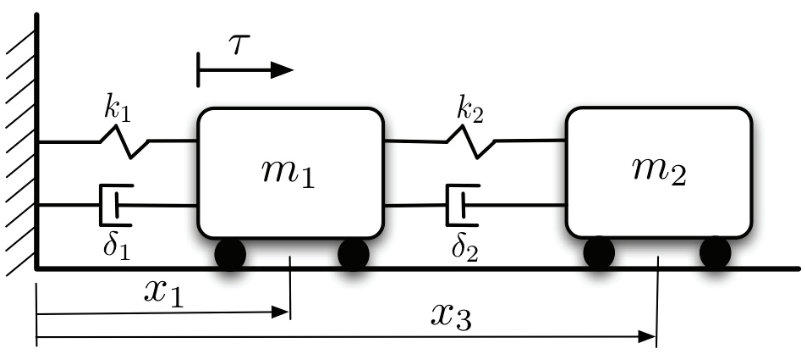

Fig. 2. Mass-spring-damper mechanical system.

with

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad C = \begin{pmatrix} \delta_1 + \delta_2 & -\delta_2 \\ -\delta_2 & \delta_2 \end{pmatrix}, \quad G = \begin{pmatrix} (k_1 + k_2)x_1 - k_2 x_3 \\ k_2(x_3 - x_1) \end{pmatrix}, \quad u = \begin{pmatrix} \tau \\ 0 \end{pmatrix},$$

where $x_1 = q_1$, $x_3 = q_2$. Consider that parameters k_i, δ_i, and m_i, for $i = 1, 2$, are known. Note also that the system is underactuated, and only one control input is driving the system at mass m_1. Therefore, we aim to control the position of mass 1 (x_1), and consider that the action of the second mass is a disturbance. Hence, the model of the controlled system is again given by equation (1), but now with $M = m_1$, $C = \delta_1$, $G = k_1 q$. If we denote $x_1 = q$, $x_2 = \dot{q}$, and $x = (x_1, x_2, x_3, x_4) = (x_1, \dot{x}_1, x_3, \dot{x}_3)$ (see figure 2), then

$$\Gamma(x, \dot{x}; \theta) = \Phi(x, \dot{x})\theta + \gamma = k_2(x_1 - x_3) + \delta_2(x_2 - x_4),$$

where x_3 and x_4 are the solutions of the system

$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = -\frac{k_2}{m_2}(x_3 - x_1) - \frac{\delta_2}{m_2}(x_4 - x_2),$$

groups the effect of uncertainty and disturbance terms $\Phi\theta + \gamma$ of equation (1).
Now denote as $e_1 = x_1 - q_r$, $\hat{e}_2 = \hat{x}_2 - \dot{q}_r$, then the nominal control input τ_0 is proposed as equation (3), that is,

$$\tau_0 = -m_1 \left[K_p e_1 + K_v \hat{e}_2 - \ddot{q}_r(t)\right] + k_1 x_1 + \delta_1 \hat{x}_2, \tag{23}$$

where K_p and K_v are positive constants. Because the velocity is not measured, in (23) we have used the estimation $\hat{x}_2 = \hat{e}_2 + \dot{q}_r$, delivered by the observer given by (8).
With an adequate selection of the constants K_p and K_v we can guarantee that the perturbation $\Gamma(\cdot)$ in (1) is bounded (see Section 2 and (Khalil, 2002)). Therefore, from equation (16), $u_{eq} = \Gamma(\cdot)$.
Using the filter (17), we can recover an estimation of the disturbance, denoted as x_f. Therefore, the redesigned control will be $\Delta_\tau = m_1 x_f$ which, added to (23), adjusts the nominal control input to attenuate the effect of the disturbance Γ.
A numerical simulation was performed with plant parameter values $k_1 = 10\left[k_g m/sec^2\right]$, $k_2 = 20\left[k_g m/sec^2\right]$, $\delta_1 = \delta_2 = 0.1\left[k_g m/sec\right]$, $m_1 = 1\,[kg]$, and $m_2 = 4\,[kg]$. The observer parameter values were set to $c_1 = 2$, $c_2 = 2$, and $c_0 = 3$, with controller gains $K_p = K_v = 10$, and filter frequencies $\omega_c = 500[rad/sec]$. In this simulation the nominal control τ_0 was applied from 0 to 15 sec. The additional control term Δ_τ is activated from 15 to 30 sec. The aim is to track the reference signal $q_r(t) = 0.25\sin(t)$.
Figure 3 shows the response of this controlled system.
Figures a) and b) show the convergence of the observer state to the plant state, in spite of disturbances produced by the mass m_2. Figure c) shows the disturbance identified by this observer. The response of the closed-loop system is presented in Figures d), e), and f). Here we see a tracking error when the additional control term Δ_τ is not present (from 0 to 15 seconds). However, when this term is incorporated to the control signal, at $t = 15$ sec, the tracking error tends to zero. It is important to note that, contrary to typical sliding mode controllers, the control input (Figure 3.f) does not contain high frequency components of large amplitude.

5.2 An industrial robot

This is an example of the application of the first observer (Section 3.1) to a real system.
In this section we show the application of the described technique to control the first two joints of a Selective Compliant Assembly Robot Arm (SCARA), shown in figure 4, used in the manufacturing industry, and manufactured by Sony®.
In this experiment we have an extreme situation because all parameters are unknown. The control algorithm was programed in a PC using the Matlab® software, and the control signals are applied to the robot via a data acquisition card for real-time PC-based applications, the DSpace® 1104. The desired trajectory, which was the same for both joints, is a sinusoidal signal given by $q_r(t) = \sin(t)$.

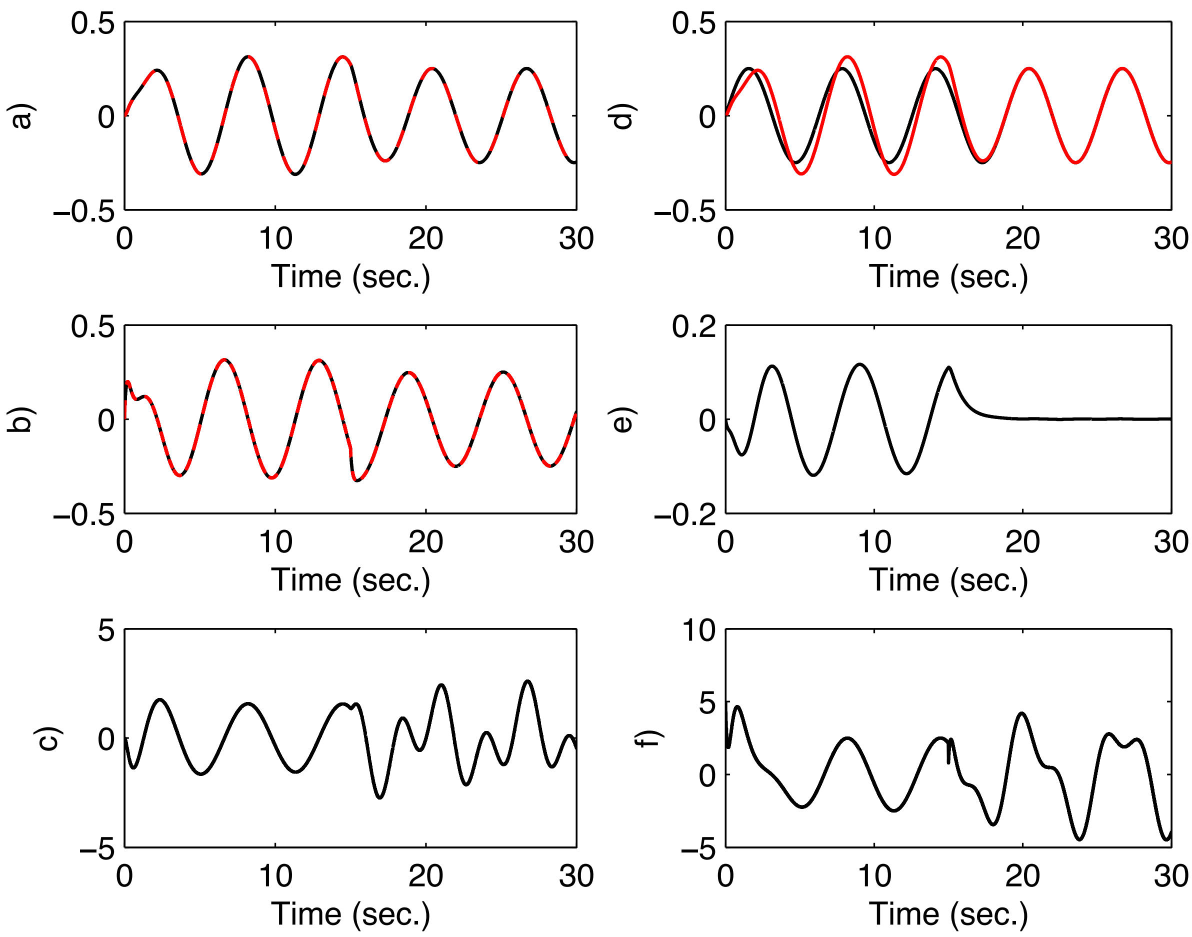

Fig. 3. Response of the closed-loop MSD system. a) x_1 (red) and $\hat{x}_1 = \hat{e}_1 + q_r$ (black); b) x_2 (red) and $\hat{x}_2 = \hat{e}_2 + \dot{q}_r$ (black), c) identified disturbance, x_f, d) reference q_r (black) and position x_1 (red); e) error $e_1 = x_1 - q_r$; f) control $\tau = \tau_0 + \Delta_\tau$.

In the design of the observer (8) the following matrices were selected,

$$C_0 = \begin{bmatrix} 300 & 0 \\ 0 & 300 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}, \quad M^{-1} = \begin{bmatrix} 55.549 & 0 \\ 0 & 55.549 \end{bmatrix}.$$

A cut-off frequency $\omega_{ci} = 75$ rad/seg was selected for the filter(17). The control law is given by the controller (22), where

$$K_p = \begin{bmatrix} 668 & 0 \\ 0 & 391 \end{bmatrix}, \; K_v = \begin{bmatrix} 379 & 0 \\ 0 & 49 \end{bmatrix}.$$

Note that a nominal value of matrix M was used. Differences between nominal and the actual matrix $M(q)$ are supposed to be included in the perturbation term, as well as the Coriolis, centrifugal, and friction forces, external disturbances, parametric variations and coupling effects.

The perturbation terms $\xi_i(\cdot)$ for $i = 1, 2$ that correspond to perturbations present in the two joints are displayed in Figure 5.

Fig. 4. A SCARA industrial robot.

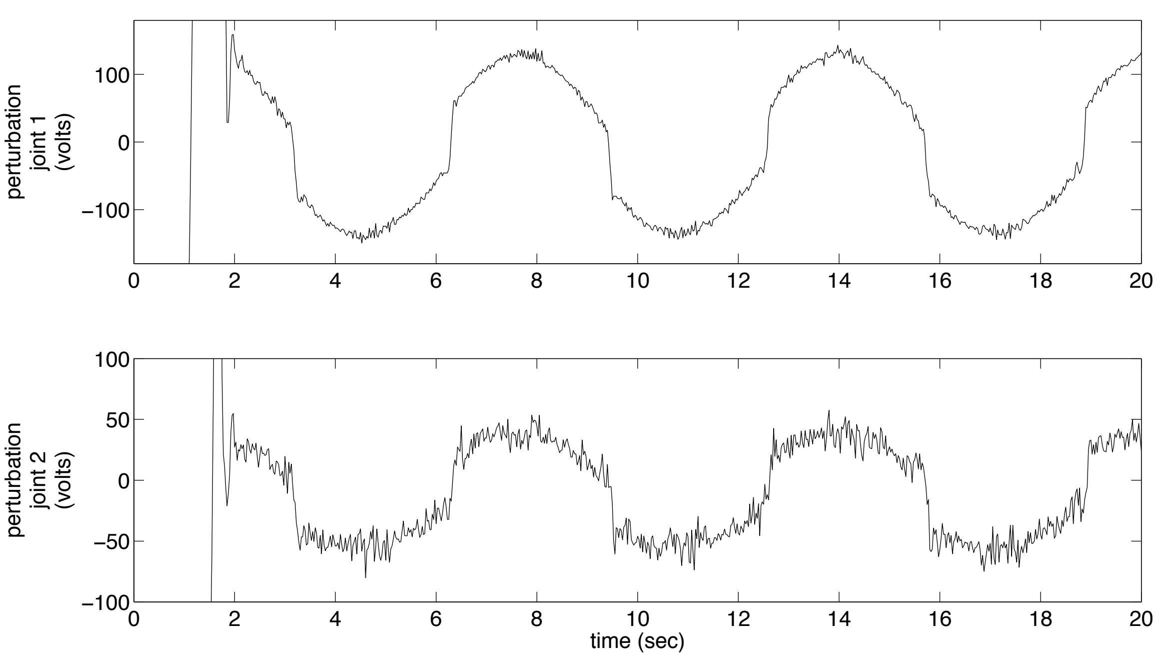

Fig. 5. Identified perturbation terms in the joints of an industrial robot. Up: joint 1 perturbation. Down: joint 2 perturbation.

To verify the observer performance, the observation errors $e_i = \theta_i - \hat{\theta}_i$, for $i = 1, 2$, are displayed in Figure 6, showing small steady-state values.

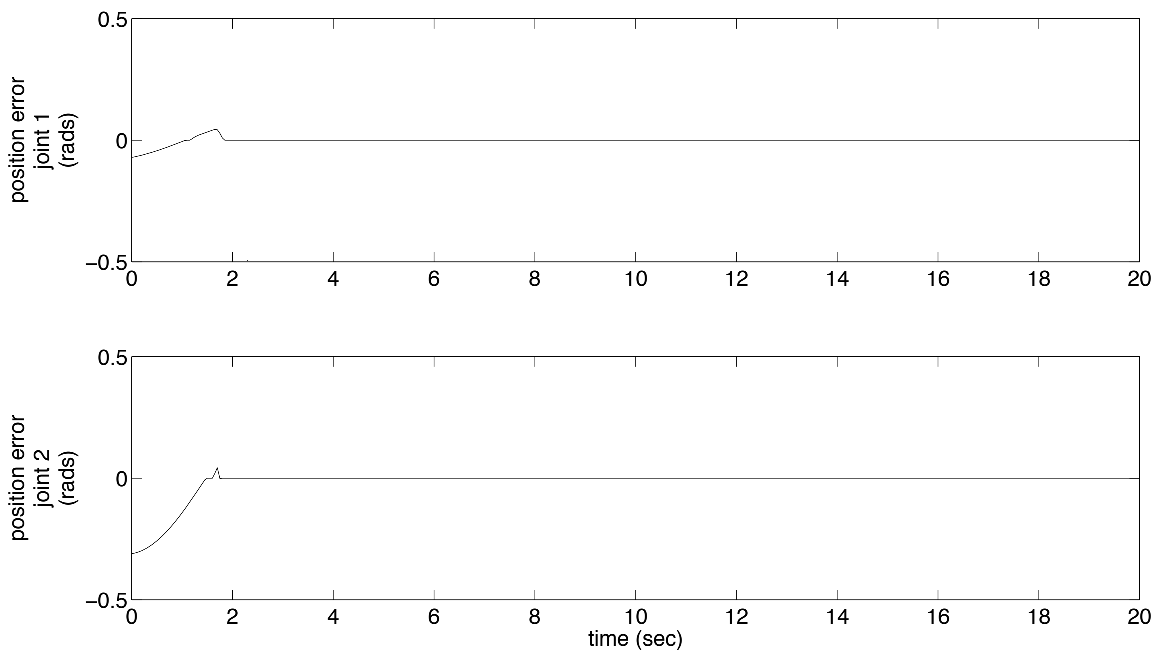

Fig. 6. Observation position errors of the industrial robot.

Figure 7 shows the system output and the reference. Control inputs for joints 1 and 2 are displayed in Figure 8.
Although these control inputs exhibit high frequency components with small amplitude, they do not produce harmful effects on the robot. Also, it is interesting to note that the control input levels remain in the dynamic range allowed by the robot driver, that is, between -12 V and $+12$ V.

5.3 Two synchronized mechanical systems

This example illustrates the practical performance of the proposed technique, using the augmented observer given by (19). It refers to a basic problem of synchronization.
Synchronization means correlated or corresponding-in-time behavior of two or more processes (Arkady et al., 2003). In some situations the synchronization is a natural phenomenon; in others, an interconnection system is needed to obtain a synchronized behavior or improve its transient characteristics. Hence, the synchronization becomes a control objective and the synchronization obtained in this way is called controlled synchronization (Blekhman et al., 1997). Some important works in this topic are given by (Dong & Mills, 2002; Rodriguez & Nijmeijer, 2004; Soon-Jo & Slotine, 2007).
In this subsection we present a simple application of the control technique to synchronize two mechanisms connected in the basic configuration, called master-slave (see figure 9).
The master system is the MSD described in Section 5.1, manufactured by the company ECP®, model 210, with only the first mass activated. The slave is a torsional system from the same company, with the first and third disks connected. The master sends its position x to the slave, and the synchronization objective is to make the slave track the master state, that is, the

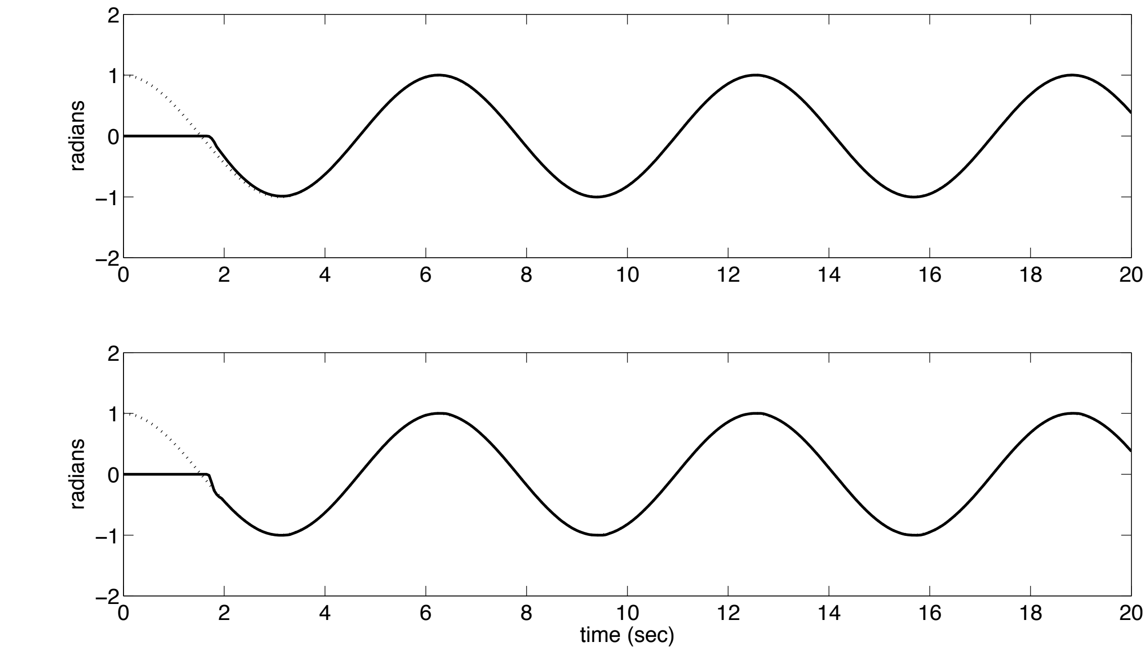

Fig. 7. Reference signal (dotted line) and position (solid line) for each joint of the industrial robot. Up: joint 1. Down: joint 2.

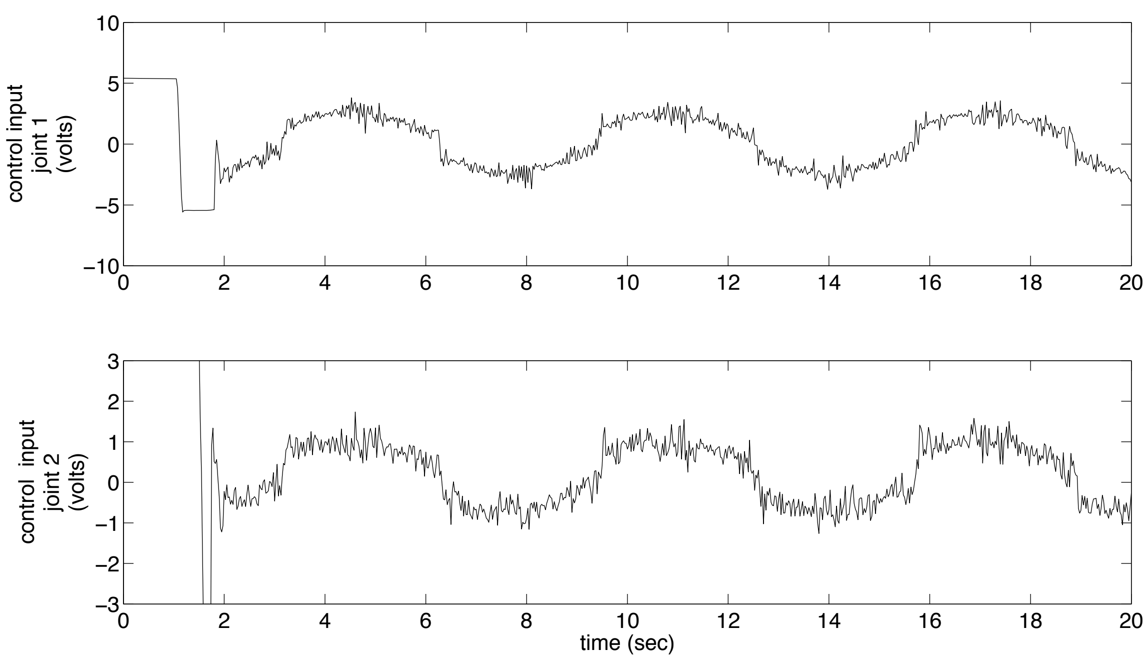

Fig. 8. Control input for each joint of the industrial robot.

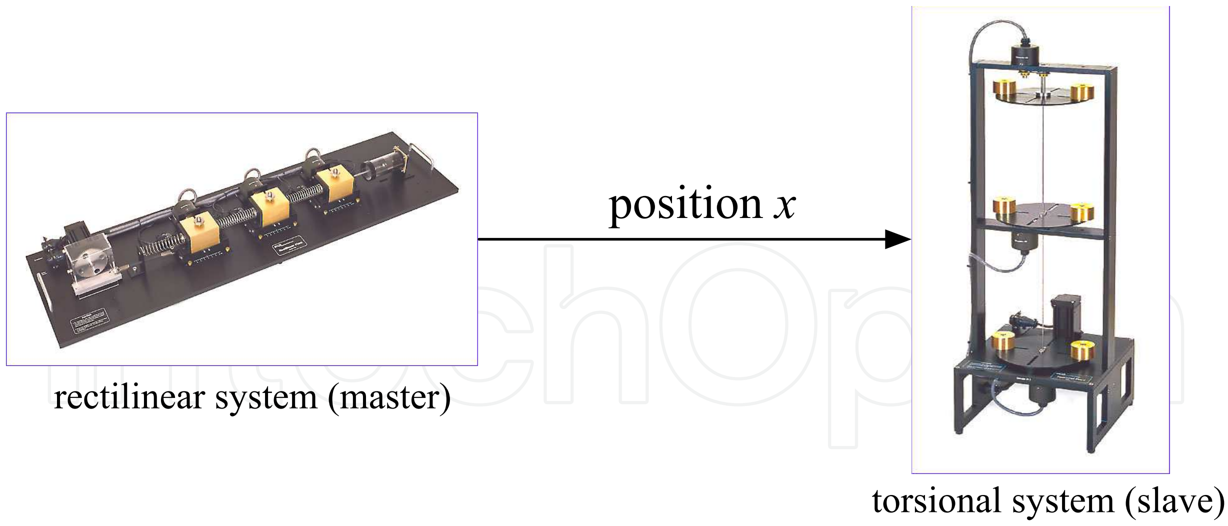

Fig. 9. Two synchronized mechanisms in a master/slave configuration. The master is the rectilinear system, model 210, from ECP®. The slave is the torsional system, model 205, from the same company.

angular position θ and velocity $\dot{\theta}$ of the torsional system must follow the position x and the velocity $\dot{x}$ of the master, respectively. The relation between the two states is 1cm of the master corresponds to 1rad of the slave.
The rectilinear system is modeled by

$$m\ddot{x} + c_m\dot{x} + k_m x + \gamma_m(t) = F(t),$$

where x is the position of the mass; m, c_m, and k_m are the mass, damping, and spring coefficients, respectively, and F is an external force driving the system. The torsional system is described as

$$J\ddot{\theta}_1 + c_t\dot{\theta}_1 + k_t(\theta_1 - \theta_2) + \gamma_t(t) = \tau_0 + \Delta_\tau,$$

where θ_1 and θ_2 are the angular positions of the first and third disks, respectively; J, c_t, and k_t are the inertia, damping, and spring coefficients of the first disk. γ_m and γ_t are external disturbances possibly affecting the systems. The force driving the MSD system is set as $F(t) = 1.5\sin(1.5\pi t)$. All positions are available, but the velocities are estimated with the second observer (19) (see Section 3.2).
The nominal values of the coefficients are given in Table 1.

System	Parameter	Value	Units
MSD	m	1.27	kg
	k_m	200	N/m
	c_m	2.1	N/m/sec
Torsional	J	0.0108	Kg-m^2
	c_t	0.007	N-m/rad/sec
	k_t	1.37	N-m/rad
Observer	$c_{11}, c_{12}, c_{21}, c_{22}$	500	
	c_{01}	50	
	c_{02}	100	

Table 1. Parameter values for the synchronization example.

If we define the synchronization error as

$$e_1 = x - \theta, \quad e_2 = \dot{x} - \dot{\theta},$$

the control objective es to make $e = (e_1, e_2)$ converge to zero.
Let us consider the nominal control

$$\tau_0 = -J(k_p e_1 + k_v \hat{e}_2) + c_t \hat{\dot{\theta}}_1 + k_t \theta_1 - k_t \theta_2,$$

where $\hat{\dot{\theta}}_1$ and $\hat{e}_2$ are the estimated velocity and the estimated velocity error obtained from the observer. From the last equations it is possible to get the synchronization error dynamics as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -k_p e_1 - k_v e_2 + \Delta u - \xi, \end{aligned}$$

where $\Delta u = J^{-1} \Delta_\tau$ and

$$\xi = (J c_t - k_v)\epsilon_2 + J^{-1}\gamma_t(t) - m^{-1}\left(c_m \dot{x} + k_m x - F(t) + \gamma_m(t)\right),$$

with $\epsilon_2 = e_2 - \hat{e}_2$.
We have then formulated this synchronization problem in the same framework allowing us to design a robust controller. Therefore, we can use one of the observers described previously, and use a redesign control $\Delta_\tau = J\hat{\xi}$.
We describe the results obtained from this controller to synchronize these devices. Figure 10 shows its performance, using the augmented observer (19).

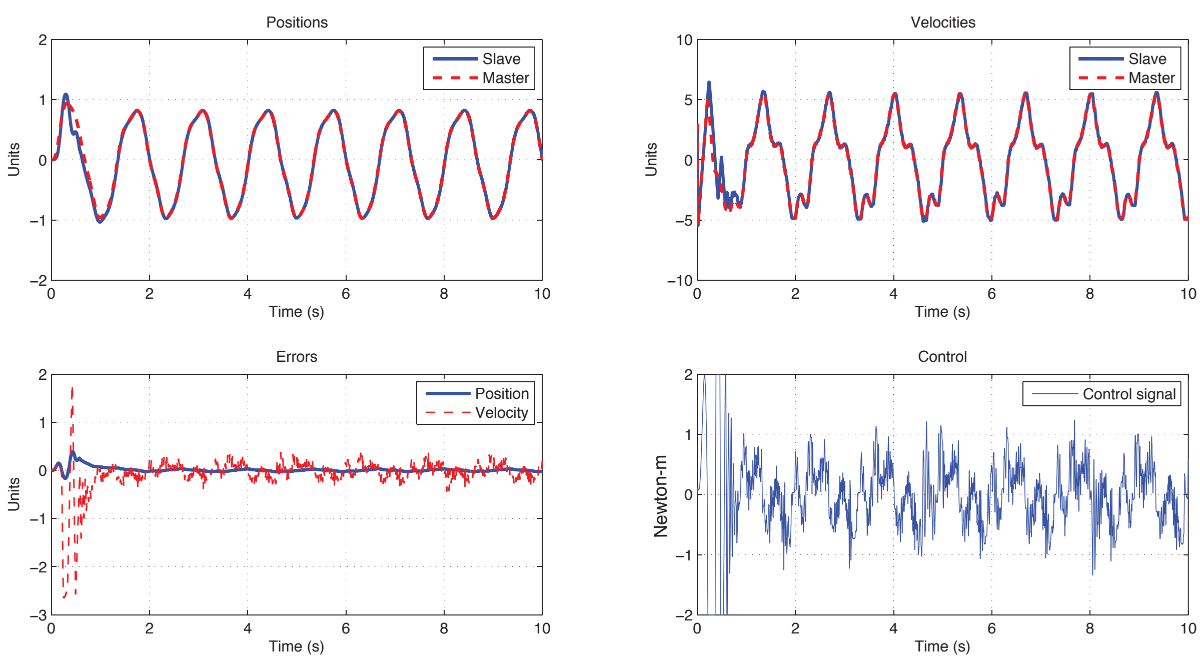

Fig. 10. Responses of the synchronized mechanisms (Figure 9). One unit corresponds to 1 cm (1 rad) for the position, or 1 cm/sec (1 rad/sec) for the velocity, of the master (slave) system.

This figure shows how the slave (torsional) system synchronizes with the master (rectilinear) system in about 1 sec. In 2 sec the synchronization error (position and velocity) is very small.

The control input designed for the slave is saturated at ± 2 N-m, and after 1 sec maintains its values between -1 and $+1$ N-m. This is accomplished even under the presence of the disturbance introduced by the third disk, which is not modeled.

6. Conclusions

A robust control structure for uncertain Lagrangian systems with partial measurement of the state has been presented. This control structure allows us to solve tracking and regulation problems and guarantees the convergence to a small neighborhood of the reference signal, in spite of nonvanishing disturbances affecting the plant.
This technique makes use of robust, discontinuous observers with a simple structure. An important property of these observers is its ability to estimate the disturbances acting on the plant, which can be conveniently incorporated in the control signal to increase the robustness of the controller and decrease the steady-state tracking error. The observer structure can even be built with conventional analog circuits, as it is described in (Alvarez et al., 2009). An adequate tuning of the observer parameters guarantees the convergence to the reference signal in an operation region large enough to cover practical situations.
The numerical simulations and the experimental results described in this chapter exhibited a good performance of the proposed technique, and the control signal showed values inside practical ranges.
An interesting and important problem that has been intensively studied recently is the synchronization of dynamical systems. Synchronization of mechanical systems is important as soon as two or more mechanical systems have to cooperate. The control technique described in this chapter has been applied to the simplest configuration, that is, the master/slave synchronization, exhibiting a good performance. This same control strategy, based on robust observers, can be also successfully applied to synchronize arrays of mechanical systems, connected in diverse configurations. A more detailed application can be found in (Alvarez et al., 2010).

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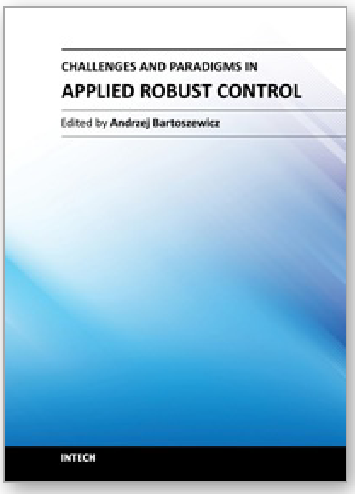

Challenges and Paradigms in Applied Robust Control

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The main objective of this book is to present important challenges and paradigms in the field of applied robust control design and implementation. Book contains a broad range of well worked out, recent application studies which include but are not limited to H-infinity, sliding mode, robust PID and fault tolerant based control systems. The contributions enrich the current state of the art, and encourage new applications of robust control techniques in various engineering and non-engineering systems.