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Observer-Based Robust Control of Uncertain Fuzzy Models with Pole Placement Constraints

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1. Introduction

Practical systems are often modelled by nonlinear dynamics. Controlling nonlinear systems are still open problems due to their complexity nature. This problem becomes more complex when the system parameters are uncertain. To control such systems, we may use the linearization technique around a given operating point and then employ the known methods of linear control theory. This approach is successful when the operating point of the system is restricted to a certain region. Unfortunately, in practice this approach will not work for some physical systems with a time-varying operating point. The fuzzy model proposed by Takagi-Sugeno (T-S) is an alternative that can be used in this case. It has been proved that T-S fuzzy models can effectively approximate any continuous nonlinear systems by a set of local linear dynamics with their linguistic description. This fuzzy dynamic model is a convex combination of several linear models. It is described by fuzzy rules of the type *If-Then* that represent local input output models for a nonlinear system. The overall system model is obtained by "blending" these linear models through nonlinear fuzzy membership functions. For more details on this topic, we refer the reader to (Tanaka & al 1998 and Wand & al, 1995) and the references therein.

The stability analysis and the synthesis of controllers and observers for nonlinear systems described by T-S fuzzy models have been the subject of many research works in recent years. The fuzzy controller is often designed under the well-known procedure: Parallel Distributed Compensation (PDC). In presence of parametric uncertainties in T-S fuzzy models, it is necessary to consider the robust stability in order to guarantee both the stability and the robustness with respect to the latter. These may include modelling error, parameter perturbations, external disturbances, and fuzzy approximation errors. So far, there have been some attempts in the area of uncertain nonlinear systems based on the T-S fuzzy models in the literature. The most of these existing works assume that all the system states are measured. However, in many control systems and real applications, these are not always available. Several authors have recently proposed observer based robust controller design methods considering the fact that in real control problems the full state information is not always available. In the case without uncertainties, we apply the separation property to design the observer-based controller: the observer synthesis is designed so that its dynamics are fast and we independently design the controller by imposing slower dynamics. Recently, much effort has been devoted to observer-based control for T-S fuzzy models. (Tanaka & al, 1998) have studied the fuzzy observer design for T-S fuzzy control systems. Nonetheless, in

the presence of uncertainties, the separation property is not applicable any more. In (El Messousi & al, 2006), the authors have proposed sufficient global stability conditions for the stabilization of uncertain fuzzy T-S models with unavailable states using a robust fuzzy observer-based controller but with no consideration to the control performances and in particular to the transient behaviour.
From a practical viewpoint, it is necessary to find a controller which will specify the desired performances of the controlled system. For example, a fast decay, a good damping can be imposed by placing the closed-loop poles in a suitable region of the complex plane. Chilali and Gahinet (Chilali & Gahinet, 1996) have proposed the concept of an LMI (Linear Matrix Inequality) region as a convenient LMI-based representation of general stability regions for uncertain linear systems. Regions of interest include α-stability regions, disks and conic sectors. In (Chilali & al 1999), a robust pole placement has been studied in the case of linear systems with static uncertainties on the state matrix. A vertical strip and α-stability robust pole placement has been studied in (Wang & al, 1995, Wang & al, 1998 and Wang & al, 2001) respectively for uncertain linear systems in which the concerned uncertainties are polytopic and the proposed conditions are not LMI. In (Hong & Man 2003), the control law synthesis with a pole placement in a circular LMI region is presented for certain T-S fuzzy models. Different LMI regions are considered in (Farinwata & al, 2000 and Kang & al, 198), for closed-loop pole placements in the case of T-S fuzzy models without uncertainties.
In this work, we extend the results of (El Messoussi & al, 2005), in which we have developed sufficient robust pole placement conditions for continuous T-S fuzzy models with measurable state variables and structured parametric uncertainties.
The main goal of this paper is to study the pole placement constraints for T-S fuzzy models with structured uncertainties by designing an observer-based fuzzy controller in order to guarantee the closed-loop stability. However, like (Lo & Li, 2004 and Tong & Li, 2002), we do not know the position of the system state poles as well as the position of the estimation error poles. The main contribution of this paper is as follows: the idea is to place the poles associated with the state dynamics in one LMI region and to place the poles associated with the estimation error dynamics in another LMI region (if possible, farther on the left). However, the separation property is not applicable unfortunately. Moreover, the estimation error dynamics depend on the state because of uncertainties. If the state dynamics are slow, we will have a slow convergence of the estimation error to the equilibrium point zero in spite of its own fast dynamics. So, in this paper, we propose an algorithm to design the fuzzy controller and the fuzzy observer separately by imposing the two pole placements. Moreover, by using the H_∞ approach, we ensure that the estimation error converges faster to the equilibrium point zero.
This chapter is organized as follows: in Section *2*, we give the class of uncertain fuzzy models, the observer-based fuzzy controller structure and the control objectives. After reviewing existing LMI constraints for a pole placement in Section *3*, we propose the new conditions for the uncertain augmented T-S fuzzy system containing both the fuzzy controller as well as the observer dynamics. Finally, in Section *4*, an illustrative application example shows the effectiveness of the proposed robust pole placement approach. Some conclusions are given in Section *5*.

2. Problem formulation and preliminaries

Considering a T-S fuzzy model with parametric uncertainties composed of *r* plant rules that can be represented by the following fuzzy rule:

Plant rule *i* :

$$\text{If } z_1(t) \text{ is } M_{1i} \text{ and } \ldots \text{and } z_v(t) \text{ is } M_{vi} \text{ Then } \begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t), \\ y(t) = C_i x(t) \qquad i = 1,\ldots,r \end{cases} \tag{1}$$

The structured uncertainties considered here are norm-bounded in the form:

$$\begin{aligned} \Delta A_i &= H_{ai}\Delta_{ai}(t)E_{ai}, \\ \Delta B_i &= H_{bi}\Delta_{bi}(t)E_{bi}, \; i = 1,\ldots,r \end{aligned} \tag{2}$$

Where $H_{ai}, H_{bi}, E_{ai}, E_{bi}$ are known real constant matrices of appropriate dimension, and $\Delta_{ai}(t), \Delta_{bi}(t)$ are unknown matrix functions satisfying:

$$\begin{aligned} &\Delta_{ai}^t(t)\Delta_{ai}(t) \le I, \\ &\Delta_{bi}^t(t)\Delta_{bi}(t) \le I \qquad i = 1,\ldots,r \end{aligned} \tag{3}$$

$\Delta_{ai}^t(t)$ is the transposed matrix of $\Delta_{ai}(t)$ and I is the matrix identity of appropriate dimension. We suppose that pairs (A_i, B_i) are controllable and (A_i, C_i) are observable. M_{ij} indicates the j^{th} fuzzy set associated to the i^{th} variable $z_i(t)$, r is the number of fuzzy model rules, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input vector, $y(t) \in R^l$ is the output vector, $A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$ and $C_i \in \Re^{l\times n}$. $z_1(t),\ldots,z_v(t)$ are premise variables.

From (1), the T-S fuzzy system output is :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\left[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)\right] \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{cases} \tag{4}$$

where $h_i(z(t)) = \dfrac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$ and $w_i(z(t)) = \prod_{j=1}^{v} \mu_{M_{ij}}(z_j(t))$

Where $\mu_{M_{ij}}(z_j(t))$ is the fuzzy meaning of symbol M_{ij}.

In this paper we assume that all of the state variables are not measurable. Fuzzy state observer for T-S fuzzy model with parametric uncertainties (1) is formulated as follows:

Observer rule *i*:

$$\text{If } z_1(t) \text{ is } M_{1i} \text{ and } \ldots \text{and } z_v(t) \text{ is } M_{vi} \text{ Then } \begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i u(t) - G_i(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C_i\hat{x}(t) \qquad i = 1,\ldots,r \end{cases} \tag{5}$$

The fuzzy observer design is to determine the local gains $G_i \in \Re^{n\times l}$ in the consequent part.

Note that the premise variables do not depend on the state variables estimated by a fuzzy observer.

The output of (5) is represented as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t)) \{ A_i \hat{x}(t) + B_i u(t) - G_i (y(t) - \hat{y}(t)) \} \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(z(t)) C_i \hat{x}(t) \end{cases} \tag{6}$$

To stabilize this class of systems, we use the PDC observer-based approach (Tanaka & al, 1998). The PDC observer-based controller is defined by the following rule base system:
Controller rule i :

$$\text{If } z_1(t) \text{ is } M_{1i} \text{ and } \ldots \text{and } z_\nu(t) \text{ is } M_{\nu i} \text{ Then } u(t) = K_i \hat{x}(t) \qquad i = 1, \ldots, r \tag{7}$$

The overall fuzzy controller is represented by:

$$u(t) = \frac{\sum_{i=1}^{r} w_i(z(t)) K_i \hat{x}(t)}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t)) K_i \hat{x}(t) \tag{8}$$

Let us denote the estimation error as:

$$e(t) = x(t) - \hat{x}(t) \tag{9}$$

The augmented system containing both the fuzzy controller and observer is represented as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \bar{A}(z(t)) \times \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} \bar{A}(z(t)) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \bar{A}_{ij} \\ \bar{A}_{ij} &= \begin{bmatrix} (A_i + \Delta A_i) + (B_i + \Delta B_i) K_j & -(B_i + \Delta B_i) K_j \\ \left(\Delta A_i + \Delta B_i K_j \right) & \left(A_i + G_i C_j - \Delta B_i K_j \right) \end{bmatrix} \end{aligned} \tag{11}$$

The main goal is first, to find the sets of matrices K_i and G_i in order to guarantee the global asymptotic stability of the equilibrium point zero of (10) and secondly, to design the fuzzy controller and the fuzzy observer of the augmented system (10) separately by assigning both "observer and controller poles" in a desired region in order to guarantee that the error between the state and its estimation converges faster to zero. The faster the estimation error will converge to zero, the better the transient behaviour of the controlled system will be.

3. Main results

Given (1), we give sufficient conditions in order to satisfy the global asymptotic stability of the closed-loop for the augmented system (10).

Lemma 1: The equilibrium point zero of the augmented system described by (10) is globally asymptotically stable if there exist common positive definite matrices P_1 and P_2, matrices W_i, V_j and positive scalars $\varepsilon_{ij} \succ 0$ such as

$$\begin{aligned} &\Pi_{ii} \le 0, \ i = 1,...,r \\ &\Pi_{ij} + \Pi_{ji} \le 0, \ i < j \le r \end{aligned} \tag{12}$$

And

$$\begin{aligned} &\Sigma_{ii} \le 0, \ i = 1,...,r \\ &\Sigma_{ij} + \Sigma_{ji} \le 0, \ i < j \le r \end{aligned} \tag{13}$$

with

$$\Pi_{ij} = \begin{bmatrix} D_{ij} & P_1 E_{ai}^t & V_j^t E_{bi}^t & B_i & H_{bi} \\ E_{ai} P_1 & -0.5\varepsilon_{ij} I & 0 & 0 & 0 \\ E_{bi} V_j & 0 & -0.5\varepsilon_{ij} I & 0 & 0 \\ B_i^t & 0 & 0 & -\varepsilon_{ij} I & 0 \\ H_{bi}^t & 0 & 0 & 0 & -\varepsilon_{ij} I \end{bmatrix} \quad \Sigma_{ij} = \begin{bmatrix} D_{ij}^* & K_j^t E_{bi}^t & P_2 H_{ai} & P_2 H_{bi} & K_j^t \\ E_{bi} K_j & -\varepsilon_{ij}^{-1} I & 0 & 0 & 0 \\ H_{ai}^t P_2 & 0 & -\varepsilon_{ij}^{-1} I & 0 & 0 \\ H_{bi}^t P_2 & 0 & 0 & -0.5\varepsilon_{ij}^{-1} I & 0 \\ K_j & 0 & 0 & 0 & -\varepsilon_{ij}^{-1} I \end{bmatrix}$$

$$\begin{aligned} D_{ij} &= A_i P_1 + P_1 A_i^t + B_i V_j + V_j^t B_i^t + \varepsilon_{ij} H_{ai} H_{ai}^t + \varepsilon_{ij} H_{bi} H_{bi}^t \\ D_{ij}^* &= P_2 A_i + A_i^t P_2 + W_i C_j + C_j^t W_i^t + \varepsilon_{ij}^{-1} K_j^t E_{bi}^t E_{bi} K_j \end{aligned}$$

Proof: using theorem *7* in (Tanaka & al, 1998), property (3), the separation lemma (Shi & al, 1992)) and the Schur's complement (Boyd & al, 1994), the above conditions (12) and (13) hold with some changes of variables. Let us briefly explain the different steps…
From (11), in order to ensure the global, asymptotic stability, the sufficient conditions must be verified:

$$\exists X = X^t > 0 : M_D(\overline{A}, X) = \overline{A}_{ij} X + X \overline{A}_{ij}^t < 0 \tag{14}$$

Let: $X = \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix}$ where *0* is a zero matrix of appropriate dimension. From (14), we have:

$$M_D(\overline{A}, X) = M_D^1 + M_D^2 \tag{15}$$

With $M_D^1 = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}$ where

$$D_1 = A_i X_{11} + X_{11} A_i^t + B_i K_j X_{11} + X_{11} K_j^t B_i^t \tag{16}$$

and

$$D_2 = A_i X_{22} + X_{22} A_i^t + G_i C_j X_{22} + X_{22} C_j^t G_i^t \tag{17}$$

From (15),

$$M_D^2 = \begin{bmatrix} \Delta_1 & X_{11}\Delta A_i^t + X_{11}K_j^t\Delta B_i^t - B_iK_jX_{22} - \Delta B_iK_jX_{22} \\ \Delta A_iX_{11} + \Delta B_iK_jX_{11} - X_{22}K_j^tB_i^t - X_{22}K_j^t\Delta B_i^t & \Delta_2 \end{bmatrix}$$

where $\Delta_1 = \Delta A_iX_{11} + X_{11}\Delta A_i^t + \Delta B_iK_jX_{11} + X_{11}K_j^t\Delta B_i^t$ and $\Delta_2 = -\Delta B_iK_jX_{22} - X_{22}K_j^t\Delta B_i^t$

From (15), we have:

$M_D^2 = \Sigma_1 + \Sigma_2 + \Sigma_3$ with $\Sigma_1 = \begin{bmatrix} 0 & -B_iK_jX_{22} - \Delta B_iK_jX_{22} \\ -X_{22}K_j^tB_i^t - X_{22}K_j^t\Delta B_i^t & 0 \end{bmatrix}$,

$\Sigma_2 = \begin{bmatrix} 0 & X_{11}\Delta A_i^t + X_{11}K_j^t\Delta B_i^t \\ \Delta A_iX_{11} + \Delta B_iK_jX_{11} & 0 \end{bmatrix}$ and $\Sigma_3 = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}$

Let $X_{11} = P_1$, $X_{11} = P_2^{-1}$. From the previous equation and (2), we have:

$$\begin{aligned} \Sigma_1 = &\begin{bmatrix} 0 & 0 \\ 0 & -P_2^{-1}K_j^t \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ B_i^t & 0 \end{bmatrix} + \begin{bmatrix} 0 & B_i \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & -K_jP_2^{-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -P_2^{-1}K_j^tE_{bi}^t \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ \Delta_{bi}^tH_{bi}^t & 0 \end{bmatrix} \\ &+ \begin{bmatrix} 0 & H_{bi}\Delta_{bi} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & -E_{bi}K_jP_2^{-1} \end{bmatrix} \end{aligned} \quad (18)$$

And,

$$\begin{aligned} \Sigma_2 = &\begin{bmatrix} 0 & 0 \\ H_{ai}\Delta_{ai} & 0 \end{bmatrix} \times \begin{bmatrix} E_{ai}P_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} P_1E_{ai}^t & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & \Delta_{ai}^tH_{ai}^t \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ H_{bi}\Delta_{bi} & 0 \end{bmatrix} \times \begin{bmatrix} E_{bi}K_jP_1 & 0 \\ 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} P_1K_j^tE_{bi}^t & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & \Delta_{bi}^tH_{bi}^t \\ 0 & 0 \end{bmatrix} \end{aligned} \quad (19)$$

And finally:

$$\begin{aligned} \Sigma_3 = &\begin{bmatrix} H_{ai}\Delta_{ai} & H_{bi}\Delta_{bi} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} E_{ai}P_1 & 0 \\ E_{bi}K_jP_1 & 0 \end{bmatrix} + \begin{bmatrix} P_1E_{ai}^t & P_1K_j^tE_{bi}^t \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \Delta_{ai}^tH_{ai}^t & 0 \\ \Delta_{bi}^tH_{bi}^t & 0 \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 \\ 0 & -H_{bi}\Delta_{bi} \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & E_{bi}K_jP_2^{-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & P_2^{-1}K_j^tE_{bi}^t \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & -\Delta_{bi}^tH_{bi}^t \end{bmatrix} \end{aligned} \quad (20)$$

From (18), (19) and (20) and by using the separation lemma (Shi & al, 1992)), we finally obtain:

$$M_D^2 \le \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix} \quad (21)$$

Where:

$$\begin{aligned} T_1 = &\varepsilon_{ij}^{-1}B_iB_i^t + \varepsilon_{ij}^{-1}H_{bi}\Delta_{bi}\Delta_{bi}^tH_{bi}^t + \varepsilon_{ij}^{-1}P_1E_{ai}^tE_{ai}P_1 + \varepsilon_{ij}^{-1}P_1K_j^tE_{bi}^tE_{bi}K_jP_1 \\ &+\varepsilon_{ij}H_{ai}\Delta_{ai}\Delta_{ai}^tH_{ai}^t + \varepsilon_{ij}H_{bi}\Delta_{bi}\Delta_{bi}^tH_{bi}^t + \varepsilon_{ij}^{-1}P_1E_{ai}^tE_{ai}P_1 + \varepsilon_{ij}^{-1}P_1K_j^tE_{bi}^tE_{bi}K_jP_1 \end{aligned}$$

and

$$T_2 = \varepsilon_{ij} P_2^{-1} K_j^t K_j P_2^{-1} + \varepsilon_{ij} P_2^{-1} K_j^t E_{bi}^t E_{bi} K_j P_2^{-1} + \varepsilon_{ij} H_{ai} \Delta_{ai} \Delta_{ai}^t H_{ai}^t$$
$$+\varepsilon_{ij} H_{bi} \Delta_{bi} \Delta_{bi}^t H_{bi}^t + \varepsilon_{ij} H_{bi} \Delta_{bi} \Delta_{bi}^t H_{bi}^t + \varepsilon_{ij}^{-1} P_2^{-1} K_j^t E_{bi}^t E_{bi} K_j P_2^{-1}$$

From (15), (16), (17) and (21), we have:

$$M_D(\overline{A}, X) \le \begin{bmatrix} D_1 + T_1 & 0 \\ 0 & D_2 + T_2 \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \tag{22}$$

In order to verify (14), we must have:

$$\begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} < 0 \tag{23}$$

Which implies:

$$\begin{cases} R_1 < 0 \\ R_2 < 0 \end{cases} \tag{24}$$

First, from (24), by using (3), using the Schur's complement (Boyd & al, 1994) as well as the introduction of the new variable: $V_i = K_j P_1$:

$$R_1 < 0$$
$$\Leftrightarrow \begin{bmatrix} D_{ij} & P_1 E_{ai}^t & V_j^t E_{bi}^t & B_i & H_{bi} \\ E_{ai} P_1 & -0.5\varepsilon_{ij} I & 0 & 0 & 0 \\ E_{bi} V_j & 0 & -0.5\varepsilon_{ij} I & 0 & 0 \\ B_i^t & 0 & 0 & -\varepsilon_{ij} I & 0 \\ H_{bi}^t & 0 & 0 & 0 & -\varepsilon_{ij} I \end{bmatrix} < 0 \tag{25}$$

Where I is always the identity matrix of appropriate dimension and $D_{ij} = A_i P_1 + P_1 A_i^t + B_i V_j + V_j^t B_i^t + \varepsilon_{ij} H_{ai} H_{ai}^t + \varepsilon_{ij} H_{bi} H_{bi}^t$

Then, from (24), by using (3), using the Schur's complement (Boyd & al, 1994) as well as the introduction of the new variable: $W_i = P_2 G_i$:

$$R_2 < 0$$
$$\Leftrightarrow \begin{bmatrix} D_{ij}^* & K_j^t E_{bi}^t & P_2 H_{ai} & P_2 H_{bi} & K_j^t \\ E_{bi} K_j & -\varepsilon_{ij}^{-1} I & 0 & 0 & 0 \\ H_{ai}^t P_2 & 0 & -\varepsilon_{ij}^{-1} I & 0 & 0 \\ H_{bi}^t P_2 & 0 & 0 & -0.5\varepsilon_{ij}^{-1} I & 0 \\ K_j & 0 & 0 & 0 & -\varepsilon_{ij}^{-1} I \end{bmatrix} < 0 \tag{26}$$

Where $D_{ij}^{*} = P_2 A_i + A_i^t P_2 + W_i C_j + C_j^t W_i^t + \varepsilon_{ij}^{-1} K_j^t E_{bi}^t E_{bi} K_j$

Thus, conditions (12) and (13) yield for all *i, j* from (25) and (26) and by using theorem *7* in (Tanaka & al, 1998) which is necessary for LMI relaxations.

Remark *1*: In lemma *1*, the positive scalars ε_{ij} are optimised unlike (Han & al, 2000), (Lee & al, 2001), (Tong & Li, 2002), (Chadli & El Hajjaji, 2006). We do not actually need to impose them to solve the set of LMIs. The conditions are thus less restrictive.

Remark *2*: Note that it is a two-step procedure which allows us to design the controller and the observer separately. First, we solve (12) for decision variables $(P_1, K_j, \varepsilon_{ij})$ and secondly, we solve (13) for decision variables (P_2, G_i) by using the results from the first step. Furthermore, the controller and observer gains are given by: $G_i = P_2^{-1} W_i$ and $K_j = V_j P_1^{-1}$, respectively, for $i, j = 1, 2, ..., r$.

Remark *3*: From lemma 1 and (10), the location of the poles associated with the state dynamics and with the estimation error dynamics is unknown. However, since the design algorithm is a two-step procedure, we can impose two pole placements separately, the first one for the state and the second one for the estimation error. In the following, we focus in the robust pole placement.

We hereafter give sufficient conditions to ensure the desired pole placements by using the LMI conditions of (Chilali & Gahinet (1996) and (Chilali & al, 1999) to the case of uncertain T-S fuzzy systems with unavailable state variables. Let us recall the definition of an LMI region and pole placement LMI constraints.

Definition 1 (Boyd & al, 1994)**:** A subset *D* of the complex plane is called an LMI region if there exists a symmetric matrix $\alpha = [\alpha_{kl}] \in \Re^{m \times m}$ and a matrix $\beta = [\beta_{kl}] \in \Re^{m \times m}$ such as:

$$D = \left\{ z \in C : f_D(z) = \alpha + \beta z + \beta^t \overline{z} < 0 \right\} \tag{27}$$

Definition 2 (Chilali and Gahinet, 1996)**:** Let *D* be a subregion of the left-half plane. A dynamical system described by: $\dot{x} = Ax$ is called *D*-stable if all its poles lie in *D*. By extension, A is then called *D*-stable.

From the two previous definitions, the following theorem is given.

Theorem 1 (Chilali and Gahinet , 1996)**:** Matrix *A* is *D*-stable if and only if there exists a symmetric matrix $X > 0$ such as

$$M_D(A, X) = \alpha \otimes X + \beta \otimes AX + \beta^t \otimes XA^t < 0 \tag{28}$$

where $\otimes$ denotes the Kronecker product.

From (10) and (11), let us define: $T_{ij} = (A_i + \Delta A_i) + (B_i + \Delta B_i) K_j$ and $S_{ij} = A_i + G_i C_j - \Delta B_i K_j$.

We hereafter give sufficient conditions to guarantee that $\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) T_{ij}$ and $\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) S_{ij}$ are D_T-stable and D_S-stable respectively in order to impose the dynamics of the state and the dynamics of the estimation error.

Lemma 2: Matrix $\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) T_{ij}$ is D_T-stable if and only if there exist a symmetric matrix $P_1 > 0$ and positive scalars $\mu_{ij} \succ 0$ such as

$$\begin{aligned} &\Omega_{ii} \le 0,\ i = 1,...,r, \\ &\Omega_{ij} + \Omega_{ji} \le 0,\ i < j \le r. \end{aligned} \tag{29}$$

With

$$\begin{aligned} &\Omega_{ij} = \begin{pmatrix} E_{ij} & \left(\beta^t \otimes P_1 E_{ai}^t\right) & \left(\beta^t \otimes V_j E_{bi}^t\right) \\ \left(\beta \otimes E_{ai} P_1\right) & -\mu_{ij} I & 0 \\ \left(\beta \otimes E_{bi} V_j\right) & 0 & -\mu_{ij} I \end{pmatrix} \\ &E_{ij} = \xi_{ij} + \mu_{ij}\left(I \otimes H_{ai} H_{ai}^t\right) + \mu_{ij}\left(I \otimes H_{bi} H_{bi}^t\right) \\ &\xi_{ij} = \alpha \otimes P_1 + \beta \otimes A_i P_1 + \beta^t \otimes P_1 A_i^t + \beta \otimes B_i V_j + \beta^t \otimes V_j^t B_i^t \\ &V_j = K_j P_1 \end{aligned} \tag{30}$$

Proof: Using theorem *1*, matrix T_{ij} is D_T-stable if and only if there exists a symmetric matrix $X > 0$ such that:

$$M_{D_T}(T_{ij}, X) = \alpha \otimes X + \beta \otimes T_{ij} X + \beta^t \otimes X {T_{ij}}^t < 0 \tag{31}$$

$$\begin{aligned} M_{D_T}(T_{ij}, X) = &\alpha \otimes X + \beta \otimes A_i X + \beta^t \otimes X A_i^t + \beta \otimes B_i K_j X + \beta^t \otimes X K_j^t B_i^t + \beta \otimes H_{ai} \Delta_{ai} E_{ai} X \\ &+ \beta^t \otimes X E_{ai}^t \Delta_{ai}^t H_{ai}^t + \beta \otimes H_{bi} \Delta_{bi} E_{bi} K_j X + \beta^t \otimes X K_j^t E_{bi}^t \Delta_{bi}^t H_{bi}^t \end{aligned} \tag{32}$$

Let $X = P_1$ and $V_j = K_j P_1$:

$$\begin{aligned} M_{D_T}(T_{ij}, X) = &\xi_{ij} + (I \otimes H_{ai} \Delta_{ai})(\beta \otimes E_{ai} P_1) + (\beta^t \otimes P_1 E_{ai}^t)(I \otimes \Delta_{ai}^t H_{ai}^t) + (I \otimes H_{bi} \Delta_{bi})(\beta \otimes E_{bi} V_j) \\ &+ (\beta^t \otimes V_j^t E_{bi}^t)(I \otimes \Delta_{bi}^t H_{bi}^t) \end{aligned} \tag{33}$$

where

$$\xi_{ij} = \alpha \otimes P_1 + \beta \otimes A_i P_1 + \beta^t \otimes P_1 A_i^t + \beta \otimes B_i V_j + \beta^t \otimes V_j^t B_i^t \tag{34}$$

Using the separation lemma (Shi & al, 1992) and (3), we obtain:

$$\begin{aligned} M_{D_T}(T_{ij}, X) \le &\xi_{ij} + \mu_{ij}(I \otimes H_{ai} H_{ai}^t) + \mu_{ij}^{-1}(\beta^t \otimes P_1 E_{ai}^t)(\beta \otimes E_{ai} P_1) \\ &+ \mu_{ij}(I \otimes H_{bi} H_{bi}^t) + \mu_{ij}^{-1}(\beta^t \otimes V_j^t E_{bi}^t)(\beta \otimes E_{bi} V_j) \end{aligned} \tag{35}$$

Thus, matrix T_{ij} is D_T-stable if:

$$\begin{aligned} &\xi_{ij} + \mu_{ij}(I \otimes H_{ai} H_{ai}^t) + \mu_{ij}(I \otimes H_{bi} H_{bi}^t) + \mu_{ij}^{-1}(\beta^t \otimes P_1 E_{ai}^t)(\beta \otimes E_{ai} P_1) \\ &+ \mu_{ij}^{-1}(\beta^t \otimes V_j^t E_{bi}^t)(\beta \otimes E_{bi} V_j) \prec 0 \end{aligned} \tag{36}$$

Where, of course, $\mu_{ij} \in \Re\ \forall i, j$

By using the Schur's complement (Boyd & al, 1994),

$$\begin{pmatrix} E_{ij} & \left(\beta^t \otimes P_1 E_{ai}^t\right) & \left(\beta^t \otimes V_j E_{bi}^t\right) \\ \left(\beta \otimes E_{ai} P_1\right) & -\mu_{ij} I & 0 \\ \left(\beta \otimes E_{bi} V_j\right) & 0 & -\mu_{ij} I \end{pmatrix} \prec 0, \\ E_{ij} = \xi_{ij} + \mu_{ij}\left(I \otimes H_{ai} H_{ai}^t\right) + \mu_{ij}\left(I \otimes H_{bi} H_{bi}^t\right). \tag{37}$$

Thus, conditions (29) easily yield for all *i, j*.

Lemma 3: Matrix $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))S_{ij}$ is D_S-stable if and only if there exist a symmetric matrix $P_2 > 0$, matrices W_i, K_j and positive scalars $\lambda_{ij} \succ 0$ such as

$$\Phi_{ii} \le 0, \ i = 1,...,r \\ \Phi_{ij} + \Phi_{ji} \le 0, \ i < j \le r \tag{38}$$

with

$$\Phi_{ij} = \begin{pmatrix} R_{ij} + \lambda_{ij}(\beta^t \otimes K_j^t E_{bi}^t)(\beta \otimes E_{bi} K_j) & I \otimes P_2 H_{bi} \\ I \otimes H_{bi}^t P_2 & -\lambda_{ij} I \end{pmatrix} \\ R_{ij} = \alpha \otimes P_2 + \beta \otimes P_2 A_i + \beta^t \otimes A_i^t P_2 + \beta \otimes W_i C_j + \beta^t \otimes C_j^t W_i^t \\ W_i = P_2 G_i \tag{39}$$

Proof: Same lines as previously can be used to prove this lemma.
Let:

$$M_{D_S}(S_{ij}, X) = \alpha \otimes X + \beta \otimes A_i X + \beta^t \otimes X A_i^t + \beta \otimes G_i C_j X + \beta^t \otimes X C_j^t G_i^t \\ -\beta^t \otimes X K_j^t E_{bi}^t (I \otimes \Delta_{bi}^t H_{bi}^t) - (I \otimes \Delta_{bi} H_{bi})(\beta \otimes E_{bi} K_j X) < 0 \tag{40}$$

Using the separation lemma (Shi & al, 1992), by pre- and post- multiplying by $I \otimes X^{-1}$, we obtain:

$$\alpha \otimes X^{-1} + \beta \otimes (X^{-1} A_i) + \beta^t \otimes (A_i^t X^{-1}) + \beta \otimes (X^{-1} G_i C_j) + \beta^t \otimes (C_j^t G_i^t X^{-1}) \\ +\lambda_{ij}(\beta^t \otimes K_j^t E_{bi}^t)(\beta \otimes E_{bi} K_j) + 1 / \lambda_{ij}(I \otimes X^{-1} H_{bi})(I \otimes H_{bi}^t X^{-1}) < 0 \tag{41}$$

Where, of course, $\lambda_{ij} \in \Re \ \forall i, j$

Thus, by using the Schur's complement (Boyd & al, 1994) as well as by defining $P_2 = X^{-1}$:

$$\Phi_{ij} = \begin{pmatrix} \alpha \otimes P_2 + \beta \otimes P_2 A_i + \beta^t \otimes A_i^t P_2 + \beta \otimes P_2 G_i C_j + \beta^t \otimes C_j^t G_i^t P_2 + \lambda_{ij}(\beta^t \otimes K_j^t E_{bi}^t)(\beta \otimes E_{bi} K_j) & I \otimes P_2 H_{bi} \\ I \otimes H_{bi}^t P_2 & -\lambda_{ij} I \end{pmatrix} < 0 \tag{42}$$

By using $W_i = X^{-1} G_i$, conditions (38) easily yield for all *i, j*. The lemma proof is given.

Remark 4: Any kind of LMI region (disk, vertical strip, conic sector) may be easily used for D_S and D_T .

From lemma *2* and lemma *3*, we have imposed the dynamics of the state as well as the dynamics of the estimation error. But from (10), the estimation error dynamics depend on the state. If the state dynamics are slow, we will have a slow convergence of the estimation error to the equilibrium point zero in spite of its own fast dynamics. So in this paper, we add an algorithm using the H_∞ approach to ensure that the estimation error converges faster to the equilibrium point zero.

We know from (10) that:

$$\begin{aligned}\dot{e}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\left(A_i + G_iC_j - \Delta B_iK_j\right)e(t) \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))S_{ij}\left(\Delta A_i + \Delta B_iK_j\right)x(t)\end{aligned} \tag{43}$$

This equation is equivalent to the following system:

$$\begin{bmatrix}\dot{e}\\ e\end{bmatrix} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\left(\begin{bmatrix}A_i + G_iC_j - \Delta B_iK_j & \Delta A_i + \Delta B_iK_j\\ I & 0\end{bmatrix}\begin{bmatrix}e\\ x\end{bmatrix}\right) \tag{44}$$

The objective is to minimize the L_2 gain from $x(t)$ to $e(t)$ in order to guarantee that the error between the state and its estimation converges faster to zero. Thus, we define the following H_∞ performance criterion under zero initial conditions:

$$\int_0^\infty \{e^t(t)e(t) - \gamma^2 x^t(t)x(t)\}dt < 0 \tag{45}$$

where $\gamma \in \Re^{+*}$ has to be minimized. Note that the signal $x(t)$ is square integrable because of lemma *1*.

We give the following lemma to satisfy the H_∞ performance.

Lemma 4: If there exist symmetric positive definite matrix P_2, matrices W_i and positive scalars $\gamma \succ 0$, $\beta_{ij} \succ 0$ such as

$$\begin{aligned}&\Gamma_{ii} \le 0,\ i = 1,...,r\\ &\Gamma_{ij} + \Gamma_{ji} \le 0,\ i < j \le r\end{aligned} \tag{46}$$

With

$$\Gamma_{ij} = \begin{bmatrix} Z_{ij} & P_2H_{bi} & P_2H_{ai} & -\beta_{ij}K_j^tE_{bi}^tE_{bi}K_j\\ H_{bi}^tP_2 & -\beta_{ij}I & 0 & 0\\ H_{ai}^tP_2 & 0 & -\beta_{ij}I & 0\\ -\beta_{ij}K_j^tE_{bi}^tE_{bi}K_j & 0 & 0 & U_{ij}\end{bmatrix}$$

$$Z_{ij} = P_2 A_i + A_i^t P_2 + W_i C_j + C_j^t W_i^t + I + \beta_{ij} K_j^t E_{bi}^t E_{bi} K_j$$

$$U_{ij} = -\gamma^2 I + \beta_{ij} K_j^t E_{bi}^t E_{bi} K_j + \beta_{ij} E_{ai}^t E_{ai}$$

Then, the dynamic system:

$$\begin{bmatrix} \dot{e} \\ e \end{bmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \begin{bmatrix} A_i + G_i C_j - \Delta B_i K_j & \Delta A_i + \Delta B_i K_j \\ I & 0 \end{bmatrix} \begin{bmatrix} e \\ x \end{bmatrix} \tag{47}$$

satisfies the H_∞ performance with a L_2 gain equal or less than γ (44) .

Proof: Applying the bounded real lemma (Boyd & al, 1994), the system described by the following dynamics:

$$\dot{e}(t) = \left(A_i + G_i C_j - \Delta B_i K_j \right) e(t) + \left(\Delta A_i + \Delta B_i K_j \right) x(t) \tag{48}$$

satisfies the H_∞ performance corresponding to the L_2 gain γ performance if and only if there exists $P_2 = P_2^T > 0$:

$$\begin{aligned} &(A_i + G_i C_j - \Delta B_i K_j)^t P_2 + P_2 (A_i + G_i C_j - \Delta B_i K_j) \\ &+ P_2 (\Delta A_i + \Delta B_i K_j)(\gamma^2 I)^{-1} (\Delta A_i + \Delta B_i K_j)^t P_2 + I \prec 0 \end{aligned} \tag{49}$$

Using the Schur's complement, (Boyd & al, 1994) yields

$$\underbrace{\begin{bmatrix} J_{ij} & P_2 \Delta A_i + P_2 \Delta B_i K_j \\ \Delta A_i^t P_2 + K_j^t \Delta B_i^t P_2 & -\gamma^2 I \end{bmatrix}}_{\Theta_{ij}} \prec 0 \tag{50}$$

where

$$J_{ij} = P_2 A_i + A_i^t P_2 + P_2 G_i C_j + C_j^t G_i^t P_2 - P_2 \Delta B_i K_j - K_j^t \Delta B_i^t P_2 + I \tag{51}$$

We get:

$$\Theta_{ij} = \begin{bmatrix} P_2 A_i + A_i^t P_2 + P_2 G_i C_j + C_j^t G_i^t P_2 + I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \underbrace{\begin{bmatrix} -P_2 \Delta B_i K_j - K_j^t \Delta B_i^t P_2 & P_2 \Delta A_i + P_2 \Delta B_i K_j \\ \Delta A_i^t P_2 + K_j^t \Delta B_i^t P_2 & 0 \end{bmatrix}}_{\Delta_{ij}} \tag{52}$$

By using the separation lemma (Shi & al, 1992) yields

$$\Delta_{ij} \le \beta_{ij} \begin{bmatrix} K_j^t E_{bi}^t E_{bi} K_j & -K_j^t E_{bi}^t E_{bi} K_j \\ -K_j^t E_{bi}^t E_{bi} K_j & K_j^t E_{bi}^t E_{bi} K_j + E_{ai}^t E_{ai} \end{bmatrix} + \beta_{ij}^{-1} \begin{bmatrix} P_2 H_{bi} \Delta_{bi} \Delta_{bi}^t H_{bi}^t P_2 + P_2 H_{ai} \Delta_{ai} \Delta_{ai}^t H_{ai}^t P_2 & 0 \\ 0 & 0 \end{bmatrix} \tag{53}$$

With substitution into Θ_{ij} and defining a variable change: $W_i = P_2 G_i$, yields

$$\Theta_{ij} \le \begin{bmatrix} Q_{ij} & -\beta_{ij} K_j^t E_{bi}^t E_{bi} K_j \\ -\beta_{ij} K_j^t E_{bi}^t E_{bi} K_j & -\gamma^2 I + \beta_{ij} K_j^t E_{bi}^t E_{bi} K_j + \beta_{ij} E_{ai}^t E_{ai} \end{bmatrix} \tag{54}$$

where

$$\begin{aligned} \mathrm{Q_{ij}} &= \mathrm{R_{ij}} + \beta_{ij}^{-1} \mathrm{P_2 H_{bi} \Delta_{bi} \Delta_{bi}^t H_{bi}^t P_2} + \varepsilon_{ij}^{-1} \mathrm{P_2 H_{ai} \Delta_{ai} \Delta_{ai}^t H_{ai}^t P_2}, \\ \mathrm{R_{ij}} &= \mathrm{P_2 A_i} + \mathrm{A_i^t P_2} + \mathrm{W_i C_j} + \mathrm{C_j^t W_i^t} + \mathrm{I} + \beta_{ij} \mathrm{K_j^t E_{bi}^t E_{bi} K_j}. \end{aligned} \tag{55}$$

Thus, from the following condition

$$\begin{bmatrix} Q_{ij} & -\beta_{ij} K_j^t E_{bi}^t E_{bi} K_j \\ -\beta_{ij} K_j^t E_{bi}^t E_{bi} K_j & -\gamma^2 I + \beta_{ij} K_j^t E_{bi}^t E_{bi} K_j + \beta_{ij} E_{ai}^t E_{ai} \end{bmatrix} \prec 0 \tag{56}$$

and using the Schur's complement (Boyd & al, 1994), theorem *7* in (Tanaka & al, 1998) and (3), condition (46) yields for all *i,j*.

Remark 5: In order to improve the estimation error convergence, we obtain the following convex optimization problem: minimization γ under the LMI constraints (46).

From lemma *1*, *2*, *3* and *4* yields the following theorem:

Theorem 2: The closed-loop uncertain fuzzy system (10) is robustly stabilizable via the observer-based controller (8) with control performances defined by a pole placement constraint in LMI region D_T for the state dynamics, a pole placement constraint in LMI region D_S for the estimation error dynamics and a L_2 gain γ performance (45) as small as possible if first, LMI systems (12) and (29) are solvable for the decision variables $(P_1, K_j, \varepsilon_{ij}, \mu_{ij})$ and secondly, LMI systems (13), (38) , (46) are solvable for the decision variables $(P_2, G_i, \lambda_{ij}, \beta_{ij})$. Furthermore, the controller and observer gains are $K_j = V_j P_1^{-1}$ and $G_i = P_2^{-1} W_i$, respectively, for $i, j = 1, 2, ..., r$.

Remark 6: Because of uncertainties, we could not use the separation property but we have overcome this problem by designing the fuzzy controller and observer in two steps with two pole placements and by using the H_∞ approach to ensure that the estimation error converges faster to zero although its dynamics depend on the state.

Remark 7: Theorem *2* also proposes a two-step procedure: the first step concerns the fuzzy controller design by imposing a pole placement constraint for the poles linked to the state dynamics and the second step concerns the fuzzy observer design by imposing the second pole placement constraint for the poles linked to the error estimation dynamics and by minimizing the H_∞ performance criterion (18). The designs of the observer and the controller are separate but not independent.

4. Numerical example

In this section, to illustrate the validity of the suggested theoretical development, we apply the previous control algorithm to the following academic nonlinear system (Lauber, 2003):

$$\begin{cases} \dot{x}_1(t) = \left(\cos^2(x_2(t)) - \dfrac{1}{1+x_1^2(t)} \right) x_2(t) + \left(1 + \dfrac{1}{1+x_1^2(t)} \right) u(t) \\ \dot{x}_2(t) = b\left(1 + \dfrac{1}{1+x_1^2(t)} \right) \sin(x_2(t)) - 1.5x_1(t) - 3x_2(t) \\ \qquad + \left(a\cos^2(x_2(t)) - 2 \right) u(t) \\ y(t) = x_1(t) \end{cases} \tag{57}$$

$y \in \Re$ is the system output, $u \in \Re$ is the system input, $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^t$ is the state vector which is supposed to be unmeasurable. What we want to find is the control law u which globally stabilizes the closed-loop and forces the system output to converge to zero but by imposing a transient behaviour.

Since the state vector is supposed to be unmeasurable, an observer will be designed.

The idea here is thus to design a fuzzy observer-based robust controller from the nonlinear system (57). The first step is to obtain a fuzzy model with uncertainties from (57) while the second step is to design the fuzzy control law from theorem 2 by imposing pole placement constraints and by minimizing the $H\infty$ criterion (46). Let us recall that, thanks to the pole placements, the estimation error converges faster to the equilibrium point zero and we impose the transient behaviour of the system output.

First step:

The goal is here to obtain a fuzzy model from (57).

By decomposing the nonlinear term $\dfrac{1}{1+x_1^2(t)}$ and integring nonlinearities of $x_2(t)$ into incertainties, then (20) is represented by the following fuzzy model:

Fuzzy model rule *1*:

$$If\ x_1(t)\ is\ M_1\ then \begin{cases} \dot{x} = (A_1 + \Delta A_1)x + (B_1 + \Delta B_1)u \\ y = Cx \end{cases} \tag{58}$$

Fuzzy model rule *2*:

$$If\ x_1(t)\ is\ M_2\ then \begin{cases} \dot{x} = (A_2 + \Delta A_2)x + (B_2 + \Delta B_2)u \\ y = Cx \end{cases} \tag{59}$$

where

$$A_1 = \begin{pmatrix} 0 & 0.5 \\ -1.5 & -3 + \dfrac{1+m}{2}b \end{pmatrix}, \quad B_1 = \begin{pmatrix} 1 \\ \dfrac{a}{2} - 2 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 0.5 \\ -1.5 & -3 + (1+m)b \end{pmatrix}, \quad B_2 = \begin{pmatrix} 2 \\ \dfrac{a}{2} - 2 \end{pmatrix},$$

$$H_{ai} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, H_{bi} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, E_{b1} = E_{b2} = 0.5|a|$$

$$E_{a1} = \begin{pmatrix} 0 & 0.5 \\ 0 & \dfrac{1-m}{2}|b| \end{pmatrix}, \quad E_{a2} = \begin{pmatrix} 0 & 0.5 \\ 0 & (1-m)|b| \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

m=-0.2172, b=-0.5, a=2 and *i=1,2*

Second step:
The control design purpose of this example is to place both the poles linked to the state dynamics and to the estimation error dynamics in the vertical strip given by: $(\alpha_1 \quad \alpha_2) = (-1 \quad -6)$. The choice of the same vertical strip is voluntary because we wish to compare results of simulations obtained with and without the H_∞ approach, in order to show by simulation the effectiveness of our approach.
The initial values of states are chosen: $x(0) = [-0.2 \quad -0.1]$ and $\hat{x}(0) = [0 \quad 0]$.
By solving LMIs of theorem 2, we obtain the following controller and observer gain matrices respectively:

$$K_1 = [-1.95 \quad -0.17], K_2 = [-1.36 \quad -0.08], G_1 = [-7.75 \quad -80.80]^t, G_2 = [-7.79 \quad -82.27]^t \quad (60)$$

The obtained H_∞ criterion after minimization is:

$$\gamma = 0.3974 \quad (61)$$

Tables 1 and 2 give some examples of both nominal and uncertain system closed-loop pole values respectively. All these poles are located in the desired regions. Note that the uncertainties must be taken into account since we wish to ensure a global pole placement. That means that the poles of (10) belong to the specific LMI region, whatever uncertainties (2), (3). From tables *1* and *2*, we can see that the estimation error pole values obtained using the H_∞ approach are more distant (farther on the left) than the ones without the H_∞ approach.

	With the H_∞ approach		Without the H_∞ approach	
	Pole 1	Pole 2	Pole 1	Pole 2
$A_1 + B_1K_1$	-1.8348	-3.1403	-1.8348	-3.1403
$A_2 + B_2K_2$	-2.8264	-3.2172	-2.8264	-3.2172
$A_1 + G_1C_1$	-5.47 +5.99i	-5.47- 5.99i	-3.47 + 3.75i	-3.47- 3.75i
$A_2 + G_2C_2$	-5.59 +6.08i	-5.59 - 6.08i	-3.87 + 3.96i	-3.87 - 3.96i

Table 1. Pole values (nominal case).

	With the H_∞ approach		Without the H_∞ approach	
	Pole 1	*Pole 2*	*Pole 1*	*Pole 2*
$A_1 + H_{a1}E_{a1} + (B_1 + H_{b1}E_{b1})K_1$	*-2.56 + .43i*	*-2.56 - 0.43i*	*-2.56+ 0.43i*	*-2.56 - 0.43i*
$A_2 + H_{a2}E_{a2} + (B_2 + H_{b2}E_{b2})K_2$	*-3.03 +0.70i*	*-3.032- 0.70i*	*-3.03 + 0.70i*	*-3.03 - 0.70i*
$A_1 - H_{a1}E_{a1} + (B_1 + H_{b1}E_{b1})K_1$	*-2.58 +0.10i*	*-2.58- 0.10i*	*-2.58 + 0.10i*	*-2.58 - 0.10i*
$A_2 - H_{a2}E_{a2} + (B_2 + H_{b2}E_{b2})K_2$	*-3.09 +0.54i*	*-3.09-0.54i*	*-3.09 + 0.54i*	*-3.09 - 0.54i*
$A_1 + G_1C_1 - H_{b1}E_{b1}K_1$	*-5.38+5.87i*	*-5.38 - 5.87i*	*-3.38 + 3.61i*	*-3.38 - 3.61i*
$A_2 + G_2C_2 - H_{b2}E_{b2}K_2$	*-5.55 +6.01i*	*-5.55 - 6.01i*	*-3.83 + 3.86i*	*-3.83 - 3.86i*

Table 2. Pole values (extreme uncertain models).

Figures *1* and *2* respectively show the behaviour of error $e_1(t)$ and $e_2(t)$ with and without the H_∞ approach and also the behaviour obtained using only lemma 1. We clearly see that the estimation error converges faster in the first case (with H_∞ approach and pole placements) than in the second one (with pole placements only) as well as in the third case (without H_∞ approach and pole placements). At last but not least, Figure *3* and *4* show respectively the behaviour of the state variables with and without the H_∞ approach whereas Figure 5 shows the evolution of the control signal. From Figures 3 and 4, we still have the same conclusion about the convergence of the estimation errors.

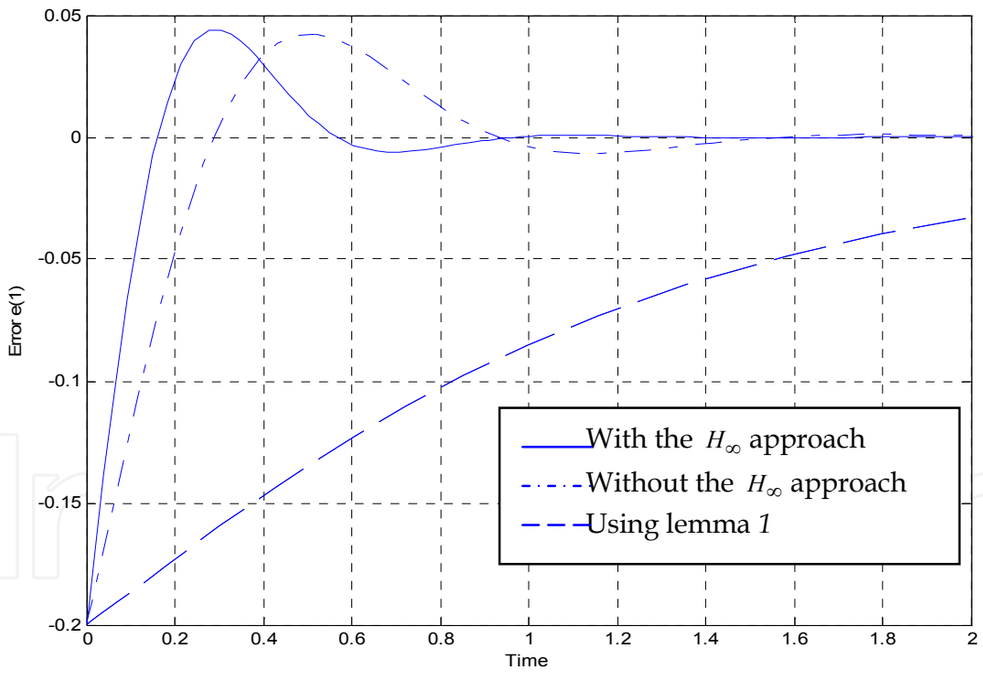


Fig. 1. Behaviour of error $e_1(t)$.

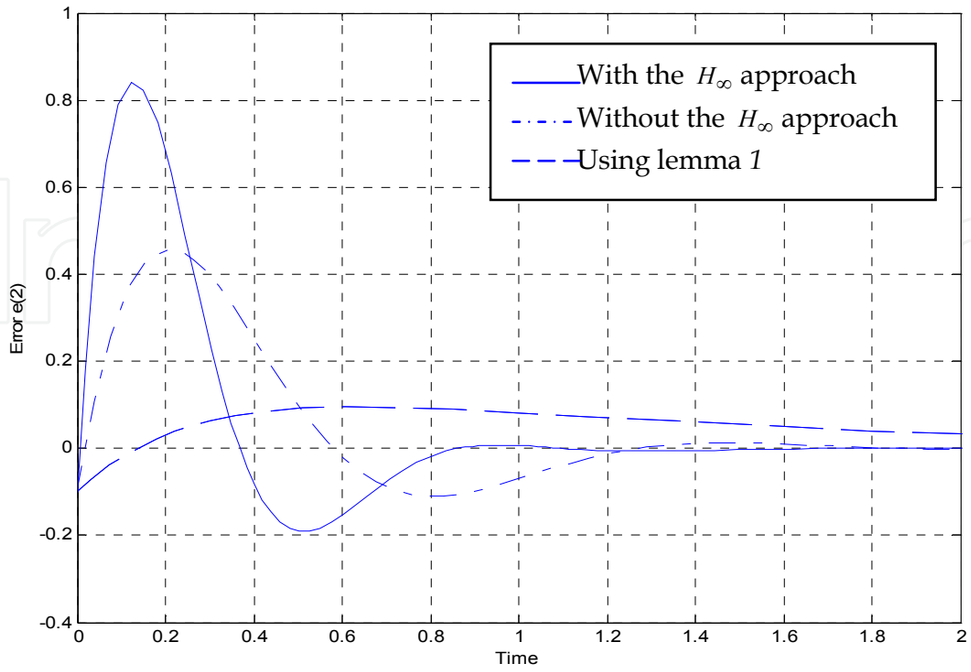


Fig. 2. Behaviour of error $e_2(t)$.

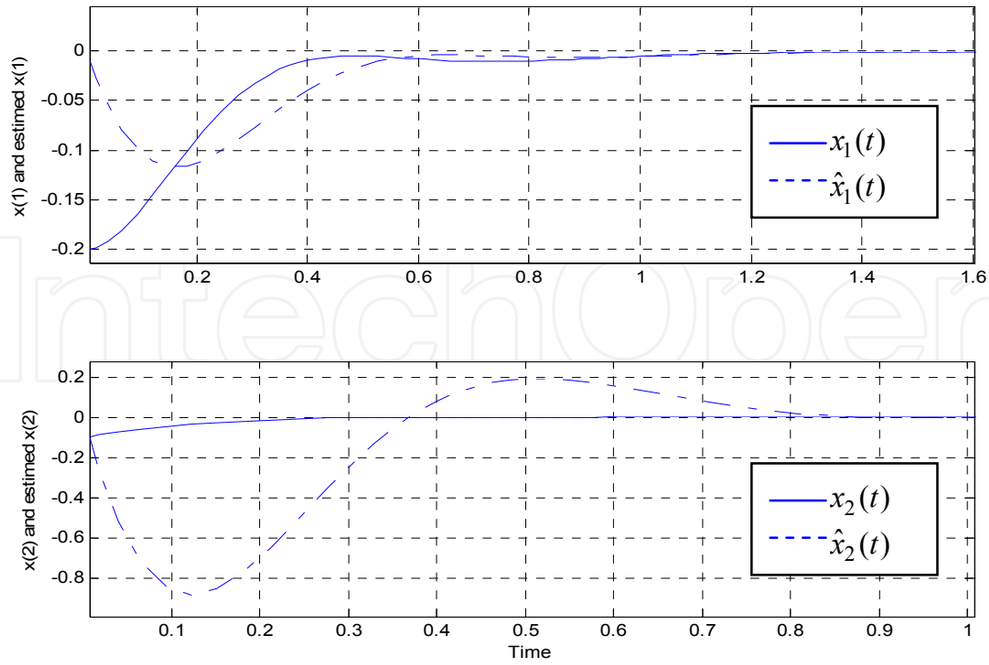


Fig. 3. Behaviour of the state vector and its estimation with the H_∞ approach.

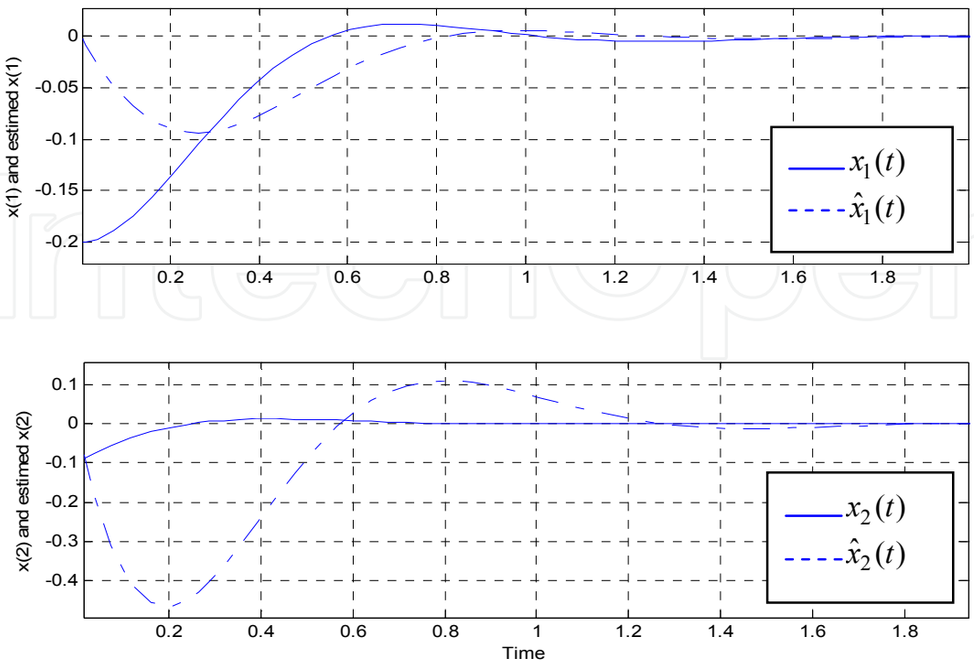


Fig. 4. Behaviour of the state and its estimation without the H_∞ approach.

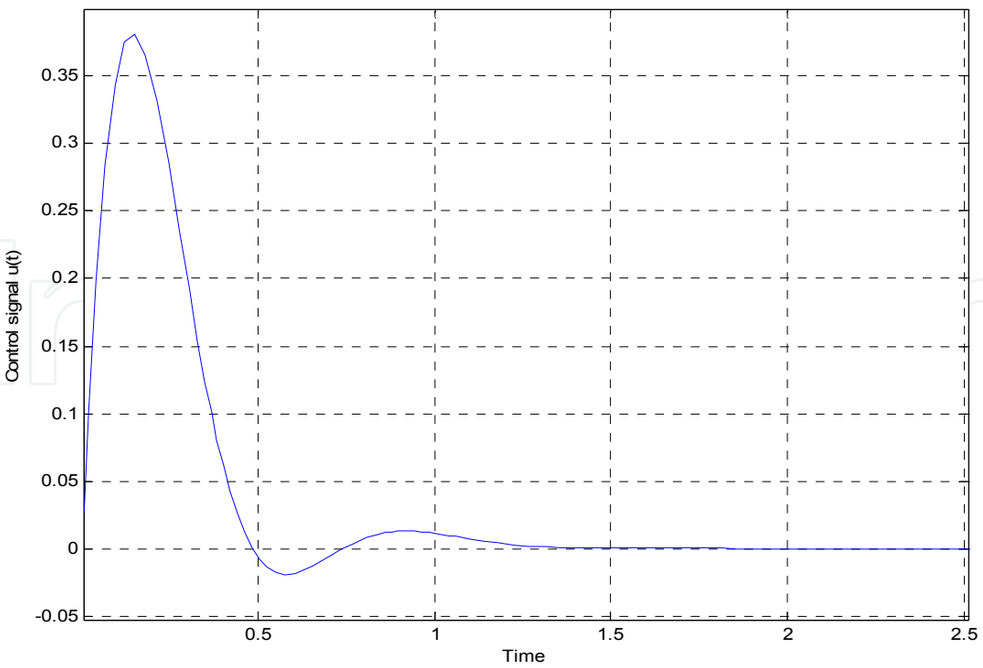


Fig. 5. Control signal evolution *u(t)*.

5. Conclusion

In this chapter, we have developed robust pole placement constraints for continuous T-S fuzzy systems with unavailable state variables and with parametric structured uncertainties. The proposed approach has extended existing methods based on uncertain T-S fuzzy models. The proposed LMI constraints can globally asymptotically stabilize the closed-loop T-S fuzzy system subject to parametric uncertainties with the desired control performances. Because of uncertainties, the separation property is not applicable. To overcome this problem, we have proposed, for the design of the observer and the controller, a two-step procedure with two pole placements constraints and the minimization of a H_∞ performance criterion in order to guarantee that the estimation error converges faster to zero. Simulation results have verified and confirmed the effectiveness of our approach in controlling nonlinear systems with parametric uncertainties.

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