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Description of Two Functions I and J Characterizing the Interior Ground Inertia of a Traditional Greenhouse - A Theoretical Model Using the Green's Functions Theory

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1. Introduction

In this chapter, we are presenting a precise model translating the influence of the ground inertia in the thermal behaviour of a greenhouse without vegetation. This work takes into account all the real mechanisms of exchanges (solar conduction, convection, radiation, thermal inertia) between the various elements of the system (cover, interior air, ground), but does not take into account the mass transfers (diffusions of moisture in the ground, evapotranspiration). We sought here to define a model constituting a core of a procedure on which new extensions will be based. We show via the Green Functions Theory (GFT) that the model's differential equations are reduced to a system of integral equations on the ground surface. These equations implicitly take into account the heat propagation in the ground. This model carefully describes in details the exchanges between the ground and the interior of the greenhouse. It aims also at defining the evolution of the greenhouse internal air temperature as well as that of the superficial temperature of its ground according to the following external data (power, exterior temperature). The mathematical study is completed by a numerical simulation on an isolated greenhouse.

One of the delicate problems in the study of thermal behaviours of the horticultural greenhouses is the modelling of their thermal inertia, which comes mainly from the ground. Indeed, the implementation of knowledge's model is an effective mean to accurately envisage the thermal behaviour of a greenhouse over long periods. Theoretical, numerical and experimental studies were the subject of many former publications. From these principal works we retain

- a simplified model which is based on a total heat balance by holding account particularly of absorbed solar radiation, and conductive losses through the wall of the greenhouse [1-6]
- another model is limited to the heat balances of the interior air and the cover, we retain of this model that the case of day and night were treated separately [7-8]
- then, another model taking into account the heterogeneity of the interior greenhouse ground, this one is subdivided into ten homogeneous layers of different conductivities [9]

- finally another model whose objective is to bring a contribution to the study of a process of setting out-freezing of a greenhouse by a technique of water sprinkling. A distinction is thus made in the choice of the system of equations according to the three following cases: summer (days and nights) and winter days, normal winter nights and winter nights with setting out-freezing.

The cover is considered at uniform temperature in the first two cases, will be subdivided into three layers in the last case to take into account of the layer of the formed ice. The ground is constituted of seven homogeneous layers in all cases [10-13]

Other modelling [14] lead to a too simplistic analytical resolution or being based on a one-dimensional approach by section of the ground's behaviour, so they do not make it possible to simulate the real behaviour [15]. The theoretical models, although often partial, contain many unknown parameters or difficult to determine experimentally.

A rigorous modelling of the interaction ground-greenhouse requires the solution of a differential equation with complex conditions of surface. The current models deal with this problem either numerically by discretizing the basement in the form of some layers [28-29], or by admitting that it behaves overall like a virtual thermal mass whose heat capacity and time-constant are given by the experiments [30].

However, the equation of propagation in the ground has an analytical solution which is obtained by GFT.

Our objective in this work is to show that this exact solution makes it possible to obtain the evolution's equation of the surface temperature $T_{si}(t)$ of the ground's interior of the greenhouse, according to the total power absorbed by the ground and to the temperature of the exterior air of the greenhouse. Two functions characteristic of the ground's behaviour appear in this equation (hereafter in the text) and we show starting from their properties at what the approximation of the virtual thermal mass consists of. The limits of this approximation appear clearly, we thus show how to correct and compare the two results in both cases.

In section (2), we establish the evolution's equation of the greenhouse's interior air, it acts here as a simplified model (greenhouse without vegetation) where solar energy is only absorbed by the ground and where the phenomena of evapo-transpirations do not intervene. Initially, we are concerned to establish a model taking into account the mechanisms of exchange by radiation, convection and conduction. In this model, we were able to control all the physical parameters in the case where it is possible to validate experimentally and quantitatively to separate the respective influences from these various modes and to determine in a reliable and univocal way the parameters suitable for each one of them: conduction, convection, radiation on the one hand and mass transfer (evapo-transpiration, condensation) on the other hand.

2. Setting in equation

2.1 Study of the heat balance of the greenhouse's internal air

Our system consists of three essential elements: the cover, internal air and the ground, the thermal behaviour of the internal air of the greenhouse, which we consider well ventilated, translating the evolution of the interior temperature $T_i(t)$, obeys to the following equation

$$\frac{dT_i(t)}{dt} = \frac{S_i H_{si}}{V_i \rho_i c_i} (T_{si}(t) - T_i(t)) + \frac{S_c H_{ci}}{V_i \rho_i c_i} (T_{ci}(t) - T_i(t)) + \frac{D_v(t)}{V_i} (T_e(t) - T_i(t)) \quad (1)$$

The internal air exchanges its heat, by convection, with the surface of the ground of average temperature $T_{si}(t)$ and the surface of internal face of the cover, which average temperature is $T_{ci}(t)$, by two convection coefficients respectively H_{si} and H_{ci} . There is also a heat exchange between the internal and external air via openings of air renewal of volume throughput $D_v(t)$, S_i : surface of the internal ground of the greenhouse, S_c : that of the cover, and V_i : the volume of the greenhouse see Figure 2.

All the temperatures are evaluated, thereafter, compared to a temperature of reference T_0 which is that of the ground taken at a depth superior to the effect of annual thermal skin. The latter is stable in a given area (evaluated at a depth of 2 meters), and practically equalizes at the annual average atmospheric temperature.

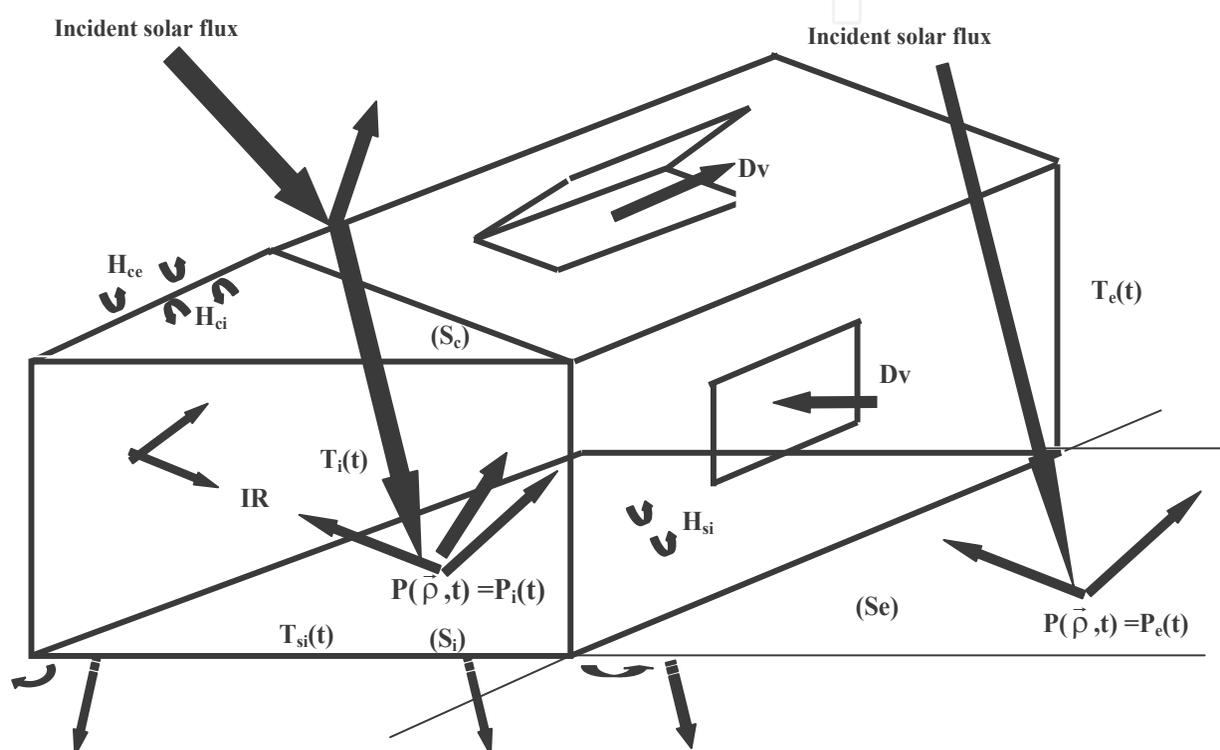


Fig. 1. Lay-out of the balance-sheet of heat exchange

2.2 Heat balance of the cover

The thickness of the cover is very low and the temperatures are slowly variable. We admit in his thickness a temperature profile constantly linear what amounts to neglecting its thermal inertia. The cover exchanges with its medium surrounding two fluxes Φ_e and Φ_i such as:

Φ_{ci} : heat flux exchanged by the internal face of the cover with the internal medium of the greenhouse.

Φ_{ce} : heat flux exchanged by the external face of the cover with the external medium of the greenhouse.

$$\Phi_{ci} = H_{ci}(T_i(t) - T_{ci}(t)) + H_{irc}(T_{si}(t) - T_{ci}(t)) \quad (2)$$

$$\Phi_{ce} = H_{ce}(T_e(t) - T_{ce}(t)) + H_{irc}(T_e'(t) - T_{ce}(t)) + a_c p_r \quad (3)$$

These two fluxes are composed of a convective part and of a linearized radiative part because the temperatures oscillate slightly around the absolute annual average temperature of reference T_0 . Indeed, the radiative power exchanged with the outside face of the cover is written as:

$$S_c \phi_{ir,ce} = \varepsilon_c S_{atm} F_{atm \rightarrow ce} \varepsilon_{atm} \sigma T_{atm}^4 + \varepsilon_c S_e F_{se \rightarrow ce} \varepsilon_s \sigma T_{se}^4 - \varepsilon_c S_c F_{ce \rightarrow atm} \sigma T_{ce}^4$$

Knowing that: $\varepsilon_{atm} = 1$, $T_{atm}^4 = \varepsilon_e T_e^4$ and by neglecting the effects of the external ground on the cover, the preceding expression is reduced:

$$S_c \phi_{ir,ce} = \varepsilon_c \sigma \left(S_{atm} F_{atm \rightarrow ce} \varepsilon_e T_e^4 - S_c F_{ce \rightarrow atm} T_{ce}^4 \right)$$

The relation of reciprocity:

$$S_{atm} F_{atm \rightarrow ce} = S_c F_{ce \rightarrow atm} = S_c \text{ because } F_{ce \rightarrow atm} = 1$$

Let's make the change of variable:

$$T_e'^4 = \varepsilon_e T_e^4 \quad (4)$$

Consequently, the expression of $\phi_{ir,ce}$ becomes:

$$\phi_{ir,ce} = \varepsilon_c \sigma \left(T_e'^4 - T_{ce}^4 \right)$$

$$\text{Let, } \begin{cases} T_e' = T_0 + \tilde{T}_e' \\ T_{ce} = T_0 + \tilde{T}_{ce} \end{cases}$$

With \tilde{T}_e' , \tilde{T}_{ce} are the fluctuations corresponding to each of two temperatures and T_{ce} , finally we obtain:

$\Phi_{ir,ce} = H_{irc} (T_e'(t) - T_{ce}(t))$ with: $H_{irc} = 4\varepsilon_c \sigma T_0^4$, we took off the hats on the temperatures only to reduce the writing.

The temperature of the cover is written: $T_c(y) = a y + b$ with a and b two constants determined by the boundary conditions

$$T_c(y) = \frac{(T_{ce} - T_{ci})}{e} y + T_{ci} \quad (5)$$

The boundary conditions on the surface of the cover are written:

$$-k_c \frac{\partial T_c}{\partial y} \Big|_{y=0} = H_{ci} (T_i(t) - T_{ci}(t)) + H_{irc} (T_{si}(t) - T_{ci}(t)) \quad (6)$$

$$k_c \frac{\partial T_c}{\partial y} \Big|_{y=e} = H_{ce} (T_e(t) - T_{ce}(t)) + H_{irc} (T_e'(t) - T_{ce}(t)) + a_c P_r \quad (7)$$

We found previously the expression of $T_c(y)$, then

$$\frac{\partial T_c}{\partial y} = \frac{T_{ce} - T_{ci}}{e} \tag{8}$$

By injecting the expression (8), in the system of equations (6) and (7), we obtain a new system of equation ; ((6'), (7')) of unknown factors $T_{ci}(t)$ and $T_{ce}(t)$:

$$\left(\frac{k_c}{e} + H_{ci} + H_{irc}\right)T_{ci}(t) - \frac{k_c}{e}T_{ce}(t) = H_{ci}T_i(t) + H_{irc}T_{si}(t) \tag{6'}$$

$$-\frac{k_c}{e}T_{ci}(t) + \left(\frac{k_c}{e} + H_{ce} + H_{irc}\right)T_{ce}(t) = \left(H_{ce} + \varepsilon_s^4 H_{irc}\right)T_e(t) + a_c p_r \tag{7'}$$

We deduce, starting from this system, the expressions of $T_{ci}(t)$ and $T_{ce}(t)$ taking the following forms respectively

$$T_{ci}(t) = \lambda_i T_i(t) + \lambda_{si} T_{si}(t) + \lambda_e T_e(t) + \lambda_r \tag{9}$$

$$T_{ce}(t) = \beta_i T_i(t) + \beta_{si} T_{si}(t) + \beta_e T_e(t) + \beta_r \tag{10}$$

With :

$$\lambda_i = \frac{H_{ci} \frac{e}{k_c} \left(\frac{k_c}{e} + H_{ce} + H_{irc}\right)}{\text{Deno}}, \lambda_r(t) = \frac{a_c p_r}{\text{Deno}}, \lambda_{si} = \frac{H_{irc} \frac{e}{k_c} \left(\frac{k_c}{e} + H_{ce} + H_{irc}\right)}{\text{Deno}},$$

$$\lambda_e = \frac{(H_{ce} + H_{irc} \varepsilon_s^4)}{\text{Deno}}$$

$$\text{Deno} = \frac{e}{k_c} \left(\frac{k_c}{e} + H_{ci} + H_{irc}\right) \left(\frac{k_c}{e} + H_{ce} + H_{irc}\right) - \frac{k_c}{e}$$

$$\beta_i = \frac{H_{ci}}{\text{Deno}}, \beta_e = \frac{\frac{e}{k_c} \left(\frac{k_c}{e} + H_{ci} + H_{irc}\right) (H_{ce} + \varepsilon_s^4 H_{irc})}{\text{Deno}}$$

$$\beta_{si} = \frac{H_{irc}}{\text{Deno}}, \beta_r = \frac{\frac{e}{k_c} \left(\frac{k_c}{e} + H_{ci} + H_{irc}\right) a_c p_r}{\text{Deno}}$$

Let's introduce the expression of $T_{ci}(t)$ into the equation (1), this latter takes the following form

$$\frac{dT_i(t)}{dt} = G_{si} T_{si}(t) - G_i T_i(t) + G_e T_e(t) + G_r(t) \tag{11}$$

With:

$$G_{si} = \frac{S_i H_{si} + S_c H_{ci} \lambda_{si}}{V_i \rho_i C_i}, G_i = \frac{S_i H_{si} + S_c H_{ci} (1 - \lambda_i) + D_v \rho_i C_i}{V_i \rho_i C_i}$$

$$G_e = \frac{S_c H_{ci} \lambda_e + D_v \rho_i C_i}{V_i \rho_i C_i}, G_r(t) = \frac{S_c H_{ci} \lambda_r(t)}{V_i \rho_i C_i}$$

The transformed of Laplace of the equation (11), leads to

$$\bar{T}_i(p) = \frac{T_i(0)}{p + G_i} + G_{si} \frac{\bar{T}_{si}(p)}{p + G_i} + G_e \frac{\bar{T}_e(p)}{p + G_i} + \frac{\bar{G}_r(p)}{p + G_i} \quad (12)$$

With, $T_i(0) \equiv T_0$: initial field of temperature, whose original is:

$$T_i(t) = T_0 e^{-G_i t} + G_{si} \int_0^t T_{si}(\tau) e^{-G_i(t-\tau)} d\tau + G_e \int_0^t T_e(\tau) e^{-G_i(t-\tau)} d\tau + \int_0^t G_r(\tau) e^{-G_i(t-\tau)} d\tau \quad (13)$$

In this integral equation, the temperature T_0 represents the field of initial temperature, $T_e(t)$ is a field of temperature which translates the influence of the exterior climatic conditions on the temperature of the internal air of the greenhouse, on the other hand one will show that $T_{si}(t)$ depends functionally on $T_i(t)$.

The hour is taken as unit of time, the temporal evolution of the internal temperature is typically during few seconds, consequently we can neglect $\frac{dT_i(t)}{dt}$, then the expression of $T_i(t)$ takes the following form

$$T_i(t) = \frac{G_{si}}{G_i} T_{si}(t) + \frac{G_e}{G_i} T_e(t) + \frac{G_r(t)}{G_i} \quad (14)$$

2.3 Heat balance of the ground

We suppose that the basement of the greenhouse is homogeneous, isotropic and of thermal properties (voluminal heat ρc and thermal conductivity k_s) constant, we note $p(\vec{s}, t)$ the absorbed power per square meter in an unspecified point \vec{s} of the surface of the ground. This power comes primarily from the absorption of the direct and indirect solar radiation, as it can include other phenomena like precipitations and evaporation. We neglect the variations coming from the phenomena of shade, variation of the surface quality, etc.

We can write: $p(\vec{s}, t) = p_i(t)$ si $\vec{s} \in (S_i)$
 $p(\vec{s}, t) = p_e(t)$ si $\vec{s} \in (S_e)$

The field of temperature $T(\vec{r}, t)$ in a point $\vec{r}(x, y, z)$ at the moment t inside the ground obeys to the following equation

$$\frac{\partial T(\vec{r}, t)}{\partial t} - a \Delta T(\vec{r}, t) = \frac{p(\vec{s}, t)}{\rho c} \delta(z) \text{ in } (D) \quad (15)$$

The field $T(\vec{r}, t)$ must check on the surface of the ground the following conditions

2.3.1 Condition on the surface of the interior greenhouse ground (S_i)

$$-k_s \frac{\partial T(\vec{r},t)}{\partial Z} \Big|_{Z=0} = H_{si} (T(\vec{r},t) - T_i(t)) + H_{iri} (T(\vec{r},t) - T_{ci}(t)) \Big|_{Z=0} \tag{16}$$

By replacing T_{ci}(t) by his expression we obtains

$$-\frac{\partial T(\vec{r},t)}{\partial Z} \Big|_{Z=0} = \frac{(H_{si} + H_{iri}(1-\lambda_{si}))}{k_s} \left(T(\vec{r},t) - \frac{(H_{si} + \lambda_i H_{iri})T_i + H_{iri}\lambda_e T_e}{H_{si} + H_{iri}(1-\lambda_{si})} \right) \Big|_{Z=0} + \frac{H_{iri}\lambda_{si}}{k_s} (T(\vec{r},t) - T_{si}(t)) - \frac{H_{iri}\lambda_r}{k_s} \Big|_{Z=0} \tag{17}$$

Let

$$h'_i = \frac{H_{si} + H_{iri}(1-\lambda_{si})}{k_s} \tag{18}$$

$$T_2(t) = \frac{(H_{si} + \lambda_i H_{iri})T_i(t) + \lambda_e H_{iri} T_e(t)}{H_{si} + H_{iri}(1-\lambda_{si})} \tag{19}$$

$$h_i^0 = \frac{\lambda_{si} H_{iri}}{k_s} \tag{20}$$

$$R_{si}(t) = H_{iri}\lambda_r(t) \tag{21}$$

The equation (16) takes thus the following form

$$-\frac{\partial T(\vec{r},t)}{\partial Z} \Big|_{Z=0} = h'_i (T(\vec{r},t) - T_2(t)) + h_i^0 (T(\vec{r},t) - T_{si}(t)) - \frac{R_{si}(t)}{k_s} \tag{22}$$

2.3.2 Condition on the surface of the external greenhouse ground (S_e)

$$-k_s \frac{\partial T(\vec{r},t)}{\partial Z} \Big|_{Z=0} = H_{se} (T(\vec{r},t) - T_e(t)) + H_{ire} (T(\vec{r},t) - T'_e(t)) \Big|_{Z=0} \tag{23}$$

By using the expression (4) we obtain

$$-\frac{\partial T(\vec{r},t)}{\partial Z} \Big|_{Z=0} = \frac{H_{se} + H_{ire}}{k_s} \left(T(\vec{r},t) - \frac{\left(H_{se} + H_{ire} \epsilon_e^{\frac{1}{4}} \right) T_e(t)}{H_{se} + H_{ire}} \right) \Big|_{Z=0} \tag{24}$$

Let

$$h'_e = \frac{H_{se} + H_{ire}}{k_s} \tag{25}$$

$$T_1(t) = \frac{(H_{se} + H_{ire} \epsilon_e^{\frac{1}{4}}) T_e(t)}{H_{se} + H_{ire}} \quad (26)$$

The equation (24) is written then

$$-\left. \frac{\partial T(\vec{r}, t)}{\partial Z} \right|_{z=0} = h'_e (T(\vec{r}, t) - T_1(t)) \Big|_{z=0} \quad (27)$$

2.3.3 Resolution of the problem by Green's functions theory

Let's consider $G(\vec{r}, \vec{r}', t)$ (Green's function) translating the response in temperature of the medium to an impulse of heat into a given point, the language generally employed consists to talk about effect into \vec{r} corresponding of the cause in \vec{r}' . In addition Green's functions obey to the reciprocity's relation of the cause and the effect, if the cause is produced in \vec{r} , the effect will be identical in \vec{r}' with the proviso of respecting the same interval of time between the moment when the cause occurs t and that when the effect occurs ($t_0 = 0$) selected arbitrarily as origin of time.

The Green's function is a particular solution of the heat's equation

$$\frac{\partial G(\vec{r}, \vec{r}', t)}{\partial t} - a \Delta G(\vec{r}, \vec{r}', t) = \delta(\vec{r}' - \vec{r}) \delta(z) \quad (28)$$

With the initial condition

$$G(\vec{r}, \vec{r}', t=0) = 0 \quad (29)$$

And checking the condition on the surface of the ground ($S_e \cup S_i$)

$$-\left. \frac{\partial G(\vec{r}, \vec{r}', t)}{\partial z} \right|_{z=0} = h'_i G(\vec{r}, \vec{r}', t) \quad (30)$$

The Green's function has as expression [22]

$$G(\vec{r}, \vec{r}', t) = \frac{e^{-\frac{(x-x')^2 + (y-y')^2}{4at}}}{4\pi at} \times \left[\frac{e^{-\frac{(z-z')^2}{4at}} + e^{-\frac{(z+z')^2}{4at}}}{\sqrt{4\pi at}} - h'_i e^{h'_i(z+z') + ah_i^2 t} \operatorname{erfc}\left(\frac{z+z'}{2\sqrt{at}} + h'_i \sqrt{at}\right) \right] \quad (31)$$

The Laplace's transformation of two equations (15) and (28) is

$$p\bar{T}(\vec{r}, p) - T(\vec{r}, 0) - a \Delta \bar{T}(\vec{r}, p) = \frac{\bar{P}(\vec{s}, p)}{\rho c} \delta(z) \quad (32)$$

$$p\bar{G}(\bar{r},\bar{r}',p) - a \Delta\bar{G}(\bar{r},\bar{r}',p) = \delta(\bar{r}-\bar{r}')\delta(z) \tag{33}$$

Let's multiply the equations (32) and (33) respectively by $\bar{G}(\bar{r},\bar{r}',p)$ and $\bar{T}(\bar{r},p)$, we obtain

$$p\bar{T}(\bar{r},p)\bar{G}(\bar{r},\bar{r}',p) - T(\bar{r},0)\bar{G}(\bar{r},\bar{r}',p) - a \bar{G}(\bar{r},\bar{r}',p)\Delta\bar{T}(\bar{r},p) = \frac{\bar{G}(\bar{r},\bar{r}',p)\bar{p}(\bar{s},p)}{\rho c} \delta(z) \tag{34}$$

$$p\bar{T}(\bar{r},p)\bar{G}(\bar{r},\bar{r}',p) - T(\bar{r},0)\bar{G}(\bar{r},\bar{r}',p) - a \bar{G}(\bar{r},\bar{r}',p)\Delta\bar{T}(\bar{r},p) = \frac{\bar{G}(\bar{r},\bar{r}',p)\bar{p}(\bar{s},p)}{\rho c} \delta(z) \tag{35}$$

Let's make the subtraction between the equations (35) and (34), we obtains

$$\begin{aligned} \bar{T}(\bar{r},p)\delta(\bar{r}-\bar{r}')\delta(z) &= \bar{G}(\bar{r},\bar{r}',p)\left(T_0(\bar{r}) + \frac{\bar{p}(\bar{s},p)}{\rho c} \delta(z)\right) \\ &- a \left(\bar{T}(\bar{r},p) \Delta\bar{G}(\bar{r},\bar{r}',p) - \bar{G}(\bar{r},\bar{r}',p) \Delta\bar{T}(\bar{r},p)\right) \end{aligned} \tag{36}$$

Let's integrate this equation on all the field (D), we obtain the field of temperature $\bar{T}(\bar{r}',p)$ then the original $T(\bar{r}',t)$, single solution of the equation (36)

$$\begin{aligned} T(\bar{r}',t) &= \int_0^t d\tau \iint_{(S=S_e \cup S_i)} \frac{p(\bar{s},t)}{\rho c} G(\bar{r},\bar{r}',t-\tau) dS + \iiint_{(D)} G(\bar{r},\bar{r}',t) T_0(\bar{r}) d^3r \\ &+ a \int_0^t d\tau \iint_{(S=S_e \cup S_i)} \left(G(\bar{r},\bar{r}',t-\tau) \bar{\nabla} T(\bar{S},\tau) - T(\bar{r},\tau) \bar{\nabla} G(\bar{r},\bar{r}',t-\tau)\right) dS \end{aligned} \tag{37}$$

(S) being the meeting of (S_i) and (S_e), if we take account of the boundary conditions on the surface of the ground, satisfying the conditions (22), (27) and (30) and from the initial condition we obtain

$$\begin{aligned} T(\bar{r}',t) &= \iiint_{(D)} G(\bar{r},\bar{r}',t) T_0(\bar{r}) d^3r + \int_0^t d\tau \iint_{(S_i)} \frac{p_i(\bar{s},t)}{\rho c} G(\bar{r},\bar{r}',t-\tau) dS_i \\ &+ \int_0^t d\tau \iint_{(S_e)} \frac{p_e(\bar{s},t)}{\rho c} G(\bar{r},\bar{r}',t-\tau) dS_e + \\ &+ a \int_0^t d\tau \iint_{(S_i)} \left(G(\bar{r},\bar{r}',t-\tau) \frac{\partial T(\bar{r},\tau)}{\partial z} - T(\bar{r},\tau) \frac{\partial G(\bar{r},\bar{r}',t-\tau)}{\partial z}\right) dS_i \\ &+ a \int_0^t d\tau \iint_{(S_e)} \left(G(\bar{r},\bar{r}',t-\tau) \frac{\partial T(\bar{r},\tau)}{\partial z} - T(\bar{r},\tau) \frac{\partial G(\bar{r},\bar{r}',t-\tau)}{\partial z}\right) dS_e \\ T(\bar{r}',t) &= \underbrace{\iiint_{(D)} G(\bar{r},\bar{r}',t) T_0(\bar{r}) d^3r}_{(1)} + \underbrace{\int_0^t d\tau \iint_{(S_i)} \frac{p_i(t-\tau)}{\rho c} G(\bar{r},\bar{r}',\tau) dS_i}_{(2)} \\ &+ \underbrace{\int_0^t d\tau \iint_{(S_e)} \frac{p_e(t-\tau)}{\rho c} G(\bar{r},\bar{r}',\tau) dS_e}_{(3)} + a \underbrace{\int_0^t d\tau \iint_{(S_i)} G(\bar{r},\bar{r}',\tau) h_i (T(\bar{r},t-\tau) - T_2(t-\tau)) dS_i}_{(4)} + \end{aligned} \tag{38}$$

$$\begin{aligned}
& + a \int_0^t d\tau \iint_{(S_i)} \underbrace{G(\vec{r}, \vec{r}', t-\tau) h_i^0 (T_{si}(\tau) - T(\vec{r}, \tau))}_{(5)} dS_i + \\
& + a \int_0^t d\tau \iint_{(S_e)} \underbrace{\left(\underbrace{(h_i' - h_e')}_{(b)} G(\vec{r}, \vec{r}', t-\tau) T(\vec{r}, \tau) + h_e' T_1(\tau) G(\vec{r}, \vec{r}', t-\tau) \right)}_{(6)} dS_e + \quad (39) \\
& + a \int_0^t d\tau \iint_{(S_i)} \underbrace{\frac{R_{si}(\tau)}{k_s} G(\vec{r}, \vec{r}', t-\tau)}_{(7)} dS_i
\end{aligned}$$

With $\vec{r}' \in (D)$

- $T_0(\vec{r}', t)$ is the field of temperature in (D) due only to the initial condition given by the term (1).
- $T_{Si}(\vec{r}', t)$ is the field of temperature in the under-ground, due only to the exchanges on internal surface of the greenhouse given by the terms (2, 4, 5, 7).
- $T_{Se}(\vec{r}', t)$ is the field of temperature in the under-ground, due only to the exchanges with exterior surface of the greenhouse represented by the terms (3, 6).

We can break up the field of temperature in the form

$T(\vec{r}', t) = T_0(\vec{r}', t) + T_{Si}(\vec{r}', t) + T_{Se}(\vec{r}', t)$ varies with exterior surface of the greenhouse represented by the terms (3, 6).

We note that the term (a), in (4) comes owing to the fact that the surface temperature $T_{Si}(t)$ is not completely homogeneous, the term (b) in (6) disappears since at the exterior of the greenhouse the difference ($h_i' \approx h_e'$) is negligible. These two terms, generally very weak, they could be treated as a perturbation and the expression of $T(\vec{r}', t)$ can be written as

$$\begin{aligned}
T(\vec{r}', t) = & \iiint_{(D)} G(\vec{r}, \vec{r}', t) T_0(\vec{r}) d^3r + \int_0^t d\tau \iint_{(S_i)} \frac{P_i(t-\tau)}{\rho c} G(\vec{r}, \vec{r}', \tau) dS_i \\
& + \int_0^t d\tau \iint_{(S_e)} \frac{P_e(t-\tau)}{\rho c} G(\vec{r}, \vec{r}', \tau) dS_e + \quad (40) \\
& + a \int_0^t d\tau \iint_{(S_i)} G(\vec{r}, \vec{r}', \tau) h_i' (T(\vec{r}, t-\tau) - T_2(t-\tau)) dS_i \\
& + a \int_0^t d\tau \iint_{(S_e)} h_e' T_1(\tau) G(\vec{r}, \vec{r}', t-\tau) dS_e + a \int_0^t d\tau \iint_{(S_i)} \frac{R_{si}(\tau)}{k_s} G(\vec{r}, \vec{r}', t-\tau) dS_i
\end{aligned}$$

In fact, the knowledge of the field $T(\vec{r}', t)$ in the domain (D) is useless, what we really need is to know the surface temperature of the interior ground of the greenhouse. Indeed, let \vec{r}' be close to $\vec{s} \in (S_i)$, in this case the co-ordinates z' of \vec{r}' become null, and if we make the average on (S_i) , the equation (40) gives us the field of surface temperature $T_{Si}(t)$

$$\begin{aligned}
T_{Si}(t) & = \frac{1}{S_i} \iint_{S_i} T(\vec{r}', t) dS_i \\
& = \frac{1}{S_i} \iint_{S_i} dS_i \iiint_{(D)} G(\vec{r}, \vec{r}', t) T_0(\vec{r}) d^3r +
\end{aligned}$$

$$\begin{aligned}
 & + \int_0^t \left(\frac{p_i(t-\tau) + R_{si}(t-\tau)}{\rho C} + ah'_i T_2(t-\tau) \right) d\tau \frac{1}{S_i} \iint_{(Si)} dS_i \iint_{(Si)} G(\vec{r}, \vec{r}', \tau) dS_i \\
 & + \int_0^t \left(\frac{p_e(t-\tau)}{\rho C} + ah'_e T_1(t-\tau) \right) d\tau \frac{1}{S_i} \iint_{(Si)} dS_i \iint_{(Se)} G(\vec{r}, \vec{r}', \tau) dS_e
 \end{aligned} \tag{41}$$

We can show that

$$\begin{aligned}
 \frac{1}{S_i} \iint_{S_i} dS_i \iint_{S_i} G(\vec{S}, \vec{S}', \tau) dS_i & = I(\tau)J(\tau) \\
 \frac{1}{S_i} \iint_{(Si)} dS_i \iint_{(Se)} G(\vec{S}, \vec{S}', \tau) dS_e & = I(\tau)(1-J(\tau))
 \end{aligned}$$

with $I(\tau) = \frac{1}{\sqrt{\pi a \tau}} - h'_i e^{h_i^2 a \tau} \operatorname{erfc}(h'_i \sqrt{a \tau})$

$$J(\tau) = \left[1 - \operatorname{erfc}\left(\frac{L}{2\sqrt{a\tau}}\right) + \frac{2}{L} \sqrt{\frac{a\tau}{\pi}} \left(e^{-\left(\frac{L}{2\sqrt{a\tau}}\right)^2} - 1 \right) \right]$$

and

$$* \left[1 - \operatorname{erfc}\left(\frac{\ell}{2\sqrt{a\tau}}\right) + \frac{2}{\ell} \sqrt{\frac{a\tau}{\pi}} \left(e^{-\left(\frac{\ell}{2\sqrt{a\tau}}\right)^2} - 1 \right) \right]$$

The expression of $T_{si}(t)$ becomes

$$\begin{aligned}
 T_{si}(t) & = \frac{1}{S_i} \iint_{S_i} dS_i \iiint_{(D)} G(\vec{r}, \vec{r}', t) T_0(\vec{r}) d^3r + \int_0^t \left(\frac{P_i(t-\tau)}{\rho c} + ah'_i T_2(t-\tau) \right) I(\tau)J(\tau) d\tau \\
 & + \int_0^t \left(\frac{p_e(t-\tau)}{\rho c} + ah'_e T_1(t-\tau) \right) I(\tau)(1-J(\tau)) d\tau
 \end{aligned} \tag{42}$$

With

$$P_i(t-\tau) = p_i(t-\tau) + R_{si}(t-\tau) \tag{43}$$

The field of temperature can be broken up in the following form

$$T_{si}(t) = T_{si}^0(t) + T_{si,si}(t) + T_{si,se}(t) \tag{44}$$

With

$$T_{si}^0(t) = \frac{1}{S_i} \iint_{S_i} dS_i \iiint_{(D)} G(\vec{r}, \vec{r}', t) T_0(\vec{r}) d^3r \tag{45}$$

Representing the contribution of the initial field of temperature; it tends towards zero when t tends towards the infinite one. So, we admit that the field of temperature in all the ground

is initially uniform and is equal to the temperature of reference $T_0(\bar{r})$. This approximation does not affect the precision to the beginning and over a limited time.

$$T_{si,si}(t) = \int_0^t \left(\frac{P_i(t-\tau)}{\rho c} + ah'_i T_2(t-\tau) \right) I(\tau) J(\tau) d\tau \quad (46)$$

being the field of temperature of internal greenhouse surface due to the exchanges with the basement or due to the contribution of the basement

$$T_{si,se}(t) = \int_0^t \left(\frac{P_e(t-\tau)}{\rho c} + ah''_e T_1(t-\tau) \right) I(\tau) (1 - J(\tau)) d\tau \quad (47)$$

Generally, the greenhouses are of average and of large dimensions, consequently the contribution of $T_{si,se}(t)$ in the expression of $T_{si}(t)$ is negligible except in the case of the

greenhouses of very small dimensions ($L \approx \ell \approx 3m$).

Consequently

$$T_{si}(t) = T_{si}^0(t) + \int_0^t \left(\frac{P_i(t-\tau)}{\rho c} + ah'_i T_2(t-\tau) \right) I(\tau) J(\tau) d\tau \quad (48)$$

By replacing $T_2(t-\tau)$ by its expression, $T_{si}(t)$ takes the following form

$$\begin{aligned} T_{si}(t) = & T_{si}^0(t) + \frac{1}{\rho c} \int_0^t P_i(t-\tau) I(\tau) J(\tau) d\tau \\ & + \frac{H'_i}{\rho c} \int_0^t T_i(t-\tau) I(\tau) J(\tau) d\tau + \frac{H''_i}{\rho c} \int_0^t T_e(t-\tau) I(\tau) J(\tau) d\tau \end{aligned} \quad (49)$$

with $H'_i = H_{si} + H_{iri}\lambda_i$ and $H''_i = H_{iri}\lambda_e$

We developed a fast algorithm of resolution of this type equation.

2.4 Study of the characteristic functions $I(\tau)$ et $J(\tau)$

Equation (49) shows that the two functions $I(\tau)$ and $J(\tau)$ characterize entirely and rigorously the thermal inertia of the ground and the interaction of this one with the entire system (cover, interior ground and interior air).

These two functions are positive monotonous and tend towards zero when τ tends towards infinity, consequently they can be approximated numerically, with a good precision, by a series of exponential decreasing of time allowing a fast calculation of the product of convolution. The function $I(\tau)$ presents a singularity at the term $\tau^{-1/2}$ in the vicinity of zero fortunately this singularity can be integrated. We introduce the function

$I_1(\alpha) = e^\alpha \text{erfc}(\sqrt{\alpha})$ with $\alpha = h_i'^2 a \tau$ in addition we saw

$$I(\tau) = \frac{1}{\sqrt{\pi a \tau}} - h'_i \exp(h_i'^2 a \tau) \text{erfc}(h'_i \sqrt{a \tau}) \quad , \tau \in [0, +\infty [$$

We can write then for $\alpha \in [0, +\infty[$

$$I(\alpha) = h_i' \left(\frac{1}{\sqrt{\pi\alpha}} - e^\alpha \operatorname{erfc}(\sqrt{\alpha}) \right) \tag{50}$$

It is noticed finally that

$$I(\alpha) = -h_i' \frac{dI_1(\alpha)}{d\alpha} \tag{51}$$

Let $u = \exp(-\alpha)$ $u \in [0,1]$

$I_1(u)$ is a strictly decreasing positive function and tends towards zero when u tends towards unity; in addition its graph is not obviously linear, the numerical analysis of the graph shows that we can approach this function with a quadratic average on the interval $[0,1]$ by polynomials of type : au^b .

The approximate expression of $I_1(u)$ is written:

$$I_1(u)_{\text{app}} = a u^b + c u^d \tag{52}$$

Calculation gives:

$$a = 0,4269, \quad b = 4,676, \quad c = 0,499, \quad d = 0,1659.$$

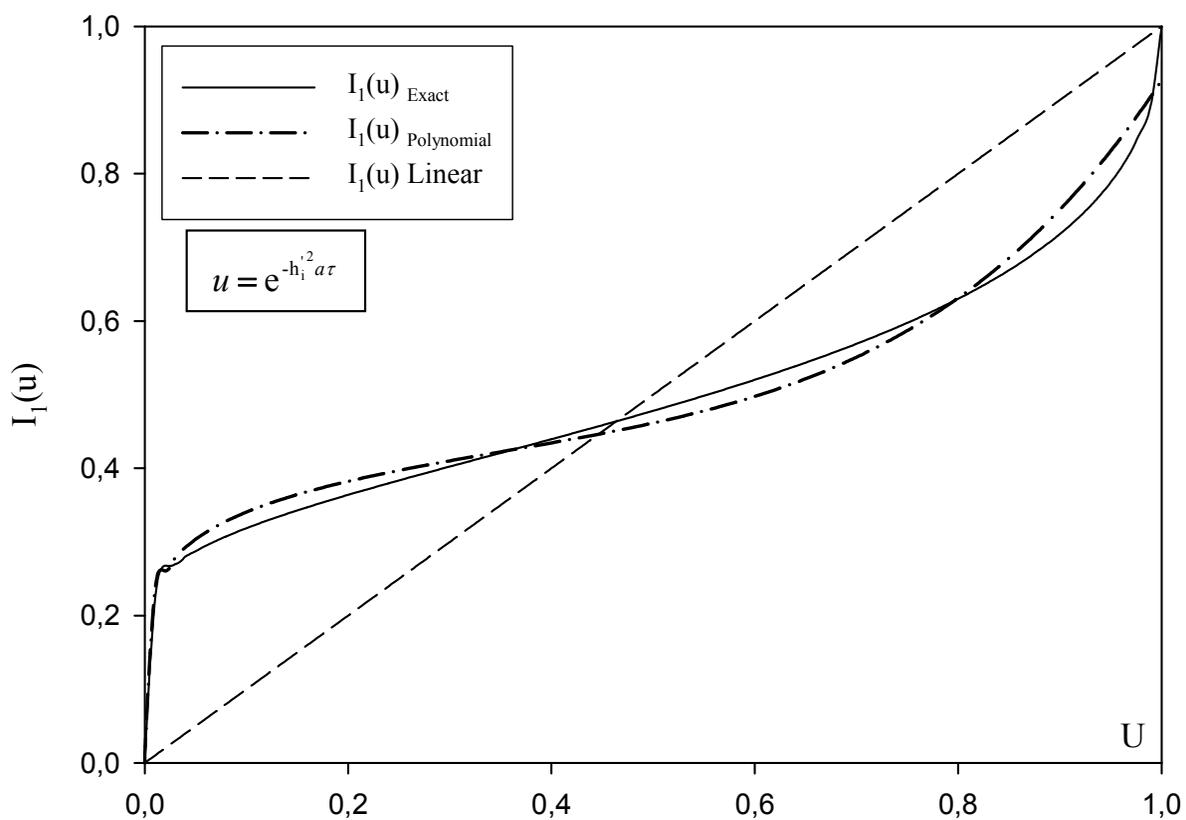


Fig. 2. Evolution of the inertia's function I_1 with its various approximations

These numbers without dimensions are thus defined once for all, we notice that exact $I_1(u)$ and its approximate expression coincide well in Figure 2 and the function $I(u)$ can be then approximated in Figure 3, by a polynomial of following this form:

$$I(u)_{app} = h'_i (1,9962 u^{4,676} + 0,0828 u^{0,1659}) \tag{53}$$

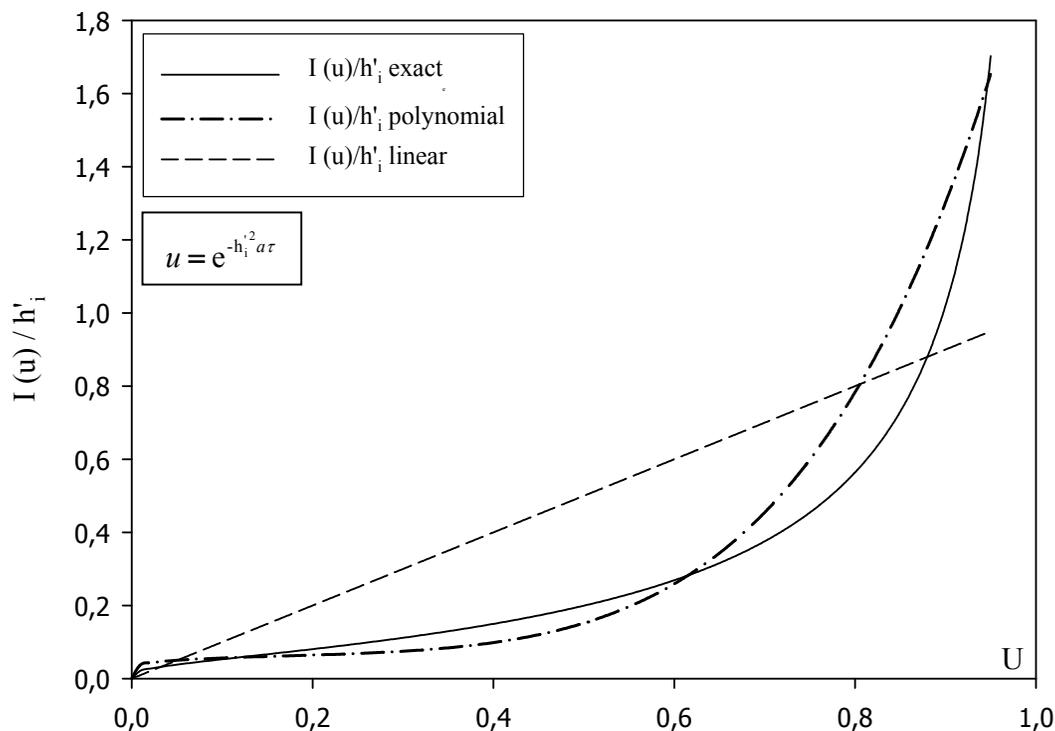


Fig. 3. Evolution of the inertia's function $I(u)$ with its various approximations

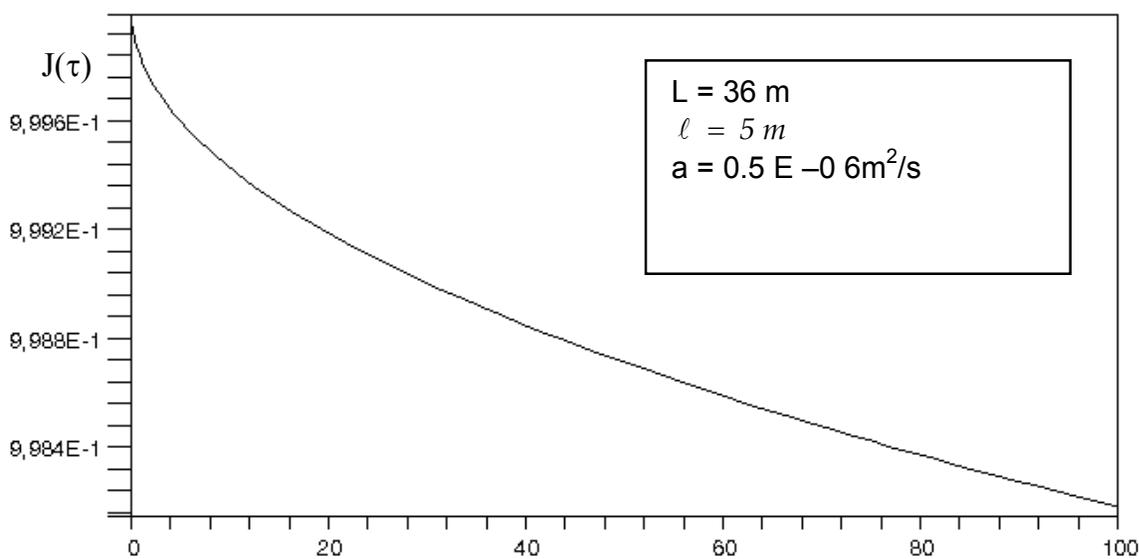


Fig. 4. Lay-out of the balance-sheet of heat exchange

The typical values of thermal conductivity (k_s) and thermal diffusivity (a) for a ground are respectively about 1 and $0,5 \cdot 10^{-6}$, it is obvious that for large-sized greenhouses, the effect of the surrounding ground is so negligible that we cannot measure it. It is obvious to admit that $J(\tau)$ remains practically equal to the unit Figure 4, except for greenhouses of very small sizes (L et $\ell \leq 3 \text{ m}$) where the effect can be perceptible.

3. Discussion of the numerical model

We carried out the numerical simulation on a tunnel greenhouse with a plastic cover (polyethylene) with a simple cover, isolated, of volume 354 m^3 (length 36 meters and 5 meters broad), placed on a ground of thermal diffusivity ($a = 0,5 \cdot 10^{-6} \text{ Wm}^2/\text{J}$) and of thermal conductivity ($k_s = 1 \text{ W/m } ^\circ\text{C}$), we took the function of inertia ($J(\tau) = 1$) because the studied greenhouse is practically of great dimension.

The equations (13) and (49) that appear in their products of convolution, climatic data, exterior temperature and total solar power, contain parameters depending on the place and season.

Indeed, we took for our numerical simulation

$$\text{Exterior temperature: } T_e(t) = -5 \cos\left(\frac{2 \pi t}{24}\right)$$

$$\text{Total solar power: } p_i(t) = 280 \left(\cos\left(\frac{2 \pi t}{24}\right)\right)^2$$

In addition, the tableau1 appearing below gathers the thermo-physical constants of the air, ground and cover which we used in this simulation.

$\epsilon_{ce} = 0.95$	$\rho_s C_s = 2 \cdot 10^6 \text{ JK}^{-1}\text{m}^{-1}$	$\rho_i = 1.117 \text{ kgm}^{-3}$
$t_c = 0.65$	$\epsilon_{si} = 0.95$	$C_i = 1006 \text{ Jkg}^{-1} \text{ K}^{-1}$
$k_c = 1.5 \text{ Wm}^{-1} \text{ K}^{-1}$	$T_{si0} = 9^\circ\text{C}$	$T_{i0} = 279.15 \text{ K}$
$\epsilon_{ci} = 0.95$	$k_s = 1 \text{ Wm}^{-1}\text{K}^{-1}$	
$a_c = 0.31$	$a = 0,5 \cdot 10^{-6} \text{ m}^2\text{s}^{-1}$	
$h_{ce} = 0.1 \text{ Wm}^{-2} \text{ K}^{-1}$		
$h_{ci} = 0.3 \text{ Wm}^{-2}\text{K}^{-1}$		
$L = 36 \text{ m}$		
$\ell = 5 \text{ m} \quad \ell = 5 \text{ m}$		

Table 1. Table of the entered parameters of the digital simulation

4. Interpretation of the results

Figure 5 shows the superimposed evolutions of incident solar flux and those exchanged with the external and internal face of the greenhouse's cover.

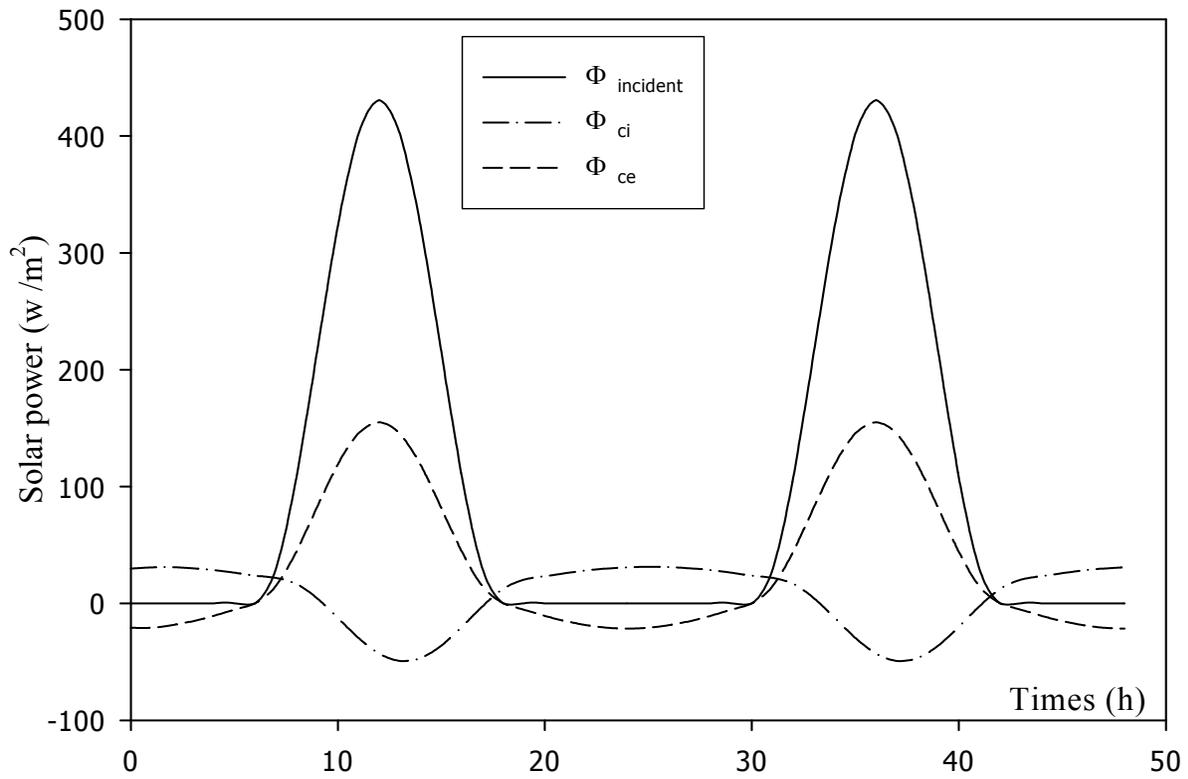


Fig. 5. Temporal evolution of the heat fluxes, incident and exchanged with internal and external faces of the greenhouse's cover

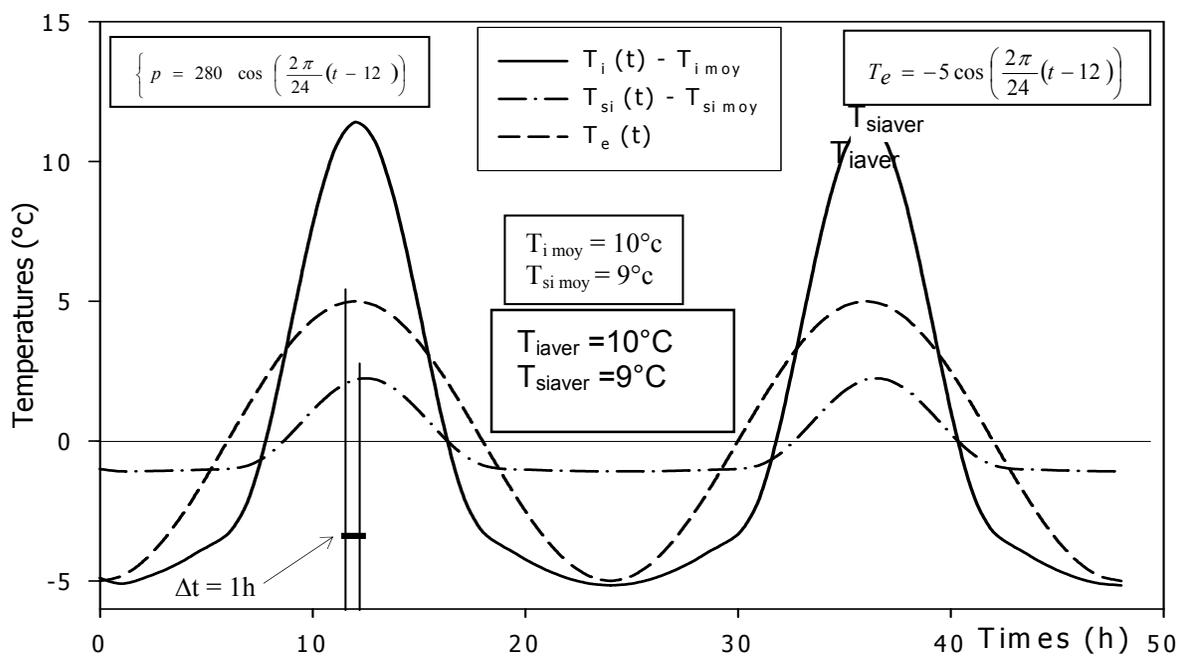


Fig. 6. Temporal evolution of temperatures, of the internal air, internal ground and the external air of the greenhouse for an air renewal' flux null

We note during the day, that the exchanged flux with the exterior face of the cover ϕ_{ce} is more important than that exchanged with the interior face ϕ_{ci} , that is foreseeable because of the absorption of a part of the incident heat flux by the cover.

During the night, the incident heat flux becomes null, consequently the interior air and ground must radiate now towards the exterior, it is the night radiation, therefore ϕ_{ci} becomes more important than ϕ_{ce} , but remain the two weak contributions.

We deferred in Figure 6, the evolutions of the exterior temperature and that of the interior ground of a closed greenhouse.

We note that the effect of inertia of the ground and the absorption of the heat of the day by its surface appears in the form of a rise in temperature of the order of 2°C and a phase shift of the order of one hour with the interior air of the greenhouse.

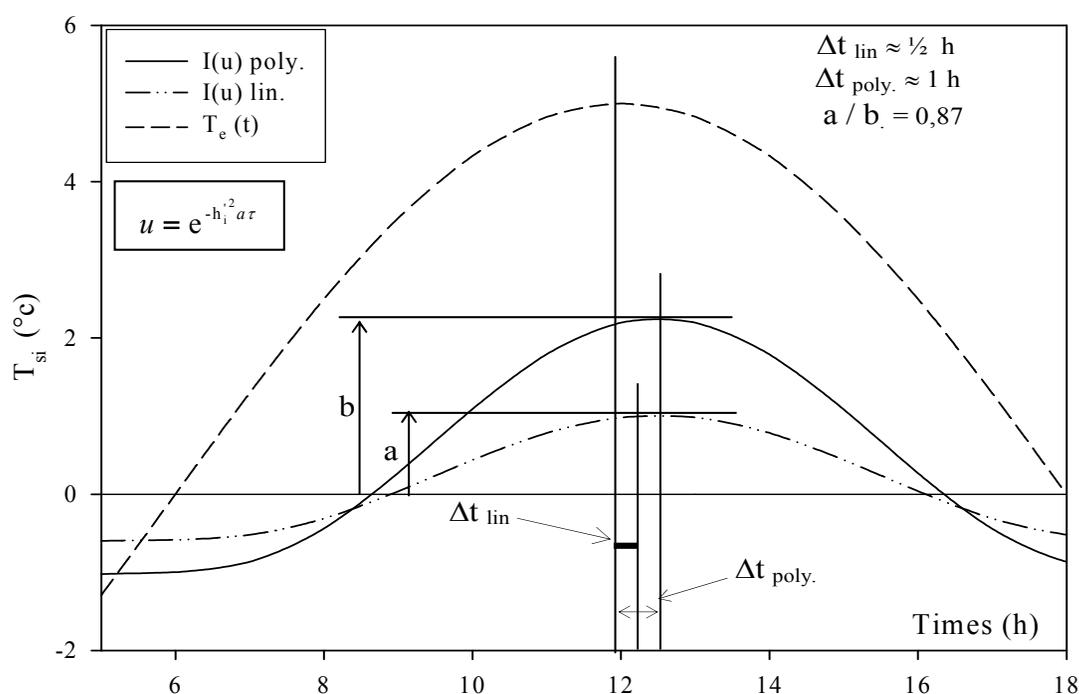


Fig. 7. Influence of the approximation's nature of the inertia's function $I(\tau)$ on the temperature's evolution of the greenhouse's internal ground

We also note the presence of the night radiation; indeed, the ground behaves like a tank of heat which was recovered and stored along the day, this heat takes part in the stabilization of the temperature of the internal air in a level higher than that of the exterior air of the greenhouse. Consequently, during the night the ground presents a thermal inertia in front of the internal air and presents also a thermal inertia compared to the exterior temperature.

The thermal inertia of the ground is characterized by the two functions $I(\tau)$ and $J(\tau)$, this latter is practically equal to unity. In order to materialize this characterization, we studied the impact of the approximation's nature of $I(\tau)$ on the temperature's evolution of the interior ground of the greenhouse compared to the exterior temperature.

For this reason, we visualized in Figure 7 the curves of the temperature's evolution of the interior ground, respectively for a polynomial and linear approximations of the function of inertia $I(\tau)$, we also deferred in the same graph the exterior ambient temperature of the air.

Going through the linear approximation which is coarse towards a more exact polynomial approximation, we announce the following remarks:

- an increase in the amplitude of the ground's temperature
- an increase in the phase shift compared to the exterior temperature
- a remarkable rise in the thermal mass (see Figure 8)

We notice for the polynomial approximation the materialisation of thermal inertia, consequently, we can affirm that the polynomial approximation is more correct because it is closer to the exact function that is proved moreover by Figures (3) and (4).

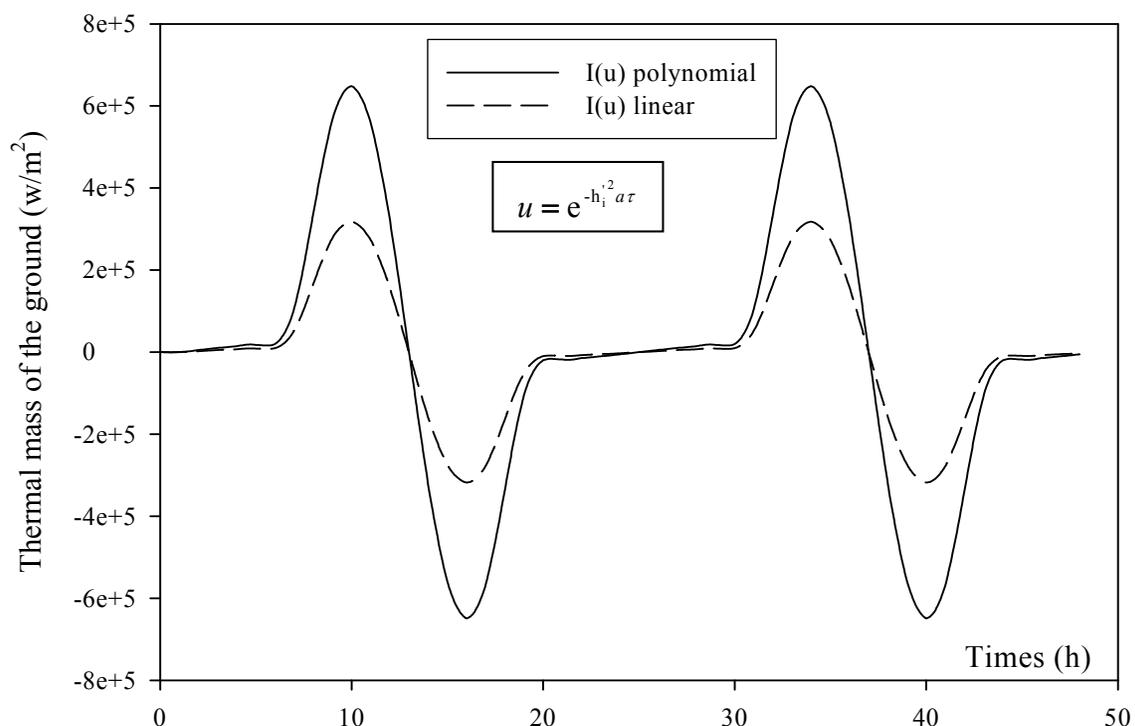


Fig. 8. Influence of the approximation's nature of the inertia's function $I(\tau)$ on the evolution of the thermal mass of the greenhouse's internal ground $(\frac{\rho_s C_s}{h_i} \frac{d\Gamma_{si}}{dt})$

Figure 9 shows the temperature's evolution of the greenhouse's interior air according to time for various debits of air's renewal. The continuous air's renewal obviously clearly lowers the maximum temperature of the day, as it also lowers the minimal temperature of the night.

Consequently, the greenhouse's internal air becomes increasingly dependant on the exterior conditions in particular the exterior temperature.

We can say that the increase of air's renewal's debit, gradually eliminates the effect of thermal inertia of the interior air vis-à-vis to the exterior, the exterior temperature remains at a lower limit that we cannot practically exceed.

Figure 10 presents the influence of the cover's temperature on the evolution of the internal air temperature of the greenhouse. Since the cover's thickness is very low in the order of $180 \mu m$, its thermal conductivity which is inversely proportional to the thickness is very important, consequently, the temperatures of the interior and exterior faces of the cover are finding practically the same ones.

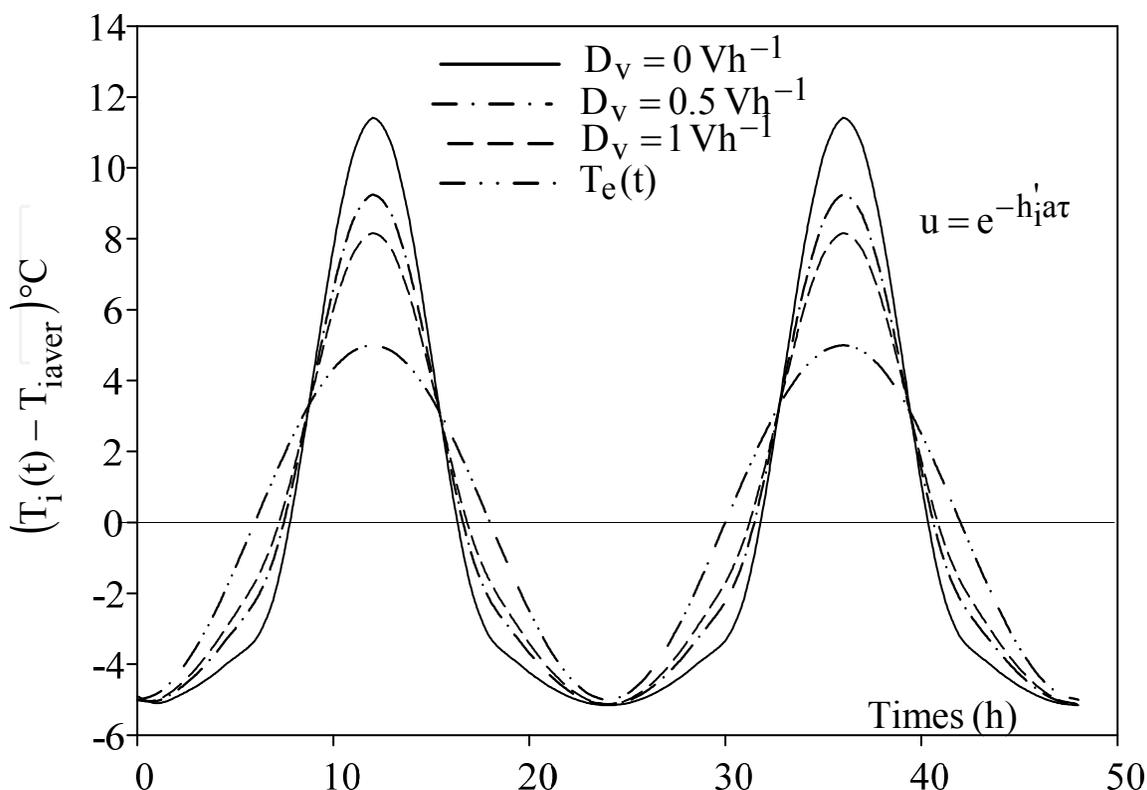


Fig. 9. Temperature's temporal evolution of the greenhouse's internal air for various renewal's flux of air

The transmissions of the external effects towards the interior of the greenhouse are carried out through the cover, what explains the important role of the latter.

During the day, the absorption of part of the incident heat flux appreciably increases the temperature of the cover beyond the temperature of the interior air of the greenhouse. During the night, when there is absence of the incident heat flux, there remains only the conduction of the exterior temperature which dominates the other modes of transfer of heat, which generates a reduction of the cover temperature under the interior air temperature. We summarize the explanation of these two observations by the fact that the cover does not have a thermal inertia.

5. Conclusion and prospects

In this study, we noticed that the ground behaves approximately like a thermal mass. We consider here a simplified model (greenhouse without vegetation) where solar energy is absorbed only by the ground where the phenomena of evaporation and transpiration do not intervene.

This model shows that it is possible to envisage the general behaviour of a naked greenhouse and can without difficulty, be supplemented to hold account in particular of the phenomena of evapo-transpiration in the case of a cultivated greenhouse.

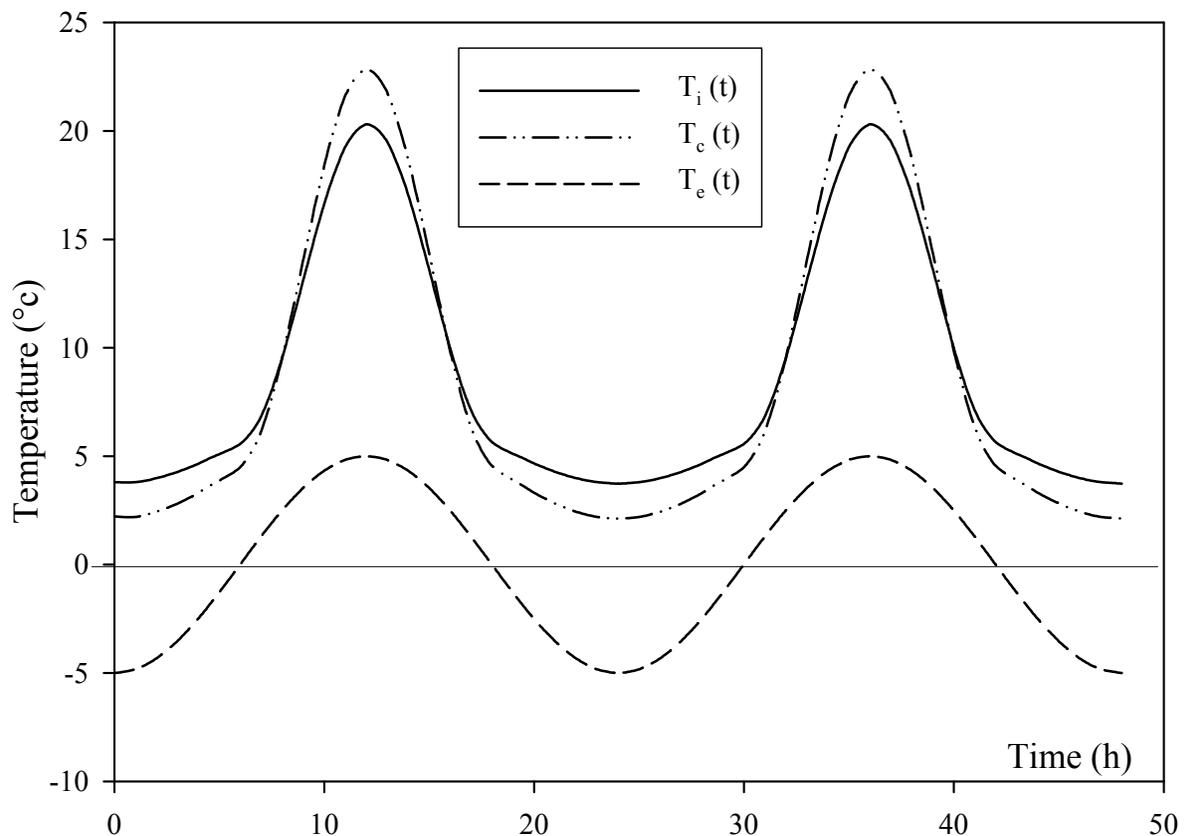


Fig. 10. Temporal evolution of the temperatures, of the internal and external air of the greenhouse and of the cover

However, the interest of the agricultural greenhouses is to increase the production period as well as the output, but requires for the periods of the unfavourable climatic conditions, the use of an expensive heating.

Consequently, the heating of the greenhouses by the integration of a significant storage unit of heat can prove to be interesting to spread out the calendar of production. This is why we highlighted theoretically and experimentally [24, 32] the interest of an underground thermal storage of short and long duration by establishing a mathematical model taking account of all the physical parameters intervening in the system.

The ground, indeed, is able to absorb the solar contributions of the greenhouse which are surplus by playing the role of a thermal wheel of inertia. But the presence of a battery of exchanger buried can play a double function, diurnal cooling of summer or nocturnal reheating of winter by providing all the year an air flow practically to the desired temperature $T_i(t)$.

Realized in the form of a battery of vertical exchangers with air buried in the internal ground of the greenhouse, this storage unit and destocking can constitute an alternative to the problem of the strong thermal amplitudes of a traditional greenhouse (considerable loss of energy during the opening).

The unification of the theory (used for this model as for the battery of exchangers) will make it possible, easy, to integrate a unit of heating in the internal atmosphere of the greenhouse. Finally, the following stage of this work consists to confirm these numerical results on an experimental greenhouse and to find industrial partners.

6. Nomenclature

a	thermal diffusivity of the ground [m^2/s]
a_c	absorption coefficient of the cover infra-red radiation
c_i	heat capacity of the air [J/kgK]
c_s	specific heat of the ground [J/kgK]
D_v	flow of air renewal [m^3/h]
erfc	Error
H_{ci}	coefficient of heat exchange by convection between the cover and the internal air [$\text{W}/\text{m}^2\text{K}$]
H_{ce}	coefficient of heat exchange by convection between the cover and the external air [$\text{W}/\text{m}^2\text{K}$]
H_{si}	coefficient of heat exchange by convection between the ground and the internal air [$\text{W}/\text{m}^2\text{K}$]
H_{se}	coefficient of heat exchange by convection between the ground and the external air [$\text{W}/\text{m}^2\text{K}$]
H_{IRc}	coefficient of the linearized cover infrared exchange [$\text{W}/\text{m}^2\text{K}$]
H_{IRi}	coefficient of the linearized internal ground infrared exchange [$\text{W}/\text{m}^2\text{K}$]
H_{IRe}	coefficient of the linearized external ground infrared exchange [$\text{W}/\text{m}^2\text{K}$]
k	thermal conductivity of the ground [W/mK]
k_c	thermal conductivity of the cover [W/mK]
e	thickness of the cover [m]
L	length of the greenhouse [m]
ℓ	width of the greenhouse [m]
P	average power absorbed by the ground [W]
P_i	average power absorbed by the internal ground [W]
P_e	average power absorbed by the external ground [W]
P_r	average power radiation [W]
T_0	annual average temperature of reference [K]
V_i	volume of the greenhouse [m^3]
S_i	surface of the internal ground [m^2]
S_e	surface of the external ground [m^2]
S_c	surface of the cover [m^2]
$T_i(t)$	temperature of the greenhouses internal air [K]
$T_e(t)$	temperature of the greenhouses external air [K]
$T_{ci}(t)$	temperature of the internal face of the cover [K]
$T_{si}(t)$	surface average temperature of the greenhouses internal ground [K]
$T_{se}(t)$	surface average temperature of the greenhouses external ground [K]
$T_{i\text{aver}}$	average temperature of the greenhouses internal air for the time interval of simulation [K]
$T_{si\text{aver}}$	average temperature of the greenhouses internal ground for the time interval of simulation [K]
Δt	time dephasing [h]
Δt_{lin}	time dephasing corresponding to a linear approximation of the function of inertia $I(u)$ [h]
Δt_{poly}	time dephasing corresponding to a polynomial approximation of the function of inertia $I(u)$ [h]

T_e' virtual temperature of the sky [K]
 $T(\vec{r}, t)$ The field of temperature inside the ground [K]

7. Indexes

c cover
 e exterior
 i interior
 s ground
 se exterior ground
 si interior ground
 ce exterior face of the cover
 ci interior face of the cover
 atm vault of heaven
 IRc infrared exchange with the cover

Greek Symbols

ε_a emission's total factor of the ambient air [-]
 λ dimensionless coefficient [-]
 ρ density [kg/m³]
 δ distribution of Dirac
 σ Constant of Stefan-Boatman [W/m²K⁴]

8. Appendix 1

1- The expression $\frac{1}{S_i} \iint_{S_i} dS_i \iint_{S_i} G(\vec{r}, \vec{r}', \tau) dS_i$ equals to $I(\tau)J(\tau)$, indeed, on the level of the ground we have $z = z' = 0$ then the expression of the Green's function takes the following form:

$$G(\vec{r}, \vec{r}', \tau) = \frac{e^{-\frac{(x-x')^2 - (y-y')^2}{4a\tau}}}{4\pi a\tau} \left[\frac{2}{\sqrt{4\pi a\tau}} - h'_i e^{ah'^2_i \tau} \operatorname{erfc}(h'_i \sqrt{a\tau}) \right] \quad (1)$$

Let

$$I(\tau) = \frac{1}{\sqrt{\pi a\tau}} - h'_i e^{ah'^2_i \tau} \operatorname{erfc}(h'_i \sqrt{a\tau}) \quad (2)$$

$$\begin{aligned} \iint_{S_i} G(\vec{r}, \vec{r}, \tau) dS_i &= \iint_{S_i} \frac{e^{-\frac{(x-x')^2 - (y-y')^2}{4a\tau}}}{4\pi a\tau} I(\tau) dS_i = \\ &= \frac{I(\tau)}{4\pi a\tau} \int_0^L e^{-\frac{(x-x')^2}{4a\tau}} dx \int_0^l e^{-\frac{(y-y')^2}{4a\tau}} dy \end{aligned} \quad (3)$$

Let $u = \frac{x - x'}{2\sqrt{a\tau}}$ and $v = \frac{y - y'}{2\sqrt{a\tau}}$

We can write $\begin{cases} x = x' + 2u\sqrt{a\tau} \\ y = y' + 2v\sqrt{a\tau} \end{cases}$ what gives $\begin{cases} dx = 2u\sqrt{a\tau} \\ dy = 2v\sqrt{a\tau} \end{cases}$

The equation (3) becomes:

$$\begin{aligned} \iint_{Si} G(\bar{r}, \bar{r}', \tau) dSi &= \frac{I(\tau)}{4} \left(\frac{2}{\sqrt{\pi}} \int_{\frac{-x'}{2\sqrt{a\tau}}}^{\frac{L-x'}{2\sqrt{a\tau}}} e^{-u^2} du \right) \left(\frac{2}{\sqrt{\pi}} \int_{\frac{-y'}{2\sqrt{a\tau}}}^{\frac{\ell-y'}{2\sqrt{a\tau}}} e^{-v^2} dv \right) \\ &= \frac{I(\tau)}{4} \left[\operatorname{erf}\left(\frac{L-x'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{x'}{2\sqrt{a\tau}}\right) \right] \left[\operatorname{erf}\left(\frac{\ell-y'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{y'}{2\sqrt{a\tau}}\right) \right] \end{aligned} \quad (4)$$

Let

$$J(x', y', \tau) = \frac{1}{4} \left[\operatorname{erf}\left(\frac{L-x'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{x'}{2\sqrt{a\tau}}\right) \right] \left[\operatorname{erf}\left(\frac{\ell-y'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{y'}{2\sqrt{a\tau}}\right) \right] \quad (5)$$

Finally we obtain :

$$\iint_{Si} G(\bar{r}, \bar{r}', \tau) dSi = I(\tau) J(x', y', \tau) \quad (6)$$

$$\frac{1}{Si} \iint_{Si} \left(\iint_{Si} G(\bar{r}, \bar{r}', \tau) dSi \right) dSi = \frac{1}{Si} \iint_{Si} I(\tau) J(x', y', \tau) dSi = \frac{I(\tau)}{Si} \iint_{Si} J(x', y', \tau) dSi \quad (7)$$

$$\begin{aligned} \frac{1}{Si} \iint_{Si} J(x', y', \tau) dSi &= \frac{1}{4L\ell} \int_0^L \left[\operatorname{erf}\left(\frac{L-x'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{x'}{2\sqrt{a\tau}}\right) \right] \\ &dx' \int_0^\ell \left[\operatorname{erf}\left(\frac{\ell-y'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{y'}{2\sqrt{a\tau}}\right) \right] dy' \end{aligned} \quad (8)$$

$$\int_0^L \left[\operatorname{erf}\left(\frac{L-x'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{x'}{2\sqrt{a\tau}}\right) \right] dx' = \int_0^L \operatorname{erf}\left(\frac{L-x'}{2\sqrt{a\tau}}\right) dx' + \int_0^L \operatorname{erf}\left(\frac{x'}{2\sqrt{a\tau}}\right) dx'$$

$$\begin{cases} u = \frac{L-x'}{2\sqrt{a\tau}} \\ v = \frac{x'}{2\sqrt{a\tau}} \end{cases} \quad \text{what gives} \quad \begin{cases} dx' = -2\sqrt{a\tau} du \\ dx' = 2\sqrt{a\tau} du \end{cases}$$

This leads to

$$\int_0^L \left[\operatorname{erf}\left(\frac{L-x'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{x'}{2\sqrt{a\tau}}\right) \right] dx' = -2\sqrt{a\tau} \int_{\frac{L}{2\sqrt{a\tau}}}^0 \operatorname{erf}(u) du + 2\sqrt{a\tau} \int_0^{\frac{L}{2\sqrt{a\tau}}} \operatorname{erf}(v) dv$$

in addition we have $\operatorname{erf}(r) = \frac{2}{\sqrt{\pi}} \int_0^r e^{-u^2} du$ and $\operatorname{erf}(u) + \operatorname{erfc}(u) = 1$

We obtain then

$$\begin{aligned} \int_0^L \left[\operatorname{erf}\left(\frac{L-x'}{2\sqrt{a\tau}}\right) + \operatorname{erf}\left(\frac{x'}{2\sqrt{a\tau}}\right) \right] dx' &= 4\sqrt{a\tau} \int_0^{\frac{L}{2\sqrt{a\tau}}} \operatorname{erf}(u) du \\ &= 4\sqrt{a\tau} \int_0^{\frac{L}{2\sqrt{a\tau}}} (1 - \operatorname{erfc}(u)) du \\ &= 4\sqrt{a\tau} \left[u - \operatorname{ierfc}(u) \right]_0^{\frac{L}{2\sqrt{a\tau}}} \\ &= 4\sqrt{a\tau} \left[u - \left(u \operatorname{erfc}(u) - \frac{1}{\sqrt{\pi}} e^{-u^2} \right) \right]_0^{\frac{L}{2\sqrt{a\tau}}} \\ &= 4\sqrt{a\tau} \left(\frac{L}{2\sqrt{a\tau}} - \frac{L}{2\sqrt{a\tau}} \operatorname{erfc}\left(\frac{L}{2\sqrt{a\tau}}\right) + \frac{1}{\sqrt{\pi}} e^{-\left(\frac{L}{2\sqrt{a\tau}}\right)^2} - \frac{1}{\sqrt{\pi}} \right) \\ &= 2L \left[1 - \operatorname{erfc}\left(\frac{L}{2\sqrt{a\tau}}\right) + \frac{2}{L} \sqrt{\frac{a\tau}{\pi}} \left(e^{-\left(\frac{L}{2\sqrt{a\tau}}\right)^2} - 1 \right) \right] \end{aligned}$$

There after

$$\begin{aligned} J(\tau) &= \frac{1}{S_i} \iint_{S_i} J(x', y', \tau) dS \\ &= \left[1 - \operatorname{erfc}\left(\frac{L}{2\sqrt{a\tau}}\right) + \frac{2}{L} \sqrt{\frac{a\tau}{\pi}} \left(e^{-\left(\frac{L}{2\sqrt{a\tau}}\right)^2} - 1 \right) \right] * \\ & \quad * \left[1 - \operatorname{erfc}\left(\frac{\ell}{2\sqrt{a\tau}}\right) + \frac{2}{\ell} \sqrt{\frac{a\tau}{\pi}} \left(e^{-\left(\frac{\ell}{2\sqrt{a\tau}}\right)^2} - 1 \right) \right] \end{aligned} \quad (9)$$

Finally

$$\frac{1}{S_i} \iint_{S_i} \left(\iint_{S_i} G(\vec{r}, \vec{r}', \tau) dS_i \right) dS_i = \frac{I(\tau)}{S_i} \iint_{S_i} J(x', y', \tau) dS = I(\tau) J(\tau) \tag{10}$$

2- The factor $\frac{1}{S_i} \iint_{(S_i)} dS_i \iint_{(S_e)} G(\vec{r}, \vec{r}', \tau) dS_e = I(\tau) (1 - J(\tau))$

Indeed, we have $(S) = (S_i) \cup (S_e)$

Then $\iint_{S_e} G(\vec{r}, \vec{r}', \tau) dS_e = \iint_S G(\vec{r}, \vec{r}', \tau) dS - \iint_{S_i} G(\vec{r}, \vec{r}', \tau) dS_i$
 in addition

$$\begin{aligned} \iint_S G(\vec{r}, \vec{r}', \tau) dS &= \iint_S \left(\frac{e^{-\frac{(x-x')^2}{4a\tau}} e^{-\frac{(y-y')^2}{4a\tau}}}{4\pi a\tau} I(\tau) \right) dS \\ &= \frac{I(\tau)}{\pi} \int_{-\infty}^{+\infty} e^{-\frac{(x-x')^2}{4a\tau}} \frac{dx}{2\sqrt{a\tau}} \int_{-\infty}^{+\infty} e^{-\frac{(y-y')^2}{4a\tau}} \frac{dy}{2\sqrt{a\tau}} \\ &= \frac{I(\tau)}{\pi} \int_{-\infty}^{+\infty} e^{-u^2} du \int_{-\infty}^{+\infty} e^{-v^2} dv = \frac{I(\tau)}{\pi} \left(\int_{-\infty}^{+\infty} e^{-u^2} du \right)^2 = I(\tau) \end{aligned}$$

Consequently

$$\iint_{S_e} G(\vec{r}, \vec{r}', \tau) dS_e = I(\tau) (1 - J(x, y, \tau)) \tag{11}$$

By using the equation (9), we obtain

$$\frac{1}{S_i} \iint_{(S_i)} dS_i \iint_{(S_e)} G(\vec{r}, \vec{r}', \tau) dS_e = I(\tau) (1 - J(\tau)) \tag{12}$$

9. Appendix 2

Approximate forms of the functions of inertia $I(\tau)$ and $J(\tau)$:

We have

$$T_{si}(t) = T_{si}^0(t) + \frac{1}{\rho c} \int_0^t P_i(t-\tau) I(\tau) J(\tau) d\tau + \frac{H_i}{\rho c} \int_0^t T_i(t-\tau) I(\tau) J(\tau) d\tau + \frac{H_e}{\rho c} \int_0^t T_e(t-\tau) I(\tau) J(\tau) d\tau \tag{1}$$

In order to facilitate the discretization of this expression, we must approach the two functions of inertia $I(\tau)$ and $J(\tau)$.

In addition, the expression of $I(\tau)$ can be written:

$$I(\tau) = \frac{1}{\sqrt{\pi a \tau}} - h_i' \exp(h_i'^2 a \tau) \operatorname{erfc}(h_i' \sqrt{a \tau}) \quad , \quad \tau \in [0, +\infty [\tag{2}$$

Let $\alpha = h_i'^2 a \tau$ then $I(\alpha) = \frac{h_i'}{\sqrt{\pi \alpha}} - h_i' e^\alpha \operatorname{erfc}(\sqrt{\alpha})$

Let $I_1(\alpha) = e^\alpha \operatorname{erfc}(\sqrt{\alpha})$ we find then :

$$I(\alpha) = -h_i' \frac{dI_1(\alpha)}{d\alpha} \quad (3)$$

however we have

$$I_1(\alpha)_{\text{app}} = 0.4269 e^{-4.676\alpha} + 0.499 e^{-0.1659\alpha} \quad (4)$$

finally

$$\begin{cases} I(\alpha) = h_i' \left(\frac{1}{\sqrt{\pi \alpha}} - e^\alpha \operatorname{erfc}(\sqrt{\alpha}) \right) \\ I(\alpha)_{\text{app}} = h_i' \left(1.9962 e^{-4.676\alpha} + 0.0828 e^{-0.1659\alpha} \right) \end{cases}$$

Or according to τ
$$\begin{cases} I(\tau) = \frac{1}{\sqrt{\pi a \tau}} - h_i' e^{h_i'^2 a \tau} \operatorname{erfc}(h_i' \sqrt{a \tau}) \\ I(\tau)_{\text{app}} = h_i' \left(1.9962 e^{-4.676 h_i'^2 a \tau} + 0.0828 e^{-0.1659 h_i'^2 a \tau} \right) \end{cases}$$

In addition we saw that

$$J(\tau) = \left[1 - \operatorname{erfc}\left(\frac{L}{2\sqrt{a\tau}}\right) + \frac{2}{L} \sqrt{\frac{a\tau}{\pi}} \left(e^{-\left(\frac{L}{2\sqrt{a\tau}}\right)^2} - 1 \right) \right] \left[1 - \operatorname{erfc}\left(\frac{\ell}{2\sqrt{a\tau}}\right) + \frac{2}{\ell} \sqrt{\frac{a\tau}{\pi}} \left(e^{-\left(\frac{\ell}{2\sqrt{a\tau}}\right)^2} - 1 \right) \right] \quad (5)$$

We represented $J(\tau)$, with $\tau \in [0, +\infty[$, we noted that $J(\tau)$ practically evolve in the vicinity of the unit, consequently, we can approximate the product IJ by:

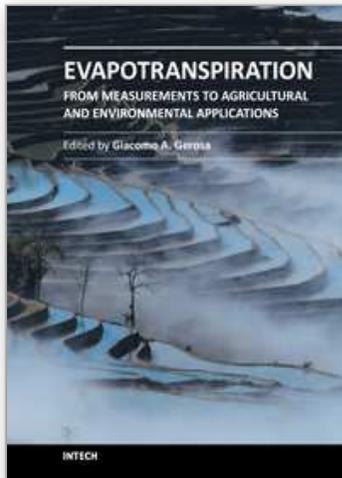
$$I(\tau)J(\tau)_{\text{app}} = h_i' \left(1.9962 e^{-4.676 h_i'^2 a \tau} + 0.0828 e^{-0.1659 h_i'^2 a \tau} \right) \quad (6)$$

10. References

- [1] Boulard, T. et Baille, A., " Analysis of thermal performance of a greenhouse as a solar collector", *Energy in Agriculture*, 6, 17, pp. 17-26, (1986).
- [2] Jolliet, O., "Modélisation du comportement thermique d'une serre horticole", Thèse de docteur Sciences, école Polytechnique Fédérale de Lausanne, (1988).
- [3] Comary, Y. et Nicolas, C., "La thermique des serres", Collection de la direction des études et recherches d'Electricité de France. Ed. Eyrolles (1985).

- [4] Monteil, C., "Contribution informatique à l'analyse énergétique des serres agricoles", Thèse de docteur ingénieur, Institut National Polytechnique de Toulouse (1985).
- [5] Chabaane, D.E., "Influence d'un écran thermique sur le microclimat nocturne d'une serre agricole", Thèse de docteur de 3^{ème} cycle, Université de Perpignan (1986).
- [6] Kabbaj, R., "Modélisation de la ventilation statique d'une serre, Rapport de Diplôme Approfondie en Energétique, Université de Perpignan (1988).
- [7] Georgios K. Spanomitsios, "Temperature Control and Energy Conservation in a Plastic Greenhouse", Journal of Agricultural Engineering Research Volume 80, Issue 3 , November pp. 251-259 (2001)
- [8] G. L. A. M. Swinkels, P. J. Sonneveld and G. P. A. Bot "Improvement of Greenhouse Insulation with Restricted Transmission Loss through Zigzag Covering Material", Journal of Agricultural Engineering Research Volume 79, Issue 1 , pp. 91-97 (2001).
- [9] M. Trigui, S. Barrington and L. Gauthier, "A Strategy for Greenhouse Climate Control, Part II: Model Validation" , Journal of Agricultural Engineering Research ,Volume 79, Issue 1, pp.99-105 (2001).
- [10] Maklouf, S., " Expérimentation et modélisation d'une serre solaire à air avec stockage par chaleur latente assisté par pompe à chaleur de déshumidification", Thèse de docteur ès-Sciences, Université de Nice (1988).
- [11] De Halleux, D., "Modèle dynamique des échanges énergétiques des serres : Etude théorique et expérimentale, Thèse de doctorat de l'Université de Gembloux (1989).
- [12] Walker, J.N., "*Predicting temperature in ventilated greenhouse*", Transactions of the ASAE, 8, 3, pp.445-448 (1965).
- [13] Gag, A., Bartoli, R., Filelle, J.F., Bethery, J., Gérard, A. Et Feldmann, C., " *Échanges thermiques dans les serres et abris*, bulletin Technique du génie rural, n°113 (1973).
- [14] Boulard, T., "Caractérisation et modélisation du climat des serres : application à la climatisation estivale", Thèse de Doctorat, E.N.S.A.M., Montpellier (1996).
- [15] Kimball, B.A., " Simulation of the energy balance of a greenhouse, agric.Meteorol., n°11, pp.243-260 (1973).
- [16] Arinze, E.A. Schoenau, G.J. Et Besant, R.W., "*A dynamic thermal performance simulation model of an energy conserving greenhouse with thermal storage*", Transactions of the ASAE, 27, 2, pp.508-519 (1984).
- [17] Bot, G.P.A., "Greenhouse climate : from physical processes to a dynamic model", Ph. D. Thesis, Agricultural University, Wageningen (1983).
- [18] Kindelan, M., " Dynamic modelling of greenhouse environment", Transaction of the ASAE, 23, 5, pp.1232-1239 (1980).
- [19] Kittas, C. , "Contribution théorique et expérimentale à l'étude du bilan d'énergie des serres", Thèse de Docteur-ingénieur, Université de Perpignan (1980).
- [20] Boulard, T. , Draoui, B. et Neirac, F., "Analyse du bilan thermodynamique d'une serre horticole - Application à la maîtrise du microclimat -", Colloque de la société Française des thermiciens (S.F.T.), Pau, Mai 1993, pp 632-647.
- [21] Issanchou, G., "Modélisation à la mise au point d'un logiciel de thermique appliqué à l'ingénierie des serres", Thèse de Doctorat de l'Université de Perpignan (1991).
- [22] Carslaw, H.S. and Jaeger, J.P., "Conduction of heat in solids". Oxford, University Press (1959).
- [23] Bbernard , R., Menguy , G. et Schwartz, M., "Le rayonnement solaire - Conversation thermique applications", éd. Technique et Documentation, Paris 1979.

- [24] Ben Younes , R., "Prévision numérique et expérimentale de la réponse intrinsèque d'un échangeur bitubulaire enterré en régime continu", Thèse d'Université de Valenciennes, (1993).
- [25] Ben Younes , R., Desmons , J.Y., Randriatsarafara, J.E., Khaine , D., Khaine, A. et Le Ray (M.), "Etude Théorique et Numérique de l'intégration d'une unité de stockage à air souterrain dans une Serre Agricole". 8^{èmes} Journées Internationales de Thermique, Marseille, (J.I.T.H.).97, Vol.2 pp 415 - 423, 7 - 10 juillet (1997).
- [26] Desmons, J.Y., Ben Younes, R., Khaine, D. Et Le Ray, M. " Simulation du comportement thermique d'une serre traditionnelle - Etude de l'inertie du sol à l'aide d'un modèle théorique utilisant les fonctions de Green". Acte du Congrès S.F.T.94, Thermique et Transports, Paris, pp. 480- 485 (1994).
- [27] Ayeb, H., Ben Younes, R. Et Elouragini, S., "Mise en évidence de deux fonctions caractérisant l'inertie thermique du sol intérieur d'une serre traditionnelle - Nouvelle méthode analytique et numérique". 7^{ème} Colloque National de Recherche en Physique de la Société Tunisienne de Physique (STP), Hammamet 21 - 24 décembre (2003).
- [28] Ben Younes, R. , Desmons, J.Y. Et Randriatsarafara, J.E. " Prévision théorique et numérique du comportement thermique d'une serre en Physique de la Société Tunisienne e Physique (traditionnelle" PD20. 6^{ème} Colloque National de Recherche STP), Hammamet 19 - 20 - 21 Mai (1999).
- [29] R. Ben Younes, H. Ayeb And J.Y. Desmons. "Mathematical formulation and numerical resolution of the thermal behavior of a Greenhouse conpled with a unit of underground storage", International Journal of Heat and Technology Vol 23 N°2 december 2005.
- [30] Ben Younes, R., "Numerical simulation of the thermal behaviour of a tradional greenhouse by using the theory of the green's functions", Numerical heat Transfer, Part A, 56, pp.914-929, (2009).
- [31] De Brichambaut , C.P. et VAUGE, C., Le gisement solaire "évaluation de la ressource" , p.52, édition Technique et Documentation (Lavoisier), 1982.
- [32] Météo France ,: Direction Régionale, Villeneuve d'Ascq, Lille (1995).
- [33] Le Ray, M Cours "Energétique du Bâtiment", I.S.T.V., Université de Valenciennes, French.
- [34] Desmons, J.Y. et Ben Younes, R. "Prévision à long terme de la réponse d'un stockage de chaleur sensible dans le sol", Int. J.Heat Mass Transfer Vol 40 N 13 pp. 319 - 334, (1997).



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This book represents an overview of the direct measurement techniques of evapotranspiration with related applications to the water use optimization in the agricultural practice and to the ecosystems study. Different measuring techniques at leaf level (porometry), plant-level (sap-flow, lysimetry) and agro-ecosystem level (Surface Renewal, Eddy Covariance, Multi layer BREB), are presented with detailed explanations and examples. For the optimization of the water use in agriculture, detailed measurements on transpiration demands of crops and different cultivars, as well as results of different irrigation schemes and techniques (i.e. subsurface drip) in semi-arid areas for open-field, greenhouse and potted grown plants are presented. Aspects on ET of crops in saline environments, effects of ET on groundwater quality in xeric environments as well as the application of ET to climatic classification are also depicted. The book provides an excellent overview for both, researchers and students who intend to address these issues.

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