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Analysis of Mixed Convection in a Lid Driven Trapezoidal Cavity

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1. Introduction

Convection is the heat transfer mechanism affected by the flow of fluids. The amount of energy and matter are conveyed by the fluid can be predicted through the convective heat transfer. The convective heat transfer splits into two branches; the natural convection and the forced convection. Forced convection regards the heat transport by induced fluid motion which is forced to happen. This induced flow needs consistent mechanical power. But natural convection differs from the forced convection through the fluid flow driving force which happens naturally. The flows are driven by the buoyancy effect due to the presence of density gradient and gravitational field. As the temperature distribution in the natural convection depends on the intensity of the fluid currents which is dependent on the temperature potential itself, the qualitative and quantitative analysis of natural convection heat transfer is very difficult. Numerical investigation instead of theoretical analysis is more needed in this field.

Two types of natural convection heat transfer phenomena can be observed in the nature. One is that external free convection that is caused by the heat transfer interaction between a single wall and a very large fluid reservoir adjacent to the wall. Another is that internal free convection which befalls within an enclosure. Mathematically, the tendency of a particular system towards natural convection relies on the Grashof number, ($Gr = \frac{g\beta\nabla TL^3}{\nu^2}$), which is a ratio of buoyancy force and viscous force. The parameter β is the rate of change of density with respect to the change in temperature (T), and ν is viscosity. Thus, the Grashof number can be thought of as the ratio of the upwards buoyancy of the heated fluid to the internal friction slowing it down. In very sticky, viscous fluids, the fluid movement is restricted, along with natural convection. In the extreme case of infinite viscosity, the fluid could not move and all heat transfer would be through conductive heat transfer.

Forced convection is often encountered by engineers designing or analyzing heat exchangers, pipe flow, and flow over flat plate at a different temperature than the stream (the case of a shuttle wing during re-entry, for example). However, in any forced convection situation, some amount of natural convection is always present. When the natural convection is not negligible, such flows are typically referred to as mixed convection.

When analyzing potentially mixed convection, a parameter called the Richardson number ($Ri = Gr/Re^2$) parametizes the relative strength of free and forced convection. The Richardson number is the ratio of Grashof number and the square of the Reynolds number, which represents the ratio of buoyancy force and inertia force, and which stands in for the contribution of natural convection. When $Ri >> 1$, natural convection dominates and when $Ri << 1$, forced convection dominates and when $Ri = 1$, mixed convection dominates.

The thermo-fluid fields developed inside the cavity depend on the orientation and the geometry of the cavity. Reviewing the nature and the practical applications, the enclosure phenomena can loosely be organized into two classes. One of these is enclosure heated from the side which is found in solar collectors, double wall insulations, laptop cooling system and air circulation inside the room and the another one is enclosure heated from below which is happened in geophysical system like natural circulation in the atmosphere, the hydrosphere and the molten core of the earth.

Nomenclature

h convective heat transfer coefficient ($W/m^2 K$)
q" Heat Flux (W/m^2)
C_P specific heat at constant pressure ($J/kg K$)
g gravitational acceleration (m/s^2)
k thermal conductivity of the fluid ($W/m K$)
Nu Nusselt number, hW/k
Pr Prandtl number, v/α
Gr Grashof number, $g\beta\Delta TW^3/v^2$
Re Reynolds number, U_0W/v
Ri Richardson number, Gr/Re^2
A Aspect Ratio, H/W
R length of the inclined sidewalls (m)
T temperature of the fluid, (°C)
u velocity component at x-direction (m/s)
U dimensionless velocity component at X-direction
v velocity component at y-direction (m/s)
V dimensionless velocity component at Y-direction
W length of the cavity, (m)
x distance along the x-coordinate
X distance along the non-dimensional x-coordinate
Y distance along the non-dimensional y-coordinate

Greak symbols

α thermal diffusivity of the fluid (m^2/s)
β volumetric coefficient of thermal expansion (K^{-1})
γ inclination angle of the sidewalls of the cavity

θ dimensionless temperature ,$(T_H-T_C)/\Delta T$
μ dynamic viscosity of the fluid (Pa s)
v kinematic viscosity of the fluid (m^2/s)
ρ density of the fluid (kg/m^3)
Φ rotational angle of the cavity

Subscript

a average value
v value of cold temperature
c_H value of hot temperature

1.1 Flow within enclosure

The flow within an enclosure consisting of two horizontal walls, at different temperatures, is an important circumstance encountered quite frequently in practice. In all the applications having this kind of situation, heat transfer occurs due to the temperature difference across the fluid layer, one horizontal solid surface being at a temperature higher than the other. If the upper plate is the hot surface, then the lower surface has heavier fluid and by virtue of buoyancy the fluid would not come to the lower plate. Because in this case the heat transfer mode is restricted to only conduction. But if the fluid is enclosed between two horizontal surfaces of which the upper surface is at lower temperature, there will be the existence of cellular natural convective currents which are called as Benard cells. For fluids whose density decreases with increasing temperature, this leads to an unstable situation. Benard [1] mentioned this instability as a "top heavy" situation. In that case fluid is completely stationary and heat is transferred across the layer by the conduction mechanism only. Rayleigh [2] recognized that this unstable situation must break down at a certain value of Rayleigh number above which convective motion must be generated. Jeffreys [3] calculated this limiting value of *Ra* to be 1708, when air layer is bounded on both sides by solid walls.

1.1.1 Tilted enclosure

The tilted enclosure geometry has received considerable attention in the heat transfer literature because of mostly growing interest of solar collector technology. The angle of tilt has a dramatic impact on the flow housed by the enclosure. Consider an enclosure heated from below is rotated about a reference axis. When the tilted angle becomes 90^o, the flow and thermal fields inside the enclosure experience the heating from side condition. Thereby convective currents may pronounce over the diffusive currents. When the enclosure rotates to 180^o, the heat transfer mechanism switches to the diffusion because the top wall is heated.

1.1.2 LID driven enclosure

Flow and heat transfer analysis in lid-driven cavities is one of the most widely studied problems in thermo-fluids area. Numerous investigations have been conducted in the past on lid-driven cavity flow and heat transfer considering various combinations of the imposed temperature gradients and cavity configurations. This is because the driven cavity configuration is encountered in many practical engineering and industrial applications. Such configurations can be idealized by the simple rectangular geometry with regular boundary conditions yielding a well-posed problem. Combined forced-free convection flow in lid-driven cavities or enclosures occurs as a result of two competing mechanisms. The

first is due to shear flow caused by the movement of one of the walls of the cavity while the second is due to buoyancy flow produced by thermal non homogeneity of the cavity boundaries. Understanding these mechanisms is of great significance from technical and engineering standpoints.

1.2 Application

Air-cooling is one of the preferred methods for the cooling of computer systems and other electronic equipments, due to its simplicity and low cost. It is very important that such cooling systems should be designed in the most efficient way and the power requirement for the cooling should be minimized. The electronic components are treated as heat sources embedded on flat surfaces. A small fan blows air at low speeds over the heat sources. This gives rise to a situation where the forced convection due to shear driven flow and the natural convection due to buoyancy driven flow are of comparable magnitude and the resulting heat transfer process is categorized as mixed convection. Mixed convection flow and heat transfer also occur frequently in other engineering and natural situations. One important configuration is a lid-driven (or shear- driven) flow in a differentially heated/cooled cavity, which has applications in crystal growth, flow and heat transfer in solar ponds [5], dynamics of lakes [6], thermal-hydraulics of nuclear reactors [7], industrial processes such as food processing, and float glass production [8]. The interaction of the shear driven flow due to the lid motion and natural convective flow due to the buoyancy effect is quite complex and warrants comprehensive analysis to understand the physics of the resulting flow and heat transfer process.

1.3 Motivation behind the selection of problem

Two dimensional steady, mixed convection heat transfers in a two-dimensional trapezoidal cavity with constant heat flux from heated bottom wall while the isothermal moving top wall has been studied numerically. The present study is based on the configuration of Aydin and Yang [27] where the isothermal heat source at the bottom wall is replaced by a constant flux heat source, which is physically more realistic. The main attribute for choosing the trapezoidal shape cavity is to enhance the heat transfer rate as it could be said intuitionally due to its extended cold top surface. The inclination angle of the sidewalls of the trapezoid has been changed (30°, 45° and 60°) to get the maximum heat transfer in terms of maximum Nusselt number. Then the trapezoid has been rotated (30°, 45° and 60°) and the results have been studied. The tilted position of the enclosure shows a significant influence on the heat transfer. Results are obtained for both the aiding and opposing flow conditions by changing the direction of the lid motion. This study includes additional computations for cavities at various aspect ratios, A, ranging from 0.5 to 2 and their effects on the heat transfer process is analyzed in terms of average Nusselt number. Contextually the present study will focus on the computational analysis of the influence of inclination angle of the sidewalls of the cavity, rotational angle of the cavity, Aspect ratio, direction of the lid motion and Richardson number.

1.4 Main objectives of the work

The investigation is carried out in a two dimensional lid driven trapezoidal enclosure filled with air. The inclined side walls are kept adiabatic and the bottom wall of the cavity is kept at uniform heat flux. The cooled top wall having constant temperature will move with a constant velocity. The specific objectives of the present research work are as follows:

a. To study the variation of average heat transfer in terms of Nusselt number with the variation of Richardson number at different aspect ratios of the rectangular enclosure and compare it with the established literature.
b. To find out the optimum configuration by changing the inclination angle of the side walls of the trapezoidal cavity by analyzing the maximum heat transfer.
c. To study the variation of average heat transfer in terms of Nusselt number with the variation of Richardson number of the optimum trapezoidal cavity.
d. To study the variation of average heat transfer in terms of Nusselt number at different aspect ratios of the optimum trapezoidal cavity.
e. To study the variation of average heat transfer in terms of Nusselt number with the variation of Richardson number at different aspect ratios of the optimum trapezoidal enclosure by changing the rotation angle for both aiding and opposing flow conditions..
f. To analyze the flow pattern inside the trapezoidal enclosures in terms of Streamlines and isotherms.

2. Literature review

There have been many investigations in the past on mixed convective flow in lid-driven cavities. Many different configurations and combinations of thermal boundary conditions have been considered and analyzed by various investigators. Torrance et al. [9] investigated mixed convection in driven cavities as early as in 1972. Papaniclaou and Jaluria [10-13] carried out a series of numerical studies to investigate the combined forced and natural convective cooling of heat dissipating electronic components, located in rectangular enclosures, and cooled by an external through flow of air. The results indicate that flow patterns generally consists of high of low velocity re-circulating cells because of buoyancy forces induced by the heat source. Koseff and Street [14] studied experimentally as well as numerically the recirculation flow patterns for a wide range of Reynolds (*Re*) and Grashof (*Gr*) numbers. Their results showed that the three dimensional features, such as corner eddies near the end walls, and Taylor- Gortler like longitudinal vortices, have significant effects on the flow patterns for low Reynolds numbers. Khanafer and Chamakha [15] examined numerically mixed convection flow in a lid-driven enclosure filled with a fluid-saturated porous medium and reported on the effects of the Darcy and Richardson numbers on the flow and heat transfer characteristics.

G. A. Holtzman et. al [16] have studied laminar natural convection in isosceles triangular enclosures heated from below and symmetrically cooled from above. This problem is examined over aspect ratios ranging from 0.2 to 1.0 and Grashoff numbers from 10^3 to 10^5. Its is found that a pitchfork bifurcation occurs at a critical Grashoff number for each of the aspect ratios considered, above which the symmetric solutions are unstable to finite perturbations and asymmetric solutions are instead obtained. Results are presented detailing the occurance of the pitchfork bifurcation in each of the aspect ratios considered, and the resulting flow patterns are described. A flow visualization study is used to validate the numerical observations. Difference in local values of the Nusselt number between asymmetric and symmetric solutions are found to be more than 500 percent due to the shifting of the buoyancy- driven cells. The phenomenon of natural convection in trapezoidal enclosures where upper and lower walls are not parallel, in particular a triangular geometry, is examined by H. Asan, L. Namli [17] over a parameter domain in which the aspect ratio of the enclosure ranges from 0.1 to 1.0, the Rayleigh number varies between 10^2

to 10^5 and Prandtl number correspond to air and water. It is found that the numerical experiments verify the flow features that are known from theoretical asymptotic analysis of this problem (valid for shallow spaces) only over a certain range of the parametric domain.
Moallemi and Jang [18] numerically studied mixed convective flow in a bottom heated square driven cavity and investigated the effect of Prandtl number on the flow and heat transfer process. They found that the effects of buoyancy are more pronounced for higher values of Prandtl number. They also derived a correlation for the average Nusselt number in terms of the Prandtl number, Reynolds number, and Richardson number. Mohammad and Viskanta [19] performed numerical investigation and flow visualization study on two and three-dimensional laminar mixed convection flow in a bottom heated shallow driven cavity filled with water having a Prandtl number of 5.84. They concluded that the lid motion destroys all types of convective cells due to heating from below for finite size cavities. They also implicated that the two-dimensional heat transfer results compare favorably with those based on a three-dimensional model for $Gr/Re < 1$. Later, Mohammad and Viskanta [20] experimentally and numerically studied mixed convection in shallow rectangular bottom heated cavities filled with liquid Gallium having a low Prandtl number of 0.022. They found that the heat transfer rate is rather insensitive to the lid velocity and an extremely thin shear layer exists along the major portion of the moving lid. The flow structure consists of an elongated secondary circulation that occupies a third of the cavity.
Mansour and Viskanta [21] studied mixed convective flow in a tall vertical cavity where one of the vertical sidewalls, maintained at a colder temperature than the other, was moving up or downward thus assisting or opposing the buoyancy. They observed that when shear assisted the buoyancy a shear cell developed adjacent to the moving wall while the buoyancy cell filled the rest of the cavity. When shear opposed buoyancy, the heat transfer rate reduced below that for purely natural convection. Iwatsu et al. [22] and Iwatsu and Hyun [23] conducted two-dimensional and three-dimensional numerical simulation of mixed convection in square cavities heated from the top moving wall. Mohammad and Viskanta [24] conducted three-dimensional numerical simulation of mixed convection in a shallow driven cavity filled with a stably stratified fluid heated from the top moving wall and cooled from below for a range of Rayleigh number and Richardson number.
Prasad and Koseff [25] reported experimental results for mixed convection in deep lid-driven cavities heated from below. In a series of experiments which were performed on a cavity filled with water, the heat flux was measured at different locations over the hot cavity floor for a range of Re and Gr. Their results indicated that the overall (i.e. area-averaged) heat transfer rate was a very weak function of Gr for the range of Re examined ($2200 < Re < 12000$). The data were correlated by Nusselt number vs Reynolds number, as well as Stanton number vs Reynolds number relations.
They observed that the heat transfer is rather insensitive to the Richardson number. Hsu and Wang [26] investigated the mixed convective heat transfer where the heat source was embedded on a board mounted vertically on the bottom wall at the middle in an enclosure. The cooling air flow enters and exits the enclosure through the openings near the top of the vertical sidewalls. The results show that both the thermal field and the average Nusselt number depend strongly on the governing parameters, position of the heat source, as well as the property of the heat-source-embedded board.
Aydin and Yang [27] numerically studied mixed convection heat transfer in a two-dimensional square cavity having an aspect ratio of 1. In their configuration the isothermal sidewalls of the cavity were moving downwards with uniform velocity while the top wall

was adiabatic. A symmetrical isothermal heat source was placed at the otherwise adiabatic bottom wall. They investigated the effects of Richardson number and the length of the heat source on the fluid flow and heat transfer. Shankar et al. [28] presented analytical solution for mixed convection in cavities with very slow lid motion. The convection process has been shown to be governed by an inhomogeneous biharmonic equation for the stream function. Oztop and Dagtekin [29] performed numerical analysis of mixed convection in a square cavity with moving and differentially heated sidewalls. Sharif [30] investigates heat transfer in two-dimensional shallow rectangular driven cavity of aspect ratio 10 and Prandtl number 6.0 with hot moving lid on top and cooled from bottom. They investigated the effect of Richardson number and inclination angle. G. Guo and M. A. R. Sharif [31] studied mixed convection in rectangular cavities at various aspect ratios with moving isothermal sidewalls and constant heat source on the bottom wall. They plotted the streamlines and isotherms for different values of Richardson number and also studied the variation of the average Nu and maximum surface temperature at the heat source with Richardson number with different heat source length. They simulated streamlines and isotherms for asymmetric placements of the heat source and also the effects of asymmetry of the heating elements on the average Nu and the maximum source length temperature.

3. Physical model

The physical model considered here is shown in figure 1 and 2, along with the important geometric parameters. It consists of a trapezoidal cavity filled with air, whose bottom wall and top wall are subjected to hot T_H and cold T_C temperatures respectively while the side walls are kept adiabatic. Two cases of thermal boundary conditions for the top moving wall have been considered here. The first case is (figure 1) when the moving cold wall is moving in the positive x direction (opposing flow condition). In that case the shear flow caused by moving top wall opposes the buoyancy driven flow caused by the thermal non-homogeneity of the cavity boundaries. The second case is (figure 2) when the moving cold wall is moving in the negative x direction (aiding flow condition). In that case the shear flow assists the buoyancy flow. The cavity height is H, width of the bottom hot wall is W, is inclined at angle Φ with the horizontal reference axis. γ is the inclination angle of the sidewalls of the cavity. The flow and heat transfer phenomena in the cavity are investigated for a series of Richardson numbers (*Ri*), aspect ratio (A=H/W), rotation angle of the cavity Φ.

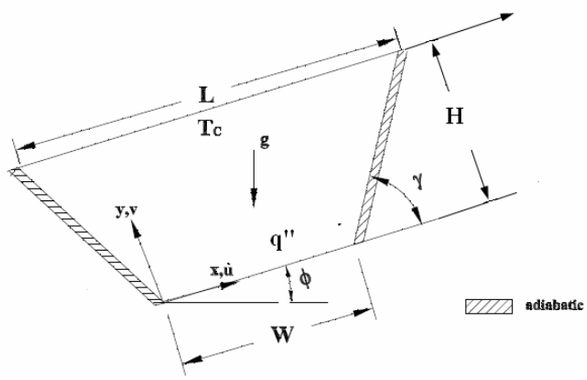


Fig. 1. Schematic diagram of the physical system considering opposing flow condition

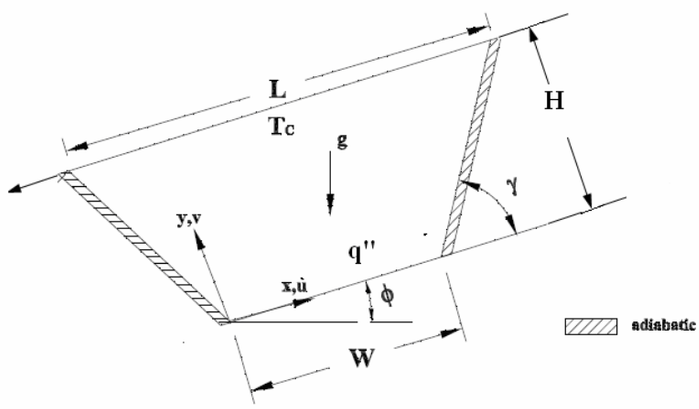


Fig. 2. Schematic diagram of the physical system considering aiding flow condition

3.1 Mathematical model

Using the Boussinesq approximation and neglecting the viscous dissipation effect and compressibility effect the dimensionless governing equations for two dimensional laminar incompressible flows can be written as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Gr}{\mathrm{Re}^2}\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

The dimensionless variables are as follows:

$$X=x/W,\ Y=y/W,\ \theta=(T_H-T_C)/\Delta T,\ \Delta T=q''W/k,\ U=u/U_0,\ V=v/U_0,\ P=p/\rho U_o^2$$

The dimensionless parameters, appearing in Eqs. (1)-(4) are Reynolds number $Re= U_0W/\nu$, the Prandtl number $Pr=\nu/a$, the Grashof number $Gr=\frac{g\beta\nabla TL^3}{\nu^2}$. The ratio of Gr/Re^2 is the mixed convection parameter and is called Richardson number Ri and is a measure of the relative strength of the natural convection and forced convection for a particular problem. If $Ri<<1$ the forced convection is dominant while if $Ri>>1$, then natural convection is dominant. For problems with $Ri\sim1$ then the natural convection effects are comparable to the forced convection effects.

The boundary conditions for the present problem are specified as follows:

Top wall: $U=U_0$, $V=0$, $\theta=0$

Bottom wall: U=V=0, θ=1
Right and Left wall: U=V=0,

$$\partial\theta/\partial X = 0$$

Non-dimensional heat transfer parameter Nusselt number is stated as:

3.2 Numerical method

Firstly the problem is defined as a two dimensional enclosure. Control Volume based finite volume method (FVM) is to be used to discretize the governing differential equations. The pressure- velocity coupling in the governing equations is achieved using the well known SIMPLE method for numerical computations. The set of governing equations are to be solved sequentially. A second order upwind differencing scheme is to be used for the formulation of the coefficients in the finite-volume equations. As the sides of the trapezoidal cavity are not parallel, the present numerical techniques will descretize the computational domain into unstructured triangular elements.
In order to obtain the grid independence solution, a grid refinement study is performed for the trapezoidal cavity (A=1) under constant heat flux condition keeping, Re=400, Pr=0.71, Ri= 1.0.n It is found in figure 3 that 5496 regular nodes are sufficient to provide accurate results. This grid resolution is therefore used for all subsequent computations for A≤1. For taller cavities with A>1, a proportionately large number of grids in the vertical direction is used.

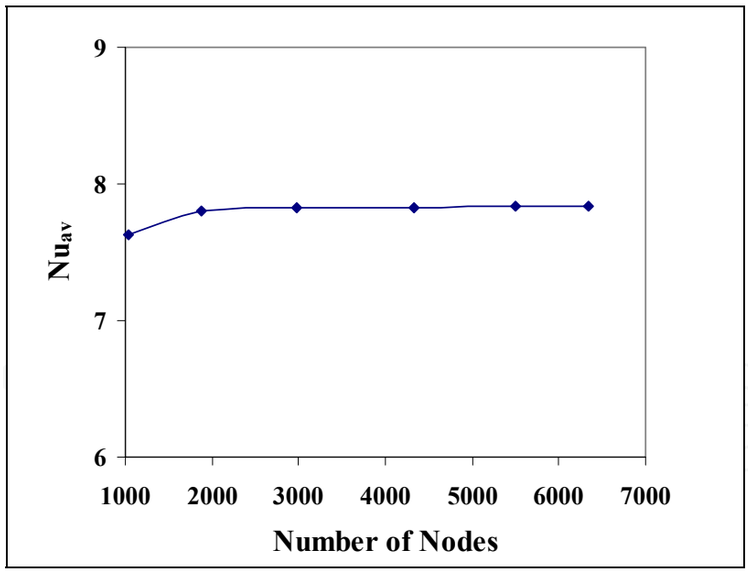


Fig. 3. Grid sensitivity test for trapezoidal cavity at Ri=1.0, Re=400 and A=1

The convergence criterion was defined by the required scaled residuals to decrease 10^{-5} for all equations except the energy equations, for which the criterion is 10^{-8}.
The computational procedure is validated against the numerical results of Iwatsu et el.[22] for a top heated moving lid and bottom cooled square cavity filled with air (*Pr*=0.71). A 60×60 mesh is used and computations are done for six different *Re* and *Gr* combinations. Comparisons of the average Nusselt number at the hot lid are shown in Table 1. The general

agreement between the present computation and that of Iwatsu et al. [22] is seen to be very well with a maximum discrepancy of about 3.9%.

Re	*Gr*= 10^2			*Gr*= 10^4			*Gr*= 10^6		
	Present	Iwatsu et al.	Diff. %	Present	Iwatsu et al.	Diff. %	Present	Iwatsu et al.	Diff. %
400	3.97	3.84	3.3	3.75	3.62	3.5	1.18	1.22	3.2
1000	6.25	6.33	1.2	6.32	6.29	0.47	1.70	1.77	3.9

Table 1. Comparison of the computed average Nusselt number at the hot plate

The computational procedure has also been validated against the numerical results of Guo and Sharif [31] shown in the figure 4.

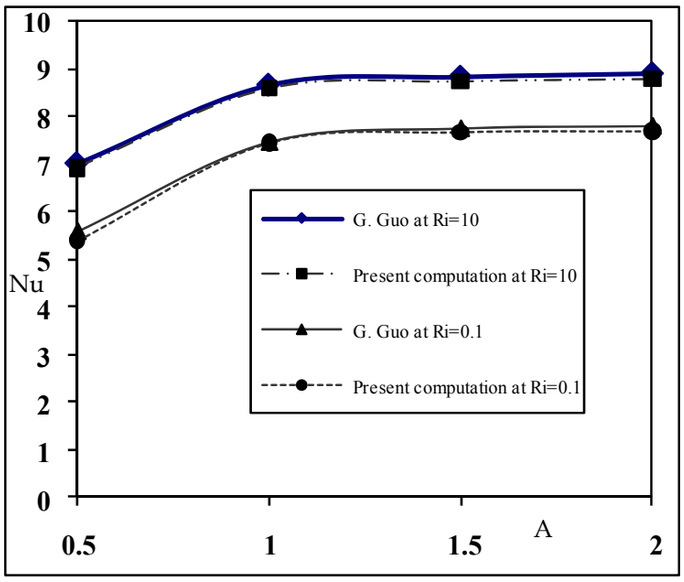


Fig. 4. Variation of the average Nusselt number with different aspect ratio at Ri=10, Re=100 and ε =0.6

Figure 4, reveals that the Average Nusselt numbers in the present study have excellent agreement with those obtained by Guo and Sharif [31] having a maximum discrepancy of about 2.3% Therefore, it can be concluded that the numerical code used in this analysis can solve the present problem with reasonable agreement.

4. Results and disscussion

Numerical results are presented in order to determine the effects of the inclination angle of the side walls, Richardson number *Ri*, Reynolds number *Re*, Aspect ratio *A*, the rotational angle of the cavity *Φ* on mixed convection flow in trapezoidal enclosure. The inclination angle of the sidewalls of the trapezoidal enclosure has been changed from 30º to 60º with an interval of 15º. The values of Richardson number varies from 0.1 to 10, Aspect ratio, *A* changes from 0.5 to 2.0 taking Rotational angle 30º, 45 º, 60º for two different Reynolds numbers 400 and 600.

4.1 Effect of inclination angle

In this study the effect of inclination angle of the adiabatic sidewalls has been observed first. The inclination angle of the side walls has been changed to 30°, 45 ° and 60°. The Richardson number has been changed from 0.1 to 10. The optimum inclination angle has been selected based on the average Nusselt number which is a non dimensional parameter that indicates the rate of heat transfer between the hot and cold walls. The results are obtained both for *Re*=400 and *Re*=600.

Figure 5-7 reveals the impact of varying inclination angles of the sidewalls of the trapezoidal cavity. These figures show the contours of streamlines and isotherms at different Richardson numbers. For small values of *Ri* number, it can be seen that the shear effect due to the movement of the top wall is dominant. The fluid flow is characterized by a primary re-circulating eddy of the size of the cavity generated by the movement of the top lid. The isothermal contour maps are clustered near the bottom and top walls resulting in steep temperature gradient there. In each case as the Richardson number increases the convection current becomes more dominant resulting in stronger flow field. Again at γ=45° (figure 6), the flow field is stronger than the γ=30° and γ=60° (figure 5 and figure 7), which is an indication of better heat transfer. The isothermal plots also complies with the flow field, showing minimum value of the maximum isotherms at γ=45°.

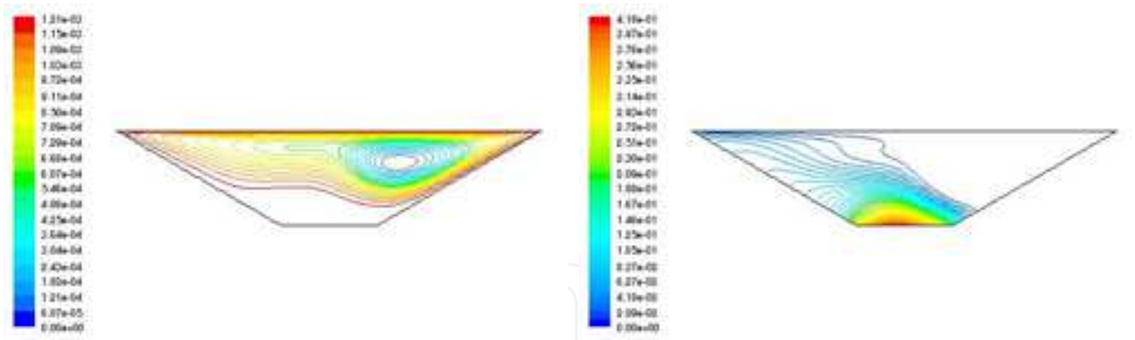
From figure 8-9 the average value of the Nusselt number with respect to the Richardson number has been plotted. Here it can be seen that Nusselt number at γ=45° dominates the other two cases i.e. γ=30° and γ=60°, showing better heat transfer. So it is clearly visible that trapezoid having the inclination angle γ=45° gives better heat transfer and consequently it can be taken as the optimum inclination angle.

4.2 Effect of rotational angle, Φ

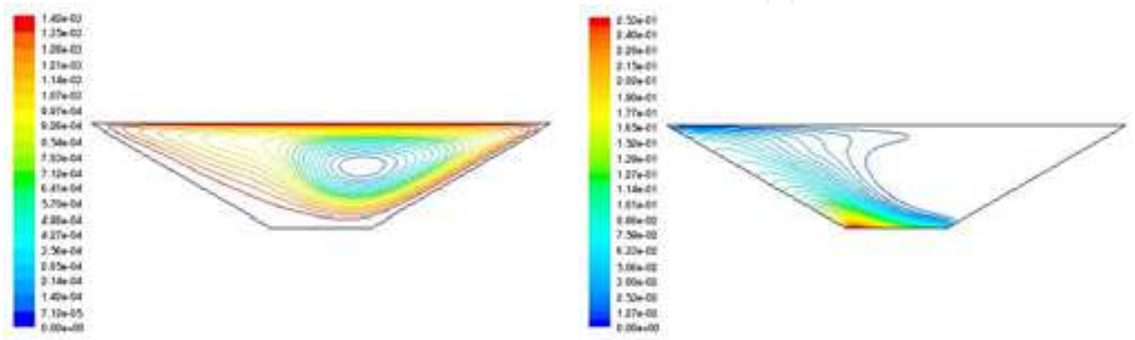
Next the effect of rotational angle, Φ has been studied. When studying the effect of rotational angel, two distinct cases have been taken into consideration. They are aiding and opposing flow condition. The first one is when the shear driven flow opposes the convective flow and in that case the top moving lid is moving in the positive direction at a specified rotational angle [figure 1]. The second condition is the aiding flow condition where the shear driven flow aids the natural convective flow and the moving top lid moves in the opposite direction unlike the first case [figure 2]. Both these cases have been studied for a rotational angle for Φ=30°, 45° and 60° and their heat transfer characteristics has been studied in terms of streamlines and isothermal plots.

Unlike Φ=0°, when the buoyancy is acting only in the y direction, as the rotational angle Φ changes, the flow field changes significantly. In opposing flow condition the shear driven flow opposes the natural convective flow, At low Richardson number (*Ri*<1)the forced convection is dominating, creating a single circulation at the right corner of the top moving lid [figure 10-12].

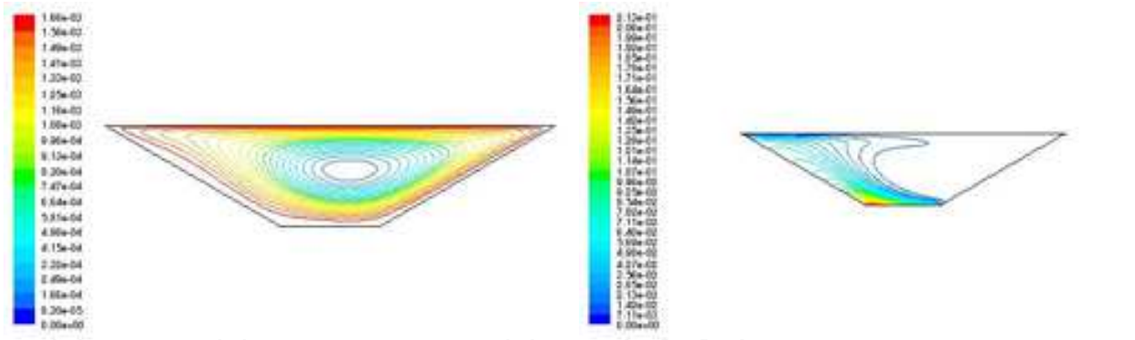
As the Richardson number increases (*Ri*>1), natural convection becomes dominating creating a large circulation at the bottom of the cavity. This large circulation causing by natural convection goes bigger and stronger as *Ri* number increases as well as squeezes the upper circulation, resulting an opposing effect. If we observe the isothermal plots, it changes accordingly with streamlines. As *Ri* number increases, the isothermal lines changes significantly indicating that the convection is the dominating heat transfer for the specified case. The shear driven circulation at the upper right side becomes smaller and smaller as the *Ri* number increases because of dominating natural convection.



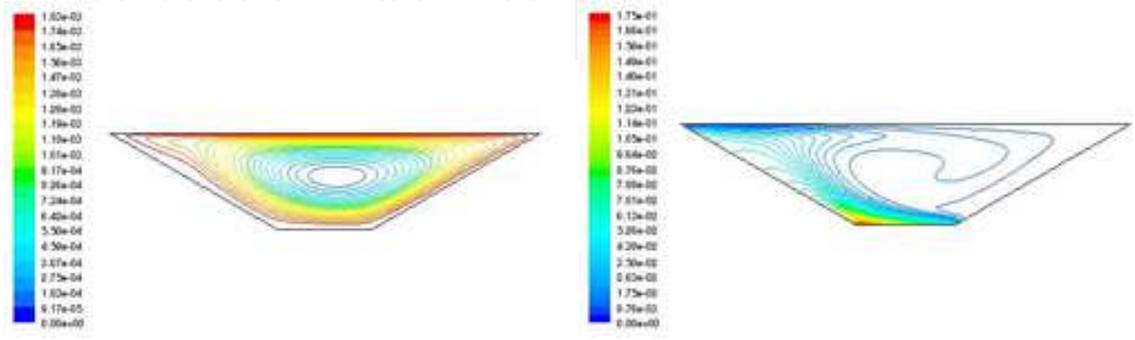
R_i = 0.1



R_i = 1

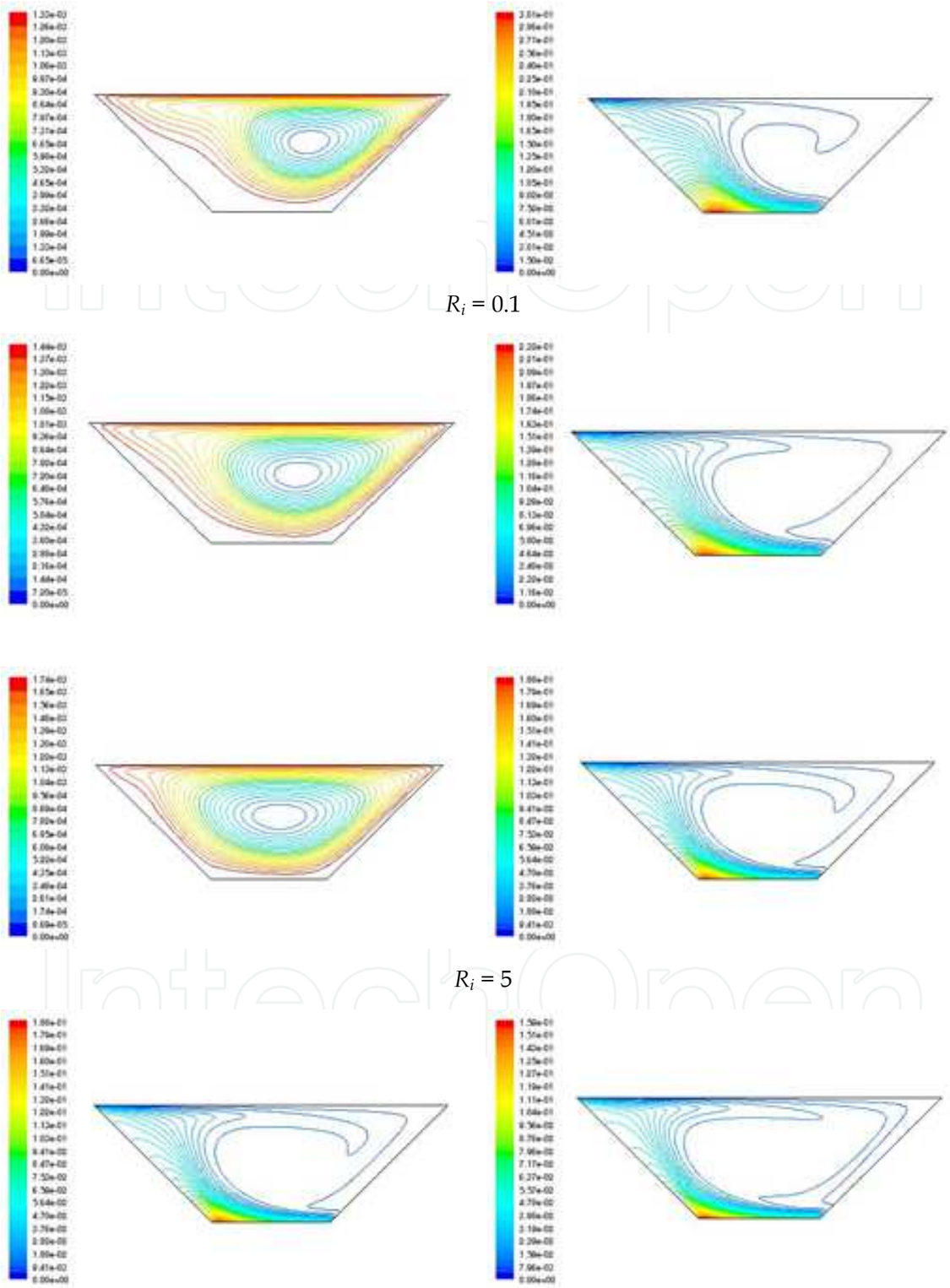


R_i = 5



R_i = 10

Fig. 5. Contours of streamlines and isotherms at Re=400, A=1.0 and γ=30°

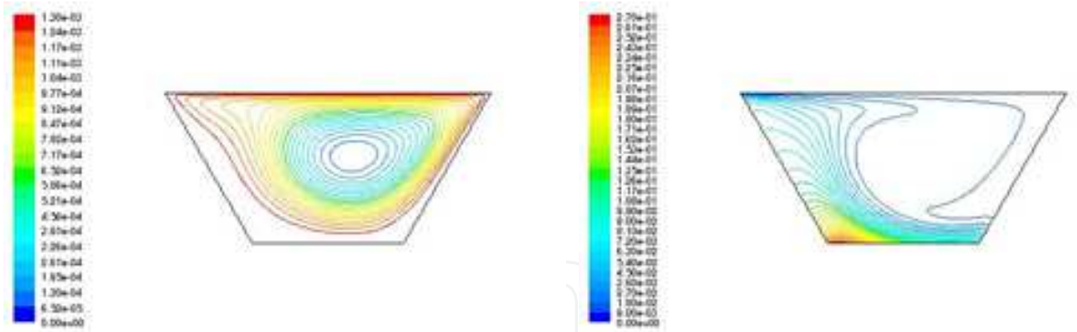


R_i = 0.1

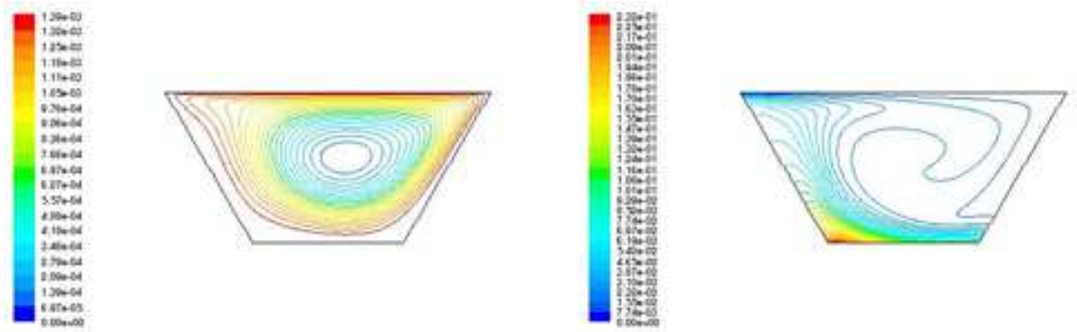
R_i = 5

R_i = 10

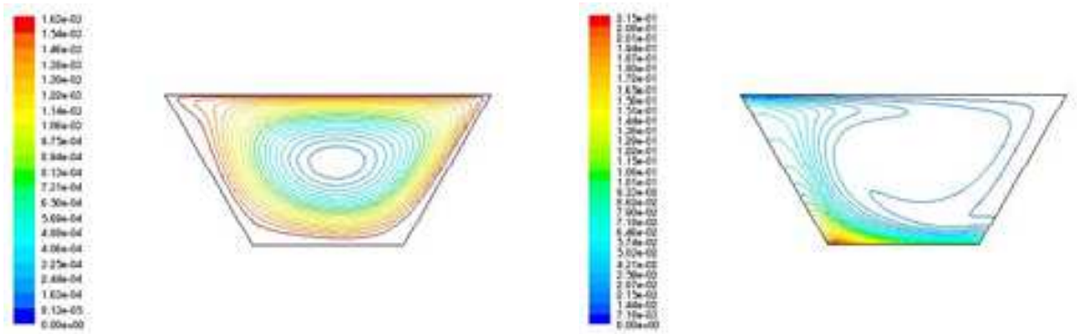
Fig. 6. Contours of streamlines and isotherms at Re=400, A=1.0 and γ=45°



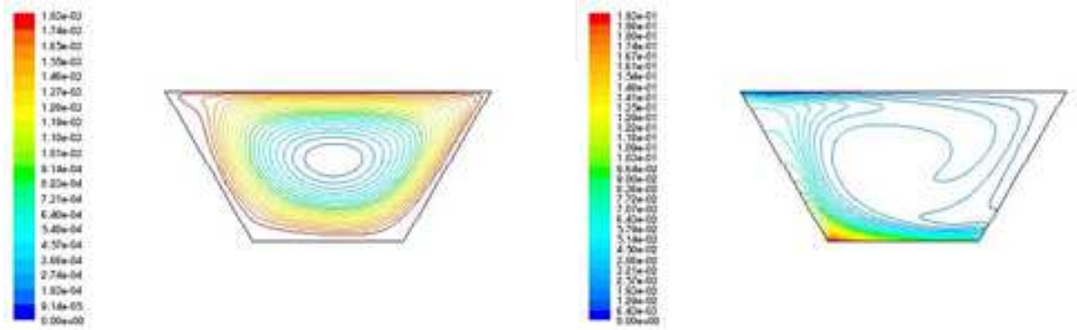
R_i = 0.1



R_i = 1



R_i = 5



R_i = 10

Fig. 7. Contours of streamlines and isotherms at Re=400, A=1.0 and γ=60°

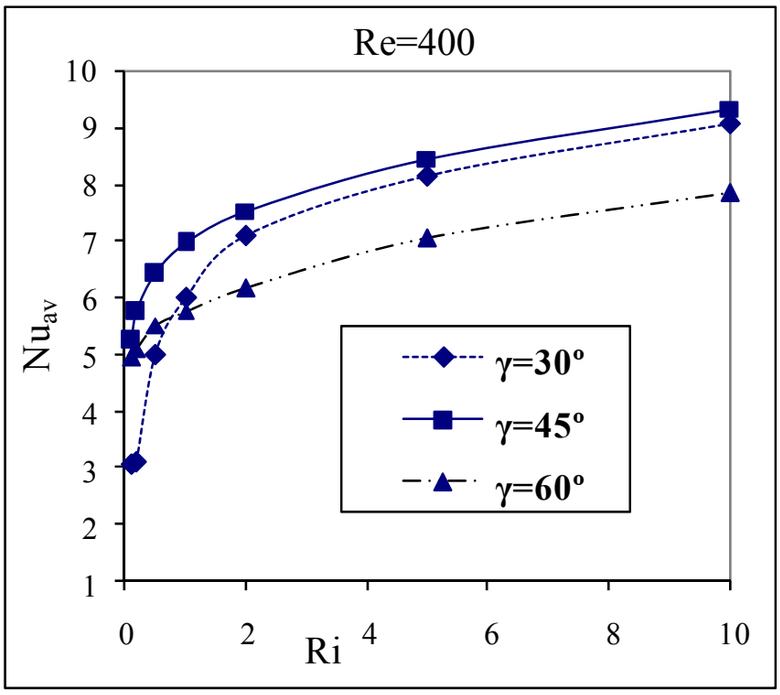


Fig. 8. Average Nusselt number, Nu_{av} vs Richardson number at Re=400, A=1

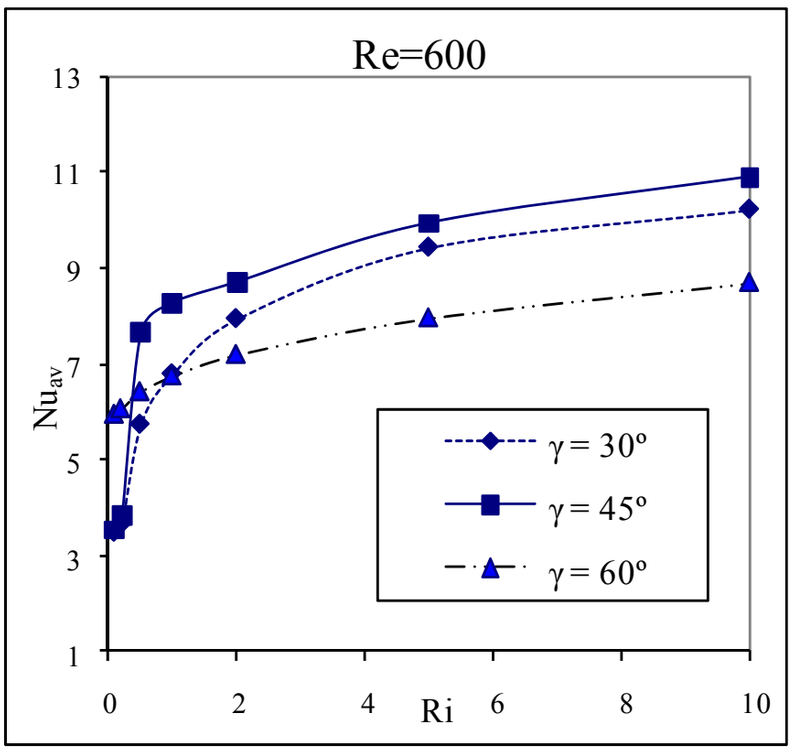
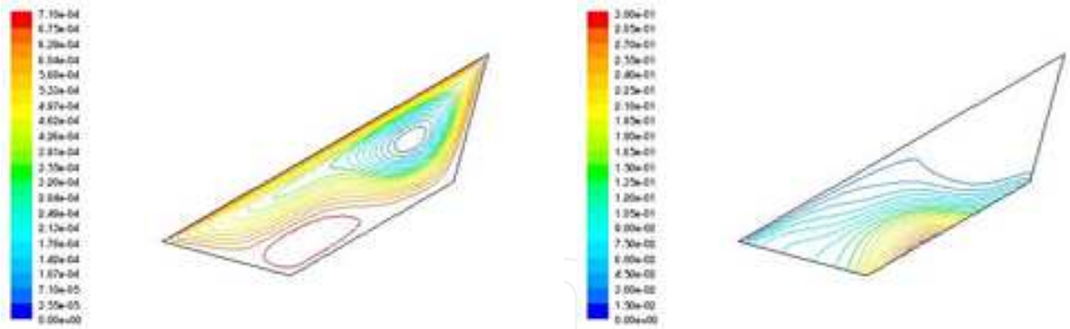
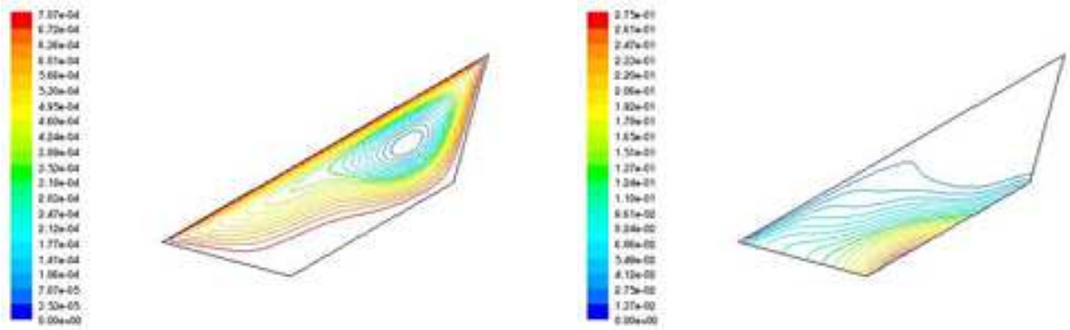


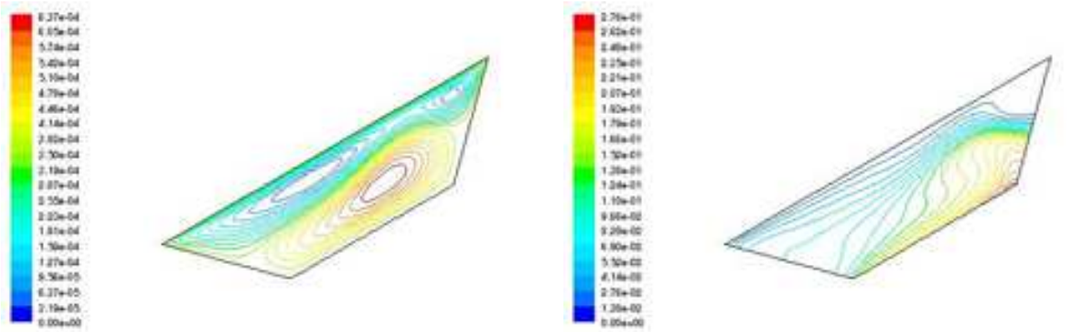
Fig. 9. Average Nusselt number, Nu_{av} vs Richardson number at Re=600, A=1



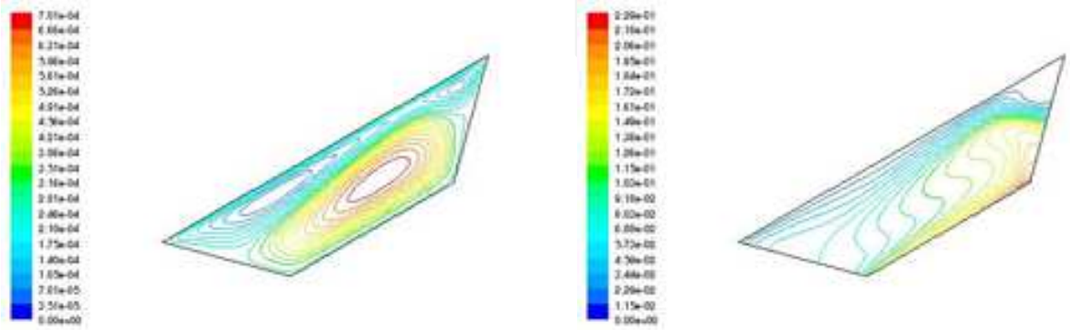
R_i = 0.1



R_i = 1

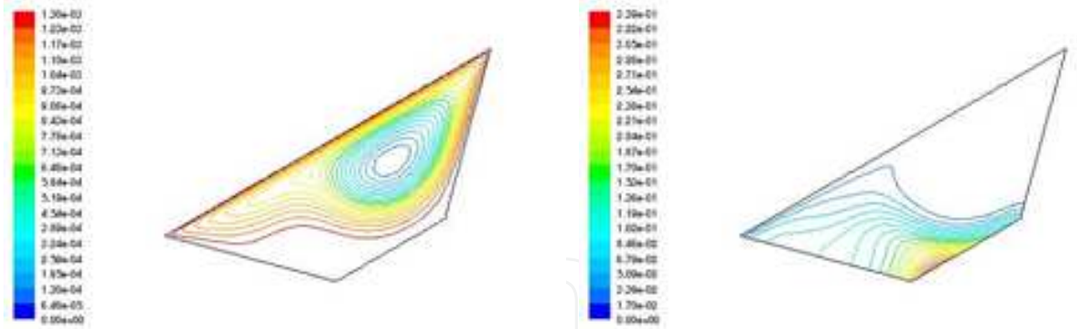


R_i = 5

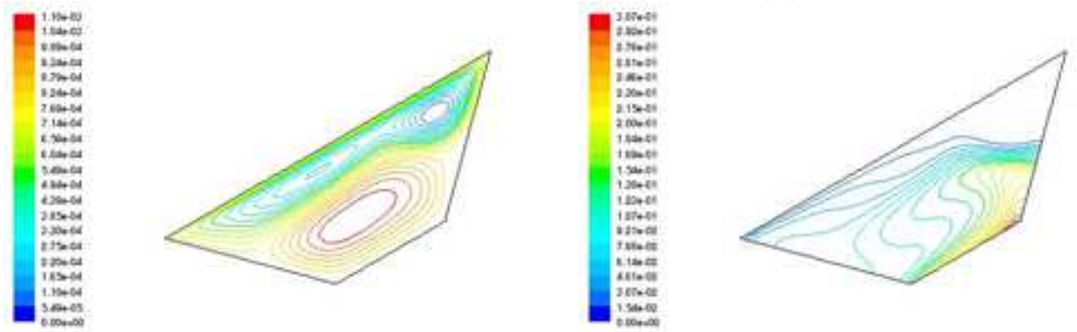


R_i = 10

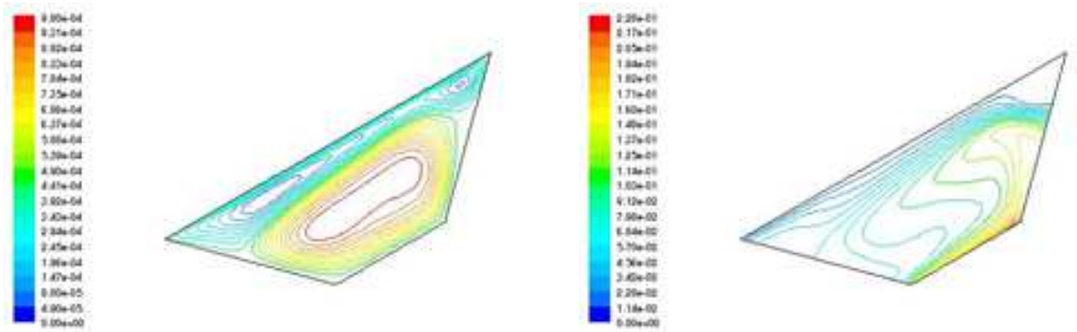
Fig. 10. Contours of streamlines and isotherms at Re=400, A=0.5 and Φ=30°, opposing flow



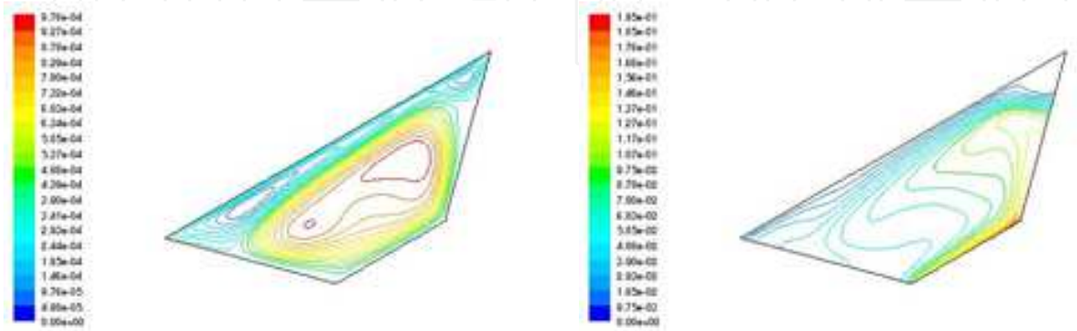
R_i = 0.1



R_i = 1

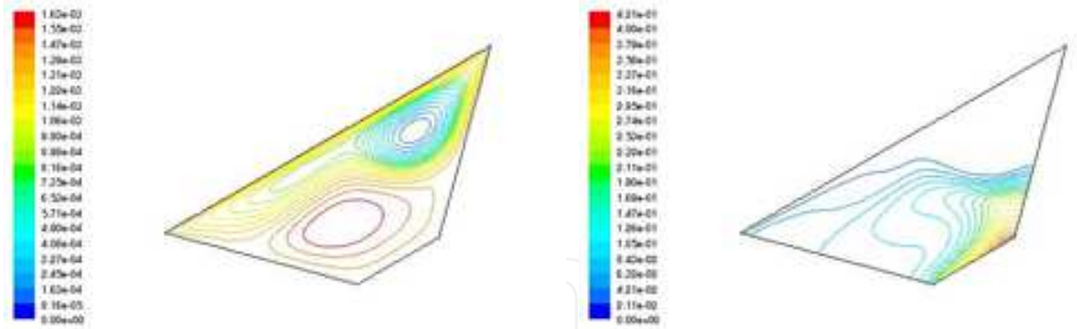


R_i = 5

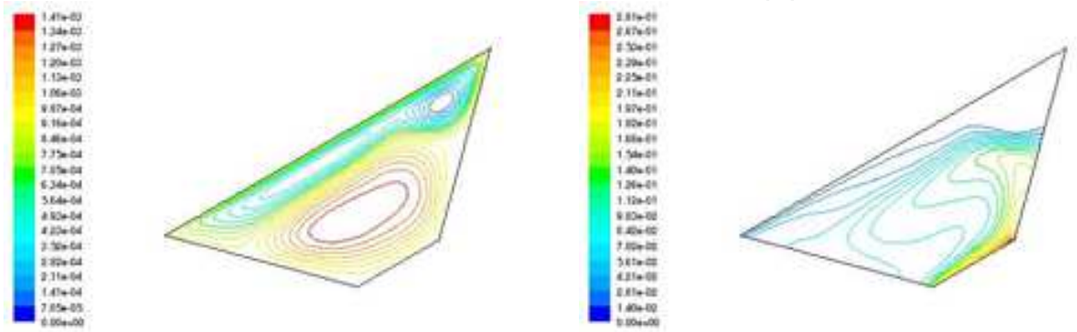


R_i = 10

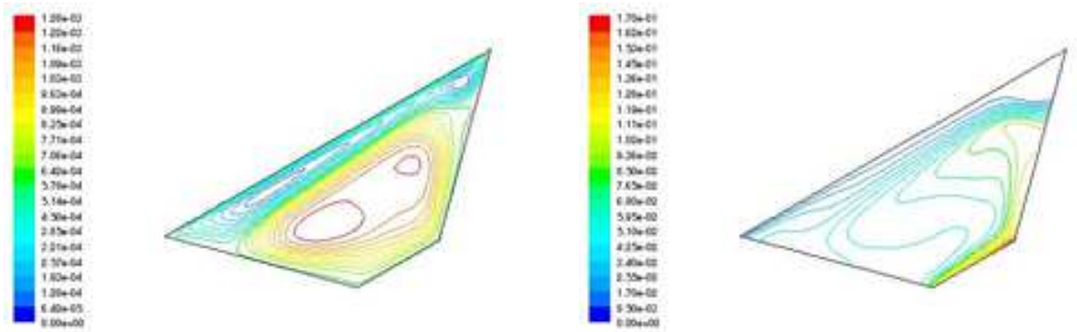
Fig. 11. Contours of streamlines and isotherms at Re=400, A=1 and Φ=30°, opposing flow



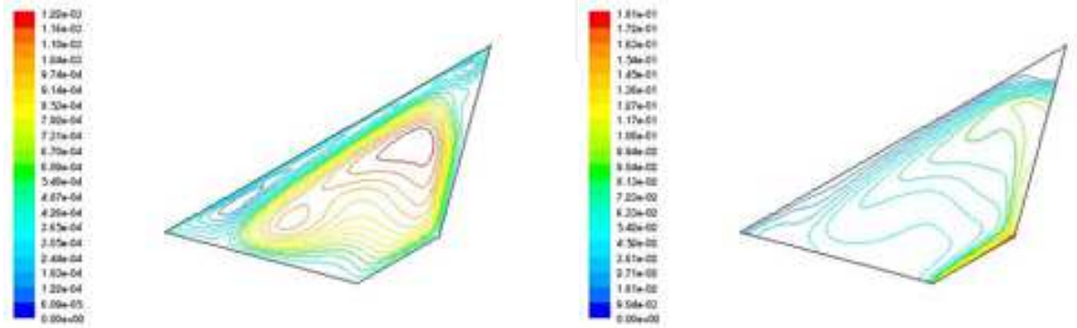
$R_i = 1$



$R_i = 1$

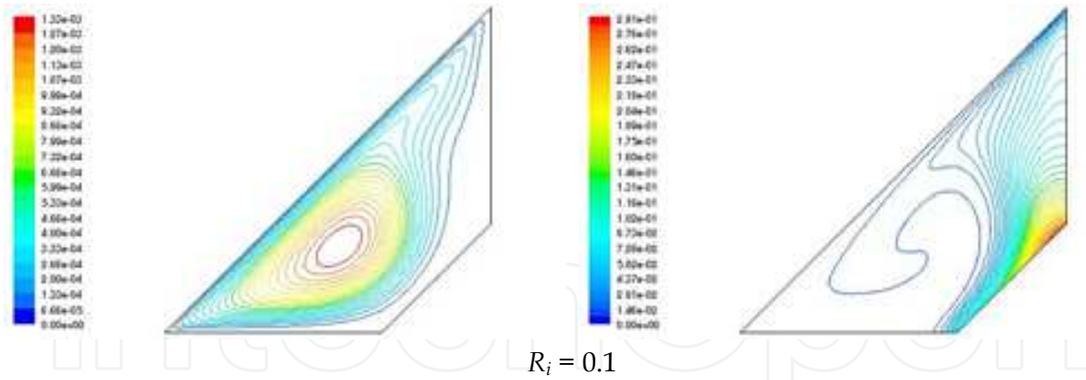


$R_i = 1$

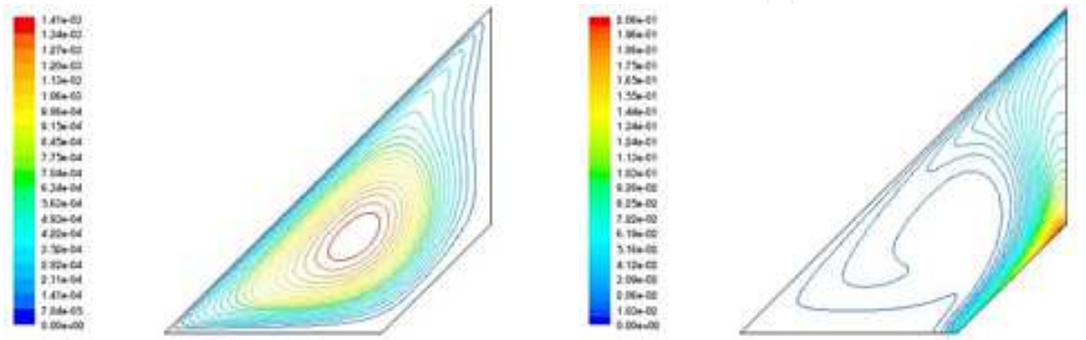


$R_i = 1$

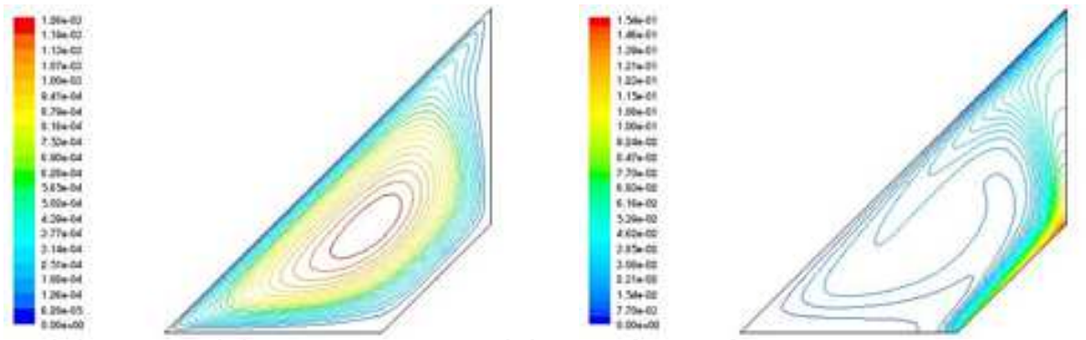
Fig. 12. Contours of streamlines and isotherms at Re=400, A=1.5 and Φ=30°, opposing flow



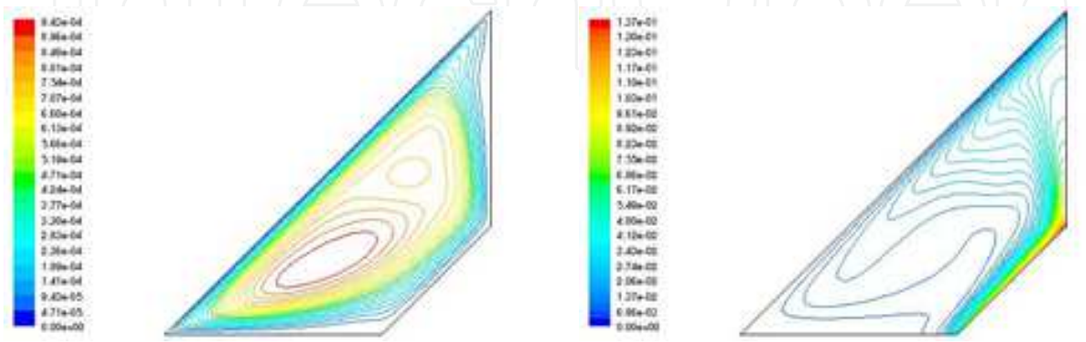
R_i = 0.1



R_i = 1



R_i = 5



R_i = 10

Fig. 13. Contours of streamlines and isotherms at Re=400, A=1 and Φ=45°, aiding flow

In the case of aiding flow, condition when the forced convection aids the natural convection a different scenario has been observed [e.g. figure 13]. In all the cases, a single circulation of the size of the cavity has been observed. Unlike the opposing flow condition, in that case the natural convection aids the shear driven flow from the smaller value of *Ri* number, resulting a much stronger convective current. As the *Ri* number increases, the convection flow fields become more and more stronger resulting better and better heat transfer. The isotherms changes significantly as the Richardson number increases and gives the minimum value at higher *Ri* number.

As the aspect ratio, *A* increases the convective flow fields become more and more stronger. As cavity volume increases with aspect ratio and more volume of cooling air is involved in cooling the heat source leading to better cooling effect. The effect of aspect ratio at different rotational angle has also been studied. In all the cases, it can be observed that Nu_{av} increases with increasing aspect ratio for all rotational angles, leading to better heat transfer.

Fig. 14-18 shows a comparative analysis of aiding and opposing flow conditions. There it can be seen that, the aiding flow condition always dominates the opposing flow condition in terms of Nu_{av} , which indicates better heat transfer at all rotational angle. The aiding flow condition provides stronger convective currents, which has been visible in the study, as the natural convection aids the shear driven flow. As a result the maximum value of the isotherms is lower in case of aiding flow condition, indicating lower temperature. But in opposing condition the natural convection opposes the shear driven flow, providing weak convective currents.

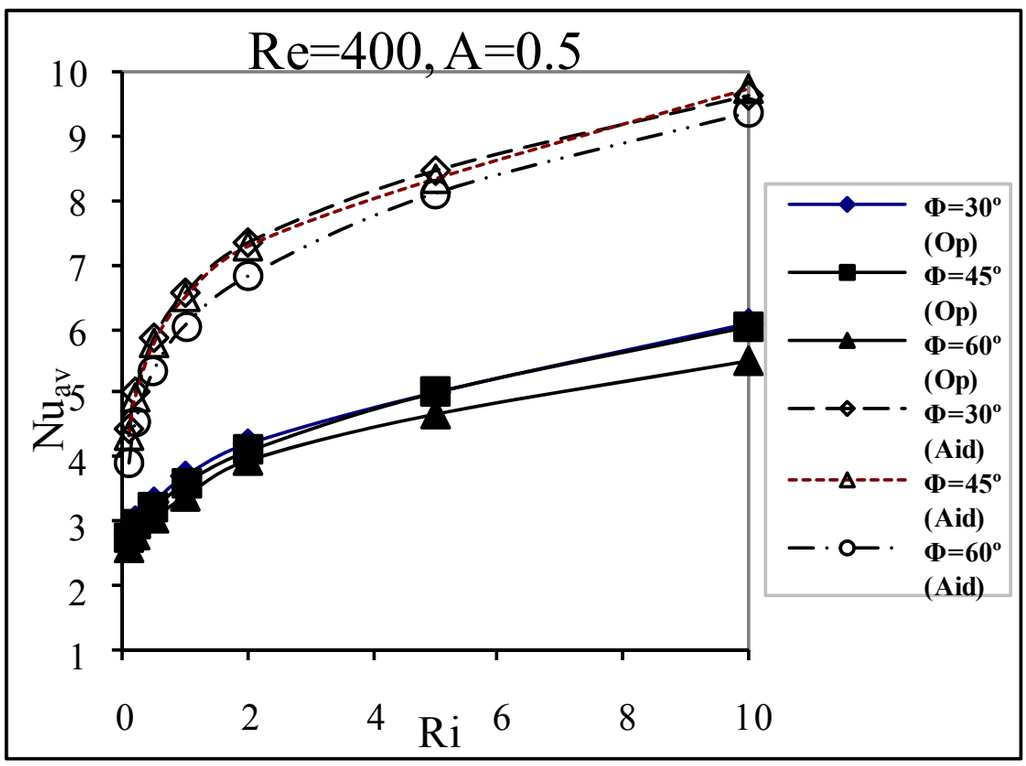


Fig. 14. Variation of Nu_{av} with Ri at A=0.5, Re=400

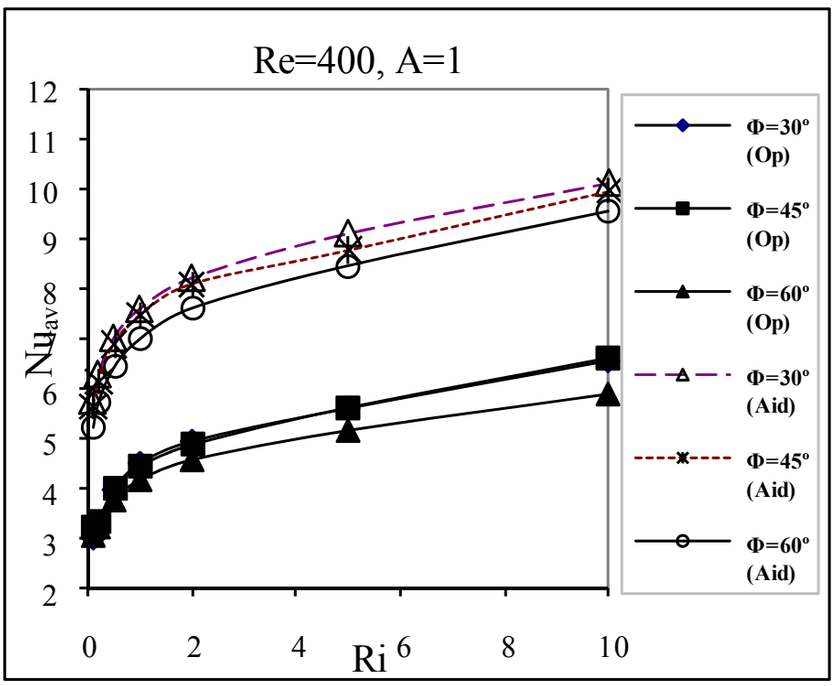


Fig. 15. Variation of Nu_{av} with Ri at A=1, Re=400

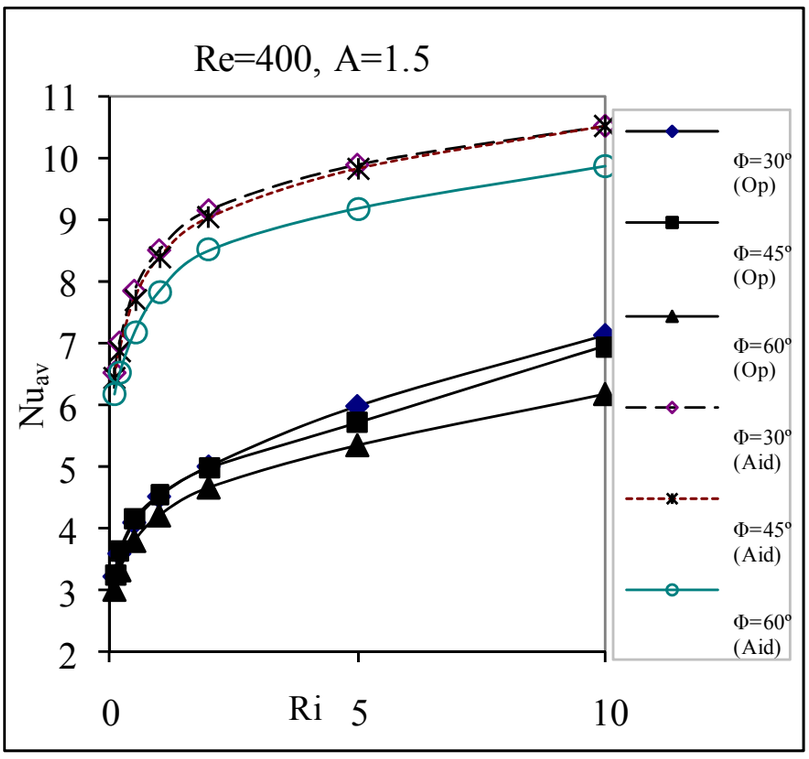


Fig. 16. Variation of Nu_{av} with Ri at A=1.5, Re=400

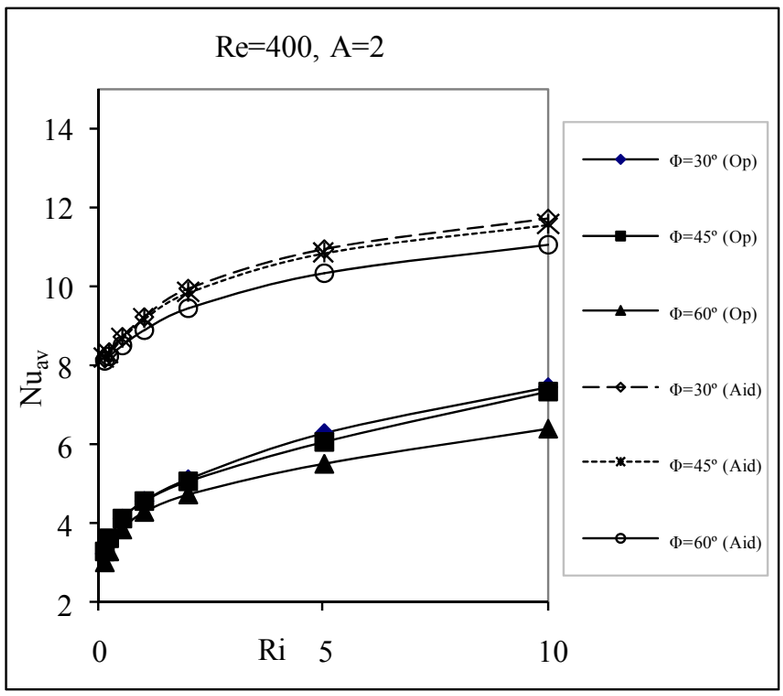


Fig. 17. Variation of Nu_{av} with Ri at A=2, Re=400

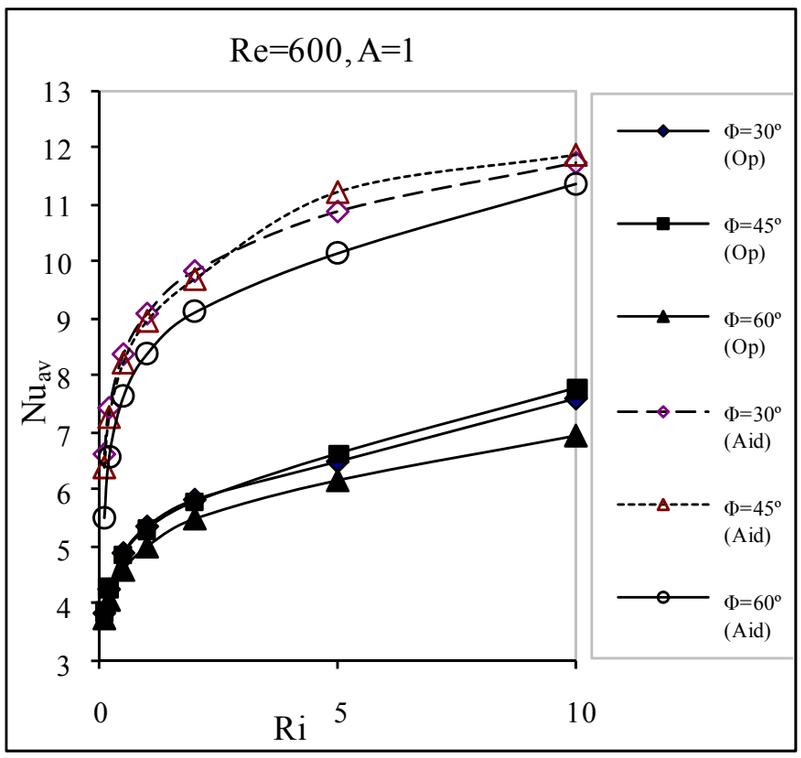


Fig. 18. Variation of Nu_{av} with Ri at A=1, Re=600

The Nu_{av} is also sensitive to rotational angle, figure 14-18. At Re=400 it can be seen that, Nusselt number decreases as the rotational angle, Φ increases. Nu_{av} increases marginally at $\Phi=30^o$ from $\Phi=45^o$ but at $\Phi=60^o$, Nu_{av} drops significantly for all the aspect ratios, figure 14-17.The flow fields also changes accordingly. At Re=600, the maximum heat transfer has been obtained at $\Phi=45^o$, in terms of average Nusselt number, figure 18. Nu_{av} increases marginally at $\Phi=45^o$ from $\Phi=30^o$ but drops significantly at $\Phi=60^o$, indicating poor heat transfer at $\Phi=60^o$.

4.3 Effect of Richardson number, *Ri*

The value of the Richardson number, $Ri=Gr/Re^2$ provides a measure of the importance of buoyancy driven natural convection relative to the lid driven forced convection. When the Buoyancy effects are relatively small, $Ri<1$, the gross flow features are similar to those of a conventional non-stratified fluid at comparable values of Re. The main circulation fills the entire cavity of the size of the cavity generated by the movement of the top wall. Minor cells may be visible near the bottom corners. The streamlines and isotherms indicated that the hydrodynamic and thermal boundary layers are not developed fully at low Richardson number. The isothermal lines are mostly undistorted and horizontal lines except the large recirculation area inside the cavity at low Richardson number. In the large recirculation zone temperature gradients are very weak. This implies that, due to the vigorous actions of the mechanically driven circulations, fluids are well mixed; consequently, temperature differences in much of this interior region are very small.

When $Ri>1$, natural convection begins to dominate the forced convection. The Buoyancy assists the core flow and thus the convection current becomes more and more strong with increasing Richardson number. As Richardson number increases, the main circulation occupies the whole cavity and it become more symmetrical inside the cavity. If we see the isothermal plots, we can see that as the Richardson number increases the isothermal lines becomes more and more denser at the upper cold lid. The crowded streamlines and isothermal lines indicate that the hydrodynamic and thermal boundary layers have been developed along the hot wall and cold wall, respectively, reflecting rigorous heat transfer rate occurred. Consequently the maximum temperature reduces due to this large heat transfer rate.

The average Nusselt number as a function of Richardson number has been plotted in figure 19-20 for different Reynolds number. It can be observed that as the Richardson number increases the average Nusselt number increases accordingly for all the aspect ratios. When $Ri<1$, Nu_{av} grows only slightly with increasing Ri. After Ri is more than 1, Nu_{av} is found to increase more rapidly. Since Re is kept constant the forced convection effect remains invariant as Ri increases for a particular case. When $Ri>1$, the natural convection aids more and more in the heat transfer process in addition to the forced convection which results in more rapid increase of $Nu_{av.}$

4.4 Effect of aspect ratio, *A*

Changing the aspect ratio, A (A=H/W) causes a change in heat transfer characteristics. In order to investigate the convection heat transfer at different aspect ratios, computations has been done for cavities at aspect ratios of 0.5, 1, 1.5 and 2.0. Keeping Reynolds number fixed at 400 and 600 the Richardson number has been changed from 0.1 to 10. If we compare the flow fields at different aspect ratios from 0.5 to 2.0, it can be revealed that in the convection region adjacent to the heat source, the isotherms become thinner and denser producing higher temperature gradients with increasing aspect ratio. The streamlines becomes stronger as the aspect ratio increases. This is due to the fact that the cavity volume increases with

aspect ratio and more volume of cooling air is involved in cooling the heat source leading to better cooling effect.

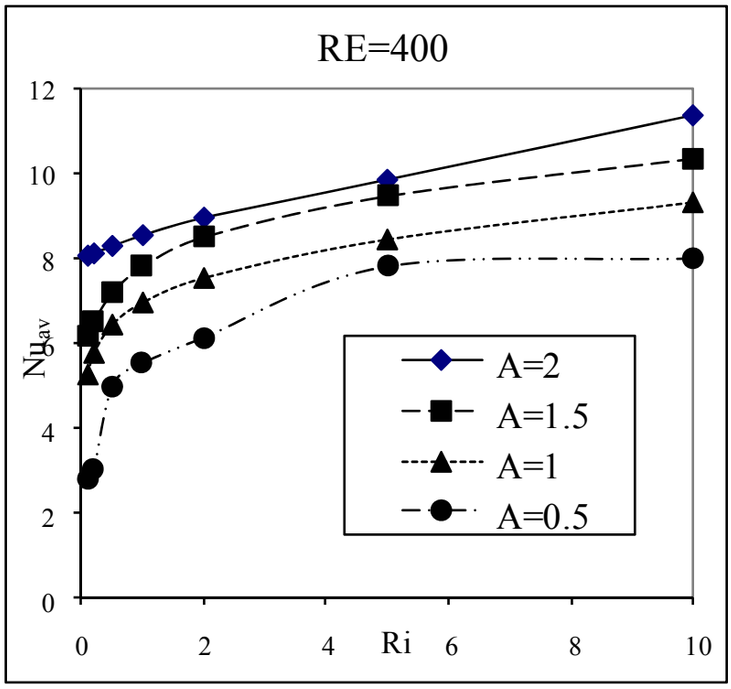


Fig. 19. Variation of Nu_{av} with Ri at Re=400 and Φ=0°

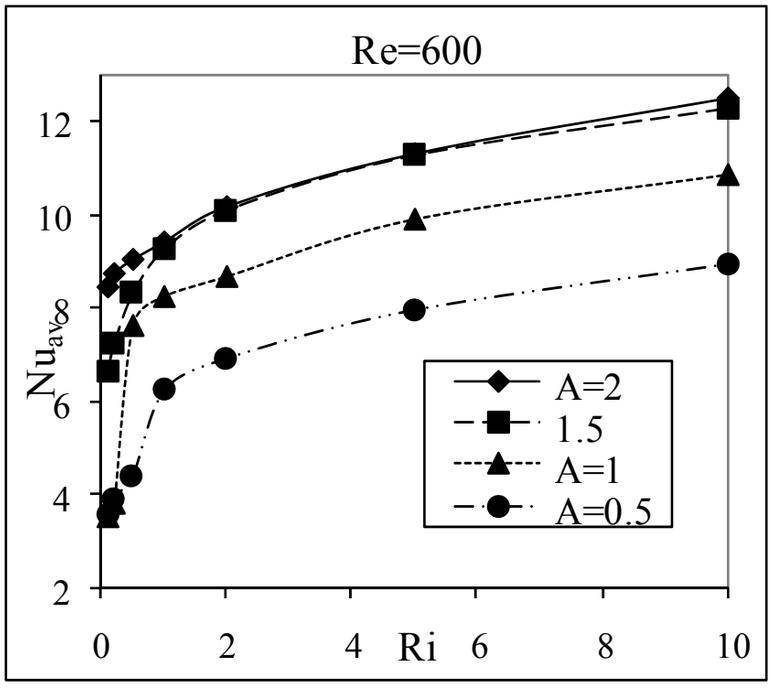


Fig. 20. Variation of Nu_{av} with Ri at Re=600 and Φ=0°

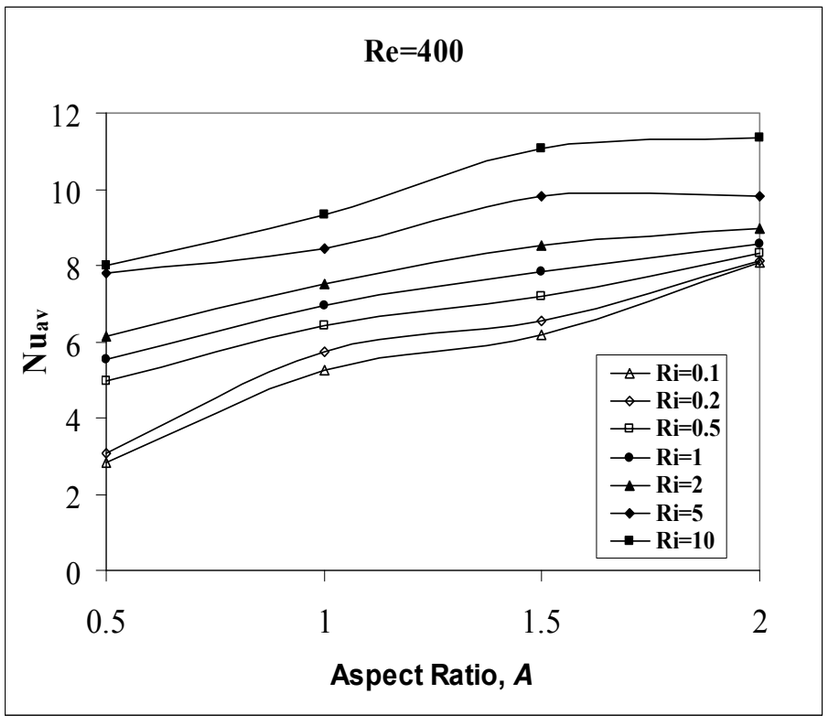


Fig. 21. Variation of Nu_{av} with A at Re=400 and $\Phi=0^o$

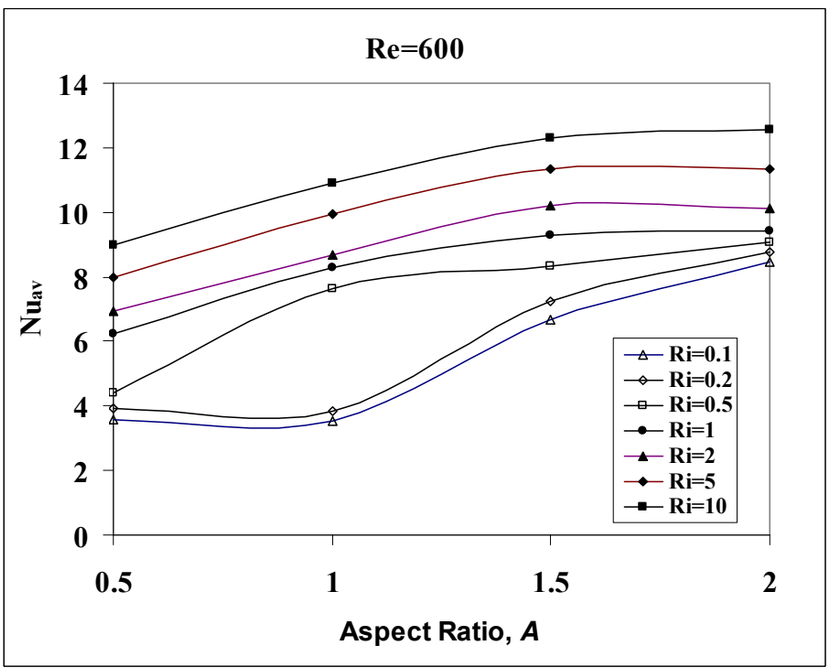


Fig. 22. Variation of Nu_{av} with A at Re=600 and $\Phi=0^o$

The average Nusselt number at the heat source surface has been plotted in figure 21-22 for a range of *Ri* and aspect ratios. For a particular aspect ratio, the Nu_{av} increases with increasing

Ri. As a result, the maximum temperature decreases monotonously which can be recognized from the isothermal plots. As the aspect ratio increases from 0.5 to 1 the Nu_{av} increases for a particular *Ri*.
At higher Reynolds number i.e. Re=600, with increasing aspect ratio some secondary eddy at the bottom surface of the cavity has been observed. This is of frictional losses and stagnation pressure. As the *Ri* increases, natural convection dominates more and the bottom secondary eddies blends into the main primary flow. For A>1.5 the variation is almost flat indicating that the aspect ratio does not play a dominant role on the heat transfer process at that range.

4.5 Effect of Reynolds number, *Re*

This study has been done at two different Reynolds numbers. They are Re=400 and Re=600. With a particular case keeping *Ri* and *A* constant, as the Reynolds number increases the convective current becomes more and more stronger and the maximum value of the isotherms reduces. As we know $Ri=Gr/Re^2$. *Gr* is square proportional of *Re* for a fixed *Ri*. So slight change of Re and Ri causes huge change of Gr. Gr increases the buoyancy force. As buoyancy force is increased then heat transfer rate is tremendously high. So changes are very visible to the change of *Re*. From figure 19-20, it can be observed that as the *Re* increases the average Nusselt number also increases for all the aspect ratios.

5. Conclusion

Two dimensional steady, mixed convection heat transfer in a two-dimensional trapezoidal cavity with constant heat flux from heated bottom wall while the isothermal moving top wall in the horizontal direction has been studied numerically for a range of Richardson number, Aspect ratio, the inclination angle of the side walls and the rotational angle of the cavity. A number of conclusions can be drawn form the investigations:

- The optimum configuration of the trapezoidal enclosure has been obtained at $\gamma=45^o$, as at this configuration the Nu_{av} was maximum at all Richardson number.
- As the Richardson number increases the Nu_{av} increases accordingly at all Aspect ratios, because at higher Richardson number natural convection dominates the forced convection.
- As Aspect Ratio increases from 0.5 to 2.0, the heat transfer rate increases. This is due to the fact that the cavity volume increases with aspect ratio and more volume of cooling air is involved in cooling the heat source leading to better cooling effect.
- The direction of the motion of the lid also affects the heat transfer phenomena. Aiding flow condition always gives better heat transfer rate than opposing flow condition. Because at aiding flow condition, the shear driven flow aids the natural convective flow, resulting a much stronger convective current that leads to better heat transfer.
- The Nu_{av} is also sensitive to rotational angle Φ. At Re=400 it can be seen that, Nusselt number decreases as the rotational angle, *Φ* increases. Nu_{av} increases marginally at $\Phi=30$ from $\Phi=45^o$ but at $\Phi=60^o$, Nu_{av} drops significantly for all the aspect ratios.

6. Further recommandations

The following recommendation can be put forward for the further work on this present research.

1. Numerical investigation can be carried out by incorporating different physics like radiation effects, internal heat generation/ absorption, capillary effects.
2. Double diffusive natural convection can be analyzed through including the governing equation of concentration conservation.
3. Investigation can be performed by using magnetic fluid or electrically conducting fluid within the trapezoidal cavity and changing the boundary conditions of the cavity's wall.
4. Investigation can be performed by moving the other lids of the enclosure and see the heat transfer effect.
5. Investigation can be carried out by changing the Prandtl number of the fluid inside the trapezoidal enclosure.
6. Investigation can be carried out by using a porous media inside the trapezoidal cavity instead of air.

7. References

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Convection and Conduction Heat Transfer

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The convection and conduction heat transfer, thermal conductivity, and phase transformations are significant issues in a design of wide range of industrial processes and devices. This book includes 18 advanced and revised contributions, and it covers mainly (1) heat convection, (2) heat conduction, and (3) heat transfer analysis. The first section introduces mixed convection studies on inclined channels, double diffusive coupling, and on lid driven trapezoidal cavity, forced natural convection through a roof, convection on non-isothermal jet oscillations, unsteady pulsed flow, and hydromagnetic flow with thermal radiation. The second section covers heat conduction in capillary porous bodies and in structures made of functionally graded materials, integral transforms for heat conduction problems, non-linear radiative-conductive heat transfer, thermal conductivity of gas diffusion layers and multi-component natural systems, thermal behavior of the ink, primer and paint, heating in biothermal systems, and RBF finite difference approach in heat conduction. The third section includes heat transfer analysis of reinforced concrete beam, modeling of heat transfer and phase transformations, boundary conditions-surface heat flux and temperature, simulation of phase change materials, and finite element methods of factorial design. The advanced idea and information described here will be fruitful for the readers to find a sustainable solution in an industrialized society.

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