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Statistical Mechanics That Takes into Account Angular Momentum Conservation Law - Theory and Application

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1. Introduction

The fundamental problem of statistical mechanics is obtaining an ensemble average of physical quantities that are described by phase functions (classical physics) or operators (quantum physics). In classical statistical mechanics the ensemble density of distribution is defined in the phase space of the system. In quantum statistical mechanics the space of functions that describe microscopic states of the system play a role similar to the classical phase space. The probability density of the system detection in the phase space must be normalized. It depends on external parameters that determine the macroscopic state of the system.

An in-depth study of the statistical mechanics foundations was presented in the works of A.Y. Khinchin (Khinchin, 1949, 1960). For classical statistical mechanics an invariant set was introduced. It would be mapped into itself by transforming with the Hamilton equations. The phase point of the isolated system remains during the process of the motion at the invariant set at all times. If the system is in the stationary equilibrium state, this invariant set has a finite measure. The Ergodic hypothesis asserts that in this case the probability $dP(\mathbf{R})$ to detect this system at any point \mathbf{R} of the phase space is:

$$dP(\mathbf{R}) = \frac{\varphi_{\Sigma}(\mathbf{R})d\Gamma}{\Omega_{\Sigma}[(2\pi\hbar)^{3N} N!]} \quad (1)$$

where Ω_{Σ} - the measure (phase volume) of the invariant set Σ ; $\varphi_{\Sigma}(\mathbf{R})$ - the characteristic function of the invariant set, which is equal to one if the point \mathbf{R} belongs to this set, and is equal to zero in all other points of the phase space; $d\Gamma = \prod_{i=1}^N d\mathbf{p}_i d\mathbf{r}_i$ - the phase space volume element. The number of distinguishable states in a phase space volume element $d\Gamma$ is $[(2\pi\hbar)^{3N} N!]^{-1}$. The system that will be under consideration is a collection of N structureless particles. The averaged value of a phase function $F(\mathbf{R})$ is $\bar{F} = \int_{\Gamma} F(\mathbf{R})dP(\mathbf{R})$. Here the integral goes over all phase space Γ . This is microcanonical distribution. A characteristic function often would be presented as $\delta[f(\mathbf{R}) - z]$, where $f(\mathbf{R})$ is a phase function and z is it's fixed value.

A hypersurface in a hyperspace is a set with zero measure. Therefore the invariant set is determined as a thin layer that nearly envelops the hypersurface in the phase space. The

determining equations of this hypersurface are the equalities that fix the values of controllable motion integrals. A controllable motion integral is a phase function, the value of which does not vary with the motion of the system and can be measured. An isolated system universally has the Hamiltonian that does not depend on the time explicitly, and is the controllable motion integral. A fixed value of the Hamiltonian is the energy of the system. The kinetic energy of majority of systems is a positive definite quadric form of all momenta. It determines a closed hypersurface in the subspace of momenta of the phase space. If motions of all particles are finite, the hypersurface of the fixed energy is closed and the layer that envelops it has the finite measure. Then this hypersurface can determine the invariant set of the system. A finiteness of motions of particles as a rule is provided by enclosing the system in an envelope that reflects particles without changing their energy, if the system is considered as isolated. It is common in statistical mechanics to consider the layer enveloping the energy hypersurface as the invariant set. But A.Y. Khinchin (Khinchin, 1949) shows that other controllable integrals of the system, if they exist, must be taken into account. In the general case an isolated system can have another two vector controllable integrals. That is the total momentum of the system, and the total angular momentum relative to the system's mass centre. The total momentum is a sum of all momenta of particles. If the volume of the system is bounded by an external field or an envelope, the total momentum does not conserve. In the absence of external fields the total momentum conservation cannot make particle motions finite. Therefore the total momentum cannot be a controllable motion integral that determines the invariant set.

The angular momentum is another case. A vector of angular momentum relative to the mass center always is conserved in an isolated system. If this vector is nonzero, a condition should exist that provides a limitation of a gas expansion area. For example, nebulas do not collapse because they rotate, and do not scatter because of the gravitation. In the system of charged particles in a uniform magnetic field the conservation of the angular momentum provides a limitation of a gas expansion area (confinement of plasma). If a gas system is enclosed into envelope, and total system has nonzero angular momentum, the vector of the angular momentum should be conserved. However an envelope can have the non-ideal form and surface. That is the cause of the failure to consider the angular momentum of the gas as a controllable motion integral (Fowler, & Guggenheim, 1939). But if the cylindrical envelope rotates and the gas rotates with the same angular velocity deviations of the angular momentum of the gas from the fixed value as the result of reflections of particles from the envelope should be small and symmetric with respect to a sign. These fluctuations are akin to energy fluctuations for a system that is in equilibrium with a thermostat. Therefore the angular momentum conservation in specific cases can determine the invariant set and the thermodynamical natures of the system together with the energy conservation. Taking into account all controllable motion integrals is the necessary condition of the validity of the Ergodic hypothesis (Khinchin, 1949).

There is a contradiction in physics at the present time. Firstly, it has been proven that in the equilibrium state a system spin can exist only if the system is rigid and can rotate as a whole (Landau, & Lifshitz, E.M., 1980a). Therefore a gas, which supposed not be able to rotate as a whole, cannot have any angular momentum and spin. Based on this reasoning R.P. Feynman proves that an electron gas cannot have diamagnetism (the Bohr – van Leeuwen theorem) (Feynman, Leighton, & Sands, 1964). On the other hand, it is well known that density of a gas in a rotating centrifuge is non-uniform. This effect is used for the separation

of isotopes (Cohen, 1951). The experiment by R. Tolman, described in the book (Pohl, 1960), is a proof of the existence of the electron gas angular momentum. In this experiment a coil was rotated and then sharply stopped. An electrical potential was observed that generated a moment of force, which decreased to zero the angular momentum of electron gas.

The contradiction described above requires creation of statistical mechanics for non-rigid systems taking into account the nonzero angular momentum conservation. This statistical mechanics differs from common one in many respects. If the angular momentum relative to the axis that passes through the mass centre conserves, the system is spatially inhomogeneous. This means that passage to the thermodynamical limit makes no sense, a spatial part of the system is not a subsystem that similar to the total system, specific quantities such as densities or susceptibilities have no physical meaning.

The microcanonical distribution is seldom used directly when the computations and the justifications of thermodynamics are done. The more usable Gibbs distribution can be deduced from microcanonical one (Krutkov, 1933; Zubarev, 1974). The Gibbs assembly describes a system that is in equilibrium with environment. These systems do not have motion integrals because they are non-isolated. All elements of the Gibbs assembly must have equal values of parameters that are determined by the equilibrium conditions. In usual thermodynamics this parameters are the temperature and the chemical potential. The physical interpretation of these parameters is getting by statistical mechanics. A rotating system can be in equilibrium only with rotating environment. The equilibrium condition in this case is apparent. That is equality of the both angular velocities of the system and of the environment. The Gibbs assembly density of distribution and thermodynamical functions in the case of a rotating classical system will be obtained in the second section of this work. It was done (Landau, & Lifshitz E.M., 1980a) but an object, to which this distribution is applied, is incomprehensible, because an angular velocity of an equilibrium gas has not been determined.

In quantum statistical mechanics the invariant set is the linear manifold of the microscopic states of the system in which the commutative operators that correspond to the controllable motion integrals have fixed eigenvalues. The phase volume of system in this case is the dimension of the manifold, if this dimension is limited. It directly determines the number of distinguishable microstates of the system that are accessible and equiprobable. The role of the angular momentum conservation in quantum statistical mechanics is similar to one in classical statistical mechanics. The method of computing this phase volume will be also proposed in the second section of this work. The Gibbs assembly density of distribution and thermodynamical functions in the case of a rotating quantum system also will be obtained.

In the third section of this work statistical mechanics of an electron gas in a magnetic field is considered. This question was investigated by many during the last century. Many hundreds experimental and theoretical works were summarized in the treatises (Lifshits, I.M. et al., 1973; Shoenberg, 1984). However, together with successful theoretical explanations of many experimental effects some paradoxes and discrepancies with observed facts remain unaccounted.

“Finally, it is shown that the presence of free electrons, contrary to the generally adopted opinion, will not give rise to any magnetic properties of the metals”. This sentence ends a short report on the presentation “Electron Theory of Metals” by N. Bohr, given at the meeting of the Philosophical Society at Cambridge. It was well-known that a charged particle in a uniform magnetic field moves in a circular orbit with fixed centre in such a way

that the time average value of the magnetic moment, generated by this motion, is directed opposite to the magnetic field and equal to the derivative of the kinetic energy with respect to the magnetic field. N. Bohr computed the magnetic moment of an electron gas by statistical mechanics with the density of distribution that is determined only by a Hamiltonian. Zero result of this theory (Bohr – van Leeuwen theorem) is the first paradox. Many attempts of derivation and explanation of this were summarized in the treatise (van Vleck, 1965). The most widespread explanation was that the magnetization generated by the electrons moving far from the bound is cancelled by the near-boundary electrons that reflect from the bound. But this explanation is not correct because, when formulae are derived in statistical mechanics, any peculiarities of the near-boundary states shall not be taken into account. Another paradox of the common theory went unnoticed. It is well known that a uniform magnetic field restricts an expanse of a charged particles gas in the plane perpendicular to the field. But from common statistical mechanics it follows that the gas uniformly fills all of the bounded area. The diamagnetism of some metals also was left non-explained.

L.D. Landau (Landau, 1930) explained the diamagnetism of metals as a quantum effect. He solved the quantum problem of an electron in a uniform magnetic field. The cross-section of the envelope perpendicular to the magnetic field is a rectangle with the sides $2L_x$ and $2L_y$. The solutions are determined by three motion integrals. The first is energy that takes the values $\varepsilon_{np} = \hbar\omega_c(n + 1/2) + p^2/2m$, where ω_c is the cyclotron frequency $\omega_c = eH/m$, $(-e)$ is the charge and m is the mass of an electron, H is the magnetic induction, n is a positive integer or zero. The second is the z -component of the momentum p . The third motion integral is the Cartesian coordinate of the centre of the classical orbit. It takes the values $y_j = (\pi\hbar/eHL_x)j$, where $j = 0, \pm 1, \pm 2, \dots \pm eHL_xL_y/\pi\hbar$. The thermodynamical potential with this energy spectrum, when the spin degeneracy is taken into account, is:

$$\Omega = -k_B T \int dp \frac{L_z}{2\pi\hbar} \sum_{n=0}^{\infty} \frac{eHS}{\pi\hbar} \ln \left[1 + \exp \left(\frac{\mu - \varepsilon_{np}}{k_B T} \right) \right] \quad (2)$$

Here k_B is the Boltzmann constant, T is the temperature, $S = 4L_xL_y$, μ is the chemical potential. If in this formula the summation over n is changed to the integration, the result Ω_0 does not depend on H , and the magnetic moment $\mathfrak{M} = -(\partial\Omega_0/\partial H) = 0$. That agrees to the classical and paradoxical Bohr – van Leeuwen theorem. L.D. Landau uses the Euler – Maclaurin summation formula in the first order and obtains the amendment that depends on the magnetic field. In the limit $T \rightarrow 0$ the thermodynamical potential has appearance (Abrikosov, 1972):

$$\Omega = \Omega_0 + V \frac{e^2 H^2 p_F}{24\pi^2 \hbar m}, \quad (3)$$

where $p_F = \sqrt{2m\zeta} = (3\pi^2)^{1/3} (N/V)^{1/3} \hbar$, and ζ is the Fermi energy, $V = 4L_xL_yL_z$ is the volume. This result cannot be correct because the magnetic moment does not depend on the Plank constant \hbar and thus it cannot be a quantum effect. This problem is simpler for a two-dimensional gas. In this case the common formula of the thermodynamical potential has the form:

$$\Omega_{2D} = -k_B T \sum_{n=0}^{\infty} \frac{eHS}{\pi \hbar} \ln \left[1 + \exp \left(\frac{\mu - \varepsilon_n}{k_B T} \right) \right]. \quad (4)$$

When $T \rightarrow 0$, this sum can be computed without to change the summation to the integration.

$$\Omega_{2D} = -\frac{\zeta^2 m S}{2\pi \hbar^2} + \frac{e^2 H^2 S}{8\pi m}. \quad (5)$$

It is suggested that the Fermi level is filled. In this case the magnetic moment does not depend not only on the Plank constant, but also on the number of the electrons. Therefore the fundamental formula for the thermodynamical potential is incorrect.

In the third section of this work the diamagnetism of an electron gas is investigated with taking into account the conservation of zero value of the total angular momentum in classical and quantum statistical mechanics. The paradoxes described above are eliminated; however many other theories should be reconsidered.

2. Statistical mechanics of rotating gas

For the computation of average values of macroscopic quantities it is necessary to derive a formula of the phase volume as a function of macroscopic parameters. This function is called “structural function” by Khinchin (Khinchin, 1949) and “number of accessible states (or complexions)” by Fowler (Fowler, & Guggenheim, 1939). It determines the normalizing factor in the probability density of the microcanonical distribution (1). In usual theory this function is essential to the derivation of formulae that connect statistical physics with thermodynamics.

The system that will be considered is a collection of N structureless particles. If forces of interaction between particles manifest themselves only at distances considerably smaller than the average distance between particles, the interaction energy of particles is essential only in a small fraction of the phase volume. Therefore, the interaction of particles can be neglected or be taken into account as a perturbation in calculating the phase volume and the average values (Uhlenbeck & Ford, 1963). Otherwise, if particles interact by a long-range force, this interaction needs to be considered using a mean field method. This is a model of an ideal gas under an external field. Meanwhile, this external field can be also a periodical crystal field. In the commonly considered cases the Hamiltonian and other phase functions of the system can be presented as the sum of identical terms, each of which depends on the coordinates and momenta of a single particle. Such phase function is said to be a summatory function.

For integration characteristic functions over a phase space the method by Krutkov will be used. The main idea of this method is to make the Laplace transformation of the δ – function with respect the value of the summatory function. Then the product of the N like exponents from the terms of the summatory function would be integrated over variables of the phase space. The inverse transformation would be made by the saddle-point method with using the large parameter N .

Let us write several equalities with a characteristic function. If the system can be divided into two independent subsystems described by non-overlapping groups of phase variables, so that $\Sigma = \Sigma_1 \otimes \Sigma_2$, $\mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2$, and the determining functions possess the values independently, then

$$\Omega_{\Sigma} = \Omega_{\Sigma_1} \cdot \Omega_{\Sigma_2}, \quad \varphi_{\Sigma}(\mathbf{R}) = \varphi_{\Sigma_1}(\mathbf{R}_1) \cdot \varphi_{\Sigma_2}(\mathbf{R}_2),$$

$$dP(\mathbf{R}) = dP_1(\mathbf{R}_1) \cdot dP_2(\mathbf{R}_2) = \left[\varphi_{\Sigma_1}(\mathbf{R}_1) / \Omega_1 (2\pi\hbar)^{3N_1} \right] \cdot \left[\varphi_{\Sigma_2}(\mathbf{R}_2) / \Omega_2 (2\pi\hbar)^{3N_2} \right] d\Gamma_1 d\Gamma_2 \quad (6)$$

Here the multiplier $(N!)^{-1}$ is not taken into account because it cannot be introduced logically in classical statistical mechanics. Considering the fact that a density of distribution for a system is equal to the product of densities of distribution for subsystems, a conclusion is drawn in the treatise (Landau, & Lifshitz, E.M., 1980a) that the logarithm of the density of distribution should be an additive motion integral and, hence, it should be a linear combination of the additive controllable motion integrals, such as the energy, the momentum and the angular momentum. However, as it follows from the formula (6) this is incorrect for the microcanonical distribution, since the logarithm of the characteristic function is meaningless. A system in a thermostat does not have any motion integrals. If the invariant set is determined by some conservation laws, its characteristic function is

$$\varphi_{\Sigma}(\mathbf{R}) = \prod_i \varphi_i(\mathbf{R}), \quad (7)$$

where $\varphi_i(\mathbf{R})$ is the characteristic function that is determined by the conservation law number i . Let us denote a set, at which the phase function $A(\mathbf{R})$ is equal to a , by Σ_A^a , its characteristic function by $\varphi_A^a(\mathbf{R})$, and its measure by Ω_A^a . It is supposed that this measure is limited and is not equal to zero. Then

$$\int_{\Gamma} \varphi_A^a(\mathbf{R}) d\Gamma = \Omega_A^a, \quad \int_{\Gamma} \varphi_A^a(\mathbf{R}) f(\mathbf{R}, A(\mathbf{R})) d\Gamma = \int_{\Gamma} \varphi_A^a(\mathbf{R}) f(\mathbf{R}, a) d\Gamma, \quad (8)$$

$$\int_A \int_{\Gamma} \varphi_A^a(\mathbf{R}) f(a, \mathbf{R}) d\Gamma da = \int_A \int_{\Gamma} \varphi_A^a(\mathbf{R}) f(A(\mathbf{R}), \mathbf{R}) d\Gamma da = \int_{\Gamma} f(A(\mathbf{R}), \mathbf{R}) d\Gamma$$

Here A is the range of values of the function $A(\mathbf{R})$. The prevalent formula $\varphi_A^a = \Delta a \delta(A - a)$ (Landau, & Lifshitz, E.M., 1980a; Uhlenbeck, & Ford, 1963) satisfies to the equalities (7) and (8) but does not satisfy to the equality (6). It is more frequently considered the separation on subsystems that conserves the total value of the function. If $A(\mathbf{R}) = A_1(\mathbf{R}_1) + A_2(\mathbf{R}_2) = a$, then $\varphi_A^a = \int_A \varphi_{A_1}^{a-x} \varphi_{A_2}^x dx$, and the prevalent formula is correct.

2.1 Classical statistical thermodynamics of rotating gas

The formula for average values, when the conservation of the angular momentum is taken into account, has the form:

$$\bar{F} = \left[(2\pi\hbar)^{3N} N! \right]^{-1} \frac{\Delta E \Delta \bar{L}}{\Omega} \int_{\Gamma} F(\mathbf{R}) \delta(H(\mathbf{p}, \mathbf{r}) - E) \delta(L(\mathbf{p}, \mathbf{r}) - \bar{L}) d\Gamma \quad (9)$$

$$\Omega = \Delta E \Delta \bar{L} \int_{\Gamma} \delta(H(\mathbf{p}, \mathbf{r}) - E) \delta(L(\mathbf{p}, \mathbf{r}) - \bar{L}) d\Gamma$$

In these formulae the axis \mathbf{Z} is parallel to the angular momentum $\bar{\mathbf{L}}$. The angular momentum of the gas is $L(\mathbf{p}, \mathbf{r}) = \sum_{i=1}^N l(\mathbf{p}_i, \mathbf{r}_i)$. The effective Hamiltonian of the gas with the fixed angular momentum in the cylindrical coordinates can be obtained from the usual formula by substitution $l_1 = \bar{L} - \sum_{i=2}^N l_i$ with reduction of the quadratic form to the standard appearance. It is:

$$\mathcal{H}(\mathbf{p}, \mathbf{r}) = \frac{1}{2m} \sum_{i=2}^N \left(p_{z,i}^2 + p_{r,i}^2 + \frac{l_i^2}{r_i^2} + U(\sqrt{r_i^2 + z_i^2}) \right) + \frac{\bar{L}^2}{2m \sum_{i=1}^N r_i^2}, \quad (10)$$

where $U(\sqrt{r^2 + z^2})$ is potential energy that confines the particles in bounded volume, and the second term of the Hamiltonian is the potential energy of the centrifugal force that leads to a collapse of rotating nebula into disk. This Hamiltonian is not a summatory function. Therefore the Krutkov's method cannot be used for the subsequent computations.

Let us consider equilibrium of a gas with a rotating rigid body. The rigid body can be determined as the body in which the rotatory degree of freedom can not transfer energy and angular momentum to the internal degrees of freedom. This possibility arises when this body is a cylindrical rotating envelope with non-ideal surface filled by a gas. The state of the gas is characterized by two parameters: the temperature T and the angular velocity ω . For introducing the statistical parameter that corresponds to the thermodynamical temperature it is necessary to deduce the canonical Gibbs distribution for a system that is in equilibrium with a thermostat. That can be done, for example, by a method developed by Krutkov (Krutkov, 1933; Zubarev, 1974). The conditions of the equilibrium between the rotating envelope and the gas are apparent. Those are the equalities of the temperature and the angular velocity. Let us determine the angular velocity of a gas. An angular velocity of a particle is a stochastic quantity with an average value $\bar{\omega}$. The sum $\sum_{i=1}^N \omega_i$ is the Gaussian random variable with the average value $N\bar{\omega} = \omega_g$. That is the angular velocity of a gas. The conditions of the equilibrium between the rotating envelope and the gas are the equality of the temperatures and

$$\omega_g = \omega. \quad (11)$$

The total system can be considered as motionless if it will be described in the rotating reference frame, when the right part of the equality (11) is zero. The hollow cylinder is the envelope, the thermostat, and it keeps the gas spin. It should be named "termospinostat". The potential energy of the centrifugal force $U_{cf} = (-m\omega^2 r^2/2)$ is added in the Hamiltonian of the gas particle in the rotating reference frame (Landau, & Lifshitz, E.M., 1980a). The average angular momentum of the gas $m\omega \overline{\sum_{i=1}^N r_i^2} = \omega N m \bar{r}_i^2 = \bar{L}$ depends on the angular velocity nonlinearly because the gas moment of inertia $\bar{I}(\omega) = N m \bar{r}_i^2$ is the function of the angular velocity. This function can be obtained from the Gibbs distribution for a gas in the system of reference that rotates with the angular velocity ω :

$$\begin{aligned} dP_G(\mathbf{R}) &= \frac{d\Gamma}{(2\pi\hbar)^{3N} N! Z_N} \exp \left[\frac{1}{k_B T} \left(-\frac{1}{2m} \sum_{i=1}^N \left(p_{z,i}^2 + p_{r,i}^2 + \frac{l_i^2}{r_i^2} \right) + \frac{\omega^2 m}{2} \sum_{i=1}^N r_i^2 - U_0 \right) \right] \\ Z_N &= \left\{ (2\pi\hbar)^{3N} (N!) \right\}^{-1} \int_{\Gamma} \exp \left[\frac{1}{k_B T} \left(-\frac{1}{2m} \sum_{i=1}^N \left(p_{z,i}^2 + p_{r,i}^2 + \frac{l_i^2}{r_i^2} \right) + \frac{\omega^2 m}{2} \sum_{i=1}^N r_i^2 - U_0 \right) \right] d\Gamma = \quad (12) \\ &\left\{ (2\pi\hbar)^{3N} (N!) \right\}^{-1} \left\{ \frac{h_z}{\omega^2} \sqrt{(2\pi k_B T)^5 m} \left[1 - \exp \left(-\frac{\omega^2 m \mathcal{R}^2}{2k_B T} \right) \right] \right\}^N = \exp \left[-\frac{1}{k_B T} F \left(T, \frac{\omega^2}{2} \right) \right], \end{aligned}$$

where h_z and \mathcal{R} are the dimensions of the envelope, and $U_0 = Nm\mathcal{R}^2\omega^2/2$ is an appending constant that does the energy positive. Going to the thermodynamics (it rather can be entitled by “thermospindynamics”) it is naturally to consider $\sigma = \omega^2/2$ as an external parameter and the moment of inertia $I = m\sum_{i=1}^N r_i^2$ as a characteristic of the gas. Then

$$\bar{I} = Nm\bar{r}^2 = -\partial F/\partial \sigma. \quad (13)$$

The formulae of the isotopes separation (Cohen, 1951) can be obtained from the distribution (12). If $m(\omega\mathcal{R})^2 \ll k_B T$ the formula (12) can be presented as:

$$F \approx F_0 + \frac{1}{2}N\sigma m\mathcal{R}^2 - \frac{N}{24} \frac{\sigma^2 m^2 \mathcal{R}^4}{k_B T} = F_0 + \bar{I}_0 \sigma - \frac{N}{24} \frac{\sigma^2 m^2 \mathcal{R}^4}{k_B T} \quad (14)$$

$$F_0 = -Nk_B T \ln \left[\frac{e\pi h_z \mathcal{R}^2}{N} \left(\frac{mk_B T}{2\pi\hbar^2} \right)^{3/2} \right], \quad \bar{I}_0 = \frac{Nm\mathcal{R}^2}{2},$$

where F_0 - is the free energy of the ideal gas that does not rotate. Hence it follows that the parameters h_z , \mathcal{R}^2 , and correspondingly P_z , P_s should be introduced instead of the volume V and the pressure P . Other thermodynamical equations are changed also. The parameter of expansion in the formula (14) can be of the order of unity when $m = 10^{-25} \text{ kg}$, $\mathcal{R} = 1 \text{ m}$, $T = 100 \text{ K}$, $\omega = 100 \text{ s}^{-1}$.

2.2 Quantum statistical thermodynamics of rotating gas

The characteristic function of the invariant set that takes into account conservation of the angular momentum in quantum statistical mechanics can be presented as a set of diagonal elements of the operator:

$$\hat{\phi}(E, \bar{L}) = (2\pi)^{-2} \int_0^{2\pi} \exp \left[(i\tau + \mathcal{G})(\hat{\mathcal{H}} - E) \right] d\tau \int_0^{2\pi} \exp \left[(i\alpha + \eta)(\hat{L} - \bar{L}) \right] d\alpha \quad (15)$$

in the space of microstates of the system. Here $\hat{\mathcal{H}} = \sum_{i=1}^N \hat{h}_i$ is Hamiltonian of gas; $\hat{h}\hat{L} = \hbar \sum_{i=1}^N \hat{l}_i$ is the operator of the total angular momentum of gas; E , $\hbar\bar{L}$ are values of these quantities for the considered macroscopic state; \mathcal{G} , η are real numbers which will be defined below. As usually, let us assume that energies of one-particle states, and, hence, both eigenvalues of the operator $\hat{\mathcal{H}}$ and gas energy E , are expressed by the dimensionless positive integers. This formula would be generalized by the transition to representation of secondary quantization. In this representation function of one-particle states are eigenfunctions of the one-particle Hamiltonian \hat{h}_i and angular momentum. These functions are numbered using index Ψ which consists of a pair of quantum numbers (t, l) , where $\varepsilon(t)$ is energy and $\hbar l(\Psi)$ is the angular momentum at the state Ψ . Let us suppose that only two quantum numbers determine the state. Both the production and annihilation operators are determined by the kind of statistics. The operator (15) should be replenished

by multiplier that describes the conservation of the particle number. Then the measure of the invariant set for N -particle system is:

$$C(N, E, \bar{L}; \chi, \vartheta, \eta) = (2\pi)^{-3} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \exp[-N(i\phi + \chi) - E(i\tau + \vartheta) - \bar{L}(i\alpha + \eta)] \times \\ \times \text{Sp} \left\{ \exp \left[\sum_{\Psi} [((i\phi + \chi) + (i\tau + \vartheta))\varepsilon(\Psi) + (i\alpha + \eta)l_z(\Psi)] \begin{Bmatrix} a_{\Psi}^+ a_{\Psi} \\ b_{\Psi}^+ b_{\Psi} \end{Bmatrix} \right] \right\} d\phi d\tau d\alpha = \quad (16) \\ (2\pi)^{-3} \oint \oint \oint x^{-N-1} y^{-E-1} z^{-\bar{L}-1} \prod_{\Psi} (1 \pm xy^{\varepsilon(\Psi)} z^{l(\Psi)})^{\pm 1} dx dy dz .$$

Here the following variables are entered:

$$x = \exp(i\phi + \chi), \quad y = \exp(i\tau + \vartheta), \quad z = \exp(i\alpha + \eta). \quad (17)$$

Thus, integrals are rearranged into integrals along contours which enclose the origin of coordinates. If $\bar{L} \neq 0$ then the axis Z is parallel to the angular momentum \bar{L} and $\bar{L} > 0, \eta > 0$. The lower operator or sign should be taken for the Bose statistics, and upper ones should be taken for the Fermi statistics. The expression of the measure of the invariant set in the case of quantum statistics is obtained from the apparent formula of the characteristic function (15). This expression is similar to the initial one in the Darwin - Fowler method (Fowler, & Guggenheim, 1939), where it was substantiated as a mathematical device. These contour integrals can be computed by the saddle-point method, when $\bar{L} \neq 0$. The saddle-points determine the values of χ, ϑ, η . It can be shown that taking into account the conservation of the angular momentum does not change statistic mechanics of this model of an ideal gas when $\bar{L} = 0$.

Let us describe a quantum gas in a termosinstat with the temperature T and the angular velocity ω . The potential of the centrifugal force would be introduced in the Hamiltonian only in classical statistical mechanics (Landau, & Lifshitz, E.M., 1980a). Other method should be used for quantum theory. Wave functions in a system of reference that rotates with the angular velocity ω depend on the time, and should be determined from the Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m} \left(\hat{p}_z^2 + \hat{p}_r^2 + \frac{\hbar^2 \hat{l}^2}{r^2} \right) \Psi. \quad (18)$$

Dependence of the wave function on the time should be $\psi = \exp\{i[zp_z/\hbar + l(\varphi - \omega t)]\}$.

Then $\psi = \exp\{i[p_z/\hbar + l(\varphi - \omega t)]\} J_{|l|} \left(r \sqrt{2m(\varepsilon - \hbar\omega l) - p_z^2/\hbar} \right)$ and $J_{|l|}(x)$ is the Bessel function.

The boundary condition $\psi(\mathcal{R}) = 0$ determines the energy spectrum:

$$\varepsilon_{lv} = \frac{p_z^2}{2m} + \frac{\hbar^2}{2m\mathcal{R}^2} j^2(|l|, \nu) - \hbar\omega l, \quad (19)$$

where $j(|l|, \nu)$ is the null of the Bessel function $J_{|l|}(x)$ with number in the order of increasing ν . When $\nu > 1$, $j(|l|, \nu) \approx \pi(\nu + |l|/2)$ and $j(|l|, 1) \approx |l|$. This spectrum is quasicontinuous

because a distance between nearest-neighbor levels is proportional to \mathcal{R}^{-2} . The lowest level has value $(-m\omega^2\mathcal{R}^2/2)$ when $l = (m\mathcal{R}^2\omega/\hbar)$. Then the reference point of energy should be altered by this value. It conforms to the appearance of the centrifugal force potential in the classical system. Energies of states with $l > 0$ are lower than the ones of states with equal $|l|$ and $l < 0$. Then in the gas the part of particles with $l > 0$ should be more than half, and as result a circular current of the probability density should exist. This describes rotating of the system. With increasing argument modulus of extremes of the Bessel functions decrease. If the values of energy ε and positive angular momentum are fixed the value of the null number ν in the rotating system should be lower than this value in the motionless system. Therefore, the gas density increases with distance from the axis in rotating system.

Let us compute the thermodynamical potential $\Omega = \pm k_B T \sum_{pvl} \ln \left\{ 1 \mp \exp \left[(\mu - \varepsilon_{pvl}) / k_B T \right] \right\}$ of ideal rotating gas when $\exp(\mu/k_B T) \ll 1$. Nulls of Bessel functions would be approximated by formula $j(|l|, \nu) \approx \pi(\nu + |l|/2)$, but when $\nu = 1$ then $j(|l|, 1) \approx |l|$. The computation is performed by passing from summation over ν and l to integration over j and l . This approximation for non-rotating gas leads to the result that differs from the common result by the multiplier $(4/\pi)$. The result of computation for the rotating gas is:

$$\Omega = -V \frac{4(k_B T)^{5/2} m^{3/2}}{\pi (2\pi)^{3/2} \hbar^3} \exp\left(\frac{\mu}{k_B T}\right) \left\{ 1 - \frac{\omega^2 \mathcal{R}^2}{3} \left(\frac{m}{2k_B T}\right) + \frac{\omega^4 \mathcal{R}^4}{10} \left(\frac{m}{2k_B T}\right)^2 \right\}. \quad (20)$$

If this result is compared with that of Eq. (14), it can be shown that amendments differ only by coefficients.

3. Statistical mechanics of electron gas in magnetic field

The review of the current status of this theory is in the paper (Vagner et al., 2006). There are some inaccuracies in this problem consideration besides disregard of the angular momentum conservation. To clarify the problem, in the first subsection we consider formulations of the one-particle problem in classical and quantum mechanics and its simplest application to the statistical mechanics. For simplicity, we will restrict ourselves to the case of a two-dimensional gas on a plane perpendicular to the uniform magnetic field $\mathbf{H} = (0, 0, H)$. As will be shown the magnetization of electron gas is nonuniform. We will suppose that the magnetization is small as against the uniform field, and will not regard effect of it. Then magnetic induction H is proportional to the external magnetic field strength by the coefficient μ_0 . Where it is needed, we imply the plane to be of finite “thickness” δz , and, for example, the equations of electrodynamics are written for three-dimensional space.

3.1 Two-dimensional electron ideal gas in uniform magnetic field

This problem traditionally is considered in quasiclassical theory (Lifshitz, I.M. et al., 1973; Shoenberg, 1984). Some corrections will be inserted in this consideration in the section 3.1.1. In this section classical statistic mechanics of ideal gas will be discussed. In the next section the new correction will be obtained from the consistent quantum theory.

3.1.1 Classical statistical mechanics of ideal gas in magnetic field

The Hamiltonian of an electron in a magnetic field has the form:

$$\hbar = (1/2m)(\mathbf{p} + e\mathbf{A})^2 \quad (21)$$

where m and $(-e)$ are the mass and the charge of an electron, \mathbf{p} is momentum, and \mathbf{A} is vector potential of the magnetic field:

$$\mathbf{A} = \frac{1}{2}[\mathbf{H}\mathbf{r}] = \frac{1}{2}(-yH, xH, 0). \quad (22)$$

This Hamiltonian does not have the translation symmetry. This symmetry, seemingly, should be, if the magnetic field is uniform at an unlimited plane. But a uniform magnetic field at unlimited plane is impossible because an electrical current that generates it according to Maxwell equation should envelope a part of this plane. It is asserted (Landau, & Lifshitz, E.M. 1980b; Vagner et al., 2006) that the Hamiltonian (21) with the vector potential (22) would be converted by gauge transformation $\mathbf{A} \rightarrow \mathbf{A} + \nabla f(x, y, z)$. If the function $f = -eHxy/2$, then Hamiltonian will be in the Landau form :

$$\hbar_L = (1/2m)[(p_x - eHy)^2 + p_y^2], \quad (23)$$

and will have the translation symmetry in the direction of the axis X in return for axial symmetry. That is strange assertion because the symmetry is the physical property of the system rather than of a method of its description. In fact the transformation to the Hamiltonian (23) in classical mechanics is result of the canonical transformation of the Hamilton variables with the generating function $\Phi = p'_x x + p'_y y - eHxy/2$. Then $p_x = p'_x - (1/2)eHy$, $p_y = p'_y - (1/2)eHx$, $x' = x$, $y' = y$. Therefore the p_x, p_y (in fact p'_x, p'_y) in the Landau Hamiltonian (23) are not the momentum components in the Cartesian coordinates and the absence of the x -coordinate in this Hamiltonian does not lead to the momentum x -component conservation. In quantum mechanics the unitary transformation with operator $\exp[-ieHxy/2\hbar]$ is equivalent to this canonical transformation. The boundary is created by the line of intersection of the plane with a solenoid that generates the magnetic field. Electrons, orbits of which transverse this boundary, will be extruded from the area, and the gas will evaporate. The more realistic problem is gas in the area with a reflecting boundary.

An isolated electron has three motion integrals. Those are the angular momentum relative to the centre of area and two coordinates of the centre electron orbit:

$$l_z = xp_y - yp_x, \quad X = -\frac{p_y}{eH} + \frac{x}{2}, \quad Y = \frac{p_x}{eH} + \frac{y}{2} \quad (24)$$

Two motion integrals that have the physical importance would be created from it: energy E and squared centre electron orbit distance from the centre of area R^2 :

$$R^2(\mathbf{p}, \mathbf{r}) = X^2 + Y^2 = \frac{1}{e^2 H^2} \left[\left(p_x + \frac{eHy}{2} \right)^2 + \left(p_y - \frac{eHx}{2} \right)^2 \right];$$

$$\varepsilon(\mathbf{p}, \mathbf{r}) = \frac{e^2 H^2}{2m} \left(R^2 + \frac{2}{eH} l_z \right) = \frac{e^2 H^2}{2m} \rho^2 = \hbar(\mathbf{p}, \mathbf{r}). \quad (25)$$

Here ρ is the radius of the electron orbit. The motion integral R^2 is proportional to the Hamiltonian with opposite direction of the magnetic field. The values of R^2 and ε should be discrete by the rules of the quasiclassical quantization:

$$R_k^2 = 2\lambda^2 \left(k + \frac{1}{2} \right), \quad \varepsilon_n = \hbar\omega_c \left(n + \frac{1}{2} \right); \quad \lambda = \sqrt{\frac{\hbar}{eH}} = \sqrt{\frac{\hbar}{m\omega_c}}, \quad \omega_c = \frac{eH}{m}. \quad (26)$$

Here λ is magnetic length and ω_c is cyclotron frequency. Every energy level E_n is degenerated because the integer k that determines the position of the orbit can take any values from zero to k_1 , where k_1 is determined by the condition that electron orbit does not have common points with the boundary. That condition would be expressed by formula when the boundary is a circle with radius \mathcal{R} . Then $R_{k_{\max}} + \rho_n < \mathcal{R}$ or $\lambda(\sqrt{2k_1+1} + \sqrt{2n+1}) < \mathcal{R}$. Those are the “ordinary” states. When $k_1 < k < k_2$, where $\lambda(\sqrt{2k_2+1} - \sqrt{2n+1}) < \mathcal{R}$, the states are nominated “near-boundary state”. Their energies are not described by the formula (26). The instant magnetic moment is determined by the formula:

$$\mu_z = -\frac{e}{2} (xv_y - yv_x) = -\frac{\partial}{\partial H} \hbar(\mathbf{p}, \mathbf{r}) \quad (27)$$

The first determination is valid for any negative charged particle with and without an external magnetic field. The second equality is valid when the vector potential has the form (22), or when the equality $v_i = \partial \hbar / \partial p'_i$ is transformed by canonical transformation. For the ordinary states the averaged over the orbit magnetic moment is $\mu_z = -\varepsilon_n / H$. The trajectories of the near-boundary states are composed from arcs and envelop the all area. Their magnetic moment is positive. If in the area exists a potential field $U(r)$ the orbit centers of the ordinary states also move along equipotential lines and the energy values depend on k . The degeneration of the energy levels goes off. The angular momentum does not represent the electron motion, but rather the position of the electron orbit. From Eqs. (25-26) it follows that $l_z = (eH/2)(\rho^2 - R^2) = \hbar(n - k)$. Then $l_z > 0$ if the orbit envelops the centre, and $l_z < 0$ if the area centre locates out the orbit. If $\rho_n \ll \mathcal{R}$ a large share of the states with energy ε_n have angular momentum $l_z < 0$.

Going to consideration of the ideal gas with electron-electron collisions, let us suppose that the interaction does by a central force. Then total energy and angular momentum are conserved. It is generally believed that the area is filled by uniform and motionless positive charged background that neutralizes the electrostatic interaction. This assumption is inconsistently. If electron gas is in equilibrium with motionless background, its angular

momentum should be equal to zero. But then it should be nonuniform as is evident from the foregoing consideration. It should be regarded more comprehensively. Let us go to classical statistic mechanics for a gas of charged particles in magnetic field. The characteristic function of the total system (gas and background) is:

$$\varphi(\mathbf{R}_g + \mathbf{R}_b) = \delta(\mathcal{H}_g(\mathbf{R}_g) + \mathcal{H}_b(\mathbf{R}_b) - \varepsilon) \delta(L_g(\mathbf{R}_g)) \delta(L_b(\mathbf{R}_b)) \quad (28)$$

Here the indexes g and b denote the quantities those relating to the gas and to the background, \mathcal{H} is a Hamiltonian, ε is the value of the total energy. To provide of Gibbs distribution for the gas it is needed to integrate the function $\varphi(\mathbf{R}_g + \mathbf{R}_b)$ over the phase space of the background. Then $\delta(L_g(\mathbf{R}_g))$ is factored out from integral. The function from $\mathcal{H}_g(\mathbf{R}_g)$ can be factored from integral by the method Krutkov (Krutkov, 1933; Zubarev, 1974) as $\exp(-\mathcal{H}_g(\mathbf{R}_g)/k_B T)$. Then:

$$Z_N = \frac{1}{(2\pi\hbar)^{3N} N!} \int_{\Gamma_g} \delta(L_g(\mathbf{R}_g)) \exp(-\mathcal{H}_g(\mathbf{R}_g)/k_B T) d\Gamma_g. \quad (29)$$

The Hamiltonian $\mathcal{H}_g(\mathbf{R}_g)$ would be represented in the form:

$$\mathcal{H}_g(\mathbf{R}_g) = \sum_{i=1}^N \mathcal{H}_i = \sum_{i=1}^N \left(\frac{1}{2m} \left(p_{ri}^2 + \frac{l_{zi}^2}{r_i^2} \right) + \frac{e^2 H^2 r_i^2}{8m} \right) + \frac{eH}{2m} \sum_{i=1}^N l_{zi}. \quad (30)$$

Let us substitute Hamiltonian (30) to the formula (29) and take into account formula $\sum_{i=1}^N l_{zi} = L_g$ and the Eqs. (8). Then it is obtained:

$$Z_N = \frac{1}{(2\pi\hbar)^{3N} N!} \int_{\Gamma_g} \exp \left\{ -\frac{1}{k_B T} \sum_{i=1}^N \left(\frac{1}{2m} \left(p_{ri}^2 + \frac{l_{zi}^2}{r_i^2} \right) + \frac{e^2 H^2 r_i^2}{8m} \right) \right\} \delta(L_g) d\Gamma_g \quad (31)$$

The integration over l_{z1} leads to change $l_{z1} \rightarrow -\sum_{i=2}^N l_{zi}$ and after reduction of the quadratic form to the standard appearance the Hamiltonian of ideal gas that is collection of $N-1$ particles in harmonic potential field is obtained. Obviously, that taking into account conservation of the zero value of the angular momentum eliminates the paradoxes that were mentioned at the Introduction. The magnetic moment $\mu_z = (-\partial/\partial H)(-k_B T \ln Z_N) \neq 0$. The gas is confined by the magnetic field. But that confinement provides to the inconsistency of the model that neglects of the electrostatic interaction because the uniform background cannot neutralize it. The model that regards this interaction will be considered below.

3.1.2 Quantum problem of electron in magnetic field at bounded area

The quasiclassical description of an electron in a magnetic field would not give the correct picture of the probability density distribution and the current density. It would not also describe the alternation of the energy spectrum when a perturbation does the classical

motion nonperiodical. But that problem in quantum mechanics also is considered insufficiently. As suggested in the paper (Vagner et al., 2006) the density of the probability current of the wave function $\psi(\mathbf{r}) = \exp[i\Phi(\mathbf{r})]|\psi|$ is

$$\mathbf{j} = \frac{\hbar}{m} |\psi|^2 \left(\nabla \Phi + \frac{e}{\hbar} \mathbf{A} \right). \quad (32)$$

The eigenfunctions $\psi_n(\mathbf{r})$ cannot be chosen so that $\mathbf{j} = 0$ because any vector potential \mathbf{A} cannot be equal to a gradient of a continuous function. Then it is necessary for stationarity of the states that the current lines to be closed in the area under consideration. The boundary condition best suited to the research of this problem is null of the function on the circumference that bounds the area:

$$\psi(\mathcal{R}, \varphi) \equiv 0 \quad (33)$$

This condition retains the greatest possible symmetry. The current lines in this case are concentric circumferences. The density of the current can be zero only at separate circumferences. Therefore the magnetization is nonuniform. The magnitudes of the eigenfunctions should have the axial symmetry. The localization of the electron cannot coincide with any classical orbit because the uncertainties of values of the orbit centre coordinates (24) should satisfy to Heisenberg uncertainty relation.

The Hamiltonian is as follows:

$$\begin{aligned} \hat{h} &= \frac{1}{2m} \left[\left(\hat{p}_x - \frac{eHy}{2} \right)^2 + \left(\hat{p}_y + \frac{eHx}{2} \right)^2 \right] + U(r) = \hat{h}_0 + \frac{\omega_c}{2} \hat{l}_z, \\ \hat{h}_0 &= \frac{1}{2m} \left(\hat{p}_r^2 + \frac{\hat{l}_z^2}{r^2} \right) + \frac{m\omega_c^2 r^2}{8} + U(r). \end{aligned} \quad (34)$$

The operators \hat{h}_0 and \hat{l}_z commute with each other and each of them with the Hamiltonian \hat{h} . The potential energy $U(r)$ is created by the interaction with other electrons and with a neutralizing background. The nonuniform magnetization $\mathbf{M}(r)$ should be neglected. If the potential energy $U(r)$ also will be neglected the eigenfunctions of the Hamiltonians \hat{h} and \hat{h}_0 will have the form:

$$\begin{aligned} \Psi &= \exp(il\varphi) \psi_{|l|\alpha}(r); \quad l = 0, \pm 1, \pm 2, \dots \\ \psi_{|l|\alpha}(r) &= \frac{A}{\lambda} \left(\frac{r}{\lambda\sqrt{2}} \right)^{|l|} \exp\left(-\frac{r^2}{4\lambda^2}\right) \Phi\left(\alpha, |l| + 1; \frac{r^2}{2\lambda^2}\right). \end{aligned} \quad (35)$$

Here $\hbar l$ is eigenvalue of the operator of the angular momentum, $\Phi(a, c; x)$ is the degenerate hypergeometric function, A is normalizing factor that depends on $|l|$ and α . The eigenvalues of energy are expressed by α and l : for the Hamiltonian \hat{h}_0 that is $\varepsilon_0 = \hbar\omega_c [-\alpha + (|l| + 1)/2]$, and for \hat{h} that is $\varepsilon = \hbar\omega_c [-\alpha + (|l| + 1)/2] + \hbar\omega_c (l/2)$. The permissible values of α are determined by the boundary condition. In the common theory (Landau, & Lifshitz, E.M., 1980b; Vagner et al., 2006) that is normability of the eigenfunctions at an infinity plane. Then it

is necessary that $\alpha = -n_r$, where n_r is integer or zero. $\Phi(-n_r, |l| + 1; x) = [n_r! |l|! / (|l| + n_r)!] L_{n_r}^{|l|}(x)$, where $L_{n_r}^{|l|}(x)$ is the Laguerre polynomial. The energy spectrum of the operator \hat{h}_0 is $\varepsilon_0(n_0) = (\hbar\omega_c/2)(2n_r + |l| + 1) = (\hbar\omega_c n_0/2)$, where n_0 is integer. The energy spectrum of the operator \hat{h} is $\varepsilon(n) = (\hbar\omega_c/2)(2n_r + l + |l| + 1) = \hbar\omega_c(n + 1/2)$ where n is integer. There are two kinds of a degeneracy of energy levels of those Hamiltonians. The degeneracy of the first kind arise as result of the formulae: $n_0 = 2n_r + |l| + 1$, and $n = n_r + l$ when $l > 0$. Every level is degenerated with multiplicity that equals to its number. The perturbation $U(r)$ would be described as a power series without the linear term. Then the term that is proportional to r^2 in the spectrum of the Hamiltonian \hat{h}_0 should change the distance between levels without elimination of the degeneracy. In the spectrum of the Hamiltonian \hat{h} the level would be split. The other terms of potential field series eliminate the degeneracy in the first order of the perturbation theory. The second kind of the degeneracy is inherent only to the spectrum of the Hamiltonian \hat{h} in absence of any perturbation. Every level is degenerated with infinity multiplicity because when $l < 0$ the energy value does not depend on l . At a bounded area the multiplicity would be limited but very large and would depend on the magnetic field. That will be shown below. This degeneracy eliminates by any potential field. The modulo l would have many integer value, and when $\langle l|U|l \rangle \geq \hbar\omega_c$, if that's possible at some l , the spectrum would be quasicontinuous. That is essential because the explanations of many experimental effects rely on the discreteness of the Landau spectrum. In so doing ones do not study the model stability relative to the electrostatic interaction that is unavoidable perturbation. The other inconsistency of the common model is the large-scale negative total angular momentum of the ground state when all levels have the identical and large-scale multiplicity of degeneracy. Let us study the degeneracy eliminating by the boundary condition (33) in the absence of other perturbation. The polynomial $L_{n_r}^{|l|}(x)$ has n simple zeroes, which, if $n_r \gg 1$, are expressed by formula:

$$\zeta(n_r, |l|; i) \approx \frac{j^2(|l|, i)}{2|l| + 4n_r + 2}. \quad (36)$$

Here $j(|l|, i)$ is a null of the Bessel function $J_{|l|}(x)$ with number in the order of increasing i . Then the function $\psi_{|l|(-n_r)}$ has $(n_r + 1)$ extremes that are decrease modulo. The extreme number $(n_r + 1)$ has the sign $(-1)^{n_r}$, and further the function tends to zero asymptotically. Obviously, this function cannot satisfy to the boundary condition (33). When $\alpha = -n_r - \gamma$ the degenerate hypergeometric function has the form:

$$\begin{aligned} \Phi(-n_r - \gamma, |l| + 1; x) &= [(n_r)! |l|! / (|l| + n_r)!] L_{n_r}^{|l|}(\gamma; x) - \gamma A_{n_r}(\gamma, x) - (-1)^n \gamma(1 - \gamma) T_{n_r}(x), \\ A_{n_r}(\gamma, x) &= -(-x)^{n_r+1} [\Gamma(n_r + 1 + \gamma) \Gamma(|l| + 1)] [\Gamma(|l| + n_r + 2) \Gamma(1 + \gamma) (n_r + 1)!]^{-1}, \end{aligned} \quad (37)$$

where $L_{n_r}^{|l|}(\gamma; x)$ is the polynomial, the nulls of which are less than $\zeta(n_r, |l|; i)$ by quantities that are proportional to γ , and $T_{n_r}(x)$ is the infinite series that at $x \gg \zeta(n_r, |l|; n_r)$ is proportional to $x^{-(|l|+1+n_r)} \exp(x)$. Then the function (35) will have one more null at the large-

scale value x . This null X tends to infinity when γ tends to zero, and it would be shown that

$$\gamma \approx \frac{X^{2n_r+|l|+1}}{(|l|+n_r)!(n_r)!} \exp(-X), \quad (38)$$

when $\gamma \ll 1$. When $\gamma \rightarrow 1$, $X \rightarrow \zeta(n_r+1, |l|; n_r+1)$. Then the boundary condition (33) would be satisfied when $(\mathcal{R}^2/2\lambda^2) = X$. (In the mathematical handbook (Erdélyi, 1953) it is written that the function $\Phi(\alpha, |l|+1; x)$ at $\alpha = -n_r - \gamma$ has n_r+1 nulls, but all these nulls are determined by the formula that is like to the formula (36). Then the boundary condition would be satisfied at arbitrarily large values $(\mathcal{R}^2/2\lambda^2)$ only when energy has also large-scale value). The value of the null $\zeta(n_r+1, |l|; n_r+1)$ increases when the value of $|l|$ increases. The maximal value $|l_1|$, at which the inequality $(\mathcal{R}^2/2\lambda^2) \geq \zeta(n_r+1, |l_1|; n_r+1)$ is fulfilled, determines the number of the eigenstates of the Hamiltonian \hat{h} that have $\gamma \ll 1$ (ordinary states). It can be shown that when $|l_1| \gg n_r$ then $|l_1| \approx (\mathcal{R}^2/2\lambda^2) - (n_r+1)(\mathcal{R}/2\lambda)$. The quantum number of the energy n is equal to n_r because when $l < 0$ the energy depends on l only by $\gamma(n, l)$. This number of the ordinary states is consistent with the estimate that was obtained in classic mechanics theory in section 3.1.1 and in the work (Landau, 1930). Those ordinary states are quasi degenerated. The multiplicity of this degeneration is proportional to the magnetic field. Then the main term in the thermodynamical potential, as it was computed by Landau (Landau, 1930), should not depend on the magnetic field, and the Bohr – van Leeuwen theorem is vindicated. When $|l| > |l_1|$ and $l < 0$ the other nulls that are described by the formula (36) would satisfy the boundary condition. Those are the near-boundary states, and in this case are not restrictions on the values of γ besides $0 < \gamma < 1$. There are n near-boundary states.

In the section 3.2 it will be shown that for statistical mechanics of the electron gas in the magnetic field the Hamiltonian \hat{h}_0 has fundamental importance. Let us study the spectrum of this Hamiltonian with boundary condition (33). Then

$$\begin{aligned} \varepsilon_0(n_0, n_r) &= \frac{\hbar\omega_c}{2} [n_0 + 2\gamma(n_0, n_r)], \quad n_0 = 2n_r + |l| + 1, \\ \gamma(n_0, n_r) &\approx \frac{(\mathcal{R}^2/2\lambda^2)^{n_0}}{(n_0 - n_r - 1)!(n_r)!} \exp(-\mathcal{R}^2/2\lambda^2). \end{aligned} \quad (39)$$

The degenerate levels are transformed in zonule. It follows from formula (39) that the zonule upper edge is determined by minimum value of $(n_0 - n_r - 1)!(n_r)!$ that is roughly $[(n_0/2)!]^2$. In the vicinity of $n_r = n_0/2$ a shift of energy γ most slowly varies with n_r . It means that a density of states is the highest in the vicinity of the zonule upper edge. The zonule lower edge is shifted by $\gamma_{\min} = \gamma_{\max} [(n_0/2)!]^2 / n_0! \ll \gamma_{\max}$. The zonule width $\gamma_{\max}(n_0)$ increases with n_0 if $(\mathcal{R}^2/2\lambda^2) > (n_0/2)$. If $\gamma_{\max}(n_0) < 1/2$ then number of states on each interval of energy values of width $\hbar\omega_c/2$ is equal to n_0 , (the spin will be taken into account subsequently). For $\gamma_{\max}(n_0) > 1/2$ zonules overlap, gaps in the spectrum disappear, and the number of states in the interval becomes less than n_0 , i.e., grows of the density of states is decelerated with energy increase. That is transitive area of energy. For higher

values of energy not the greatest but other nulls, which are described by formula (36), satisfy the boundary condition. With the approximate formula for the nulls of the Bessel function $j(|l|, \nu) \approx \pi(\nu + |l|/2)$ the second in magnitude null of the function (35) with $\nu = n_r$ is $[\pi^2 n_0^2 / 8(n_0 + 2\gamma')] \approx (\pi^2 n_0 / 8)$. When it will be so that $\pi^2 n_0 / 8 \geq \mathcal{R}^2 / 2\lambda^2$ the boundary condition should be satisfied only not the greatest but other nulls of the degenerate hypergeometric function. Corresponding values of energy look like:

$$\varepsilon_0(n_0) = \frac{\hbar\omega_c}{2}(n_0 + 2\gamma') = \frac{\hbar\omega_c}{2} \left(\frac{\mathcal{R}^2}{\lambda^2} \right)^{-1} j^2(|l|, \nu) = \varepsilon_0(|l|, \nu) = \frac{\hbar^2}{2m\mathcal{R}^2} j^2(|l|, \nu) \quad (40)$$

where $\nu = n_r \leq n/2$ and γ' is not described by the formula (39). They coincide with eigenvalues energy in a circular potential well with reflecting boundaries (see formula (19)). The spectrum becomes quasicontinuous as distances between the nearest levels are proportional to \mathcal{R}^2 . The density of states does not depend on energy, as well as for two-dimensional gas of free particles. The function (35) would be expanded in series over the Bessel functions (Erdélyi, 1953) and the first term is proportional to $J_{|l|}((r/\hbar)\sqrt{2\varepsilon m})$. The parameter of this expansion is $(n_0)^{-1/2}$. Hence, the wave functions in this approximation also coincide with free electron wave functions. This form of the spectrum would be illustrated by the classical consideration. If the formula (34) would be considered as the classical Hamiltonian, the ultimate energy for which the classical accessible area is determined by the parabolic potential is equal to $m\omega_c^2 \mathcal{R}^2 / 8$. That within a factor ξ that is close to 1 coincides with the energy of the transitive area and with energy of the transition to quasicontinuous spectrum $(2/\pi^2)m\omega_c^2 \mathcal{R}^2$.

These results can be described as energy spectrum breakdown into two bands. A spectrum lower part is denoted as a magnetic band, and the upper one will be denoted as a conduction band. Bands are not separated by a gap or sharp boundary, but far from transitive area the density of states and wave functions differ substantially. Fine structure of the density of states in the lower part of magnetic band represents the narrow zonule separated by gaps. The total width of the allowed zonule and gap is equal to $\hbar\omega_c/2$. Number of states in an interval with number n_0 is equal to n_0 . In the transitive area gaps disappear and in the conduction band the spectrum is quasicontinuous. The maximum of magnitude of a wave function in the magnetic band is localized within a ring of width about $\lambda\sqrt{2}$ and about $R = \lambda\sqrt{2n_0}$ in radius. The density of states in the magnetic band, averaged over the interval $\hbar\omega_c/2$, is $2n_0/\hbar\omega_c = 4\varepsilon_0/(\hbar\omega_c)^2$ and grows with energy. In the transitive area this grows is decelerated, and in the conduction band the density of states does not depend on energy and is equal to $m\mathcal{R}^2/2\hbar^2$ (without spin consideration). It is believed that the magnetic band is finished when $n_0 = \mathcal{R}^2/4\lambda^2 = n_b$ or $\varepsilon_0 = m\omega_c^2 \mathcal{R}^2 / 8 = \varepsilon_b$ as would be expected from the quasiclassical consideration. Then the density of states will be continuous.

3.2 Statistical mechanics of electron gas in uniform magnetic field with regard for electrostatic interaction

The quantum-mechanical average value of the magnetic moment in the ordinary eigenstate of the Hamiltonian \hat{h} in absence of the potential energy $U(r)$ is:

$$\mu_z = -(\partial \varepsilon(n, l) / \partial H) = -\{(\hbar e / m)(n + \gamma) + (\hbar e H / m)(\partial \gamma / \partial H)\} \quad (41)$$

It is a negative quantity because the positive term that proportional to $(\partial \gamma / \partial H)$ is small. In the paper (Landau, 1930) were taken into account only ordinary states. Then the quantum-statistical average value of the gas magnetic moment must be:

$$\mathfrak{M} = -\sum_{nl} \frac{\partial \varepsilon(n, l)}{\partial H} n(n, l) = -\sum_{nl} \frac{\partial \varepsilon(n, l)}{\partial H} \left\{ D(H) \left[1 + \exp \left(\frac{\varepsilon(n, l) - \mu}{k_B T} \right) \right]^{-1} \right\} < 0 \quad (42)$$

Here $n(n, l)$ is the average occupation number of the state $\Psi(n, l)$, $D(H)$ is the multiplicity of degeneracy that in this case does not depend on the state energy and depend on the magnetic field. But in this work the magnetic moment of the gas was computed as

$$\begin{aligned} \mathfrak{M} &= -\frac{\partial \Omega}{\partial H} = k_B T \frac{\partial}{\partial H} \sum_{nl} \left\{ D(H) \ln \left[1 + \exp \left(\frac{\mu - \varepsilon(n, l)}{k_B T} \right) \right] \right\} = \\ &k_B T \sum_{nl} \left\{ \frac{\partial D(H)}{\partial H} \ln \left[1 + \exp \left(\frac{\mu - \varepsilon(n, l)}{k_B T} \right) \right] \right\} - \sum_{nl} \frac{\partial \varepsilon(n, l)}{\partial H} n(n, l). \end{aligned} \quad (43)$$

When this result is compared with the formula (42) it is apparent that the thermodynamical potential Ω is determined incorrectly.

Let us obtain the density of distribution for an electron gas that is at equilibrium with thermostat, which is described by classical mechanics. The conservation of the zero value of the angular momentum also will be taken into account. The characteristic function of this system is:

$$\begin{aligned} \varphi &= (2\pi)^{-3} I_E \int_0^{2\pi} \int_0^{2\pi} \exp \left[\left(N - \sum_{\Psi} a_{\Psi}^+ a_{\Psi} \right) (i\phi + \chi) + \left(\sum_{\Psi} l_z(\Psi) a_{\Psi}^+ a_{\Psi} \right) (i\alpha + \eta) \right] d\phi d\alpha \\ I_E &= 2\pi \int_0^{\infty} \varphi_1(\varepsilon) \varphi_2(E - \varepsilon) d\varepsilon = \\ &\int_0^{\infty} \delta[\mathfrak{H}_{th} - (E - \varepsilon)] \int_0^{2\pi} \exp \left[\left(\varepsilon - \sum_{\Psi} [\varepsilon_0(\Psi) + (\hbar \omega_c / 2) l_z(\Psi)] a_{\Psi}^+ a_{\Psi} \right) (i\tau + \vartheta) \right] d\tau d\varepsilon. \end{aligned} \quad (44)$$

This formula is obtained on a basis of the properties of characteristic functions that was described in the formulae (6 – 8), and the quantum characteristic function (see formula (15)). Here E is a total energy, ε is the energy of the electron gas, \mathfrak{H}_{th} is the Hamiltonian of the thermostat that is a summatory function of the classical variables. Values of energy are expressed by the dimensionless positive integers, and the angular momentum is measured in \hbar . The expression of the Hamiltonian \hat{h} as the sum $\hat{h}_0 + (\omega_c / 2) \hat{l}_z$ is taken into account. Then the second equality in (44) would be rewritten as:

$$I_E = \int_0^{\infty} \delta[\mathfrak{H}_{th} - (E - \varepsilon)] \int_0^{2\pi} \exp \left[\left(\varepsilon - \sum_{\Psi} \varepsilon_0(\Psi) a_{\Psi}^+ a_{\Psi} \right) (i\tau + \vartheta) \right] d\tau d\varepsilon \quad (45)$$

by using the formula (8). Let us generalize the Krutkov method (Krutkov, 1933; Zubarev, 1974) for this case. We calculate the Laplace transformation with respect the total energy E . To do this let us multiply the function $(2\pi)^{-1} I_E$ (formulae (44 - 45) by $\exp(-\mathcal{G}E)$ and integrate over E between 0 and ∞ . Then the integrations over ε and τ can be performed, and the result is:

$$I_E = \int_0^\infty \delta[\mathcal{H}_{th} - (E - \varepsilon)] \int_0^{2\pi} \exp\left[\left(\varepsilon - \sum_{\Psi} \varepsilon_0(\Psi) a_{\Psi}^+ a_{\Psi}\right)(i\tau + \mathcal{G})\right] d\tau d\varepsilon \quad (46)$$

where $\mathcal{L}\{f(E), \mathcal{G}\} = \mathcal{L}_{\mathcal{G}}[f(E)]$ is Laplace transform of a function $f(E)$. The integration over the thermostat phase space leads to the formula:

$$(2\pi)^{-1} [t_{th}(\mathcal{G})]^{N_{th}} \exp\left(-\mathcal{G} \sum_{\Psi} \varepsilon_0(\Psi) a_{\Psi}^+ a_{\Psi}\right), \quad t_{th}(\mathcal{G}) = \int_{\Gamma_1} \exp(-\mathcal{G} h_{th}) d\Gamma_1 \quad (47)$$

where Γ_1 is the phase space of a thermostat particle and h_{th} is its Hamiltonian and N_{th} is the number of thermostat particles. As result of inverse Laplace transformation we obtain:

$$\int_{\Gamma_{th}} \int_0^\infty \varphi_1(\varepsilon) \varphi_2(E - \varepsilon) d\varepsilon d\Gamma_{th} = (2\pi)^{-1} \int_{a-i\infty}^{a+i\infty} \exp\left[N_{th} \left(\mathcal{G} \frac{E}{N_{th}} + \ln t(\mathcal{G})\right)\right] \exp\left[-\mathcal{G} \sum_{\Psi} \varepsilon_0(\Psi) a_{\Psi}^+ a_{\Psi}\right] d\mathcal{G} \quad (48)$$

This integration can be performed by the saddle-point method because N_{th} is a large-scale number. The saddle-point $\mathcal{G}_0 = (k_B T)^{-1}$ is determined by the first exponent, and the second exponent that depend on variables-operators of the quantum subsystem can be factored from integral at $\mathcal{G} = (k_B T)^{-1}$ as well as in the classical case. The result should be substituted into (44). Then the non-normalized statistical operator is obtained:

$$\hat{\rho} = \exp\left[-(k_B T)^{-1} \sum_{\Psi} \varepsilon_0(\Psi) a_{\Psi}^+ a_{\Psi}\right] \int_0^{2\pi} \exp\left[\left(N - \sum_{\Psi} a_{\Psi}^+ a_{\Psi}\right)(i\phi + \chi)\right] d\phi \times \int_0^{2\pi} \exp\left[\left(\sum_{\Psi} I_z(\Psi) a_{\Psi}^+ a_{\Psi}\right)(i\alpha + \eta)\right] d\alpha. \quad (49)$$

Here the first multiplier is the common formula of the statistical operator for the quantum Gibbs distribute (Landau, & Lifshitz, E.M., 1980a; Zubarev, 1974). The second multiplier would be computed by the Darwin - Fowler method (Fowler, & Guggenheim, 1939) like as in the formula (16) and describes the conservation of the particle number. If the grand canonical ensemble is considered, then the statistical operator of particle number

$$\hat{\rho}_N = \exp\left[-\mu (k_B T)^{-1} \sum_{\Psi} a_{\Psi}^+ a_{\Psi}\right] \quad (50)$$

would be obtained from this multiplier by the Krutkov method. The last multiplier in formula (49) cannot be computed by those methods because it does not have any large-scale parameter. This multiplier imposes constraints on ensembles that the total angular momentum equal to zero. If Hamiltonian and an operator that should be averaged have the

commutative term that is proportional to the total angular momentum operator, this term should be eliminated when the averaging is performed. That is the reason for the change ε to ε_0 in the Hamiltonian $\sum_{\Psi} \varepsilon_0 (\Psi) a_{\Psi}^{\dagger} a_{\Psi}$ in the formula (48).

The model that will be considered below is described by the Hamiltonian:

$$\hat{\mathcal{H}} = \sum_{i=1}^N \hat{h}_{0i}; \quad \hat{h}_0 = \frac{1}{2m} \left(\hat{p}_r^2 + \frac{\hat{l}_z^2}{r^2} \right) + \frac{m\omega_c^2 r^2}{8} + U(r) \quad (51)$$

Here $U(r)$ is potential of the self-consistent electric field that describes interaction of an electron with other electrons and neutralizing background that have the form of circle with radius \mathcal{R} .

The electron density in the magnetic field should be distributed in such a way as to shield the external potential $m\omega_c^2 r^2 / 8$. This shielding cannot be perfect. The current that would be generated by a residual potential would be compensated by the diffusion current that generated by an inhomogeneity of the electron density. Because the kinetic energy in the Hamiltonian (51) has the standard form, electrons density distribution and self-consistent potential can be calculated by the density functional method, described in the work (March, 1983). In this approximation, energy of the ground state of electron gas is presented in the form of functional of the gas density, i.e., number of particles per unit of area $\rho(r)$:

$$E_0[\rho(\mathbf{r})] = \left(\pi \hbar^2 / 2m \right) \int_C \rho^2(\mathbf{r}) d\mathbf{r} + \left(e^2 / 8\pi \epsilon_0 \right) \iint_{CC} \rho(\mathbf{r}) \rho(\mathbf{r}') |\mathbf{r} - \mathbf{r}'|^{-1} d\mathbf{r} d\mathbf{r}' - \left(e^2 / 4\pi \epsilon_0 \right) \int_C \rho(\mathbf{r}) [u(\mathbf{r}) + u_0] d\mathbf{r} + \left(m\omega_c^2 / 8 \right) \int_C \rho(\mathbf{r}) r^2 d\mathbf{r}. \quad (52)$$

Here $\pi \hbar^2 \rho^2 / 2m$ is density of kinetic energy of two-dimensional degenerated Fermi gas (each state is supposed to be twice degenerated), the second addend describes energy of electrons Coulomb interaction, ϵ_0 is vacuum inductivity, the third addend is energy of electrostatic interaction with the neutralizing background, the last addend described gas potential energy in the effective harmonic potential depending on magnetic field. The neutralizing background has the charge density $e\rho_0 Y(\mathcal{R} - r)$ and generate the potential $eu(\mathbf{r})$. This potential would be expressed by the elliptical integrals and is approximated by the parabola and a repulsing bound. The step function $Y(x) = 1$ at $x > 0$ and $Y(x) = 0$ at $x \leq 0$. The constant eu_0 would be assigned so that the potential energy in the effective Hamiltonian would be zero at $r = 0$. All integrations are made over the area C , which is a circle of radius \mathcal{R} . The functional $E_0[\rho(\mathbf{r})]$ should be minimized under the following supplementary condition:

$$\int_C \rho(\mathbf{r}) d\mathbf{r} = \pi \mathcal{R}^2 \rho_0 \quad (53)$$

The quadratic term of the residual potential is of chief interest. Therefore the minimization would be performed in the quadratic approximation. The test function has the form:

$$\rho(r) = \rho_0 + \alpha \mathcal{R}^{-2} - \beta \mathcal{R}^{-4} r^2, \quad (54)$$

and as result we obtain:

$$\alpha = \frac{(m\omega_c \mathcal{R}^2)^2 a_0}{12\pi\hbar^2 \mathcal{R}} \left(1 - \frac{4a_0}{3\mathcal{R}}\right), \quad \beta = \frac{(m\omega_c \mathcal{R}^2)^2 a_0}{6\pi\hbar^2 \mathcal{R}} \left(1 - \frac{4a_0}{3\mathcal{R}}\right), \quad (55)$$

where $a_0 = 4\pi e_0 \hbar^2 / me^2$ is Bohr radius. The Lagrange multiplier ζ , which has meaning of the chemical potential or the Fermi energy is obtained:

$$\zeta = (\pi\hbar^2 \rho_0 / m) + m\omega_c^2 \mathcal{R} a_0 / 12 = \zeta_0 + m\omega_c^2 \mathcal{R} a_0 / 12, \quad (56)$$

where ζ_0 is the Fermi energy of two-dimensional electron gas in absence of a magnetic field. The residual potential energy has the form $(4a_0/3\mathcal{R})(m\omega_c^2 r^2/8)$. It is expressed conveniently by new frequency $\omega_r = \omega_c \sqrt{4a_0/3\mathcal{R}}$. Then one-particle effective Hamiltonian will have the form that was study in the previous subsection:

$$\hat{h}_r = \frac{1}{2m} \left(\hat{p}_r^2 + \frac{\hat{l}_z^2}{r^2} \right) + \frac{m\omega_r^2 r^2}{8} \quad (57)$$

The Coulomb interaction and the electron density inhomogeneous that are neglected commonly decrease the frequency $\omega_c \rightarrow \omega_r$ or another way the effective charge $e \rightarrow e_r$ by factor $\sqrt{4a_0/3\mathcal{R}} \approx 10^{-4}$. This factor does not depend on the electron density if this density satisfies to the condition of the degeneration. The operator of the total magnetic moment $\hat{\mathcal{M}} = -(e_r/2m) \sum_i^N (\hat{l}_i + e_r \hbar r_i^2/2)$. When the quantum-statistical average is computed, $\sum_i^N \hat{l}_i = 0$, and $\mathcal{M} = \langle \hat{\mathcal{M}} \rangle = -\langle \sum_i^N \partial \hat{h}_r / \partial H \rangle$. The energy magnetic splitting over spin does not alter. Hence it should be considered separately.

The electron gas always interacts with electromagnetic field that in the ordinary circumstance has zero temperature. This leads to the fact that the gas passes to the ground state by the spontaneous photon irradiation. The role of statistical mechanics is that it imposes a constraint on a value of the angular momentum in the ground state. The energy of the ground state with zero angular momentum would be computed by the spectrum of the one-particle states that was described in the previous subsection. The energy levels, the degeneracy multiplicity, and the boundary of the magnetic band are determined only the orbital motion. Then the energy of the magnetic band is:

$$E_{mb} = \hbar\omega_r \sum_{n=1}^{n_b} (n + 2\gamma_{n||}) n \approx \frac{\hbar\omega_r n_b^3}{3} = \frac{e^4 a_0^2}{108\hbar^2 m} \mathcal{R}^4 H^4 \quad (58)$$

The number of states in the magnetic band equals to:

$$N_b = n_b(n_b + 1) \approx n_b^2 = (e^2 a_0 / 12\hbar^2) \mathcal{R}^3 H^2 \quad (59)$$

The quantity n_b is discrete; therefore the formulae (58 - 59) describe a smoothed function. The distance between the neighboring spikes in E_{mb} is $\hbar\omega_r n_b^2$. If N_b is less than the number of electrons $N = \pi\mathcal{R}^2 \rho_0$ the energy of the ground state with a glance the formula (56) is:

$$E_0 = E_{mb} + \frac{m\mathcal{R}^2}{\hbar^2} \int_{\varepsilon_b}^{\zeta} \varepsilon d\varepsilon - \frac{m\mathcal{R}^2}{\hbar^2} \left(\frac{e\hbar}{2m} H \right)^2 =$$

$$\frac{m\mathcal{R}^2 \zeta_0^2}{2\hbar^2} + \frac{\pi\rho_0 a_0 e^2}{12m} \mathcal{R}^3 H^2 - \frac{a_0^2 e^4}{864\hbar^2 m} \mathcal{R}^4 H^4 - \frac{e^2}{4m} \mathcal{R}^2 H^2. \quad (60)$$

Here the first term is the energy of the electron gas in the absence of a magnetic field. The second term that makes the main addend in the magnetic moment is the product of the Fermi energy of the electron gas ζ_0 and the additional term in it, which depends on the magnetic field (formula (56)). The third term in the formula (60) is created by the addends E_{mb} , the squared additional term of the Fermi energy, and the negative addend in the result of integration $-(m\mathcal{R}^2 \varepsilon_b^2 / 2\hbar^2)$. The last term in this formula describes the energy lowering by the spin polarization. It always is smaller than the second term. The magnetic moment is:

$$\mathfrak{M} = -\frac{\partial(E_0 - E_p)}{\partial H} - \frac{E_p}{H} = -\frac{\pi\rho_0 a_0 e^2}{6m} \mathcal{R}^3 H + \frac{a_0^2 e^4}{216\hbar^2 m} \mathcal{R}^4 H^3 + \frac{e^2}{4m} \mathcal{R}^2 H \quad (61)$$

where $E_p = -(e\mathcal{R}H)^2 / 4m$. The first term is the diamagnetic moment of the orbital motion in the states of the conduction band. It is proportional to the electron density and third power of the radius as distinct from the result of the Landau theory (see to formula (5)). This formula would be obtained by other way. Let us consider the density of electric current (formula (32)) in the ground state with zero angular momentum:

$$j_\varphi = -\frac{e_r \hbar}{2m} \left\langle \sum_{i=1}^N \left\{ \left[\frac{\hat{l}_i}{r_i} + \frac{e_r H r_i}{2\hbar} \right] \delta(\mathbf{r}_i - \mathbf{r}) + \delta(\mathbf{r}_i - \mathbf{r}) \left[\frac{\hat{l}_i}{r_i} + \frac{e_r H r_i}{2\hbar} \right] \right\} \right\rangle \quad (62)$$

The magnetic moment is:

$$\mathfrak{M} = \pi \int_0^{\mathcal{R}} j_\varphi(r) r^2 dr = -\frac{e_r \hbar}{m} \sum_{nl} n(n, l) l \pi \int_0^{\mathcal{R}} |\psi_{nl}(r)|^2 r dr - \frac{\pi e_r^2 H}{2m} \int_0^{\mathcal{R}} \sum_{nl} |\psi_{nl}(r)|^2 n(n, l) r^3 dr \quad (63)$$

The first term equals to zero because $2\pi \int_0^{\mathcal{R}} |\psi_{nl}(r)|^2 r dr \equiv 1$ and $\sum_{nl} n(n, l) l = 0$. Let us change in the second addend $\sum_{nl} |\psi_{nl}(r)|^2 n(n, l) = \rho(r)$. Then the integration gives both the first and the second terms of the formula (61). If the sum $\sum_{nl} n(n, l) l$ will be computed in the common theory, the linear with respect to the magnetic field terms in the formula (63) will cancel.

The orbital diamagnetic susceptibility decreases with increasing of the magnetic field. This function also has a spikes that is caused by the addend $\partial^2 E_{mb} / \partial H^2$. When N_b will be equal to $N = \pi \mathcal{R}^2 \rho_0$ the diamagnetic susceptibility will be zero. When the magnetic field is stronger, the Fermi energy remains on the level $n_F = \sqrt{N}$ of the magnetic band and increases proportionally to magnetic field. The magnetic moment remains constant. The paramagnetic moment also does not increase because the density of states on the Fermi level decreases linearly. The value of the magnetic induction whereby the saturation of the magnetic moment will start is:

$$H_s = \frac{\hbar}{e} \left(\frac{12\pi\rho_0}{a_0\mathcal{R}} \right)^{1/2} = 5.15 \cdot 10^{-10} \left(\frac{\rho_0}{\mathcal{R}} \right)^{1/2} \text{ T} \quad (64)$$

If $\rho_0 = 10^{19} \text{ m}^{-2}$, $\mathcal{R} = 0.1 \text{ m}$, then $H_s = 5.15 \text{ T}$ and $\mathcal{M}_s = -2.92 \cdot 10^{-2} \text{ J} \cdot \text{T}^{-1}$.

4. Conclusion

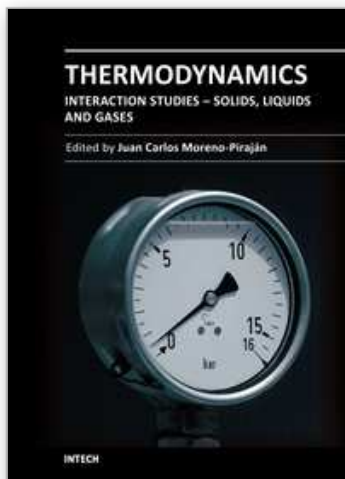
The fundamental theory of statistical mechanics requires taking into account the law of the angular momentum conservation. The fulfilment of this requirement does not introduce any essential alterations into statistical thermodynamics, when the angular momentum of the system equal zero and the system Hamiltonian is a positive definite quadric form of all momenta. An equilibrium isolated system would have nonzero angular momentum only if an attraction of particles can resist centrifugal forces, as it is in nebulas. A gas can be in equilibrium with a rotating envelope that is a termospinostat. The condition of this equilibrium is the equality of the average value of sum of particles angular velocities to the angular velocity of the envelope. The Gibbs density of distribution and the thermodynamical functions are generalized for this case. If a system has the angular momentum equal to zero, the conservation of this value is important only when the Hamiltonian or/and an averaged quantity depend on the angular momentum. The problem of an electron gas in a uniform magnetic field is considered with taking into account the conservation of the zero value of the angular momentum. This consideration eliminates the paradoxical statement of the conventional theory that diamagnetic moment of the gas equals zero in classical as well as quantum physics (the Bohr – van Leeuwen theorem). The new formulae for the magnetic moment of the electron gas are obtained. It also leads to the effect of confinement of two-dimensional gas of charged particles by magnetic field. This results in effect of a non-uniform density of a gas, which decreases with distance from a center according both to classical as well as quantum theory. Then the model of noninteracting charged particles does not have areas of application. Many theories should be reconsidered, if they are founded on this model and on the statistical mechanics which does not take into account the angular momentum conservation law.

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