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On the Optimal Allocation of the Heat Exchangers of Irreversible Power Cycles

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1. Introduction

Thermal engines are designed to produce mechanical power, while transferring heat from an available hot temperature source to a cold temperature reservoir (generally the environment). The thermal engine will operate in an irreversible power cycle, very often with an ideal gas as the working substance. Several power cycles have been devised from the fundamental one proposed by Carnot, such as the Brayton, Stirling, Diesel and Otto, among others. These ideal cycles have generated an equal number of thermal engines, fashioned after them. The real thermal engines incorporate a number of internal and external irreversibilities, which in turn decrease the heat conversion into mechanical power.
A standard model is shown in Fig. 1 (Aragón-González et al., 2003), for an irreversible Carnot engine. The temperatures of the hot and cold heat reservoirs are, respectively, T_H and T_L. But there are thermal resistances between the working fluid and the heat reservoirs; for that reason the temperatures of the working fluid are T_1 and T_2, for the hot and cold isothermal processes, respectively, with $T_1 < T_H$ and $T_L < T_2$. There is also a heat loss $\dot{Q}_{leak}$ from the hot reservoir to the cold reservoir and there are other internal irreversibilities (such as dissipative processes inside the working fluid). This Carnot-like model was chosen because of its simplicity to account for three main irreversibilities above, which usually are present in real heat engines.
On the other hand, the effectiveness of heat exchangers (ratio of actual heat transfer rate to maximum possible heat transfer rate), influence over the power cycle thermal efficiency. For a given transfer rate requirement, and certain temperature difference, well-designed heat exchangers mean smaller transfer surfaces, lesser entropy production and smaller thermal resistances between the working fluid and the heat reservoirs. At the end all this accounts for larger power output from the thermal engine.
Former work has been made to investigate the influence of finite-rate heat transfer, together with other major irreversibilities, on the performance of thermal engines. There are several parameters involved in the performance and optimization of an irreversible power cycle; for instance, the isentropic temperature ratio, the allocation ratio of the heat exchangers and the cost and effectiveness ratio of these exchangers (Lewins, 2000; Aragón-González et al., 2008 and references there included). The allocation of the heat exchangers refers to the distribution of the total available area for heat transfer, between the hot and the cold sides of an irreversible power cycle. The irreversible Carnot cycle has been optimized with respect to the allocation ratio of the heat exchangers (Bejan, 1988; Aragón-González et al., 2009).

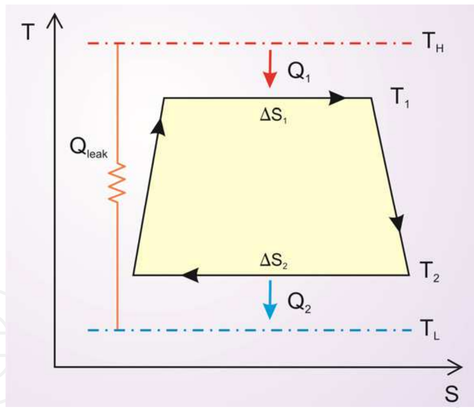

Fig. 1. A Carnot cycle with heat leak, finite rate heat transfer and internal dissipations of the working fluid.

1.1 Heat exchangers modelling in power cycles

Any heat exchanger solves a typical problem, to get energy from one fluid mass to another. A simple or composite wall of some kind divides the two flows and provides an element of thermal resistance between them. There is an enormous variety of configurations, but most commercial exchangers reduce to one of three basic types: a) the simple parallel or counterflow configuration; b) the shell-and-tube configuration; and c) the cross-flow configuration (Lienhard IV & Lienhard V, 2004). The heat transfer between the reservoirs and the hot and cold sides is usually modeled with single-pass counterflow exchangers; Fig. 2. It is supposed a linear relation with temperature differences (non radiative heat transfer), finite one-dimensional temperature gradients and absence of frictional flow losses. For common well-designed heat exchangers these approximations capture the essential physics of the problem (Kays & London, 1998).
Counterflow heat exchangers offer the highest effectiveness and lesser entropy production, because they have lower temperature gradient. It is well-known they are the best array for single-pass heat exchanging. It has also been shown they offer an important possibility, to achieve the heating or cooling strategy that minimizes entropy production (Andresen, B. & Gordon J. M., 1992). For the counterflow heat exchanger in Fig. 2, the heat transfer rate is

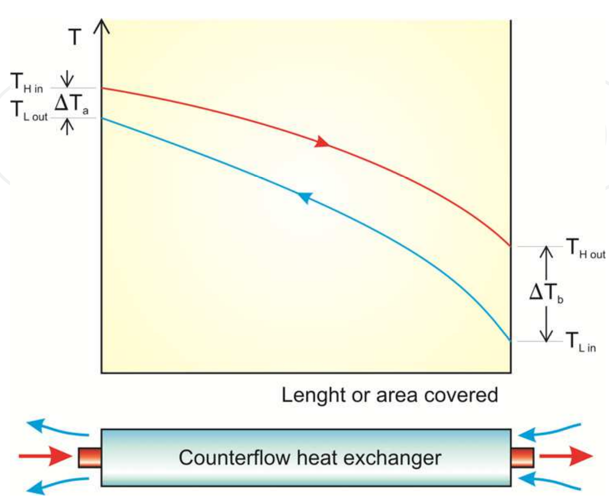

Fig. 2. Temperature variation through single-pass counterflow heat exchanger, with high and low temperature streams.

(Lienhard IV & Lienhard V, 2004):

$$q = UA\ \Delta T_{mean} \tag{1}$$

where U (W/m²K) is the overall heat transfer coefficient, A (m²) is the heat transfer surface and ΔT_{mean} is the logarithmic mean temperature difference, LMTD (K) (see Fig. 2).

$$LMTD = \Delta T_{mean} = \frac{\Delta T_a - \Delta T_b}{\ln \frac{\Delta T_a}{\Delta T_b}} \tag{2}$$

For an isothermal process exchanging heat with a constant temperature reservoir, as it happens in the hot and cold sides of the irreversible Carnot cycle in Fig. 1, it appears the logarithmic mean temperature difference is indeterminate (since $\Delta T_a = \Delta T_b$). But applying L'Hospital's rule it is easily shown:

$$LMTD = \Delta T_a = \Delta T_b\ . \tag{3}$$

For the Brayton cycle (Fig. 3) with external and internal irreversibilities which has been optimized with respect to the total inventory of the heat transfer units (Aragón-González G. et al., 2005), the hot and cold sides of the cycle have:

$$LMTD_H = \frac{T_3 - T_{2s}}{\ln \frac{T_H - T_{2s}}{T_H - T_3}} \text{ and } LMTD_L = \frac{T_{4s} - T_1}{\ln \frac{T_{4s} - T_L}{T_1 - T_L}} \tag{4}$$

The design of a single-pass counterflow heat exchanger can be greatly simplified, with the help of the effectiveness-NTU method (Kays and London, 1998). The heat exchanger effectiveness (ε) is defined as the ratio of actual heat transfer rate to maximum possible heat transfer rate from one stream to the other; in mathematical terms (Kays & London, 1998):

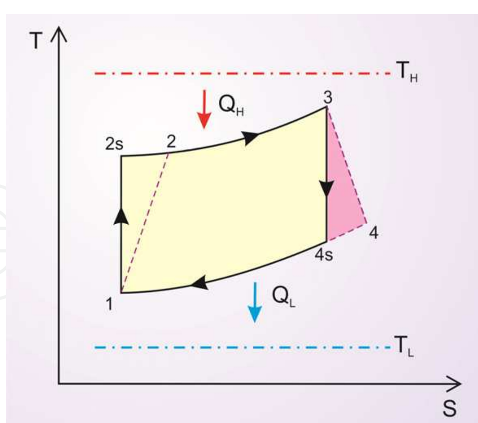

Fig. 3. A Brayton cycle with internal and external irreversiblities.

$$\varepsilon = \frac{C_H\left(T_{Hin} - T_{Hout}\right)}{C_{min}\left(T_{Hin} - T_{Lin}\right)} = \frac{C_L\left(T_{Lout} - T_{Lin}\right)}{C_{min}\left(T_{Hin} - T_{Lin}\right)} \tag{5}$$

it follows that:

$$q_{actual} = \varepsilon\, C_{min} (T_{Hin} - T_{Lin}) \tag{6}$$

The number of transfer units (NTU) was originally defined as (Nusselt, 1930):

$$NTU = \frac{UA}{C_{min}}; \tag{7}$$

where C_{min} is the smaller of $C_L = (\dot{m}\, c_p)_L$ and $C_H = (\dot{m}\, c_p)_H$, both in (W/K); with $\dot{m}$ the mass flow of each stream and c_p its constant-pressure specific heat. This dimensionless group is a comparison of the heat rate capacity of the heat exchanger with the heat capacity rate of the flow. Solving for ε gives:

$$\varepsilon = \frac{1 - e^{-\left(1 - \frac{C_{min}}{Cmax}\right)NTU}}{1 - \frac{C_{min}}{C_{max}} e^{-\left(1 - \frac{C_{min}}{Cmax}\right)NTU}} \tag{8}$$

Equation (8) is shown in graphical form in Fig. 4. Entering with the ratio C_{min}/C_{max} and $NTU = UA/C_{min}$ the heat exchanger effectiveness ε can be read, and with equation (6) the actual heat transfer rate is obtained.
When one stream temperature is constant, as it happens with both temperature reservoirs in the hot and cold sides of the irreversible Carnot and Brayton cycles, the capacity rate ratio C_{min}/C_{max} is equal to zero. This heat exchanging mode is called "single stream heat exchanger", and the equation (8) reduces to:

$$\varepsilon_{singlestream} = 1 - e^{-NTU} \tag{9}$$

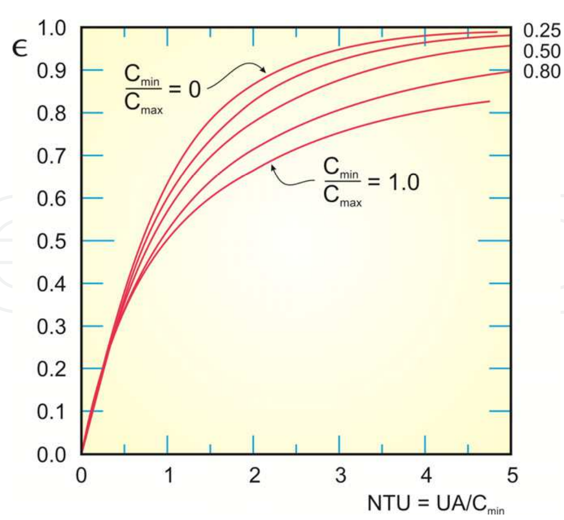

Fig. 4. Effectiveness of counterflow heat exchangers is a function of NTU and C_{min}/C_{max}.

The following sections will be dedicated to the optimal allocation of counterflow heat exchangers which are coupled in the hot-cold sides of irreversible Carnot-like and Brayton-like cycles (Fig. 1 and Fig. 3, respectively).

2. The optimal allocation of the heat exchangers for a Carnot-like cycle

The optimal allocation of the heat exchangers of irreversible power cycles was first analyzed for A. Bejan (Bejan, 1988). He optimized the power for the endoreversible Carnot cycle and found that the allocation (size) of the heat exchangers is balanced. Furthermore, Bejan also found for the model of Carnot the optimal isentropic temperature ratio $x = T_2/T_1$ by a double maximization of the power. He obtained the optimal ratio: $x_{mp}=\sqrt{\mu}$; $\mu = T_L/T_H$; which corresponded to the efficiency to maximum power proposed previously for Novikov-Chambadal-Curzon-Ahlborn (Bejan, 1996 and Hoffman et al., 1997):

$$\eta_{CNCA} = 1 - \sqrt{\mu} \tag{10}$$

The equation (10) was also found including the time as an additional constraint (see Aragón et al., 2006; and references there included). Recently in (Aragón-González et al., 2009), the model of the Fig. 1 has been optimized with respect to x and to the allocation ratio φ of the heat exchangers of the hot and cold side for different operation regimes (power, efficiency, power efficient, ecological function and criterion $\dot{\Omega}(x,\varphi)$). Formerly, the maximum power and efficiency have been obtained in (Chen, 1994; Yan, 1995; and Aragón et al., 2003). The maximum ecological function has been analyzed in general form in (Arias-Hernández et al., 2003). In general, these optimizations were performed with respect to only one characteristic parameter: x including sometimes also time (Aragón-González et al., 2006)). However, (Lewins, 2000) has considered the optimization of the power generation with respect to other parameters: the allocation, cost and effectiveness of the heat exchangers of the hot and cold sides (Aragón-González et al., 2008; see also the reviews of Durmayas et al., 1997; Hoffman et al., 2003). Also, effects of heat transfer laws or when a property is independent of the heat transfer law for this Carnot model, have been discussed in several works (Arias-Hernández et al., 2003; Chen et al., 2010; and references there included), and so on.

Moreover, the optimization of other objective functions has been analyzed: $\dot{\Omega}$ criterion (Sanchez-Salas et al., 2002), ecological coefficient of performance (ECOP), (Ust et al., 2005), efficient power (Yilmaz, 2006), and so on.

In what follows, Carnot-like model shown in Fig. 2 will be considered, it satisfies the following conditions (Aragón-González (2009)): The working fluid flows through the system in stationary state. There is thermal resistance between the working fluid and the heat reservoirs. There is a heat leak rate from the hot reservoir to the cold reservoir. In real power cycle leaks are unavoidable. There are many features of an actual power cycle which fall under that kind of irreversibility, such as the heat lost through the walls of a boiler, a combustion chamber, or a heat exchanger and heat flow through the cylinder walls of an internal combustion engine, and so on. Besides thermal resistance and heat leak, there are the internal irreversibilities. For many devices, such as gas turbines, automotive engines, and thermoelectric generator, there are other loss mechanisms, i.e. friction or generators losses, and so on, which play an important role, but are hard to model in detail. Some authors use the compressor (pump) and turbine isentropic efficiencies to model the internal loss in the gas turbines or steam plants. Others, in Carnot-like models, use simply one constant greater than one to describe the internal losses. This constant is associated with the entropy produced inside the power cycle. Specifically, this constant makes the Claussius inequality to become equality:

$$\frac{\dot{Q}_2}{T_2} - I\,\frac{\dot{Q}_1}{T_1} = 0 \tag{11}$$

where $\dot{Q}_i$ (i = 1, 2) are the heat transfer rates and $I = \Delta S_2/\Delta S_1 \geq 1$ (Chen, 1994). The heat transfer rates $\dot{Q}_H$, $\dot{Q}_L$ transferred from the hot-cold reservoirs are given by (Bejan, 1988):

$$\dot{Q}_H = \dot{Q}_1 + \dot{Q}; \quad \dot{Q}_L = \dot{Q}_2 + \dot{Q} \tag{12}$$

where the heat leak rate $\dot{Q}$ is positive and $\dot{Q}_1$, $\dot{Q}_2$ are the finite heat transfer rates, between the reservoirs T_H, T_L and the working substance. By the First Law and combining equations (11) and (12), the power P, heat transfer rate $\dot{Q}_H$ and thermal efficiency are given by:

$$\begin{aligned} P &= \dot{Q}_H - \dot{Q}_L = \dot{Q}_1 - \dot{Q}_2 = \dot{Q}(1 - Ix); \\ \dot{Q}_H &= \dot{Q}_1 + \dot{Q} = \frac{P}{1 - Ix} + \dot{Q} \\ \eta &= \frac{P}{f(x)P + \dot{Q}} \end{aligned} \tag{13}$$

where $x = \frac{T_2}{T_1}$ is the internal isentropic temperature ratio and $f(x) = \frac{1}{1 - Ix}$ is always positive. The entropy-generation rate and the entropy-generation rate multiplied by the temperature of the cold side gives a function Σ (equations (13)):

$$\begin{aligned} S_{gen} &= \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} > 0 \\ \Sigma &= T_L S_{gen} = T_L\left(\frac{\dot{Q}_H - P}{T_L} - \frac{\dot{Q}_H}{T_H}\right) = \dot{Q}_H(1 - \mu) - P \\ \Sigma &= g(x)P + \dot{Q}(1 - \mu) \end{aligned} \tag{14}$$

where g(x) = f(x)(xI - μ) is also positive. The ecological function (Arias-Hernández et al., 2003), if T_L is considered as the environmental temperature, and the efficient power (Yilmaz, 2006) defined as power times efficiency, are given by

$$E = P - \Sigma = (1 - g(x))P - \dot{Q}\,(1 - \eta) \tag{15}$$

$$P_\eta = \eta P$$

and g(x) should be less than one (Arias-Hernández et al. (2003)). This is fulfilled if and only if E > 0 (see conditions on it in subsection 2.3). Finally, the $\dot{\Omega}$ criterion states a compromise between energy benefits and losses for a specific job and for the Carnot model discussed herein, it is expressed as (Sanchez-Salas et al., 2002):

$$\dot{\Omega} = \frac{2\eta - \eta_{max}}{\eta} P \tag{16}$$

where η_{max} is a constant.

2.1 The fundamental optimal relations of the allocation and effectiveness of the heat exchangers

The relevance of the optimization partial criterion obtained in (Aragón-González et al. (2009)) is that can also be applied to any parameter z different from x, and to any objective function that is an algebraic combination of the power and/or efficiency (as long as the objective function has physical meaning and satisfies the equations (20) and (21) below). In particular, for all the objective functions $P(x,z)$, $\eta(x,z)$, $E(x,z)$, $P_\eta(x,z)$, $\dot{\Omega}(x,z)$ and also for other characteristic parameters (not only these presented in (Aragón-González et al. (2009)). In what follows, let z be any characteristic parameter of the power plant different to x and the following operation regimes will be considered:

$$G(x,z)=P(x,z),\ \eta(x,z),\ E(x,z),\ P_\eta(x,z),\dot{\Omega}(x,z) \tag{17}$$

(power, efficiency, ecological function, efficient power, and $\dot{\Omega}$ criterion, respectively). Assuming, the parameter z can be any characteristic parameter of the cycle different to x. Thus, if $z \neq x$ and z_{mp} is the point in which the power P achieves a maximum value, then:

$$\left.\frac{\partial P}{\partial z}\right|_{z_{mp}} = 0 \text{ and } \left.\frac{\partial^2 P}{\partial z^2}\right|_{z_{mp}} < 0 \tag{18}$$

and, from the third equation of (13):

$$\frac{\partial \eta}{\partial z} = \frac{\dot{Q}\left(\frac{\partial P}{\partial z}\right)}{\left[f(x)P + \dot{Q}\right]^2} \tag{19}$$

since $\dot{Q}$ does not depend of the variable z. Thus,

$$\left.\frac{\partial \eta}{\partial z}\right|_{z_{me}} = 0 \Leftrightarrow \left.\frac{\partial P}{\partial z}\right|_{z_{mp}} = 0 \tag{20}$$

where z_{me} is the point in which the efficiency η achieves its maximum value. This implies that their critical values are the same $z_{mp} = z_{me}$ (necessary condition). The sufficiency condition is obtained by:

$$\left.\frac{\partial^2 \eta}{\partial Z^2}\right|_{z_{mp}=z_{me}} = \frac{\dot{Q}\left(\left.\frac{\partial^2 P}{\partial z^2}\right|_{z_{mp}=z_{me}}\right)}{\left[f(x)P + \dot{Q}\right]^2} < 0 \tag{21}$$

The optimization described by the equations (18)-(21) can be applied to the operation regimes given by equations (17) (the operation regime Σ (equation (14)) does not have a global minimum as was shown in (Aragón-González et al., 2009)). Thus, if z_{mec}, $z_{mP\eta}$, $z_{m\Omega}$ are the values in which the objective functions $E(x,z)$, $P_\eta(x,z)$, $\dot{\Omega}(x,z)$ reach their maximum value, then: $z_{mp} = z_{me} = z_{mec} = z_{mp\eta} = z_{m\Omega}$. Furthermore, the optimization performed, with respect to x, is invariant to the law of heat transfer no matter the operation regime G(x,z).

As an illustration, only two design rules corresponding to z will be considered. The first rule is when the constrained internal conductance, which is applied to the allocation of the heat exchangers from hot and cold sides with the same overall heat transfer coefficient U by unit of area A in both ends (see equations (1) and (3)). Thus,

$$\begin{aligned} \alpha + \beta &= \gamma \\ \frac{\alpha}{U} + \frac{\beta}{U} &= A \end{aligned} \tag{22}$$

where γ is a constant, α, β are the thermal conductances on the hot and cold sides, respectively, and in parametrizing as:

$$\varphi = \frac{\alpha}{UA}; \quad 1 - \varphi = \frac{\beta}{UA} \tag{23}$$

The second rule corresponds to total constrained area. Now, the total area A is fixed, but when distributed it has different overall heat transfer coefficients (different effectiveness) on hot and cold sides (see equations (1) and (3)). Then,

$$A = A_H + A_L = \frac{\alpha}{U_H} + \frac{\beta}{U_L} \tag{24}$$

where A_H and A_L are heat transfer areas on hot and cold sides, and U_H and U_L are overall heat transfer coefficients on the hot and cold sides, respectively. In parametrizing again:

$$\varphi^* = \frac{\alpha}{U_H A}; \quad 1 - \varphi^* = \frac{\beta}{U_L A} \tag{25}$$

The following criterion can be established:

Criterion 1. If x is fixed, $z \neq x$ is a characteristic parameter arbitrary of the irreversible Carnot cycle, the law of heat transfer is any, including the heat leak, and the objective functions are $G(x,z) = P(x,z), \eta(x,z), E(x,z), P_\eta(x,z), \dot{\Omega}(x,z)$. Then, the objective function G(x,z) reaches his maximum in: $z_{mG}=z_{mP}=z_{me}=z_{mec}=z_{mP\eta}=z_{m\Omega}$. In particular, if $z=\varphi$ or φ^*, the thermal conductances, overall heat transfer coefficients and areas are given either by the equations (22) or (25), and G(x,z) represent any operation regime given by the equation (17), then, $z_{mG} = \varphi_{mG}$ or φ^*_{mG}, and are given by:

$$\begin{aligned} \varphi_{mG} &= \frac{1}{1 + \sqrt{I}} \\ \text{or} \quad \varphi^*_{mG} &= \frac{\sqrt{R_U}}{\sqrt{I} + \sqrt{R_U}} \end{aligned} \tag{26}$$

where $R_U = \frac{U_L}{U_H}$. Consequently, the optimal area ratio of the heat exchangers is given by:

$$\frac{A_L}{A_H} = \sqrt{I} \tag{27}$$

or the optimal distribution of the heat exchangers areas is:

$$A_H^* = \frac{A}{1+\sqrt{I\frac{U_H}{U_L}}};\ A_L^* = \frac{A}{1+\sqrt{\frac{U_L}{IU_H}}} \tag{28}$$

Indeed, it is enough to choose as objective function G(x,z), the power and the transfer heat law by conduction, since they are the algebraically simplest. For the first design rule (equation (23)), the dimensionless power output, $p = \frac{P}{UAT_H}$ is:

$$p = \frac{(1 - Ix)\left(1 - \frac{\mu}{x}\right)}{\frac{1}{\varphi} + \frac{I}{1-\varphi}} \tag{29}$$

or for the second design rule, the dimensionless power $p^* = \frac{P}{AU_HT_H}$ is given by:

$$p^* = \frac{(1 - Ix)\left(1 - \frac{\mu}{x}\right)}{\frac{1}{\varphi^*} + \frac{I}{(1-\varphi^*)R_U}} \tag{30}$$

In optimizing p or p* with respect to φ or φ* (respectively), the equations (26) are derived and combining the equations (23) or (25) and (26), the equations (27) and (28) are obtained; from these equations, the Eq. (26) or Eq. (27) are derived.

2.2 Efficiencies to maximum G(x, φ)

In this section, the efficiencies to maximum G(x, φ) are calculated by the substitution of the optimal value φ_{mG} given by the first equation (26) that doesn't depend on the aforesaid operation regimes G and neither on the transfer heat law. The power as objective function and the transfer heat law by conduction are newly chosen. The optimal values of x will be adapted or extended from the current literature for each objective function. The efficiencies to maximum G, for numerical values given, will be compared.

Now, if all heat transfer rates are assumed to be linear in temperature differences, from the equation (29), the dimensionless power is: $p = \frac{(1 - Ix)\left(1 - \frac{\mu}{x}\right)}{\left(1+\sqrt{I}\right)^2}$.

The optimization of p, η, $e = \frac{E}{UAT_H}$ with respect to x has been discussed in (Aragón-González et al., 2000; 2003; 2008), and are given by:

$$\begin{aligned} x_{mP} &= \sqrt{\frac{\mu}{I}}; \\ x_{me} &= \frac{I\mu + \left(1+\sqrt{I}\right)\sqrt{\mu L(1-\mu)(C - I\mu)}}{CI}; \\ x_{mec} &= \sqrt{\frac{\mu(1+\mu)}{2I}}; \end{aligned} \tag{31}$$

For the objective functions $p_\eta = \frac{P_\eta}{UAT_H}$, $\dot{\omega} = \frac{\dot{\Omega}}{UAT_H}$, they have been calculated only numerically in (Yilmaz, 2006; Sanchez-Salas et al., 2002). However, they can be calculated analytically. For instance, the optimal value $x_{mp\eta}$ for maximum efficient power p_η is obtained by:

$$p_\eta = \frac{p^2}{f(x)p + L(1-\mu)}; \quad \frac{\partial p_\eta}{\partial x} = 0 \tag{32}$$

and from the equation (32), the following cubic equation is obtained:

$$\left[\frac{\partial p}{\partial x}\left(2\left(L(1-\mu) + \frac{p}{1-Ix}\right)(1-Ix)^2 - p(1-Ix)\right) - p^2 I\right]_{x_{mp\eta}} = 0 \tag{33}$$

then, the root with physical meaning is chosen; the solution is too large to be presented here. This value of $x_{p\eta}$ extends to one presented in (Yilmaz, 2006). Similarly, a closed form for $x_{m\Omega}$ which extends to one presented in (Sanchez-Salas et al., 2002) is equally calculated, which is:

$$x_{m\Omega} = \sqrt{\frac{Ix_{me}^2 + \mu}{2I}} \tag{34}$$

Using equations (13), (30), (32) and (33) the efficiencies η_{mG} (η_{mp}, η_{max}, η_{mec}, $\eta_{mp\eta}$ and $\eta_{m\Omega}$) to maximum $G = P, \eta, E, P_\eta$ and $\dot{\Omega}$ are given by:

$$\eta_{mG} = \frac{1 - Ix_{mG}}{1 + \frac{L(1-\mu)\left(\sqrt{I}+1\right)^2}{1 - \frac{\mu}{x_{mG}}}} \tag{35}$$

where $L = K/UA$, being K the thermal conductance of the heat leak $\dot{Q} = K(T_H - T_L)$. For numerical values of (Aragón-González et al., 2009): $I = 1.235$ and $L = 0.01$ the efficiencies η_{mG} can be contrasted. Fig. 5 shows the behavior between, η_{mE}, $\eta_{mp\eta}$, $\eta_{m\Omega}$ with respect to η_{mp} and η_{max}, versus μ; it can be seen that the following inequality is satisfied: $\eta_{mp} \leq \eta_{mec}$, and $\eta_{p\eta}$ $\eta_{m\Omega} \leq \eta_{max}$.

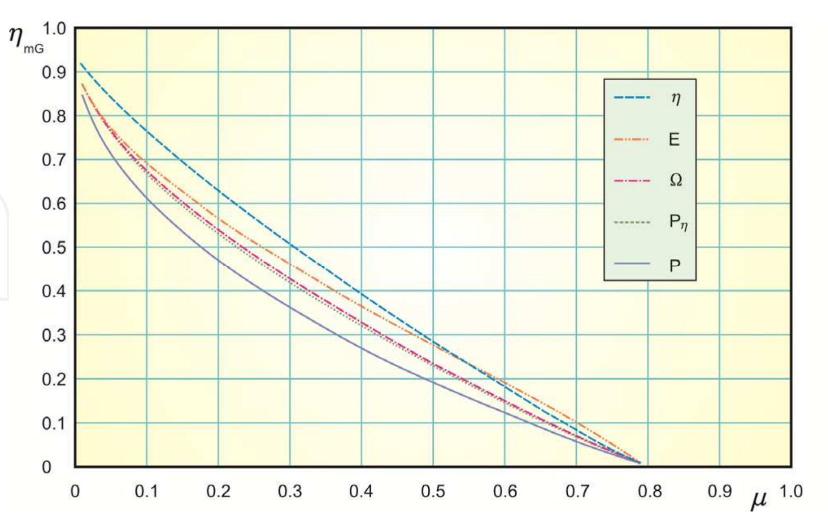

Fig. 5. Behaviors of η_{mp}, η_{max}, η_{mE}, $\eta_{mp\eta}$, $\eta_{m\Omega}$, with respect to μ if I = 1.235 and L = 0.01.

A completely analogous analysis can be performed by the substitution of the second equation of (26) (or any other optimal parameter, v. gr. costs per unit heat transfer) in the equation (30) for the power (or some other equation corresponding to the power including costs as was proposed in (Aragón-González et al., 2008)) for the same transfer heat law. Also, this can be made applying the same objective function (power) and only changing the heat transfer law. But this is not covered by this chapter's scope.

2.3 The ecological coefficient of performance (ECOP)

In (Ust et al. (2005)) was indicated that the ecological function may take negative values, due to the loss rate of availability term, $T_0 S_{gen}$ (T_0 is the environment temperature), can be greater than the actual power output. In general, the ecological function takes only positive values for efficiencies greater than the half of the Carnot efficiency η_C. Indeed, from the equation (15) for Σ the following is obtained:

$$\begin{aligned} \Sigma &= P\left(\frac{\eta_C - \eta}{\eta}\right) > 0 \\ E &= \left(\frac{2\eta - \eta_C}{\eta}\right) P > 0 \Leftrightarrow \eta > \frac{\eta_C}{2} \end{aligned} \tag{36}$$

where $T_0 = T_L$ has been taken.

Then, a new ecological objective function was proposed in (Ust et al. (2005)), in order to identify the effect of loss rate of availability on power output. This objective function has always positive values just like the power, efficiency and efficient power, and is an algebraic expression only of the efficiency η. The objective function was called the ecological coefficient of performance (ECOP) and is defined as the power output per unit loss rate of availability, i.e. $\text{ECOP} = \frac{P}{\Sigma} > 0$; where $T_L = T_0$ have been supposed. A performance analysis upon the Carnot-like model, using the ECOP criterion as objective function, was carried out by Ust et al. Their optimization was performed only for x and taking some numerical optimal values of φ^* (Figure 3 (d) of Ust et al., 2005). It was found (see Figure 4 of Ust et al. 2005) that the optimal conditions to maximum ECOP and efficiency η coincides, although their functional forms are different. This can be shown applying the optimization described in the subsection 2.1 (equations (18) to (21)) and in a form more general. Indeed, let $z = x$, φ or φ^* (or any other parameter); from the first equation of (36), the ECOP can be wrote:

$$\text{ECOP} = \frac{\eta}{\eta_C - \eta} > 0 \tag{37}$$

and as

$$\begin{aligned} \left.\frac{\partial \text{ECOP}}{\partial z}\right|_{z_{mEcop}} &= \left.\frac{\frac{\partial \eta}{\partial z}\text{ECOP}}{(\eta_C - \eta)}\right|_{z_{mEcop}} = 0 \Leftrightarrow \left.\frac{\partial \eta}{\partial z}\right|_{z_{me}} = 0 \\ \left.\frac{\partial^2 \text{ECOP}}{\partial z^2}\right|_{z_{mEcop}} &= \frac{\left.\frac{\partial^2 \eta}{\partial z^2}\right|_{z_{mEcop}} (1 + \text{ECOP})}{(\eta_C - \eta)} < 0 \end{aligned} \tag{38}$$

Furthermore, the following criterion can be established and can be shown in simpler form:

Criterion 2. The ECOP criterion reaches its maximum if $\eta = \eta_{max}$. Moreover, the optimal conditions are the same for the ECOP and η objective functions, independently of the heat transfer law. In particular, the second equation of (31) is fulfilled. Indeed, it is enough to write (37) as:

$$ECOP = \frac{1}{\frac{\eta_C}{\eta} - 1}; \qquad \frac{1}{\frac{\eta_C}{\eta} - 1} \leq \frac{1}{\frac{\eta_C}{\eta_{max}} - 1} \tag{39}$$

since $\eta \leq \eta_{max}$, i.e. the maximum is reached if $\eta = \eta_{max}$ and as none heat transfer explicit law has been used, then, it is satisfied for any the heat transfer law and for any characteristic parameter. And clearly the second equation of (31) is fulfilled. From the Criterion 2 follows that the optimal conditions are the same to maximum ECOP and efficiency η. Thus the work of (Ust et al., 2005) has been extended to any characteristic parameter and to any heat transfer law. Nevertheless, the optimal conditions coincide for ECOP and η independently of the heat transfer law. They contain different thermodynamic meanings: the efficiency gives information about the necessary fuel consumption in order to produce a certain power level whereas ECOP gives information about the entropy generation (loss rate of availability). If the parameters are x and φ^*, a discussion is presented in (Ust et al., 2005). Also, (Ust et al., 2005) have noted that the actual power equals to the theoretical power output minus the loss rate of availability, i.e. $P = P_{actual} - T_0 S_{gen}$ (cfr. with (Sonntag et al., 2003) for more details). They have concluded that the loss rate of availability has been considered twice in the ecological function. It will need interpretation in order to understand what it means thermodynamically.

3. The optimal allocation of the heat exchangers for a Brayton-like cycle

A. Bejan (Bejan, 1988) optimized the power for the endoreversible Brayton cycle and found that the allocation (size) of the heat exchangers is balanced. The Brayton endoreversible model discussed for him corresponds to the cycle 1-2s-3-4s of the Fig. 3. Formerly, H. Leff (Leff, 1987)) was focused on the reversible Brayton cycle and obtained that the efficiency to maximum work corresponds to the CNCA efficiency (equation (10) with one $\mu^* = T_1/T_3$, see Fig. 3. In (Wu et al; 1991) a non-isentropic Brayton model was analyzed and found that the isentropic temperatures ratio (pressure ratio), that maximizes the work, is the same as a CNCA-like model (Aragon-González et al., 2000; 2003). In (Swanson, 1991) the endoreversible model was optimized by log-mean temperature difference for the heat exchangers in hot and cold sides and assumed that it was internally a Carnot cycle. In (Chen et. al., 2001) the numerical optimization for density power and distribution of a heat exchangers for the endoreversible Brayton cycle is presented. Other optimizations of Brayton-like cycles can be found in the following reviews (Durmayas et al. 1997; Hoffman et al. 2003). Recent optimizations of Brayton-like models were made in (Herrera et al., 2006; Lewins, 2005; Ust, 2006; Wang et al., 2008).

For the isentropic Brayton cycle (1-2s-3-4s-1) its efficiency is given by (Fig. 3):

$$\eta = 1 - x \tag{40}$$

where $x = \varepsilon^{1 - \frac{1}{\gamma}}$, with $\varepsilon = p_{2s}/p_1$ the pressure ratio (maximum pressure divided by minimum pressure) and $\gamma = c_p/c_v$, with c_p and c_v being the constant-pressure and the constant-volume specific heats. Furthermore, the following temperature relations are satisfied:

$$T_{2s} = \frac{T_1}{x};\ T_{4s} = T_3 x \tag{41}$$

where x is given by the equation (40). If a non-isentropic Brayton cycle, without external irreversibilities (see 1-2-3-4 cycle in Fig. 3) is considered, with isentropic efficiencies of the turbine and compressor η_1 and η_2, respectively, and from here the following temperature relations are obtained (Aragón-González et al., 2000):

$$\begin{gathered}\eta_1 = \frac{T_3 - T_4}{T_3 - T_{4s}};\ \eta_2 = \frac{T_{2s} - T_1}{T_2 - T_1};\\ T_2 = T_1\left(1 + \frac{1 - x}{\eta_2 x}\right);\ T_4 = T_3(1 - \eta_1(1 - x))\end{gathered} \tag{42}$$

Now, if we consider the irreversible Brayton cycle of the Fig. 3, the temperature reservoirs are given by the constant temperatures T_H and T_L. In this cycle, two single-pass counterflow heat exchangers are coupled to the cold-hot side reservoirs (Fig. 2 and Fig. 3). The heat transfer between the reservoirs and the working substance can be calculated by the log mean temperature difference LMTD (equation (2)). The heat transfer balances for the hot-side are (equations (1) and (6)):

$$Q_H = U_H\ A_H\,LMTD_H = mc_p\left(T_3 - T_2\right);\ Q_L = U_L\ A_L\ LMTD_H = mc_p\left(T_4 - T_1\right) \tag{43}$$

where $LMTD_{H.L}$ are given by the equations (4). The number of transfer units NTU for both sides are (equation (7)):

$$N_H = \frac{U_H\ A_H}{mc_p} = \frac{T_3 - T_2}{LMTD_H};\quad N_L = \frac{U_L\ A_L}{mc_p} = \frac{T_4 - T_1}{LMTD_L} \tag{44}$$

Then, its effectiveness (equation (9)):

$$\varepsilon_H = 1 - e^{-N_H} = \frac{T_3 - T_2}{T_H - T_2};\quad \varepsilon_L = 1 - e^{-N_L} = \frac{T_4 - T_1}{T_4 - T_L} \tag{45}$$

As the heat exchangers are counterflow, the heat conductance of the hot-side (cold side) is $U_H A_H$ ($U_L A_L$) and the thermal capacity rate (mass and specific heat product) of the working substance is C_W. The heat transfer balances results to be:

$$Q_H = C_W\varepsilon_H\left(T_H - T_2\right) = C_W\left(T_3 - T_2\right);\quad Q_L = C_W\varepsilon_L\left(T_4 - T_L\right) = C_W\left(T_4 - T_1\right) \tag{46}$$

The temperature reservoirs T_H and T_L are fixed. The expressions for the temperatures T_2 and T_4, including the isentropic efficiencies η_1 and η_2, the effectiveness ε_H and ε_L and $\mu = T_L/T_H$ are obtained combining equations (41), (42), and (45):

$$T_2 = \frac{\left[\varepsilon_L \mu x^{-1} + \varepsilon_H\left(1 - \varepsilon_L\right)\right]\left(\frac{1 - x}{\eta_2} + x\right)}{\left[\varepsilon_L + \varepsilon_H\left(1 - \varepsilon_L\right)\right]} T_H,\quad T_4 = \frac{\left[\varepsilon_H x + \varepsilon_L \mu\left(1 - \varepsilon_H\right)\right]\left(\frac{1}{x} - \frac{(1 - x)\eta_1}{x}\right)}{\left[\varepsilon_L + \varepsilon_H\left(1 - \varepsilon_L\right)\right]} T_H \tag{47}$$

And, the dimensionless expressions, $q = Q / C_W T_H$, for the hot-cold sides are:

$$q_H = \varepsilon_H\left(1 - \frac{T_2}{T_H}\right); \quad q_L = \varepsilon_L\left(\frac{T_4}{T_H} - \mu\right) \tag{48}$$

From the first law of the Thermodynamic, the dimensionless work $w = W/C_W T_H$ of the cycle is given by:

$$w = \varepsilon_H\left[1 - \frac{T_2}{T_H}\right] - \varepsilon_L\left[\frac{T_4}{T_H} - \mu\right] \tag{49}$$

and substituting the equations (47), the following analytical relation is obtained:

$$w = \varepsilon_H\left[1 - \frac{\varepsilon_L \mu x^{-1} + \varepsilon_H(1 - \varepsilon_L)}{\varepsilon_L + \varepsilon_H(1 - \varepsilon_L)}\left(\frac{1 - x}{\eta_2} + x\right)\right] - \varepsilon_L\left[\frac{\varepsilon_H x + \varepsilon_L \mu(1 - \varepsilon_H)}{\varepsilon_L + \varepsilon_H(1 - \varepsilon_L)}\left(\frac{1}{x} - \frac{(1 - x)\eta_1}{x}\right) - \mu\right] \tag{50}$$

This relation will be focused on the analysis of the optimal operating states. There are three limiting cases: isentropic [$\varepsilon_H = \varepsilon_L = \eta_1 = \eta_2 = 1$]; non-isentropic [$\varepsilon_H = \varepsilon_L = 1$, $0 < \eta_1, \eta_2 < 1$]; and endoreversible [$\eta_1 = \eta_2 = 1$, $0 < \varepsilon_H, \varepsilon_L < 1$]. Nevertheless, only the endoreversible cycle is relevant for the allocation of the heat exchangers (see subsection 3.2). However, conditions for regeneration for the non-isentropic cycle are analyzed in the following subsection.

3.1 Conditions for regeneration of a non-isentropic Brayton cyle for two operation regimes

J. D. Lewins (Lewins, 2005) has recognized that the extreme temperatures are subject to limits: a) the environmental temperature and; b) in function of the limits on the adiabatic flame or for metallurgical reasons. The thermal efficiency η (see equation (40)) is maximized without losses, if the pressure ratio ε_p grows up to the point that the compressor output temperature reaches its upper limit. These results show that there is no heat transferred in the hot side and as a consequence the work is zero. The limit occurs when the inlet temperature of the compressor equals the inlet temperature of the turbine; as a result no heat is added in the heater/combustor; then, the work vanishes if $\varepsilon_p = 1$. Therefore at some intermediate point the work reaches a maximum and this point is located close to the economical optimum. In such condition, the outlet temperature of the compressor and the outlet temperature of the turbine are equal ($T_{2s} = T_{4s}$; see Fig. 3). If this condition is not fulfilled ($T_{2s} \neq T_{4s}$), it is advisable to couple a heat regeneration in order to improve the efficiency of the system if $T_{2s} < T_{4s}$ (Lewins, 2005). A similar condition is presented when internal irreversibilities due to the isentropic efficiencies of the turbine (η_1) and compressor (η_2) are taken into account (non-isentropic cycle): $T_2 < T_4$ (see Fig. 3 and equation (20) of (Zhang et al., 2006)).

The isentropic cycle corresponds to a Brayton cycle with two coupled reversible counterflow heat exchangers (1-2s-3-4s in Fig. 3). The supposition of heat being reversibly exchanged (in a balanced counterflow heat exchanger), is an equivalent idealization to the supposed heat transfer at constant temperature between the working substance of a Carnot (or Stirling) isentropic cycle, and a reservoir of infinite heat capacity. In this cycle $C_W T_H = mc_p T_3$, $T_H = T_3$, $T_H = T_3$ and $T_L = T_1$, then,

$$w = (1 - x) - \left(\frac{1}{x} - 1\right)\mu^*; \quad q_H = 1 - x\mu^*; \quad q_L = x - \mu^* \tag{51}$$

For maximum work:

$$x_{mw} = \sqrt{\mu^*}\ ; \quad \eta_{CNCA} = 1 - \sqrt{\mu^*} \tag{52}$$

where $\mu^* = \frac{T_1}{T_3}$ and η_{CNCA} corresponds to the CNCA efficiency (equation (10)). Furthermore, in condition of maximum work:

$$\frac{T_1}{T_{2s}} = x_{mw}\ ; \quad \frac{T_1}{T_{4s}} = \frac{T_1}{T_3}\frac{T_3}{T_{4s}} = \frac{x_{mw}^2}{x_{mw}} = x_{mw} \tag{53}$$

so $T_{2s} = T_{4s}$. In other conditions of operation, when $T_{2s} < T_{4s}$, a regenerator can be coupled to improve the efficiency of the cycle. An example of a regenerative cycle is provided in (Sontagg et al., 2003).

On the other hand, the efficiency of the isentropic cycle can be maximized by the following criterion (Aragón-González et al., 2003).

Criterion 3. Let $\eta = \frac{w}{q_H} = 1 - \frac{q_L}{q_H}$. Suppose that $\frac{\partial^2 q_H}{\partial x^2} < 0$ and $\frac{\partial^2 q_L}{\partial x^2} = 0$, for some x. Then, the maximum efficiency η_{max} is given by:

$$\eta_{max} = \frac{\frac{\partial w}{\partial x}\Big|_{x=x_{me}}}{\frac{\partial q_H}{\partial x}\Big|_{x=x_{me}}} = 1 - \frac{\frac{\partial q_L}{\partial x}\Big|_{x=x_{me}}}{\frac{\partial q_H}{\partial x}\Big|_{x=x_{me}}} \tag{54}$$

where x_{me} is the value for which the efficiency reaches its maximum.

Criterion 3 hypothesis are clearly satisfied: $\frac{\partial^2 q_H}{\partial x^2} < 0$ and $\frac{\partial^2 q_L}{\partial x^2} = 0$ for some x (Fig. 6). Thus, the maximum efficiency η_{max} is given by the equation (54):

$$1 - \frac{\frac{1}{\mu}}{\frac{1}{x_{me}^2}} = 1 - \frac{x_{me}^2}{\mu} \tag{55}$$

In solving, $x_{me} = \mu$ and $\eta_{max} = 1 - \mu$ which corresponds to the Carnot efficiency; the other root, $x_{me} = 0$, is ignored. And the work is null for $x_{me} = \mu$; as a consequence the added heat is also null (Fig. 6). Now regeneration conditions for the non-isentropic cycle will be established.

Again $C_W T_H = mc_p T_3$, $T_H = T_3$ and $T_L = T_1$ (cycle 1-2-3-4 in Fig. 3) and T_2 and T_4 are given by the equations (42). Thus, using equations (42) and the structure of the work in the equation (51), the work w and the heat q_H are:

$$\begin{aligned} w &= \eta_1(1 - x) - \frac{1}{\eta_2}\left(\frac{1}{x} - 1\right)\mu^*; \\ q_H &= \left[1 - \left(1 + \frac{(1 - x)}{\eta_2 x}\right)\mu^*\right] \end{aligned} \tag{56}$$

Maximizing,

$$T_{4s} = IT_{2s}\ ; \ x_{NI} = \sqrt{I\mu^*} \ \text{and} \ \eta_{NI} = 1 - \frac{I\eta_2\left(1 - \mu^*\right) + \sqrt{I\mu^*} - 1}{\sqrt{I}\left(\sqrt{I}\eta_2\left(1 - \mu^*\right) + \sqrt{\mu^*}\left(\sqrt{I\mu^*} - 1\right)\right)} \tag{57}$$

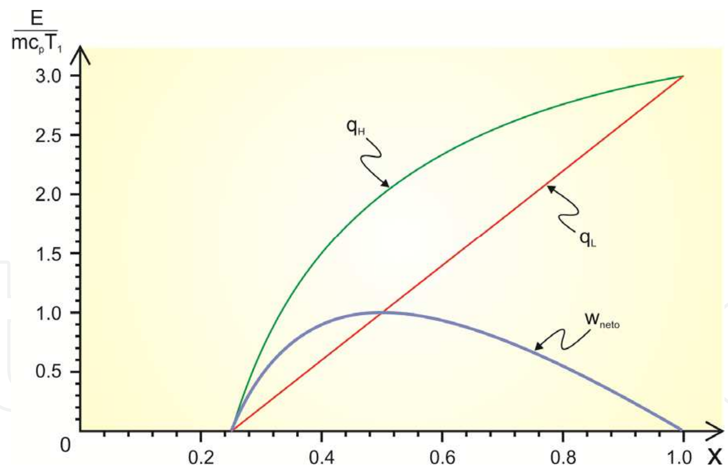

Fig. 6. Heat and work qualitative behavior for μ=0.25.

where $I = 1/\eta_1\eta_2$ and η_{NI} is the efficiency to maximum work of the non-isentropic cycle. Furthermore, the hypotheses from the Criterion 3 are fulfilled (the qualitative behavior of w and q_H is preserved, Fig. 6). In solving the resulting cubic equation, the maximum efficiency, its extreme value and the inequality that satisfies are obtained:

$$\begin{aligned} \eta_{max} &= 1 - \frac{\eta_2}{\mu\eta_1}\left(\frac{\eta_1\mu + \sqrt{\eta_1\mu(1-\mu)((1-\eta_2)\mu + \eta_2(1-\eta_1))}}{\mu(1-\eta_2)+\eta_2}\right)^2 \\ x_{me} &= \frac{\eta_1\mu + \sqrt{\eta_1\mu(1-\mu)(1-\eta_2)\mu + \eta_2(1-\eta_1)}}{\eta_1(\mu(1-\eta_2)+\eta_2)} \\ I\mu &\le x_{me} \le x_{mw} \end{aligned} \tag{58}$$

Now, following (Zhang et al., 2006), in a Brayton cycle a regenerator is used only when the temperature of the exhaust working substance, leaving the turbine, is higher than the exit temperature in the compressor ($T_4 > T_2$). Otherwise, heat will flow in the reverse direction decreasing the efficiency of the cycle. This point can be directly seen when $T_4 < T_2$, because the regenerative rate is smaller than zero and consequently the regenerator does not have a positive role. From equations (42) the following relation is obtained:

$$T_4 = T_3(1 - \eta_1(1-x)) > T_1\left(1 + \frac{1}{\eta_2}\left(\frac{1}{x} - 1\right)\right) = T_2 \tag{59}$$

which corresponds to a temperature criterion which is equivalent to the first inequality of:

$$x > x_{min} = \frac{-\beta + \sqrt{\beta^2 + 4I\mu}}{2}; \ \beta = \left(\tfrac{1}{\eta_1} - 1\right) + \left(I - \tfrac{1}{\eta_1}\right)\mu > 0 \tag{60}$$

Indeed, from the equation (59):

$$\begin{aligned} &x^2 + \beta x - I\mu > 0 \\ &x > x_{min} = \frac{-\beta + \sqrt{\beta^2 + 4I\mu}}{2} > 0 \end{aligned} \tag{61}$$

the inequality is fulfilled since $\sqrt{\beta^2 + 4I\mu} > \beta$. The other root is clearly ignored. Therefore, if $x \le x_{min}$, a regenerator cannot be used. Thus, the first inequality of (60) is fulfilled.

Criterion 3. If the cycle operates either to maximum work or efficiency, a counterflow heat exchanger (regenerator) between the turbine and compressor outlet is a good option to improve the cycle. For other operating regimes is enough that the inequality (61) be fulfilled. When the operating regime is at maximum efficiency the inequality of (61) is fulfilled. Indeed,

$$\begin{gathered} x_{me} > x_{min} \\ x_{me} - x_{min} = \eta_1\mu + \beta + \left(\frac{\sqrt{\beta^2 + 4I\mu}}{2} - \frac{\sqrt{\eta_1\mu(1-\mu)((1-\eta_2)\mu + \eta_2(1-\eta_1))}}{\eta_1(\mu(1-\eta_2) + \eta_2)} \right) \\ \left(\frac{\sqrt{\beta^2 + 4I\mu}}{2} \right)^2 - \left(\frac{\sqrt{\eta_1\mu(1-\mu)((1-\eta_2)\mu + \eta_2(1-\eta_1))}}{\eta_1(\mu(1-\eta_2) + \eta_2)} \right)^2 = ... \\ \eta_1^2\left((\eta_2(1-\mu) + \mu)\left(\beta^2(\eta_2(1-\mu) + \mu) + 4I\mu^2\right) + 4\eta_2\mu(1-\mu)\right) > 0 \\ \frac{\sqrt{\beta^2 + 4I\mu}}{2} > \frac{\sqrt{\eta_1\mu(1-\mu)((1-\eta_2)\mu + \eta_2(1-\eta_1))}}{\eta_1(\mu(1-\eta_2) + \eta_2)} \end{gathered} \tag{62}$$

where the following elementary inequality has been applied: If a, b > 0, then $a < b \Leftrightarrow a^2 < b^2$. If the operating regime is at maximum work, the proof is completely similar to the equations (62). An example of a non-isentropic regenerative cycle is provided in (Aragón-González et al., 2010).

3.2. Optimal analytical expressions

If the total number of transfer units of both heat exchangers is N, then, the following parameterization of the total inventory of heat transfer (Bejan, 1988) can be included in the equation (50):

$$N_H + N_L = N; \quad N_H = yN \text{ and } N_L = (1-y)N \tag{63}$$

For any heat exchanger $N = \frac{UA}{C}$, where U is the overall heat-transfer coefficient, A the heat-transfer surface and C the thermal capacity. The number of transfer units in the hot-side and cold-side, N_H and N_L, are indicative of both heat exchangers sizes. And their respective effectiveness is given by (equation (9)):

$$\varepsilon_H = 1 - e^{-yN}; \quad \varepsilon_L = 1 - e^{-(1-y)N} \tag{64}$$

Then, the work w (equation (50)) depends only upon the characteristics parameters x and y. Applying the extreme conditions: $\frac{\partial w}{\partial x} = 0$; $\frac{\partial w}{\partial y} = 0$, the following coupled optimal analytical expressions for x and y, are obtained:

$$\begin{gathered} x_{NE} = \sqrt{\frac{(z_1 - z)(Cz - B)}{(z-1)(Az_1 - Bz)}\mu}; \\ y_{NE} = \frac{1}{2} + \frac{1}{2N}\ln\left(\frac{Ax - B\mu}{Bx - C\mu}\right) \end{gathered} \tag{65}$$

where $z_1 = e^N$; $z = e^{yN}$; $A = \eta_1\eta_2 e^N + 1 - \eta_2$; $B = e^N(\eta_1\eta_2 + 1 - \eta_2)$ and $C=e^N-\eta_2 + \eta_1\eta_2$.
The equations (65) for x_{NE} and y_{NE} cannot be uncoupled. A qualitative analysis and its asymptotic behavior of the coupled analytical expressions for x_{NE} and y_{NE} (equations (65)) have been performed (Aragón-González (2005)) in order to establish the bounds for x_{NE} and y_{NE} and to see their behaviour in the limit cases. Thus the following bounds for x_{NE} and y_{NE} were found:

$$0 < x_{NI} \leq x_{NE} < 1; \quad 0 < y_{NE} < \tfrac{1}{2} \tag{66}$$

where x_{NI} is given by the equation (57). The inequality (66) is satisfied because of $1 < I \leq \frac{(z_1 - z)(Cz - B)}{(z - 1)(Az_1 - Bz)}$. If $I = 1$ ($\eta_1 = \eta_2 = 100\%$), the following values are obtained: $x_{NE} = x_{CNCA} = \sqrt{\mu}$; $y_{NE} = y_E = ½$ which corresponds to the endoreversible cycle. In this case necessarily: $\varepsilon_H = \varepsilon_L = 1$. Thus, the equations (65) are one generalization of the endoreversible case [$\eta_1 = \eta_2 = 1$, $0 < \varepsilon_H, \varepsilon_L < 1$]. The optimal allocation (size) of the heat exchangers has the following asymptotic behavior: $\lim_{N\to\infty} y_{NE} = 1/2$; $\lim_{\eta_1,\eta_2\to 1} y_{NE} = 1/2$. Also, x_{NE} has the following asymptotic behavior: $\lim_{N\to\infty} x_{NE} = x_{NI}$; $\lim_{N\to\infty} \eta_{NE} = \eta_{NI}$. Thus, the non-isentropic [$\varepsilon_H = \varepsilon_L = 1$, $0 < \eta_1, \eta_2 < 1$] and endoreversible [$\eta_1 = \eta_2 = 1$, $0 < \varepsilon_H, \varepsilon_L < 1$] cycles are particular cases of the cycle herein presented. A relevant conclusion is that the allocation always is unbalanced ($y_{NE} < ½$).
Combining the equations (65), the following equation as function only of z, is obtained:

$$\sqrt{\mu}\left(\frac{Bz_1 - Cz^2}{Az_1 - z^2B}\right) = \sqrt{\frac{(z_1 - z)(Cz - B)}{(z - 1)(Az_1 - Bz)}} \tag{67}$$

which gives a polynomial of degree 6 which cannot be solved in closed form. The variable *z* relates (in exponential form) to the allocation (unbalanced, $\varepsilon_H < \varepsilon_L$) and the total number of transfer units N of both heat exchangers. To obtain a closed form for the effectiveness ε_H, ε_L, the equation (67) can be approximated by:

$$\sqrt{\mu}\left(\frac{Bz_1 - Cz^2}{Az_1 - z^2B}\right) = \left(\frac{1}{2} + \frac{1}{2}H\right) \tag{68}$$

with $H = \frac{(z_1 - z)(Cz - B)}{(z - 1)(Az_1 - Bz)}$; and using the linear approximation: $\sqrt{H}=1 + \tfrac{1}{2}(H-1)+O\left((H-1)^2\right)$.
It is remarkable that the non-isentropic and endoreversible limit cases are not affected by the approximation and remain invariant within the framework of the model herein presented. Thus, this approximation maintains and combines the optimal operation conditions of these limit cases and, moreover, they are extended. The equation (68) is a polynomial of degree 4 and it can be solved in closed form for z with respect to parameters: μ or N, for realistic values for the isentropic efficiencies (Bejan (1996)) of turbine and compressor: $\eta_1 = \eta_2 = 0.8$ or 0.9, but it is too large to be included here. Fig. 7 shows the values of z (z_{mp}) with respect to μ. Using the same numerical values, Fig. 8 shows that the efficiency to maximum work η_{NE}, with respect to μ, can be well approached by the efficiency of the non-isentropic cycle η_{NI} (equation (57)) for a realistic value of N = 3 and isentropic efficiencies of 90%. The behavior of y_{NE} with respect to the total number of transfer units N of both heat exchangers, with the same numerical values for the isentropic efficiencies of turbine and compressor and μ = 0.3, are

presented in Fig. 9. When the number of heat transfer units, N, is between 2 to 5, the allocation for the heat exchangers y_{NE} is approximately 2 - 8% or 1 - 3%, less than its asymptotic value or ½, respectively.

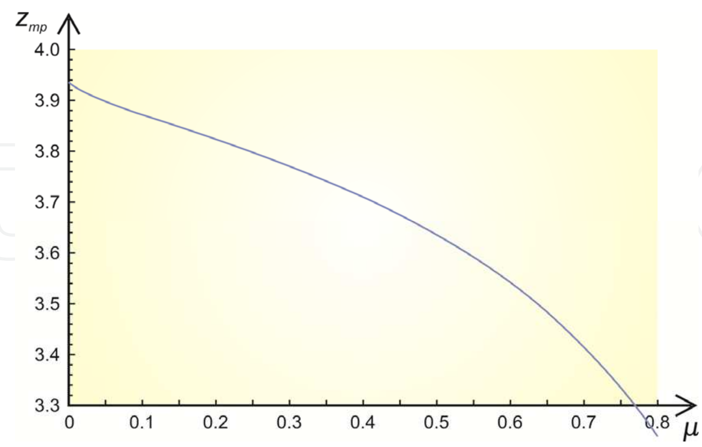

Fig. 7. Behaviour of $z(z_{mp})$ versus μ, if $\eta_1 = \eta_2 = 0.8$ and N=3.

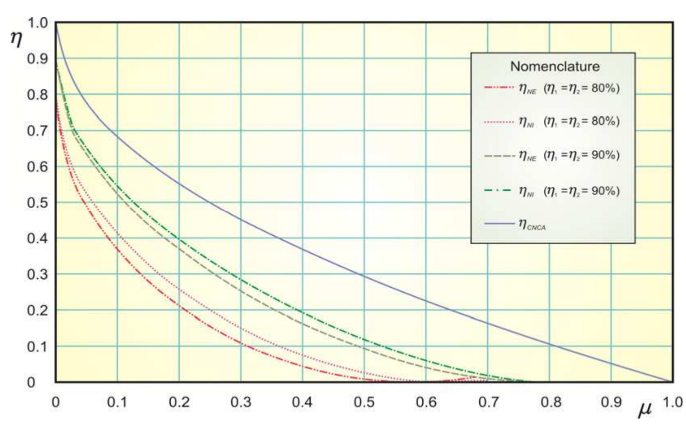

Fig. 8. Behaviour of η_{NE}, η_{NI} and η_{CNCA} versus μ, if $\eta_1 = \eta_2 = 0.8$ or 0.9 and N=3.

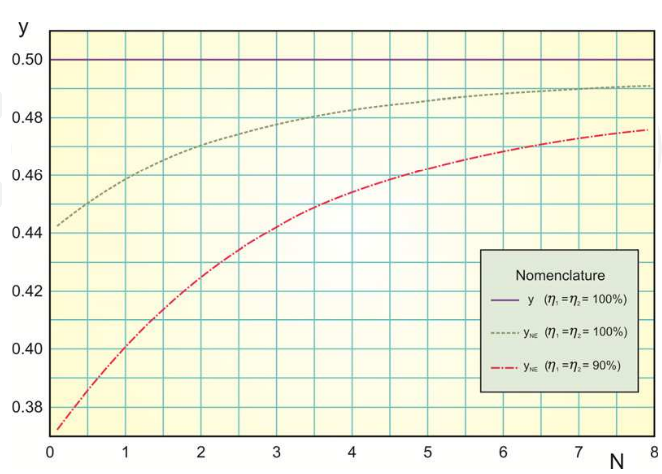

Fig. 9. Behavior of y_{NE} versus N, when $\eta_1 = \eta_2 = 0.8$ or 0.9 and $\mu = 0.3$.

This result shows that the size of the heat exchanger in the hot side decreases. Now, if the Carnot efficiency is 70% the efficiency η_{NE} is approximately 25 - 30% or 10 - 15%, when the

number of heat transfer units N is between 2 and 5 and the isentropic efficiencies are $\eta_1 = \eta_2 = 0.9$ or 0.8 respectively, as is shown in Fig. 9.
Now, if $\eta_1 = \eta_2 = 0.8$ (I = 1.5625) ; $y_{NE} = 0.45$ then $N \cong 3.5$ (see Fig. 9) and for the equations (64): $\varepsilon_H = 0.74076$ and $\varepsilon_L = 0.80795$. Thus, one cannot assume that the effectiveness are the same: $\varepsilon_H = \varepsilon_L < 1$; whilst $I > 1$. Current literature on the Brayton-like cycles, that have taken the same less than one effectiveness and with internal irreversibilities, should be reviewed. To conclude, $\varepsilon_H = \varepsilon_L$ if and only if the allocation is balanced ($y = ½$) and the unique thermodynamic possibility is: optimal allocation balanced ($y_{NE} = y_E = ½$); that is $\varepsilon_H = \varepsilon_L$. And $\varepsilon_H<\varepsilon_L$ if and only if I>1 there is internal irreversibilities.

4. Conclusions

Relevant information about the optimal allocation of the heat exchangers in power cycles has been described in this work. For both Carnot-like and Brayton cycles, this allocation is unbalanced. The expressions for the Carnot model herein presented are given by the Criterion 1 which is a strong contribution to the problem (following the spirit of Carnot's work): to seek invariant optimal relations for different operation regimes of Carnot-like models, independently from the heat transfer law. The equations (26)-(28) have the above characteristics. Nevertheless, the optimal isentropic temperatures ratio depends of the heat transfer law and of the operation regime of the engine as was shown in the subsection 2.2 (Fig. 5). Moreover, the equations (26) can be satisfied for other objective functions and other characteristic parameter: For instance, algebraic combination of power and/or efficiency and costs per unit heat transfer; as long as these objective functions and parameters have thermodynamic sense. Of course, the objective function must satisfy similar conditions to the equations (20) and (21). But this was not covered by this chapter's scope.
The study performed for the Brayton model combined and extended the optimal operation conditions of endoreversible and non-isentropic cycles since this model provides more realistic values for efficiency to maximum work and optimal allocation (size) for the heat exchangers than the values corresponding to the non-isentropic or the endoreversible operations. A relevant conclusion is that the allocation always is unbalanced ($y_{NE} < ½$). Furthermore, the following correlation can be applied between the effectiveness of the exchanger heat of the hot and cold sides:

$$\varepsilon_H = \frac{1 - \frac{1}{z_{NE}}}{1 - z_{NE}e^{-N}}\varepsilon_L \tag{69}$$

where z_{NE} is calculated by the equation (68) and shown in Fig. 7, which can be used in the current literature on the Brayton-like cycles. In subsection 3.1 the problem of when to fit a regenerator in a non-isentropic Brayton cycle was presented and criterion 3 was established. On the other hand, the qualitative and asymptotic analysis proposed showed that the non-isentropic and endoreversible Brayton cycles are limit cases of the model of irreversible Brayton cycle presented which leads to maintain the performance conditions of these limit cases according to their asymptotic behavior. Therefore, the non-isentropic and endoreversible Brayton cycles were not affected by our analytical approximation and remained invariant within the framework of the model herein presented. Moreover, the optimal analytical expressions for the optimal isentropic temperatures ratio, optimal allocation (size) for the heat exchangers, efficiency to maximum work and maximum work obtained can be more useful than those we found in the existing literature.

Finally, further work could comprise the analysis of the allocation of heat exchangers for a combined (Brayton and Carnot) cycle with the characteristics and integrating the methodologies herein presented.

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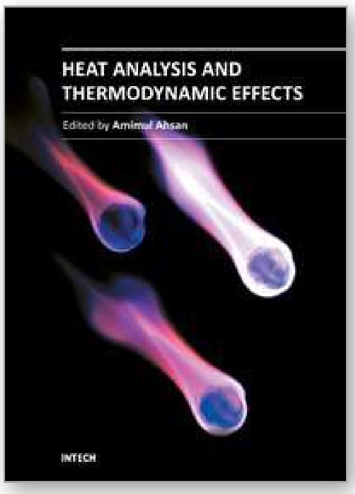

Heat Analysis and Thermodynamic Effects

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The heat transfer and analysis on heat pipe and exchanger, and thermal stress are significant issues in a design of wide range of industrial processes and devices. This book includes 17 advanced and revised contributions, and it covers mainly (1) thermodynamic effects and thermal stress, (2) heat pipe and exchanger, (3) gas flow and oxidation, and (4) heat analysis. The first section introduces spontaneous heat flow, thermodynamic effect of groundwater, stress on vertical cylindrical vessel, transient temperature fields, principles of thermoelectric conversion, and transformer performances. The second section covers thermosyphon heat pipe, shell and tube heat exchangers, heat transfer in bundles of transversely-finned tubes, fired heaters for petroleum refineries, and heat exchangers of irreversible power cycles. The third section includes gas flow over a cylinder, gas-solid flow applications, oxidation exposure, effects of buoyancy, and application of energy and thermal performance index on energy efficiency. The forth section presents integral transform and green function methods, micro capillary pumped loop, influence of polyisobutylene additions, synthesis of novel materials, and materials for electromagnetic launchers. The advanced ideas and information described here will be fruitful for the readers to find a sustainable solution in an industrialized society.