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# Stabilization of Fuzzy Takagi - Sugeno Descriptor Models; Application to a Double Inverted Pendulum

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## 1. Introduction

Takagi-Sugeno (TS) fuzzy models (Takagi & Sugeno 1985) have been widely used in the context of control and observation of nonlinear models, see for example (Tanaka & Wang 2001) and the references therein. They consist in a collection of linear models blended together with nonlinear membership functions. Their capability to represent exactly in a compact set of the state variables a nonlinear model makes them attractive for control and observation (Taniguchi et al. 2001). The stability and the stabilization of such models (including performances and/or robustness considerations) are mainly investigated through Lyapunov functions (Chen et al. 2000, Joh et al. 1997, Tanaka et al. 1996, Tanaka et al. 1998, Tong et al. 2002, Tuan et al. 2001, Zhao 1995). These ones are most of the time quadratic ones, nevertheless interesting results can also be found using piecewise quadratic functions (Feng 2003, Johansson et al. 1999) or non quadratic Lyapunov functions (Blanco et al. 2001, Guerra & Vermeiren 2004, Tanaka et al. 2001). At last, TS fuzzy descriptors have been studied for the stabilization and the observation points of view and some results are given in (Guerra et al. 2004, Tanaka & Wang 2001, Taniguchi et al. 2000). Most of the time an interesting way to solve the different problems addressed is to write the obtained conditions in a LMI form (Linear Matrix Inequalities) (Boyd et al. 1994).

There is a systematic way, called the sector nonlinearity approach (Tanaka & Wang 2001) to go from a nonlinear model affine in the control to a Takagi Sugeno fuzzy model. The number of linear models  $r$  of the nonlinear TS fuzzy model grows exponentially, i.e. in  $2^{nl}$ , with  $nl$  the number of nonlinearities to be treated (Tanaka & Wang 2001, Taniguchi et al. 2001). In the stabilization framework, the conditions of stabilization will only depend on the linear models of the TS models, i.e. the nonlinear membership functions blending the linear models together are not used. Of course, this remains in conservative results. Let us also point out that the number of conditions to be solved in the LMI problems is directly related to the number  $r$  of linear models of the TS fuzzy model. We can emphasize that the more  $r$  is big, the more the results obtained will be conservative. Thus it is of high interest to obtain a TS representation of nonlinear models with a reduced number of rules. Another remark can be formulated. For some models, mechanical ones for example, a descriptor form can be interesting to keep a TS model structure closed to the nonlinear one. In some cases, we will show that it allows reducing the number of rules and therefore it can improve, in an interesting way, the results. Nevertheless using this

specific descriptor structure, it is then necessary to derive conditions of stabilization. Some results can be found in (Tanaka & Wang 2001, Taniguchi et al. 2000). Hence, this work focuses on continuous TS fuzzy models using a descriptor form and the stabilization results will be studied through a quadratic Lyapunov function.

The chapter is organized as follows. The first part gives the notations used and the material necessary to derive the results. It includes some specific matrix properties and basic properties of TS models. The second part presents the statement of the problem using TS models in a descriptor form and an example to show their interest. The third part gives the main result obtained for stabilization. The goal is to use matrix properties in order to reduce the conservatism of the basic conditions. A first academic example is given to show the different results. At last the application of this methodology to the double-inverted pendulum is presented.

## 2. Notations and Material

Let us consider positive scalar functions  $h_i(\cdot) \geq 0$ ,  $i \in \{1, \dots, r\}$  and  $v_k(\cdot) \geq 0$   $k \in \{1, \dots, e\}$  satisfying the convex sum property:

$$\sum_{i=1}^r h_i(\cdot) = 1, \quad \sum_{k=1}^e v_k(\cdot) = 1 \quad (1)$$

With such functions and some matrices of appropriate dimensions  $Y_i$  we define the

following notations:  $Y_h = \sum_{i=1}^r h_i(z(t)) Y_i$ ,  $Y_{hh} = \sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) Y_{ij}$ ,  $Y_v = \sum_{k=1}^e v_k(z(t)) Y_k$ ,

$$Y_{vh} = \sum_{k=1}^e \sum_{i=1}^r v_k(z(t)) h_i(z(t)) Y_{ik}, \text{ and so on}$$

As usual, a star (\*) in a symmetric matrix indicates a transpose quantity. Congruence of a symmetric definite positive matrix  $P = P^T > 0$  with a full rank matrix  $Y$  corresponds to the following quantity:  $YPY^T > 0$ .

We will also use the following lemma. It is a slightly modified version of a property given in (Peaucelle et al. 2000) and also used in the context of Takagi-Sugeno fuzzy models stabilization (Guerra et al. 2003).

*Lemma 1:*

Let  $P$ ,  $Y$ ,  $\Gamma$ ,  $\Phi$  and  $\Psi$  be matrices of appropriate dimensions the two following properties are equivalent.

$$P^T \Gamma^T + \Gamma P + Y < 0 \quad (2)$$

$$\text{It exists } \Phi \text{ and } \Psi \text{ such that: } \begin{bmatrix} \Phi^T \Gamma^T + \Gamma \Phi + Y & (*) \\ P - \Phi + \Psi^T \Gamma^T & -\Psi - \Psi^T \end{bmatrix} < 0 \quad (3)$$

*Proof:*

(3) implies (2): the result is obtained using the congruence with  $[I \quad \Gamma]$ .

(2) implies (3): As  $P^T \Gamma^T + \Gamma P + Y < 0$ , it always exists an enough small  $\varepsilon^2$  such that:

$$P^T \Gamma^T + \Gamma P + Y + \frac{\varepsilon^2}{2} \Gamma^T \Gamma < 0 \quad (4)$$

Using the Schur's complement, (4) is equivalent to:

$$\begin{bmatrix} P^T \Gamma^T + \Gamma P + Y & \varepsilon^2 \Gamma^T \\ \varepsilon^2 \Gamma & -2\varepsilon^2 I \end{bmatrix} < 0 \quad (5)$$

If we choose  $\Phi = P$  and  $\Psi = \varepsilon^2 I$ , then the first inequality of (2) holds.

*Remark 1:*

If  $\Phi$  or  $\Psi$  are under constraint, the equivalence is not more true.

Most of the LMI problems encountered for TS stabilization can be resumed in the following way. For a given  $k$ , with  $\Upsilon_{ij}^k$ ,  $i, j \in \{1, \dots, r\}$  expressions being independent from time, find the best conditions, i.e. in the sense of reducing the conservatism, to the problem:

$$\sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) (\Upsilon_{ij}^k + \Upsilon_{ji}^k) < 0 \quad (6)$$

Several results are available, going from the very simple one (Tanaka et al. 1998):

$$\Upsilon_{ii}^k \leq 0, \quad \Upsilon_{ij}^k + \Upsilon_{ji}^k \leq 0, \quad i, j \in \{1, \dots, r\}, \quad j > i \quad (7)$$

to very specific matrix transformations (Kim & Lee 2000, Teixeira et al. 2003, Liu & Zhang 2003). Let us point out that whatever the relaxations are they can be used on the  $\Upsilon_{ij}^k$ . We will just give the one of (Liu & Zhang 2003) that seems to be a good compromise between complexity and number of variables involved in the LMI problem. The work presented in (Teixeira et al, 2003) can also be quoted, but it implies a serious increase of the number of variables involved in the problem.

*Lemma 2* (Liu & Zhang 2003):

With  $\Upsilon_{ij}^k$ ,  $i, j \in \{1, \dots, r\}$  matrices of appropriate dimension, (6) holds if there exist matrices  $Q_{ii}^k > 0$  and  $Q_{ij}^k = (Q_{ji}^k)^T$ ,  $i, j \in \{1, \dots, r\}$   $j > i$  such that the following conditions are satisfied:

$$\Upsilon_{ii}^k + Q_{ii}^k < 0 \quad (8)$$

$$\Upsilon_{ij}^k + \Upsilon_{ji}^k + Q_{ij}^k + Q_{ji}^k \leq 0 \quad (9)$$

$$Q^k = \begin{bmatrix} Q_{11}^k & (*) & & (*) \\ Q_{12}^k & Q_{22}^k & & \\ \vdots & & \ddots & (*) \\ Q_{1r}^k & \dots & Q_{(r-1)r}^k & Q_{rr}^k \end{bmatrix} > 0 \quad (10)$$

The models under consideration in this chapter are the so-called Takagi-Sugeno's ones (Takagi & Sugeno 1985). They correspond to linear models blended with nonlinear functions (12). They can represent exactly a large class of affine nonlinear models in compact region of the state space (Tanaka & Wang 2001, Taniguchi et al. 2001). From a nonlinear model with  $x(t)$  the state,  $u(t)$  the input vector and  $y(t)$  the output vector:

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u(t) \\ y(t) = h(x(t)) \end{cases} \quad (11)$$

there exists a systematic way called the sector nonlinearity approach (Tanaka & Wang 2001) to put it into a TS form (see example 1 hereinafter):

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(z(t)) (A_i x(t) + B_i u(t)) = A_z x(t) + B_z u(t) \\ y(t) = \sum_{i=1}^r h_i(z(t)) (C_i x(t)) = C_z x(t) \end{cases} \quad (12)$$

with  $r$  the number of linear models,  $z(t)$  a vector which depends linearly or not on the state,  $h_i(z(t)) \geq 0$ ,  $i \in \{1, \dots, r\}$  nonlinear functions verifying the convex sum property (1).

The number  $r$  of linear models grows exponentially according to the number of nonlinearities to be treated in the model (11) (Tanaka et al. 1998). Note also that the TS representation of (11) is not unique (Taniguchi et al. 2001).

In order to stabilize this kind of models, classically the control law used is the Parallel Distributed Compensation (PDC) (Wang et al. 1996). The expression of this control law is given by:

$$u(t) = -\sum_{i=1}^r h_i(z(t)) F_i x(t) = -F_z x(t) \quad (13)$$

Basic results of stabilization of TS models with a PDC control law can be found in (Wang et al. 1996).

At last looking at the problem (6), the results do not depend on the nonlinear functions  $h_i(z(t))$  and then can lead to a strong conservatism. Thus it is of high interest to find new ways to reduce this conservatism. One way can be to use other Lyapunov functions (Feng 2003, Guerra & Vermeiren 2004, Johansson et al. 1999). The way explored in this chapter is to use a descriptor form of TS fuzzy models.

### 3. Statement of the Problem

Let us consider a fuzzy descriptor model as (Taniguchi & al. 2001):

$$\sum_{k=1}^e v_k(z(t)) E_k \dot{x}(t) = \sum_{i=1}^r h_i(z(t)) (A_i x(t) + B_i u(t)), \text{ or:} \quad (14)$$

$$\begin{cases} E_v \dot{x}(t) = A_h x(t) + B_h u(t) \\ y(t) = C_h x(t) \end{cases}$$

In the following we suppose that the problem is always well formulated, hypothesis 1.

*Hypothesis 1:*

$$\text{For all } z(t), \sum_{k=1}^e v_k(z(t)) E_k \neq 0 \quad (15)$$

Defining  $x^*(t) = [x^T(t), \dot{x}^T(t)]^T$ , the system (14) can be written as:

$$\begin{cases} E^* \dot{x}^*(t) = A_{hv}^* x^*(t) + B_h^* u(t) \\ y(t) = C_h^* x^*(t) \end{cases} \quad (16)$$

where:

$$E^* = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, A_{ik}^* = \begin{bmatrix} 0 & I \\ A_i & -E_k \end{bmatrix}, B_i^* = \begin{bmatrix} 0 \\ B_i \end{bmatrix}, C_i^* = [C_i \quad 0].$$

Consider a modified PDC control law (Taniguchi et al. 2000):

$$u(t) = -\sum_{i=1}^r \sum_{k=1}^e h_i(z(t)) v_k(z(t)) F_{ik} x(t) \quad (17)$$

then, introducing (17) in (16), with  $F_{hv}^* = [F_{hv} \quad 0]$  leads to:

$$\begin{cases} E^* \dot{x}^*(t) = (A_{hv}^* - B_h^* F_{hv}^*) x^*(t) \\ y(t) = C_h^* x^*(t) \end{cases} \quad (18)$$

According to the work of (Taniguchi & al. 2001) the following theorem conditions ensure the fuzzy descriptor to be quadratically stable.

*Theorem 1:*

The fuzzy descriptor model (18) is quadratically stable if there exists a common matrix  $X$  such that:

$$E^* X = X^T E^* \geq 0 \quad (19)$$

$$(A_{hv}^* - B_h^* F_{hv}^*)^T X + X^T (A_{hv}^* - B_h^* F_{hv}^*) < 0 \quad (20)$$

Proof:

It is straightforward considering the following Lyapunov candidate function:  $V(x^*(t)) = x^{*T}(t) E^{*T} X x^*(t)$ .

The goal is now to propose LMI conditions for ensuring to find  $X$  and the gains  $F_{ik}$ . Let us define:  $X = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix}$ , condition (19) implies:  $X_1 = X_1^T \geq 0$  and  $X_2 = 0$ . Then condition (20) can be written as:

$$\begin{bmatrix} 0 & A_h^T - F_{hv}^T B_h^T \\ I & -E_v^T \end{bmatrix} \begin{bmatrix} X_1 & 0 \\ X_3 & X_4 \end{bmatrix} + \begin{bmatrix} X_1 & X_3^T \\ 0 & X_4^T \end{bmatrix} \begin{bmatrix} 0 & I \\ A_h - B_h F_{hv} & -E_v \end{bmatrix} < 0 \quad (21)$$

With  $X_4$  non-singular, we have:  $X^{-1} = \begin{bmatrix} X_1^{-1} & 0 \\ -X_4^{-1} X_3 X_1^{-1} & X_4^{-1} \end{bmatrix} = \begin{bmatrix} P_1 & 0 \\ P_3 & P_4 \end{bmatrix}$ , and after congruence

with  $X^{-T}$  we obtain:

$$\begin{bmatrix} P_1 & P_3^T \\ 0 & P_4^T \end{bmatrix} \begin{bmatrix} 0 & A_h^T - F_{hv}^T B_h^T \\ I & -E_v^T \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_h - B_h F_{hv} & -E_v \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_3 & P_4 \end{bmatrix} < 0 \quad (22)$$

Let us rewrite (22) as:

$$\begin{bmatrix} P_3^T + P_3 & (*) \\ P_4^T + A_h P_1 - B_h F_{hv} P_1 - E_v P_3 & -P_4^T E_v^T - E_v P_4 \end{bmatrix} < 0 \quad (23)$$

Let us define with  $M_{ik} = F_{ik} P_1$ :

$$\Upsilon_{ij}^k = \begin{bmatrix} P_3^T + P_3 & (*) \\ P_4^T + A_i P_1 - B_i M_{jk} - E_k P_3 & -P_4^T E_k^T - E_k P_4 \end{bmatrix} \quad (24)$$

The following theorem gives the result.

*Theorem 2:*

Let us consider TS the fuzzy descriptor model (18), the  $\Upsilon_{ij}^k$  defined in (24). The TS fuzzy descriptor with control law (17) is quadratically stable if there exists matrices:  $P_1 = P_1^T > 0$ ,  $P_3$ ,  $P_4$  regular,  $M_{ik}$ , such that for each  $k \in \{1, \dots, e\}$  and  $i, j \in \{1, \dots, r\}$ ,  $j > i$  the conditions given equation (7) hold. Moreover the gains of the control law are given by  $F_{ik} = M_{ik} P_1^{-1}$ .

*Remark 2:*

As stated previously, any usual relaxation can be used. For example with the one presented before (Liu & Zhang 2003) the result will be: the TS fuzzy descriptor with control law (17) is quadratically stable if there exists matrices:  $P_1 = P_1^T > 0$ ,  $P_3$ ,  $P_4$  regular,  $M_{ik}$ ,  $Q_{ii}^k > 0$  and  $Q_{ij}^k = (Q_{ji}^k)^T$  such that for each  $k \in \{1, \dots, e\}$  and each  $i, j \in \{1, \dots, r\}$ ,  $j > i$  the conditions given equations (8), (9) and (10) hold. Note also that the number of LMI conditions obtained in both case (excepted (10)) is:  $e \cdot \frac{r \cdot (r+1)}{2}$ .

*Remark 3:*

These first conditions include those presented in (Taniguchi et al. 2000).

To show the interest of this descriptor form formulation we will study a first academic example.

*Example 1:*

Consider the following nonlinear model with  $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$  the state vector:

$$E(x(t)) \cdot \dot{x}(t) = A \cdot \dot{x}(t) + B(x_1(t)) \cdot u(t) \quad (25)$$

with: 
$$A = \begin{bmatrix} -1 & -1 \\ 2 & 6 \end{bmatrix}, E(x(t)) = \begin{bmatrix} \frac{1}{1+x_1^2(t)} & -1 \\ 1 & \frac{1}{1+x_2^2(t)} \end{bmatrix} \text{ and } B(x_1(t)) = \begin{bmatrix} 2 + \cos(x_1) \\ 1 \end{bmatrix}.$$

A TS descriptor in the form of (14) can be obtained. For that, note that there are two nonlinearities in  $E(x(t))$  that leads to  $e = 4$  and one in the right side of (25) which gives

$r = 2$ . To explicit the way to obtain a TS form, we consider the function  $f(x) = \frac{1}{1+x^2}$ . For  $x \in \mathfrak{R}$  it is easy to check that  $f(x)$  belongs to  $[0,1]$  then we can write:

$f(x) = w_1(x) \cdot 1 + w_2(x) \cdot 0$  with  $w_1(x) = \frac{1}{1+x^2}$  and  $w_2(x) = \frac{x^2}{1+x^2}$ . Note that the  $w_i(x)$   $i = 1, 2$  are positive functions and satisfy the convex sum property:  $w_1(x) + w_2(x) = 1$ . Thus using that decomposition for  $E(x(t))$  leads to four models, i.e.:

$$E(x) = w_1(x_1)w_1(x_2) \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} + w_1(x_1)w_2(x_2) \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + w_2(x_1)w_1(x_2) \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} + w_2(x_1)w_2(x_2) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

. Of course, this way of taking into account the nonlinearities can be applied to all bounded nonlinearities (Morère 2001, Tanaka et al. 1998). Then the number of conditions

involved in the LMI problem is  $e \cdot \frac{r \cdot (r+1)}{2} = 12$ .

The conditions of theorem 2 give a solution using MATLAB LMI toolbox, this ensures the stabilization of the TS descriptor.

Considering now a classical TS model (12) for (25) will impose to invert  $E(x(t))$ , i.e.:

$$\dot{x}(t) = E^{-1}(x(t))A \cdot \dot{x}(t) + E^{-1}(x(t))B(x_1(t)) \cdot u(t) \quad (26)$$

Note that 
$$E^{-1}(x(t)) = \begin{bmatrix} \frac{1+x_1^2}{(1+x_1^2) \cdot (1+x_2^2) + 1} & \frac{(1+x_1^2) \cdot (1+x_2^2)}{(1+x_1^2) \cdot (1+x_2^2) + 1} \\ \frac{-(1+x_1^2) \cdot (1+x_2^2)}{(1+x_1^2) \cdot (1+x_2^2) + 1} & \frac{1+x_2^2}{(1+x_1^2) \cdot (1+x_2^2) + 1} \end{bmatrix},$$
 then after some easy but

fastidious calculus it can be shown that the four nonlinearities:  $1+x_1^2$ ,  $1+x_2^2$ ,  $\frac{1}{(1+x_1^2) \cdot (1+x_2^2) + 1}$  and  $2 + \cos(x_1)$  have to be treated to obtain a TS model. This will give a

TS model with  $r = 2^4 = 16$  linear models and then a LMI problem with  $136$  LMI! For

example, considering a compact set of the state variable:  $x_1(t), x_2(t) \in [-6, 6]$  no solution was obtained, even using the relaxation of (Liu & Zhang 2003).

This example clearly shows that keeping the TS form close to the nonlinear model can be helpful. The goal of the next section is to try to reduce the conservatism of the conditions obtained in the theorem 2.

#### 4. Main Result

With the change of variable:  $M_{hv} = F_{hv}P_1$ , let us rewrite (22) in the following way:

$$\begin{bmatrix} P_1 & P_3^T \\ 0 & P_4^T \end{bmatrix} \begin{bmatrix} 0 & A_h^T \\ I & -E_v^T \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_h & -E_v \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_3 & P_4 \end{bmatrix} + \begin{bmatrix} 0 & -M_{hv}^T B_h^T \\ -B_h M_{hv} & 0 \end{bmatrix} < 0 \quad (27)$$

Let us consider the property described in lemma 1, equation (2), i.e.  $P^T \Gamma^T + \Gamma P + Y < 0$ ,

with  $P = \begin{bmatrix} P_1 & 0 \\ P_3 & P_4 \end{bmatrix}$ ,  $\Gamma = \begin{bmatrix} 0 & I \\ A_h & -E_v \end{bmatrix}$  and  $Y = \begin{bmatrix} 0 & -M_{hv}^T B_h^T \\ -B_h M_{hv} & 0 \end{bmatrix}$  then using the equivalence

with (3) and  $\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{bmatrix}$ ,  $\Psi = \begin{bmatrix} \Psi_1 & \Psi_2 \\ \Psi_3 & \Psi_4 \end{bmatrix}$ , condition (22) is satisfied if:

$$\begin{bmatrix} \Phi_3^T + \Phi_3 & (*) & (*) & (*) \\ \Phi_4^T + A_h \Phi_1 - E_v \Phi_3 - B_h M_{hv} & \Phi_2^T A_h^T + A_h \Phi_2 - \Phi_4^T E_v^T - E_v \Phi_4 & (*) & (*) \\ P_1 - \Phi_1 + \Psi_3^T & -\Phi_2 + \Psi_1^T A_h^T - \Psi_3^T E_v^T & -\Psi_1^T - \Psi_1 & (*) \\ P_3 - \Phi_3 + \Psi_4^T & P_4 - \Phi_4 + \Psi_2^T A_h^T - \Psi_4^T E_v^T & -\Psi_3^T - \Psi_2 & -\Psi_4^T - \Psi_4 \end{bmatrix} < 0 \quad (28)$$

Note that the expression (28), due to the term  $B_h M_{hv}$  will be at least a triple sum:

$\sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^e h_i(z) h_j(z) v_k(z)$ . As  $\Phi$  and  $\Psi$  are unspecified matrices, their degrees of freedom

can be extended to this triple sum in the following way:

$$\Phi = \begin{bmatrix} \Phi_{1hv} & \Phi_{2hv} \\ \Phi_{3hh} & \Phi_{4hh} \end{bmatrix} = \sum_{i=1}^r h_i(z) \begin{bmatrix} \sum_{k=1}^e v_k(z) \Phi_{1ik} & \sum_{k=1}^e v_k(z) \Phi_{2ik} \\ \sum_{j=1}^r h_j(z) \Phi_{3ij} & \sum_{j=1}^r h_j(z) \Phi_{4ij} \end{bmatrix} \quad (29)$$

$$\Psi = \begin{bmatrix} \Psi_{1hv} & \Psi_{2hv} \\ \Psi_{3hh} & \Psi_{4hh} \end{bmatrix} = \sum_{i=1}^r h_i(z) \begin{bmatrix} \sum_{k=1}^e v_k(z) \Psi_{1ik} & \sum_{k=1}^e v_k(z) \Psi_{2ik} \\ \sum_{j=1}^r h_j(z) \Psi_{3ij} & \sum_{j=1}^r h_j(z) \Psi_{4ij} \end{bmatrix} \quad (30)$$

*Remark 4:* If the fuzzy descriptor shares the same input matrices, i.e.  $B_i = B$ ,  $i \in \{1, \dots, r\}$

then, of course,  $\Phi = \begin{bmatrix} \Phi_{1v} & \Phi_{2v} \\ \Phi_{3h} & \Phi_{4h} \end{bmatrix}$  and  $\Psi = \begin{bmatrix} \Psi_{1v} & \Psi_{2v} \\ \Psi_{3h} & \Psi_{4h} \end{bmatrix}$ .

A new expression for (28) is then:

$$\begin{bmatrix} \Phi_{3hh}^T + \Phi_{3hh} & (*) & (*) & (*) \\ \begin{pmatrix} \Phi_{4hh}^T + A_h \Phi_{1hv} \\ -E_v \Phi_{3hh} - B_h M_{hv} \end{pmatrix} & \begin{pmatrix} \Phi_{2hv}^T A_h^T + A_h \Phi_{2hv} \\ -\Phi_{4hh}^T E_v^T - E_v \Phi_{4hh} \end{pmatrix} & (*) & (*) \\ P_1 - \Phi_{1hv} + \Psi_{3hh}^T & -\Phi_{2hv} + \Psi_{1hv}^T A_h^T - \Psi_{3hh}^T E_v^T & -\Psi_{1hv}^T - \Psi_{1hv} & (*) \\ P_3 - \Phi_{3hh} + \Psi_{4hh}^T & P_4 - \Phi_{4hh} + \Psi_{2hv}^T A_h^T - \Psi_{4hh}^T E_v^T & -\Psi_{3hh}^T - \Psi_{2hv} & -\Psi_{4hh}^T - \Psi_{4hh} \end{bmatrix} < 0 \quad (31)$$

Let us define:

$$\Upsilon_{ij}^k = \begin{bmatrix} \Phi_{3ij}^T + \Phi_{3ij} & (*) & (*) & (*) \\ \Phi_{4ij}^T + A_i \Phi_{1jk} - E_k \Phi_{3ij} - B_i M_{jk} & \Phi_{2jk}^T A_i^T + A_i \Phi_{2jk} - \Phi_{4ij}^T E_k^T - E_k \Phi_{4ij} & (*) & (*) \\ P_1 - \Phi_{1jk} + \Psi_{3ij}^T & -\Phi_{2jk} + \Psi_{1jk}^T A_i^T - \Psi_{3ij}^T E_k^T & -\Psi_{1jk}^T - \Psi_{1jk} & (*) \\ P_3 - \Phi_{3ij} + \Psi_{4ij}^T & P_4 - \Phi_{4ij} + \Psi_{2jk}^T A_i^T - \Psi_{4ij}^T E_k^T & -\Psi_{3ij}^T - \Psi_{2jk} & -\Psi_{4ij}^T - \Psi_{4ij} \end{bmatrix} \quad (32)$$

*Theorem 3:*

Let us consider the fuzzy descriptor model (18) and the  $\Upsilon_{ij}^k$  defined in (32). The fuzzy descriptor with control law (17) is quadratically stable if there exist matrices  $P_1 = P_1^T > 0$ ,  $P_3$ ,  $P_4$ ,  $M_{ik}$ ,  $\Phi$  and  $\Psi$  defined in (29) and (30) such that for each  $k \in \{1, \dots, e\}$  and  $i, j \in \{1, \dots, r\}$ ,  $j > i$  the conditions (7) (or considering also matrices  $Q_{ii}^k > 0$  and  $Q_{ij}^k = (Q_{ji}^k)^T$ ,  $i, j \in \{1, \dots, r\}$ ,  $j > i$  the conditions (8), (9) and (10)) are satisfied. Moreover the gains of the control law are given by:  $F_{ik} = M_{ik} P_1^{-1}$ .

*Lemma 3:*

For any fuzzy descriptor (14), if the conditions of theorem 2 are satisfied then those of theorem 3 are also satisfied.

*Proof:*

In the proof, the exponent <sup>(1)</sup> stands for the first approach (theorem 2), the exponent <sup>(2)</sup> stands for the second one (theorem 3). Suppose that the conditions of theorem 2 are satisfied. Then there exists  $P_1 = P_1^T > 0$ ,  $P_3$ ,  $P_4$ ,  $M_{ik}$ ,  $Q_{ii}^{k(1)} > 0$  and  $Q_{ij}^{k(1)} = (Q_{ji}^{k(1)})^T$  satisfying (8), (9) and (10) (if no relaxation is chosen the proof follows the same path). We keep the same matrices  $P_1 = P_1^T > 0$ ,  $P_3$ ,  $P_4$ ,  $M_{ik}$ , for the theorem 3 and fix  $Q_{ii}^{k(2)} = \begin{bmatrix} Q_{ii}^{k(1)} & 0 \\ 0 & \varepsilon^2 I \end{bmatrix}$ ,

$Q_{ij}^{k(2)} = \begin{bmatrix} Q_{ij}^{k(1)} & 0 \\ 0 & 0 \end{bmatrix}$ . Then directly,  $Q^{k(2)} > 0$  if and only if:  $\begin{bmatrix} Q^{k(1)} & 0 \\ 0 & \varepsilon^2 I_{n \times r} \end{bmatrix} > 0$  which is clearly satisfied as  $Q^{k(1)} > 0$ . Fix also:  $\Phi_{1ik} = P_1$ ,  $\Phi_{2ik} = 0$ ,  $\Phi_{3ij} = P_3$ ,  $\Phi_{4ij} = P_4$ ,  $\Psi_{2ik} = \Psi_{3ij} = 0$ ,  $\Psi_{1ik} = \Psi_{4ij} = \varepsilon^2 I$ ,  $i, j \in \{1, \dots, r\}$ ,  $j > i$ ,  $k \in \{1, \dots, e\}$ , (32) can be written as:

$$\Upsilon_{ij}^{k(2)} = \begin{bmatrix} P_3 + P_3^T & (*) & 0 & (*) \\ A_i P_1 - B_i M_{jk} - E_k P_3 + P_4^T & -E_k P_4 - P_4^T E_k^T & (*) & (*) \\ 0 & \varepsilon^2 A_i^T & -2\varepsilon^2 I & 0 \\ \varepsilon^2 I & -\varepsilon^2 E_k^T & 0 & -2\varepsilon^2 I \end{bmatrix} \quad (33)$$

$$\text{or defining: } \Gamma_i^k = \begin{bmatrix} 0 & I \\ A_i & -E_k \end{bmatrix}; \Upsilon_{ij}^{k(2)} = \begin{bmatrix} \Upsilon_{ij}^{k(1)} & (*) \\ \varepsilon^2 (\Gamma_i^k)^T & -2\varepsilon^2 I_{2n} \end{bmatrix} \quad (34)$$

Then:

$$\Upsilon_{ii}^{k(2)} + Q_{ii}^{k(2)} = \begin{bmatrix} \Upsilon_{ii}^{k(1)} + Q_{ii}^{k(1)} & (*) \\ \varepsilon^2 (\Gamma_i^k)^T & -\varepsilon^2 I_{2n} \end{bmatrix} \quad (35)$$

$$\Upsilon_{ij}^{k(2)} + \Upsilon_{ji}^{k(2)} + \mathcal{Q}_{ij}^{k(2)} + \mathcal{Q}_{ji}^{k(2)} = \begin{bmatrix} \Upsilon_{ij}^{k(1)} + \Upsilon_{ji}^{k(1)} + \mathcal{Q}_{ij}^{k(1)} + \mathcal{Q}_{ji}^{k(1)} & (*) \\ \varepsilon^2 (\Gamma_i^k + \Gamma_j^k)^T & -4\varepsilon^2 I_{2n} \end{bmatrix} \quad (36)$$

Applying Schur's complement leads to:

$$(35) \Leftrightarrow \Upsilon_{ii}^{k(1)} + \mathcal{Q}_{ii}^{k(1)} + \varepsilon^2 \Gamma_i^k (\Gamma_i^k)^T < 0 \quad (37)$$

$$(36) \Leftrightarrow \Upsilon_{ij}^{k(1)} + \Upsilon_{ji}^{k(1)} + \mathcal{Q}_{ij}^{k(1)} + \mathcal{Q}_{ji}^{k(1)} + \frac{\varepsilon^2}{4} (\Gamma_i^k + \Gamma_j^k) (\Gamma_i^k + \Gamma_j^k)^T < 0 \quad (38)$$

As (8), (9) are verified for the theorem 2, then it always exists an enough small  $\varepsilon^2$  such that (37) and (38) are satisfied.

To show the interest of this new result we will study a second academic example.

*Example 2:*

This example is constructed in a way that theorem 2 conditions fail to obtain a solution.

Consider the following nonlinear model in a descriptor form with  $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$  the state vector:

$$E(x(t)) \cdot \dot{x}(t) = A(x_1(t)) \cdot \dot{x}(t) + B(x_1(t)) \cdot u(t) \quad (39)$$

The different matrices are given by:

$$A(x_1(t)) = \begin{bmatrix} -28.7 & 45.2 \\ -14.7 - \frac{\sin(x_1(t))}{x_1(t)} \cdot 47.4 & -19.9 \end{bmatrix}, \quad E(x(t)) = \begin{bmatrix} 48.9 - \cos(x_1(t) - x_2(t)) \cdot 41.8 & 33.5 \\ -0.1 & -20.7 \end{bmatrix}$$

$$\text{and } B(x_1(t)) = \begin{bmatrix} -40 \cdot \frac{\sin(x_1(t))}{x_1(t)} \\ 5 \end{bmatrix}.$$

All the nonlinearities being bounded, following the same path as presented for the example 1, we obtain the TS model in a descriptor form:

$$\begin{cases} E_v \dot{x}(t) = A_h x(t) + B_h u(t) \\ y(t) = C_h x(t) \end{cases} \quad (40)$$

$$\text{With: } A_1 = \begin{bmatrix} -28.7 & 45.2 \\ -4.272 & -19.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -28.7 & 45.2 \\ -62.1 & -19.9 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 8.8 \\ 5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -40 \\ 5 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 91.8 & 33.5 \\ -0.1 & -20.7 \end{bmatrix}$$

and  $E_2 = \begin{bmatrix} 8.2 & 33.5 \\ -0.1 & -20.7 \end{bmatrix}$ . Using theorem 3 conditions allows obtaining the following solution.

Matrices :  $P_1 = \begin{bmatrix} 4.117 & -0.249 \\ -0.249 & 1.058 \end{bmatrix}$ ,  $P_3 = \begin{bmatrix} -5.196 & -0.729 \\ 1.254 & -59.511 \end{bmatrix}$  and  $P_4 = \begin{bmatrix} 217.55 & 18.175 \\ 1966.3 & -1206.9 \end{bmatrix}$ , and for the gains:

$$\begin{aligned} F_{11} &= [-1.445 \quad -7.69], & F_{12} &= [-0.913 \quad -2.708] \\ F_{21} &= [-2.49 \quad -20.65], & F_{22} &= [-0.99 \quad -4.85] \end{aligned} \quad (41)$$

The non linear model (39) and the obtained control law (17) with the gains (41) have been implemented using the MATLAB/SIMULINK software. An example of simulation is presented figure 1. Considering the initial condition vector  $x(0) = [-5 \quad 5]^T$ , the convergence of the state vector and the evolution of the control signal are presented figure 1.

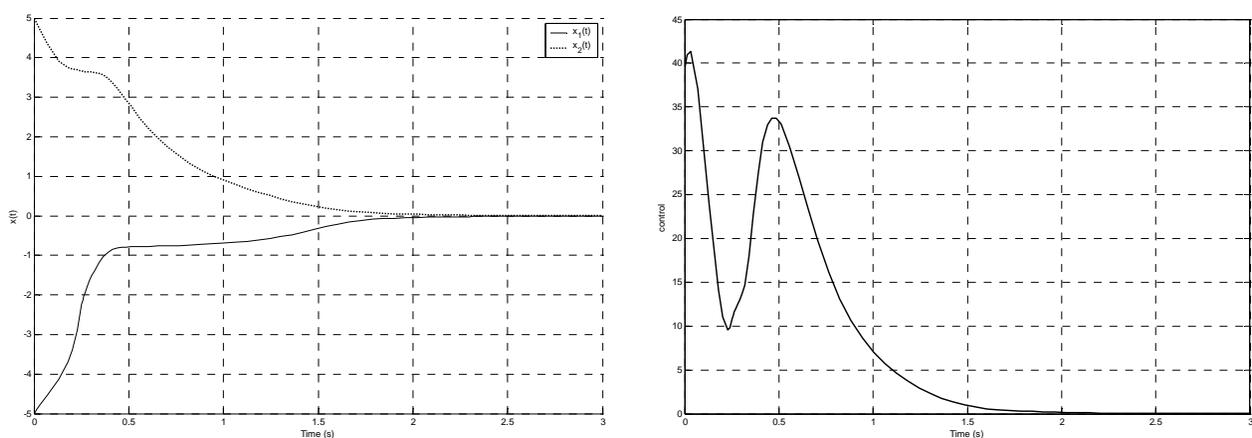


Figure 1. Stabilization of the non linear model (39) using control law (17) and initial conditions  $x(0) = [-5 \quad 5]^T$ .

## 5. Application to a Double-Inverted Pendulum

We consider the well-known double-inverted pendulum application. It is composed with a cart with two poles. Both of the poles are free in rotation around their axis as shown figure 2. The goal is to keep the angles  $\theta(t)$  and  $\beta(t)$  - respectively the angle between the first pole and the vertical and the second one and the vertical - around 0 and to ensure the tracking of the cart position  $p(t)$ . The different variables useful and the values chosen for the model description are resumed table 1.

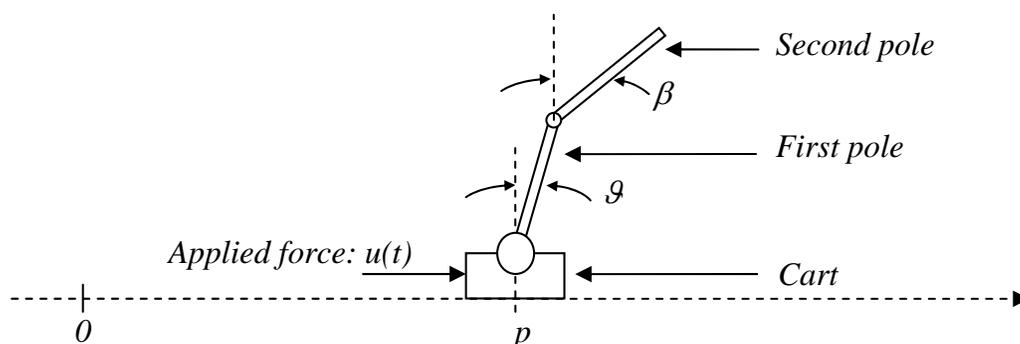


Figure 2. Representation of a double-inverted pendulum on a cart

Notation	value	Description
$M$	30 Kg	Mass of the cart
$J_1$	$5 \cdot 10^{-3} \text{ Kg} \cdot \text{m}^2$	Inertia of the first pole
$m_1$	0.2 Kg	Mass of the first pole
$l_1$	10 cm	Half-length of the first pole
$k_1$	$0.1 \text{ N} \cdot \text{s} \cdot \text{m}^{-1}$	First joint friction (viscous)
$J_2$	$8 \cdot 10^{-3} \text{ Kg} \cdot \text{m}^2$	Inertia of the second pole
$m_2$	0.3 Kg	Mass of the second pole
$l_2$	15 cm	Half-length of the second pole
$k_2$	$0.1 \text{ N} \cdot \text{s} \cdot \text{m}^{-1}$	Second joint friction (viscous)
$g$	$9.81 \text{ m} \cdot \text{s}^{-2}$	Gravity
$f$	$1 \text{ m} \cdot \text{s}^{-1}$	Friction
$u(t)$		Force to apply on the cart
$\theta(t)$		Angle for the first pole
$\beta(t)$		Angle for the second pole
$p(t)$		Position of the cart

Table 1. Variables useful for the double-inverted pendulum

According to the Euler Lagrange equations the following model can be obtained (Morère 2001):

$$(M + m_1 + m_2) \cdot \ddot{p} + (m_1 + 2 \cdot m_2) \cdot l_1 \cdot \cos(\theta) \cdot \ddot{\theta} + m_2 \cdot l_2 \cdot \cos(\beta) \cdot \ddot{\beta} = -f \cdot \dot{p} + (m_1 + 2 \cdot m_2) \cdot l_1 \cdot \dot{\theta}^2 \cdot \sin(\theta) + m_2 \cdot l_2 \cdot \dot{\beta}^2 \cdot \sin(\beta) + u \quad (42)$$

$$(m_1 + 2 \cdot m_2) \cdot l_1 \cdot \cos(\theta) \cdot \ddot{p} + (m_1 \cdot l_1^2 + 4 \cdot m_2 \cdot l_1^2 + J_1) \cdot \ddot{\theta} + 2 \cdot m_2 \cdot l_1 \cdot l_2 \cdot \cos(\theta - \beta) \cdot \ddot{\beta} = -2 \cdot m_2 \cdot l_1 \cdot l_2 \cdot \dot{\beta}^2 \sin(\theta - \beta) + (m_1 + 2 \cdot m_2) \cdot g \cdot l_1 \cdot \sin(\theta) - k_1 \cdot \dot{\theta} \quad (43)$$

$$m_2 \cdot l_2 \cdot \cos(\beta) \cdot \ddot{p} + 2 \cdot m_2 \cdot l_1 \cdot l_2 \cdot \cos(\theta - \beta) \cdot \ddot{\theta} + (m_2 \cdot l_2^2 + J_2) \cdot \ddot{\beta} = + 2 \cdot m_2 \cdot l_1 \cdot l_2 \cdot \dot{\theta}^2 \cdot \sin(\theta - \beta) + m_2 \cdot g \cdot l_2 \cdot \sin(\beta) - k_2 \cdot \dot{\beta} \quad (44)$$

With the state vector  $x(t) = [\dot{p} \quad \dot{\theta} \quad \dot{\beta} \quad \ddot{p} \quad \ddot{\theta} \quad \ddot{\beta}]^T$  we have the following descriptor form of the double-inverted pendulum:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & M + m_1 + m_2 & (m_1 + 2m_2)l_1 \cos(\theta) & m_2 l_2 \cos(\beta) \\ 0 & 0 & 0 & (m_1 + 2 \cdot m_2)l_1 \cos(\theta) & m_1 l_1^2 + 4m_2 l_1^2 + J_1 & 2m_2 l_1 l_2 \cos(\theta - \beta) \\ 0 & 0 & 0 & m_2 l_2 \cos(\beta) & 2m_2 l_1 l_2 \cos(\theta - \beta) & m_2 l_2^2 + J_2 \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{\theta} \\ \dot{\beta} \\ \ddot{p} \\ \ddot{\theta} \\ \ddot{\beta} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -f & 0 & 0 \\ 0 & (m_1 + 2m_2)gl_1 \frac{\sin(\theta)}{\theta} & 0 & 0 & -k_1 & 0 \\ 0 & 0 & m_2gl_2 \frac{\sin(\beta)}{\beta} & 0 & 0 & -k_2 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \beta \\ \dot{x} \\ \dot{\theta} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ H_4 \\ H_5 \\ H_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u \quad (45)$$

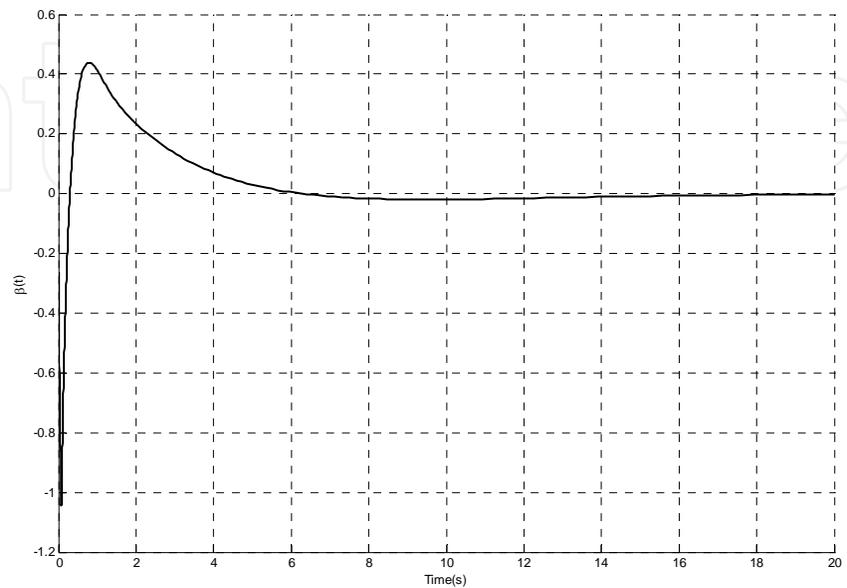
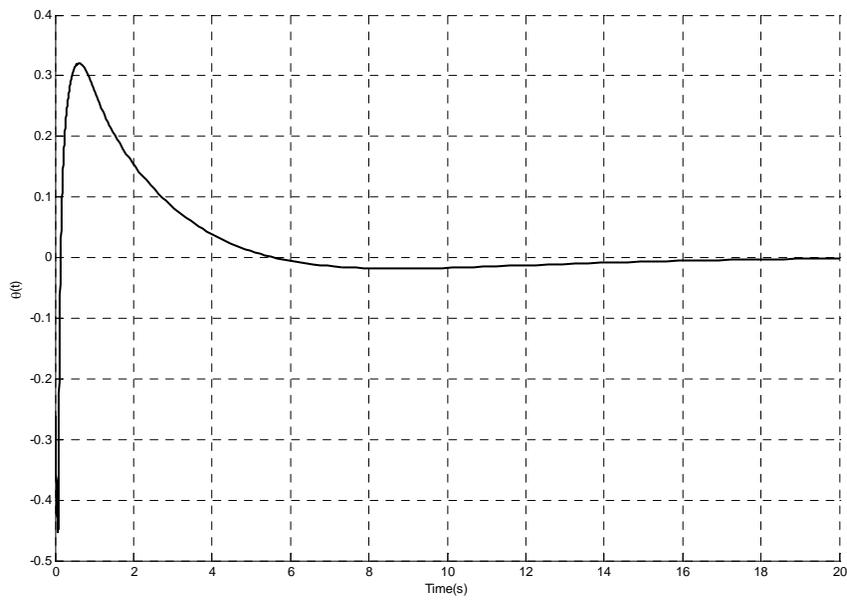


Figure 3. Simulation of the double-inverted pendulum. Upper figure: first pole angle  $\Theta(t)$ , lower figure: second pole angle  $\beta(t)$

With:  $H_4 = (m_1 + 2m_2)l_1\dot{\theta}^2 \sin(\theta) + m_2l_2\dot{\beta}^2 \sin(\beta)$ ,  $H_5 = -2m_2l_1l_2\dot{\beta}^2 \sin(\theta - \beta)$  and  
 $H_6 = 2m_2l_1l_2\dot{\theta}^2 \sin(\theta - \beta)$

Or in a compact writing:

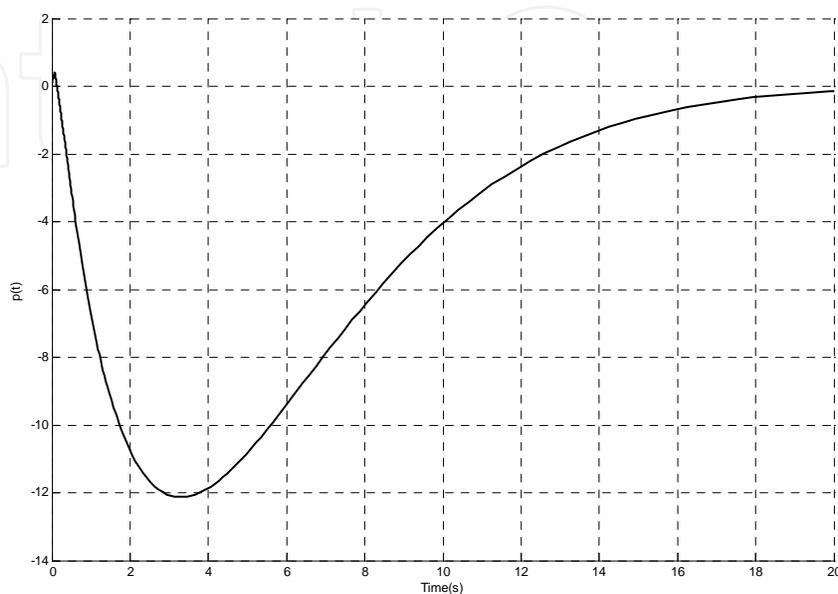
$$\begin{aligned} E_v \dot{x}(t) &= A_h x(t) + B_h u(t) + H \\ y(t) &= C_h x(t) \end{aligned} \tag{46}$$

*Remark 5:* In this simplified version the term  $H$  is neglected. In a more general framework, this is due to the fact that for output stabilization, i.e. with an observer, there exists a separation principle in the case where the premise vector  $z(t)$  is measurable. As  $\theta(t)$  and  $\beta(t)$  are measurable, neglecting  $H$  allows using this principle. Let us also say that several ways to use a complete TS descriptor model are possible. One is to consider the  $H$  part as bounded uncertainties and then use robust conditions of stabilization; nevertheless this was not the purpose of this chapter.

For  $E_v$ , it is necessary to take into account three nonlinearities:  $\cos(\theta)$ ,  $\cos(\beta)$  and  $\cos(\theta - \beta)$ . That means  $e = 2^3 = 8$  functions  $v_k$ . For  $A_h$ , 2 nonlinearities  $\frac{\sin(\theta)}{\theta}$  and  $\frac{\sin(\beta)}{\beta}$ , that means  $r = 2^2 = 4$  functions  $h_i$ . Conditions of theorem 2 were then performed to obtain the control law.

An example of simulation is presented in the next figures with the initial condition:  $\theta(0) = -20^\circ$  and  $\beta(0) = -15^\circ$ .

Figure 3 shows the evolution of the angles  $\theta(t)$  and  $\beta(t)$ , figure 4 the evolution of the cart position  $p(t)$  and the control law. The three first figures are simulated on 20s, the last figure, the control law, is zoomed on the first 0.25s.



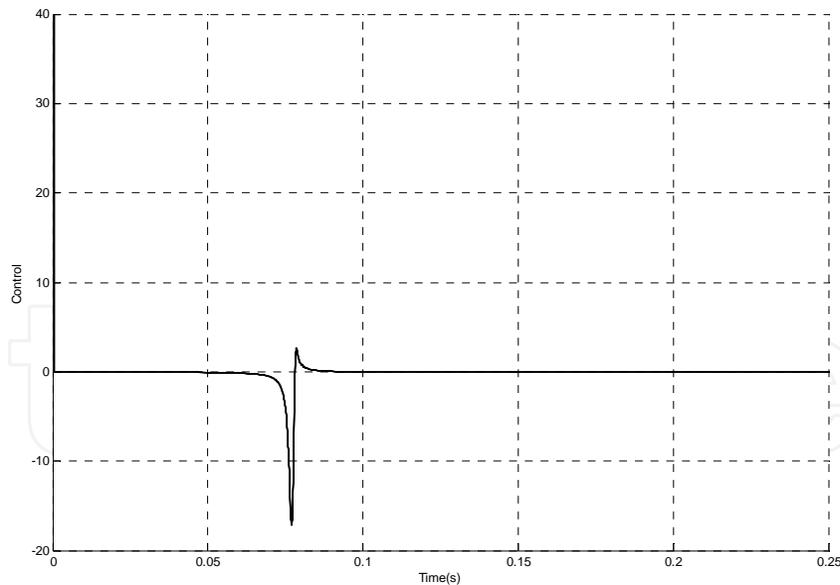


Figure 4. Simulation of the double-inverted pendulum. Upper figure: cart position  $p(t)$ , lower figure: control law evolution (zoomed on the first 0.25s)

## 6. Conclusion

The chapter focused on TS fuzzy models in descriptor form stabilization. As for classical TS models, they can be obtained in a systematic way using the sector nonlinearity approach. Their main interest is to remain close to the nonlinear model. Thus, in some cases the number of models involved can be highly reduced in comparison to a classical TS model representing the same nonlinear model. Hence, the results obtained with the conditions of stabilization can allow reducing in a large way the conservatism of preceding classical results. Nevertheless, if we want to outperform basic conditions of stabilization for TS models in the descriptor form, a way is to use specific matrix transformation. The application of matrix transformation allowed outperforming the results. At last we presented the application to the well-known double inverted pendulum in simulation.

To go further, robustness can be easily introduced in the different conditions. It can be done using classical bounded uncertainties. The regulator problem could be also investigated.

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### **Cutting Edge Robotics**

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This book is the result of inspirations and contributions from many researchers worldwide. It presents a collection of wide range research results of robotics scientific community. Various aspects of current research in robotics area are explored and discussed. The book begins with researches in robot modelling & design, in which different approaches in kinematical, dynamical and other design issues of mobile robots are discussed. Second chapter deals with various sensor systems, but the major part of the chapter is devoted to robotic vision systems. Chapter III is devoted to robot navigation and presents different navigation architectures. The chapter IV is devoted to research on adaptive and learning systems in mobile robots area. The chapter V speaks about different application areas of multi-robot systems. Other emerging field is discussed in chapter VI - the human- robot interaction. Chapter VII gives a great tutorial on legged robot systems and one research overview on design of a humanoid robot. The different examples of service robots are showed in chapter VIII. Chapter IX is oriented to industrial robots, i.e. robot manipulators. Different mechatronic systems oriented on robotics are explored in the last chapter of the book.

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