

We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

185,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Entransy Dissipation Theory and Its Application in Heat Transfer

Mingtian Xu
Shandong University
P. R. China

1. Introduction

In this chapter, our recent work on the entransy dissipation theory and its application in heat convection and the heat exchanger design is reviewed. In that work, the thermodynamic basis of the entransy dissipation theory is established. It is shown that the entransy is a state variable and the second law of thermodynamics can be described by the entransy and entransy dissipation. Then this entransy dissipation theory is applied to the convective heat transfer and heat exchanger design. The local expression of the entransy dissipation rate is obtained. The extremum principle of entransy dissipation rate is proposed. The expressions of the entransy dissipation caused by heat conduction and fluid friction in heat exchanger are derived. The optimization design of heat exchanger is discussed.

1.1 The second law of thermodynamics in terms of entransy

The second law of thermodynamics is one of the most important fundamental laws in physics, which originates from the study of the efficiency of heat engine and places constraints upon the direction of heat transfer and the attainable efficiencies of heat engines (Kondepudi & Prigogine, 1998). The concept of entropy introduced by Clausius for mathematically describing the second law of thermodynamics has stretched this law across almost every discipline of science. However, in the framework of the classical thermodynamics the definition of entropy is abstract and ambiguous, which was noted even by Clausius (Clausius, 1865). This has induced some controversies for statements related to the entropy. Recently, Bertola and Cafaro found that the principle of minimum entropy production is not compatible with continuum mechanics (Bertola & Cafaro, 2008). Herwig showed that the assessment criterion for heat transfer enhancement based on the heat transfer theory contradicts the ones based on the second law of thermodynamics (Herwig, 2010). The entropy generation number defined by Bejan (Bejan, 1988) is not consistent with the exchanger effectiveness which describes the heat exchanger performance (Guo et al., 2009b). Shah and Skiepko (2004) found that the heat exchanger effectiveness can be maximum, minimum or in between when the entropy generation achieves its minimum value for eighteen kinds of heat exchangers, which does not totally conform to the fact that the reduction of entropy generation leads to the improvement of the heat exchanger performance. These findings signal that the concepts of entropy and entropy generation may not be perfect for describing the second law of thermodynamics.

Although there has been effort to modify the expression of the second law of thermodynamics (Bizarro, 2008; Ben-Amotz & Honig, 2003, 2006) and to improve the classical thermodynamics by considering the Carnot construction cycling in a finite time (den Broek, 2005; Esposito & Lindenberg, 2009; Esposito et al., 2010), the eminent position of entropy in thermodynamics has not been questioned. Recently, Guo et al. (2007) defined two new physical quantities called entransy and entransy dissipation for describing the heat transfer ability and irreversibility of heat conduction, respectively. Guo et al. (2009a) have introduced a dimensionless method for the entransy dissipation and defined an entransy dissipation number which can serve as the heat exchanger performance evaluation criterion. Based on the concept of entransy dissipation, an equivalent thermal resistance of heat exchanger was defined which is consistent with the exchanger effectiveness (Guo et al., 2010). Cheng and Liang (2011) defined the entransy flux and entransy function for the thermal radiation in enclosures with opaque surfaces, and the minimum principle of radiative entransy loss was established. Chen et al. (2011) proposed an entransy dissipation rate minimization approach for the disc cooling system and the influence of various system parameters on the entransy dissipation rate of the cooling system has been investigated.

Although the concepts of entransy and entransy dissipation have been applied to heat transfer and demonstrate some advantages in comparison with the entropy and entropy generation, how to define these concepts from the thermodynamic point of view is still an open question. In this section we place the concepts of entransy and entransy dissipation on the solid thermodynamic basis.

1.2 Carnot's theorem in terms of entransy

We start with the Carnot cycle. In this cycle, the heat engine absorbs heat Q_1 from the hot reservoir with the temperature T_1 (absolute temperature is always assumed in the following discussion), converts part of heat to work W and discards the rest of the heat to the cold reservoir with the temperature T_2 . For this cycle, Carnot's theorem states that (Kondepudi & Prigogine, 1998)

$$Q_1 / T_1 \leq Q_2 / T_2 \quad (1)$$

where the equality and inequality correspond to the reversible and irreversible heat engines, respectively. The efficiency of a reversible engine is defined as

$$\eta = 1 - \frac{T_2}{T_1} \quad (2)$$

Carnot's theorem dictates that reversible engines have the maximum efficiency (Kondepudi & Prigogine, 1998).

Equivalently, Inequality (1) can be rewritten as follows

$$Q_1(T_1 - T_2) \geq (Q_1 - Q_2)T_1 \quad (3)$$

Inspired by Inequality (3), we define $E = Q_1(T_1 - T_2)$ as the entransy gained by the heat engine from the hot reservoir in the Carnot cycle. From this definition, one can see that the larger the amount of heat Q_1 and the temperature difference between the hot and cold

reservoir, the greater the entransy obtained by the heat engine in the Carnot cycle. Obviously, the larger entransy means higher ability for heat engine to perform work. Guo et al. (2007) shown that the entransy can describe the heat conduction ability. Therefore, we may say that the entransy defined here quantifies the energy transfer ability including the ability to deliver work and transfer heat.

To obtain the efficiency of a reversible heat engine, firstly we know that $W = Q_1 - Q_2$, which is the statement of the law of conservation of energy. We define $E_W = (Q_1 - Q_2)T_1 = WT_1$ and regard it as the entransy consumed by the heat engine for delivering work W to the system's exterior. For the reversible heat engine, Inequality (3) reduces to:

$$Q_1(T_1 - T_2) = (Q_1 - Q_2)T_1 = WT_1 \quad (4)$$

which says that the entransy gained by the reversible heat engine in the Carnot's cycle is completely converted to work W . While for the irreversible heat engine, Inequality (3) becomes

$$Q_1(T_1 - T_2) > (Q_1 - Q_2)T_1 \quad (5)$$

which tells us that only part of the entransy obtained by the heat engine in the Carnot's cycle is utilized to deliver work W , the rest is consumed by the irreversibility in the heat engine. In order to quantify the irreversibility occurring in the heat engine, we define the entransy generation in parallel with the entropy generation in the following way:

$$E_g = (Q_1 - Q_2)T_1 - Q_1(T_1 - T_2) \quad (6)$$

While $E_{diss} = -E_g$ is called the entransy dissipation which represents the entransy consumed by the irreversibility in heat engine. Since the reversible heat engine converts the total entransy gained during the Carnot cycle to perform work, we may define the efficiency of the heat engine in terms of entransy as follows

$$\eta_E = \frac{WT_1}{Q_1(T_1 - T_2)} = \frac{E_W}{E} \quad (7)$$

Notice that the reversible heat engine achieves the maximum value of η_E which is equal to 100%. This is the statement of Carnot's theorem in terms of entransy.

1.3 The second law of thermodynamics

Carnot's theorem has played a pivotal role on the development of the classical thermodynamics. By generalizing Inequality (1) to an arbitrary cycle, Clausius introduced the concept of entropy which is a physical quantity as fundamental and universal as energy. In this section, we aim to generalize Inequality (3) to an arbitrary cycle. Firstly, Inequality (3) can be rewritten as follows:

$$Q_1\Delta T + T_1\Delta Q \geq 0 \quad (8)$$

where $\Delta T = T_1 - T_2$, $\Delta Q = -(Q_1 - Q_2)$. Note that since the system delivers energy to its exterior, therefore $\Delta Q < 0$.

An arbitrary cycle can be decomposed into a group of Carnot's cycles denoted as $C_i (i = 1, 2, \dots, n)$ (Kondepudi & Prigogine, 1998). Applying Inequality (8) to the i -th Carnot's cycle yields:

$$Q_{a,i}\Delta T_i + \Delta Q_i T_{h,i} \geq 0 \quad (9)$$

where $Q_{a,i}$ is the heat absorbed from the hot reservoir with the temperature $T_{h,i}$, $\Delta Q_i = -(Q_{a,i} - Q_{r,i})$ ($Q_{r,i}$ is the heat discarded into the cold reservoir), $\Delta T_i = T_{h,i} - T_{c,i}$ ($T_{c,i}$ is the temperature of the cold reservoir). If the number of Carnot's cycles under consideration tends to the infinity, the temperature difference between the hot and cold reservoirs approaches to infinitesimal, Inequality (9) becomes

$$\oint QdT + T\delta Q \geq 0 \quad (10)$$

For a reversible cycle, there is no heat conduction, Inequality (10) reduces to

$$\oint QdT + T\delta Q = 0$$

Equation (11) suggests that we can define a quantity E called the entransy as follows

$$dE = QdT + T\delta Q \quad (11)$$

which only depends on the initial and final states of a reversible process. Thus the entransy is a state variable. If E_A and E_B are values of this variable in the initial state A and final state B , respectively, we have

$$E_B - E_A = \int_A^B QdT + T\delta Q \quad (12)$$

If the temperature remains fixed, it follows from Eq. (12) that for a reversible flow of heat δQ , the change in entransy is $T\delta Q$, while the change in entropy is $\delta Q/T$. For this case we get the following relationship between the entransy and entropy,

$$dE = T^2 dS \quad (13)$$

Consider an irreversible process I which starts from the equilibrium state A and ends at the equilibrium state B . In order to form a cycle, we add a reversible process R from the state B to state A . Then from Inequality (10), we have

$$\int_{A_I}^{B_I} QdT + T\delta Q > \int_{A_R}^{B_R} QdT + T\delta Q \quad (14)$$

The subscripts R and I represent the reversible and irreversible processes, respectively. The application of Eq. (12) on Inequality (14) yields

$$\int_{A_I}^{B_I} QdT + T\delta Q > E_B - E_A \quad (15)$$

Thus in parallel with the entropy generation we can define the entransy generation E_g of the irreversible process as follows

$$E_g = (E_B - E_A) - \int_{A_I}^{B_I} QdT + T\delta Q < 0 \quad (16)$$

Note that for the reversible process $E_g = 0$. We define $E_{diss} = -E_g$ as the entransy dissipation. Therefore, the entransy dissipation quantifies the entransy consumed by the irreversibility in the irreversible process. For an irreversible process, the second law of thermodynamics states that the entransy generation is always negative or the irreversible process always decreases the system's ability to do work and transfer heat. From the entransy's definition (12), it is evident that the entransy is an extensive quantity. Subsequently, Inequality (16) is also valid under the local equilibrium assumption.

We may express the system's change in entransy as a sum of two parts

$$dE = d_e E + d_i E, \text{ and } d_i E \leq 0 \quad (17)$$

In which $d_e E$ is the entransy change due to exchange of matter and energy with the exterior of the system and $d_i E$ is the entransy generation produced by the irreversible processes occurring in the system. For the closed systems that exchange energy, but not matter, we have

$$d_e E = QdT + T\delta Q \text{ and } d_i E \leq 0 \quad (18)$$

For open systems that exchange both matter and energy:

$$d_e E = QdT + T\delta Q + (d_e E)_{matter} \text{ and } d_i E \leq 0 \quad (19)$$

Finally, for the isolated systems, we have

$$d_e E = 0, \text{ and } d_i E \leq 0 \quad (20)$$

Therefore, in the isolated systems the entransy never increases, namely the energy transfer ability can not increase.

1.4 Entansy dissipation due to heat conduction

Now let us consider the heat conduction process discussed by Kondepudi & Prigogine (1998). The system under consideration is an isolated system and consists of two parts, each having a well-defined temperature. Let the temperatures of two parts be T_1 and T_2 ($T_1 > T_2$), respectively. δQ is the amount of heat flow from the hotter part to colder part in a time period dt . Since this isolated system does not exchange entransy with its exterior, $d_e E = 0$. Assume the volume of each part is constant, thus $dW = 0$. The energy change in each part is solely due to the flow of heat: $dU_i = \delta Q_i$ ($i = 1, 2$). In accordance with the first law, the heat gained by one part is equal to the heat lost by the other. Therefore, $-\delta Q_1 = \delta Q_2 = \delta Q$. The total change in entransy $d_i E$ of the system is the sum of the changes of entransy in each part due to the flow of heat

$$d_i E = -T_1 \delta Q + T_2 \delta Q = -(T_1 - T_2) \delta Q \quad (21)$$

In terms of the rate of flow of heat $\delta Q / \delta t$, the rate of entransy generation can be written as follows

$$\frac{d_i E}{dt} = -(T_1 - T_2) \frac{\delta Q}{\delta t} \quad (22)$$

The rate of heat flow $J_Q = \delta Q / \delta t$ is given by the Fourier law of heat conduction $J_Q = k(T_1 - T_2)$, in which k is the coefficient of heat conductivity, therefore

$$\frac{d_i E}{dt} = -k(T_1 - T_2)^2 \leq 0 \quad (23)$$

Accordingly, the rate of entransy dissipation is written as

$$\frac{d_i E_{diss}}{dt} = k(T_1 - T_2)^2 \geq 0 \quad (24)$$

This equation is consistent with the expression of the entransy dissipation function obtained by Guo et al. (2007) from the heat conduction equation. Due to the flow of heat from the hot part to cold part, the temperatures of both parts eventually become equal, and the entransy dissipation rate tends to zero. Then the system reaches the equilibrium state. Therefore, the entransy dissipation rate must vanish at the state of equilibrium, which is called as the principle of minimum entransy dissipation rate. This is the counterpart of the principle of the minimum entropy production. Generally, for three-dimensional steady heat conduction without heat source, the entransy dissipation rate is expressed as

$$\frac{d_i E_{diss}}{dt} = \int_V \frac{1}{2} k (\nabla T)^2 dV \quad (25)$$

where V is the volume of the heat conduction medium. The principle of minimum entransy dissipation rate is mathematically formulated as

$$\delta \int_V \frac{1}{2} k (\nabla T)^2 dV = 0 \quad (26)$$

which is consistent with the least entransy dissipation principle established by Guo et al. (2007). By the variational method, Eq. (26) is equivalent to the following Euler-Lagrange equation:

$$\frac{\partial}{\partial x} \left(\frac{\partial F}{\partial (\frac{\partial T}{\partial x})} \right) + \frac{\partial}{\partial y} \left(\frac{\partial F}{\partial (\frac{\partial T}{\partial y})} \right) + \frac{\partial}{\partial z} \left(\frac{\partial F}{\partial (\frac{\partial T}{\partial z})} \right) = 0 \quad (27)$$

with

$$F = \frac{1}{2} k (\nabla T)^2 \quad (28)$$

Substituting Eq. (28) into Eq. (27) gives

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (29)$$

which is exactly the governing equation of the steady heat conduction without heat sources based on the Fourier law. While Bertola and Cafaro (2008) found that the principle of minimum entropy production can not lead to the governing equation of the steady Fourier heat conduction. In this point, the principle of minimum entransy dissipation rate demonstrates an obvious advantage.

1.5 Concluding remarks

Note that the concept of entropy can be replaced with entransy for describing the second law of thermodynamics. In comparison with entropy, the entransy has a definite physical meaning and the principle of minimum entransy dissipation rate avoids the contradiction between the principle of minimum entropy production and the classical Fourier heat conduction theory. Therefore, we anticipate that the concepts of entransy and entransy dissipation may help us to gain more profound insight on thermodynamics in particular and on science in general.

2. The application of entransy dissipation theory in heat convection

In this section, the entransy dissipation theory developed in Section 1 are applied to analyze the heat convection.

2.1 Introduction

The entropy and entropy generation can help us to deeply understand the momentum and heat transfer (Bejan, 1982; Herwig, 2010). Bejan (1982) realized that in order to improve the performance of the heat transfer enhancement or thermal insulation equipments, one need to reduce the entropy generation rate. Similarly, according to the definition of entransy dissipation given in Section 1.3, it is required to minimize the entransy dissipation rate for achieving the best heat transfer enhancement and thermal insulation. Therefore, it is of great value to derive the expression of the local rate of entransy dissipation rate for heat convection. Xu et al. (2009) have managed to get an expression of the local rate of entransy dissipation rate for heat convection. However, it lacks the theoretical basis in this derivation. In Section 2.2 with the help of the second law of thermodynamics in terms of entransy and entransy dissipation established in Section 1.3 we will make the derivation more rigorously.

2.2 Local thermodynamic entransy dissipation in heat convection

The infinitesimal element as shown in Fig. 1 is an open thermodynamic system, where $[v_x, v_y]^T$ is the velocity, $[q_x, q_y]^T$ is the heat flux. For this system, we assume that the thermodynamic state is irrelevant with the position, but relevant with time. By the second law of thermodynamics for the open system expressed as Eqs. (17) and (19) the rate of local thermodynamic entransy dissipation generation \dot{E}_g per unit volume in the infinitesimal element is expressed as

$$\begin{aligned} \dot{E}_g dxdy = & (T + \frac{\partial T}{\partial x} dx - T) q_x dy + (T + \frac{\partial T}{\partial y} dy - T) q_y dx \\ & + T(q_x - q_x - \frac{\partial q_x}{\partial x} dx) dy + T(q_y - q_y - \frac{\partial q_y}{\partial y} dy) dx \\ & + (e + \frac{\partial e}{\partial x} dx)(v_x + \frac{\partial v_x}{\partial x} dx)(\rho + \frac{\partial \rho}{\partial x} dx) dy \\ & + (e + \frac{\partial e}{\partial y} dy)(v_y + \frac{\partial v_y}{\partial y} dy)(\rho + \frac{\partial \rho}{\partial y} dy) dx \\ & - \rho v_x edy - \rho v_y edx + \frac{\partial(\rho e)}{\partial t} dxdy \end{aligned} \quad (30)$$

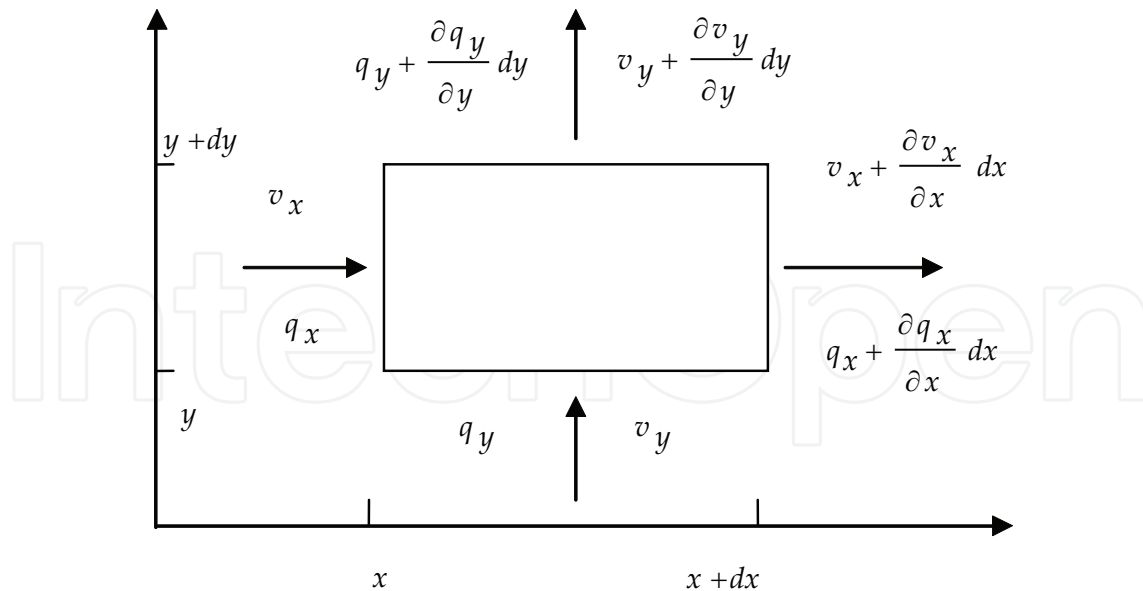


Fig. 1. Two-dimensional infinitesimal element for heat convection

where e is the specific entransy, ρ is the density of fluid. The first four terms in the right side of Eq. (30) account for the entransy exchanged with the environment through the element boundary, the following four terms represent the entransy transported by convection, the last term stands for the variation of the entransy with respect to time. Rearranging Eq. (30) yields,

$$\begin{aligned} \dot{E}_g = & (q_x \frac{\partial T}{\partial x} + q_y \frac{\partial T}{\partial y}) - T(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y}) + \rho(\frac{\partial e}{\partial t} + v_x \frac{\partial e}{\partial x} + v_y \frac{\partial e}{\partial y}) \\ & + e[\frac{\partial \rho}{\partial t} + \rho(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}) + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y}] \end{aligned} \quad (31)$$

Making use of the following mass balance equation,

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + \rho(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}) = 0 \quad (32)$$

where ρ is the density of fluid, Eq. (31) is simplified to

$$\dot{E}_g = \mathbf{q} \cdot \nabla T - T \nabla \cdot \mathbf{q} + \rho \frac{De}{Dt} \quad (33)$$

where $D/Dt = \partial/\partial t + v_x \partial/\partial x + v_y \partial/\partial y$ is the material derivative. By the following canonical relation of thermodynamics,

$$\rho \frac{De}{Dt} = -T \rho \frac{Du}{Dt} + \frac{PT}{\rho} \frac{D\rho}{Dt} \quad (34)$$

where u is the specific internal energy, P is the pressure, and the energy balance equation for heat convection

$$\rho \frac{Du}{Dt} = -\nabla \cdot \mathbf{q} - P(\nabla \cdot \mathbf{v}) + \mu \Phi \quad (35)$$

where μ is the viscosity, Φ is the dissipation function and can be expressed as

$$\Phi = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2\right] + \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 \quad (36)$$

By Eqs. (34) and (35), Eq. (33) becomes

$$\dot{E}_g = \mathbf{q} \cdot \nabla T - \mu T \Phi \quad (37)$$

The application of the Fourier law, $\mathbf{q} = -k \cdot \nabla T$, on Eq. (37) yields,

$$\dot{E}_{diss} = -\dot{E}_g = k(\nabla T)^2 + \mu T \Phi \quad (38)$$

where k is the thermal conductivity. Thus we obtained the formula for calculating the local entransy dissipation rate in heat convection. It is evident that the expression of the local entransy dissipation rate in the three-dimensional case shares the same form as Eq. (38).

From Eq. (38), one can see that if the temperature and velocity fields in heat conduction problems are known, the local entransy dissipation rate can be obtained exactly. Unfortunately, it is seldom to obtain the exact solution of heat convection problems. In many cases only the heat transfer correlations are available. Therefore it is of importance to develop expressions of the entransy dissipation rate by the knowledge of heat transfer correlations for heat convection. In this section we attempt to establish such a formula for internal flow in a duct with arbitrary cross section. Consider a flow passage enclosed by two cross sections with distance dx . The temperatures of the duct wall and fluid inside the duct are $T + \Delta T$ and T , respectively. The heat transfer rate from the duct wall to the fluid per unit length is \dot{q} . The fluid inside the duct flows with friction in x -direction, thus the pressure gradient $dP/dx \leq 0$. In this heat transfer process, assume there are two irreversibilities. One is the heat conduction from duct wall to the fluid. The other irreversibility is flow friction. As discussed in Section 1.4 the entransy dissipation rate per unit length $(\partial \dot{E}_{diss} / \partial x)_T$ induced by the heat conduction is

$$\left(\frac{\partial \dot{E}_{diss}}{\partial x}\right)_T = \dot{q} \Delta T \quad (39)$$

Assume the fluid is incompressible. The entransy dissipation rate per unit length $(\partial \dot{E}_{diss} / \partial x)_f$ induced by the flow friction is expressed as

$$\left(\frac{\partial \dot{E}_{diss}}{\partial x}\right)_f = -\frac{\dot{m} T}{\rho} \left(\frac{dP}{dx}\right) \quad (40)$$

Where ρ is the density of the fluid, \dot{m} is the mass flow rate. Thus the total entransy dissipation rate per unit length $\partial \dot{E}_{diss} / \partial x$ is

$$\frac{\partial \dot{E}_{diss}}{\partial x} = \dot{q} \Delta T - \frac{\dot{m} T}{\rho} \left(\frac{dP}{dx}\right) \quad (41)$$

The Stanton number correlation reads

$$S_t = \frac{\dot{q}}{p\Delta T c_p R} \quad (42)$$

with

$$R = \dot{m} / A \quad (43)$$

where A is the cross sectional area of the duct, c_p is the specific heat at constant pressure, p is the wetted perimeter. And the friction factor correlation is

$$f = -\frac{\rho D}{2R^2} \frac{dP}{dx} \quad (44)$$

Where D is the hydraulic diameter. By Eqs. (42) and (44), Eq. (41) arrives at

$$\frac{\partial \dot{E}_{diss}}{\partial x} = \frac{q^2}{4\dot{m}c_p} \frac{D}{S_t} + \frac{2\dot{m}T}{\rho^2} \frac{f}{DA^2} \quad (45)$$

Eq. (45) is thus the formula for calculating the derivative of the entransy dissipation rate with respect to the position variable for the forced convection in duct flow. Note that the higher Stanton number means the smaller entransy dissipation rate, while a high friction factor leads to the increase of the entransy dissipation rate.

2.3 Entransy dissipation for external flow

Consider an external steady flow that a solid body with arbitrary shape is suspended in a uniform stream with velocity U_∞ and absolute temperature T_∞ . For the convenience of analysis, the stream tube with cylindrical shape and large enough volume around the solid body as shown in Fig.2 is taken as the thermodynamic system. The surface temperature of the solid body is T_w . In the regions near the stream tube surface and the external part, the influence of the solid body on the flow is neglected. There is no heat transfer between the solid surface and its external environment. The irreversible dissipation is only caused by the heat conduction and flow friction. For the considered open thermodynamic system, the mass and energy conservations are expressed as,

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m} \quad (46)$$

$$\dot{m}h_{in} + \iint_A \dot{q}ds - \dot{m}h_{out} = 0 \quad (47)$$

where the subscripts "in" and "out" respectively indicate the inlet and outlet, h is the specific enthalpy, \dot{q} is the local heat transfer rate between the solid body and the stream, A is the surface area of the solid body, \dot{m} is the mass flow rate. For the heat conduction across the nonzero temperature difference $T_w - T_\infty$, the entransy dissipation rate $\dot{E}_{diss,T}$ is written as

$$\dot{E}_{diss,T} = \iint_A \dot{q}(T_w - T_\infty)dA$$

where A is the surface area of the solid body. For the flow friction, the entransy dissipation rate $\dot{E}_{diss,P}$ is

$$\dot{E}_{diss,P} = -\frac{\dot{m}T_{\infty}}{\rho_{\infty}}(P_{out} - P_{in})$$

where ρ_{∞} is the density of the fluid, P is the pressure. Here we assume the fluid is incompressible. Therefore, the total entransy dissipation rate is written as

$$\dot{E}_{diss} = \iint_A \dot{q}(T_w - T_{\infty})dA - \frac{\dot{m}T_{\infty}}{\rho_{\infty}}(P_{out} - P_{in}) \quad (48)$$

Since

$$\dot{m} = A_{tube}\rho_{\infty}U_{\infty}$$

and from the force balance on the control volume, we have

$$F_D = A_{tube}(P_{in} - P_{out})$$

where F_D is the drag force, A_{tube} is the cross sectional area of the cylindrical tube. Therefore,

$$\dot{E}_{diss} = \iint_A \dot{q}(T_w - T_{\infty})dA + T_{\infty}F_DU_{\infty} \quad (49)$$

If the body temperature is uniform, then the temperature difference $T_w - T_{\infty}$ is constant, thus Eq. (49) becomes

$$\dot{E}_{diss} = (T_w - T_{\infty})\iint_A \dot{q}dA + T_{\infty}F_DU_{\infty} \quad (50)$$

Since

$$\iint_A \dot{q}dA = Q = \bar{h}A(T_w - T_{\infty})$$

where \bar{h} is the average heat transfer coefficient based on A , Eq. (50) arrives at

$$\dot{E}_{diss} = \bar{h}A(T_w - T_{\infty})^2 + T_{\infty}F_DU_{\infty} \quad (51)$$

Note that when the body-ambient temperature difference is fixed, the only way to minimize the heat transfer contribution to the entransy dissipation rate is by decreasing the thermal conductance $\bar{h}A$.

The other extreme is that the heat transfer rate \dot{q} is uniform around the solid body. In this case Eq. (50) becomes

$$\dot{E}_{diss} = \dot{q}^2 \iint_A \frac{1}{h}dA + T_{\infty}F_DU_{\infty} \quad (52)$$

where h is the local heat transfer coefficient. Note that in order to reduce the heat transfer entransy dissipation rate, we must increase the heat transfer coefficient. In other words, the enhancement of the heat transfer from the solid body to the external fluid can decrease the entransy dissipation rate.

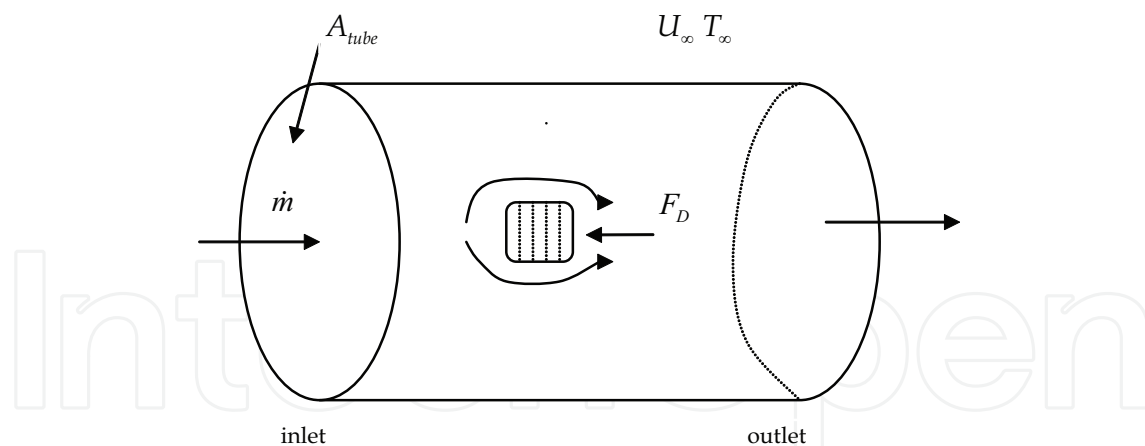


Fig. 2. Schematic diagram for external flow

2.4 Concluding remarks

In this section we have derived the expressions of the entransy dissipation rate for heat convection. These results lay a foundation for the application of entransy dissipation theory in the heat convection problems.

3. From the extremum principle of entransy dissipation to balance equations of fluid mechanics

In this section, our work (Xu & Cheng 2010) is reviewed. In this work, the extremum principle of entransy dissipation is developed by using the entransy dissipation theory developed in Section 1. We further show that this principle is compatible with the Fourier law and Newton's law of viscosity.

3.1 Introduction

The modeling and optimization design of heat exchanger based on the second law of thermodynamics has attracted lots of attention in past decades. In the early 1951, McClintock (1951) recognized that the irreversibility in heat exchanger could not be neglected in heat exchanger design. Bejan's creative work led to the establishment of the minimization entropy generation approach which is widely applied in optimization designs of thermal equipments (Bejan, 1982, 1988, 1995, 1996). However some paradoxes and contradiction appear when the minimization entropy generation approach is applied to the heat transfer problems (Hesselgreaves, 2000; Bertola & Cafaro, 2008). Although a non-dimensionalisation method for the entropy generation in heat exchanger was proposed by Hesselgreaves (2000) that avoids the 'entropy generation paradoxes', it has induced other contradictions.

In fact the thermodynamic basis of the minimization entropy generation approach is the minimum entropy production principle which reads "A steady state has the minimum rate of entropy production with respect to other possible states with the same boundary condition". Actually even for this principle itself, there are lots of controversies (Bertola & Cafaro, 2008; Jaynes, 1980; Mamedov, 2003; Landauer 1975; Ziman, 1956). Landauer (1975) found that the principle of the minimum entropy production is not valid even for some simple thermodynamic systems such as electric resistances. A variational principle developed by Ziman (1956) demonstrates that the entropy production assumes the

maximum value in the steady state for transport process. Bertola and Cafaro (2008) found that generally the principle of minimum entropy production does not agree with the conventional Fourier law as well as mass, energy and momentum balance equations in continuum mechanics. In order to remedy the defects of the principle of the minimum entropy production, Guo et al. (2007) developed the extremum principle of entransy dissipation in which only the irreversibility due to heat conduction under finite temperature difference was considered. In Section 3.2, we review the work which extends the current extremum principle of entransy dissipation by taking into account the influence of fluid friction and demonstrate the advantage of this principle over the principle of minimum entropy production in dealing with fluid flow problems (Xu & Cheng, 2010).

3.2 Extremum principle over entransy dissipation and balance equations of fluid mechanics

In order to seek some clues for establishing the extremum principle of entransy dissipation, we first consider natural convection problems, such as the Rayleigh-Benard convection. In this problem, when the temperature difference increases and reaches some critical point, the stationary fluid loses its stability and begins to move. In terms of the entransy, the Rayleigh-Benard convection can be viewed as the entransy related to conduction is converted to the entransy for driving the movement of fluid. The entransy consumed by performing work can be quantified as $-\mu\Phi\delta T$ according to the derivation in Section 2.2. The entransy dissipation associated with the heat conduction in the Rayleigh-Benard convection is quantified by $-\mathbf{q}\cdot\nabla T$. Therefore, we obtain the following extremum principle of entransy dissipation

$$\int_V [-\mathbf{q}\cdot\delta(\nabla T) - \mu\Phi\delta T] dV = 0 \quad (53)$$

The application of the Fourier law on Eq. (53) gives

$$\delta \int_V \left[\frac{1}{2} k (\nabla T)^2 - \mu \Phi T \right] dV = 0 \quad (54)$$

This is the principle of minimum entransy dissipation. Note that in this principle, the entransy dissipation comes from two irreversibilities: one is the heat conduction under finite temperature difference, another is the work against the fluid friction.

According to fluid mechanics the governing equation of incompressible fluid under steady state with constant shear viscosity and thermal conductivity reads

$$\frac{\partial v_i}{\partial x_i} = 0 \quad (55)$$

$$\rho v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \mu \frac{\partial}{\partial x_j} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) = 0 \quad (56)$$

$$\rho c_v v_j \frac{\partial T}{\partial x_j} - k \frac{\partial^2 T}{\partial x_j^2} = \frac{1}{2} \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (57)$$

where $\mathbf{x} = (x_1, x_2, x_3)^T$ is the position vector, $\mathbf{v} = (v_1, v_2, v_3)^T$ is the velocity field, p is the pressure. The density of entropy production is given by:

$$\sigma = \frac{k}{T^2} \frac{\partial T}{\partial x_i} \frac{\partial T}{\partial x_i} + \frac{1}{2} \frac{\mu}{T} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \frac{\partial v_k}{\partial x_k} \delta_{ij} \right) \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \frac{\partial v_k}{\partial x_k} \delta_{ij} \right) \quad (58)$$

Now we consider the one-dimensional steady incompressible shear flow, $\mathbf{v} = (0, v(x), 0)^T$, the continuity equation, momentum balance equation and energy conservative equation reduce to

$$\frac{\partial v}{\partial y} = 0 \quad (59)$$

$$\mu \frac{d^2 v}{dx^2} = -\frac{\partial p}{\partial x} \quad (60)$$

$$\frac{d^2 T}{dx^2} + \frac{\mu}{k} \left(\frac{dv}{dx} \right)^2 = 0 \quad (61)$$

From Eq. (59), one can see that the continuity equation is satisfied automatically for the flow under consideration. By Eq.(54), we define the following function

$$F = \frac{1}{2} k \frac{\partial T}{\partial x_i} \frac{\partial T}{\partial x_i} - \frac{\mu T}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - 2\lambda T \frac{\partial v_i}{\partial x_i} \quad (62)$$

In order to satisfy the continuity equation, the Lagrange multiplier λ is introduced in Eq. (62). From the variational method, Eq. (54) is equivalent to the following Euler-Lagrange equations

$$\frac{\partial}{\partial x_i} \left(\frac{\partial F}{\partial \left(\frac{\partial T}{\partial x_i} \right)} \right) - \frac{\partial F}{\partial T} = 0 \quad (63)$$

$$\frac{\partial}{\partial x_i} \left(\frac{\partial F}{\partial \left(\frac{\partial v_j}{\partial x_i} \right)} \right) - \frac{\partial F}{\partial v_j} = 0 \quad (64)$$

Substituting Eq. (62) into Eqs. (63) and (64) yields

$$k \frac{\partial^2 T}{\partial x_i^2} + \frac{\mu}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) + 2\lambda \frac{\partial v_i}{\partial x_i} = 0 \quad (65)$$

$$\mu \frac{\partial T}{\partial x_i} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) + \mu T \left(\frac{\partial^2 v_i}{\partial x_i \partial x_j} + \frac{\partial^2 v_j}{\partial x_i \partial x_j} \right) = -\lambda \frac{\partial T}{\partial x_i} - \frac{\partial \lambda}{\partial x_i} T \quad (66)$$

The application of the continuity equation (59) on Eq. (65) gives

$$k \frac{\partial^2 T}{\partial x_i^2} + \frac{\mu}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) = 0 \quad (67)$$

For the one-dimensional steady shear flow, Eqs. (65) and (66) become

$$\mu \frac{\partial T}{\partial x} \frac{\partial v}{\partial x} + \mu T \frac{\partial^2 v}{\partial x^2} = -\lambda \frac{\partial T}{\partial x} - \frac{\partial \lambda}{\partial x} T \quad (69)$$

$$k \frac{\partial^2 T}{\partial x^2} + \mu \left(\frac{\partial v}{\partial x} \right)^2 = 0 \quad (70)$$

By setting $\lambda = p = -\mu dv / dx$, Eqs. (69) and (70) reduce to Eqs. (60) and (61), respectively. Therefore, the extremum principle of entransy dissipation yields the same governing equation of the one-dimensional steady shear flow as given in fluid mechanics. Since Eqs. (60) and (61) are determined by the Fourier law and Newton's law of viscosity, we may say that the extremum principle of entransy dissipation is compatible with these two classical laws. While the principle of the minimum entropy production contradicts with the standard balance equations of the one-dimensional steady shear flow (Bertola & Cafaro, 2008). In this point, the extremum principle of entransy dissipation demonstrates the obvious advantage.

3.3 Concluding remarks

In summary, a new type of extremum principle of entransy dissipation is established by taking into account the viscosity resistance of the fluid. The analysis shows that this extremum principle of entransy dissipation gives rise to the standard governing equations of one-dimensional steady shear flow in fluid mechanics. Therefore, the extremum principle of entransy dissipation is consistent with the Fourier law and Newton's law of viscosity.

4. The application of entransy dissipation theory in heat exchanger design

In this section the expressions of the entransy dissipation related to the irreversibility in heat exchanger are developed. The entransy dissipation number which can be employed to evaluate the performance of heat exchangers is defined by a non-dimensionalisation method for the entransy dissipation. The optimization of heat exchanger design is analyzed.

4.1 Introduction

With the sharp decline of fossil fuels such as petroleum and coal, to use energy efficiently is one of effective ways to face the increasing energy demand. Heat exchanger as an important device in thermal system is widely applied in power engineering, petroleum refineries, chemical industries, and so on. Hence, it is of great importance to develop technologies which enable us to reduce the unnecessary energy dissipation and improve the performance of heat exchanger.

The evaluation criteria for heat exchanger performance are generally classified into two groups: the first is based on the first law of thermodynamics; the second is based on the

combination of the first and second law of thermodynamics. The heat transfer in heat exchangers usually involves the heat conduction under finite temperature difference, the fluid friction under finite pressure drop and fluid mixing. These processes are characterized as irreversible non-equilibrium thermodynamic processes. Hence, in recent decades the study of the second group has attracted a lot of attention (Yilmaz et al., 2001). Inspired by the principle of the minimum entropy production advanced by Prigogine (1976), Bejan (1982, 1995) developed the entropy generation minimization (EGM) approach to heat exchanger optimization design. In this approach Bejan (1982) took into account two types of the irreversibilities in heat exchanger, namely the heat conduction under the stream-to-stream temperature difference and the frictional pressure drop that accompanies the circulation of fluid through the apparatus. Therefore, the total entropy production rate denoted by \dot{S}_{gen} is the sum of entropy productions associated with heat conduction and fluid friction. However, among all the variational principles in thermodynamics, Prigogine's minimum entropy generation principle is still the most debated one as mentioned in Section 3.1. Accordingly, the entropy generation minimization approach, widely applied to modeling and optimization of thermal systems that owe their thermodynamic imperfection to heat transfer, mass transfer, and fluid flow irreversibilities, demonstrates some inconsistencies and paradoxes in applications of heat exchanger designs (Hesselgreaves, 2000).

Recently based on the concept of entransy Guo et al. (2010), the heat transfer efficiency and equivalent thermal resistance in a heat exchanger were defined. Wang et al. (2009) derived an entransy transfer equation describing the entransy transfer processes of a multi-component viscous fluid subjected to heat transfer by conduction and convection, mass diffusion and chemical reactions. Chen and Ren (2008) defined a ratio of temperature difference to heat flux as the generalized thermal resistance of convective heat transfer processes, and developed the minimum thermal resistance theory for convective heat transfer optimization, it was found that the minimum thermal resistance principle is equivalent to the entransy dissipation extremum principle. Xia et al. (2009) studied the optimum parameter distributions in two-fluid flow heat exchanger by using optimal control theory under the fixed heat load condition and taking the entransy dissipation minimization as the optimization objective. Liu et al. (2009) investigated the applicability of the extremum principles of entropy generation and entransy dissipation for heat exchanger optimization, and found that the former is better for the heat exchanger optimization when it works in the Brayton cycle, while the latter gives better results when heat exchanger is only for the purpose of heating and cooling. Recently the entransy dissipation extremum principle was extended to the radiative heat transfer by Wu and Liang (2008). Guo et al. (2009) introduced a non-dimensionalisation method for the entransy dissipation in heat exchanger and an entransy dissipation number which can be used to evaluate the heat exchanger performance was defined.

However, all the work mentioned above is based on the entransy defined by Guo et al. (2007) which is only related to the heat conduction in heat exchanger. In Section 1 we have presented a new definition of entransy which not only describes the ability to transfer heat, but also the ability to perform work. Subsequently, the entransy dissipation can quantify not only the irreversibility associated with the heat conduction, but also the irreversibility induced by fluid friction. In this section, we first derive the expressions of the entransy dissipation induced by heat conduction, fluid friction and fluid mixing in heat exchanger. Then a non-dimensionalisation of the total entransy dissipation will be introduced.

4.2 Entropy dissipation in heat exchanger

According to the definition of entropy, we have the following relationship between the entropy and enthalpy

$$de = Tdh \quad (71)$$

where e and h are the specific entropy and enthalpy, respectively. Assume that the heat exchanger under consideration is an adiabatic open system. By making use of Eq. (71) and the following relationship

$$h = mc_p T \quad (72)$$

the entropy dissipation rate induced by heat conduction in the heat exchanger is written as

$$\dot{E}_1 = \int_o^i (\dot{m} T dh)_1 = \int_o^i (\dot{m} c_p T dT)_1 = \frac{1}{2} (\dot{m} c_p)_1 (T_{1,i}^2 - T_{1,o}^2) \quad (73)$$

where \dot{E}_1 is the entropy dissipation rate of the fluid 1, c_p is the specific heat of fluid at constant pressure and assumed to be a constant, the subscript 1 denotes the fluid 1, the subscripts i and o denote the inlet and outlet, respectively. Similarly, for the fluid 2 we have

$$\dot{E}_2 = \int_o^i (\dot{m} T dh)_2 = \int_o^i (\dot{m} c_p T dT)_2 = \frac{1}{2} (\dot{m} c_p)_2 (T_{2,i}^2 - T_{2,o}^2) \quad (74)$$

Therefore, the total entropy dissipation rate induced by heat conduction in the heat exchanger is expressed as

$$\dot{E}_T = \dot{E}_1 + \dot{E}_2 = \frac{1}{2} (\dot{m} c_p)_1 (T_{1,i}^2 - T_{1,o}^2) + \frac{1}{2} (\dot{m} c_p)_2 (T_{2,i}^2 - T_{2,o}^2) \quad (75)$$

Now we consider the entropy dissipation rate caused by flow friction in heat exchangers. Assume that the fluid flow in the heat exchanger is driven by the finite pressure drop between the inlet and outlet, and the flow is stationary and adiabatic. Then from the definition of entropy, we have

$$de = -T dp / \rho \quad (76)$$

where p is the pressure. Then the entropy dissipation rate induced by flow friction is written as

$$\dot{E}_p = \int_i^o \dot{m} dg_0 = - \int_i^o \dot{m} T / \rho dp \quad (77)$$

If the fluid is the ideal gas, applying its state equation in Eq. (77) gives

$$\dot{E}_p = - \int_i^o \dot{m} T^2 R / p dp \quad (78)$$

where R is the ideal gas constant. If replacing the temperature in Eq. (78) with the logarithmic mean temperature, we obtain

$$\dot{E}_p = -T_{lm}^2 \int_i^o \dot{m} R / p dp \quad (79)$$

where $T_{lm} = (T_o - T_i) / (\ln T_o - \ln T_i)$. Similarly, for the incompressible fluid we have

$$\dot{E}_p = \frac{\dot{m} T_{lm} \Delta p}{\rho} \quad (80)$$

where $\Delta p = p_i - p_o$. If both fluids in the heat exchanger are idea gases, the entransy dissipation rate induced by fluid friction is written as

$$\dot{E}_p = -\dot{m}_1 \left(\frac{T_{1,o} - T_{1,i}}{\ln T_{1,o} - \ln T_{1,i}} \right)^2 R \ln \frac{p_{1,o}}{p_{1,i}} - \dot{m}_2 \left(\frac{T_{2,o} - T_{2,i}}{\ln T_{2,o} - \ln T_{2,i}} \right)^2 R \ln \frac{p_{2,o}}{p_{2,i}} \quad (81)$$

When both fluids in the heat exchanger are incompressible, we obtain

$$\dot{E}_p = \frac{\dot{m}_1 \Delta p_1}{\rho_1} \frac{T_{1,o} - T_{1,i}}{\ln T_{1,o} - \ln T_{1,i}} + \frac{\dot{m}_2 \Delta p_2}{\rho_2} \frac{T_{2,o} - T_{2,i}}{\ln T_{2,o} - \ln T_{2,i}} \quad (82)$$

When the fluid 1 is idea gas, fluid 2 is impressible, we have

$$\dot{E}_p = -\dot{m}_1 \left(\frac{T_{1,o} - T_{1,i}}{\ln T_{1,o} - \ln T_{1,i}} \right)^2 R \ln \frac{p_{1,o}}{p_{1,i}} + \frac{\dot{m}_2 \Delta p_2}{\rho_2} \frac{T_{2,o} - T_{2,i}}{\ln T_{2,o} - \ln T_{2,i}} \quad (83)$$

It is well known that when two or more streams of dissimilar fluids are mixing, the irreversibility occurs, which is detrimental to the performance of heat exchanger. In the cross flow exchanger, the mixing of thermal dissimilar fluids (with different temperatures) frequently happens. In the following we focus on the entransy dissipation rate induced by this type of mixing. Assume that n streams of thermal dissimilar fluids enter into the heat exchanger from its inlet. Their temperatures are denoted as $T_{j,i} (j = 1, 2, \dots, n)$. In the flow process from the inlet to outlet, the streams of thermal dissimilar fluids are adequately mixing. At the outlet, they have the same temperature T_o . Assume that the flow is stationary and the mixing process is adiabatic. Then making use of the law of energy conservation, the entropy balance equation and Eq. (71), we obtain

$$\dot{m}_o = \sum_{j=1}^n \dot{m}_{j,i} \quad (84)$$

$$\dot{E}_m = \dot{m}_o e_o - \sum_{j=1}^n (\dot{m} e)_{j,i} = \sum_{j=1}^n \dot{m}_j \Delta e_j \quad (85)$$

The similar derivation as leading to Eq. (72) yields

$$\dot{E}_m = \sum_{j=1}^n \frac{1}{2} (\dot{m} c_p)_j (T_{j,i}^2 - T_o^2) \quad (86)$$

Finally, the total entransy dissipation rate caused by heat conduction, fluid friction and fluid mixing is expressed as

$$\dot{E} = \dot{E}_T + \dot{E}_p + \dot{E}_m \quad (87)$$

4.3 Entransy dissipation number

In this section our work (Guo et al. 2009, Li et al. 2011) is reviewed. In this work, a dimensionless method of the entransy dissipation of the heat exchanger is established. Subsequently a quantity called the entransy dissipation number of heat exchanger is defined. Guo et al. (2009) examined the validity of the entransy dissipation number as the performance evaluation criterion of heat exchanger. Furthermore, for the water-water balanced counter-flow heat exchanger Li et al. (2011) took as the minimum overall entransy dissipation number as an objective function, under certain assumptions it was proved that there is a corresponding optimum in duct aspect ratio or mass velocity since the variation in the duct aspect ratio or mass velocity has opposing effects on the two types of entransy dissipations caused by heat conduction under finite temperature difference and the flow friction under finite pressure drop, respectively. We also develop analytically expressions for the optimal duct aspect ratio and mass velocity of a heat exchanger that are useful for design optimization.

Now we consider a water-water balanced counter-flow heat exchanger. Assume that both the hot and cold fluids are incompressible. The inlet temperatures and pressures of the hot and cold fluids are denoted as $T_{1,i}$, $P_{1,i}$ and $T_{2,i}$, $P_{2,i}$ respectively. Similarly the outlet temperatures and pressures are $T_{1,o}$, $P_{1,o}$ and $T_{2,o}$, $P_{2,o}$. For the balanced heat exchanger, the heat capacity rate ratio satisfies condition $C^* = (\dot{m}c)_2 / (\dot{m}c)_1 = 1$ (where \dot{m} is the mass flow rate). For the one-dimensional heat exchanger considered in this section, the usual assumptions such as steady flow, no heat exchange with environment, and ignoring changes in kinetic and potential energies as well as the longitudinal conduction are made. In the heat exchanger, there mainly exist two kinds of irreversibility: the first is heat conduction under finite temperature differences and the second is flow friction under finite pressure drops.

Note that according to the definition of entransy given in Section 1 the maximum entransy in a heat exchanger is $Q(T_{1,i} - T_{2,i})$, where Q is the actual heat transfer rate, $(T_{1,i} - T_{2,i})$ is the maximum temperature difference in the heat exchanger. Therefore the entransy dissipation rate caused by heat conduction under a finite temperature difference can be non-dimensionalized in the following way (Guo et al. 2009):

$$\dot{E}_T^* = \frac{\dot{E}_T}{Q(T_{1,i} - T_{2,i})} = \frac{\dot{E}_T}{\varepsilon (\dot{m}c)_1 (T_{1,i} - T_{2,i})^2} \quad (88)$$

where ε is the heat exchanger effectiveness which is defined as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate. As discussed by Guo et al (2009), the entransy dissipation number can be employed as the performance evaluation criterion of heat exchangers which avoids the "entropy generation paradox".

From Eq. (82), the entransy dissipation due to flow friction under a finite pressure drop is expressed as

$$\dot{E}_p = \frac{\dot{m}_1 \Delta P_1}{\rho_1} \frac{T_{1,o} - T_{1,i}}{\ln T_{1,o} - \ln T_{1,i}} + \frac{\dot{m}_2 \Delta P_2}{\rho_2} \frac{T_{2,o} - T_{2,i}}{\ln T_{2,o} - \ln T_{2,i}} \quad (89)$$

where ΔP_1 and ΔP_2 refer to the pressure drops of the hot and cold water, respectively; ρ_1 and ρ_2 are their corresponding densities. Putting in dimensionless form leads to

$$\begin{aligned} \dot{E}_p^* = & \frac{-\Delta P_1}{(\rho c)_1 (T_{1,i} - T_{2,i})} \frac{1}{\ln \frac{T_{2,i}}{T_{1,i}} \left[1 - (1 - \varepsilon) \frac{T_{2,i} - T_{1,i}}{T_2} \right]} + \\ & \frac{\Delta P_2}{(\rho c)_2 (T_{1,i} - T_{2,i})} \frac{1}{\ln \frac{T_{1,i}}{T_{2,i}} \left[1 + (1 - \varepsilon) \frac{T_{2,i} - T_{1,i}}{T_{1,i}} \right]} \end{aligned} \quad (90)$$

which is called the entransy dissipation number due to flow friction. Assuming that the heat exchanger behaves as a nearly ideal heat exchanger, then $(1 - \varepsilon)$ is considerably smaller than unity (Bejan, 1982). For a water-water heat exchanger under usual operating conditions, the inlet temperature difference between hot and cold water, $\Delta T = T_{1,i} - T_{2,i}$, is less than 100K, hence $\Delta T/T_i < 100 / 273 \approx 0.366$ ($i = 1, 2$). Therefore, Eq. (90) can be simplified to

$$\dot{E}_p^* = \frac{\Delta P_1}{(\rho c)_1 (T_{1,i} - T_{2,i})} \frac{1}{\ln \frac{T_{1,i}}{T_{2,i}}} + \frac{\Delta P_2}{(\rho c)_2 (T_{1,i} - T_{2,i})} \frac{1}{\ln \frac{T_{1,i}}{T_{2,i}}} \quad (91)$$

Accordingly, the overall entransy dissipation number becomes

$$\dot{E}^* = \dot{E}_T^* + \dot{E}_p^* = (1 - \varepsilon) + \frac{\Delta P_1}{(\rho c)_1 (T_{1,i} - T_{2,i})} \frac{1}{\ln \frac{T_{1,i}}{T_{2,i}}} + \frac{\Delta P_2}{(\rho c)_2 (T_{1,i} - T_{2,i})} \frac{1}{\ln \frac{T_{1,i}}{T_{2,i}}} \quad (92)$$

For a typical water-water balanced heat exchanger, the number of heat transfer units Ntu can be introduced, which approaches infinity as the effectiveness tends to unity. Since $C^* = 1$, the effectiveness is (Bejan, 1982):

$$\varepsilon = \frac{Ntu}{1 + Ntu} \quad (93)$$

where the number of heat transfer units is defined as

$$Ntu = \frac{UA}{\dot{m}c_p}$$

here U is the overall heat transfer coefficient, and A is the heat transfer area. Assume that the heat conduction resistance of the solid wall can be neglected, compared with the convective

heat transfer, then it is appropriate to replace U with the convective heat transfer coefficient h . Therefore,

$$\frac{1}{UA} = \frac{1}{(hA)_1} + \frac{1}{(hA)_2} \quad (94a)$$

or

$$\frac{1}{Ntu} = \frac{1}{Ntu_1} + \frac{1}{Ntu_2} \quad (94b)$$

where h_1 and h_2 are the convective heat transfer coefficients of the hot and cold fluids, respectively, and $Ntu_i = (hA)_i / (\dot{m}c)_i (i = 1, 2)$. In the nearly ideal heat exchanger limit, $Ntu \gg 1$, that is (Bejan, 1982)

$$1 - \varepsilon \approx \frac{1}{Ntu} \quad (95)$$

From Eq.(95) the overall entransy dissipation number is

$$\dot{E}^* = \left[\frac{1}{Ntu_1} + \frac{\Delta P_1}{(\rho c)_1 (T_{1,i} - T_{2,i})} \frac{1}{\ln \frac{T_{1,i}}{T_{2,i}}} \right] + \left[\frac{1}{Ntu_2} + \frac{\Delta P_2}{(\rho c)_2 (T_{1,i} - T_{2,i})} \frac{1}{\ln \frac{T_{1,i}}{T_{2,i}}} \right] \quad (96)$$

The two terms on the right of Eq. (96) correspond to the entransy dissipations of two sides of heat transfer surfaces. For each side, the entransy dissipation number can be written as follows:

$$\dot{E}_i^* = \frac{1}{Ntu_i} + \frac{P_i}{(\rho c)_i (T_{1,i} - T_{2,i}) \ln \frac{T_{1,i}}{T_{2,i}}} \frac{\Delta P_i}{P_i} \quad (i = 1, 2) \quad (97)$$

It is evident that the first term accounts for the entransy dissipation from the heat conduction under finite temperature difference and the second for the entransy dissipation from flow friction under finite pressure drop. For simplicity, we now use \dot{E}^* instead of \dot{E}_i^* to denote the entransy dissipation number for each side of the heat exchanger surface.

4.4 Parameter optimization

Theoretically, the exchanger effectiveness increases when the irreversible dissipation in the heat exchanger decreases. From Section 1.3 one can see that the entransy dissipation can be used to describe the irreversible dissipations, therefore we seek optimums in duct aspect ratio and mass velocity by minimizing the entransy dissipation number \dot{E}^* based on Eq. (97). Although the entransy dissipation number on one side of a heat transfer surface can be expressed as the sum of the contributions of the heat conduction under the finite temperature difference and flow friction under the finite pressure drop, the effects of these two factors on heat exchanger irreversibility are strongly coupled through the geometric

parameters of the heat exchanger tube residing on that side. Therefore, based on entransy dissipation minimization, it is possible to obtain optimal geometric parameters of the heat exchanger such as the optimal duct aspect ratio.

Recall the definition of the Stanton number $St((Re)_D, Pr)$ and friction factor $f((Re)_D)$:

$$Ntu = \frac{4L}{D} St \quad (98)$$

$$\frac{\Delta P}{P} = f \frac{4L}{D} \frac{G^2}{2\rho P} \quad (99)$$

where $G = \dot{m} / A$ is the mass velocity, L is the flow path length and D is the duct hydraulic diameter. Introducing the dimensionless mass velocity, $G_* = G / \sqrt{2\rho P}$

$$\tau^2 = \frac{P}{(\rho c)(T_{1,i} - T_{2,i}) \ln \frac{T_{1,i}}{T_{2,i}}}$$

and substituting Eqs. (98) and (99) into Eq. (97), we obtain

$$\dot{E}^* = \frac{1}{\frac{4L}{D} St} + \tau^2 f \frac{4L}{D} G_*^2 \quad (100)$$

Clearly, the duct aspect ratio $4L / D$ has opposing effects on the two terms of the right side of Eq. (100). Therefore, there exists an optimal duct aspect ratio to minimize the entransy number. When the Reynolds number and mass velocity are fixed, minimizing the entransy dissipation number leads to the following expression for this optimum:

$$\left(\frac{4L}{D} \right)_{opt} = \frac{1}{\tau G_* (fSt)^{1/2}} \quad (101)$$

The corresponding minimum entransy dissipation number is

$$\dot{E}_{min}^* = 2\tau G_* \left(\frac{f}{St} \right)^{1/2} \quad (102)$$

From Eqs.(101) and (102), one can see that the optimal duct aspect ratio decreases as the mass velocity G_* increases, and the minimum entransy dissipation number is directly proportional to the dimensionless mass velocity. Note that the minimum entransy dissipation number is also dependent on the Reynolds number via f and St . However, the impact of the Reynolds number on the minimum entransy dissipation number is very weak since for many heat transfer surfaces the ratio of the friction factor to the Stanton number does not have a significant change as the Reynolds number varies (Bejan, 1982). Therefore, the minimum entransy dissipation number is mainly determined by the selected dimensionless mass velocity. Obviously, the smaller the mass velocity, the longer the working fluid remains on the heat transfer surface and the lower the irreversible dissipations in the heat exchanger.

In designing a heat exchanger, the heat transfer area is an important consideration when it accounts for most of the total cost of a heat exchanger. Thus in the following, we discuss design optimization of the heat exchanger with a fixed heat transfer area.

From the definition of the hydraulic diameter, the heat transfer area for one side is

$$A = \frac{4L}{D} A_c$$

where A_c is the duct cross-section area. This expression can be put in dimensionless form as

$$A_* = \frac{4L}{D} G_*^{-1} \quad (103)$$

where A_* is the dimensionless heat transfer area, $A_* = (2\rho P)^{1/2} A / \dot{m}$. Substituting Eq. (103) into Eq. (100) yields

$$\dot{E}^* = \frac{1}{A_* St} G_*^{-1} + \tau^2 f A_* G_*^3 \quad (104)$$

Obviously, the dimensionless mass velocity has an opposing effect on the two terms of the right side of Eq.(104). Thus there exists an optimal dimensionless mass velocity which allows the entropy dissipation number to reach a minimum value when A_* and Reynolds number $(Re)_D$ are given. Solving this optimization problem yields:

$$G_{*,opt} = \left(\frac{1}{3 A_*^2 \tau^2 f St} \right)^{1/4} \quad (105)$$

$$\dot{E}_{min}^* = 4 \left(\frac{\tau^2 f}{27 A_*^2 St^3} \right)^{1/4} \quad (106)$$

Eqs. (105) and (106) respectively give the optimal dimensionless mass velocity and the minimum entropy dissipation number under fixed A_* and Reynolds number $(Re)_D$. From these two equations, the larger heat transfer area clearly corresponds to the smaller mass velocity and lower entropy dissipation rates. Hence, to reduce the irreversible dissipation occurring in a heat exchanger, the largest-possible heat transfer area should be adopted under the allowable conditions.

If \dot{E}^* and $(Re)_D$ are given, the minimum heat transfer area is

$$A_{*,min} = \frac{16}{3^{3/2}} \frac{\tau f^{1/2}}{\dot{E}^{*2} St^{3/2}} \quad (107)$$

with

$$\left(\frac{4L}{D} \right)_{opt} = \frac{4}{3} \frac{1}{\dot{E}^* St} \quad (108)$$

From Eqs. (107) and (108), one can see that a lower entropy dissipation rate corresponds to larger heat transfer area or duct aspect ratio. Eqs. (106) and (107) are identical, providing an

expression for the minimum attainable value for the product $A_*^{1/2}\dot{E}^*$ under the given Reynolds number.

4.5 Concluding remarks

As exemplified by the water-water counter-flow heat exchanger, the present work shows that there exists an optimal duct aspect ratio for heat exchangers under the fixed Reynolds number and mass velocity when the entransy dissipation number is taken as the performance evaluation criterion. Furthermore, the formula for the optimal duct aspect ratio was obtained analytically. Under constraints of the fixed heat transfer area (or duct volume) and Reynolds number, it was shown that there is an optimal dimensionless mass velocity; for which an analytical expression was also given. The results indicated that to reduce irreversible dissipations in heat exchangers, largest-possible heat transfer areas and lowest-possible mass velocities should be adopted. This conclusion is in agreement with numerical results obtained by design optimization of the shell-and-tube heat exchanger based on the entransy dissipation number as the objective function (Guo et al., 2010).

From the results obtained in this study, it can be seen that the traditional heat exchanger design optimizations based on total cost as an objective function usually sacrifice heat exchanger performance. This issue has been demonstrated by numerical results (Guo et al., 2009). Guo et al. (2009) found that a little improvement in heat exchanger performance can lead to large gains in terms of energy saving and environmental protection. Hence, in heat exchanger design, reduction in total cost and improvement in heat exchanger performance should be treated equally. The present work will be useful to drive new research in this direction.

5. Acknowledgements

The support of our research by National Basic Research Program of China (Project No. 2007CB206900) is greatly appreciated.

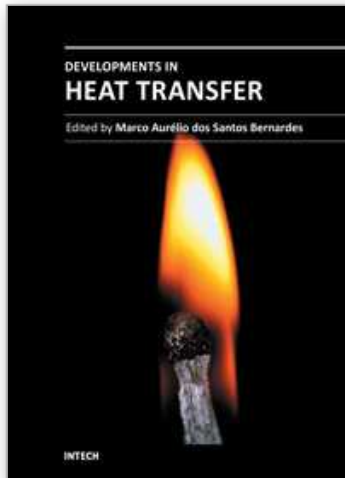
6. References

- Bejan, A. (1982). *Entropy Generation through Heat and Fluid Flow*. New York, Wiley
- Bejan, A. (1988). *Advanced Engineering Thermodynamics*. New York, Wiley
- Bejan, A. (1995). *Entropy Generation Minimization*. New York, CRC Press
- Bejan, A. (1996). Entropy generation minimization: the new thermodynamics of finite-size devices and finite-time processes. *J Appl Phys*, Vol. 79, pp.1191-1218
- Ben-Amotz, D. & Honig, J. M. (2006). Average entropy dissipation in irreversible mesoscopic processes. *Phys Rev Lett*, Vol. 96, pp. 020602.
- Ben-Amotz, D. & Honig, J. M. (2003). Rectification of thermodynamic inequalities. *J Chem Phys*, Vol.118, pp. 5932.
- Bertola, V. & Cafaro, E. (2008). A critical analysis of the minimum entropy production theorem and its application to heat and fluid flow. *Int J Heat Mass Transfer*, Vol. 51, pp.1907-1912.
- Bizarro, J. P. S. (2008). Entropy production in irreversible processes with friction. *Phys Rev E*, Vol. 78, pp. 021137

- Chen, L.; Wei, S. & Sun, F. (2011). Constructal entransy dissipation rate minimization of a disc. *Int J Heat Mass Transfer*, Vol. 54, pp. 210.
- Chen, Q. & Ren, J. X. (2008). Generalized thermal resistance for convective heat transfer and its relation to entransy dissipation. *Chinese Science Bulletin*, Vol. 53, pp. 3753-3761
- Cheng, X. & Liang, X. (2011). Entransy flux of thermal radiation and its application to enclosures with opaque surfaces. *Int J Heat Mass Transfer*, Vol. 54, pp.269.
- Cheng, X. G. (2004). Entransy and its Application in Heat Transfer Optimization. PhD thesis, Tsinghua University, Beijing
- Clausius, R. (1865). The Mechanical Theory of Heat-With its Applications to the Steam Engine and to Physical Properties of Bodies. London: John van Voorst, 1 Paternoster Row, MDCCCLXVII.
- den Broek, C. V. (2005). Thermodynamic efficiency at maximum power. *Phys Rev Lett*, Vol. 95, pp.190602.
- Esposito, M. & Lindenberg, K. (2009). Universality of efficiency at maximum power. *Phys Rev Lett*, Vol.102, pp.130602.
- Esposito, M. et al. (2010). Efficiency at maximum power of low-dissipation Carnot engines. *Phys Rev Lett*, Vol. 105, pp. 150603.
- Guo, J. F.; Cheng, L. & Xu, M. T. (2009a). Entransy dissipation number and its application to heat exchanger performance evaluation. *Chinese Sci Bull*, Vol. 54, pp. 2708.
- Guo, J. F.; Xu, M. T. & Cheng, L. (2009b). The application of field synergy number in shell-and-tube heat exchanger optimization design. *Appl Energ*, Vol. 86, pp. 2079-2087
- Guo, J. F.; Li, M. X. & Xu M. T. (2010). The application of entransy dissipation theory in optimization design of heat exchanger, *Proceedings of the 14th International Heat Transfer Conference, IHTC14-23348*, Washington, DC, USA, August 8-13, 2010
- Guo, Z. Y.; Zhu, H. Y. & Liang, X. G. (2007). Entransy-A physical quantity describing heat transfer ability. *Int J Heat Mass Transfer*, Vol. 50, pp. 2545.
- Guo, Z. Y. et al. (2010). Effectiveness-thermal resistance method for heat exchanger design and analysis. *Int J Heat Mass Transfer*, Vol. 53, pp. 2877.
- Hesselgreaves, J. E. (2000). Rationalisation of second law analysis of heat exchanger. *Int. J. Heat Mass Transfer*, Vol. 43, pp. 4189-4204
- Herwig, H. (2010). The Role of Entropy Generation in Momentum and Heat Transfer, *Proceedings of the 14th International Heat Transfer Conference, IHTC14-23348*, Washington, DC, USA, August 8-13, 2010
- Jaynes, E. T. (1980). The minimum entropy production principle. *Annu Rev Phys Chem*, Vol. 31, pp. 579-601
- Kondepudi, D & Prigogine, I. (1998). *Modern Thermodynamics-From Heat Engines to Dissipative Structures*, John Wiley & Sons, Chichester
- Landauer R. (1975). Inadequacy of entropy and entropy derivatives in characterizing the steady state. *Phys Rev A*, Vol. 12, pp.636-630
- Li, X. F.; Guo, J. F., Xu, M. T. & Cheng, L. (2011). Entransy dissipation minimization for optimization of heat exchanger design. *Chinese Sci. Bull.* (Accepted)
- Liu, X. B.; Meng, J. A. & Guo, Z. Y. (2009). Entropy generation extremum and entransy dissipation extremum for heat exchanger optimization. *Chinese Science Bulletin*, Vol. 54, pp. 943-947

- Mamedov, M. M. (2003). On the incorrectness of the traditional proof of the principle of minimum production. *Technical Physics Letters*, Vol. 29, pp. 69-71
- McClintock, F. A. (1951). The design of heat exchangers for minimum irreversibility. *ASME Paper*, No. 51-A-108, 1951, presented at the 1951 ASME Annual Meeting
- Prigogine, I. (1967). *Introduction to Thermodynamics of Irreversible Processes*, Third ed., Wiley, New York, pp. 76-77.
- Shah, R. K. & Skiepko, T. (2004). Entropy generation extrema and their relationship with heat exchanger effectiveness-number of transfer unit behavior for complex flow arrangement. *ASME J Heat Transfer*, Vol. 126, pp. 994.
- Wang, S. P.; Chen, Q. L. & Zhang, B. J. (2009). An equation of entransy transfer and its application, *Chinese science bulletin*, Vol.54, pp. 3572-3578.
- Wu, J. & Liang, X. G. (2008). Application of entransy dissipation extremum principle in radiative heat transfer optimization. *Science in China Series E: Technological Sciences*, Vol. 51, pp. 1306-1314
- Xia, S. J.; Chen, L. G. & Sun, F. R. (2009). Optimization for entransy dissipation minimization in heat exchanger. *Chinese Science Bulletin*, Vol.54, pp. 3587-3595
- Xu, M. T. & Cheng L. (2010). From the extremum principle of entransy dissipation to steady balance equations of fluid mechanics, *Proceedings of the 14th International Heat Transfer Conference*, IHTC14-23348, Washington, DC, USA, August 8-13, 2010
- Xu, M. T.; Guo, J. F. & Chen, L. (2009). Application of entransy dissipation theory in heat convection. *Front. Energy Power Eng China*, Vol.3, pp. 402-405
- Xu, M. T. (2011). The thermodynamic basis of entransy and entransy dissipation. *Energy*, Vol. 36, 4272-4277
- Yilmaz, M.; Sara, O. N. & Karsli, S. (2001). Performance Evaluation Criteria for Heat Exchangers Based on Second Law Analysis. *Exergy, an International Journal*, Vol.1, pp. 278-294.
- Ziman, J. M. (1956). The general variational principle of transport theory. *Can J Phys*, Vol. 34, pp.1256-1273

IntechOpen



Developments in Heat Transfer

Edited by Dr. Marco Aurelio Dos Santos Bernardes

ISBN 978-953-307-569-3

Hard cover, 688 pages

Publisher InTech

Published online 15, September, 2011

Published in print edition September, 2011

This book comprises heat transfer fundamental concepts and modes (specifically conduction, convection and radiation), bioheat, entransy theory development, micro heat transfer, high temperature applications, turbulent shear flows, mass transfer, heat pipes, design optimization, medical therapies, fiber-optics, heat transfer in surfactant solutions, landmine detection, heat exchangers, radiant floor, packed bed thermal storage systems, inverse space marching method, heat transfer in short slot ducts, freezing and drying mechanisms, variable property effects in heat transfer, heat transfer in electronics and process industries, fission-track thermochronology, combustion, heat transfer in liquid metal flows, human comfort in underground mining, heat transfer on electrical discharge machining and mixing convection. The experimental and theoretical investigations, assessment and enhancement techniques illustrated here aspire to be useful for many researchers, scientists, engineers and graduate students.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Mingtian Xu (2011). Entransy Dissipation Theory and Its Application in Heat Transfer, Developments in Heat Transfer, Dr. Marco Aurelio Dos Santos Bernardes (Ed.), ISBN: 978-953-307-569-3, InTech, Available from: <http://www.intechopen.com/books/developments-in-heat-transfer/entransy-dissipation-theory-and-its-application-in-heat-transfer>

INTECH
open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen