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Carbon Bio-Economics and Forests:
Getting the BESF Out of Climate Policy

Valny Giacomelli Sobrinho
Universidade Federal de Santa Maria
Brazil

1. Introduction

One of the most remarkable debates in the ongoing climate policy refers to carbon removal through biological terrestrial sinks. Since nearly 40 per cent of the planet's surface is covered with forests or forested areas (Ciesla, 1997), forestland stands out as one of the major terrestrial carbon sinks.

Nonetheless, ever since energy-efficiency-CDM (Clean Development Mechanism) was established by Article 12 of the Kyoto Protocol, in 1997, during the COP-3 (3rd Conference of the Parties signing the United Nations Framework Convention on Climate Change – UNFCCC), in Kyoto, Japan, forest-rich countries have complained that energy projects either saving or removing carbon emissions from fossil fuels would largely favour industrialised nations. Only at the COP-5, in 1999, in Bonn, Germany, Latin American, Asian and African countries made their point of including, in the Kyoto Protocol, the so-called "forestry-CDM", for reforestation and afforestation projects. Such an amendment was though very cautiously endorsed by countries like Germany and the United Kingdom (Moura-Costa & Aukland, 2001). Due to geo-political imbalances caused by differences in fossil-fuel prices across a few industrialised nations, they have strongly campaigned for reducing emissions at *source* – like in energy-efficiency-CDM projects – against mitigating them by *sinks* – like through avoiding deforestation (Fearnside, 2001).

Later on, forestry-CDM was blamed for favouring only planted (unnatural) forests and disregarding any effort towards conservation of natural woodlands. Therefore, at the COP-13, in 2007, in Bali, Indonesia, forest-rich countries demanded that the protection of natural forests, by avoiding deforestation, had also to be rewarded. Thenceforth, at the following COP's (COP-14, in 2008, in Poland; COP-15, in 2009, in Denmark; and COP-16, in 2010, in Mexico), forest-rich countries had been arguing that avoiding deforestation was the cheapest and fastest way of curbing carbon emissions and combating climate change. On top of this argument, labelled REDD (Reduced Emissions from Deforestation and forest Degradation), a REDD+ one was added, at COP-15, to include, in the protection strategy (ecosystem conservation and damage prevention), the enhancement of forest stocks.

Although, unlike CDM, the REDD mechanism is still under construction, the current state of affairs concerning the role played by forests in the climate policy comes down to the clash between forest plantations (forestry-CDM) and natural forests (REDD and REDD+). First and foremost, vegetation sinks, such as forests, are often claimed to "buy time" or play a "bridging role" until cleaner technologies become available to greatly curb future

anthropogenic CO_2 emissions (Kirschbaum, 2003). However, changes in spatial relations (*where-flexibility*), like those splitting forestland into natural and unnatural forests, imply changes in temporal relations (*when-flexibility*) as well (Martínez-Alier, 2002). Insofar as the time and growth speed of the economic output demands additional producing territories, the bio-geochemical time underlying environmental processes is increasingly overlooked. The greater the lag between the economy's faster and nature's slower production times, the larger the ecological imbalances (credit or debt) accruing over time.

The model (BESF – Bio-Economic model for carbon Sequestration by Forests) presented here not only can address the trade-off between forestry-CDM and REDD, but can also be applied to countries or regions with different endowments of forestland – both unnatural (**u**) and natural forests (**v**). It thus highlights the link between the spatial distribution (λ) of instant emissions across sinks (exports Z) and the demand for emissions over time (imports *M*) caused by economic growth (*k*).

As far as emissions given off by economic activities must be removed, it is demonstrated how the geographical distribution of ecological sinks (forestland) – *where-flexibility* – can influence the rhythm of economic growth over time – *when-flexibility* (Giacomelli Sobrinho, 2009). The usual *when-where-flexibility* argument relies on spatially uneven endowments of removing sinks both to lower the monetary costs of carbon removal or mitigation and to slow down mitigation investments. However it can be shown that, by and large, the more uneven the distribution of carbon sinks (the greater λ) is in the present, the longer it takes for the ecological-economic system to lessen the increasing biophysical cost of mitigation over time, as recorded by the mean long-run growth of the biophysical overshoot rate (Eq. (17)).

In this regard, the main objective of the model is to ground economic growth (emission source), translated by an emission supply (removal demand) function ($\hat{h}_t$), on its ecological limits, expressed by an emission demand (removal supply) function ($\hat{G}(X_t)$) sustained by forestland (forest sinks). Whereas $\hat{h}_t$ is fuelled by the economic growth rate (*k*), $\hat{G}(X_t)$ ultimately depends on the availability of forestland, split into natural and unnatural (plantations) forests. Therefore *k* and λ are supposed to be linked by an underlying ecological variable, guiding changes in both emission supply (ln *k*) and demand (ln λ). This invisible variable (ε), labelled the *bio-economic exchange rate*, works as a *shadow price*, which is found through dynamic optimisation methods (Fig. 1).

Because both *k* and λ are functions of ε, they can come together like in Fig. 2. That picture translates, at the macroscopic level, the effects of changes in $\hat{h}_t$ and $\hat{G}(X_t)$ triggered at the microscopic level. If an upper boundary (K_h) to the emissions arising from human economy could be signalled to their micro-economic sources ($\hat{h}_t$), biomass (forest) sinks would have to supply corresponding removing stocks ($\hat{G}(X_t)$) to counterbalance emission outflows. Macro-economically, whenever this ecological balance holds for any change in emissions, then ε = 1; whenever it does not, then 0 < ε < 1 (ecological credit) or ε > 1 (ecological debt). However, a point such as **P**, in Fig. 2, does not necessarily mean that ε = 1.

Long-run macro-bio-economic equilibrium might as well occur either with ecological credit or debt. In the former case, the ecological buffer to economic growth is greater than in the latter. Although this might sound environmentally friendly, ecological credit means exporting the ecological burden (bio-capacity overshoot) to elsewhere, whereas ecological debt implies carrying the ecological burden within an economy's boundaries over time (imports of bio-capacity).
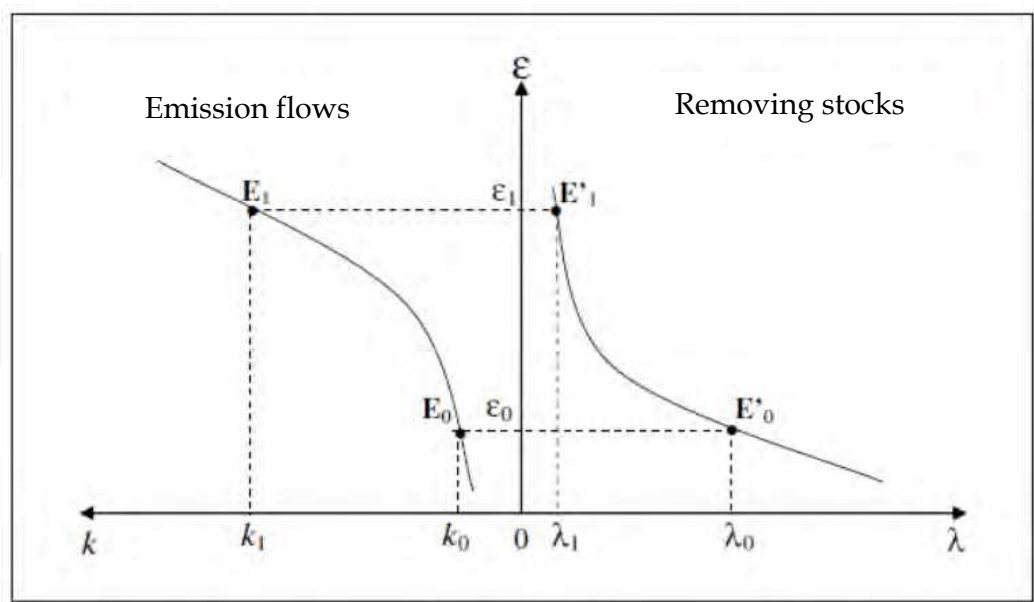
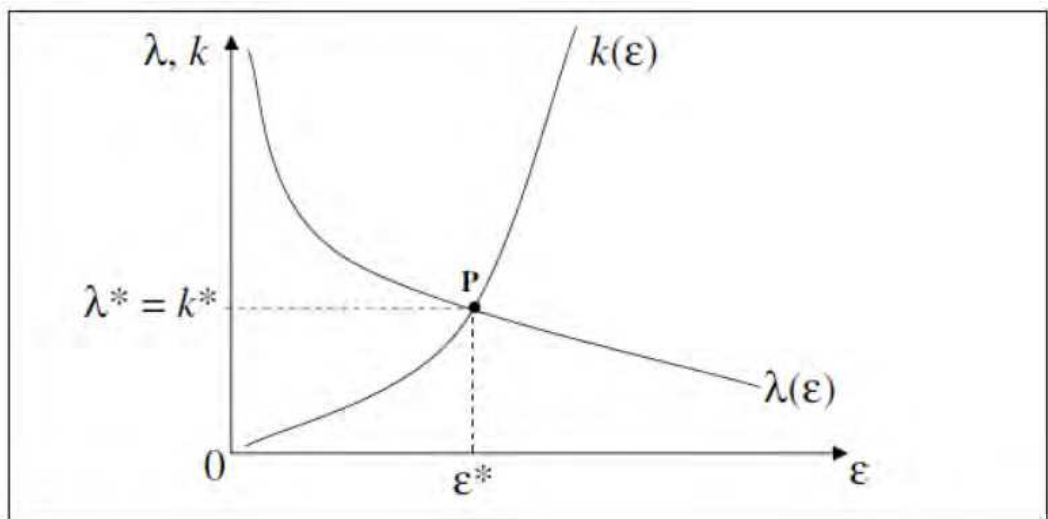Fig. 1. Adjustment of *k* and λ by the bio-economic exchange rate (ε)

Fig. 2. Long-run macro-bio-economic equilibrium

Last but not least, none of the ecological imbalances are properly caught by monetary measures. Mainly because money takes on the function of value reserve, it allows not only for carrying wealth over time, but also for splitting up buying and selling, thereby setting its holder free to decide *when* to use the purchasing power of money. Moreover, money is the only commodity that is (not) demanded when its price – i.e., its purchasing power – goes up (down). However, the more (less) it is demanded, the less (more) it is spent. As fewer (more) goods and services are consumed, the lower (higher) their prices turn out to be; then again, the purchasing power (price) of money increases (decreases), thereby reinforcing (discouraging) the demand for it. In other words, the utility (use value) of money only depends on its own *exchange value* (Carvalho et al., 2007). As Soddy (1934, p. 24) defines it: "Money now is the *nothing* you get for *something* before you can get *anything*".

Such a unique feature makes money and monetary prices unreliable guides for the biophysical reality and its ecological commodities (natural resources and environmental services). Therefore, mindful of these shortcomings, the BESF model does not make use of any monetary measure. Yet, the only methodological purpose in that is to point out how the economic analysis can fully give up money values and still get a reliable picture of real life problems.

2. Methodology

2.1 Theoretical background

Bio-economic models, such as forestry and fishery, are concerned with the age and size (Clark, 2010) of their biomass stocks (trees and fish). Whereas age mostly matters to biologically biased forestry models, size prevails in economically driven fishery ones. Whereas forestry models enhance the maximum sustainable yield (MSY) provided by even-aged stocks to be harvested at the end of every rotation period, fishery models highlight the maximum economic yield (MEY) (rent) to be earned under regulated competition and even before the MSY can take place. Because fishermen would like to maximize the difference between economic revenue and cost, they cannot wait until that biological maximum happens. Therefore fishery models follow most population studies, which, by making use of autonomous differential equations, leave aside time and take, instead, the size of stocks into closer account. After all, the growth of a biological population, like fisheries, depends rather on its initial stock than on the instant of time in which it began to be studied (Simon & Blume, 2004).

Anyway, despite the emphasis being placed either on the age or size of biomass stocks, both forestry and fishery models are evenly source-biased and output-driven. Both of them care about natural stocks (trees and fish) provided by natural sources (forests and oceans). None of them is concerned with the *environmental service* (input) or waste-sinking capacity provided by the natural pool in which the resource stock grows. It has been learned, though, that any sustainable and successful achievement in environmental planning is only supposed to come out if the management of both sources (environmental management) and sinks (environmental policy) take place altogether.

Whereas, in forestry and fishery models, the stock harvested ($\hat{h}_t$) becomes the source of economic gains (revenue), in the BESF model, it means the *environmental cost* of storing into forest biomass the emissions from the atmospheric pool given off by the production of the economic output. Thus, in the latter model, the biomass stock is an *input* rendering an environmental service (emission removal), instead of an *output* yielding biological (MSY) and economic (MEY) gains. However, as fruitfully demonstrated by input-output methods, the output is ultimately limited by the provisioning of minimum needed inputs. Therefore, whenever the supply of inputs is overlooked, caring about the output has usually proven frustrating in the long run.

Because in the BESF model the optimal stock stands for the environmental cost of removing emissions, it is supposed to be smaller than in standard bio-economic models, in which resource stocks are used as sources of revenue. Therefore MEY, in the former case, is expected to be slightly smaller than in the latter. Notwithstanding, MEY is still the greatest possible, given now the constraint of supplying ecological services (emission removal) with the least possible use of natural resource stocks (forest biomass). This constraint is needed because the availability of forest biomass is limited by the supply of forestland.

On the other hand, the availability of forest biomass depends, at the micro-level, on the demand for removing forest stocks ($\hat{h}_t$), which, by its turn, is, at the macro-level, set by the rate (k) of carbon emissions from economic growth. Thus, in this low carbon economy, the removal push comes, primarily and exogenously, from the *macro-economic* level.

Next, as long as emissions from economic growth have to be removed by forests, a *micro-economic* demand for forest stocks comes out. Then, to meet this demand, removing forest stocks have to be supplied ($\hat{G}(X_t)$), provided the supply of emissions (demand for removal) from economic growth would meet an upper boundary (K_h) somewhere. Such an upper boundary depends ultimately on λ – a variable indicating the current distribution of forestland as between natural and unnatural forests.

2.2 Assumptions

The model BESF departs from a geometric framework (Fig. 3), algebraically described (Table 2) by an emission-removal matrix (Klaassen & Amann, 1992) containing biophysical (PIOT – Physical Input-Output Table) instead of monetary (MIOT – Monetary Input-Output Table) figures (Hubacek & Giljum, 2003). Fig. 3 reminds an architectonic array in which the stability of the blocks building the horizontal upper beam relies on the weight that the blocks piling upon the column can support. The "height" of the building block **M** grows with time ($t = 1,…, m$) and sets the vertical weight to be borne; the "length" of the upper beam extends with the number of j ($j = 1,…, n$) sinks providing increasingly higher biomass stocks and depends on the "width" of **Z**. The balance between each other is regulated by a spherical ε (the bio-economic exchange rate). If **M** is too "high", it causes ε to flatten (i.e., its value is positively high) and the upper beam to bend downwards. Conversely, if **Z** is too "wide", it squeezes ε (i.e., its value is positively low), thereby lifting the rightmost end of the upper beam and increasing the pressure upon its supporting column.

When the supporting column is subject to any additional pressure, it means that the amount of emission removal transferred to other sinks (**Z**) must be reduced. This turns the width of **Z** smaller and blows ε out back, thereby causing its value to rise (depreciate). Conversely, the "height" of the supporting column owes to the transfer of emission removals over time (**M**). The larger these transfers, the lower the biophysical value (**V**) of the j removing sinks, which will increasingly become saturated. Then, when **M** is too high, it must be reduced, by pumping ε up again and making its value drop (appreciate). Just like in standard international trade models, excess imports (**M**) make the exchange rate (ε) go up (depreciate); excess exports (**Z**) make it go down (appreciate).

From	To	
	Human	Non-human
Human	(2) economy – economy standard economics **economic commodities**	(1) economy – ecology waste sinks **ecological commodities (environmental services)**
Non-human	(3) ecology – economy energy and material sources **ecological commodities (natural resources)**	(4) ecology – ecology standard ecology **ecological commodities**

Table 1. Augmented input-output model. Source: Adapted from Daly (1968, p. 401)
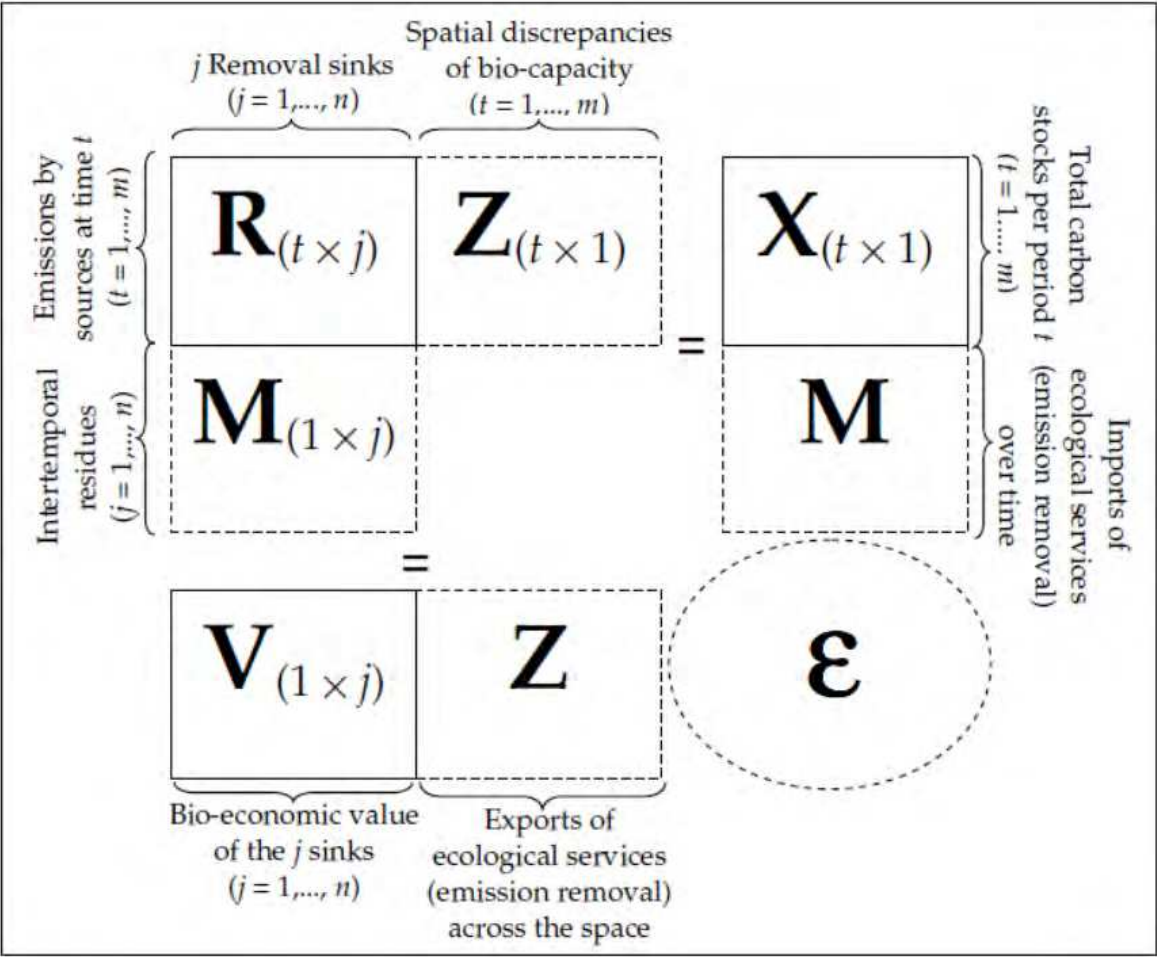
(*) Dotted shapes stand for underlying distributional variables not only recording emission-removal surpluses within the source-sink system, but also guiding its structural balance.

Fig. 3. Geometric framework of the BESF model*

The mechanics of such a structure (Fig. 3) falls back on a few underlying assumptions:

a. The emission-removal matrix (Table 2) includes forestland only, represented by silvicultural plantations, unnatural forests or lower biomass stock forests (**u**) and natural or higher biomass stock forests (**v**). To each forest sort a production bio-technology or bio-technological strategy is assigned: forestry-CDM (mitigation) to **u** and REDD (conservation or prevention) to **v**;

b. The emission-removal matrix (Table 2) is a PIOT (Physical Input-Output Table) rather than a MIOT (Monetary Input-Output Table) array (Hubacek & Giljum, 2003). When the economy-environment relationship is to be assessed, the physical measurement of material flows (input-output) is more useful than the monetary one (Dietzenbacher, 2005). After all, monetary prices hardly bear, if any, correlation with either energy or mass content (input) of the output produced (Ayres, 2004);

c. The link between the economic activity and land-use greatly draws on the assumptions underlying the *ecological footprint* method (Wackernagel & Rees, 1996). Whenever the avoidance, storage or removal of carbon emissions from economic growth is assigned to forest sinks, then some land appropriation is needed to supply forest carbon stocks. In these circumstances, economic growth can be translated into availability of forestland;

d. The framework depicted by Fig. 3 and Table 2 points to cell (1) in Daly's (1968) augmented input-output model (Table 1). Any cell like (1), (3) and (4) in Table 1 holds the "biophysical foundations of economics" (Daly, 1968, p. 401). By following Soddy (1934), whatever bigger world including both economic (cell 2) and ecological commodities (cells 1, 3 and 4), it could not rely on an invention like money which concerns not what is given up for it, but merely what is received in exchange for it. In this regard, the only "price" in the BESF model, which is represented by ε – the bio-economic exchange rate (Førsund & Nævdal, 1998) –, is dimensionless, although it can, through k and λ, be respectively translated into either percentage economic growth rates or carbon-equivalent tonnes and hectares of land;

e. Both **u** and **v** forests are taken as *sinks* (Fearnside, 2001).

2.3 Variables and equations

Algebraically, Fig. 3 is described by Table 2 and Eqs. (1) through (15), in Table 3.

t periods (emission sources)	j removal sinks* ($\mathbf{u} < \mathbf{v}$)		X	Z	λ
	u	**v**			
1	x_{11}	x_{12}	X_1	Z_1	λ_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	x_{m1}	x_{m2}	X_m	Z_m	λ_m
V	V_1	V_2	$V = X$	Z	λ
M	M_1	M_2	M	ε	
k	k_1	k_2	k		

(*) $j = \mathbf{u}$ = the smallest biomass stock sink; $j = \mathbf{v}$ = the largest biomass stock sink. However large j may be, sinks must always be displayed on an increasing biomass stock order.

Table 2. Emission-removal algebraic matrix

Variable	Description	Equation	
u	Vector of the smallest biomass stock sink	$\mathbf{u} = (x_{11}, ..., x_{m1}) = (u_1, ..., u_m)$	(1)
v	Vector of the largest biomass stock sink	$\mathbf{v} = (x_{12}, ..., x_{m2}) = (v_1, ..., v_m)$	(2)
x_{tj}	Emissions by sources at time t to be stored at (removed by) sink j	—	
X_t	Total removing stock at time t	$X_t = \sum_{j=1}^{n} x_{tj}$	(3)
X	Total removing stock through time	$X = \sum_{t=1}^{m} X_t$	(4)
Z_t	Exports of removing capacity at time t by the largest biomass stock sink (**v**)	$Z_t = x_{t2} - x_{t1}$	(5)

Z	Total exports over time	$$Z = \sum_{t=1}^{m} Z_t = \Delta V = V_2 - V_1 \qquad (6)$$
λ_t	Bio-diversity ratio of the largest (**v**) to the smallest (**u**) biomass stock sink	$$\lambda_t = \frac{\mathbf{v}}{\mathbf{u}} = \frac{v_t}{u_t} \qquad (7)$$
λ	Source-sink system's bio-diversity ratio	$$\lambda - 1 = \frac{Z}{V_1} \qquad (8)\text{a}$$ $$\ln \lambda = \frac{Z}{V_j} \qquad (8)\text{b}$$
V_j	Bio-economic value of sink j given by its supply, in the long-run, of biomass stocks for emission removal	$$V_j = \sum_{t=1}^{m} x_{tj} \qquad (9)$$
V	Total spatial bio-economic value	$$V = \sum_{j=1}^{n} V_j \qquad (10)$$
M_j	Imports (indebtedness) of removing stock needs (environmental services) caused by overshooting economic growth rates over time at sink j	$$M_j = x_{mj} - x_{1j} \qquad (11)$$
M	Total spatial imports (across all sinks j)	$$M = \sum_{j=1}^{n} M_j = \Delta X = X_m - X_1 \qquad (12)$$
k_j	Economic growth rate at sink j	$$k_j = \frac{x_{mj}}{x_{1j}} \qquad (13)$$
k	Source-sink system's economic growth rate	$$k - 1 = \frac{M}{X_1} \qquad (14)\text{a}$$ $$\ln k = \frac{M}{X_t} \qquad (14)\text{b}$$
k	Source-sink system's economic growth rate	$$k = \frac{X_t}{X_{t-1}} \qquad (14)\text{c}$$
ε	Bio-economic exchange rate	$$\varepsilon = \frac{M}{Z} = \frac{\ln k}{\ln \lambda} \times \frac{X_t}{V_j} \qquad (15)$$
ψ_t	†Biophysical overshoot rate at time t	$$\psi_t = \frac{k_t}{k^*} \qquad (16)$$
ψ	Long-run overshoot rate	$$\psi = \sqrt[m]{\prod_{t=1}^{m} \psi_t} \qquad (17)$$

(†) k* = optimal value for k.

Table 3. Variables and equations of the BESF model

Eq. (15) demands an important remark. Although $0 < \lambda_t < 1$ (Eq. (7)) and $0 < k_j < 1$ (Eq. (13)), thereby rendering $Z_t < 0$ (Eq. (5)) and $M_j < 0$ (Eq. (11)), there must always be $\varepsilon > 0$. Whereas for continuous calculations (Eqs. (8)*b* and (14)*b*), $Z < 0$ and $M < 0$ whenever $0 < \lambda < 1$ and $0 < k < 1$, for discrete figures (Eqs. (5), (6), (11) and (12)), the calculations must always turn both $\Delta V > 0$ (Eq. (6)) and $\Delta X > 0$ (Eq. (12)). The reason why it has to be so is that the bio-economic exchange rate (ε) does *not* seek to rule the *direction*, either across the space (Z) or over time (M), of the emission removal transfers. Rather, it cares about the *amounts* transferred across the space and over time.

Furthermore, when $0 < \lambda_t < 1$, it can be inferred, from Eq. (7), that $\mathbf{u} > \mathbf{v}$. Although this is an acceptable assumption when international removal trade is at sight, it cannot hold any longer when deforestation (i.e., falling $\mathbf{v}$) is strictly forbidden. In this case, the only allowed range for λ-values might at best be $\lambda \geq 1$, with any fall in λ thereby implying enhancement of forest stocks through greater use of forestry-CDM techniques, such as Sustainable Forest Management (SFM).

Because λ is primarily modified by the economic growth rate, k is the variable triggering ecological overshoot. Although k and λ are exogenously set at the macro-economic level, both of them can be optimised to check how much their observed values have been close to or far from the optimal (*) ones. When it comes to k, that distance is meant to be the overshoot rate at each period (Eq. (16)). The geometric mean of all k-distances gives the long-run overshoot rate (Eq. (17)).

Of course, for any given k_t in Eq. (16), ψ_t becomes smaller as k^* grows. Paradoxically, this sounds as if economic growth could be the solution for ecological overshoot. Yet, in an emission removing economy, larger values for k mean that more forestland is needed to store increasing amounts of emissions from economic growth. Thus, the cost of maintaining a high k would be an abrupt fall of λ.

A falling λ means either shrinking natural forests ($\mathbf{v}$) by increasing deforestation or causing unnatural forests ($\mathbf{u}$) to rise by enhancing their stocks through forestry-CDM techniques, such as Sustainable Forest Management (SFM). When it comes to international trade of removing stocks, a rising $\mathbf{u}$ means exporting deforestation, by causing $\mathbf{v}$ to fall, in exchange for imports of environmental services, such as emission removal. Should that be likely, the long-run overshoot rate (ψ) would go down too, along with λ.

Whereas a falling $\lambda \rightarrow 0$ implies higher *where-flexibility* in favour of unnatural forests ($\mathbf{u} > \mathbf{v}$), a *natural where-flexibility* ($\mathbf{u} < \mathbf{v}$) occurs with a growing $\lambda \rightarrow +\infty$. Whereas the latter case implies higher overshoot rates, the former yields *lower* ones. This is so because, when the picture changes from $\mathbf{u} < \mathbf{v}$ to $\mathbf{u} > \mathbf{v}$, exports $Z = \mathbf{v} - \mathbf{u}$ become highly negative, whereas imports M become lowly negative. Since Z indicates how much of the removing capacity depends on somewhere else's forests, $Z < 0$ means that the provision of removing capacity has greatly been switched over to the sink (economy) with the smallest biomass stock ($\mathbf{u}$). Over time, a shift like this implies reducing the transfers into the future of non-removed emissions – which is meant by a low $M < 0$. Hence, by making reduce both imports (M) – over time – and exports (Z) – across the space – of bio-capacity, this scenario can be paralleled with increasing autarky in trade, when hardly does any commerce take place.

2.4 Parameter

Table 2 and Eqs. (1) through (15) concern changes in removing stocks across the space and over time. However, they fail to set an upper boundary both to emissions and to the supply

of removing stocks. After all, ε is essentially affected by the ratio energy (*k*) to land (λ), which points out, in each period, how much emissions from economic growth can be sustained by every hectare of ecologically productive land (Wackernagel & Rees, 1996). Hence, there ought actually to be an upper boundary that balances the effect of two opposite forces. On one hand, the push for economic growth (measured by *k*) raises the demand for removing stocks; on the other hand, the supply of these stocks is constrained by biophysical limits given by existing forests (λ).

Unless the macro-level signs emitted by *k* and λ can be caught at the micro-level of economic activities, removing stock changes, recorded by Table 2, will meet no boundary at all. This bridge is, of course, supposed to, first and foremost, lie on *k*, for, by modifying λ, it is the variable triggering ecological overshoot. Even so, what still remains is how to rationally set that non-existing upper boundary.

Like in standard bio-economic (forestry and fishery) models, it is assumed that (removing) stocks grow by following a logistic pattern. Accordingly, they are supposed to reach an upper level beyond which stock losses outstrip stock growth. As the BESF model is rather concerned with emission flows than with output stocks, it claims for an upper limit (K_h) to the growth of emissions ($\hat{h}_t$) rather than to that of stocks ($\hat{G}(X_t)$). However, by all means, there is a removing stock level (X_T) associated with those maximum emissions (K_h) at some terminal time *T*.

Rationally, K_h can be found when two ideal equilibrium conditions are achieved at the same time *T*:

a. *Maximum economic efficiency* ($k_j = k$): whenever the rates of economic growth or return across the sinks even off, either *conservation* (investment on natural forest sinks) or *mitigation* (investment on unnatural forest sinks) can be indifferently traded off for one another (Common, 1996);

b. *Perfect ecological efficiency* (ε = 1): when every waste generated (emitted) is removed, any allocation and redistributive move across the sinks turns out to be over (Ayres, 1999, 2004).

Theoretically, these conditions stand for both the economic and ecological sustainability of the source-sink system. The terminal stock level (X_T), though, represents the time interval required to get *k* stable ($k_j = k$) and ε = 1. Therefore it means the "bio-economic cost" of achieving a stable state of sustainability. Such a cost is called the "bio-economic carrying capacity". It thus rather translates a loss to be incurred than a target to be complied with (Giacomelli Sobrinho & Schneider, 2008).

Mathematically, both conditions can be found by vector algebra (Eqs. (18) and (19)). The data used in the calculations (Table 4) were collected from FAO (2011) and are related to biomass forest stocks in Austria (AUT) and Brazil (BRA).

$$\begin{bmatrix} 1 & \bar{k}^{-1} \\ \bar{k} & 1 \end{bmatrix} \begin{bmatrix} 377 \\ -64927 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} 1 & \bar{k}^{-1} \\ \bar{k} & 1 \end{bmatrix} \begin{bmatrix} 377 \\ \bar{v}_{2010-T} \end{bmatrix} = \begin{bmatrix} 65304 \\ \bar{X}_{2010-T} \end{bmatrix} \tag{19}$$

with the bar over the letter standing for mean values.

Mean values from available time series can be worked out to feed in Eqs. (18) and (19). Arguably, mean values can cover a wider range of the relevant time horizon than instantaneous values could. From solving Eq. (18), it is found that $\bar{k}$ = 172.22. By substituting

this into Eq. (19), it comes out that $\bar{v}_{2010-T} = 11194310.34$ and $\bar{X}_{2010\text{-}T} = 11259237.28$. When $\bar{X}_{2010-T}$ is used to obtain $\hat{h}_t$ (Eq. (23)),the corresponding value of $\hat{h}_t$ is then called K_h.

Time t (in years)	Forest stocks* j (in ktC)		X	Z	λ	λ_t/λ_{t-1}
	u (AUT)	**v** (BRA)				
1990	339	68119	68458	67780	200.94	—
2000	375	65304	65679	64929	174.14	0.8666
2005	399	63679	64078	63280	159.60	0.9165
2010	393	62607	63000	62214	159.31	0.9982
Mean	**377**	**64927**	**65304**		**173.50**	**0.9255†**

Source: FAO (2011, pp. 123 and 126)

(*) Here, vectors **u** and **v** do not necessarily stand, respectively, for *unnatural* and *natural* forests. (†) Geometric mean.

Table 4. Biomass stocks in Austria's and Brazil's forests

2.5 Functions and hypotheses

Structurally, the model BESF takes into account the interplay of macro and micro-economic tiers. Changes in k and λ taking place at the *macro-bio-economic* level affect, correspondingly, the *micro-bio-economic* demand ($\hat{h}_t$) and supply ($\hat{G}(X_t)$) of emission removal. Whereas *micro-bio-economics* pinpoints removing stock levels yielding MSY (biological equilibrium), MEY (restricted access equilibrium, RA), economic rent dissipation (open access equilibrium, OA) and the steady-state (SS) equilibrium ($\hat{G}(X_t) = 0$), *macro-bio-economics* shows how the rate of economic growth (k) affects forestland use (λ) and overuse (ψ_t), as recorded by ε (Fig. 2 and
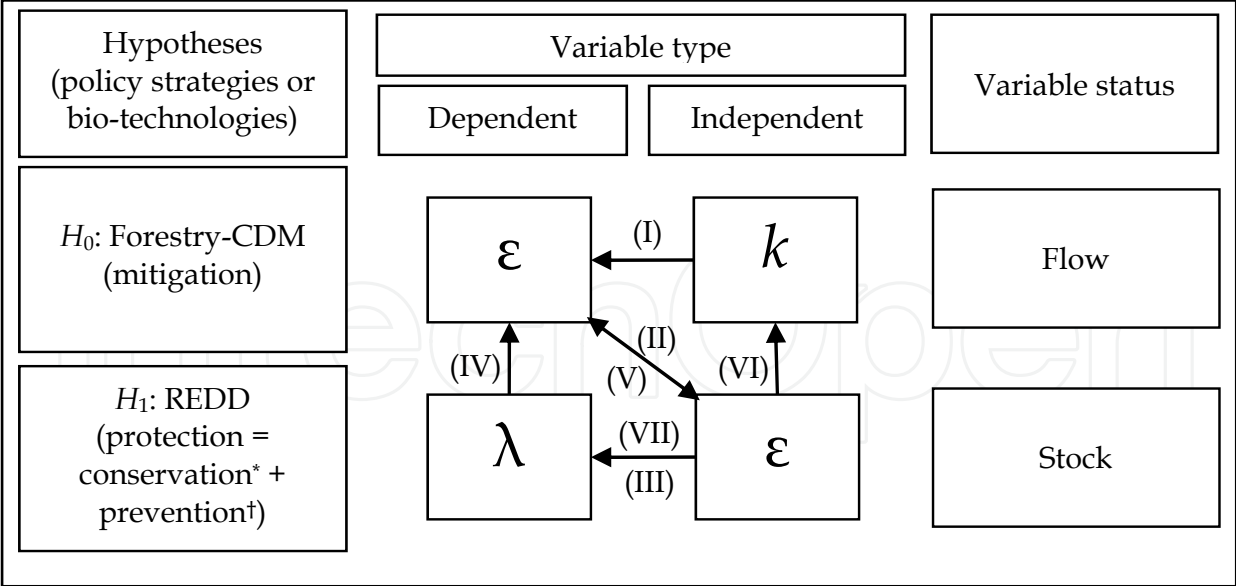
(*) Enhancement of stocks or stock maintenance approach (REDD+). (†) Avoided deforestation (REDD). (‡) Arrows indicate the direction of the causation flows; roman numerals describe the circuit of causation. It is worth noticing that, at the two last steps (VI and VII) of this circuit, both k and λ are turned into dependent variables, yet, for drawing reasons only, k still appears to be an independent one.

Fig. 4. Causation flows‡ between the macro-bio-economic variables of the BESF model

Fig. 4). In other words, whereas, through the parameter K_h, *micro-bio-economics* sets limits to the *supply* of emission removal, thereby ultimately determining λ at the macro-level, *macro-bio-economics*, by the interplay of k and λ, informs to the micro-level activities the changes in demand for removal the economy is allowed to grasp.

2.5.1 Micro-bio-economic removal demand or emission supply function ($\widehat{h}_t$)

The estimation of the removal demand function draws on the *Permanent Income Hypothesis* (PIH), put forward by the American economist Milton Friedman, in the late 1950's. His basic intuition was that "individuals would wish to smooth consumption and not let it fluctuate with short-run fluctuations in income" (Meghir, 2004, p. F293). In an emission-saving (low carbon) economy, consumption can be replaced by emissions released, whereas income arises from emission savings or removals.

Actually, the PIH divided both consumption and income into a permanent and a transitory component. The permanent income is thought of as the mean income regarded as permanent by consumers, which in turn depends on their horizon and foresightedness. On the other hand, "the transitory component consists of unforeseen additions or subtractions to income, which are supposed to cancel out[1] over the period considered and to be uncorrelated with the permanent income" (Houthakker, 1958, p. 397). Shortly, the PIH claims that consumption is planned over a fairly long period, on the basis of expected income (removal) during that period ($E(X_t)$) and that consumption plans (demand for emission removal, h_t) are not supposed to change because income (removing stocks, X_t) in a particular year falls short of or exceeds expectations ($E(X_t)$) (Houthakker, 1958). Thus, the *observed* demand for emission removal ("measured consumption") is given by:

$$h_t = X_t - E(X_t), \quad \text{for } X_t > E(X_t) \tag{20a}$$

$$h_t = E(X_t) - X_t, \quad \text{for } X_t < E(X_t) \tag{20b}$$

where $E(X_t)$ is some function of X over time t (X_p). $E(X_t)$ can be obtained from Table 8 as follows:

a. By regression of X_t over time, it is found out there to be a *cubic* relationship guiding (permanent) removal consumption over time (X_p):

$$X_{p(t)} = 0.001859t^3 - 0.141161t^2 + 5.113131t$$

t-stat.	6.286	-7.282	16.856
sig. t	0.0000	0.0000	0.0000	(21)

b. By regression of X_p (Eq. (21)) on observed X_t, then Eq. (22) comes out:

$$E(X_t) = 0.993992X_t$$

t-stat.	88.184
sig. t	0.0000	(22)

---

[1] This rule, however, does not apply to Eqs. (20)*a* and (20)*b*, which must always render positive outcomes. The reason is that these equations account for the biophysical rather than the monetary worth of consumption. According to the First Law of Thermodynamics (*material and energy balance*), in the biophysical world, nothing can be ruled out at all. As Soddy (1934, p. 96) adds on, "so long as physical tokens exist it is not possible to make them less than zero. But by book-keeping this obvious limitation can be got round, and in figures it is just as easy to count in negative numbers as in positive (...)".

Eq. (22) holds mean values for Eq. (21). In this case, as $E(X_t) < X_t$, then Eq. (20)a applies (see footnote 1). The results are displayed in Table 8. Now, by running the regression of the h_t values given by Eq. (20)a on the observed X_t, the demand function for emission removal ($\hat{h}_t$) can be finally estimated:

$$\hat{h}_t = 0.00601205X_t$$
$$\text{t-stat.} \quad 85216.783$$
$$\text{sig. t} \quad 0.0000 \tag{23}$$

2.5.2 Micro-bio-economic removal supply or emission demand function ($\widehat{G}(X_t)$)

By substituting $\bar{X}_{2010-T}$ = 11259237.28, found through Eqs. (18) and (19), in Eq. (23), the parameter K_h = 67690.53 arises. This is the *upper boundary* needed to establish, on the supply side, a logistic pattern of growth for the emission removing stocks, thereby indicating that, on the demand side, an upper level of emissions is supposed to be met somewhere, at a certain point in time.

K_h is next used in a conditioned optimisation programme (Eqs. (24) through (26)) to find $g(v(X_t))$ – the *logistic growth rate for emission removing stocks* (Eqs. (25)a and (25)b):

Objetive-function:

$$MIN \sum_t S_t = \left[g(v(X_t)) - \hat{h}_t \right] \tag{24}$$

Constraints:

I.
$$\hat{h}_t = 0.00601205X_t \tag{23}$$

II. (Boyce & DiPrima, 2006, p. 45)
$$g(v(X_t)) = \frac{K_h \hat{h}_1}{\hat{h}_1 + \left[(K_h - \hat{h}_1)e^{-b_1 X_t} \right]} \tag{25)a}$$

$$g(v(X_t)) = \frac{67690.53 \times 0.127}{0.127 + \left[(67690.53 - 0.127)e^{-0.01738464X_t} \right]}$$

$$g(v(X_t)) = \frac{8596.70}{0.127 + 67690.40e^{-0.01738464X_t}} \tag{25)b}$$

III.
$$g(v(X_t)) \geq \hat{h}_t \ , \tag{26}$$

where S_t = instantaneous surplus arising from the gap between removal growth rates ($g(v(X_t))$) and removal consumption rates ($\hat{h}_t$). In Eq. (25)b, $\hat{h}_1$ = 0.127 is taken out of Table 8, when t = 1 (1960), whereas that of b_1, another parameter of the logistic function (Eq. (25)a), is provided by the optimisation programme above, run in version 22.8 of GAMS (General Algebraic Modelling System). At last, the condition laid down by Eq. (26) ensures that emission removal rates will never be smaller than the emission removal demanded. The optimal values for $g(v(X_t))$ are also displayed in Table 8.

Next, by knowing the optimal $g(v(X_t))$ at each period, it is then possible figure out how much removal should be periodically supplied ($G(X_t)$). Yet, that amount is formerly up to the relationship between future ($\hat{F}(X_t)$) and currently observed (X_t) needs of emission removing stocks (Eq. (27)).

$$G(X_t) = \hat{F}(X_t) - X_t \tag{27}$$

Variable $\hat{F}(X_t)$, by its turn, results from an emission *outflow-inflow* ratio (Eq. (29)), in which the *outflow* component (Eq. (28)) is also supposed to follow a logistic growth pattern. It is worth noticing that the *hat notation* for variable $\hat{F}(X_t)$ does not mean it is an estimate in the statistical sense, but rather that $\hat{F}(X_t)$ is inferred from an *optimal* and *non-observable* logistic rate of growth for emission removing stocks.

a. Emission *outflow* (rate of demand for removing stocks):

$$\frac{d\hat{h}_t}{dX_t} = \hat{h}_t(K_h - \hat{h}_t) \tag{28}$$

b. Emission *inflow* (rate of supply of removing stocks):

$$g(v(X_t)) = \frac{K_h \hat{h}_1}{\hat{h}_1 + [(K_h - \hat{h}_1)e^{-b_1 X_t}]} \tag{25}a$$

$$\hat{F}(X_t) = \frac{d\hat{h}_t/dX_t}{g(v(X_t))} = \frac{\hat{h}_t(K_h - \hat{h}_t)}{g(v(X_t))} \tag{29}$$

By feeding in Eq. (27) the values provided by Eq. (29) and the observed X_t, displayed in Table 8, it is possible to arrive at the amount of $G(X)$ per period. By carrying out the regression of these so calculated values on the observed X_t ones, from Table 8, the supply function of emission removal can then be estimated (Eq. (30)).

$$\hat{G}(X_t) = -0.008164X_t^2 + 1.087793X_t + 30.775753$$

t-stat.	-29.275	27.579	24.765
sig. t	0.0000	0.0000	0.0000

$$\tag{30}$$

2.5.3 Macro-bio-economic removal supply or emission demand function ($\hat{\lambda}(\hat{\varepsilon})$)

The estimation of macro-bio-economic functions requires knowing the behaviour of ε for every given (observed) k and λ. As shown by Fig. 3, ε is an underlying variable, the role of which resembles very much that of a *shadow price*, informing, along an optimal path, the marginal bio-economic value of the asset (forestland) at time t (Clark, 2010).

Although ε is not an observable variable, it can, through Table 2 and Eqs. (1) through (15), be inferred from *observed* k and λ. This amounts to say that ε is a function of both k (Method I) and λ (Method II) – or, in symbols, $\hat{\varepsilon}(k)$ and $\hat{\varepsilon}(\lambda)$. Yet, these estimates are just intermediate steps to obtain their inverse functions, namely, $\hat{k}(\hat{\varepsilon})$ and $\hat{\lambda}(\hat{\varepsilon})$. The latter are those that actually account, respectively, for macro-bio-economic demand (supply) and supply (demand) of removal (emissions). As they are *long-run* (mean) functions, they carry no time (t) index.

To get $\hat{\varepsilon}(\lambda)$, it is first needed to hold fixed *observed* $k = \bar{k} = 1.03742$ (Table 8) and, by recalling Eq. (14)c, apply it evenly to every period, starting at $t = 1$. In this way, new values of X_t (X'_t) will be arrived at. The rationale behind this trick is to check, for every single period, the impact of λ itself onto ε, thereby tearing the effect of λ apart from that of k.

Although λ values cannot be obtained from economic data, they can be retrieved from forestry data, such as those displayed in Table 4. Yet, even so, data for λ are available for only four periods, thereby rendering unlikely to know the evolution of λ over time. Therefore, to ground the calculations on, at least, a slice of reality, the geometric mean of the changes in λ throughout 1990-2010 (in the last line and column of Table 4) is also supposed to hold all over the 1960-2007 period, for which economic data are available. As the latter is made up of $t = 48$ time periods, the (geometric) mean growth of λ (g_λ) throughout is given by:

$$g_\lambda = \sqrt[48]{0.9255} = 0.9984 \qquad (31)$$

By taking the natural logarithm of the outcome of Eq. (31) and multiplying the result by 100, it is found the rate of (de-)growth of λ from 1960 through 2007 (Eq. (32)).

$$\ln g_\lambda = \ln 0.9984 = -0.00161242$$

$$g_\lambda\% = \ln g_\lambda \times 100 = -0.161242\% \qquad (32)$$

By observing the evolution of λ, in Table 4, it can be seen that this variable has been falling over time. Hence, should the starting point for λ be its least value, during the 2005-2010 period (Table 4), its earlier values ought to be found by increasingly raising $\lambda_{2005\text{-}2010} = 159.31$ by $g_\lambda\% = 0.161242\%$, so as to get $\lambda_{1960} > \lambda_{2007}$.

Now, by consecutively taking the values of X'_t, yielded from holding $k = \bar{k} = 1.03742$, and using, accordingly, the corresponding λ_t's found by making them change by $g_\lambda\% = -0.161242\%$ (Eq. (32)), from $t = 1$ (1960) through $t = 48$ (2007), ε can then be calculated for every year. At last, λ and ε must be ordered pairwise, according to increasing λ figures.

For scaling reasons, $\ln \lambda$ is taken instead of λ itself. Logarithms scale down larger values of λ as compared with those too much smaller of k (Table 8). The resulting estimations for both $\hat{\varepsilon}(\lambda)$ (Method II) and $\hat{\lambda}(\hat{\varepsilon})$ are, respectively, given by Eqs. (33) and (34). Eq. (34) is, of course, the *macro-bio-economic removal supply* or *emission demand* function.

$$\hat{\varepsilon}(\ln \lambda) = 0.020497 - 0.000234 \ln \lambda \qquad (33)$$

t-stat.	10181.248	-594.029
sig. t	0.0000	0.0000

$$\ln \hat{\lambda}(\hat{\varepsilon}) = 87.546334 - 4271.091093\hat{\varepsilon} \qquad (34)$$

t-stat.	—	—
sig. t	—	—

2.5.4 Macro-bio-economic removal demand or emission supply function ($\hat{k}(\hat{\varepsilon})$)

Removal demand-side estimations, namely, $\hat{\varepsilon}(k)$ and $\hat{k}(\varepsilon)$, must follow through the same rationale guiding the former removal supply-side estimations. This time, though, it is needed to hold fixed *observed* $\lambda = \bar{\lambda} = 173.50$ (Table 4). Next, every observed k_t, in Table 8, is, consecutively pairwise (i.e., k_t and k_{t+1}; k_{t+1} and k_{t+2}; ...; k_{t+m-1} and k_{t+m}), placed into Table 2. Whenever $X_m < X_1$ or, more generally, $X_t < X_{t-1}$, thus rendering $M < 0$ (Eq. (12)), two single tricks can allow, as required, for $M > 0$. As long as $\mathbf{v} > \mathbf{u}$ and $|M_\mathbf{v}| > |M_\mathbf{u}|$, M becomes positive by applying: *a)* $M = -\sum_{j=\mathbf{u}}^{\mathbf{v}} M_j$, if $M_\mathbf{v} < 0$; and *b)* $M = \sum_{j=\mathbf{u}}^{\mathbf{v}} M_j$, if $M_\mathbf{v} > 0$, but $M_\mathbf{u} < 0$.

At last, while keeping $\lambda = \bar{\lambda} = 173.50$ throughout, Eqs. (1) through (15) are used to reckon the impact of the various observed k on the value of ε. Again, as in removal supply-side estimations, k and ε must be ordered pairwise, according to increasing k figures.

The resulting estimations for both $\hat{\varepsilon}(k)$ (Method I) and $\hat{k}(\hat{\varepsilon})$ are, respectively, given by Eqs. (35) and (36). Eq. (36) is, of course, the *macro-bio-economic removal demand* or *emission supply* function. For both solution and domain reasons, $\ln k$ is taken instead of k itself. The removal market solution requires that removal supply (Eq. (34)) equals removal demand (Eq. (36)). Therefore it is handier to the solution, if both functions can match their scales.

$$\hat{\varepsilon}(\ln k) = 3.106921(\ln k)^2 + 0.088291 \ln k + 0.010558 \tag{35}$$

t-stat.	8.930	2.678	10.456
sig. t	0.0000	0.0104	0.0000

$$\ln \hat{k}(\hat{\varepsilon}) = -45.321413\hat{\varepsilon}^2 + 5.626990\hat{\varepsilon} + 0.059203 \tag{36}$$

t-stat.	-5.484	9.378	-7.284
sig. t	0.0000	0.0104	0.0000

3. Equilibrium and scenario analysis

In this section, the *removal bio-economics* described so far is applied to Austrian (AUT) and Brazilian (BRA) economies, for a 48-year-long time period, spanning from 1960 through 2007 (Table 8). The analysis is split into *removal micro-bio-economics* and *removal macro-bio-economics*.

3.1 Micro-bio-economic analysis

By following suit equilibrium analysis in standard fisheries, five equilibrium points are spotted: *a*) *BESF* equilibrium; *b*) *restricted access* (RA) or *maximum economic yield* (MEY) equilibrium; *c*) *maximum sustainable yield* (MSY) *d*) *open access* (OA) equilibrium; and *e*)

Equili-brium	Equilibrium conditions	Optimal[†] stock (X_t^*) (ktC)[‡]	Emission savings ($\hat{G}(X_t^*)$) Eq. (30) (MtC)[‡]	Emission consump-tion ($\hat{h}_t^*$) Eq. (23) × 10^2 (tens of tC)[‡]	Rate of return[††] $\dfrac{d\hat{G}(X_t)}{dX_t}$	Rate of depletion[§] $\dfrac{d\hat{h}_t}{dX_t}$	Economic yield or rent[‡] $Y_t = \hat{G}(X_t^*) - \hat{h}_t^*$
BESF/ MEY′	$\dfrac{d\hat{G}(X_t)}{dX_t} = \dfrac{d\hat{h}_t}{dX_t}$ $0 < X_{\text{BESF}} < X_{\text{MSY}}$	29.80083293	55.94252661	17.91626076	**0.601205000**	0.601205	**38.0262658498285**
RA/ MEY″	$\dfrac{d\hat{G}(X_t)}{dX_t} = \dfrac{d\hat{h}_t}{dX_t}$ $0 < X_{\text{RA}} < X_{\text{OA}}$	29.80083303	55.94252667	17.91626082	**0.601204998**	0.601205	**38.0262658498290**
MSY	$\dfrac{d\hat{G}(X_t)}{dX_t} = 0$	66.62	**67.01**	40.05	0.000000	0.601205	26.96
OA	$\hat{G}(X_t) = \hat{h}_t$	98.05	58.95	58.95	-0.513147	0.601205	**0.00**
SS	$\hat{G}(X_t) = 0$	157.22	**0.00**	94.52	-1.479293	0.601205	-94.52

(†) Provided by GAMS 22.8. (‡) For scaling reasons, carbon units diverge. Actually, the barter ratio of $\hat{G}(X_t)$ to $\hat{h}_t$ is 1 MtC per 10 tC (Table 8). Of course, carbon rent (economic yield) can surely be calculated, but its values will not correspond to those displayed in the last column of Table 5. Even though, scale discrepancies themselves do not rule out the rationale underlying the calculations of the economic yield or rent. However, when carbon units are matched up, rent values will not exactly look like those figures appearing in the last column of Table 5. (††) "... a private profit-maximising steady-state equilibrium ... will be one in which the resource stock is maintained at a level where the rate of growth ($d\hat{G}(X_t)/dX_t$) equals the market rate of return on investment (...)" (Perman et al., 1996, p. 179). (§) Before calculating this derivative, Eq. (23) must be multiplied by 10^2, because the barter ratio of $\hat{G}(X_t)$ to $\hat{h}_t$ is 1 MtC per 10 tC or 10^6 tC ÷ 10 tC = $10^2 \times 10^3$ tC ÷ 1 tC = 10^2 ktC per tC. So, multiplying 1 tC by 10^2 gives the *unit* barter ratio of $\hat{G}(X_t)$ to $\hat{h}_t$, namely, 10^2 ktC : 1×10^2 tC = 1 ktC : 1 tC.

Table 5. Results from micro-bio-economic equilibrium analysis

steady-state (SS) equilibrium. The results are displayed in Table 5 and shown by Fig. 5. Although Y_{BESF} (MEY') is slightly smaller than Y_{RA} (MEY"), the rate of return at BESF equals the rate of depletion, whereas at RA the rate of return is slightly smaller than the rate of depletion (Table 5). As explained earlier, this was expected, because BESF is input-driven (emission saver), whereas RA is mostly concerned with the economic output arising from the consumption of emissions to be later stored in biomass stocks (emission removal).

By comparing X_t^*, in Table 5, with the observed X_t, in Table 8, it is possible to approximately know *when* each micro-bio-economic equilibrium happens. Both X_{BESF} and X_{RA} took place somewhere in-between 1967 and 1968; X_{MSY}, between 1985 and 1986; X_{OA}, between 1995 and 1996, thus, one decade after X_{MSY}; X_{SS} seems to be still to come.

The major warning coming out of this micro-bio-economic equilibrium analysis is that, given the observed growth of emissions (X_t in Table 8) in their economies, Austria and Brazil's forests have already left behind their potential to generate MEY (1967-1968). Likewise, the point of biological equilibrium (MSY) was also surpassed around 1985-1986. Thenceforth, it took only one decade longer (1995-1996) to reach the removing stock level (X_{OA}) at which emission savings just even off emission needs. Such a stock level is thought of dangerously driving to that of removal overshoot (X_{SS}), beyond which emission consumption becomes increasingly larger than emission savings. In the sink-based (environmental service) approach, X_{SS} can be compared with exhaustion or extinction, in the source-based (natural resource output) one. At present, Austrian and Brazilian economies are already in the neighbourhood of that overshoot point.
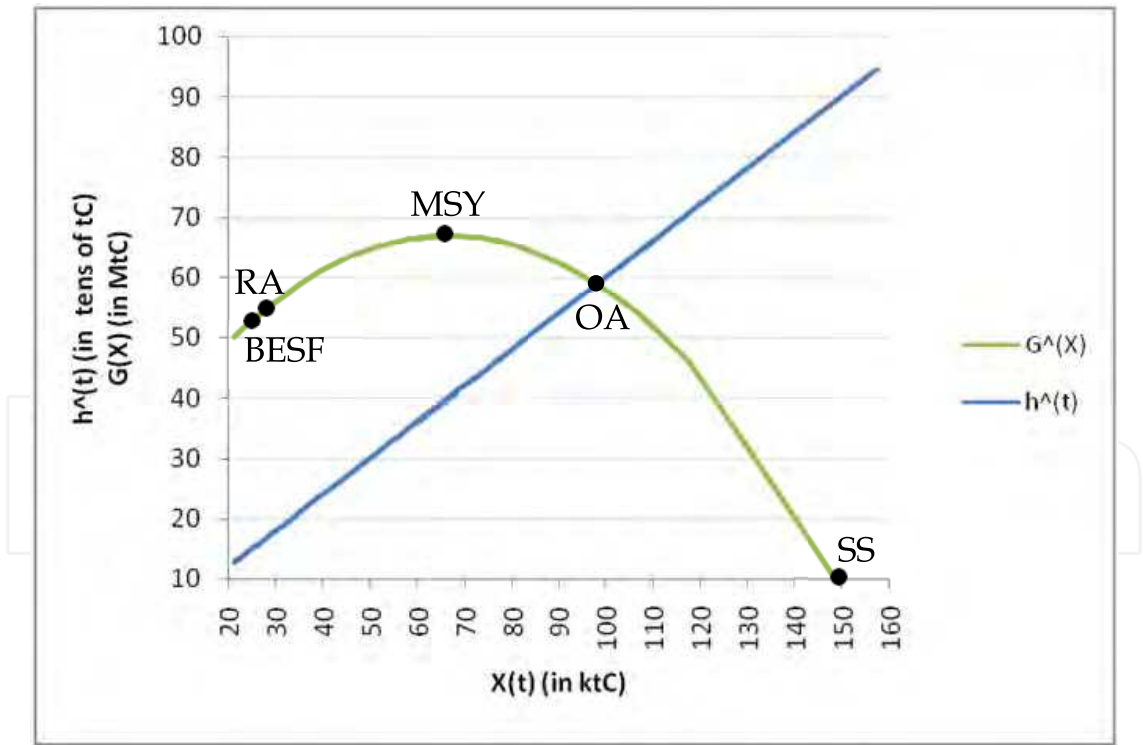

Fig. 5. Micro-bio-economic equilibrium points for Austria and Brazil's economies (1960-2007)

3.2 Macro-bio-economic analysis

In this section, five scenarios are tried out to make it clear how ε responds to the long-run overshoot rate ψ (Fig. 7). The scenarios range from an extreme situation in which λ is

minimum (Scenario 3, in Table 6) – therefore deforestation in sink **v** is maximum (Eq. (7)) – to the opposite setting in which $\lambda \to +\infty$ (Scenario 4, in Table 6) – and thus, by Eq. (7), conservation (REDD) in sink **v** is maximum. Sink **v** (Brazil's forests) commands conservation not only because it shelters the largest biomass stocks, but also because, by definition (Eq. (7)), λ is the variable guiding the exports Z (Eqs. (5) and (6)) of emission removing services across sinks and, thereupon, the *supply* of emission removing stocks (Eq. (34)).

Scenarios*	Constraints[†]	Objective-function (W or W')[‡]
1. BEE (Bio-Econ. Equil.)	$\ln \hat{\lambda} \geq \ln \hat{k}$	MIN W
2.a. Max. REDD a	$\ln \hat{\lambda} \geq \ln \hat{k}; 0 < \hat{\varepsilon} < +\infty$	MAX W
2.b. Max. REDD b	$\ln \hat{\lambda} \geq \ln \hat{k}; 0 < \hat{\varepsilon} < \hat{\varepsilon}_{\text{BEE}}$	MAX W
3. Full CDM	$\ln \hat{\lambda} \leq \ln \hat{k};$ $\hat{\lambda} \to 0$	MAX W' or MIN W'
4. Full REDD	$\ln \hat{\lambda} \geq \ln \hat{k};$ $\hat{\lambda} \to +\infty$	MAX W or MIN W
5. CDM = REDD	$\ln \hat{\lambda} \leq \ln \hat{k};$ $\ln \hat{\lambda} = 0; \hat{\lambda} = 1$	MAX W' or MIN W'

(*) Forestry-CDM is assigned to vector **u**; REDD, to vector **v**. By Eq. (7), λ rules both.
(†) Where $\ln \hat{\lambda}$ is given by Eq. (34) and $\ln \hat{k}$, by Eq. (36).
(‡) $W = \ln \hat{\lambda} - \ln \hat{k}$ and $W' = \ln \hat{k} - \ln \hat{\lambda}$

Table 6. Scenario analysis

Sce-nario	$\ln \hat{\lambda}^*$	$\ln \hat{\lambda}^*$ [†] (×10-2)	$\ln \hat{k}^*$	$\hat{k}^*$ (%)	$\hat{\varepsilon}^*$ (×10-2)	Long-run overshoot rate		
						ψ	$\ln \psi$	ψ (%)
3	-23.026	-0.2303	0.05609	5.609	2.59	0.980834	-0.01935	-1.935
5	0.000	0.0000	0.03709	3.709	2.05	0.999650	-0.00035	-0.035
1	0.037	0.0004	0.03706	3.706	2.05	0.999682	-0.00032	-0.032
—	2.000	0.0200	0.03532	3.532	2.03	1.001425	0.00142	0.142
—	3.000	0.0300	0.03442	3.442	1.98	1.002322	0.00232	0.232
—	5.000	0.0500	0.03262	3.262	1.93	1.004132	0.00412	0.412
2a	16.667	0.1667	0.02169	2.169	1.66	1.015161	0.01505	1.505
2b	18.656	0.1866	0.01976	1.976	1.61	1.017122	0.01698	1.698
4	23.016	0.2302	0.01546	1.546	1.51	1.021504	0.02128	2.128

(†) Once $\ln \hat{\lambda}^*$ (Eq. (34)) is much larger than $\ln \hat{k}^*$ (Eq. (36)), the former must be scaled down, by multiplying it by 10-2, so that both function values can be plotted together. (‡) Results provided by GAMS 22.8.

Table 7. Optimal[‡] (*) results from scenario analysis

After running the scenarios from Table 6, the outcomes displayed in Table 7 are achieved. The results from Table 7 are plotted in both Fig. 6 and Fig. 7. The curves in Fig. 6 show the optimal long-run path for economic growth rates ($\hat{k}^*$) as well as for forest and climate policy (forestry-CDM and REDD, driven by $\hat{\lambda}^*$). Whereas the removal supply function ($\ln \hat{\lambda}^*$) is highly sensitive (steeper) to changes in the bio-economic exchange rate ($\hat{\varepsilon}^*$), the removal

demand function ($\ln \hat{k}^*$) is just slightly modified (smoother) by changes in $\hat{\varepsilon}^*$. This means that economic growth rates are not supposed to considerably change in response to any movement in the bio-economic exchange rate.

As the movements of the bio-economic exchange rate ($\hat{\varepsilon}^*$) are found to offset changes in the long-run overshoot rate (ψ) (Fig. 7), it can be stated that economic growth is rather insensitive (inelastic) to ecological overshoot – large shifts in overshoot rates can only make the economic growth rate slightly change, or, conversely, small changes in the economic growth rate cause large changes in the long-run overshoot rate. On the other hand, just small changes in $\hat{\varepsilon}^*$ are suffice to bring about large changes in $\hat{\lambda}^*$ – thus rendering the bio-diversity ratio quite sensitive (elastic) to ecological overshoot. In a nutshell, this means that the longer it takes to control long-run economic growth rates in order to make long-run
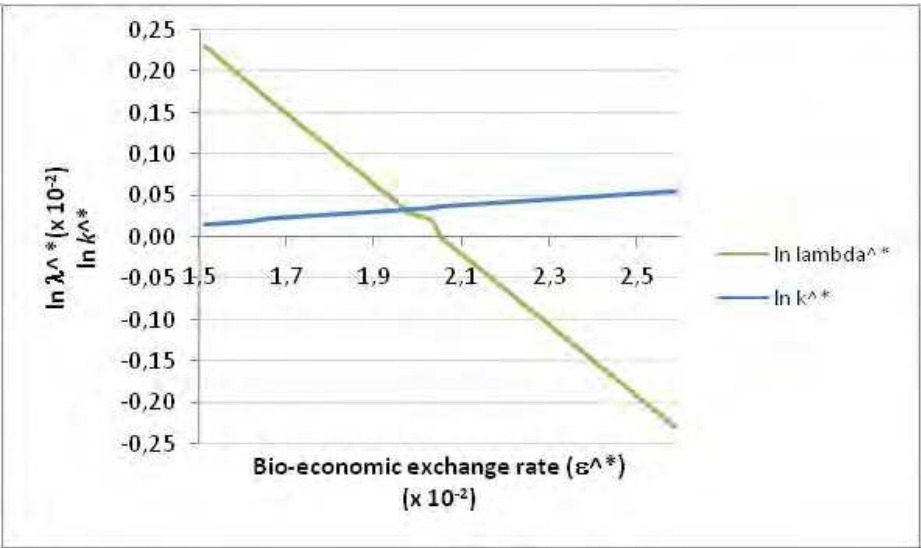

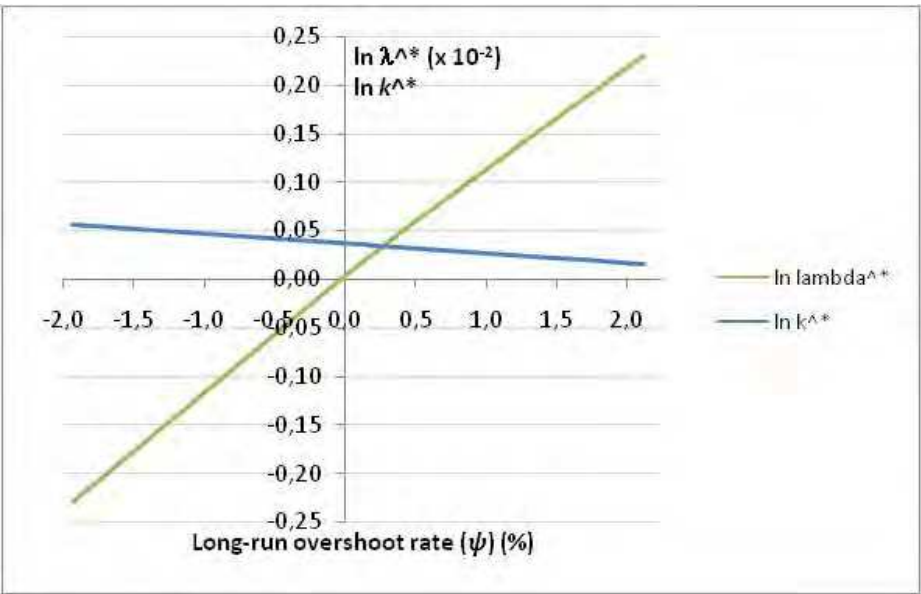Fig. 6. Bio-economic market for long-run emission removal in Austria and Brazil (1960-2007)

Fig. 7. Long-run (1960-2007) overshoot rates, optimal (*) removal supply ($\ln \hat{\lambda}^*$) and demand ($\ln \hat{k}^*$) for Austria and Brazil's economies

Time periods	Years	Obs. emissions† (ktC)	Obs. econ. growth	Estimated emissions over time (ktC)	Expected removal consumption (ktC)	Obs. removal consumption (ktC)	Estimated removal consumption (tens of tC)	Optimal‡ logistic rate (tens of tC)	Optimal‡ emission removal (in MtC)	Estimated emission removal (in MtC)
t		X_t	$k=X_t/X_{t-1}$	Eq. (21)	Eq. (22)	Eq. (20)a	Eq. (23) x 10^2	Eq. (25)b	Eq. (27)	Eq. (30)
1	1960	21.16	–	4.97	21.04	0.127	12.72	18.40	46.84	50.14
2	1961	22.07	1.04293	9.68	21.94	0.133	13.27	18.70	48.08	50.81
3	1962	23.85	1.08061	14.12	23.71	0.143	14.34	19.20	50.38	52.08
4	1963	25.21	1.05718	18.31	25.06	0.151	15.16	19.70	52.00	53.01
5	1964	26.05	1.03314	22.27	25.89	0.157	15.66	20.00	52.96	53.57
6	1965	25.75	0.98858	26.00	25.60	0.155	15.48	19.90	52.62	53.37
7	1966	28.20	1.09491	29.51	28.03	0.169	16.95	20.80	55.22	54.96
8	1967	28.90	1.02507	32.82	28.73	0.174	17.38	21.00	55.91	55.40
9	1968	32.61	1.12822	35.94	32.41	0.196	19.60	22.40	59.14	57.57
10	1969	35.12	1.07712	38.87	34.91	0.211	21.12	23.40	60.98	58.91
11	1970	39.33	1.11972	41.64	39.09	0.236	23.64	25.20	63.46	60.93
12	1971	42.14	1.07156	44.24	41.89	0.253	25.34	26.50	64.75	62.12
13	1972	46.43	1.10184	46.70	46.16	0.279	27.92	28.50	66.22	63.68
14	1973	52.44	1.12941	49.02	52.13	0.315	31.53	31.60	67.36	65.37
15	1974	54.70	1.04308	51.21	54.37	0.329	32.89	32.90	67.56	65.85
16	1975	55.96	1.02305	53.29	55.63	0.336	33.65	33.60	67.61	66.08
17	1976	58.15	1.03898	55.26	57.80	0.349	34.96	34.90	67.63	66.42
18	1977	59.67	1.02627	57.14	59.31	0.359	35.88	35.90	67.59	66.62
19	1978	63.82	1.06948	58.94	63.44	0.383	38.37	38.60	67.25	66.95
20	1979	68.04	1.06616	60.67	67.63	0.409	40.91	41.50	66.62	66.99
21	1980	65.18	0.95790	62.34	64.79	0.392	39.18	39.50	67.08	66.99
22	1981	62.06	0.95213	63.96	61.68	0.373	37.31	37.40	67.43	66.84
23	1982	61.54	0.99170	65.54	61.17	0.370	37.00	37.10	67.47	66.80
24	1983	59.52	0.96713	67.10	59.16	0.358	35.78	35.80	67.59	66.60
25	1984	60.81	1.02169	68.64	60.45	0.365	36.56	36.60	67.53	66.74

Table 8. Carbon emissions and estimation of the BESF micro-bio-economic functions, for Austria and Brazil together (1960-2007)

Time periods	Years	Obs. emissions (ktC)[†]	Obs. econ. growth	Estimated emissions over time (ktC)	Expected removal consumption (ktC)	Obs. removal consumption (ktC)	Estimated removal consumption (tens of tC)	Optimal[‡] logistic rate (tens of tC)	Optimal[‡] emission removal (in MtC)	Estimated emission removal (in MtC)
t		X_t	$k=X_t/X_{t-1}$	Eq. (21)	Eq. (22)	Eq. (20)a	Eq. (23) x 10^2	Eq. (25)b	Eq. (27)	Eq. (30)
26	1985	64.24	1.05638	70.18	63.85	0.386	38.62	38.80	67.20	66.96
27	1986	68.87	1.07211	71.73	68.46	0.414	41.41	42.10	66.47	66.97
28	1987	72.22	1.04866	73.30	71.79	0.434	43.42	44.60	65.75	66.75
29	1988	71.52	0.99031	74.89	71.09	0.430	43.00	44.10	65.92	66.81
30	1989	73.00	1.02070	76.53	72.56	0.439	43.89	45.20	65.57	66.68
31	1990	73.47	1.00636	78.22	73.03	0.441	44.17	45.60	65.45	66.63
32	1991	77.65	1.05697	79.97	77.19	0.467	46.68	49.00	64.32	66.02
33	1992	76.57	0.98604	81.80	76.11	0.460	46.03	48.10	64.63	66.20
34	1993	78.67	1.02743	83.71	78.20	0.473	47.30	49.90	64.02	65.83
35	1994	81.61	1.03734	85.72	81.12	0.490	49.06	52.50	63.10	65.18
36	1995	91.53	1.12159	87.84	90.98	0.550	55.03	62.40	59.55	61.95
37	1996	98.64	1.07765	90.08	98.04	0.593	59.30	70.60	56.70	58.64
38	1997	104.15	1.05588	92.45	103.52	0.626	62.61	77.70	54.39	55.51
39	1998	102.63	0.98540	94.95	102.01	0.617	61.70	75.70	55.03	56.43
40	1999	104.49	1.01819	97.62	103.87	0.628	62.82	78.20	54.24	55.30
41	2000	106.69	1.02102	100.44	106.05	0.641	64.14	81.30	53.30	53.90
42	2001	109.86	1.02967	103.44	109.20	0.660	66.05	85.90	51.94	51.75
43	2002	108.99	0.99212	106.63	108.34	0.655	65.53	84.60	52.31	52.35
44	2003	107.52	0.98649	110.01	106.87	0.646	64.64	82.40	52.95	53.36
45	2004	111.80	1.03982	113.60	111.13	0.672	67.21	88.80	51.10	50.35
46	2005	115.09	1.02942	117.41	114.40	0.691	69.19	94.00	49.67	47.83
47	2006	115.52	1.00377	121.45	114.83	0.694	69.45	94.70	49.49	47.49
48	2007	118.99	1.03001	125.74	118.27	0.715	71.54	100.60	47.98	44.62
Mean			**1.03742**							

Source: World Bank (2011). Notes: (†) Original data in tCO2 converted in tC after division by 3.67. (‡) GAMS 22.8.

Table 8. Carbon emissions and estimation of the BESF micro-bio-economic functions, for Austria and Brazil together (1960-2007)

overshoot rates fall, the larger the need of forestland will be to remove that additional emission burden over time. In other words, the increase of the long-run overshoot rate is rather powered by the need to increase $\hat{\lambda}^*$ than by the growth of $\hat{k}^*$ itself.

At last, from Table 7 and Fig. 7, a paradoxical proposition appears to arise. How come that the higher the economic growth rate ($\hat{k}^*$), the lower the long-run overshoot rate (ψ) turns out to be? This ironically suggests that economic growth is the solution for ecological overshoot – as most standard economic theories claim. What is meant here, though, is that, if $\hat{k}^*$ should be kept high in the long run, then $\hat{\lambda}^*$, described by a steeply down-sloped curve, would have to abruptly drop. Nevertheless, the only possible way for $\hat{\lambda}^*$ to fall that low would be when the long-run overshoot rate already were considerably negative.

4. Conclusion

Due to a remarkable *where-flexibility* (high λ_t in Table 4) between Austria and Brazil, their removal economies can fall back on a large ecological credit ($0 < \hat{\varepsilon} < 1$, as in Table 7 and Fig. 6) in the long run. By Eq. (15), this means high exports (Z) of removal services by the largest stock sink (**v** = Brazil) and low transfers of emission removal over time (*M*). Throughout the years (1960-2007), however, the maintenance of this ecological credit has cost these economies lower optimal rates of economic growth. As a result, Table 7 shows that, along the optimal path, increasing conservation (REDD) would only make things worse (scenarios 2 and 4), by exchanging smaller economic growth rates for higher long-run overshoot rates and thus causing the bio-economic exchange rate to drop (appreciate) even further.

Actually, in Austria-Brazil case, REDD alone would deepen *where-flexibility*, thereby raising the costs of removal over time, in terms of supply of forestland and removing forest stocks. Therefore, to deliver higher economic growth rates and lower long-run overshoot rates, the large ecological credit must be reduced, by raising the bio-economic exchange rate through increasing forestry-CDM. Although this conclusion might sound somewhat common place, it holds a quite interesting policy proposition, namely, that greater environmental equity in the provision of ecosystem services (lower λ and Z) might favour, instead of discouraging, economic growth.

First and foremost, this proposition means that the economy does get along with the environment, especially when ecological credit, arising from higher λ and Z (greater *where-flexibility*), prevails. Of course, whenever a forest-rich economy joins a forest-poorer one, the removal trade between them can not only bring forth ecological credit for both, but also let them enjoy higher economic growth rates. That is what is meant by biophysical foundations of economic growth. However, unlike widespread arguments towards cost-effectiveness in climate policy, the BESF model seems to point out that lesser, rather than greater, *where-flexibility* gears up investments on natural assets (sinks) providing environmental services. Otherwise, the quest for ecological credit to increase *where-flexibility* might end up deepening ecological overshoot, even though favouring economic growth. Therefore, a further step towards policy analysis would be to estimate the overshoot rate function, as close as that of Fig. 7.

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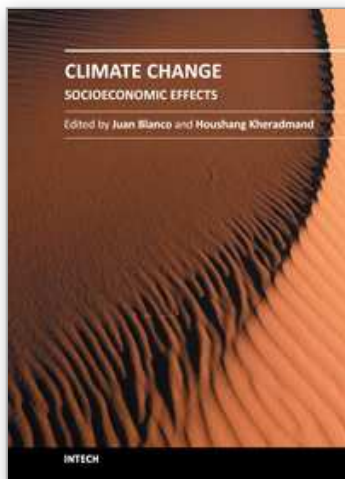
This book shows some of the socio-economic impacts of climate change according to different estimates of the current or estimated global warming. A series of scientific and experimental research projects explore the impacts of climate change and browse the techniques to evaluate the related impacts. These 23 chapters provide a good overview of the different changes impacts that already have been detected in several regions of the world. They are part of an introduction to the researches being done around the globe in connection with this topic. However, climate change is not just an academic issue important only to scientists and environmentalists; it also has direct implications on various ecosystems and technologies.

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