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Torsional Vibration of Eccentric Building Systems

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1. Introduction

The comprehensive studies conducted by a number of researchers in the past few decades and investigations of the effects of past earthquakes have shown that in buildings with non-coincident the center of mass (CM) and the center of rigidity (CR), significant coupling may occur between the translational and the torsional displacements of the floor diaphragms even when the earthquake induces uniform rigid base translations (Kuo, 1974; Chandler & Hutchinson, 1986; Cruz & Chopra, 1986; Hejal & Chopra, 1989).

In investigating the seismic torsional response of structures to earthquakes, it is customary to assume that each point of the foundation of the structure is excited simultaneously. Under this assumption, if centers of mass and rigidity of the floor diaphragms lie along the same vertical axis, a horizontal component of ground shaking will induce only lateral or translational components of motion. On the other hand, if the centers of mass and rigidity do not coincide, a horizontal component of excitation will generally induce both lateral components of motion and a rotational component about a vertical axis. Structures for which the centers of mass and rigidity do not coincide will be referred to herein as eccentric structures. Torsional actions may also be induced in symmetric structures due to the fact that, even under a purely translational component of ground excitation, all points of the base of the structure are not excited simultaneously because of the finite speed of propagation of the ground excitation, (Kuo, 1974).

This seismic torsional response leads to increased displacement at the extremes of the torsionally asymmetric building systems and may cause suffering in the lateral load-resisting elements located at the edges, particularly in the systems that are torsionally flexible. More importantly, the seismic response of the systems, especially in the torsionally flexible structure is qualitatively different from that obtained in the case of static loading at the center of mass. To account for the possible amplification in torsion produced by seismic response and accidental torsion in the elastic range, the equivalent static eccentricities of seismic forces are usually defined by building codes with simple expressions of the static eccentricity. The equivalent static eccentricities of seismic forces are proposed by researchers, (Dempsey & Irvine, 1979, Tso & Dempsey, 1980 and De la Llera & Chopra, 1994). A clear and comprehensive study of the equivalent static eccentricities that are presented by Anastassiadis et al., (1998), included a set of formulas for a one-storey scheme, allow the evaluation of the exact additional eccentricities necessary to be obtained by means of static analysis the maximum displacements at both sides of the deck, or the maximum deck rotation, given by modal analysis. A procedure to extend the static torsional provisions

of code to asymmetrical multi-storey buildings is presented by Moghadam and Tso, (2000). They have developed a refined method for determination of CM eccentricity and torsional radius for multi-storey buildings. However, the inelastic torsional response is less easily predictable, because the location of the center of rigidity on each floor cannot be determined readily and the equivalent static eccentricity varies storey by storey at each nonlinear static analysis step. The simultaneous presence of two orthogonal seismic components or the contemporary eccentricity in two orthogonal directions may have some importance, mainly in the inelastic range, (Fajfar et al., 2005). Consequently, the static analysis with the equivalent static eccentricities can be effective only if used in the elastic range. This can only be achieved, the location of the static eccentricity is necessary to change in each step of the nonlinear static procedure. It may be needed for the development of simplified nonlinear assessment methods based on pushover analysis.

Fig. 1. Damage to buildings subjected to strong earthquakes, (9-11 Research Book, 2006)

However, the seismic torsional response of asymmetric buildings in the inelastic range is very complex. The inelastic response of eccentric systems only has been investigated in an exploratory manner, and, on the whole, it has not been possible to derive any general conclusions from the data that were obtained. No work appears to have been reported concerning the torsiona1 effects induced in symmetric structures deforming into the inelastic range (Tanabashi, 1960; Koh et al., 1969; Fajfar et al., 2005).

Torsional motion is produced by the eccentricity existing between the center of mass and the center of rigidity. Some of the situations that can give rise to this situation in the building plan are:

- Positioning the stiff elements asymmetrically with respect to the center of gravity of the floor.
- The placement of large masses asymmetrically with respect to stiffness.

- A combination of the two situations described above.

Consequently, torsional-translational motion has been the cause of major damage to buildings vibrated by strong earthquakes, ranging from visible distortion of the structure to structural collapse (see Fig. 1). The purpose of this chapter is to investigate the torsional vibration of both symmetric and eccentric one-storey building systems subjected to the ground excitation.

Fig. 2. Mexico City building failure associated with the torsional-translation motion, (Earthquake Engineering ANNEXES, 2007)

2. Classification of vibration

Vibration can be classified in several ways. Some of the important classifications are as follows: Free and forced vibration: If a system, after an internal disturbance, is left to vibrate on its own, the ensuing vibration is known as free vibration. No external force acts on the system. The oscillation of the simple pendulum is an example of free vibration.

If a system is subjected to an external force (often, a dynamic force), the resulting vibration is known as forced vibration. The oscillation that arises in buildings such as earthquake is an example of forced vibration.

A building, for which the centers of mass and rigidity do not coincide, (eccentric building) will experience a coupled torsional-translational motion even when it is excited by a purely translational motion of the ground. The torsional component of response may contribute significantly to the overall response of the building, particularly when the uncoupled torsional and translational frequencies of the system are close to each other (see Fig. 2).

Failures of such structures as buildings and bridges have been associated with the torsional-translational motion.

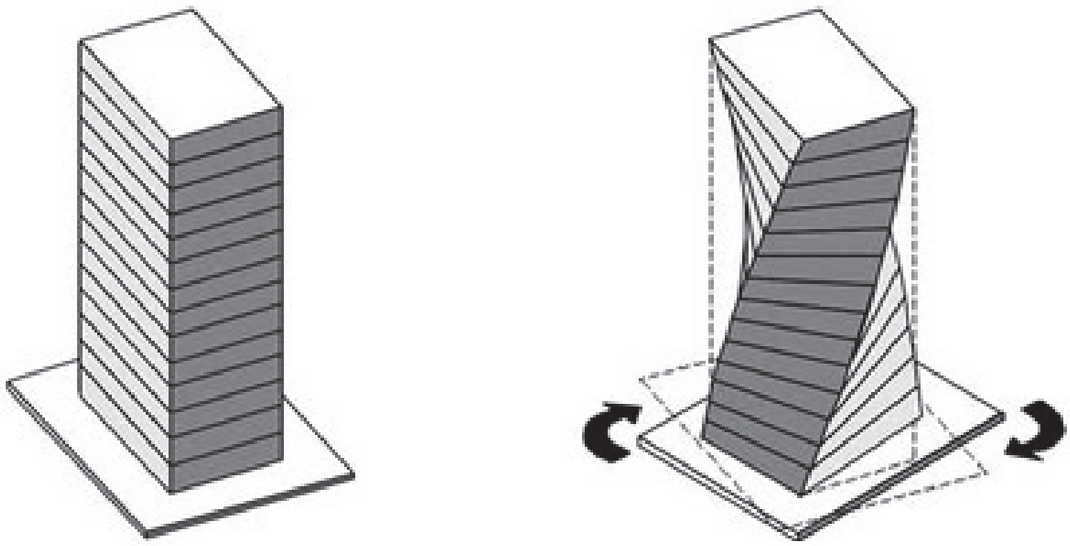

Fig. 3. Torsional vibration mode shape

2.1 Free vibration analysis

One of the most important parameters associated with engineering vibration is the natural frequency. Each structure has its own natural frequency for a series of different mode shapes such as translational and torsional modes which control its dynamic behaviour (see Fig. 3). This will cause the structures to be subjected to series structural vibrations, when they are located in environments where earthquakes or high winds exist. These vibrations may lead to serious structural damage and potential structural failure.

In buildings, both translational and torsional vibration modes arise, even if, little eccentricity in the transverse direction during earthquakes. The in-plane floor vibration mode such as arch-shaped floor vibration mode also arises during earthquakes. However these observational data are not enough at present. The causes of the torsional-translational vibration are thought as follows:

1. Input motion to the foundation has a possibility to contain the torsional component, which is the cause of the torsional vibration.
2. The torsional coupling, due to the eccentricity in both directions, is also a cause of the torsional vibration. It arises surely when the eccentricity in the transverse direction is large. However, even if the eccentricity is small, it is well-known that the strong torsional coupling also arises when the natural frequencies of the translational mode and the torsional mode approach closely to each other.
3. The eccentricity in the transverse direction is small in general, since sufficient attention is usually paid on the eccentricity to prevent the torsional vibration in the structural planning. On the other hand, the eccentricity in the longitudinal direction results often from necessity of architectural planning and/or from insufficiency of attention on the eccentricity in the structural planning, but it is also small as a necessity from the configuration of the floor plan.

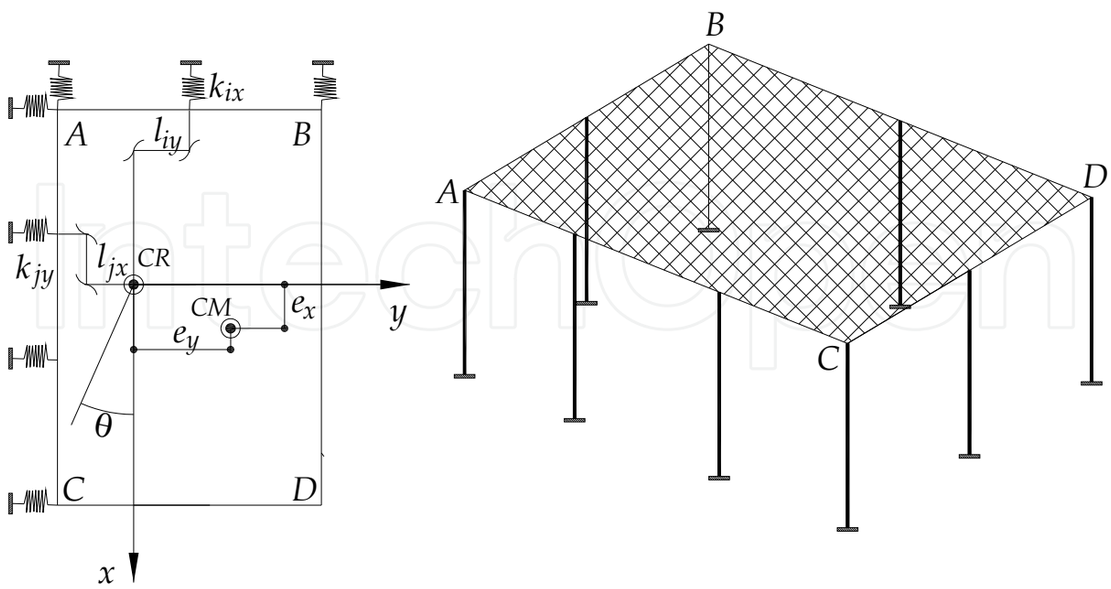

Fig. 4. Model of a one-storey system with double eccentricities

2.1.1 One-storey system with double eccentricities

The estimation of torsional-translational response of simplified procedure subjected to a strong ground motion, is a key issue for the rational seismic design of new buildings and the seismic evaluation of exacting buildings. This section is a vibration-based analysis of the simple one-storey model with double eccentricities, and it would be a promising candidate as long as buildings oscillate predominantly in the two lateral directions (Tabatabaei and Saffari, 2010).

2.1.2 Basic parameters of the model

The one-storey system, considered in this section, may be modeled as shown in Fig. 4. The center of rigidity (CR) is the point in the plan of the rigid floor diaphragm through which a lateral force must be applied in order that it may cause translational displacement without torsional rotation. When a system is subjected to forces, which will cause pure rotation, the rotation takes place around the center of rigidity, which remains fixed. The location of the center of rigidity can be determined from elementary principles of mechanic.

The horizontal rigid floor diaphragm is constrained in the two lateral directions by resisting elements (columns). Let k_{ix} and k_{jy} be the lateral stiffness of the $i-th$ and $j-th$ resisting element in x-direction and y-direction, respectively. The origin of the coordinates is taken at the center of rigidity (CR). A system for which the eccentricities, e_x and e_y are both different from zero, has three degrees of freedom. Its configuration is specified by translations x and y and rotation, θ. The positive directions of these displacements are indicated on the figure.

Applying the geometric relationships between the centers of mass and rigidity, the equations of motion of undamped free vibration of the system may be written as follows

$$m(\ddot{x} + e_y\ddot{\theta}) + K_x x = 0 \tag{1a}$$

$$m(\ddot{y} - e_x\ddot{\theta}) + K_y y = 0 \tag{1b}$$

$$I_m\ddot{\theta} + K_\theta\theta - me_x(\ddot{y} - e_x\ddot{\theta}) + me_y(\ddot{x} + e_y\ddot{\theta}) = 0 \tag{1c}$$

where

$K_x = \sum_{i=1}^{n} k_{ix}$: total translational stiffness in the x-direction (n = number of columns in x-dir),

$K_y = \sum_{j=1}^{m} k_{jy}$: total translational stiffness in the y-direction (m = number of columns in y-dir),

$K_\theta = \sum_{i=1}^{n} k_{ix}l_{iy}^2 + \sum_{j=1}^{m} k_{jy}l_{jx}^2$: total rotational stiffness, m : total mass, I_m : the mass moment of inertia of the system around the center of mass (CM), and l_{iy} and l_{jx} , be the distances of the i - th and j - th resisting element from the center of rigidity along the x and y axes, as shown in Fig. 4.

For free vibration analysis, the solution of Eqs. (1) may be taken in the form

$$x = X\sin(\omega t) \tag{2a}$$

$$y = Y\sin(\omega t) \tag{2b}$$

$$\theta = \Theta\sin(\omega t) \tag{2c}$$

where X, Y and Θ are the displacements amplitudes in x, y and θ directions, respectively. The value of ω is referred to the circular natural frequency. Substitution of Eqs. (2) into Eqs. (1) given in

$$(-m\omega^2 + K_x)X - m\omega^2 e_y\Theta = 0 \tag{3a}$$

$$(-m\omega^2 + K_y)Y + m\omega^2 e_x\Theta = 0 \tag{3b}$$

$$(-(I_m + me_x^2 + me_y^2)\omega^2 + K_\theta)\Theta - m\omega^2 e_y X + m\omega^2 e_x Y = 0 \tag{3c}$$

Eqs. (3) have a nontrivial solution only if the determinate of the coefficients of X, Y and Θ are equal to zero. This condition yields the characteristic equation of describing such a system may be taken in the form

$$\omega^6 - \left[\frac{K_x + K_y}{m} + \frac{K_\theta}{I_m} + \frac{K_x e_y^2}{I_m} + \frac{K_y e_x^2}{I_m}\right]\omega^4 + \left[\frac{K_x K_y}{m^2} + \frac{K_\theta(K_x + K_y)}{mI_m} + \frac{K_x K_y e_y^2}{mI_m} + \frac{K_x K_y e_x^2}{mI_m}\right]\omega^2 - \frac{K_x K_y K_\theta}{m^2 I_m} = 0 \tag{4}$$

where, e_x is the static eccentricity (eccentricity between mass and rigidity centers) in the x-direction and e_y is the static eccentricity in the y-direction. Now letting the following expressions,

$$\omega_x^2 = \frac{K_x}{m} \quad \omega_y^2 = \frac{K_y}{m} \quad \omega_\theta^2 = \frac{K_\theta}{I_m} \tag{5a}$$

$$\varepsilon_x = \frac{e_x}{r_m} \quad \varepsilon_y = \frac{e_y}{r_m} \quad c = \sqrt{1 + \varepsilon_x^2 + \varepsilon_y^2} \tag{5b}$$

$$\varepsilon = \frac{e}{r_m} \quad e = \sqrt{e_x^2 + e_y^2} \tag{5c}$$

and making use of the relation $I_m = m \cdot r_m^2$; , where r_m is the radius gyration of mass, Eq. (4) may be written in the following dimensionless form:

$$\left(\frac{\omega}{\omega_x}\right)^6 - \left[1 + \varepsilon_x^2 + \left(\frac{\omega_y}{\omega_x}\right)^2 \left(1 + \varepsilon_y^2\right) + \left(\frac{\omega_\theta}{\omega_x}\right)^2\right]\left(\frac{\omega}{\omega_x}\right)^4 + \left[\left(c\frac{\omega_y}{\omega_x}\right)^2 + \left(\frac{\omega_\theta}{\omega_x}\right)^2\left(1 + \left(\frac{\omega_y}{\omega_x}\right)^2\right)\right]\left(\frac{\omega}{\omega_x}\right)^2 - \left(\frac{\omega_y}{\omega_x}\right)^2\left(\frac{\omega_\theta}{\omega_x}\right)^2 = 0 \tag{6}$$

where the values of ω_x and ω_y are referred to the uncoupled circular natural frequencies of the system in x and y-directions, respectively. The value of ω_θ will be referred as the uncoupled circular natural frequency of torsional vibration. The $n\text{-}th$ squares of the coupled natural frequency ω_n are defined by three roots of the characteristic equation defined in Eq. (6). Associated with each natural frequency, there is a natural mode shape vector $\{\varphi_n\} = \{\varphi_{xn}, \varphi_{yn}, \varphi_{\theta n}\}^T$ of the one-storey asymmetric building models that can be obtained with assuming, $\{\varphi_{xn}\} = 1$, and two components as follows,

$$\{\varphi_{yn}\} = \frac{\varepsilon_y}{\varepsilon_x}\left[\frac{1 - \left(\omega_n / \omega_x\right)^2}{\left(\omega_n / \omega_x\right)^2 - \left(\omega_y / \omega_x\right)^2}\right] \tag{7a}$$

$$\{\varphi_{\theta n}\} = \frac{c}{\varepsilon_x}\left[\frac{1}{\left(\omega_n / \omega_x\right)^2} - 1\right] / r_{cr} \tag{7b}$$

where n varies from 1 to 3 and $r_{cr} = \sqrt{r_m^2 + e_x^2 + e_y^2}$, (Kuo, 1974).

As a matter of fact, the numerical results have been evaluated over a wide range of the frequency ratio ω_θ / ω_x for several different values of eccentricity parameter ε_y. A value of

$e_x/e_y = 1$ which corresponds to systems with double eccentricities along the x-axis and y-axis is considered. In the latter case, two values of ω_y/ω_x are considered. The coupled natural frequencies are summarized in Figs. 5 and 6 are also applicable to the system considered in this section for any given longitudinal distribution of motions.

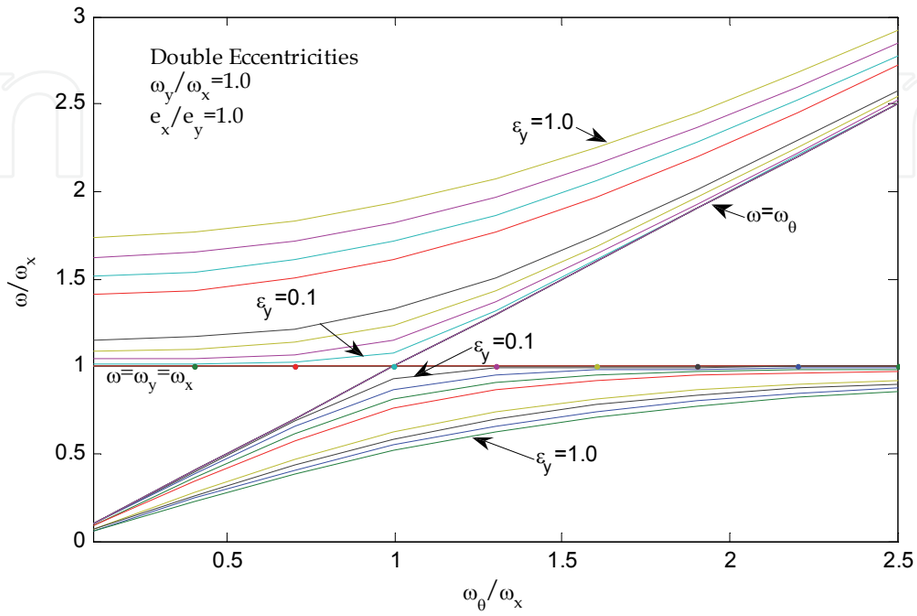

Fig. 5. The coupled natural frequency ratio for varying eccentricity parameter, ε_y of Double eccentricities system and $\omega_y/\omega_x = 1.0$

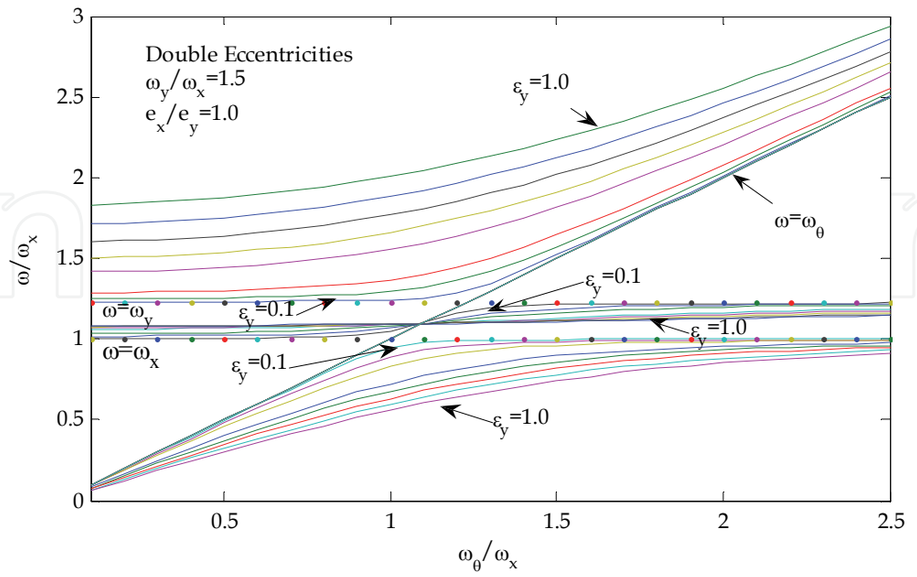

Fig. 6. The coupled natural frequency ratio for varying eccentricity parameter, ε_y of Double eccentricities system and $\omega_y/\omega_x = 1.5$

In these Figures, the uncoupled natural frequencies of the systems are represented by the straight lines corresponding to $\varepsilon_y = 0$. For the systems with double eccentricity considered in Fig. 5, these are defined by the diagonal line and the two horizontal lines. The diagonal line represents the uncoupled torsional frequency, and the horizontal lines the two uncoupled translational frequencies. As would be expected, the lower natural frequency of the coupled system is lower than either of the frequencies of the uncoupled system. Similarly, the upper natural frequency of the coupled system is higher than the upper natural frequency of the uncoupled system. The general trends of the curves for the coupled systems are typical of those obtained for other combinations of the parameters as well.

The curve for the lowest frequency always starts from the origin whereas the curve for the highest frequency starts from a value higher than the uncoupled translational frequencies of the system, depending on the value of the eccentricity. Both curves increase with the higher value of ω_θ/ω_x . For 1arge value of ω_θ/ω_x the lowest frequency approaches the value of ω_x and the highest frequency approaches the value of ω_θ . The maximum coupling effect on frequencies occurs when the value of ω_θ/ω_x is equal to unity, (Kuo, 1974).

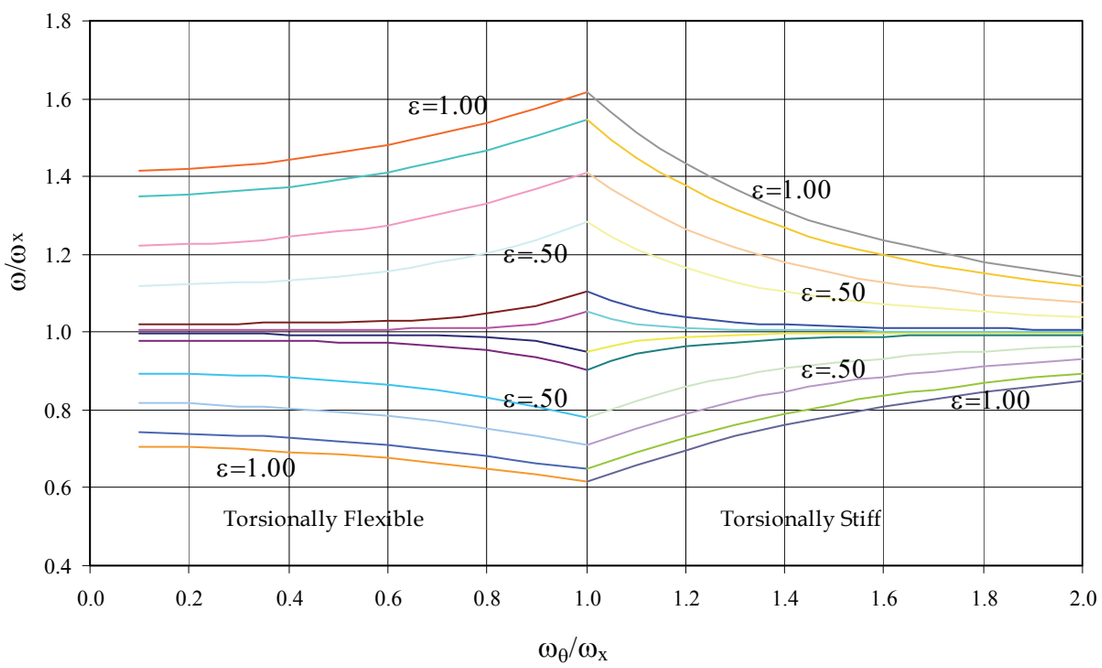

Fig. 7. Relationship between Coupled and Uncoupled Natural Frequencies

It is interesting to note that the coupled dynamic properties depend only on the four dimension 1ess parameters ε_x , ε_y , ω_θ/ω_x and ω_y/ω_x . Fig. 7 shows the relationship between the coupled and uncoupled natural frequencies, in one way torsionally coupled systems (with $\varepsilon_x = 0$), for different values of ε .

If ρ_n represents the distance positive to the left from the center of mass to the instantaneous center of rotation of the system for the modes under consideration, it can be shown that (see Fig. 8).

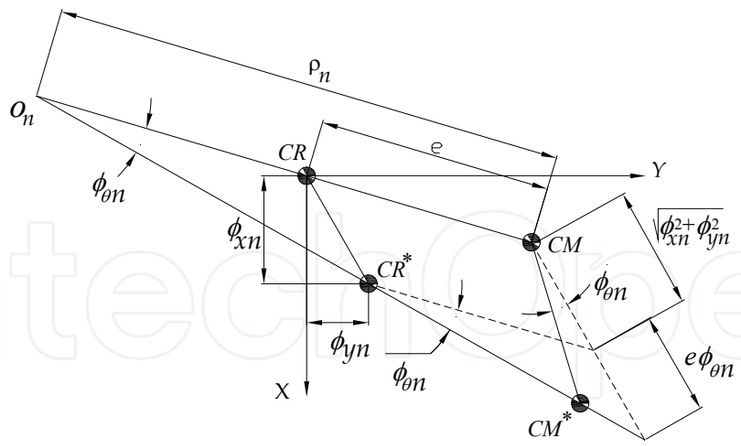

Fig. 8. CR^* and CM^* denote the new locations of the centers of rigidity and mass at any time instant, respectively (Tabatabaei and Saffari, 2010).

$$\frac{\rho_n}{e} = 1 + \frac{\varphi_{xn}}{e\varphi_{\theta n}}\sqrt{1+\left(\varphi_{yn}/\varphi_{xn}\right)^2} \tag{8}$$

The ratio of $\rho_n/e = 1$ indicates that the center of rotation is at the center of rigidity, whereas the value of $\rho_n/e = 0$ indicates that the center of rotation is at the center of mass. By making use of Eq. (7), Eq. (8) may also be related to the frequency values.

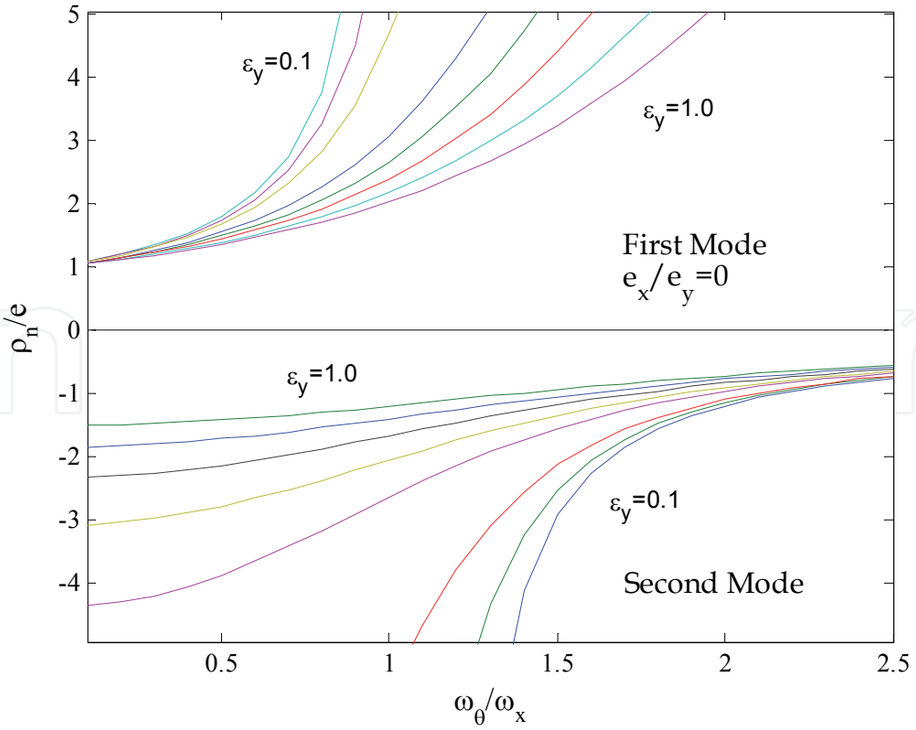

Fig. 9. Location of the center of the rotation normalized with the respect to eccentricity

For the one-storey system with eccentricity considered in Fig. 4, the locus of the associated center of rotation is plotted in Fig. 9. It should be recalled that a value of $\rho_n/e=1$ in the latter figure indicates that the center of rotation is at the center of rigidity of the system. Note that as the value of ω_θ/ω_x increases, the center of rotation shifts away from the center of rigidity for the first mode and approaches the center of mass for the higher mode for all values of eccentricity.

2.1.3 One-storey system with single eccentricity

In the particularly case of $e_x=0$, system with single eccentricity and second equation in Eq. (1) becomes independent of the others. The motion of the system in this case is coupled only in the x and θ directions. The following frequency equation is obtained from Eq. (6) by taking $\varepsilon_x=0$ and factoring out the term $(\omega^2-\omega_x^2)/\omega_y^2$, which obviously defines the uncoupled natural frequency of the system in the y direction:

$$\left(\frac{\omega}{\omega_x}\right)^4-\left[1+\varepsilon_y^2+\left(\frac{\omega_\theta}{\omega_x}\right)^2\right]\left(\frac{\omega}{\omega_x}\right)^2+\left(\frac{\omega_\theta}{\omega_x}\right)^2=0 \tag{9}$$

Numerical data have been evaluated over a wide range of the frequency ratio ω_θ/ω_x for several different values of eccentricity parameter ε_y. Two values of $\varepsilon_x/\varepsilon_y$ are considered: a value of $\varepsilon_x/\varepsilon_y=0$ which corresponds to systems with an eccentricity along the y-axis and a value of $\varepsilon_x/\varepsilon_y=1$. In the latter case, two values of ω_y/ω_x are considered. The coupled natural frequencies are summarized in Fig 10.

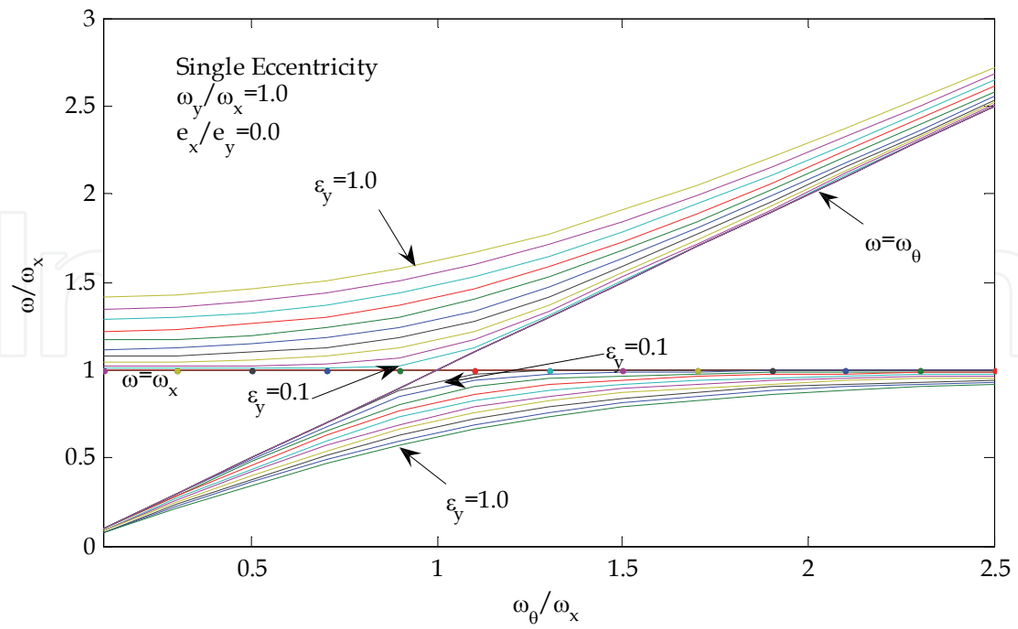

Fig. 10. The coupled natural frequency ratio for several different values of eccentricity parameter, ε_y of single eccentricity system

As a result, Figs. 5 and 10, which refer to systems with double and single eccentricities, respectively, are very similar in form. This is due to the fact that, since $\omega_y/\omega_x = 1$, the frequencies of the two uncoupled translational modes are represented by the same horizontal line, $\omega = \omega_x = \omega_y$ and this frequency value is also equal to the second natural frequency of the coupled system. As a matter of fact, it can be shown that the curves in Fig. 10 may be obtained from those in Fig. 5 by interpreting ε_y to be equal to $\varepsilon_y\sqrt{1+\left(\varepsilon_x/\varepsilon_y\right)^2}$, (Kuo, 1974).

2.1.4 Classification of torsionl behaviour

To obtain the knowledge on the torsional response of buildings, a key elastic parameter is the ratio of the two uncoupled frequencies, Ω given by

$$\Omega = \frac{\omega_\theta}{\omega_x} \tag{10}$$

where ω_x and ω_θ, are an uncoupled translational and torsional frequencies, respectively.

If Ω is greater than 1 the response is mainly translational and structure behaviour is defined "torsionally stiff"; conversely if Ω is lower than 1 the response is affected to a large degree by torsional behaviour and, then, the structure is defined as "torsionally flexible". A clear and comprehensive study of this subject is presented in Ref., (Anastassiadis et al., 1998). In the case of a torsionally stiff structure, a single translation mode controls the displacement in on direction. Thus the typical dynamic behaviour of such structure is qualitatively similar to the response obtained using static analysis, i.e. the displacements increase at the flexible edge (side is outlying to the center of rigidity), and decrease at the stiff edge (side is closed to the center of rigidity). The seismic response of torsionally flexible structure is qualitatively different from that obtained in the case of static loading at the center of mass. The main reason is that the displacement envelope of the deck depends on both the translation and the torsional modes.

2.2 Forced vibration analysis

Civil engineering structures are always designed to carry their own dead weight, superimposed loads and environmental loads such as wind or earthquake. These loads are usually treated as maximum loads not varying with time and hence as static loads. In some cases, the applied load involves not only static components, but also contains a component varying with time which is a dynamic load. In the past, the effects of dynamic loading have often been evaluated by use of an equivalent static load, or by an impact factor, or by a modification of the factor of safety.

Many developments have been carried out in order to try to quantify the effects produced by dynamic loading. Examples of structures, where it is particularly important to consider dynamic loading effects, are the construction of tall buildings, long bridges under wind-loading conditions, and buildings in earthquake zones, etc.

Typical situations, where it is necessary to consider more precisely, the response produced by dynamic loading are vibrations due to earthquakes. So it is very important to study the dynamic nature of structures.

Dynamic characteristics of damaged and undamaged buildings are, as a rule, different. This difference is caused by a change in stiffness and can be used for the detection of damage and for the determination of some damage parameters such as crack magnitude and location. In this connection, the use of vibration methods of damage diagnostics is promising. These methods are based on the relationships between the vibration characteristics (natural frequencies and mode shapes) or peculiarities of vibration system behaviour (for example, drift of building edges, the amplitudes of base shear, the resonance frequencies, etc.) and damage parameters.
Depending on the assumptions adopted, the type of analysis used the kind of the loading or excitation and the overall building characteristics, a variety of different approaches have been reported in the references to one-storey building systems.

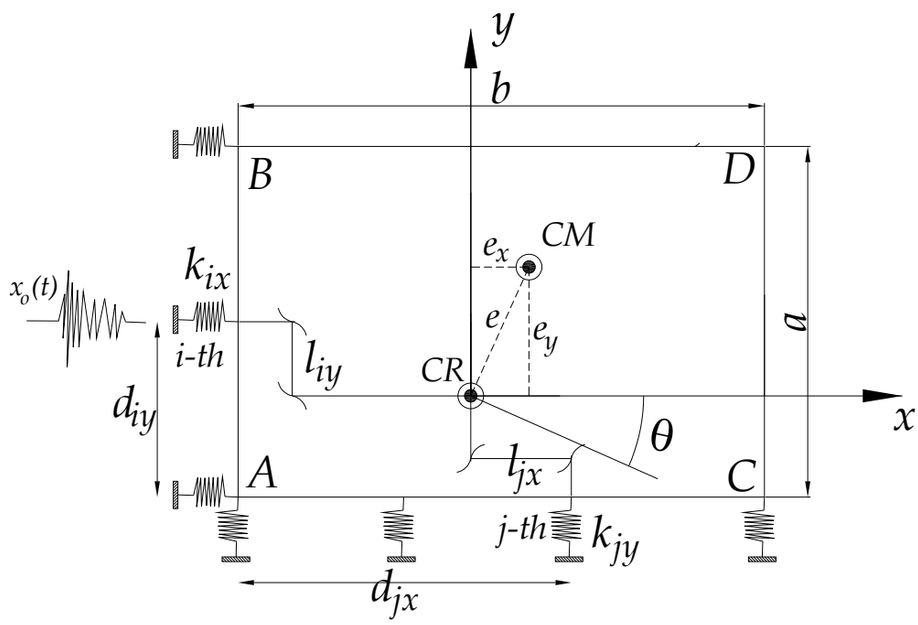

Fig. 11. Model of a one-storey system with double eccentricities subjected to ground motion

2.2.1 One-storey system with double eccentricities

In this section, the dynamic response of one-storey system to the horizontal components of ground motion is considered. At any instant of time, the ground motion is assumed to be the same at all points on the foundation. Also, the ground motion is considered to be plane shear waves propagating horizontally with a constant velocity and without change in shape. Now, equations of motion are developed for system as shown in Fig. 11 subjected to a horizontally propagating ground excitation. Let the earthquake ground motion be defined by accelerations along the two axes. Therefore, a force applied along either of the two principal axes of rigidity will cause displacement in the same direction. The principal axes of rigidity are orthogonal and pass through the center of rigidity. For building plans of the type shown in Fig. 11, where principal axes of rigidity of individual column sections are all parallel to one another, the principal axes of rigidity of the complete building are parallel to those of the individual elements. Within the range of linear behaviour, the equations of motion of the system, written about the center of rigidity of the system, are as follows

$$m\left(\ddot{x} + e_y\ddot{\theta}\right) + C_x\dot{x} + K_x x - \sum_{i=1}^{n} c_{ix}\left(\dot{x}_g\right)_i - \sum_{i=1}^{n} k_{ix}\left(x_g\right)_i = 0 \tag{11a}$$

$$m\left(\ddot{y} - e_x\ddot{\theta}\right) + C_y\dot{y} + K_y y - \sum_{j=1}^{m} c_{jy}\left(\dot{y}_g\right)_j - \sum_{j=1}^{m} k_{jy}\left(y_g\right)_j = 0 \tag{11b}$$

$$\begin{aligned} &I_m\ddot{\theta} + C_\theta\dot{\theta} + R\theta + me_y\left(\ddot{x} + e_y\ddot{\theta}\right) - me_x\left(\ddot{y} - e_x\ddot{\theta}\right) - \sum_{i=1}^{n} c_{ix}\left(\dot{x}_g\right)_i l_{iy} + \sum_{j=1}^{m} c_{jy}\left(\dot{y}_g\right)_j l_{jx} \\ &- \sum_{i=1}^{n} k_{ix}\left(x_g\right)_i l_{iy} + \sum_{j=1}^{m} k_{jy}\left(y_g\right)_j l_{jx} = 0 \end{aligned} \tag{11c}$$

where, x and y are horizontal displacements of the center of rigidity of the deck, relative to the ground, along the principal axes of rigidity of the model, x and y, and θ are the rotation of the deck about the vertical axis; $\left(x_g\right)_i$ and $\left(y_g\right)_j$ are the ground displacements at the $i-th$ and $j-th$ column supports in the x and y directions, respectively; C_x, C_y and C_θ are the total damping coefficients associated with the motions in the x, y and θ directions, respectively; c_{ix} and c_{jy} are the individual damping coefficients of the $i-th$ and $j-th$ columns in the x and y directions, respectively; k_{ix} and k_{jy} are the individual lateral stiffness of the $i-th$ and $j-th$ columns in the x and y directions, respectively; and l_{ix} and l_{jy} are the normal distances measured from the center of rigidity to the $i-th$ and $j-th$ columns in the y and x directions, respectively.

It is important to note that time is measured from the instant the ground motion reaches the first support on the left of the diagram shown in Fig. 11 and that the amplitude of the ground motion at any support is a function of both time and distance from the first, i.e.,

$$\left(\dot{x}_g\right)_i = x_g\left(t - \frac{d_{iy}}{v}\right) \quad for \quad t \geq \frac{d_{iy}}{v} \tag{12a}$$

$$= 0 \qquad for \quad t < \frac{d_{iy}}{v}$$

$$\left(\dot{y}_g\right)_j = y_g\left(t - \frac{d_{jx}}{v}\right) \quad for \quad t \geq \frac{d_{jx}}{v} \tag{12b}$$

$$= 0 \qquad for \quad t < \frac{d_{jx}}{v}$$

where v is the shear wave velocity of the ground motion; and d_{iy} and d_{jx} are the distances from the first support to the $i-th$ and $j-th$ column support in the x and y directions, respectively.

Since it is customary to specify the ground motion in terms of its acceleration, Eqs. (11) will be rewritten by making a proper coordinate transformation. Assuming that the

damping coefficients are proportional to the corresponding stiffness coefficients, and letting

$$U_x = x - \frac{\sum_{i=1}^{n} k_{ix}\left(x_g\right)_i}{K_x} \tag{13a}$$

$$U_y = y - \frac{\sum_{j=1}^{m} k_{jy}\left(y_g\right)_j}{K_y} \tag{13b}$$

$$U_\theta = \theta - \frac{1}{K_\theta}\left[\sum_{i=1}^{n} k_{ix}\left(x_g\right)_i l_{iy} - \sum_{j=1}^{m} k_{jy}\left(y_g\right)_j l_{jx}\right] \tag{13c}$$

where n and m are the total numbers of the column support in the x and y directions, respectively. Also, physically, U_x, U_y and U_θ are the displacements of the system relative to the instantaneous value of the average amplitude of the ground motion experienced by the base.

Substituting Eqs. (13) in Eqs. (11) and writing those equations, after introducing damping, in a convenient form yields

$$\left(\ddot{U}_x + e_y\ddot{U}_\theta\right) + 2\beta_x\omega_x\dot{U}_x + \omega_x^2 U_x = -\frac{\sum_{i=1}^{n} k_{ix}\left(\ddot{x}_g\right)_i}{K_x} - \frac{e_y}{K_\theta}\left[\sum_{i=1}^{n} k_{ix}\left(\ddot{x}_g\right)_i l_{iy} - \sum_{j=1}^{m} k_{jy}\left(\ddot{y}_g\right)_j l_{jx}\right] \tag{14a}$$

$$\left(\ddot{U}_y - e_x\ddot{U}_\theta\right) + 2\beta_y\omega_y\dot{U}_y + \omega_y^2 U_y = -\frac{\sum_{j=1}^{m} k_{jy}\left(\ddot{y}_g\right)_j}{K_y} + \frac{e_x}{K_\theta}\left[\sum_{i=1}^{n} k_{ix}\left(\ddot{x}_g\right)_i l_{iy} - \sum_{j=1}^{m} k_{jy}\left(\ddot{y}_g\right)_j l_{jx}\right] \tag{14b}$$

$$\begin{aligned}\ddot{U}_\theta + \frac{2}{c^2}\beta_\theta\omega_\theta\dot{U}_\theta + \frac{1}{c^2}\omega_\theta^2 U_\theta + \frac{me_y}{I_{cr}}\ddot{U}_x - \frac{me_x}{I_{cr}}\ddot{U}_y &= \frac{me_x}{I_{cr}}\frac{\sum_{j=1}^{m} k_{jy}(\ddot{y}_g)_j}{K_y} \\ -\frac{me_y}{I_{cr}}\frac{\sum_{i=1}^{n} k_{ix}(\ddot{x}_o)_i}{K_x} &- \frac{1}{K_\theta}\left[\sum_{i=1}^{n} k_{ix}\left(\ddot{x}_g\right)_i l_{iy} - \sum_{j=1}^{m} k_{jy}\left(\ddot{y}_g\right)_j l_{jx}\right]\end{aligned} \tag{14c}$$

where β_x, β_y and β_θ are the damping coefficients in fraction of the critical damping for vibrations in the x, y and θ directions, respectively.

Eqs. (14) has been expressed in the most general form. It is applicable to both symmetric and eccentric systems subjected to a ground disturbance having a finite speed of propagation. It may be reduced to the equations derived from a conventional analysis in which the speed of propagation of the ground motion is assumed to be infinite.

2.2.2 Time history response

The modal equations (14) can be solved numerically using the modal superposition method or by direct numerical integration. The modal superposition method, which involves the use of the characteristic values and functions of the system, uncouples the equations of motion so that each of the uncoupled equations may be integrated independently. Since it is based on the assumption that the structure behaves linearly, this method is applicable only to the elastic range of response. These issues have been further discussed in Ref., (Kuo, 1974).

The method of direct numerical integration, which integrates the equations of motion in their original form, may be applied to both the elastic and inelastic ranges of response. For the inelastic analysis, the material properties of the system are assumed to be linear within a small time interval, and they are modified at the end of each integration step when needed.

The modal superposition method is used to evaluate the elastic response of both symmetric and eccentric systems. Before solving Eqs. (14), it is desirable to rewrite it in matrix form. For consistency in units, the third equation is multiplied by r_{cr} to give the following set:

$$\begin{bmatrix} 1 & 0 & e_y/r_{cr} \\ 0 & 1 & -e_x/r_{cr} \\ e_y/r_{cr} & -e_x/r_{cr} & 1 \end{bmatrix} \begin{Bmatrix} \ddot{U}_x \\ \ddot{U}_y \\ r_{cr}\ddot{U}_\theta \end{Bmatrix}$$

$$+ \begin{bmatrix} 2\beta_x\omega_x & 0 & 0 \\ 0 & 2\beta_x\omega_x & 0 \\ 0 & 0 & 2\beta_\theta\omega_\theta/c^2 \end{bmatrix} \begin{Bmatrix} \dot{U}_x \\ \dot{U}_y \\ r_{cr}\dot{U}_\theta \end{Bmatrix} + \begin{bmatrix} \omega_x^2 & 0 & 0 \\ 0 & \omega_y^2 & 0 \\ 0 & 0 & \omega_\theta^2/c^2 \end{bmatrix} \begin{Bmatrix} U_x \\ U_y \\ r_{cr}U_\theta \end{Bmatrix} \tag{15}$$

$$= \begin{Bmatrix} -\sum_{i=1}^{n} k_{ix}^* \left(\ddot{x}_g\right)_i - \frac{e_y}{b}\ddot{T}_\theta \\ -\sum_{j=1}^{m} k_{jy}^* \left(\ddot{y}_g\right)_j - \frac{e_x}{b}\ddot{T}_\theta \\ \frac{e_x}{r_{cr}}\sum_{j=1}^{m} k_{jy}^* \left(\ddot{y}_g\right)_j - \frac{e_y}{r_{cr}}\sum_{i=1}^{n} k_{ix}^* \left(\ddot{x}_g\right)_i - \frac{r_{cr}}{b}\ddot{T}_\theta \end{Bmatrix}$$

where the uncoupled circular natural frequencies ω_x, ω_y and ω_θ are defined by Eq. (5a), and c is defined by Eq. (5b). Other symbols are defined as follows:

$$k_{ix}^* = \frac{k_{ix}}{K_x}, \quad k_{jy}^* = \frac{k_{jy}}{K_y} \tag{16a}$$

$$\ddot{T}_\theta = \frac{\sum_{i=1}^{n} k_{ix}^* \left(\ddot{x}_g\right)_i \eta_i - \frac{a}{b}\frac{K_y}{K_x}\sum_{j=1}^{m} k_{jy}^* \left(\ddot{y}_g\right)_j \xi_j}{\sum_{i=1}^{n} k_{ix}^* \eta_i^2 + \left(\frac{a}{b}\right)^2 \frac{K_y}{K_x}\sum_{j=1}^{m} k_{jy}^* \xi_j^2} \tag{16b}$$

$$\eta_i = \frac{l_{iy}}{b}, \quad \xi_j = \frac{l_{jx}}{a} \tag{16c}$$

Symbolically, Eq. (15) may also be written as

$$[m]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=\{f(t)\} \tag{17}$$

Now, let $[\varphi]$ be the modal matrix, formed of the three natural modes of the system, and let

$$\{U\}=[\varphi]\{Z\} \tag{18}$$

Substituting Eq. (18) into Eq. (17), multiplying the resulting equation by $[\varphi]^T$, and making use of the orthogonality of the natural modes, Eq. (17) leads to the independent modal equations as follows

$$\ddot{Z}_n+2\beta_n\omega_n\dot{Z}_n+\omega_n^2 Z_n=\frac{\{\varphi_n\}^T\{f(t)\}}{\{\varphi_n\}^T[m]\{\varphi_n\}} \tag{19}$$

where *n = 1,2,3*; ω_n is the $n\text{-}th$ coupled natural frequency of the system; β_n is the damping coefficient associated with the nth natural mode; and $\{\varphi_n\}$ is the $n\text{-}th$ natural mode.

It should be recalled that the damping coefficients are assumed to be proportional to the stiffness coefficients. For specified β_n and $f(t)$, each of the three equations may then be solved independently by a step-by-step numerical integration procedure.

There are several different methods available for integrating numerically equations of the form of Eq. (19). One of the procedures is the linear acceleration method, which is simple but sufficiently accurate for all practical purposes if the integration increment is chosen suitably.

In this method, the acceleration is assumed to vary linearly with time, t. Let $Z_n(t)$, $\dot{Z}_n(t)$ and $\ddot{Z}_n(t)$ denote, respectively, the value of Z_n and of its first two derivatives at any time t, and let Δt be the same increment between t and $t+\Delta t$. The acceleration Z_n at time $t+\Delta t$ may then be expressed in terms of all previous values at time t as follows:

$$\ddot{Z}_n(t+\Delta t)=\frac{1}{m_{eq}}\left[F_n(t+\Delta t)-2\beta_n\omega_n\dot{q}_n-\omega_n^2 q_n\right] \tag{20}$$

where

$$F_n(t+\Delta t)=\frac{\{\varphi_n\}^T\{f(t+\Delta t)\}}{\{\varphi_n\}^T[m]\{\varphi_n\}} \tag{21a}$$

$$\dot{q}_n=\dot{Z}_n(t)+\frac{\Delta t}{2}\ddot{Z}_n(t) \tag{21b}$$

$$q_n=Z_n(t)+\Delta t\dot{Z}_n(t)+\frac{\Delta t^2}{3}\ddot{Z}_n(t) \tag{21c}$$

$$m_{eq}=1+\beta_n\omega_n\Delta t+\frac{\omega_n^2\Delta t^2}{6} \tag{21d}$$

with the acceleration $\ddot{Z}_n(t+\Delta t)$ determined, the corresponding velocity and displacement are determined from

$$\dot{Z}_n(t+\Delta t) = \dot{q}_n + \frac{\Delta t}{2}\ddot{Z}_n(t+\Delta t) \tag{22}$$

$$Z_n(t+\Delta t) = q_n + \frac{\Delta t^2}{6}\ddot{Z}_n(t+\Delta t) \tag{23}$$

For specified initial values of $Z_n(0)$ and $\dot{Z}_n(0)$ the initial acceleration, $\ddot{Z}_n(0)$ may be evaluated directly from Eq. (19), i.e.

$$\ddot{Z}_n(0) = F_n(0) - 2\beta_n\omega_n\dot{Z}_n(0) - \omega_n^2 Z_n(0) \tag{24}$$

with $Z_n(0)$, $\dot{Z}_n(0)$ and $\ddot{Z}_n(0)$ known, the values of the acceleration, velocity, and displacement at any time may be determined by repeated application of Eqs. (20), (22) and (23).

Once the values of Z_n have been determined, the deformations $\{U\}$ are determined from Eq. (18), and the deformations of the individual columns are determined from the following equations:

$$U_{ix} = U_x + \eta_i b U_\theta + U_{cx} \tag{25a}$$

$$U_{jy} = U_y - \xi_j a U_\theta + U_{cy} \tag{25b}$$

where

$$U_{cx} = \sum_{i=1}^{n}\left[k_{ix}^*\left(x_g\right)_i\right] - \left(x_g\right)_i + \eta_i T_\theta \tag{26a}$$

$$U_{cy} = \sum_{j=1}^{m}\left[k_{jy}^*\left(y_g\right)_j\right] - \left(y_g\right)_j - \xi_j T_\theta \tag{26b}$$

T_θ is the corresponding displacement term of $\ddot{T}_\theta$.

Note that the terms U_{cx} and U_{cy} in Eqs. (25) are related only to the input functions, the individual stiffnesses of the system, and the geometric arrangement of the columns. They account for the difference in the displacement values of the ground motion at the locations of the columns due to the finite speed of propagation of the excitation. This fact is normally ignored in the conventional analysis. Hereafter, both quantities U_{cx} and U_{cy} are designated as U_c the displacement correction term.

2.2.3 Equations for symmetric systems

The equations are applicable to both symmetric and eccentric systems. A symmetric system may experience torsional response even under the influence of a translational motion of the

ground. This fact, which appears to have been first investigated by Newmark, can be seen clearly from Eqs. (14). In Newmark's approach, the rotational component of the response is first evaluated, and then it is combined with the translational component determined in the usual way. The method of combining the two components was not specified, (Newmark, 1969).

The method used herein consists of using Eqs. (14) with the e_x and e_y set equal to zero. In this approach, the torsional and translational effects are obtained simultaneously by solving the following set of equations:

$$\ddot{U}_x + 2\beta_x\omega_x\dot{U}_x + \omega_x^2 U_x = -\sum_{i=1}^{n} k_{ix}^{*}(\ddot{x}_o)_i \tag{27a}$$

$$\ddot{U}_y + 2\beta_y\omega_y\dot{U}_y + \omega_y^2 U_y = -\sum_{j=1}^{m} k_{jy}^{*}(\ddot{y}_o)_j \tag{27b}$$

$$\ddot{U}_\theta + 2\beta_\theta\omega_\theta\dot{U}_\theta + \omega_\theta^2 U_\theta = -\frac{1}{b}\ddot{T}_\theta \tag{27c}$$

Note that these equations are independent of one another. Accordingly, each unknown coordinate may be evaluated independently by the method described in the preceding section and the column deformations computed by use of Eqs. (25).

If the speed of propagation of the ground motion were infinite, as is normally assumed, the right-hand members of the three equations would be, respectively, $-\ddot{x}_o$, $-\ddot{y}_o$ and *0*, and no torsional response could develop. It is also interesting to note that, if only a single component of ground shaking is considered, say $\ddot{x}_o$ and if Eq. (27c) is multiplied by $b/2$, a direct comparison with the Newmark approach becomes possible.

Now, assume that the system has the same number of columns in the x and y directions and that the total stiffness of the system in either direction is equally distributed among the columns. The Newmark approach may be determined by neglecting the damping term in the Eq. (27c). For a very small transit time, t_r, which is the time required for the ground motion to traverse the longer plan dimension of the foundation, b, the Eq. (27c) may be written corresponding to the Newmark equation as follows,

$$-\frac{\ddot{x}_o t_r}{2}\left[1\Big/\sqrt{1+\frac{K_y}{K_x}\left(\frac{a}{b}\right)^2}\right] \tag{28}$$

2.2.4 Spectral response

In the time history method, there have been two major disadvantages in the use of this approach. First, the method produces a large amount of output information that can require a significant amount of computational efforts to conduct all possible design checks, as a function of time. Second, the analysis must be repeated for several different earthquake motions in order to assure that all frequencies are excited, since a response spectrum for one earthquake in a specified direction is not a smooth function.

There are computational advantages in using the spectral response method of seismic analysis for prediction of displacements and member forces in structural systems. The method involves the calculation of only the maximum values of the displacements and member forces in each mode using smooth design spectrum that are the average of several ground motions.

If the ground motions in both x and y directions are characterized by the same response spectrum, then the maximum value of the modal response function (19) may be rewritten in the following form

$$|Z_n|_{\max} = \frac{1}{\omega_n^2}\left(\Gamma_{nx}a_x + \Gamma_{ny}a_y\right)S_a(\omega_n, \beta_n) \tag{29}$$

In which a_x and a_y are the amp1itudes of the components of the ground motion along the x- and y- axes, respectively, and $S_a(\omega_n, \beta_n)$ is the spectral acceleration. Two Mode Participation Factors Γ_{nx} and Γ_{ny} may be obtained in the following form

$$\Gamma_{nx} = \varphi_{xn} \tag{30a}$$

$$\Gamma_{ny} = \varphi_{yn} \tag{30b}$$

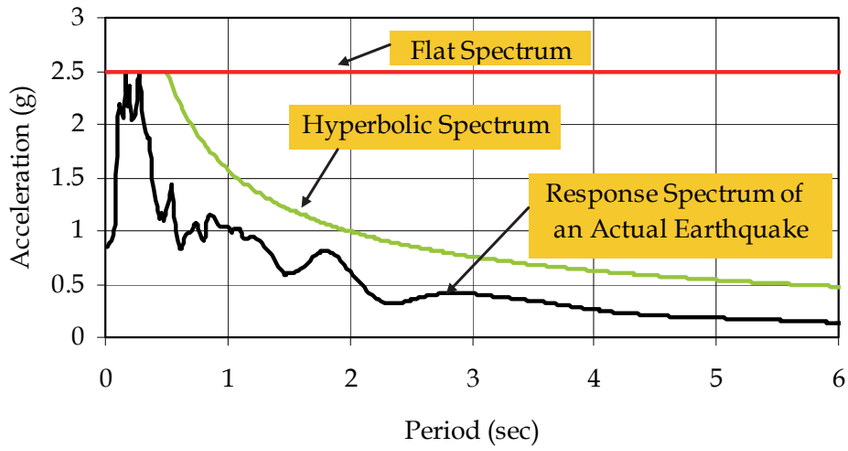

Fig. 12. Two idealized response spectrum

Finally, the most conservative method that is used to estimate a peak value of displacement or force within a structure is to use the sum of the absolute of the modal response values. This approach assumes that the maximum modal values, for all modes, occur at the same point in time. The relatively new method of modal combination is the Complete Quadratic Combination, CQC, method that was first published in 1981, (Wilson et al. 1981).

Now, the dynamic response of one-storey building model to the horizontal components of ground motion along x and y axes are investigated. For the objectives of this study it is considered the most appropriate to characterize ground motion by its response spectrum. The numerical results presented are for two idealized response spectrum (see Fig. 12), flat (or period independent) acceleration spectrum and hyperbolic acceleration spectrum (or flat velocity spectrum). The chosen spectrums have the advantage that the normalized response of the system does not depend on the periods of vibration, but only on their ratios.

For these two idealized spectrums, the normalized response does not depend on ω_n and ω_x separately but on the ratios ω_n/ω_x . The frequency ratios ω_n/ω_x and the mode shapes (φ_{xn} , φ_{xn} and $\varphi_{\theta n}$) depend on four dimensionless parameters (ω_y/ω_x , ω_θ/ω_x , ε_x and ε_y). The variation of the normalized modal responses, for a one-way coupled system ($\varepsilon_x = 0$) subjected to ground motion in the x direction, for the case of flat spectrum, are shown in Fig. 13.

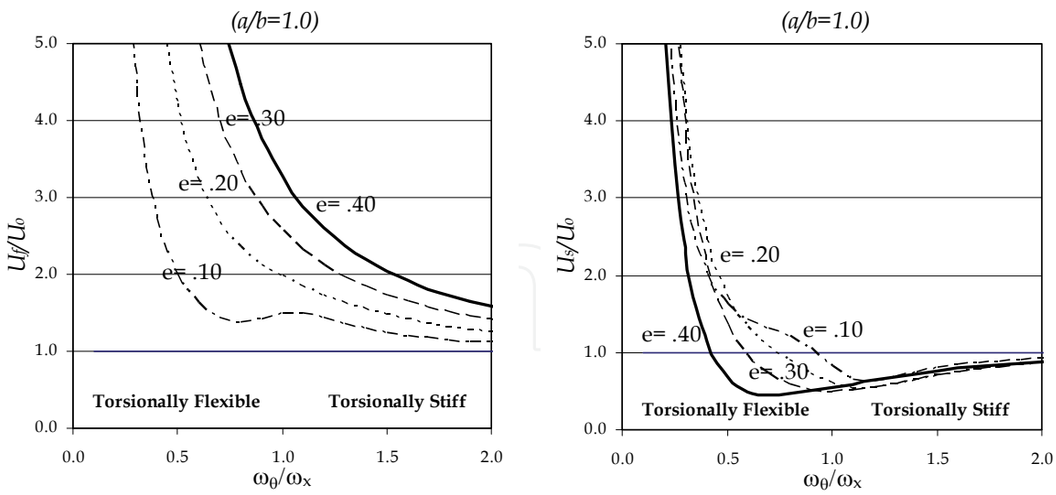

Fig. 13. Normalized displacements of the flexible and stiff edges; flat spectrum

Those for the case of hyperbolic spectrum are presented in Fig. 14. The maximum displacements of both flexible and stiff edges are calculated by modal superposition method, using complete CQC method. These values are then normalized by U_o the maximum displacement in the x direction produced by the same earthquake in an associated torsionally balanced building with stiffness and mass values similar to those of the asymmetric building but coincident centers of mass and rigidity.

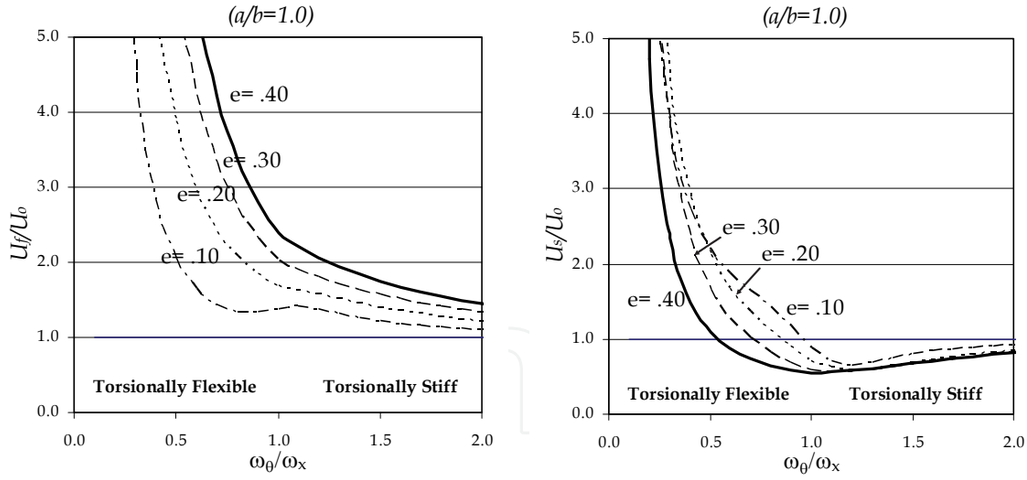

Fig. 14. Normalized displacements of the stiff and flexible edges; hyperbolic spectrum

The normalized flexible edge displacement U_f/U_o is plotted as a function of ω_θ/ω_x for different values of eccentricity $e = e_y$ and a plan aspect ratio of $a/b = 1$ in Fig. 11. In all cases flexible edge displacement in the structure is greater than the displacement of the associated symmetric structure. Of particular interest is the fact that there is a sharp increase in flexible edge displacement when ω_θ/ω_x falls below about 1.0.

It is also of interest to note that resonance between uncoupled translational and torsional frequencies, i.e., when $\omega_\theta/\omega_x = 1.0$, does not cause any significant increase in response.

Frequency resonance is not, therefore, a critical issue. Plots of normalized stiff edge displacement are shown in Fig. 13, again for different values of eccentricity $e = e_y$ and a plan aspect ratio of $a/b = 1$. Stiff edge displacement is less than 1 for $\omega_\theta/\omega_x \geq 1.0$. For $\omega_\theta/\omega_x < 1.0$, that is for torsionally flexible behaviour, stiff edge displacement starts to increase and can be, substantially, higher than 1. The results presented in Figs. 13 and 14 clearly suggest that buildings with low torsional stiffness may experience large displacements, causing distress in both structural and nonstructural components.

3. Torsional provisions in seismic codes as applied one-storey buildings

Most seismic building codes Formulate the design torsional moment at each storey as a product of the storey shear and a quantity termed design eccentricity. These provisions usually specify values of design eccentricities that are related to the static eccentricity between the center of rigidity and the center of mass. The earthquake-induced shears are applied through points located at the design eccentricities. A static analysis of the structure for such shears provides the design forces in the various elements of the structure. In some codes the design eccentricities include a multiplier on the static eccentricity to account for possible dynamic amplification of the torsion. The design eccentricities also include an allowance for accidental torsion. Such torsion is supposed to be induced by the rotational component of the ground motion and by possible deviation of the centers of rigidity and mass from their calculated positions. The design eccentricity formulae, given in building codes, can be written in two following parts:

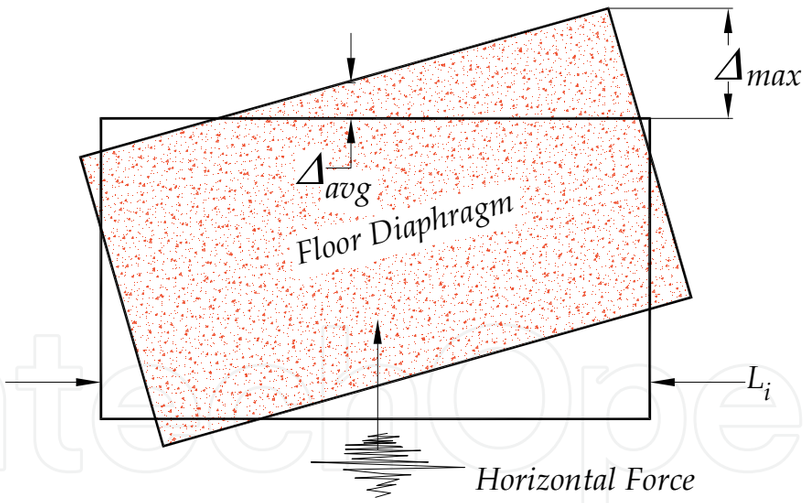

Fig. 15. Maximum and average diaphragm displacements of the structure

- The first part is expressed as some magnification factor times the structural eccentricity. This part deals with the complex nature of torsion and the effect of the simultaneous action of the two horizontal ground disturbances.
- The second term is called accidental eccentricity to account for the possible additional torsion arising from variations in the estimates of the relative rigidities, uncertain estimates of dead and live loads at the floor levels, addition of wall panels and partitions; after completion of the building, variation of the stiffness with time and, Inelastic or plastic action. The effects of possible torsional motion of the ground are also

considered to be included in this term. This terms in general a function of the plan dimension of the building in the direction of the computed eccentricity.

In Iranian code, in case of structures with rigid floors in their own plan, an additional accidental eccentricity is introduced through the effects generated by the uncertainties associate with the distribution of the mass level and/or the spatial variation of the ground seismic movement, (Iranian code 2800, 2005). This is considered for each design direction and for each level and also is related to the center of mass. The accidental eccentricity is computed with the relationship

$$e_i = \pm 0.05 L_i \tag{31}$$

where e_i is the accidental eccentricity of mass for storey *i* from its nominal location, applied in the same direction at all levels; L_i – the floor dimension perpendicular to the direction of the seismic action. If the lateral stiffness and mass are not distributed in plan and elevation, the accidental torsional effects may be accounted by multiplying an amplification factor A_j as follow

$$1.0 \le A_j = \left[\frac{\Delta_{\max}}{1.2\Delta_{avg}} \right]^2 \le 3.0 \tag{32}$$

where $\Delta_{\max}$ and Δ_{avg} are maximum and average diaphragm displacements of the structure, respectively, (see Fig. 15).

4. Conclusion

A study of free vibration characteristics of an eccentric one-storey structural model is presented. It is shown in the previous sections that the significance of the coupling effect of an eccentric system depends on the magnitude of the eccentricity between the centers of mass and of rigidity and the relative values of the uncoupled torsional and translational frequencies of the same system without taking the eccentricity into account. The coupling effect for a given eccentricity is the greatest when the uncoupled torsional frequency, ω_θ, and translational frequency, ω_x of the system are equal. As the value of ω_θ/ω_x increases, the coupling effect decreases. For small eccentricities, the motions may reasonably be considered uncoupled if the ratio of ω_θ/ω_x exceeds 2.5.

In addition, it is shown that the locus of the associated center of rotation can be formulated corresponding for a given eccentricity. Note that, for all values of eccentricity, as the value of the uncoupled natural frequencies ratio increases the center of rotation shifts away from the center of rigidity for the first mode and approaches the center of mass for the higher mode. It is also shown that, the torsional behaviour of the model assembled, using our approach, can be classified based on the nature of the instantaneous center of rotation.

It is well known that asymmetric or torsionally unbalanced buildings are vulnerable to damage during an earthquake. Resisting elements in such buildings could experience large displacements and distress. With eccentricity defined for one-storey buildings, the torsional provisions or building codes can then be applied for a seismic design or such structures.

5. Acknowledgment

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6. References

9-11 Research Book, (2006). Other Building Collapses, Available from http://911research.wtc7.net/wtc/analysis/compare/collapses.html

Anastassiadis, K., Athanatopoulos, A. & Makarios, T. (1998). Equivalent static eccentricities in the simplified methods of seismic analysis of buildings, *Earthquake Spectra*, vol. 14, No. 1, pp.1–34.

Chandler, M. & Hutchinson, G.L. (1986). Torsional Coupling Effects in the Earthquake Response of Asymmetric Buildings, *Engineering Structures*, vol. 8, pp. 222-236.

Cruz, E.F. & Chopra, A.K. (1986). Simplified Procedures for Earthquake Analysis of Buildings, *Journal of Structural Engineering*, Vol. 112, pp. 461-480.

De la Llera, J.C. & Chopra, A.K. (1994). Using accidental eccentricity in code-specified static and dynamic analysis of buildings, *Earthquake Engineering and Structural Dynamics*, vol. 23, No. 7, pp. 947–967.

Dempsey, K.M. & Irvine, H.M. (1979). Envelopes of maximum seismic response for a partially symmetric single storey building model, *Earthquake Engineering and Structural Dynamics*, vol. 7, No. 2, pp. 161–180.

Earthquake Engineering ANNEXES, (2007), European Association for Earthquake Engineering.

Fajfar P., Marusic D. & Perus I. (2005). Torsional effects in the pushover-based seismic analysis of buildings. *Journal of Earthquake Engineering*, vol. 9, No. 6, pp. 831–854.

Hejal, R. & Chopra, A.K. (1989). Earthquake Analysis of a Class of Torsionally-Coupled Buildings, Earthquake Engineering and Structural Dynamics, Vol. 18, pp. 305-323.

Iranian Code of Practice for Seismic Resistant Design of Buildings. (2005). Standard No. 2800-05, 3rd Edition.

Koh, T., Takase, H. & Tsugawa, T. (1969). Torsional Problems in Seismic Design of High-Rise Buildings, *Proceedings of the Fourth World Conference on Earthquake Engineering*, Santiago, Chile, vol. 4, pp. 71-87.

Kuo, Pao-Tsin. (1974). Torsional Effects in Structures Subjected to Dynamic Excitations of the Ground, Ph.D. Thesis, Rice University.

Moghadam, A.S. & Tso, WK. (2000). Extension of Eurocode 8 torsional provisions to multi-storey buildings, *Journal of Earthquake Engineering*, vol. 4, No. 1, pp. 25–41.

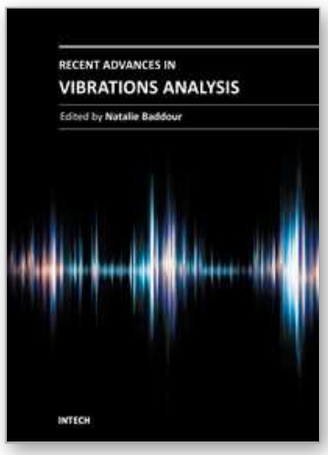
Newmark, N. M., (1969). Torsion in Symmetrical Buildings, *Proceedings of the Fourth World Conference on Earthquake Engineering*, Vol. 2, Santiago, Chile, pp. A3-19 to A3-32.

Tabatabaei, R. & Saffari, H. (2010). Demonstration of Torsional Behaviour Using Vibration-based Single-storey Model with Double Eccentricities, *Journal of Civil Engineering*, vol. 14, No. 4., pp. 557-563.

Tanabashi, R. (1960). Non-Linear Transient Vibration of Structures, *Proceedings of the Second World Conference on Earthquake Engineering*, Tokyo, Japan, vol. 2, pp. 1223.

Tso, W.K. & Dempsey, K.M. (1980). Seismic torsional provisions for dynamic eccentricity, *Earthquake Engineering and Structural Dynamics*, vol. 8, No. 3, pp. 275–289.

Wilson, E. L., Der Kiureghian, A. & Bayo, E. R. (1981). A Replacement for the SRSS Method in Seismic Analysis, *Earthquake Engineering and Structural Dynamics*, Vol. 9, pp. l87-l92.



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