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Near-Optimal Nonlinear Forwarding Strategy for Two-Hop MIMO Relaying

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1. Introduction

Relaying (1–3) has been considered as a paradigm for improving the quality of service (i.e., bit-error-rate, data rate and coverage) in wireless networks. In this work, we study a two-hop relay channel in which each node can have multiple antennas. It is well-known that utilizing multiple-input multiple-output (MIMO) links can significantly improve the transmission rate (see e.g. (4; 5) and references therein). Thus, one can expect a combination of a MIMO gain and a relaying gain in a MIMO relay link. We focus on *one-shot* transmission, where the channel is used once for the transmission of one symbol representing a message. This is often referred to as *uncoded transmission*. The main motivation for such a scenario is in considering applications requiring either low-delays or limited processing complexity.
The capacity of the MIMO relay channel is studied in (6). The work in (9) establishes the optimal *linear* relaying scheme when perfect CSI is available at the nodes. The work in (7; 8) investigates *linear* relay processing for the MIMO relay channel. In this paper, in contrast to (6–9), we study an *uncoded* system, and we propose a *nonlinear* relaying scheme which is superior to linear relaying and performs close to the theoretical bound. Our proposed scheme is based on constellation permutation (10; 11) at the relay over different streams obtained by channel orthogonalization.
We investigate a two-hop MIMO fading Gaussian relay channel consisting of a source, a relay and a destination. We assume that all three nodes have access to perfect channel state information. We propose a nonlinear relaying scheme that can operate close to the optimal performance. The proposed scheme is constructed using channel orthogonalization by employing the singular value decomposition, and permutation mapping. We also demonstrate that linear relaying can amount to a significant loss in the performance.

1.1 Organization

The remainder of the chapter is organized as follows. Section 2 first introduces the two-hop relay channel model and then explains the transmission protocol and the assumptions on the channel state information (CSI) at the nodes and finally formulates an optimization problem. Section 3 simplifies and reformulates the optimization problem introduced in the preceding section, by channel orthogonalization using SVD. Section 4 introduces a novel relaying strategy in which the relay first detects the transmitted message and employs permutation coding over different streams obtained by channel orthogonalization. This section also

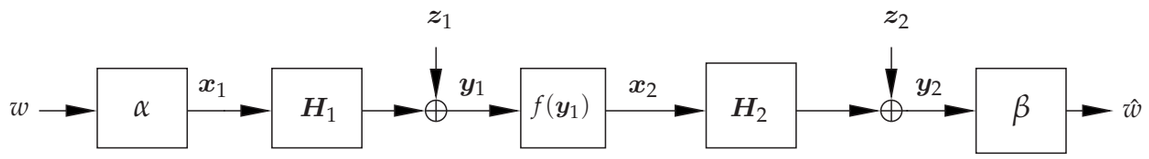

Fig. 1. Gaussian two-hop MIMO relaying.

provides some performance bounds. Section 5 finally provides some simulation results and concludes the chapter.

2. System model and problem formulation

In this section, we first introduce the two-hop Gaussian vector relay channel in detail and then formulate the general problem of finding an optimal relaying strategy for the underlying channel.

We consider Gaussian two-hop communication between a source and a destination, as illustrated in Fig. 1. The communication is assisted by a relay node located between the source and the destination. We assume that the relay node has no own information to transmit and its sole purpose is to *forward* the information received from the source to the destination. We additionally assume that all nodes may have different number of antennas. It is assumed that there is no direct communication between the source and the destination. (This is reasonable when e.g., the destination is located far away from the source or there is a severe shadow fading between the source and the destination.) The communication between the source and the relay takes place in two phases as described in the following.

First–Hop Transmission: During the first phase, the source transmits its information and the relay listens to the transmitted signal. The received signal vector at the relay, denoted by $\boldsymbol{y}_1$, is given by

$$\boldsymbol{y}_1 = \boldsymbol{H}_1 \boldsymbol{x}_1 + \boldsymbol{z}_1 \tag{1}$$

where $\boldsymbol{H}_1 \in \mathbb{C}^{[L\times M]}$ denotes the channel between the source and the relay, $\boldsymbol{x}_1 \in \mathbb{C}^{[M\times 1]}$ denotes the transmitted signal vector from the source and $\boldsymbol{z}_1 \in \mathbb{C}^{[L\times 1]}$ denotes the additive circularly symmetric Gaussian noise. The signal vector $\boldsymbol{x}_1$ is the output of the modulator α which is defined as

$$\begin{aligned} \alpha &: \mathbb{W} \longmapsto \mathbb{C}^M \\ \boldsymbol{x}_1 &= \alpha(w) \end{aligned}$$

where $w \in \mathbb{W} \triangleq \{1, 2, 3, \ldots, 2^q\}$ denotes a message to be transmitted over the channel. Some particular choices for defining α are, for example, the 2^q-QAM and 2^q-PSK modulation schemes. We assume an average power constraint at the source, such that $\mathrm{tr}\mathbb{E}\{\boldsymbol{x}_1\boldsymbol{x}_1^\dagger\} \leq P_1$.

Second–Hop Transmission: During the second phase, only the relay transmits and the source is silent. We assume that the relay uses a forwarding strategy given by the following deterministic function

$$\begin{aligned} f &: \mathbb{C}^L \longmapsto \mathbb{C}^L \\ \boldsymbol{x}_2 &= f(\boldsymbol{y}_1) \end{aligned}$$

Since the function $f(\cdot)$ is arbitrary, our model includes linear as well as nonlinear mappings. We assume an average power constraint at the relay such that $\mathrm{tr}\mathbb{E}\{\boldsymbol{x}_2\boldsymbol{x}_2^\dagger\} \leq P_2$. The received

signal at the destination, denoted by $\boldsymbol{y}_2$, is then given by

$$\boldsymbol{y}_2 = \boldsymbol{H}_2\boldsymbol{x}_2 + \boldsymbol{z}_2 \tag{2}$$

where $\boldsymbol{H}_2 \in \mathbb{C}^{[N\times L]}$ denotes the channel between the relay and the destination, $\boldsymbol{x}_2 \in \mathbb{C}^{[L\times 1]}$ denotes the transmitted signal vector from the relay and $\boldsymbol{z}_2 \in \mathbb{C}^{[N\times 1]}$ denotes the additive circularly symmetric Gaussian noise. Finally, the destination, upon receiving $\boldsymbol{y}_2$, detects the transmitted message using the function (demodulator or detector) β defined as

$$\beta : \mathbb{C}^N \longmapsto \mathbb{W}$$
$$\hat{w} = \beta(\boldsymbol{y}_2)$$

where $\hat{w} \in \mathbb{W}$ denotes the detected message at the destination.

Channel Statistics: We assume that the entries of the channel matrices $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ are i.i.d. Rayleigh fading, distributed according to $\mathcal{CN}(0,1)$. The entries of the noise vectors $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$ are assumed to be independent zero-mean circularly symmetric Gaussian noise. The covariance matrices of the noise vectors are given by $\boldsymbol{R}_{z_1z_1} = \mathbb{E}[\boldsymbol{z}_1\boldsymbol{z}_1^\dagger] = N_1\boldsymbol{I}_L$ and $\boldsymbol{R}_{z_2z_2} = \mathbb{E}[\boldsymbol{z}_2\boldsymbol{z}_2^\dagger] = N_2\boldsymbol{I}_N$, where $\boldsymbol{I}_N$ and $\boldsymbol{I}_M$ denote the identity matrices of size N and M, respectively. Additionally, we assume that the channels stay unchanged during the transmission of one block but they vary independently from one block to another.

Channel State Information (CSI): We assume that the source, the relay, and the destination know $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ perfectly. The CSI of backward channels at the relay and the destination can be obtained using training sequences and the CSI of the forward channels at the source and the relay can be obtained either using reciprocity of the links or feedback. When the channel matrices are constant or varying slowly, one can obtain accurate CSI at the nodes. Satellite MIMO link and wireless LAN are two practical examples in which this model is applicable.

2.1 Problem formulation

The goal is to minimize the *average message error probability*. Thus for a given message set $\mathbb{W}$, we need to find the triple (α^*, β^*, f^*) under the average power constraint such that

$$(\alpha^*, \beta^*, f^*) = \arg\min_{\alpha,\beta,f} \Pr\{\hat{w} \neq w\}. \tag{3}$$

We desire to find a *structured* solution to the optimization problem in (3). Imposing structure on a communication strategy results in loss of performance in general. On the other hand, a structured strategy however facilitates the design. We first utilize the channel knowledge to orthogonalize each hop using the SVD and then propose a nonlinear scheme that performs close to the theoretical bound.

3. Channel orthogonalization via SVD

In the following, we employ the singular value decomposition (SVD) to obtain an *equivalent* parallel channel for each hop. We then rewrite the optimization problem given by (3) for the equivalent channel.

Using the SVD, any channel realizations of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ can be written as

$$\boldsymbol{H}_1 = \boldsymbol{U}_1\boldsymbol{D}_1\boldsymbol{V}_1^\dagger$$
$$\boldsymbol{H}_2 = \boldsymbol{U}_2\boldsymbol{D}_2\boldsymbol{V}_2^\dagger$$

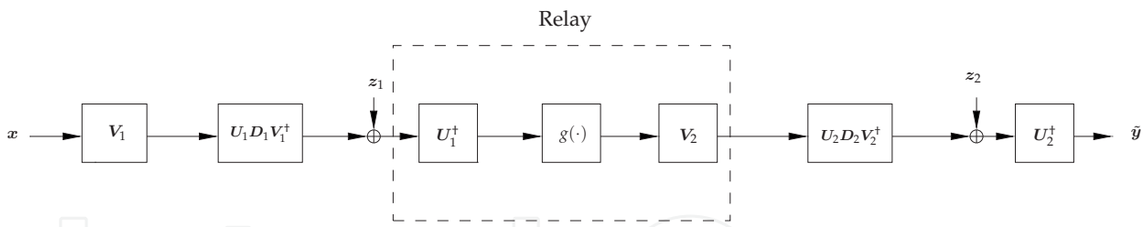

Fig. 2. Processing using the SVD of the channel matrices.

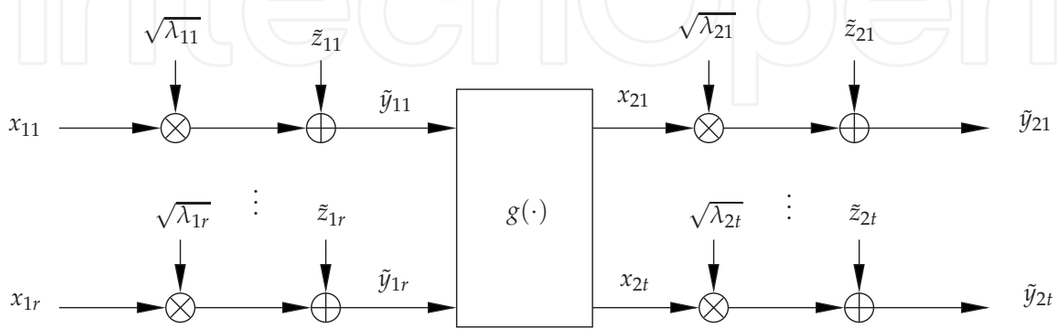

Fig. 3. Equivalent parallel channel.

where $\boldsymbol{U}_1 \in \mathbb{C}^{[L\times L]}$, $\boldsymbol{V}_1 \in \mathbb{C}^{[M\times M]}$, $\boldsymbol{U}_2 \in \mathbb{C}^{[N\times N]}$ and $\boldsymbol{V}_2 \in \mathbb{C}^{[L\times L]}$ are unitary matrices, and $\boldsymbol{D}_1 \in \mathbb{R}^{[L\times M]}$ and $\boldsymbol{D}_2 \in \mathbb{R}^{[N\times L]}$ are non-negative and diagonal matrices. Note that since $\boldsymbol{U}_1$, $\boldsymbol{V}_1$, $\boldsymbol{U}_2$ and $\boldsymbol{V}_2$ are invertible, linear operations of the form of $\boldsymbol{AG}$ or $\boldsymbol{GA}$ (where $\boldsymbol{G} \in \{\boldsymbol{U}_1, \boldsymbol{V}_1, \boldsymbol{V}_2, \boldsymbol{U}_2\}$ and $\boldsymbol{A}$ is an arbitrary matrix with an appropriate size) impose no loss of information. Thus we can preprocess the transmitted signal vectors from the source and the relay and postprocess the received signal vectors at the relay and the destination as illustrated in Fig. 2. Consequently, the received signal at the relay after the linear postprocessing is given by

$$\begin{aligned}\tilde{\boldsymbol{y}}_1 &= \boldsymbol{U}_1^\dagger \boldsymbol{y}_1 \\ &= \boldsymbol{U}_1^\dagger \boldsymbol{H}_1 \boldsymbol{V}_1 \boldsymbol{x}_1 + \boldsymbol{U}_1^\dagger \boldsymbol{z}_1 \\ &= \boldsymbol{U}_1^\dagger \boldsymbol{U}_1 \boldsymbol{D}_1 \boldsymbol{V}_1^\dagger \boldsymbol{V}_1 \boldsymbol{x}_1 + \boldsymbol{U}_1^\dagger \boldsymbol{z}_1 \\ &= \boldsymbol{D}_1 \boldsymbol{x}_1 + \tilde{\boldsymbol{z}}_1\end{aligned}$$

where the last equality follows from the identities $\boldsymbol{U}_1^\dagger \boldsymbol{U}_1 = \boldsymbol{I}_L$ and $\boldsymbol{V}_1^\dagger \boldsymbol{V}_1 = \boldsymbol{I}_M$ and the definition $\tilde{\boldsymbol{z}}_1 = \boldsymbol{U}_1^\dagger \boldsymbol{z}_1$. The random vector $\tilde{\boldsymbol{z}}_1 \sim \mathcal{CN}(0, N_1 \boldsymbol{I}_L)$ since $\boldsymbol{U}_1$ is a unitary matrix. In a similar fashion, we can obtain

$$\tilde{\boldsymbol{y}}_2 = \boldsymbol{D}_2 \boldsymbol{x}_2 + \tilde{\boldsymbol{z}}_2$$

where $\tilde{\boldsymbol{z}}_2 \triangleq \boldsymbol{U}_2^\dagger \boldsymbol{z}_2 \sim \mathcal{CN}(0, N_2 \boldsymbol{I}_M)$. See also Fig. 2. Because $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are diagonal matrices, we have

$$\begin{aligned}\tilde{y}_{1i} &= \sqrt{\lambda_{1i}} x_{1i} + \tilde{z}_{1i}, \quad i \in \{1, 2, \ldots, \min(M, L)\} \\ \tilde{y}_{2j} &= \sqrt{\lambda_{2j}} x_{2j} + \tilde{z}_{2j}, \quad j \in \{1, 2, \ldots, \min(L, N)\}\end{aligned}$$

where $\sqrt{\lambda_{1i}}$ is the ith entry on the main diagonal of $\boldsymbol{D}_1$ and $\sqrt{\lambda_{2j}}$ is the jth entry on the main diagonal of $\boldsymbol{D}_2$. The equivalent channel obtained by the SVD operation is shown in Fig. 3. The function $g(\cdot)$ in Fig. 3 denotes the forwarding strategy at the relay, defined as

$$g : \mathbb{C}^r \longmapsto \mathbb{C}^t$$
$$\boldsymbol{x}_2 = g(\tilde{\boldsymbol{y}}_1)$$

where $r = \min(M, L)$ and $t = \min(L, N)$. We consider both linear as well as nonlinear mappings. One can thus optimize the mapping according to

$$(\alpha^*, g^*(\tilde{\boldsymbol{y}}_1), \beta^*) = \underset{\left\{\alpha:\mathrm{tr}\mathbb{E}[\boldsymbol{x}_1\boldsymbol{x}_1^\dagger]\leq P_1\right\},\{g(\tilde{\boldsymbol{y}}_1):\mathrm{tr}\mathbb{E}[g(\tilde{\boldsymbol{y}}_1)g^\dagger(\tilde{\boldsymbol{y}}_1)]\leq P_2\},\beta}{\operatorname{argmin}} \Pr\{\hat{w} \neq w\}. \tag{4}$$

4. Transmission strategies and performance bounds

4.1 Lower bound on P_e

We next give a simple lower bound on the average message error probability, which we use as a benchmark to evaluate different transmission strategies in the sequel.

Lemma 1. *For the two-hop vector channel shown in Fig. 1, the average message error probability P_e is lower bounded by*

$$P_e \geq \max\{P_{e_1}, P_{e_2}\} \tag{5}$$

where P_{e_1} and P_{e_2} denote the average message error probability of the first- and the second hop, respectively.

Proof. Consider a two-hop channel where the first hop is noise-free and the second hop is identical to the original channel in Fig. 1. Denote the average error probability of this new channel by $\bar{P}_e$. It is easy to see that $P_e \geq \bar{P}_e = P_{e_2}$. In a similar manner we can obtain $P_e \geq \tilde{P}_e = P_{e_1}$, where $\tilde{P}_e$ denotes the error probability of a two-hop channel with identical first hop to that in Fig. 1 and a noise-free second hop. This yields (5). □

4.2 Linear relaying

One of the fundamental strategies in the literature is *linear* relaying, commonly known as amplify-and-forward (AF). Using AF in our setting, the relay function is given by

$$x_{2i} = g_i(\tilde{y}_{1i}) = \kappa_i \mu_i \tilde{y}_{1i}, \quad i \in \{1, ..., \min\{r, t\}\} \tag{6}$$

where $\mu_i = \sqrt{\frac{P_2}{\lambda_{1i}\mathbb{E}[x_{1i}x_{1i}^\dagger]+N_1}}$ is a power normalization factor and $0 \leq \kappa_i \leq 1$ is a power allocation factor where $\sum_{i=1}^{t} \kappa_i^2 = 1$. Note that the number of parallel channels that can be utilized is $\min\{r, t\}$, i.e., the minimum number of parallel streams of the first- and second hop. In (9), it is shown that the strategy given by (6) is optimal if the relay mapping is constrained to be linear. However as we show, AF is in general suboptimal for the underlying channel.
The received signal-to-noise ratio (SNR) of the ith stream at the destination is given by

$$\gamma_i^{AF} = \frac{\kappa_i^2 \lambda_{1i}\lambda_{2i}P_{1i}P_2}{N_1N_2 + \lambda_{1i}P_{1i}N_2 + \kappa_i^2\lambda_{2i}P_2N_1} \tag{7}$$

where $P_{1i} \triangleq \mathbb{E}[x_{1i}x_{1i}^\dagger]$. The fact that the received noise at the relay is forwarded to the destination is the main drawback of AF relaying.

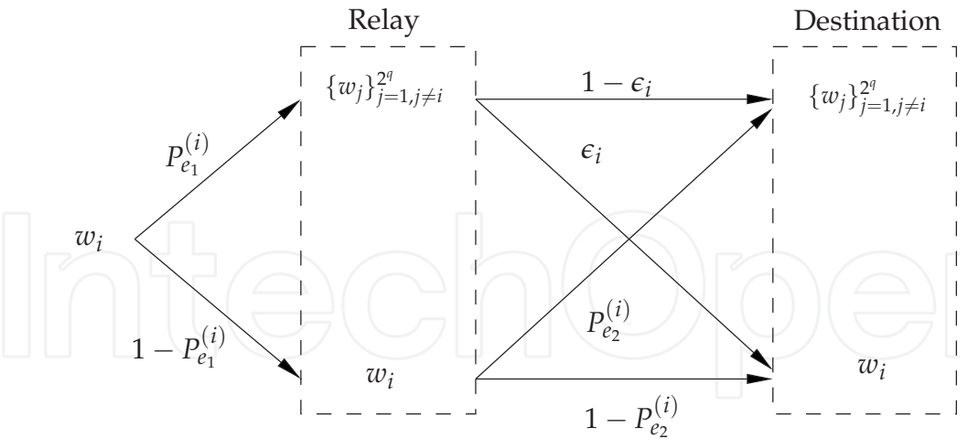

Fig. 4. Error probability transition in DF relaying.

Case $r = 1$: In order to maximize γ_i, one should choose the strongest mode (the stream with largest singular value) with full power when $r = 1$. Note that the use of weaker streams at the relay does not improve the performance of AF since all streams are transmitting the same signal, thus allocating all power to the strongest mode is the optimal solution. Therefore, the maximum possible achievable SNR for linear relaying when $r = 1$, is given by

$$\gamma_{AF}^{*} = \frac{\lambda_{11}\lambda_{21}P_1P_2}{N_1N_2 + \lambda_{11}P_1N_2 + \lambda_{21}P_2N_1} \tag{8}$$

where λ_{11} and λ_{21} are the largest eigenvalues of the first- and second hop, respectively.

4.3 Relaying via Detect-and-Forward (DF)

Another approach for forwarding the received signals is to first detect the transmitted message and then re-modulate it. That is

$$x_{2i} = g_i(\tilde{\boldsymbol{y}}_1) = \kappa_i\alpha_{ri}(\hat{\tilde{w}}) = \kappa_i\alpha_{ri}(\beta_r(\tilde{\boldsymbol{y}}_1)) \tag{9}$$

where $\hat{\tilde{w}} = \beta_r(\tilde{\boldsymbol{y}}_1)$ is the detected message and β_r denotes the detector at the relay. The modulator for generating x_{2i} is denoted by α_{ri}. We also have $\mathrm{tr}\mathbb{E}[\boldsymbol{x}_2\boldsymbol{x}_2^{\dagger}] = P_2$.
The following proposition derives a simple upper bound on the average message error probability of DF relaying.

Lemma 2. *The average message error probability is upper bounded by*

$$P_e \leq P_{e_1} + P_{e_2} - \min_{1\leq i\leq 2^q} P_{e_1}^{(i)}P_{e_2}^{(i)} \tag{10}$$

where $P_{e_1}^{(i)}$ *and* $P_{e_1}^{(i)}$ *respectively denote the ith message error probability of the first- and the second hop and* P_{e_1} *and* P_{e_1} *respectively are the average message error probabilities of the first- and the second hop.*

Proof. Consider the transmission of w_i from the source. The relay either detects the transmitted message correctly or declares another message. This is illustrated in Fig. 4. Using Fig. 4, the ith message error probability can be bounded as

$$P_e^{(i)} = (1 - P_{e_1}^{(i)})P_{e_2}^{(i)} + P_{e_1}^{(i)}(1 - \epsilon_i) \tag{11}$$

$$\leq P_{e_1}^{(i)} + P_{e_2}^{(i)} - P_{e_1}^{(i)}P_{e_2}^{(i)} \tag{12}$$

where ϵ_i denotes the detection probability of w_i at the destination when $\{w_j\}_{j=1,j\neq i}^{2^q}$ is transmitted from the relay, under the constraint that the source is transmitted w_i. The inequality in (11) follows from the fact that $0 \leq \epsilon_i \leq 1$. By taking average over all possible messages, we have

$$P_e = \sum_{i=1}^{2^q} P_e^{(i)} p(w_i) \tag{13}$$

$$\leq \sum_{i=1}^{2^q} \left(P_{e_1}^{(i)} + P_{e_2}^{(i)} - P_{e_1}^{(i)} P_{e_2}^{(i)} \right) p(w_i) \tag{14}$$

$$= P_{e_1} + P_{e_2} - \sum_{i=1}^{2^q} P_{e_1}^{(i)} P_{e_2}^{(i)} p(w_i) \tag{15}$$

$$\leq P_{e_1} + P_{e_2} - \left(\min_{1\leq i\leq 2^q} P_{e_1}^{(i)} P_{e_2}^{(i)} \right) \sum_{i=1}^{2^q} p(w_i) \tag{16}$$

$$= P_{e_1} + P_{e_2} - \min_{1\leq i\leq 2^q} P_{e_1}^{(i)} P_{e_2}^{(i)} \tag{17}$$

This completes the proof. □

Proposition 1. *DF relaying achieves the same performance as that of a single hop (i.e.,* $\max\{P_{e_1}, P_{e_2}\}$*) at high SNR when* $N \neq M$*.*

Proof. For given modulator and optimal demodulator, the error probability at the destination is upper bounded as

$$P_e^{DF} \leq P_{e_1} + P_{e_2} = \frac{a_1}{\gamma_1^{NL}} + O\left(\frac{1}{\gamma_1^{NL+1}}\right) + \frac{a_2}{\gamma_2^{LM}} + O\left(\frac{1}{\gamma_2^{LM+1}}\right)$$
$$= \begin{cases} \frac{a_1}{\gamma_1^{NL}} + O\left(\frac{1}{\gamma_1^{NL+1}}\right) & \text{if } N < M \\ \frac{a_2}{\gamma_2^{LM}} + O\left(\frac{1}{\gamma_2^{LM+1}}\right) & \text{if } M < N \end{cases} \tag{18}$$

where we used Lemma 2 and $\gamma_1 \triangleq \frac{P_1}{N_1}$, $\gamma_2 \triangleq \frac{P_2}{N_2}$, and a_1 and a_2 are two constants depending on the number of antennas and the modulation scheme.
We also have the following lower bound using Lemma 1

$$P_e \geq \max\{P_{e_1}, P_{e_2}\} = \begin{cases} \frac{a_1}{\gamma_1^{NL}} + O\left(\frac{1}{\gamma_1^{NL+1}}\right) & \text{if } N < M \\ \frac{a_2}{\gamma_2^{LM}} + O\left(\frac{1}{\gamma_2^{LM+1}}\right) & \text{if } M < N \end{cases} \tag{19}$$

Comparing (18) and (19), we see that the upper bound and lower bound meet each other at high SNR. This therefore establishes the optimality of DF at high SNR. □

Proposition 2. *DF achieves the optimal diversity order* $d^* = \min\{NL, ML\}$.

Proof. From Lemma 1, we conclude that $d^* \leq \min\{NL, ML\}$. But, from Lemma 2 we know that $P_e^{DF} \leq P_{e_1} + P_{e_2}$. Thus the diversity order is bounded as $d_{DF} \geq \min\{NL, ML\}$. Therefore, DF achieves the optimal diversity order. □

In the following we comment further on the conventional DF and a propose a novel DF relaying scheme.

Conventional DF: One way to simplify the problem is to use the same modulator over all streams. That is $\alpha_{ri} = \alpha_r$ for all streams. By doing so, with a similar argument as that in the linear relaying case, the optimal power allocation would be to use all the power on the strongest mode.

Proposition 3. *Relaying using conventional DF (i.e., transmission using the strongest mode) is optimal at high SNR when $N > M$.*

Proof. The proof follows from the observation that using only the stream with the strongest mode of the second hop, one can obtain higher diversity gain compared to the first hop for any source modulator. Since $M < N$, we have

$$P_e^{DF} \leq \frac{a_1}{\gamma_1^{LM}} + O\left(\frac{1}{\gamma_1^{LM+1}}\right), \text{ and } P_e \geq \frac{a_1}{\gamma_1^{LM}} + O\left(\frac{1}{\gamma_1^{LM+1}}\right). \tag{20}$$

This completes the proof. □

Proposed DF: A more sophisticated approach at the relay is to use different modulators over distinct streams. In the following, we propose a structured method for obtaining different modulators based on a given modulator, say α_r. Let π denote a permutation operation on a given finite sequence. For example, if $a = (1, 2, 3, 4)$ the operation $\pi(a)$ produces a different ordering of the elements in the sequence, such as $\pi_1(a) = (4, 3, 1, 2)$. In the following let $\bar{\alpha}_r$ denote the list of letters produced by the modulator α_r, in the default order. Now we construct the ith modulator using $\bar{\alpha}_r$ as

$$\bar{\alpha}_{ri} = \kappa_i \pi_i(\bar{\alpha}_r) \tag{21}$$

where κ_i is a power allocation factor used at ith stream such that $\{\kappa_i\}$ meets the power constraint $\mathrm{tr}\mathbb{E}[\boldsymbol{x}_2 \boldsymbol{x}_2^\dagger] \leq P_2$. Thus, the transmitted signal from the relay over the ith stream is given by

$$x_{2i} = g_i(\tilde{\boldsymbol{y}}_1) = \kappa_i \alpha_{ri}(\hat{\tilde{w}}) = \kappa_i \alpha_{ri}(\beta_r(\tilde{\boldsymbol{y}}_1)) \tag{22}$$

Here β_r denotes the detector used at the relay and the modulator α_{ri} is constructed using the ith permutation used over the ith stream, i.e., π_i. Now designing a relaying strategy specializes to finding the optimal permutations and the power allocation factors. That is

$$(\{\kappa_i^*\}_{i=1}^t, \{\pi_i^*\}_{i=1}^t) = \arg \min_{\{\kappa_i\}_{i=1}^t, \{\pi_i\}_{i=1}^t} \Pr\{\hat{w} \neq w\} \tag{23}$$

The proposed DF scheme includes conventional DF as a special case, by choosing $\kappa_i = 0$ for $i \neq 1$. Thus, the error probability achieved by the proposed DF scheme is upper bounded by that of conventional DF. The main advantage of the proposed scheme is that it enjoys a structured design based on a given modulator. From Proposition 3, one can conclude that this scheme does not bring any advantage at high SNR when $N > M$. However, in the following section we show that the proposed DF approach can attain considerable gain over conventional DF and linear relaying at moderate SNR's, that is, in an SNR regime where diversity gain is not a useful performance measure.

5. Numerical results and concluding remarks

In the following we present numerical results for the case when the source has only one single antenna and the relay and the destination have 10 antennas each. This scenario is of importance, for example in the uplink transmission in cellular networks, where the mobile node has only a single antenna. Under this constraint, the relay has only one incoming stream and multiple outgoing streams (see Fig. 3). Fig. 5 shows the average message error probability for three different relaying schemes; linear relaying, conventional DF relaying, the proposed DF relaying approach based on permutation mappings using two streams. We use 16-QAM as the modulator and an optimal ML detector at the relay and the destination. For the proposed scheme we use two streams in the second hop. The optimal permutation is obtained using exhaustive search. We also plotted a lower bound on the performance for any relaying scheme, using Lemma 1. Here we set $P_1 = P_2 = P$, $N_1 = N_2 = 1$. From Fig. 5, we see that linear relaying performs worst, and the proposed DF relaying scheme provides the best performance. Surprisingly, the performance of the proposed DF is very close to the lower bound.

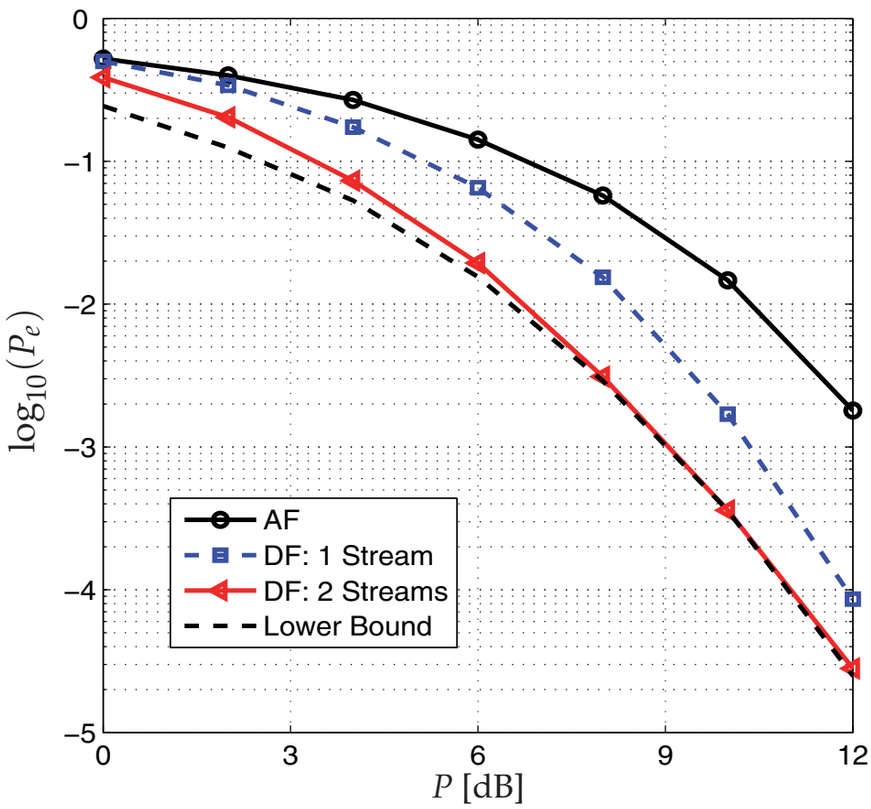

Fig. 5. Average message error probability (P_e) using 16-QAM modulation for different forwarding strategies (AF, conventional DF (i.e., one stream) and proposed DF (i.e., two streams with permuted modulations)). Here we set $P_1 = P_2 = P$, $N_1 = N_2 = 1$, number of antennas at the source is $N = 1$ and number of antennas at the relay and the destination are $L = M = 10$.

We can see from Fig. 5 that the performance of conventional DF approaches that of the proposed scheme at *high* SNR. This is in accordance with Proposition 3. However, we also see that the new scheme gives considerable gains in the low- and moderate SNR regime, and it achieves the optimal performance at lower SNR compared to conventional DF.

6. References

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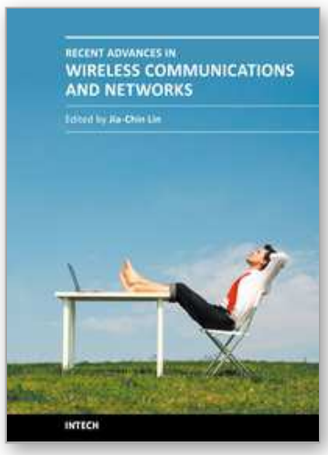

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