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Instrument Generating Function and Analysis of Persistent Economic Times Series: Theory and Application

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1. Introduction

The traditional approach to the persistence properties of time series is unit root tests, and because of the near unit root bias, the median-unbiased procedure of Andrews (1993) is widely used. In this article, we show that: To calculate half life from AR(1), the instrument generating function estimator of Phillips *et al.* (2004) is not only an asymptotically normal estimator, but also an easy-to-use alternative to Andrews (1993); to calculate half life from AR(p), we propose a FM-AR model, which is a modified version of Phillips' (1995) FM-VAR.

There are two approaches to study the persistence/convergence property of a univariate time series, for example, real exchange rate: unit root test and half life. The unit root approach to the persistence properties models the series either as trend-stationary, where innovations have no permanent effects, or difference stationary, implying that shocks have permanent effects[1]. However, reliance on unit root tests does not provide a measure of uncertainty of the estimates of finiteness or permanence of innovations because a rejection of the unit root null could still be consistent with a stationary process with highly persistent shocks. In addition, an important pitfall in using the autoregressive (AR, thereafter) model to analyze the persistence of shocks to the data is that standard estimators, such as least squares, are significantly *downwardly biased* in finite samples, especially when the true autoregressive parameter is close to but less than one. Problem of *near-unit root bias* biases empirical results in favor of stationarity.

Let y denote the log of real exchange rate, the estimation of half life begins with the autoregression below

$$y_t = \alpha y_{t-1} + u_t, \tag{1}$$

This model is the same as that used for testing whether there is a unit root in a time series — consequently, this model is often referred to as the Dickey–Fuller regression. The half-life of shocks, which is the time it takes for a unit shock to dissipate by 50%, is calculated from the AR parameter α, the formal definition is:

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[1] For example, Papell (1997).

$$\text{Half Life} = \begin{cases} \dfrac{\ln(0.5)}{\ln(\rho)}, \text{ if } 0 < \rho < 1 \\ \infty, \text{ if } \rho \geq 1 \end{cases}$$

Empirically, to compute half-life, one has to estimate coefficient α of (1), one problem of estimation is *near-unit root bias*, which biases empirical results in favor of finding stationarity. That is, standard estimators, such as least squares, are significantly *downwardly biased* in finite samples, especially when the true autoregressive parameter is close to but less than one. In this case, the process is close to being non-stationary and, as the LS estimator minimizes the regression residual variance, it will tend to make the data-generating process appear to be more stationary than it actually is by forcing the AR parameter away from unity.

As lower values of the AR parameter imply faster speeds of adjustment following a shock, this will also result in a downward bias to LS-based estimates of half-lives of shocks. In addition to the inherent difficulties in distinguishing between the stationary and random walk processes for the real interest rate.

In a nutshell, conventional procedure estimates α to characterize the persistence of time series has two main disadvantages: (i) the least squares estimates of the AR parameter in unit root regressions will be biased toward zero in small samples (Orcutt, 1948); and (ii) they have low power against plausible trend stationary alternatives (DeJong *et al.*, 1992). The downward bias in LS estimates of the AR parameter arises because there is an asymmetry in the distribution of estimators of the AR parameter in AR models (the distribution is skewed to the left, resulting in the median exceeding the mean). As a result, the median is a better measure of central tendency than the mean in least squares estimates of AR models.

The median-unbiased procedure proposed by Andrews (1993) and Andrews and Chen (1994) is usually suggested in literature to estimate (1), which combines unbiasedness with the use of point and interval estimators in achieving a more accurate estimate of the persistence of shocks to economic time series. Andrews' (1993) median unbiased estimator (MU thereafter) uses a bias correction method which delivers an impartiality property to the decision making process because there is an equal chance of under- or over- estimating the AR parameter in the unit root regression. MU is widely used in empirical studies. For example, Murray and Papell (2002), Cashin *et al.* (2004), Sekioua(2008), and Cerrato *et al* (2008). However, as mentioned by Andrews (1993), MU is merely an unbiased model selection procedure without asymptotic theory, where there lacks an explicit optimality property for it; that is, we do not know whether it is a best MU estimator. It is possible that the estimator does not fully exploit all the information in the sufficient statistics for the parameters. More importantly, the construction of confidence intervals for MU is not an easy-to-use procedure.

Recently, Phillips *et al.* (2004) proposed an instrument generating function(IGF thereafter) estimator to estimate (1), which is also an *asymptotically median unbiased* estimator and can be used to produce symmetric confidence intervals, and it has no problem of discontinuity in the confidence intervals in the transition from stationary to nonstationary cases, which also yields a *t*-ratio that has a standard normal distribution when α=1, as well as when |α|<1. This enables us to construct the confidence intervals in a conventional way easily.

In contrast to this stark dichotomy between whether shocks to the series are mean reverting (finite persistence) or not (permanent), this paper characterizes the extent of reversion by

applying the nonlinear instruments generating function (IGF thereafter) method, proposed by Phillips *et al.* (2004), to measure the estimates of the *half-life* of shocks to the series. Like AMU, the IGF estimator of Phillips *et al.* (2004) removes the downward bias of standard LS estimators. Moreover, the IGF based confidence intervals have slightly smaller coverage probabilities than AMU, and the *t*-statistic is distributed as standard normal distribution asymptotically.

Point and interval estimators are useful statistics for providing information to draw conclusions about the duration of shocks, unlike hypothesis testing, they are informative when a hypothesis is not rejected. Because the IGF estimator is shown to be asymptotically standard normal, the construction of confidence intervals is very straightforward.

For AR(p) model, impulse-response approach is used, typical examples are Murray and Papell (2002) and Sekioua (2008). To further the study, based upon the FM asymptotics of Phillips (1995), we propose a FM-AR to directly estimate the coefficients of any AR(p) process.

2. The Econometric methodology

2.1 IGF estimator for DF-AR(1)

Phillips *et al.* (2004) studies the properties of IGF estimator in which the instruments are nonlinear functions of integrated processes. Framework of Phillips *et al.* (2004) extends the analysis of So and Shin (1999)[2], providing a more general analysis of IV estimation in potentially nonstationary autoregressions and showing that the Cauchy estimator has an optimality property in the class of certain IV procedures.

For (1), Phillips *et al.* (2004) consider the IV estimator of α given by

$$\alpha = \frac{\sum_{t=1}^{n} F(y_{t-1}) y_t}{\sum_{t=1}^{n} F(Y_{t-1}) y_{t-1}} \tag{2}$$

Here, α is an IV estimator in which the instrument is generated by the IGF *F*. In its general form, the class of IV estimators that can be represented by (2) includes, of course, the conventional OLS estimator as a special case with the linear IGF $F(x) = x$. However, this paper will concentrate on IV estimators constructed with various nonlinear IGF's.

The bounded optimal IV estimator with asymptotic sign IGF has some nice properties that the conventional OLS estimator does not have. The estimator yields a *t*-ratio that has a standard normal limit distribution when $\alpha=1$, as well as when $|\alpha|<1$. This enables us to construct and interpret the confidence interval for α in a conventional way. On the other hand, of course, the *t*-ratio based on the OLS estimator has a limit normal distribution only when $|\alpha|<1$ and its limit distribution is non-Gaussian when distribution has implications for tests of a unit root. These properties are explored in So and Shin (1999), where the Cauchy estimator was first suggested, Phillips *et al.* (2004) proposed six nonlinear instrument generating functions summarized below.

Because of singularity problem, in this article, we report the IVi3 results for our empirical study, where $F(y_{t-1})=y_{t-1} \mathrm{Exp}(-|y_{t-1}|)$ is used for lagged level y_{t-1}. To control possible cross-

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[2] In recent work, So and Shin (1999) suggested the use of the Cauchy estimator, which uses the sign function as an instrumental variable, in place of the ordinary least squares (OLS) estimator in autoregressions that included both stationary and nonstationary cases.

sectional dependency among currencies, we also follow Chang(2002) to insert a variable c such that $y_{t-1}\mathrm{Exp}(-c|y_{t-1}|)$, and c is defined by

IV Estimators	Instrument generating functions, $F()$
IVh1	$\mathrm{sgn}(y_{t-1})$
IVh2	$y_{t-1}\cdot\mathbf{I}\{|y_{t-1}|\leq K\}+\mathrm{sgn}(y_{t-1})K\cdot\mathbf{I}\{|y_{t-1}|>K\}$
IVh3	$\arctan(y_{t-1})$
IVi1	$\mathrm{sgn}(y_{t-1})\cdot\mathbf{I}\{|y_{t-1}|\leq K\}$
IVi2	$y_{t-1}\cdot\mathbf{I}\{|y_{t-1}|\leq K\}$
IVi3	$y_{t-1}\mathrm{Exp}(-|y_{t-1}|)$

$$c = \frac{K}{s(\Delta y_t)\sqrt{T}}$$

where $s(\Delta y_t)$ denotes the standard deviation of Δy_t, and K is a constant fixed at 3. In addition, the recursive de-meaning procedure[3] is also applied.

Using the 0.05 and 0.95 quantile functions of α estimate, we can construct two-sided 90% confidence intervals for the true α. These confidence intervals can be used either to provide a measure of the accuracy of α or to construct the conventional exact one- or two-sided tests of the null hypothesis that $\alpha = \alpha_0$. In this paper, we use such symmetric confidence intervals only to provide a measure of the accuracy of α estimate.

2.2 IGF estimator for ADF-AR(p)

In addition, the presence of serial correlation (typical in economic time series) means that (1) will often not be appropriate. In such cases, (1) is augmented to be an AR(p) model by adding lagged first-order difference. Hence, the starting point of this analysis is the following ADF regression:

$$y_t = \alpha y_{t-1} + \sum_{k=1}^{p}\delta_k \Delta y_{t-k} + \varepsilon_t \tag{3}$$

Similarly, (3) is estimated by IGF. For augmented differenced lagged variables, instruments are themselves without IGF transformation. Subsequently, we then define the matrices below

$$\mathbf{y} = \begin{pmatrix} y_{p_1} \\ \vdots \\ y_T \end{pmatrix}, \mathbf{y}_\ell = \begin{pmatrix} y_p \\ \vdots \\ y_{T-1} \end{pmatrix}, \mathbf{X} = \begin{pmatrix} x'_{p_1} \\ \vdots \\ x'_T \end{pmatrix}, \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{p_1} \\ \vdots \\ \varepsilon_T \end{pmatrix}$$

where $x'_t = \left(\Delta y_{t-1}, \ldots\ldots \Delta y_{t-p}\right)$ collects the lagged difference terms. Then the augmented AR regression (3) can be written in matrix form as

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[3] See eq.(25) in Phillips *et al.* (2004, p.231).

$$y = y_\ell \alpha + X\beta + \varepsilon = Y\gamma + \varepsilon \tag{4}$$

where $\beta = (\alpha_1, \ldots \ldots \alpha_p)'$, $Y = (y_\ell, X)$, and $\gamma = (\alpha, \beta')'$. For equation (4), IV estimator is constructed below

$$\hat{\gamma} = \begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = \begin{bmatrix} F(y_\ell)'y_\ell & F(y_\ell)'X \\ X'y_\ell & X'X \end{bmatrix}^{-1} \begin{bmatrix} F(y_\ell)'y \\ X'y \end{bmatrix}$$

where $F(y_\ell) = \big(F(y_p), \cdots F(y_T) \big)'$.

Under the null, we have $\alpha - 1 = B_T^{-1} A_T$, where

$$\mathbf{A_T} = F(y_\ell)'\varepsilon - F(y_\ell)'X(X'X)^{-1}X'\varepsilon$$

$$= \sum_{t=1}^{T} F(y_{t-1})\varepsilon_t - \sum_{t=1}^{T} F(y_{t-1})x_t' \left(\sum_{t=1}^{T} x_t x_t' \right)^{-1} \sum_{t=1}^{T} x_t \varepsilon_t$$

$$\mathbf{B_T} = F(y_\ell)'y_\ell - F(y_\ell)'X(X'X)^{-1}X'y_\ell$$

$$= \sum_{t=1}^{T} F(y_{t-1})y_{t-1} - \sum_{t=1}^{T} F(y_{t-1})x_t' \left(\sum_{t=1}^{T} x_t x_t' \right)^{-1} \sum_{t=1}^{T} x_t y_{t-1}$$

and the variance of A_T is given by $\sigma^2 EC_T$, where

$$\mathbf{C_T} = F(y_\ell)'F(y_\ell) - F(y_\ell)'X(X'X)^{-1}X'F(y_\ell)$$

$$= \sum_{t=1}^{T} F(y_{t-1})^2 - \sum_{t=1}^{T} F(y_{t-1})x_t' \left(\sum_{t=1}^{T} x_t x_t' \right)^{-1} \sum_{t=1}^{T} x_t F(y_{t-1})$$

For differenced lagged variables, themselves are used as the instruments without IGF transformation, details are explained in Chang(2002). The half-life calculated from the value of (1) assumes that shocks to the data decay monotonically, which is inappropriate for ADF regressions represented by (3), since in general shocks to an AR(p) will not decay at a constant rate. Murray and Papell (2002) calculate the half-life from the impulse response function of an AR(p) below,

$$y_t = c + \sum_{k=1}^{p} \phi_k y_{t-k} + \varepsilon_t \tag{4}$$

From the IGF estimates of (3), coefficients of (4) can be recalculated as follows:

$$\phi_1 = \alpha + \delta_1, \phi_2 = -(\delta_1 - \delta_2), \phi_3 = -(\delta_2 - \delta_3) \ldots, \text{ and } \phi_p = -\delta_{p-1}. \tag{5}$$

When the coefficients are obtained, the k-period impulse response function (*irf* thereafter) for univariate AR(p) regression can be calculated. For convenience, we name (4) by ADF-AR(p).

(5) is widely used, for example, Murray and Papell (2002), however, not only is it subject to strict restriction, but also bias if some of right-hand-side variables of (4) are co-integrated. Therefore, instead of using (3) and (5) to indirectly calculate the coefficients of (4), we propose a Fully-Modified AR (p) to estimate (4) directly, the model is derived from Fully-Modified VAR of Phillips (1995). Section below continues the study.

2.3 Unrestricted fully-modified AR(p)

Phillips' (1995) FM-VAR is a level system regression with/without error correction terms (differenced terms). FM-VAR of Phillips (1995) generalized the asymptotics of Phillips and Hansen (1990), allowing full rank I(1) regressors and possible cointegration existed among lagged dependent variables. Mostly important, the asymptotics of FM-VAR is normal, or mixed normal, which allows us to construct confidence intervals and half life. The methodology for FM-AR(p) is illustrated below.

$$y_t = a + \alpha_1 y_{t-1} + \cdots \alpha_p y_{t-p} + u_{0t} = a + Ay_{t-} + u_{0t} \qquad (6)$$

where A is an $1 \times p$ coefficient vector and y_{t-} is a $p(p=p_1+p_2)$-dimensional vector of lagged y_t which are partitioned below:

$$H_1' y_{t-} = y_{1,t-} = u_{1t} \qquad\qquad\qquad p_1 \times 1$$

$$H_2' \Delta y_{t-} = \Delta y_{2,t-} = u_{2t} \qquad\qquad\qquad p_2 \times 1$$

Here $H=[H_1, H_2]$ is $p \times p$ orthogonal matrix and rotates the regressor space in (6) so that the model has the alternative form

$$y_t = A_1 y_{1,t-} + A_2 y_{2,t-} + u_{0t} \qquad (7)$$

Here $A_1 = AH_1$ and $A_2 = AH_2$. The form of (7) usefully separates out the I(0) and I(1) components of the regressors in (6). However, the direction (H_1) in which the regressors are stationary will not be generally known in advance, not even will the rank of the cointegrating space of the regressors. Phillips' (1995) fully-modified correction has two steps:

The first step is to correct for the serial correlation to the LS estimator $\hat{A} = \left(y_{t-}' y_{t-} \right)^{-1} y_{t-}' y_t$ of (7). The endogeneity correction is achieved by modifying the dependent variable y_t in (7) with the transformation

$$y_t^+ = y_t - \hat{\Omega}_{0y_{t-}} \hat{\Omega}_{y_{t-}y_{t-}}^{-1} \Delta y_{t-} \qquad (8)$$

In (8), $\hat{\Omega}$ denotes the kernel estimate of the long-run (*lr*) covariances of the variables denoted by the subscript:

$$\hat{\Omega}_{0y_{t-}} = lr \operatorname{cov}(u_{0t}, \Delta y_{t-}) \text{ and } \hat{\Omega}_{y_{t-}y_{t-}} = lr \operatorname{cov}(\Delta y_{t-}, \Delta y_{t-}) . \qquad (8)$$

leads us to estimate the equation below

$$y_t - \hat{\Omega}_{0y_-} \hat{\Omega}_{y_- y_-}^{-1} \Delta y_{t-} = A y_{t-} + u_{0t} - \Omega_{02} \Omega_{22}^{-1} u_{2t}$$

Phillips (1995, Pp. 1033-1034) shown that this transformation reduces to the ideal correction *asymptotically*, at least as far as the nonstationary components $y_{2,t-}$ are concerned. The stationary components $y_{1,t-}$ are present in differenced or I(-1) form in this transformation and have no effects asymptotically. Therefore, we can achieve an endogeneity correction with knowing the actual directions in which it is required or even the number of nonstationary regressors that need to be dealt with.

Secondly, the serial correction term has the form

$$\hat{\Delta}_{0y_-}^+ = \hat{\Delta}_{0y_-} - \hat{\Omega}_{0y_-} \hat{\Omega}_{y_- y_-}^{-1} \hat{\Delta}_{y_- y_-}$$

where $\hat{\Delta}_{0y_-}$ and $\hat{\Delta}_{y_- y_-}$ denote the kernel estimates of the one-sided long-run covariances

$\hat{\Delta}_{0y_-} = lr\,\mathrm{cov}_+(u_{0t}, \Delta y_{t-})$ and $\hat{\Delta}_{y_- y_-} = lr\,\mathrm{cov}_+(\Delta y_{t-}, \Delta y_{t-})$.

Combining the endoneneity and serial corrections we have the FM formula for the parameter estimates

$$\hat{A}^+ = \left(y_{t-}' y_{t-} \right)^{-1} \left(y_{t-}' y_t^+ - T \hat{\Delta}_{0y_-}^+ \right) \tag{9}$$

For (9), the limit theory of FM estimates of the stationary components of the regressors is equivalent to that of LS, while the FM estimates of the nonstationary components retain their optimality properties as derived in Phillips and Hansen (1990); that is, they are asymptotically equivalent to the MLE estimates of the co-integrating matrix.

For the finite sample property of FM-AR, Appendix offers a simulation study illustrating a good performance of this estimator in calculating half-life of near I(1) process.

3. Empirical results

To illustrate these approaches, BIS real effective exchange rates (REER thereafter) of four economies (Germany, Japan, UK, and USA) are used, samples range from 1994M1 to 2010M12. BIS' REER is CPI-based and is a broad index of monthly averages, with 2005=100. Figure 1 plots the time series plot of them, with a horizontal line of the base year index. The Y-axis also illustrates the histogram to depict the distribution property of these data. The common feature of them is an apparent behavior of stochastic trend.

In Table 1, the upper panel DF-AR(1) presents the results of (1); the lower panel ADF-AR(p) summarizes those of (5), and the BIC criterion returns unanimously 2 lags. Comparing both results, taking Germany as an example, expressed in years, its half life takes 12.55 for DF-AR(1) and substantially drops to 0.916 for ADF-AR(2); similar finding is also found in Japan. UK and USA does not have finite bounds. Apparently, the dynamic lag structure is substantial.

Table 2 of FM-AR(2) tabulates more results. Unrestricted FM-AR(2) yields finite bounds of half-lives for all four economies, most of them are roughly less than two years. To be more informative, Figure 2 graphs the impulse response plot of Germany, expressed in month; clearly, the impulse response function of Germany has a rather wide confidence interval.

	GERMANY	JAPAN	UK	USA
Mean	101.40	108.02	95.97	99.52
Median	100.91	106.55	99.93	98.83
Maximum	116.79	151.11	107.81	116.00
Minimum	90.91	79.68	75.88	86.53
Std. Dev.	5.50	14.98	8.91	7.93
Skewness	0.63	0.42	-0.68	0.31
Kurtosis	3.09	2.99	2.04	2.10
JB(Prob.)	13.71(0.001)	6.073(0.048)	23.72(0.000)	10.18(0.006)

For each currency, there are 204 observations. JB(Prob.) is the Jarque-Bera statistic for normality with probability value in the parenthesis.

Table 1. Summary statistic

DF-AR(1)	α	Std	mean HL	HL, 90% CI	
				H_L	H_U
Germany	0.995	0.028	12.55	0.95	∞
Japan	0.984	0.028	3.58	0.78	∞
UK	1.025	0.014	∞	15.59	∞
USA	1.014	0.014	∞	4.29	∞
ADF-AR(2)	$\alpha+\delta_1$	$\delta_2-\delta_1$	mean HL	HL, 90% CI	
				H_L	H_U
Germany	1.180	-0.253	0.916	0.541	∞
Japan	1.272	-0.330	1.132	0.624	∞
UK	1.162	-0.159	∞	∞	∞
USA	1.332	-0.340	∞	1.911	∞

Note. HL =half-lives. BIC suggests two lags for four economies.

Table 2. IGF estimation results, in years, 1994M1-2010M12.

	α_1	α_2	mean HL	HL, 90% CI	
				H_L	H_U
Germany	1.121	-0.156	1.789	0.376	∞
Japan	1.292	-0.335	1.525	0.356	∞
UK	1.158	-0.191	1.926	0.258	∞
USA	1.270	-0.298	2.253	0.406	∞

Table 3. FM-AR(2) estimation results, in years, 1994M1-2010M12.
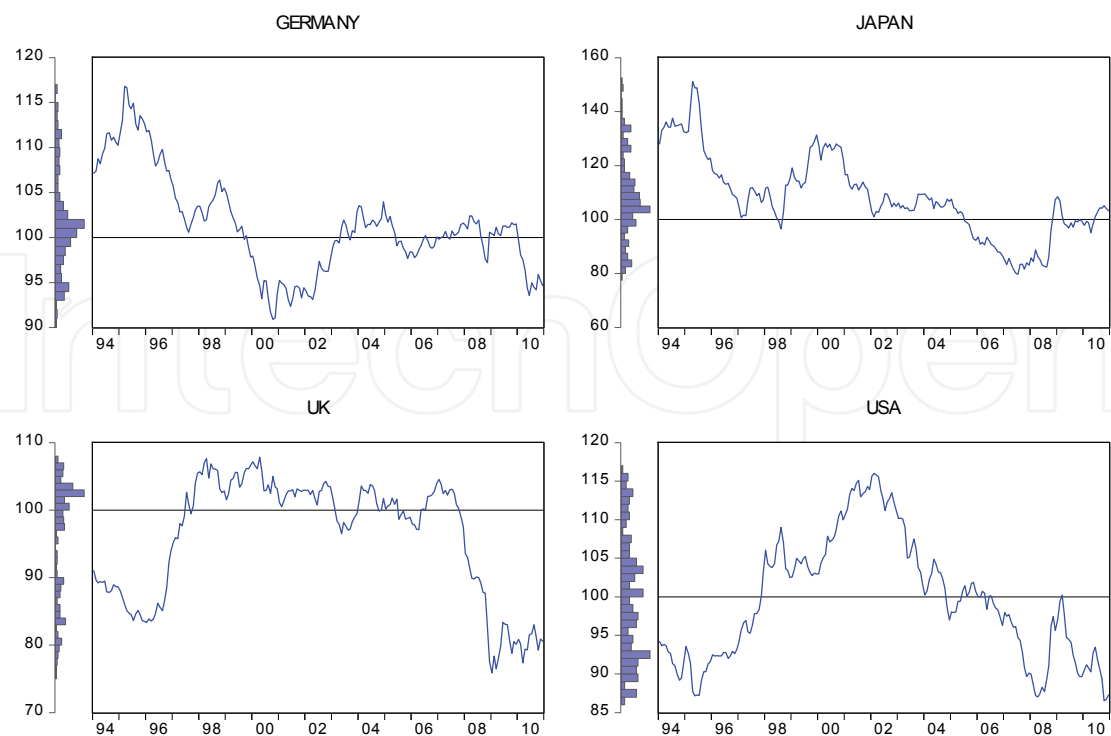
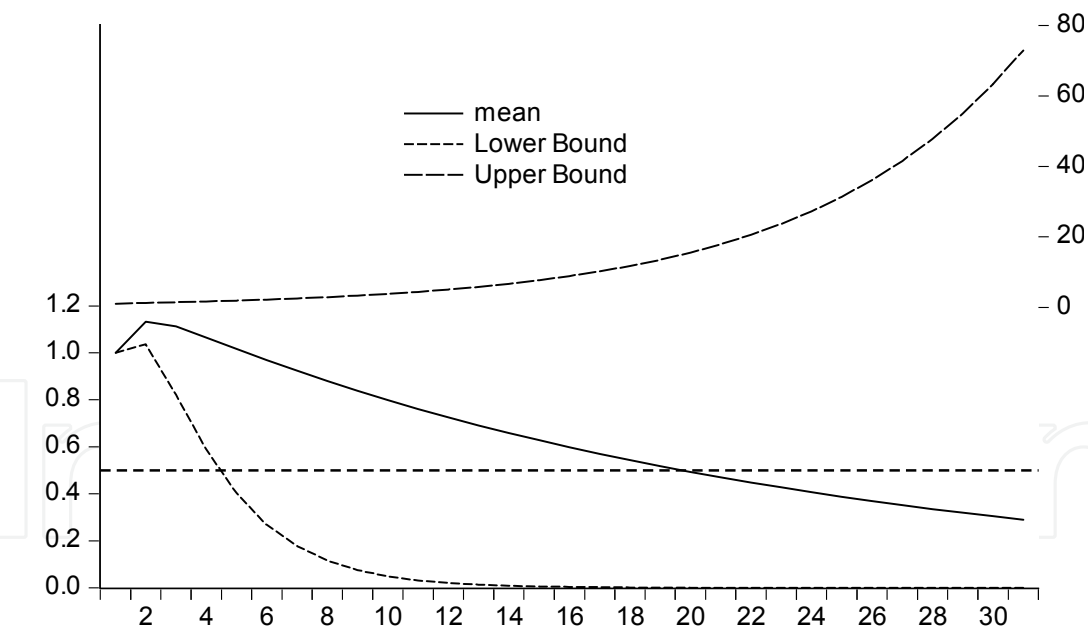Fig. 1. Time series plot of four currencies' REER

Fig. 2. Impulse-response plot of Germany, FM-AR(2), in month.

4. Conclusion

The empirical study of time series persistence uses two main approaches: unit root tests and half-life. However, reliance on unit root tests does NOT provide a measure of uncertainty of the estimates of finiteness or permanence of innovations because a rejection of the unit root null could still be consistent with a stationary process with highly persistent shocks. Because

of near-unit root bias and resulting the lack of distribution, the empirical studies generally apply Andrews' (1993) median unbiasedness method to estimate the AR(1) coefficient to investigate the persistence behavior.

For AR(1) case, this paper contributes to the literature by applying the IGF approach of Phillips *et al*. (2004) to estimate the coefficients of near unit root process, IGF estimator is proved to be normal asymptotically, hence it is very easy to construct confidence intervals. For AR(p) case, moreover, instead of recalculation, we propose a unrestricted FM-AR(p) model, a slight extension of Phillips' (1995) FM-VAR, to estimate coefficients directly.

Our empirical illustration of real effective exchange rate indicates that FM-AR(p) is a useful and easy-to-use method to examine the econometric persistence.

5. Appendix: The finite sample properties of FM-AR

This paper shows that a FM estimator can ameliorate the small sample biases that arise from near unit root bias. Our attention here is focused on the class of dynamic AR(p). A Monte Carlo simulation is used here to investigate the performance, our results indicate that FM estimator successfully reduces the small-sample bias.

Assuming $\{y_t\}_{t=0}^{\infty}$ is governed by a AR(3) time series process generated from $(\mu, \sigma^2) = (0, 1)$, which satisfies the data-generating process specified below:

$$y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \varepsilon_t \tag{1}$$

where $\varepsilon_t \sim i.i.d..$ α and β's represent the associated parameters, and σ is the standard deviation. In our simulation, we generate an AR(3) process. The summation of the three coefficients measures the degree of persistence, the vector is parameterized below

Degree of Persistence 1: $\{\beta_1, \beta_2, \beta_3\} = \{0.90, 0.085, 0.015\}$,

Degree of Persistence 2: $\{\beta_1, \beta_2, \beta_3\} = \{0.90, 0.050, 0.015\}$

Degree of Persistence 3: $\{\beta_1, \beta_2, \beta_3\} = \{0.85, 0.050, 0.015\}$

and

$T \in \{200, 400, 800, 1500, 3000\}$

where T represents the vector of sampling size that are used in practice. The DGPs are characterized by a modest change in the innovation variance but allow for drastic changes in others.

Table A1 reports the characteristics of the finite-sample distribution of both estimators of the elements of estimates. These include the deviation of the estimate from the true parameter value, or bias, as well as measures of skewness and kurtosis. I compare the bias and normality to illustrate the problem. There are several main results.

Firstly, the biases are decreasing function of sample sizes. Even in small samples around 200 and 400, the biases are in the range of 10^{-2}. The bias for FM-AR is quite small.

Secondly, the variance bias exhibits the similar conclusion. AR(3) is generated from (0,1), the empirical bias is in the range of 10^{-3}, and is a decreasing function of sample size.

Finally, the normality property of distribution is drawn from skewness and kurtosis. Unfortunately, no regular pattern is found among three parameter estimates and is related to the persistence of parameter vector designed; in general, skewness is close to zero which gives normality an acceptable condition, although the excess kurtosis (>3) is found.

As a result, FM-AR is a feasible estimator to directly estimate AR(p), whose empirical applications also calls for further studies in the future.

{DGP coefficients}	Estimates biases			Skewness			Kurtosis			Variance
T	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3	biases $\sigma^2=1$
{0.9, 0.085, 0.015}										
200	-0.027	-0.036	0.007	0.194	-0.099	-0.301	5.48	2.88	0.451	-0.029
400	-0.023	-0.030	0.011	0.186	-0.174	-0.320	5.27	3.25	4.502	-0.024
800	-0.020	-0.027	0.015	0.122	-0.25	-0.314	5.45	3.88	4.923	-0.018
1500	-0.019	-0.024	0.018	0.117	-0.337	-0.365	5.79	4.56	5.328	-0.014
3000	-0.019	-0.023	0.021	0.108	-0.399	-0.445	6.27	5.36	5.847	-0.011
{0.9, 0.05, 0.015}										
200	-0.020	-0.024	-0.016	0.060	-0.19	-0.058	5.10	3.12	4.25	-0.029
400	-0.020	-0.017	-0.014	0.030	-0.27	-0.027	5.50	3.45	4.73	-0.019
800	-0.017	-0.015	-0.014	-0.009	-0.336	-0.0005	5.63	4.06	5.01	-0.012
1500	-0.016	-0.014	-0.013	-0.038	-0.376	-0.025	5.94	4.80	5.40	-0.0073
3000	-0.015	-0.013	-0.012	-0.049	-0.432	-0.057	6.49	5.67	5.92	-0.0042
{0.85, 0.05, 0.015}										
200	-0.020	-0.0067	-0.040	0.340	-0.17	-0.09	5.6	3.04	4.74	-0.02
400	-0.016	-0.0079	-0.038	0.190	-0.15	-0.124	5.4	3.29	4.72	-0.015
800	-0.013	-0.0076	-0.038	0.110	-0.16	-0.12	5.4	3.76	5.12	-0.009
1500	-0.011	-0.0060	-0.038	0.077	-0.198	-0.108	5.6	4.36	5.54	-0.0045
3000	-0.010	-0.0058	-0.038	0.056	-0.233	-0.111	6.1	5.10	6.08	-0.0015

Table A1. Biases and skewness of the empirical distribution

6. References

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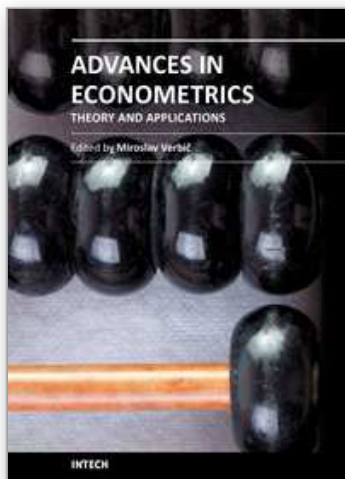
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Edited by Dr. Miroslav Verbic

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