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Whys and Wherefores of Transmissibility

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1. Introduction

The present chapter draws a general overview on the concept of transmissibility and on its potentialities, virtues, limitations and possible applications. The notion of transmissibility has, for a long time, been limited to the single degree-of-freedom (SDOF) system; it is only in the last ten years that the concept has evolved in a consistent manner to a generalized definition applicable to a multiple degree-of-freedom (MDOF) system. Such a generalization can be and has been not only developed in terms of a relation between two sets of harmonic responses for a given loading, but also between applied harmonic forces and corresponding reactions. Extensions to comply with random motions and random forces have also been achieved. From the establishment of the various formulations it was possible to deduce and understand several important properties, which allow for diverse applications that have been envisaged, such as evaluation of unmeasured frequency response functions (FRFs), estimation of reaction forces and detection of damage in a structure. All these aspects are reviewed and described in a logical sequence along this chapter.

The notion of transmissibility is presented in every classic textbook on vibrations, associated to the single degree-of-freedom system, when its basis is moving harmonically; it is defined as the ratio between the modulus of the response amplitude and the modulus of the imposed amplitude of motion. Its study enhances some interesting aspects, namely the fact that beyond a certain imposed frequency there is an attenuation in the response amplitude, compared to the input one, i.e., one enters into an isolated region of the spectrum. This enables the design of modifications on the dynamic properties so that the system becomes "more isolated" than before, as its transmissibility has decreased.

Usually, the transmissibility of forces, defined as the ratio between the modulus of the transmitted force magnitude to the ground and the modulus of the imposed force magnitude, is also deduced and the conclusion is that the mathematical formula of the transmissibility of forces is exactly the same as for the transmissibility of displacements. As it will be explained, this is not the case for multiple degree of freedom systems.

The question that arises is how to extend the idea of transmissibility to a system with N degrees-of-freedom, i.e., how to relate a set of unknown responses to another set of known responses, for a given set of applied forces, or how to evaluate a set of reaction forces from a set of applied ones. Some initial attempts were given by Vakakis et al. (Paipetis & Vakakis, 1985; Vakakis, 1985; Vakakis & Paipetis, 1985; 1986), although that generalization was still

limited to a very particular type of N degree-of-freedom system, one where a set constituted by a mass, stiffness and damper is repeated several times in the vertical direction. The works of (Liu & Ewins, 1998), (Liu, 2000) and (Varoto & McConnell, 1998) also extend the initial concept to N degrees-of-freedom systems, but again in a limited way, the former using a definition that makes the calculations dependent on the path taken between the considered co-ordinates involved, the latter by making the set of co-ordinates where the displacements are known coincident to the set of applied forces.

An application where the transmissibility seems of great interest is when in field service one cannot measure the response at some co-ordinates of the structure. If the transmissibility could be evaluated in the laboratory or theoretically (numerically) beforehand, then by measuring in service some responses one would be able to estimate the responses at the inaccessible co-ordinates.

To the best knowledge of the authors, the first time that a general answer to the problem has been given was in 1998, by (Ribeiro, 1998). Surprisingly enough, as the solution is very simple indeed. In what follows, a chronological description of the evolution of the studies on this subject is presented.

2. Transmissibility of motion

In this section and next sub-sections the main definitions, properties and applications will be presented.

2.1 Fundamental formulation

The fundamental deduction (Ribeiro, 1998), based on harmonically applied forces (easy to generalize to periodic ones), begins with the relationships between responses and forces in terms of receptance: if one has a vector F_A of magnitudes of the applied forces at co-ordinates A, a vector X_U of unknown response amplitudes at co-ordinates U and a vector X_K of known response amplitudes at co-ordinates K, as shown in Fig. 1.
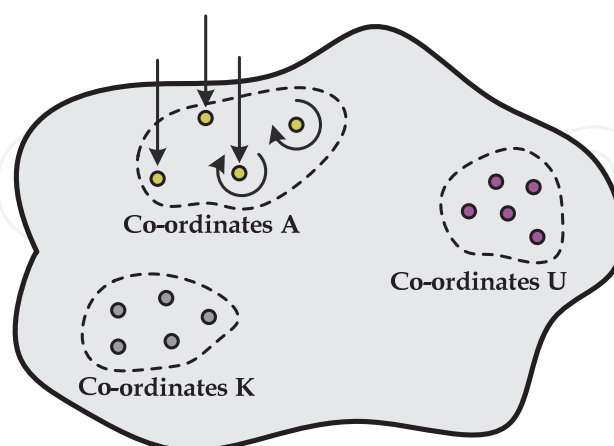

Fig. 1. System with co-ordinates *A, U, K*

One may establish the following relationships:

$$X_U = H_{UA} \, F_A \tag{1}$$

$$X_K = H_{KA} F_A \tag{2}$$

where H_{UA} and H_{KA} are the receptance frequency response matrices relating co-ordinates U and A, and K and A, respectively. Eliminating F_A between (1) and (2), it follows that

$$X_U = H_{UA} H_{KA}^+ X_K \tag{3}$$

or

$$X_U = T_{UK}^{(A)} X_K \tag{4}$$

where H_{KA}^+ is the pseudo-inverse of H_{KA}. Thus, the transmissibility matrix is defined as:

$$T_{UK}^{(A)} = H_{UA} H_{KA}^+ \tag{5}$$

Note that the set of co-ordinates where the forces are (or may be) applied (*A*) need not coincide with the set of known responses (*K*). The only restriction is that – for the pseudo-inverse to exist – the number of *K* co-ordinates must be greater or equal than the number of *A* co-ordinates.

An important property of the transmissibility matrix is that it does not depend on the magnitude of the forces, one simply has to know or to choose the co-ordinates where the forces are going to be applied (or not, as one can even choose more co-ordinates *A* if one is not sure whether or not there will be some forces there and, later on, one states that those forces are zero) and measure the necessary frequency-response-functions.

2.2 Alternative formulation

An alternative approach, developed by (Ribeiro et al., 2005) evaluates the transmissibility matrix from the dynamic stiffness matrices, where the spatial properties (mass, stiffness, etc.) are explicitly included.

The dynamic behaviour of an *MDOF* system can be described in the frequency domain by the following equation (assuming harmonic loading):

$$Z X = F \tag{6}$$

where Z represents the dynamic stiffness matrix, X is the vector of the amplitudes of the dynamic responses and F represents the vector of the amplitudes of the dynamic loads applied to the system.

From the set of dynamic responses, as defined before, it is possible to distinguish between two subsets of co-ordinates *K* and *U*; from the set of dynamic loads it is also possible to distinguish between two subsets, *A* and *B*, where *A* is the subset where dynamic loads may be applied and *B* is the set formed of the remaining co-ordinates, where dynamic loads are never applied. One can write X and F as:

$$X = \begin{Bmatrix} X_K \\ X_U \end{Bmatrix}, \quad F = \begin{Bmatrix} F_A \\ F_B \end{Bmatrix} \tag{7}$$

With these subsets, Eq. (6) can be partitioned accordingly:

$$\begin{bmatrix} \boldsymbol{Z}_{AK} & \boldsymbol{Z}_{AU} \\ \boldsymbol{Z}_{BK} & \boldsymbol{Z}_{BU} \end{bmatrix} \begin{Bmatrix} \boldsymbol{X}_K \\ \boldsymbol{X}_U \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_A \\ \boldsymbol{F}_B \end{Bmatrix} \tag{8}$$

Taking into account that co-ordinates B represent the ones where the dynamic loads are never applied, and considering that the number of these co-ordinates is greater or equal to the number of co-ordinates U, from Eq. (8) it is possible to obtain the unknown response vector:

$$\begin{matrix} \boldsymbol{F}_B = \boldsymbol{0}, & \#B \ge \#U \\ \Downarrow \\ \boldsymbol{X}_U = -\boldsymbol{Z}_{BU}^+ \, \boldsymbol{Z}_{BK} \, \boldsymbol{X}_K \end{matrix} \tag{9}$$

where $\boldsymbol{Z}_{BU}^+$ is the pseudo-inverse of $\boldsymbol{Z}_{BU}$. Therefore, this means that the transmissibility matrix can also be defined as

$$\boldsymbol{T}_{UK}^{(A)} = -\boldsymbol{Z}_{BU}^+ \, \boldsymbol{Z}_{BK} \tag{10}$$

Eq. (10) is an alternative definition of transmissibility, based on the dynamic stiffness matrices of the structure. Therefore,

$$\boldsymbol{T}_{UK}^{(A)} = \boldsymbol{H}_{UA} \, \boldsymbol{H}_{KA}^+ = -\boldsymbol{Z}_{BU}^+ \, \boldsymbol{Z}_{BK} \tag{11}$$

Taking into account that the dynamic stiffness matrix for an undamped system is described in terms of the stiffness and mass matrices, $\boldsymbol{Z} = \boldsymbol{K} - \omega^2 \boldsymbol{M}$, one can now relate the transmissibility functions to the spatial properties of the system. To make this possible, one must bear in mind that it is mandatory that both conditions regarding the number of co-ordinates be valid, i. e.,

$$\#B \ge \#U \quad \text{and} \quad \#K \ge \#A \tag{12}$$

2.3 Numerical example

An MDOF mass-spring system, presented in Fig. 2, will be used to illustrate the principal differences observed between the transmissibilities and FRFs curves. This is a six mass-spring system (designated as original system), possessing the characteristics described in Table 1.

$$\boldsymbol{X}_K^T = \left\{ X_2 \quad X_4 \quad X_6 \right\}^T \text{ and } \boldsymbol{X}_U^T = \left\{ X_1 \quad X_3 \quad X_5 \right\}^T \tag{13}$$

The number of loads can be grouped in the sub-set $\boldsymbol{F}_A$ (even if some of them are, in certain cases, null) and in subset $\boldsymbol{F}_B$.

$$\boldsymbol{F}_A^T = \left\{ F_4 \quad F_5 \quad F_6 \right\}^T \text{ and } \boldsymbol{F}_B^T = \left\{ F_1 \quad F_2 \quad F_3 \right\}^T \tag{14}$$

The subsets of known and unknown responses are assumed as:
According to Eq. (11), and considering the above-defined subsets, the transmissibility matrix is given by:

		Original System
kg	m_1	7
	m_2	7
	m_3	4
	m_4	3
	m_5	6
	m_6	8
N/m	k_1	10^5
	k_2	10^5
	k_3	4.0×10^5
	k_4	5.0×10^5
	k_5	7.0×10^5
	k_6	2.0×10^5
	k_7	8.0×10^5
	k_8	3.0×10^5
	k_9	6.0×10^5
	k_{10}	3.0×10^5
	k_{11}	5.0×10^5

Table 1. Characteristics of the original system
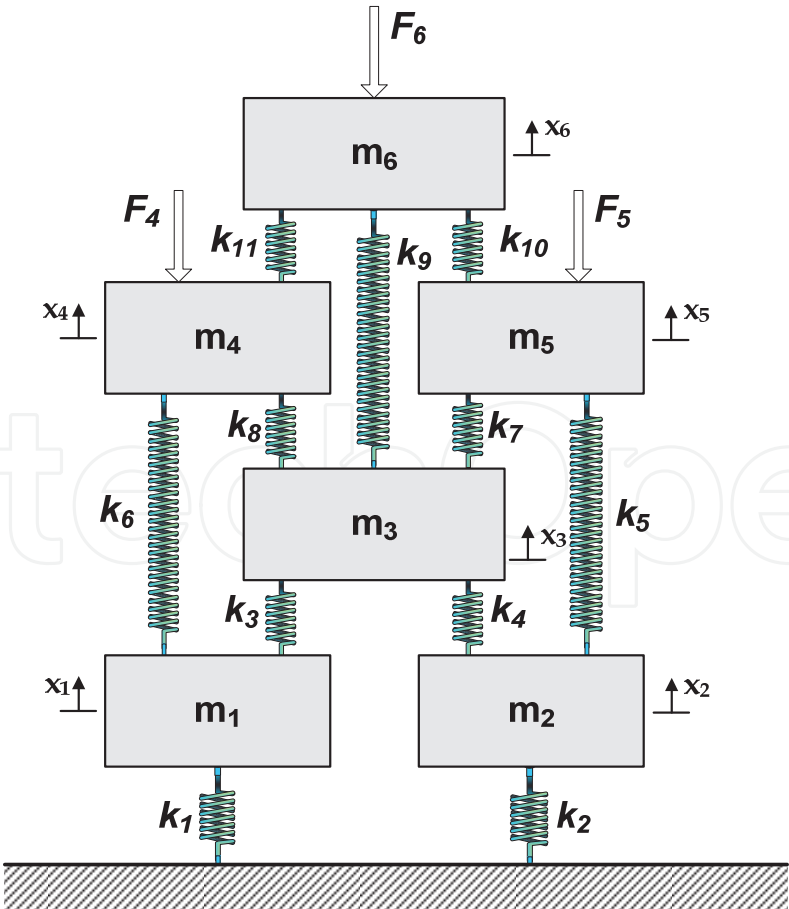
Fig. 2. Mass-spring MDOF system

$$\begin{bmatrix} T_{12}^{(A)} & T_{14}^{(A)} & T_{16}^{(A)} \\ T_{32}^{(A)} & T_{34}^{(A)} & T_{36}^{(A)} \\ T_{52}^{(A)} & T_{54}^{(A)} & T_{56}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{14} & H_{15} & H_{16} \\ H_{34} & H_{35} & H_{36} \\ H_{54} & H_{55} & H_{56} \end{bmatrix} \begin{bmatrix} H_{24} & H_{25} & H_{26} \\ H_{44} & H_{45} & H_{46} \\ H_{64} & H_{65} & H_{66} \end{bmatrix}^+$$

$$= -\begin{bmatrix} Z_{11} & Z_{13} & Z_{15} \\ Z_{21} & Z_{23} & Z_{25} \\ Z_{31} & Z_{33} & Z_{35} \end{bmatrix}^+ \begin{bmatrix} Z_{12} & Z_{14} & Z_{16} \\ Z_{22} & Z_{24} & Z_{26} \\ Z_{32} & Z_{34} & Z_{36} \end{bmatrix}$$

(15)

The characteristics of the system of Fig. 2 are presented in Table 1.

It may be noted from Fig. 3 that the maxima of the transmissibility curves occur all at the same frequencies. It can also be observed that the maxima and minima of the transmissibility curves do not coincide with the maxima and minima of the FRF curves. No simple relationships (if any) can be established between the picks and anti-picks of the transmissibilities and FRFs. Transmissibilities have a local nature and therefore they do not reflect the existence of the global properties of the system (natural frequencies and damping ratios).
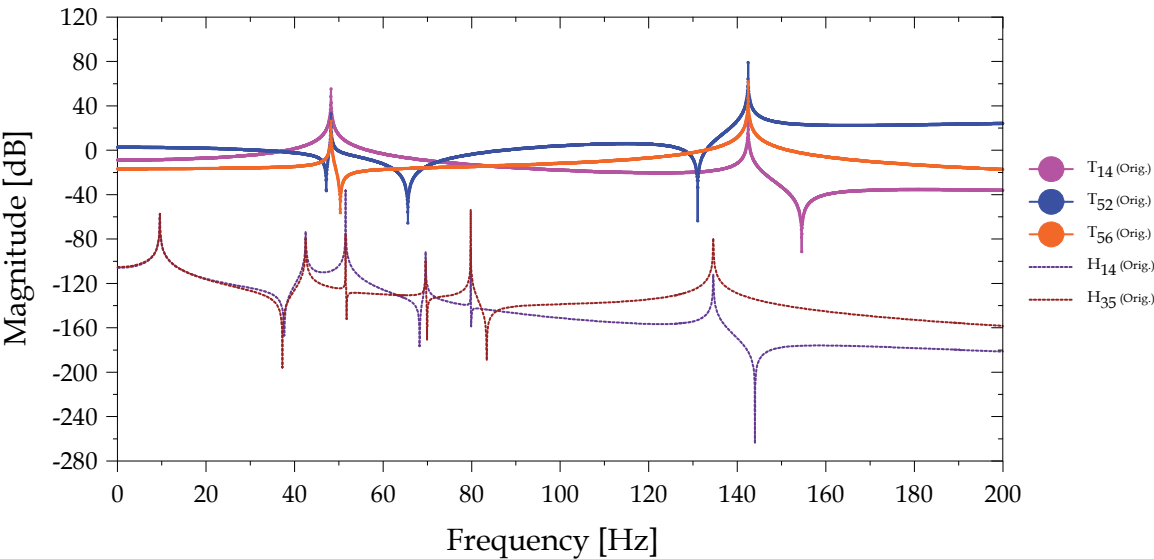

Fig. 3. Some transmissibilities and FRFs curves of the original system

2.4 Transmissibility properties

The formulation presented in section 2.1 allows us to extract some important properties for the transmissibility matrix $T_{UK}^{(A)}$. From Eq. (3) and (5) it is possible to conclude that the *transmissibility matrix is independent from the force vector* F_A (Note that F_A is eliminated between eqs. (1) and (2)). This means that any change verified in one of the force values, acting along with co-ordinates of set A, will not affect $T_{UK}^{(A)}$. This change can be due, for instance, to the alteration of mass values associated to co-ordinates A or stiffness values of springs interconnecting those co-ordinates.

Additionally, to highlight that characteristic of matrix $T_{UK}^{(A)}$, it can be verified from Eq. (10) that there is no part of matrix $\mathbf{Z}$ involving co-ordinates of set A (neither $\mathbf{Z}_{AK}$ nor $\mathbf{Z}_{AU}$). This statement reinforces the previous conclusion extracted from Eq. (5) and will lead to the formulation of two properties, as follows:

Property 1. *The transmissibility matrix does not change if some modification is made on the mass values of the system where the loads can be applied – subset A.*

Property 2. *The transmissibility matrix does not change if some modification is made on the stiffness values of springs interconnecting co-ordinates of subset A – (where the loads can be applied).*

In fact, any changes in the mass values associated to co-ordinates A and/or any changes in the stiffness values of springs interconnecting co-ordinates A, will affect the inertia forces and elastic forces, respectively, acting along those co-ordinates and thus belonging to $\boldsymbol{F}_A$.

The same MDOF system presented in Fig. 2 will be used to illustrate the transmissibility properties. In Table 2 four different modifications are made in the original system.

Situations I and II correspond to modifications on the original masses; situations III and IV correspond to modifications on stiffness. Situations I and III only involve co-ordinates A, whereas situations II and IV involve co-ordinates pertaining to both sets A and B.

Choosing, for instance, the transmissibility function $T_{52}^{(A)}$, one obtains the results presented in Figs. 4 and 5, where one can see that $T_{52}^{(A)}$ and $T_{14}^{(A)}$ remain the same only when changes are made at co-ordinates A, where the forces are applied.

2.5 Evaluation of the transmissibility from measurement responses

In 1999, (Ribeiro et al., 1999) and (Maia et al., 1999) showed how the transmissibility matrix could be evaluated directly from the measurement of the responses, rather than measuring the frequency response functions. In Eq. (4), the problem is to evaluate the $U \times K$ values of $\boldsymbol{T}_{UK}^{(A)}$ knowing $\boldsymbol{X}_U$ and $\boldsymbol{X}_K$. This can be achieved by applying various sets of forces, at a time, on co-ordinates A. Let $\boldsymbol{F}_A^{(1)}$ be the first set of applied forces (amplitudes). Then,

$$\boldsymbol{X}_U^{(1)} = \boldsymbol{T}_{UK}^{(A)} \, \boldsymbol{X}_K^{(1)} \tag{16}$$

		Original System	Situation I	Situation II	Situation III	Situation IV
kg	m_1	7	--	14	14	--
	m_2	7	--	--	--	--
	m_3	4	--	--	--	--
	m_4	3	13	--	--	--
	m_5	6	12	--	--	--
	m_6	8	--	13	13	--
			--	--	--	--
N/m	k_1	10^5	--	--	--	--
	k_2	10^5	--	--	--	--
	k_3	4.0×10^5	--	--	--	--
	k_4	5.0×10^5	--	--	--	10.0×10^5
	k_5	7.0×10^5	--	--	--	--
	k_6	2.0×10^5	--	--	--	--
	k_7	8.0×10^5	--	--	--	--
	k_8	3.0×10^5	--	--	--	--
	k_9	6.0×10^5	--	--	--	--
	k_{10}	3.0×10^5	--	--	9.0×10^5	--
	k_{11}	5.0×10^5	--	--	9.0×10^6	--

-- unchanged value

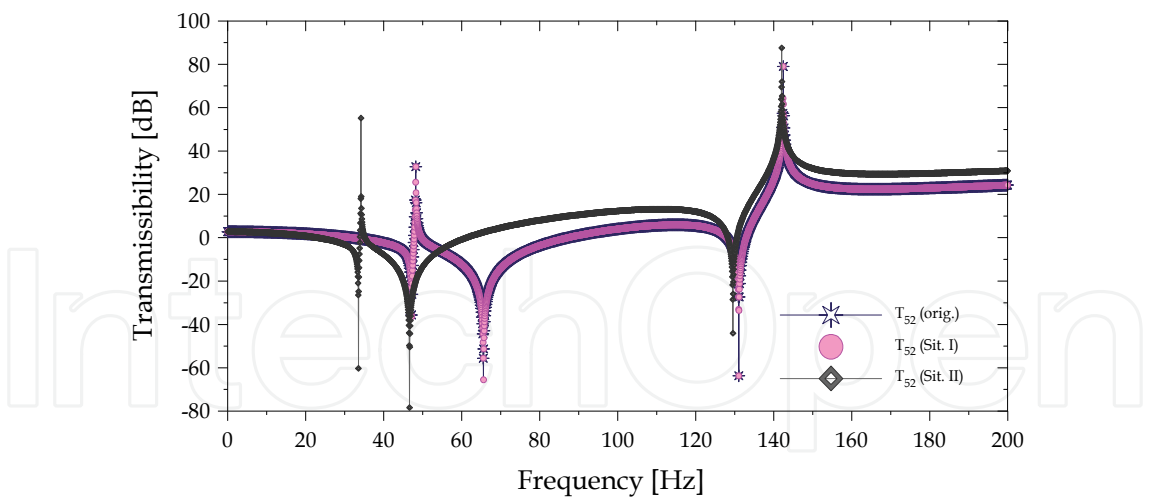Table 2. Characteristics of the modifications made in the original system

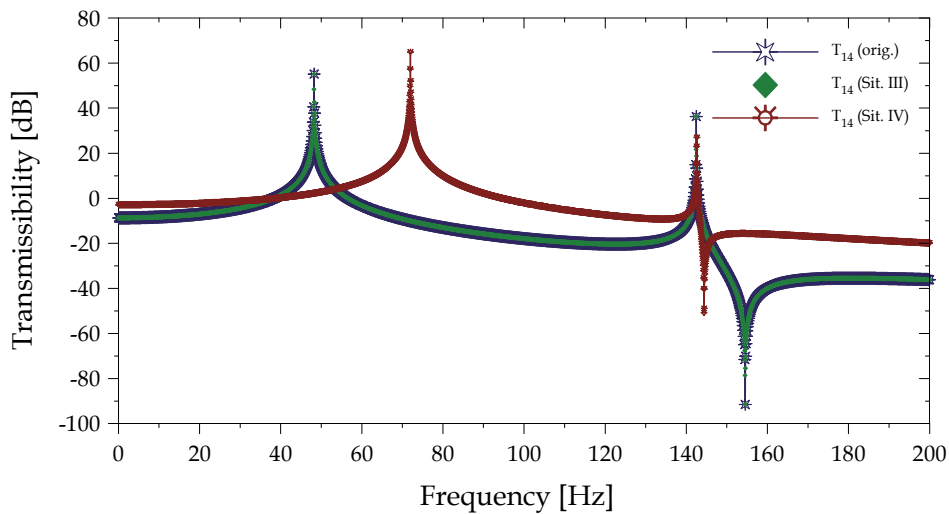Fig. 4. Transmissibility $T_{52}^{(A)}$, for the original system and the modified systems I e II

Fig. 5. Transmissibility $T_{14}^{(A)}$, for the original system and the modified systems III e IV

However, when forces change, the transmissibility matrix does not, although the responses themselves do. Therefore, if another test is performed, with a set of forces *linearly independent* of the first one – though applied at the same co-ordinates – $F_A^{(2)}$, a new set of U equations can be obtained, which are linearly independent of the first ones. If K tests are undertaken on the structure, with linearly independent forces always applied at the A set of co-ordinates, one can obtain a system of $U \times K$ equations to solve for the same number of $T_{UK}^{(A)}$ unknowns:

$$\begin{bmatrix} X_U^{(1)} & X_U^{(2)} & \cdots & X_U^{(K)} \end{bmatrix} = T_{UK}^{(A)} \begin{bmatrix} X_K^{(1)} & X_K^{(2)} & \cdots & X_K^{(K)} \end{bmatrix} \tag{17}$$

From which

$$T_{UK}^{(A)} = \begin{bmatrix} X_U^{(1)} & X_U^{(2)} & \cdots & X_U^{(K)} \end{bmatrix} \begin{bmatrix} X_K^{(1)} & X_K^{(2)} & \cdots & X_K^{(K)} \end{bmatrix}^{-1} \tag{18}$$

Note: an easy way to obtain the K sets of linearly independent forces is to apply a single force at each of the A locations at a time.

2.6 The distributed forces case

In (Ribeiro, Maia, & Silva, 2000b) the authors discussed the transmissibility between the two sets of co-ordinates U and K when a distributed force is applied to the structure. Such a force should be discretized to form the A set of applied forces. The issue was then to study what happened if one only took a subset of those co-ordinates A, which implies the reduction or condensation of the applied forces to such a subset and to ask the question: "how to study the transmissibility of responses from a set of condensed forces?". Let the set A be composed by the set C to where one wishes to condense the forces and the set D of the remaining co-ordinates, so that:

$$F_A = \begin{Bmatrix} F_C \\ F_D \end{Bmatrix} \tag{19}$$

If one wishes to condense F_A to F_C, one needs to assume some relationship between F_D and F_C, i.e, one cannot contemplate the case where all the applied forces are completely independent from each other. However, this is not a big restriction, as it seems reasonable to expect that the applied forces exhibit a more or less fixed spatial pattern along the structure. Therefore, let us assume a linear relationship between the sets of forces F_D and F_C, through the matrix P_{DC}:

$$F_D = P_{DC} F_C \tag{20}$$

If matrices H_{UA} and H_{KA} from eqs. (1) and (2) are partitioned into

$$H_{UA} = \begin{bmatrix} H_{UC} & \vdots & H_{UD} \end{bmatrix} \tag{21}$$

$$H_{KA} = \begin{bmatrix} H_{KC} & \vdots & H_{KD} \end{bmatrix} \tag{22}$$

and one has Eq. (19) into account, then eqs. (1) and (2) become

$$X_U = H_{UC} F_C + H_{UD} F_D \tag{23}$$

$$X_K = H_{KC} F_C + H_{KD} F_D \tag{24}$$

Substituting Eq. (20) in eqs. (23) and (24) and eliminating F_C, it follows that

$$X_U = \left(H_{UC} + H_{UD} P_{DC} \right) \left(H_{KC} + H_{KD} P_{DC} \right)^+ X_K \tag{25}$$

Therefore, one has now, instead of Eq. (4), another one relating X_U and X_K, through a new transmissibility matrix referred to the new subset of co-ordinates C:

$$X_U = T_{UK}^{(C)} X_K \tag{26}$$

where

$$T_{UK}^{(C)} = \left(H_{UC} + H_{UD} P_{DC} \right) \left(H_{KC} + H_{KD} P_{DC} \right)^{+} \tag{27}$$

Note: for the pseudo-inverse to exist, the number of K co-ordinates must be higher than the number of C co-ordinates. This is obviously verified, as $\#K \geq \#A$ and $\#A > \#C$. It should also be stressed that although referred to a reduced set of co-ordinates C where the forces are applied, the transmissibility matrix still relates the responses U and K and so it keeps the same size.

In (Ribeiro, Maia, & Silva, 2000a), (Ribeiro, Maia, & Silva, 2000b) and (Maia et al., 2001) the authors summarize some of the previous works and suggest some other possible applications for the transmissibility concept, namely in the area of damage detection. In this area, there has been some activity trying to use the transmissibility as defined, as well as some other variations of it, with limited results in [(Sampaio et al., 1999; 2000; 2001)], but with some promising evolution in [(Maia et al., 2007)].

An example is presented in order to illustrate the above discussion: a cantilevered beam is subjected to the loading shown in Fig. 6.
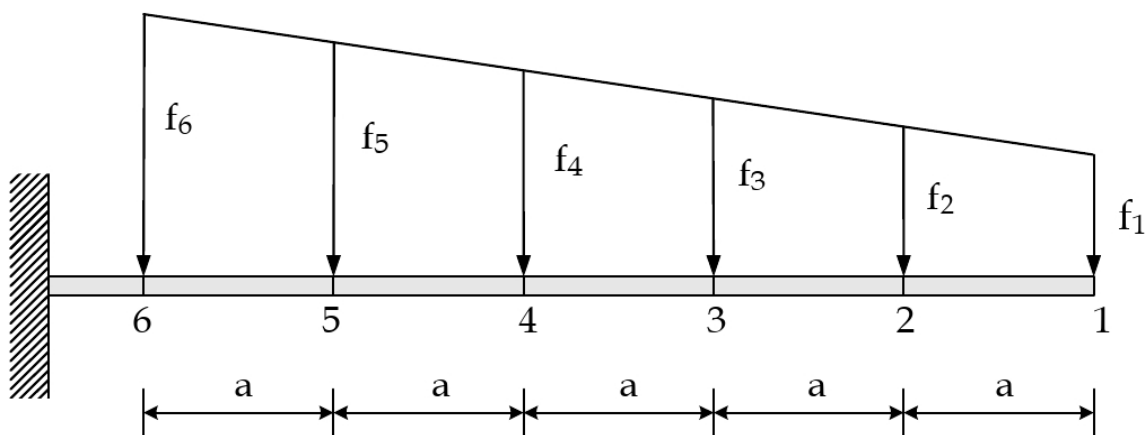
Fig. 6. Example of a loaded beam

It is assumed that forces are applied at co-ordinates 1 to 6 but the forces at co-ordinates 2 to 5 can be related to those applied at 1 and 6 through the expression:

$$F_D = P_{DC} F_C \quad \Rightarrow \quad \begin{Bmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{Bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & 1 \\ 3 & 2 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{Bmatrix} f_1 \\ f_6 \end{Bmatrix} \tag{28}$$

One can further assume that the responses at co-ordinates 1 and 2 can be measured and those at co-ordinates 4 and 5 can be computed through the transmissibility, i.e.,

$X_U = T_{UK}^{(C)} X_K$, with $X_U = \begin{Bmatrix} X_4 \\ X_5 \end{Bmatrix}$, $X_K = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$ and

$$T_{UK}^{(C)} = \left(H_{UC} + H_{UD}\, P_{DC} \right)\left(H_{KC} + H_{KD}\, P_{DC} \right)^{+} \quad \Rightarrow$$

$$T_{UK}^{(C)} = \left(\begin{bmatrix} H_{41} & H_{46} \\ H_{51} & H_{56} \end{bmatrix} + \frac{1}{5}\begin{bmatrix} H_{42} & H_{43} & H_{44} & H_{45} \\ H_{52} & H_{53} & H_{54} & H_{55} \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 3 & 2 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} \right) \left(\begin{bmatrix} H_{11} & H_{16} \\ H_{21} & H_{26} \end{bmatrix} \right.$$

$$\left. + \frac{1}{5}\begin{bmatrix} H_{12} & H_{13} & H_{14} & H_{15} \\ H_{22} & H_{23} & H_{24} & H_{25} \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 3 & 2 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} \right)^{+} \tag{29}$$

2.7 The random forces case

Often one has to deal with random forces, for instance when a structure is submitted to environmental loads. The cases that have been addressed so far were limited to harmonic or periodic forces. The generalization to random forces has been derived in [(Ribeiro et al., 2002; Fontul et al., 2004)], now in terms of power spectral densities, rather than in terms of response amplitudes. Let S_{KK} denote the auto-spectral density of the responses X_K and S_{KU} the cross-spectral density between responses X_K and X_U. Then, it can be shown (see [(Fontul, 2005)] for specific details) that both are related through the same transmissibility matrix as before (using Eq. (5), for instance):

$$S_{KU}^{T} = T_{UK}^{(A)}\, S_{KK}^{T} \tag{30}$$

2.8 Some possible applications
2.8.1 Transmissibility of motion in structural coupling

This topic has been addressed in (Devriendt, 2004; Ribeiro et al., 2004; Devriendt & Fontul, 2005). Let us consider a main structure, to which an additional structure is coupled though some coupling co-ordinates, i.e., the additional structure applies a set of forces (and moments) to the main structure. As the transmissibility between two sets of responses on the main structure does not depend on the magnitude of those forces, the transmissibility matrix of the main structure is equal to the transmissibility matrix of the total structure (main + additional). In other words, under certain conditions, the transmissibility matrix of the main structure remains unchanged, even if an additional structure is coupled to the main one. To make this property valid it is necessary to consider a sufficient number of coupled co-ordinates. Although it might be argued that a reduced number of coupling co-ordinates would hamper the results since it would not include information about some modes, it has been shown in (Devriendt, 2004; Ribeiro et al., 2004; Devriendt & Fontul, 2005) that, as long as there is enough information regarding the modes included in the frequency range of interest the minimum number of coupling co-ordinates can be reduced without deterioration of the results.

2.8.2 Evaluation of unmeasured frequency response functions

Recent papers [(Maia et al., 2008; Urgueira et al., 2008)] have explored some invariance properties of the transmissibility, namely when modifications are made in terms of masses

and/or stiffnesses at the co-ordinates where the forces are applied, to be able to estimate the new FRFs in locations that become no longer accessible. For instance, if one calculates the transmissibility matrix at some stage between two sets of responses for a given set of applied forces and later on there are some modifications at the force co-ordinates due to some added masses, the FRFs will change but the transmissibility remains the same. This allows the estimation of the new FRFs. So, initially one has $X_U^{(1)} = H_{UA}^{(1)} H_{KA}^{+\,(1)} X_K^{(1)}$ and later on one has $X_U^{(2)} = H_{UA}^{(2)} H_{KA}^{+\,(2)} X_K^{(2)}$. As the transmissibility remains unchanged, one has

$$T_{UK}^{(A)} = H_{UA}^{(1)} H_{KA}^{+\,(1)} = H_{UA}^{(2)} H_{KA}^{+\,(2)} \tag{31}$$

and one can calculate, for instance, $H_{UA}^{(2)}$, given by:

$$H_{UA}^{(2)} = T_{UK}^{(A)} H_{KA}^{(2)} \tag{32}$$

2.9 Direct transmissibility

From the definition given before one has,

$$X_U = H_{UA} H_{KA}^+ X_K = T_{UK}^{(A)} X_K \tag{33}$$

which, as explained, is a generalisation from the one degree of freedom system. However, in some cases it might be useful to divide two responses directly. In strict sense that is a transmissibility only if a single force is applied. Otherwise, one has to name it differently, like pseudo-transmissibility (e.g. (Sampaio et al., 1999)), scalar transmissibility (Devriendt et al., 2010) or direct transmissibility, which is the one we shall adopt.

Direct transmissibilities will depend on the force magnitudes (as well as location, of course). For example, dividing Eq. (33) by one of the amplitudes X_K, say X_s, one has:

$$X_U / X_s = T_{UK}^{(A)} X_K / X_s \quad \text{or} \quad \tau_{Us} = T_{UK}^{(A)} \tau_{Ks} \tag{34}$$

It is easier to understand the implications of both definitions through an example: let X_U, X_K, and F_A be given respectively by

$$X_U = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \quad X_K = \begin{Bmatrix} X_3 \\ X_4 \end{Bmatrix} \quad F_A = \begin{Bmatrix} F_5 \\ F_6 \end{Bmatrix} \tag{35}$$

The relation between X_U and X_K would be:

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{bmatrix} T_{13} & T_{14} \\ T_{23} & T_{34} \end{bmatrix} \begin{Bmatrix} X_3 \\ X_4 \end{Bmatrix} \quad \Rightarrow \quad \begin{aligned} X_1 &= T_{13}X_3 + T_{14}X_4 \\ X_2 &= T_{23}X_3 + T_{24}X_4 \end{aligned} \tag{36}$$

Dividing Eq. (36) by, say, X_3, it follows that

$$\begin{aligned} X_1/X_3 &= T_{13} + T_{14} X_4/X_3 \\ X_2/X_3 &= T_{23} + T_{24} X_4/X_3 \end{aligned} \quad \text{or} \quad \begin{aligned} \tau_{13} &= T_{13} + T_{14}\tau_{43} \\ \tau_{23} &= T_{23} + T_{24}\tau_{43} \end{aligned} \tag{37}$$

One can also write:

$$
\begin{cases} X_1 = H_{15}F_5 + H_{16}F_6 \\ X_3 = H_{35}F_5 + H_{36}F_6 \end{cases} \quad \Rightarrow \quad \tau_{13} = \frac{X_1}{X_3} = \frac{H_{15}F_5 + H_{16}F_6}{H_{35}F_5 + H_{36}F_6} \tag{38}
$$

From Eq. (38) it is clear that the direct transmissibility τ_{13} depends on the magnitudes of F_5 and F_6, unless the relation F_5/F_6 remains constant. Only in the case where there is just a single force one has a coincidence between both types of transmissibility.

Both kinds of definitions can be useful. For instance, concerning now the direct transmissibilities, one can see that from Eq. (37) one can calculate τ_{13} and τ_{23} from τ_{43}, and one can eliminate τ_{43} between both equations and establish a relationship between τ_{13} and τ_{23}, therefore allowing the evaluation of one of them from the other. Moreover and similarly to what was mentioned in section 2.5, the direct transmissibilities allow the calculation of the other ones.

To illustrate the main differences between the curves of general and the direct transmissibilities, the MDOF system presented in Fig. 2 has been used. In Fig. 7 some direct transmissibility curves are presented.
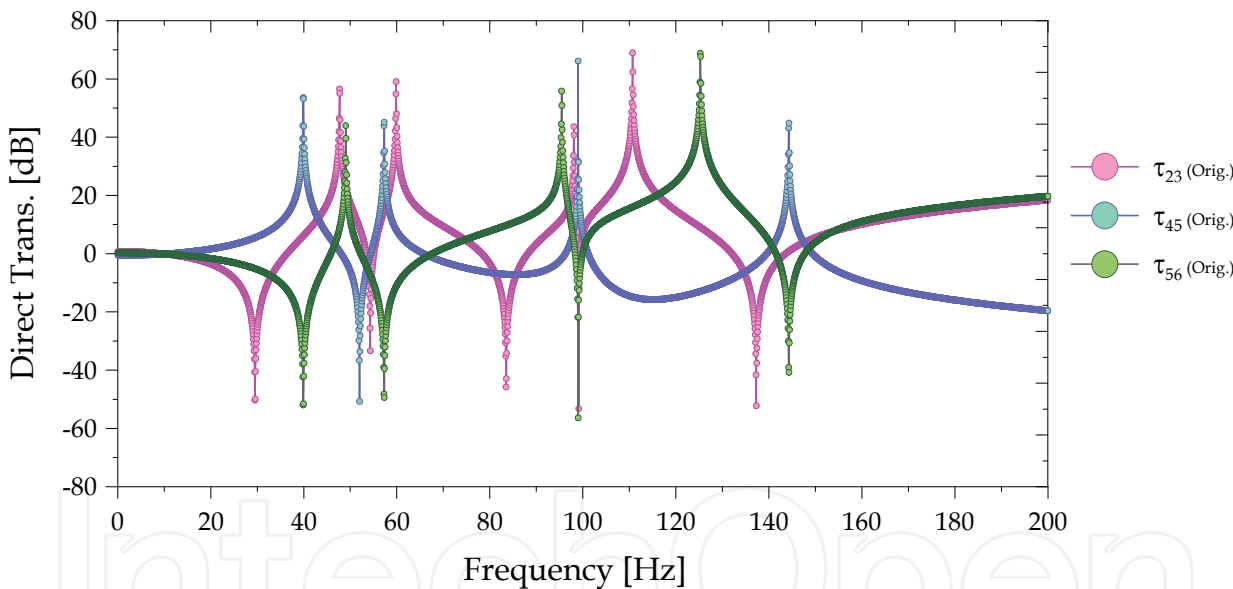
Fig. 7. Some direct transmissibility curves from the system of Fig. 2

By comparing the results of the transmissibilities of Fig. 3 with the curves obtained with the direct transmissibilities, Fig. 7, one can see that both look like FRFs, though it may be noted that in the case of the transmissibilities all the maxima occur at the same frequencies; the same is not true with the direct transmissibilities, where each curve presents distinct maxima.

2.10 Other applications

Other works have presented the possibility of using the transmissibility concept for model updating (Steenackers et al., 2007) and to identify the dynamic properties of a structure (Devriendt & Guillaume, 2007; Devriendt, De Sitter, et al., 2009; Devriendt et al., 2010).

Other recent studies have applied the transmissibility to the problem of transfer path analysis in vibro-acoustics (Tcherniak & Schuhmacher, 2009) and for damage detection (Canales et al., 2009; Devriendt, Vanbrabant, et al., 2009; Urgueira et al., 2011).

3. Transmissibility of forces

3.1 In terms of frequency response functions

Another important topic may be the prediction of the dynamic forces transmitted to the ground when a machine is working. For a single degree of freedom, the solution is well known and the transmissibility is defined as the ratio between the transmitted load (the ground reaction) and the applied one, for harmonic excitation. For an MDOF system, one has to relate the known applied loads (F_K) to the unknown reactions (F_U), Fig. 8.

The displacements at the co-ordinates of one set (the set of the reactions) are constrained, so they must also be known (possibly zero).

The inverse problem may also be of interest, i.e., to estimate the loads applied to a structure (wind, traffic, earthquakes, etc.) from the measured reaction loads. Once the load transmissibility matrix is established between the appropriate sets, the measurement of the reactions is expected to allow for the estimation of the external loads.
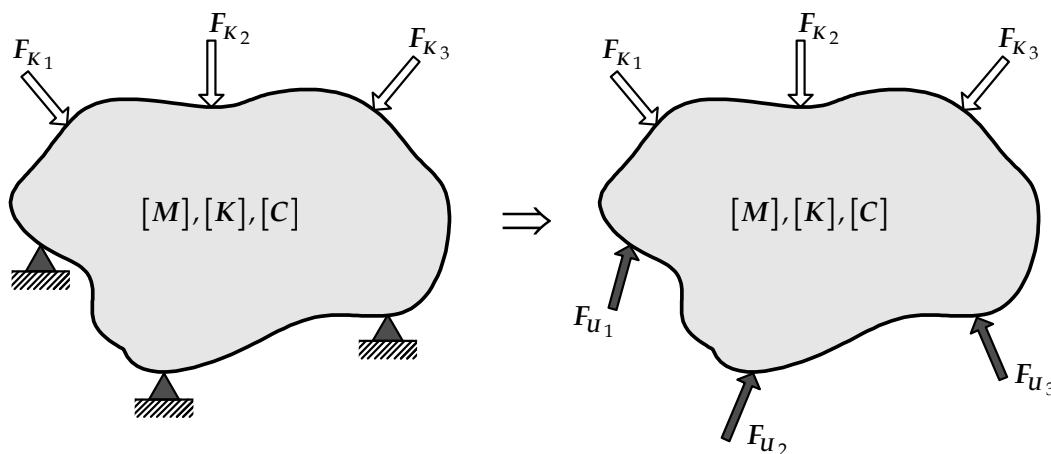
Fig. 8. Structure with applied loads and reactions in dynamic equilibrium

This topic has been addressed in (Maia et al., 2006); the force transmissibility may also be defined either in terms of FRFs or in terms of dynamic stiffnesses. Let X_K and X_U be the responses corresponding to F_K and F_U, respectively, and X_C the responses at the remaining co-ordinates; then,

$$\begin{Bmatrix} X_K \\ X_U \\ X_C \end{Bmatrix} = \begin{bmatrix} H_{KK} & H_{KU} \\ H_{UK} & H_{UU} \\ H_{CK} & H_{CU} \end{bmatrix} \begin{Bmatrix} F_K \\ F_U \end{Bmatrix} \tag{39}$$

Assuming the responses at the reactions co-ordinates as zero, i.e., $X_U = \boldsymbol{0}$, it follows that:

$$F_U = -H_{UU}^{-1} H_{UK} F_K \tag{40}$$

Therefore, the force transmissibility is defined as:

$$T_{UK} = -H_{UU}^{-1}H_{UK} \tag{41}$$

If the displacements at the co-ordinates of the reactions are not zero, or in the more general case when the two sets of loads are not the applied loads and the reactions, but any disjoint sets that encompass all the loads applied to the structure, it is easy to show (Maia et al., 2006) that:

$$F_U = T_{UK}F_K + H_{UU}^{-1}X_U \tag{42}$$

3.2 In terms of dynamic stiffness

Instead of Eq. (39) one has now:

$$\begin{Bmatrix} F_K \\ F_U \end{Bmatrix} = \begin{bmatrix} Z_{KK} & Z_{KU} & Z_{KC} \\ Z_{UK} & Z_{UU} & Z_{UC} \end{bmatrix} \begin{Bmatrix} X_K \\ X_U \\ X_C \end{Bmatrix} \tag{43}$$

Assuming fictitious loads F_C at the remaining co-ordinates and rearranging, one obtains:

$$\begin{Bmatrix} F_K \\ F_C \\ F_U \end{Bmatrix} = \begin{bmatrix} Z_{KK} & Z_{KC} & Z_{KU} \\ Z_{CK} & Z_{CC} & Z_{CU} \\ Z_{UK} & Z_{UC} & Z_{UU} \end{bmatrix} \begin{Bmatrix} X_K \\ X_C \\ X_U \end{Bmatrix} \tag{44}$$

Defining $X_E = \{X_K \quad X_C\}^T$ and $F_E = \{F_K \quad F_C\}^T$ and assuming, as before, that at the reaction co-ordinates there is no motion ($X_U = 0$), one can write:

$$\begin{aligned} F_E &= Z_{EE}X_E \\ F_U &= Z_{UE}X_E \end{aligned} \tag{45}$$

Eliminating X_E between eqs.(45), it follows that

$$F_U = Z_{UE}Z_{EE}^{-1}F_E \tag{46}$$

and the force transmissibility becomes now:

$$T_{UE} = Z_{UE}Z_{EE}^{-1} \tag{47}$$

Note that because $F_C = 0$, $F_E = \{F_K \quad 0\}^T$, and thus only the columns of T_{UE} corresponding to F_K are relevant to the transmissibility between the two sets of loads, the sub-matrix T_{UK}. One should also note that, in contrast with the SDOF system, the transmissibility of forces is different from the transmissibility of displacements.

Simply to illustrate the application of the concept, a numerical example is presented. The model is shown in Fig. 9, similar to the one of Fig. 3, where the displacements at co-ordinates 1 and 2 are now zero, i.e., $X_1 = X_2 = 0$. External forces are applied at co-ordinates 5 and 6 and the reactions happen at co-ordinates 1 and 2.
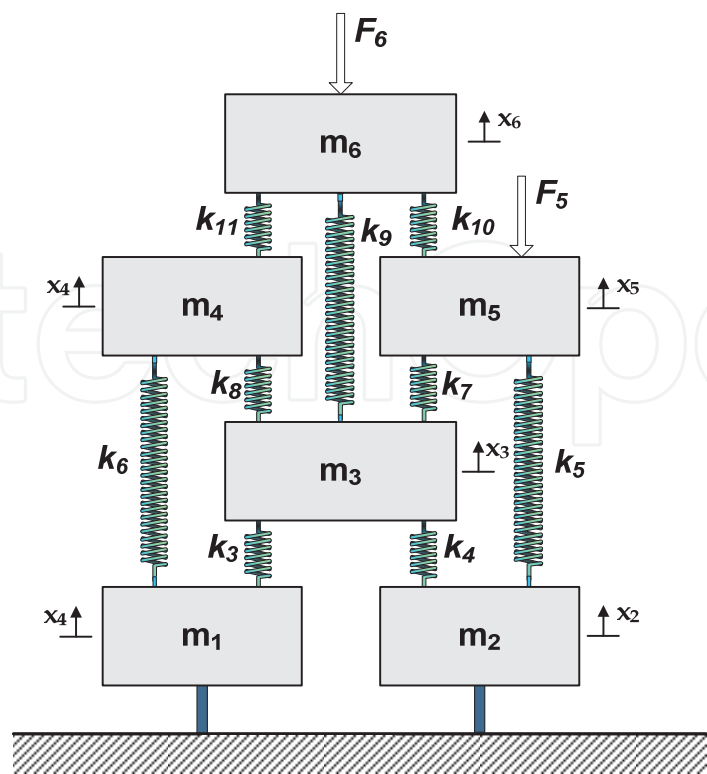
Fig. 9. Structure model in study

The force transmissibility between the two sets of loads – forces at 5 and 6 being known (set *K*) and forces at 1 and 2 being unknown (set *U*) – was computed using both described methods.

$$\boldsymbol{F}_K = \begin{Bmatrix} F_5 \\ F_6 \end{Bmatrix}, \quad \boldsymbol{F}_U = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}, \quad \boldsymbol{F}_E = \begin{Bmatrix} F_5 \\ F_6 \\ 0 \\ 0 \end{Bmatrix}, \quad \boldsymbol{X}_K = \begin{Bmatrix} X_5 \\ X_6 \end{Bmatrix}, \quad \boldsymbol{X}_U = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}, \quad \boldsymbol{X}_E = \begin{Bmatrix} X_5 \\ X_6 \\ X_3 \\ X_4 \end{Bmatrix} \tag{48}$$

Equation (41) becomes:

$$\boldsymbol{T}_{UK} = -\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}^{-1} \begin{bmatrix} H_{15} & H_{16} \\ H_{25} & H_{26} \end{bmatrix} = \begin{bmatrix} T_{H_{11}} & T_{H_{12}} \\ T_{H_{21}} & T_{H_{22}} \end{bmatrix} \tag{49}$$

where the subscript *H* means that the transmissibility has been computed using FRFs. Equation (47) becomes:

$$\boldsymbol{T}_{UE} = \begin{bmatrix} Z_{15} & Z_{16} & Z_{13} & Z_{14} \\ Z_{25} & Z_{26} & Z_{23} & Z_{24} \end{bmatrix} \begin{bmatrix} Z_{55} & Z_{56} & Z_{53} & Z_{54} \\ Z_{65} & Z_{66} & Z_{63} & Z_{64} \\ Z_{35} & Z_{36} & Z_{33} & Z_{34} \\ Z_{45} & Z_{46} & Z_{43} & Z_{44} \end{bmatrix}^{-1} = \begin{bmatrix} T_{Z_{11}} & T_{Z_{12}} & \vdots & T_{Z_{13}} & T_{Z_{14}} \\ T_{Z_{21}} & T_{Z_{22}} & \vdots & T_{Z_{23}} & T_{Z_{24}} \end{bmatrix} \tag{50}$$

from which

$$\boldsymbol{T}_{UK} = \begin{bmatrix} T_{Z_{11}} & T_{Z_{12}} \\ T_{Z_{21}} & T_{Z_{22}} \end{bmatrix} \tag{51}$$

where the subscript Z means that the transmissibility has been computed using dynamic stiffness matrices. The results obtained by using equations (49) and (51) superimpose perfectly, as expected. Two of the four transmissibilities are presented in Fig. 10 to illustrate this fact.
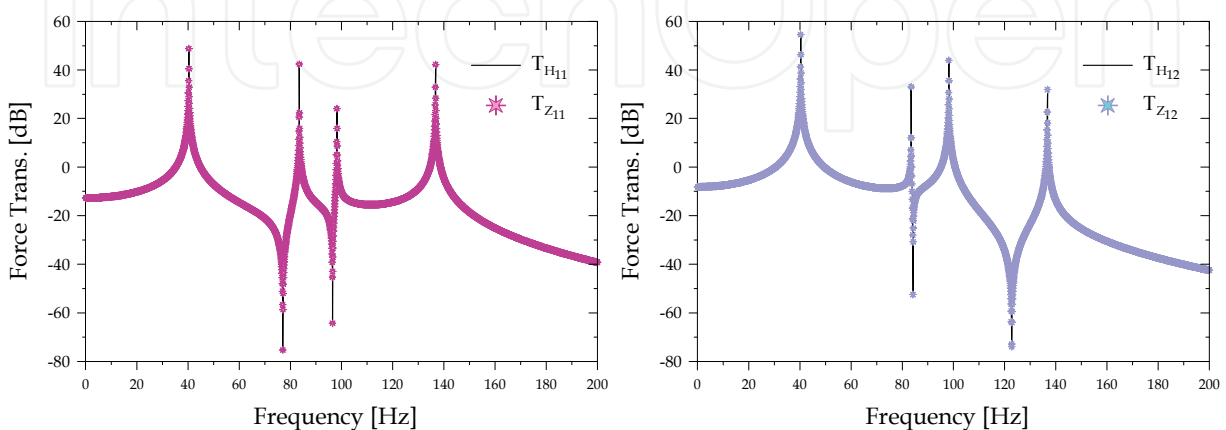

Fig. 10. Comparison between corresponding force transmissibility terms computed from FRFs ($T_{H_{11}}$ and $T_{H_{22}}$) and dynamic stiffness matrices ($T_{Z_{11}}$ and $T_{Z_{22}}$).

It may be noted from Fig. 10 that the maxima of the force transmissibility curves also occur all at the same frequencies.

4. Conclusions

The transmissibility concept for multiple degree-of-freedom systems has been developed and applied for the last ten years and the interest in this matter is continuously growing. In this paper a general overview has been given, concerning the main achievements so far and it has been shown that the various ways in which transmissibility can be defined and applied opens various possibilities for research in different domains, like system identification, structural modification, coupling analysis, damage detection, model updating, vibro-acoustic applications, isolation and vibration attenuation.

5. Acknowledgment

6. References

Canales, G., Mevel, L., Basseville, M. (2009). Transmissibility Based Damage Detection, *Proceedings of the 27th International Operational Modal Analysis Conference (IMAC XXVII)*, Orlando, Florida, U.S.A.

Devriendt, C.M.G., Fontul, M. (2005). Transmissibility in Structural Coupling: Identification of the Sufficient Set of Coupling Forces, *Proceedings of the 1st International Operational Modal Analysis Conference (IOMAC 1)*, Copenhagen, Denmark.

Devriendt, C., Vanbrabant, K., De Sitter, G., Presznzk, F., De Troyer, T., Deckers, K., Steenackers, G., Vanlanduit, S., Guillaume, P. (2009). Structural Health Monitoring in Changing Operational Conditions Using Transmissibility measurements, *Proceedings of International Conference on Engineering Dynamic (ICEDyn 2009)*, Ericeira, Portugal.

Devriendt, C. (2004). Transmissibility on Coupled Structures (Final Project Report). Instituto Superior Técnico, Portugal.

Devriendt, C., De Sitter, G., Guillaume, P. (2010). An Operational Modal Analysis Approach Based on Parametrically Identified Multivariable Transmissibilities, *Mechanical Systems and Signal Processing,* Vol. 24, No. 5, pp. 1250-1259, ISSN: 0888-3270.

Devriendt, C., De Sitter, G., Vanlanduit, S., Guillaume, P. (2009). Operational Modal Analysis in the Presence of Harmonic Excitations by the Use of Transmissibility Measurements, *Mechanical Systems and Signal Processing,* Vol. 23, No. 3, pp. 621-635, ISSN:0888-3270.

Devriendt, C., Guillaume, P. (2007). The use of Transmissibility Measurements in Output-Only Modal Analysis, *Mechanical Systems and Signal Processing*, Vol. 21, No. 7, pp. 2689-2696, ISSN: 0888-3270.

Fontul, M. (2005). Transmissibility in Multi-Degree-of-Freedom Systems (Ph. D Thesis).

Fontul, M., Ribeiro, A.M.R., Silva, J.M.M., Maia, N.M.M. (2004). Transmissibility Matrix in Harmonic and Random Processes, *Shock and Vibration*, Vol. 11, No. 5-6, pp. 563–572, ISSN: 1070-9622.

Liu, W. (2000). Structural Dynamic and Testing of Coupled Structures (Ph. D Thesis).

Liu, W., Ewins, D. J. (1998). Transmissibility Properties of MDOF Systems, *Proceedings of the 16th International Operational Modal Analysis Conference (IMAC XVI)*, Santa Barbara, California, pp. 847-854.

Maia, N.M.M., Almeida, R.A.B., Urgueira, A.P.V. (2008). Understanding Transmissibility Properties, *Proceedings of the 26th International Modal Analysis Conference (IMAC XXVI)*, ISBN: 0-912053-98-4, Orlando, Florida, U.S.A.

Maia, N.M.M., Fontul, M., Ribeiro, A.M.R. (2006). Transmissibility of Forces in Multiple-Degree-of Freedom Systems, *Proceedings of International Conference on Noise and Vibration Engineering (ISMA 2006)*, Leuven, Belgium.

Maia, N.M.M., Ribeiro, A.M.R., Fontul, M., Montalvão, D., Sampaio, R.P.C. (2007). Using the Detection and Relative Damage Quantification Indicator (DRQ) with Transmissibility, *Key Engineering Materials*, Vol. 347, pp. 455-460, ISSN: 1662-9795.

Maia, N.M.M., Silva, J.M.M., Ribeiro, A.M.R. (1999). The Transmissibility Concept in Multi-Degree-of-Freedom *Proceedings of Systems, Inverse Methods in Structural Dynamics, European Mechanics Colloquium, EUROMEC 401*, Liverpool, UK.

Maia, N.M.M., Silva, J.M.M., Ribeiro, A.M.R. (2001). The Transmissibility Concept in Multi-Degrees-of-Freedom Systems, *Mechanical Systems and Signal Processing,* Vol. 15, No. 1, pp. 129-137, ISSN: 0888-3270.

Paipetis, S.A., Vakakis, A.F. (1985). A Method of Analysis for Unidirectional Vibration Isolators with Many Degrees of Freedom, *Journal of Sound and Vibration*, Vol. 98, No. 1, pp. 13-23, ISSN: 0022-460X.

Ribeiro, A.M.R., Fontul, M., Silva, J.M.M., Maia, N.M.M. (2002). Transmissibility Matrix in Harmonic and Random Processes, *Proceedings of the International Conference on Structural Dynamics Modelling (SDM)*, Funchal, Madeira, Portugal.

Ribeiro, A.M.R., Silva, J.M.M., Maia, N.M.M., Fontul, M. (2004). Transmissibility in Structural Coupling, *Proceedings of International Conference on Noise and Vibration Engineering (ISMA 2004)*, Leuven, Belgium.

Ribeiro, A.M.R., Maia, N.M.M., Silva, J.M.M. (2000a). On the Generalisation of the Transmissibility Concept, *Mechanical Systems and Signal Processing*, Vol. 14, No. 1, pp. 29-35, ISSN: 0888-3270.

Ribeiro, A.M.R. (1998). On the Generalization of the Transmissibility Concept, *Proceedings of the NATO/ASI Conference on Modal Analysis and Testing*, Sesimbra, Portugal.

Ribeiro, A.M.R., Fontul, M., Maia, N.M.M., Silva, J.M.M. (2005). Further Developments on the Transmissibility Concept for Multiple Degree of Freedom Systems, *Scientific Bulletin of the «Politehnica» University of Timisoara, Transactions on Mechanics*, Vol. 50, No. 64, pp. 1224–6077, Timisoara, Romania.

Ribeiro, A.M.R., Maia, N.M.M., Silva, J.M.M. (1999). Experimental Evaluation of the Transmissibility Matrix, *Proceedings of the 17th International Modal Analysis Conference (IMAC XVII)*, Kissimmee FL, USA, pp. 1126-1129, ISBN: 0-912053-64-X.

Ribeiro, A.M.R., Maia, N.M.M., Silva, J.M.M. (2000b). Response Prediction From a Reduced Set of Known Responses Using the Transmissibility, *Proceedings of. the 18th International Modal Analysis Conference (IMAC XVIII)*, pp. 425-427, ISBN: 0-912053-67-4, San Antonio TX, USA.

Sampaio, R.P.C., Maia, N.M.M., Ribeiro, A.M.R., Silva, J.M.M. (1999). Damage Detection Using the Transmissibility Concept, *Proceedings of the Sixth International Congress on Sound and Vibration (ICSV 6)*, Copenhagen, Denmark.

Sampaio, R.P.C., Maia, N.M.M., Ribeiro, A.M.R., Silva, J.M.M. (2001). Transmissibility Techniques for Damage Detection, *Proceedings of 19th International Modal Analysis Conference (IMAC XIX)*, Kissimmee, Florida, USA, pp. 1524-1527.

Sampaio, R.P.C., Maia, N.M.M., Silva, J.M.M., Ribeiro, A.M.R. (2000). On the Use of Transmissibility for Damage Detection and Location, *Proceedings of the European COST F3 Conference on System Identification & Structural Health Monitoring*, Polytechnic University of Madrid, Spain, pp. 363-376.

Steenackers, G., Devriendt, C., Guillaume, P. (2007). On the Use of Transmissibility Measurements for Finite Element Model Updating, *Journal of Sound and Vibration*, Vol. 303, No. 3-5, pp. 707-722, ISSN: 0022-460X.

Tcherniak, D., Schuhmacher, A.P. (2009). Application of Transmissibility Matrix Method to NVH Source Contribution Analysis, *Proceedings of the 27th International Modal Analysis Conference (IMAC XXVII)*, Orlando, Florida, U.S.A.

Urgueira, A.P.V., Almeida, R.A.B., Maia, N.M.M. (2008). Experimental Estimation of FRFs Using the Transmissibility Concept, *Proceedings of International Conference on Modal Analysis Noise and Vibration Engineering (ISMA 2008)*, ISBN: 978-90-73802-86-5, Leuven, Belgium.

Urgueira, A.P.V., Almeida, R.A.B., Maia, N.M.M. (2011). On the use of the transmissibility concept for the evaluation of frequency response functions, *Mechanical Systems and Signal Processing*, Vol. 25, No. 3, ISSN: 0888-3270, 940-951.

Vakakis, A.F. (1985). Dynamic Analysis of a Unidirectional Periodic Isolator, Consisting of Identical Masses and Intermediate Distributed Resilient Blocks. *Journal of Sound and Vibration*, Vol. 103, No. 1, pp. 25-33, ISSN: 0022-460X.

Vakakis, A.F., Paipetis, S.A. (1985). Transient Response of Unidirectional Vibration Isolators with Many Degrees of Freedom, *Journal of Sound and Vibration*, Vol. 99, No. 4, pp. 557-562, ISSN: 0022-460X.

Vakakis, A.F., Paipetis, S.A. (1986). The Effect of a Viscously Damped Dynamic Absorber on a Linear Multi-Degree-of-Freedom System, *Journal of Sound and Vibration*, Vol. 105, No. 1, pp. 49-60, ISSN: 0022-460X.

Varoto, P.S., McConnell, K.G. (1998). Single Point vs Multi Point Acceleration Transmissibility Concepts in Vibration Testing, *Proceedings of the 12th International Modal Analysis Conference (IMAC XVI)*, pp. 83-90, ISBN 0-912053-59-3, Santa Barbara, California, USA.

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