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Control of Nonlinear Active Vehicle Suspension Systems Using Disturbance Observers

Francisco Beltran-Carbajal[1], Esteban Chavez-Conde[2], Gerardo Silva Navarro[3], Benjamin Vazquez Gonzalez[1] and Antonio Favela Contreras[4]

[1]*Universidad Autonoma Metropolitana, Plantel Azcapotzalco, Departamento de Energia, Mexico, D.F.*

[2]*Universidad del Papaloapan, Campus Loma Bonita, Departamento de Ingenieria en Mecatronica, Instituto de Agroingenieria, Loma Bonita, Oaxaca*

[3]*Centro de Investigacion y de Estudios Avanzados del I.P.N., Departamento de Ingenieria Electrica, Seccion de Mecatronica, Mexico, D.F.*

[4]*ITESM Campus Monterrey, Monterrey, N.L. Mexico*

1. Introduction

The main control objectives of active vehicle suspension systems are to improve the ride comfort and handling performance of the vehicle by adding degrees of freedom to the passive system and/or controlling actuator forces depending on feedback and feedforward information of the system obtained from sensors.

Passenger comfort is provided by isolating the passengers from the undesirable vibrations induced by irregular road disturbances and its performance is evaluated by the level of acceleration by which vehicle passengers are exposed. Handling performance is achieved by maintaining a good contact between the tire and the road to provide guidance along the track.

The topic of active vehicle suspension control system, for linear and nonlinear models, in general, has been quite challenging over the years and we refer the reader to some of the fundamental works in the vibration control area (Ahmadian, 2001). Some active control schemes are based on neural networks, genetic algorithms, fuzzy logic, sliding modes, H-infinity, adaptive control, disturbance observers, LQR, backstepping control techniques, etc. See, e.g., (Cao et al., 2008); (Isermann & Munchhof, 2011); (Martins et al., 2006); (Tahboub, 2005); (Chen & Huang, 2005) and references therein. In addition, some interesting semiactive vibration control schemes, based on Electro-Rheological (ER) and Magneto-Rheological (MR) dampers, have been proposed and implemented on commercial vehicles. See, e.g., (Choi et al., 2003); (Yao et al., 2002).

In this chapter is proposed a robust control scheme, based on the real-time estimation of perturbation signals, for active nonlinear or linear vehicle suspension systems subject to unknown exogenous disturbances due to irregular road surfaces. Our approach differs

from others in that, the control design problem is formulated as a bounded disturbance signal processing problem, which is quite interesting because one can take advantage of the industrial embedded system technologies to implement the resulting active vibration control strategies. In fact, there exist successful implementations of automotive active control systems based on embedded systems, and this novel tendency is growing very fast in the automotive industry. See, e.g., (Shoukry et al., 2010); (Basterretxea et al., 2010); (Ventura et al., 2008); (Gysen et al., 2008) and references therein.

In our control design approach is assumed that the nonlinear effects, parameter variations, exogenous disturbances and possibly input unmodeled dynamics are lumped into an unknown bounded time-varying disturbance input signal affecting a so-called differentially flat linear simplified dynamic mathematical model of the suspension system. The lumped disturbance signal and some time derivatives of the flat output are estimated by using a flat output-based linear high-gain dynamic observer. The proposed observer-control design methodology considers that, the perturbation signal can be locally approximated by a family of Taylor polynomials. Two active vibration controllers are proposed for hydraulic or electromagnetic suspension systems, which only require position measurements.

Some numerical simulation results are provided to show the efficiency, effectiveness and robust performance of the feedforward and feedback linearization control scheme proposed for a nonlinear quarter-vehicle active suspension system.

This chapter is organized as follows: Section 2 presents the nonlinear mathematical model of an active nonlinear quarter-vehicle suspension system. Section 3 presents the proposed vehicle suspension control scheme based on differential flatness. Section 4 presents the main results of this chapter as an alternative solution to the vibration attenuation problem in nonlinear and linear active vehicle suspension systems actuated electromagnetically or hydraulically. Computer simulation results of the proposed design methodology are included in Section 5. Finally, Section 6 contains the conclusions and suggestions for further research.

2. A quarter-vehicle active suspension system model

Consider the well-known nonlinear quarter-vehicle suspension system shown in Fig. 1. In this model, the sprung mass m_s denotes the time-varying mass of the vehicle-body and the unsprung mass m_u represents the assembly of the axle and wheel. The tire is modeled as a linear spring with equivalent stiffness coefficient k_t linked to the road and negligible damping coefficient. The vehicle suspension, located between m_s and m_u, is modeled by a damper and spring, whose nonlinear damping and stiffness force functions are given by

$$\mathcal{F}_k(z) = kz + k_n z^3$$

$$\mathcal{F}_c(\dot{z}) = c\dot{z} + c_n \dot{z}^2 \mathrm{sgn}(\dot{z})$$

The generalized coordinates are the displacements of both masses, z_s and z_u, respectively. In addition, $u = F_A$ denotes the (force) control input, which is applied between the two masses by means of an actuator, and $z_r(t)$ represents a bounded exogenous perturbation signal due

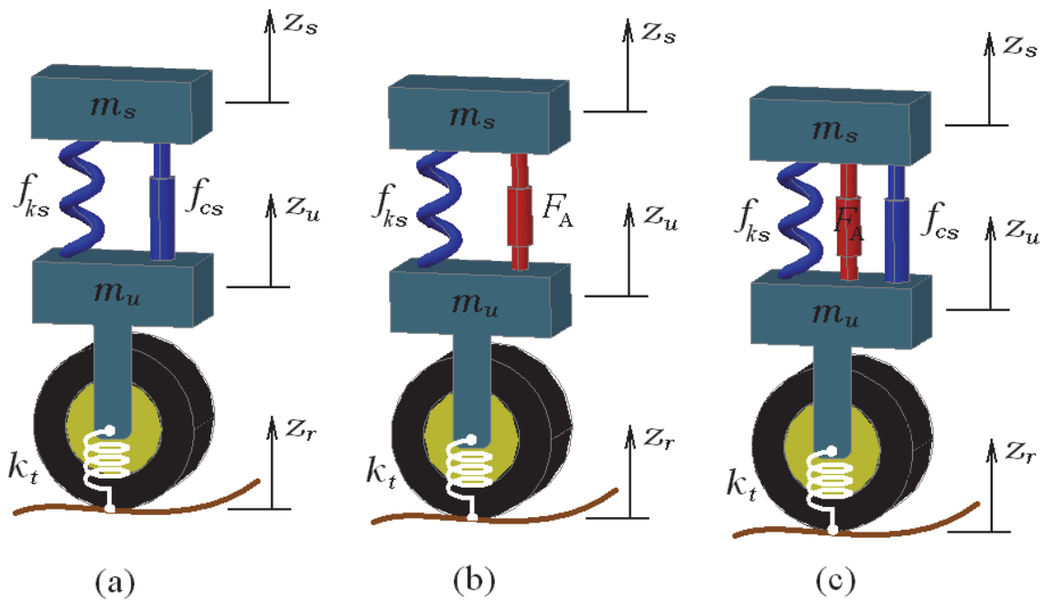

Fig. 1. Schematic diagram of a quarter-vehicle suspension system: (a) passive suspension system, (b) electromagnetic active suspension system and (c) hydraulic active suspension system.

to irregular road surfaces satisfying:

$$\begin{aligned} \|z_r(t)\|_\infty &= \gamma_1 \\ \|\dot{z}_r(t)\|_\infty &= \gamma_2 \\ \|\ddot{z}_r(t)\|_\infty &= \gamma_3 \end{aligned}$$

where

$$\begin{aligned} \gamma_1 &= \sup_{t\in[0,\infty)} |z_r(t)| \\ \gamma_2 &= \sup_{t\in[0,\infty)} |\dot{z}_r(t)| \\ \gamma_3 &= \sup_{t\in[0,\infty)} |\ddot{z}_r(t)| \end{aligned}$$

For an electromagnetic active suspension system, the damper is replaced by an electromagnetic actuator (Martins et al., 2006). In this configuration, it is assumed that $\mathcal{F}_c(\dot{z}) \approx 0$.

The mathematical model of the two degree-of-freedom suspension system is then described by the following two coupled nonlinear differential equations:

$$\begin{aligned} m_s\ddot{z}_s + \mathcal{F}_{sc} + \mathcal{F}_{sk} &= u \\ m_u\ddot{z}_u + k_t(z_u - z_r) - \mathcal{F}_{sc} - \mathcal{F}_{sk} &= -u \end{aligned} \tag{1}$$

with

$$\mathcal{F}_{sk}\left(z_s, z_u\right) = k_s(z_s - z_u) + k_{ns}\left(z_s - z_u\right)^3$$
$$\mathcal{F}_{sc}\left(\dot{z}_s, \dot{z}_u\right) = c_s(\dot{z}_s - \dot{z}_u) + c_{ns}(\dot{z}_s - \dot{z}_u)^2 \text{sgn}(\dot{z}_s - \dot{z}_u)$$

where $sgn(\cdot)$ denotes the standard signum function.
Defining the state variables as $x_1 = z_s$, $x_2 = \dot{z}_s$, $x_3 = z_u$ and $x_4 = \dot{z}_u$, one obtains the following state-space description:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\tfrac{1}{m_s}\left(\mathcal{F}_{sc} + \mathcal{F}_{sk}\right) + \tfrac{1}{m_s}u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -\tfrac{k_t}{m_u}x_3 + \tfrac{1}{m_u}\left(\mathcal{F}_{sc} + \mathcal{F}_{sk}\right) - \tfrac{1}{m_u}u + \tfrac{k_t}{m_u}z_r \end{aligned} \tag{2}$$

with

$$\mathcal{F}_{sk}\left(x_1, x_3\right) = k_s(x_1 - x_3) + k_{ns}\left(x_1 - x_3\right)^3$$
$$\mathcal{F}_{sc}\left(x_2, x_4\right) = c_s(x_2 - x_4) + c_{ns}(x_2 - x_4)^2 \text{sgn}(x_2 - x_4)$$

It is easy to verify that the nonlinear vehicle suspension system (2) is completely controllable and observable and, therefore, is differentially flat and constructible. For more details on this topics we refer to (Fliess et al., 1993) and the book by (Sira-Ramirez & Agrawal, 2004). Both properties can be used extensively during the synthesis of different controllers based on differential flatness, trajectory planning, disturbance and state reconstruction, parameter identification, Generalized PI (GPI) and sliding mode control, etc. See, e.g., (Beltran-Carbajal et al., 2010a); (Beltran-Carbajal et al., 2010b); (Chavez-Conde et al., 2009a); (Chavez-Conde et al., 2009b).
In what follows, a feedforward and feedback linearization active vibration controller, as well as a disturbance observer, will be designed taking advantage of the differential flatness property exhibited by the vehicle suspension system.

3. Differential flatness-based control

The system (2) is differentially flat, with a flat output given by

$$L = m_s x_1 + m_u x_3$$

which is constructed as a linear combination of the displacements of the sprung mass x_1 and the unsprung mass x_3.
Then, all the state variables and the control input can be parameterized in terms of the flat output L and a finite number of its time derivatives (Sira-Ramirez & Agrawal, 2004). As a matter of fact, from L and its time derivatives up to fourth order one can obtain:

$$\begin{aligned} L &= m_s x_1 + m_u x_3 \\ \dot{L} &= m_s x_2 + m_u x_4 \\ \ddot{L} &= k_t\left(z_r - x_3\right) \\ L^{(3)} &= k_t\left(\dot{z}_r - x_4\right) \\ L^{(4)} &= \tfrac{1}{m_u}u + \tfrac{k_t}{m_u}x_3 - \tfrac{1}{m_u}\left(\mathcal{F}_{sc} + \mathcal{F}_{sk}\right) - \tfrac{k_t}{m_u}z_r + k_t\ddot{z}_r \end{aligned} \tag{3}$$

Therefore, the differential parameterization of the state variables and the control input in the vehicle dynamics (2) results as follows

$$\begin{aligned} x_1 &= \tfrac{m_u}{k_t m_s}\ddot{L} + \tfrac{1}{m_s}L - \tfrac{m_u}{m_s}z_r \\ x_2 &= \tfrac{m_u}{k_t m_s}L^{(3)} + \tfrac{1}{m_s}\dot{L} - \tfrac{m_u}{m_s}\dot{z}_r \\ x_3 &= -\tfrac{1}{k_t}\ddot{L} + z_r \\ x_4 &= -\tfrac{1}{k_t}L^{(3)} + \dot{z}_r \\ u &= \tfrac{1}{b}\left(L^{(4)} + a_3 L^{(3)} + a_2\ddot{L} + a_1\dot{L} + a_0 L - \xi\left(t\right)\right) \end{aligned} \tag{4}$$

with

$$\begin{aligned} a_0 &= \tfrac{k_s k_t}{m_s m_u} \\ a_1 &= \tfrac{c_s k_t}{m_s m_u} \\ a_2 &= \tfrac{k_s}{m_s} + \tfrac{k_s + k_t}{m_u} \\ a_3 &= \tfrac{c_s}{m_s} + \tfrac{c_s}{m_u} \\ b &= \tfrac{k_t}{m_u} \end{aligned}$$

and

$$\begin{aligned} \xi\left(t\right) = &-\tfrac{k_{ns}k_t}{m_u}\left(x_1 - x_3\right)^3 - \tfrac{c_{ns}k_t}{m_u}(x_2 - x_4)^2\mathrm{sgn}(x_2 - x_4) \\ &+ k_t\ddot{z}_r + \left(\tfrac{k_t}{m_s} + \tfrac{k_t}{m_u}\right)c_s\dot{z}_r + \left(\tfrac{k_t}{m_s} + \tfrac{k_t}{m_u}\right)k_s z_r \end{aligned}$$

Now, note that from the last equation in the differential parameterization (4), one can see that the flat output satisfies the following perturbed input-output differential equation:

$$L^{(4)} + a_3 L^{(3)} + a_2\ddot{L} + a_1\dot{L} + a_0 L = bu + \xi\left(t\right) \tag{5}$$

Then, the flat output dynamics can be described by the following 4th order perturbed linear system:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ \dot{\eta}_3 &= \eta_4 \\ \dot{\eta}_4 &= -a_0\eta_1 - a_1\eta_2 - a_2\eta_3 - a_3\eta_4 + bu + \xi\left(t\right) \\ y &= \eta_1 = L \end{aligned} \tag{6}$$

To formulate the vibration control problem, let us assume, by the moment, a perfect knowledge of the perturbation term ξ, as well as the time derivatives of the flat output up to third order. Then, from (6) one obtains the following differential flatness-based controller:

$$u = \frac{1}{b}v + \underline{\frac{1}{b}\left(a_3\eta_4 + a_2\eta_3 + a_1\eta_2 + a_0\eta_1 - \xi\left(t\right)\right)} \tag{7}$$

with

$$v = -\alpha_3\eta_4 - \alpha_2\eta_3 - \alpha_1\eta_2 - \alpha_0\eta_1$$

The use of this controller yields the following closed-loop dynamics:

$$L^{(4)} + \alpha_3 L^{(3)} + \alpha_2\ddot{L} + \alpha_1\dot{L} + \alpha_0 L = 0 \tag{8}$$

The closed-loop characteristic polynomial is then given by

$$p(s) = s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \tag{9}$$

Therefore, by selecting the design parameters α_i, $i = 0, \cdots, 3$, such that the associated characteristic polynomial for (8) be *Hurwitz*, one can guarantee that the flat output dynamics be globally asymptotically stable, i.e.,

$$\lim_{t \to \infty} L(t) = 0$$

Now, the following Hurwitz polynomial is proposed to get the corresponding controller gains:

$$p_c(s) = \left(s^2 + 2\zeta_c \omega_c s + \omega_c^2\right)^2 \tag{10}$$

where $\omega_c > 0$ *and* $\zeta_c > 0$ are the natural frequency and damping ratio of the desired closed-loop dynamics, respectively.
Equating term by term the coefficients of both polynomials (9) and (10), one obtains that

$$\begin{aligned} \alpha_0 &= \omega_c^4 \\ \alpha_1 &= 4\omega_c^3 \zeta_c \\ \alpha_2 &= 4\omega_c^2 \zeta_c^2 + 2\omega_c^2 \\ \alpha_3 &= 4\omega_c \zeta_c \end{aligned}$$

On the other hand, it is easy to show that the closed-loop system (2)-(7) is L_∞-stable or bounded-input-bounded-state, that is,

$$\begin{aligned} \|x_1\|_\infty &= \frac{m_u}{m_s}\gamma_1 \\ \|x_2\|_\infty &= \frac{m_u}{m_s}\gamma_2 \\ \|x_3\|_\infty &= \gamma_1 \\ \|x_4\|_\infty &= \gamma_2 \\ \|u\|_\infty &= \left(k_{ns}\gamma_1^3\rho^2 + c_{ns}\rho\gamma_2^2 + c_s\gamma_2 + k_s\gamma_1\right)\rho + m_u\gamma_3 \end{aligned}$$

where $\rho = \frac{m_u}{m_s} + 1$.
It is evident, however, that the controller (8) requires the perfect knowledge of the exogenous perturbation signal z_r and its time derivatives up to second order, revealing several disadvantages with respect to other control schemes. Nevertheless, one can take advantage of the design methodology of robust observers with respect to unmodeled perturbation inputs, of the polynomial type affecting the observed plant, proposed by (Sira-Ramirez et al., 2008b). The proposed disturbance observer is called Generalized Proportional Integral (GPI) observer, because its design approach is the dual counterpart of the so-called GPI controllers (Fliess et al., 2002) and whose robust performance, with respect to unknown perturbation inputs, nonlinear and linear unmodeled dynamics and parametric uncertainties, have been evaluated extensively through experiments for trajectory tracking tasks on a vibrating mechanical system by (Sira-Ramirez et al., 2008a) and on a dc motor by (Sira-Ramirez et al., 2009).

4. Disturbance observer design

In the observer design process it is assumed that the perturbation input signal $\xi(t)$ can be locally approximated by a family of Taylor polynomials of $(r-1)$th degree:

$$\xi(t) = \sum_{i=0}^{r-1} p_i t^i \tag{11}$$

where all the coefficients p_i are completely unknown.
The perturbation signal could then be locally described by the following state-space based linear mathematical model:

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\vdots \\ \dot{\xi}_{r-1} &= \xi_r \\ \dot{\xi}_r &= 0 \end{aligned} \tag{12}$$

where $\xi_1 = \xi$, $\xi_2 = \dot{\xi}$, $\xi_3 = \ddot{\xi}$, $\cdots$, $\xi_r = \xi^{(r-1)}$.
An extended approximate state model for the perturbed flat output dynamics is then given by

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ \dot{\eta}_3 &= \eta_4 \\ \dot{\eta}_4 &= -a_0\eta_1 - a_1\eta_2 - a_2\eta_3 - a_3\eta_4 + \xi_1 + bu \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\vdots \\ \dot{\xi}_{r-1} &= \xi_r \\ \dot{\xi}_r &= 0 \\ y &= \eta_1 = L \end{aligned} \tag{13}$$

A Luenberger observer for the system (13) is given by

$$\begin{aligned} \dot{\widehat{\eta}}_1 &= \widehat{\eta}_2 + \beta_{r+3}(y - \widehat{y}) \\ \dot{\widehat{\eta}}_2 &= \widehat{\eta}_3 + \beta_{r+2}(y - \widehat{y}) \\ \dot{\widehat{\eta}}_3 &= \widehat{\eta}_4 + \beta_{r+1}(y - \widehat{y}) \\ \dot{\widehat{\eta}}_4 &= -a_0\widehat{\eta}_1 - a_1\widehat{\eta}_2 - a_2\widehat{\eta}_3 - a_3\widehat{\eta}_4 + \widehat{\xi}_1 + bu + \beta_r(y - \widehat{y}) \\ \dot{\widehat{\xi}}_1 &= \widehat{\xi}_2 + \beta_{r-1}(y - \widehat{y}) \\ \dot{\widehat{\xi}}_2 &= \widehat{\xi}_3 + \beta_{r-2}(y - \widehat{y}) \\ &\vdots \\ \dot{\widehat{\xi}}_{r-1} &= \widehat{\xi}_r + \beta_1(y - \widehat{y}) \\ \dot{\widehat{\xi}}_r &= \beta_0(y - \widehat{y}) \\ \widehat{y} &= \widehat{\eta}_1 \end{aligned} \tag{14}$$

The dynamical system describing the state estimation error is readily obtained by subtracting the observer dynamics (14) from the extended linear system dynamics (6). One then obtains, with $e_1 = y - \widehat{y}$ and $e_{zi} = \xi_i - \widehat{\xi}_i$, $i = 1, 2, \cdots, r$, that

$$\begin{aligned}
\dot{e}_1 &= -\beta_{r+3} e_1 + e_2 \\
\dot{e}_2 &= -\beta_{r+2} e_1 + e_3 \\
\dot{e}_3 &= -\beta_{r+1} e_1 + e_4 \\
\dot{e}_4 &= -\left(\beta_r + a_0\right) e_1 - a_1 e_2 - a_2 e_3 - a_3 e_4 + e_{z_1} \\
\dot{e}_{z_1} &= -\beta_{r-1} e_1 + e_{z_2} \\
\dot{e}_{z_2} &= -\beta_{r-2} e_1 + e_{z_3} \\
&\vdots \\
\dot{e}_{z_{r-1}} &= -\beta_1 e_1 + e_{z_r} \\
\dot{e}_{z_r} &= -\beta_0 e_1
\end{aligned} \tag{15}$$

From this expression, it is not difficult to see that the dynamics of output observation error $e_1 = y - \hat{y}$ satisfies the following differential equation:

$$\begin{aligned}
& e_1^{(r+4)} + \left(\beta_{r+3} + a_3\right) e_1^{(r+3)} + \left(\beta_{r+2} + a_2 + \beta_{r+3} a_3\right) e_1^{(r+2)} \\
& + \left(\beta_{r+1} + a_1 + \beta_{r+2} a_3 + \beta_{r+3} a_2\right) e_1^{(r+1)} \\
& + \left(\beta_r + a_0 + \beta_{r+1} a_3 + \beta_{r+2} a_2 + \beta_{r+3} a_1\right) e_1^{(r)} \\
& + \beta_{r-1} e_1^{(r-1)} + \cdots + \beta_2 \ddot{e}_1 + \beta_1 \dot{e}_1 + \beta_0 e_1 = 0
\end{aligned} \tag{16}$$

which is completely independent of any coefficients p_i, $i = 0, \cdots, r-1$, of the Taylor polynomial expansion of $\xi(t)$. This means that, the high-gain observer continuously self-updates. Therefore, as time goes on, the bounded perturbation input signal $\xi(t)$ is approximated in the form of a $(r-1)$th degree time polynomial.

Clearly, the coefficients of the associated characteristic polynomial for (16) can be adjusted, by means of a suitable specification of the design gains $\{\beta_{r+3}, \ldots, \beta_1, \beta_0\}$, sufficiently far from the imaginary axis in the left half of the complex plane, so that the output estimation error e_1 exponentially asymptotically converges to zero.

A fifth-order local mathematical model for the real-time estimation of the perturbation input signal is proposed in this chapter. Then, the characteristic polynomial for the dynamics of output observation error is simply given by

$$\begin{aligned}
p_{o1}(s) = {} & s^9 + \left(\beta_8 + a_3\right) s^8 + \left(\beta_7 + a_2 + \beta_8 a_3\right) s^7 + \left(\beta_6 + a_1 + \beta_7 a_3 + \beta_8 a_2\right) s^6 \\
& + \left(\beta_5 + a_0 + \beta_6 a_3 + \beta_7 a_2 + \beta_8 a_1\right) s^5 + \beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0
\end{aligned} \tag{17}$$

Equating the coefficients of the characteristic polynomial (17) with the corresponding ones of the following ninth-order Hurwitz polynomial:

$$p_{do1}(s) = (s + p_1)\left(s^2 + 2\zeta_1 \omega_1 s + \omega_1^2\right)^4 \tag{18}$$

one gets the observer gains as follows

$$\begin{aligned}
\beta_0 &= p_1\omega_1^8 \\
\beta_1 &= \omega_1^8 + 8p_1\zeta_1\omega_1^7 \\
\beta_2 &= 8\omega_1^7\zeta_1 + 24p_1\omega_1^6\zeta_1^2 + 4p_1\omega_1^6 \\
\beta_3 &= 24\omega_1^6\zeta_1^2 + 4\omega_1^6 + 32p_1\omega_1^5\zeta_1^3 + 24p_1\omega_1^5\zeta_1 \\
\beta_4 &= 32\omega_1^5\zeta_1^3 + 24\omega_1^5\zeta_1 + 16p_1\omega_1^4\zeta_1^4 + 48p_1\omega_1^4\zeta_1^2 + 6p_1\omega_1^4 \\
\beta_5 &= 16\omega_1^4\zeta_1^4 + 48\omega_1^4\zeta_1^2 + 6\omega_1^4 + 32p_1\omega_1^3\zeta_1^3 + 24p_1\omega_1^3\zeta_1 - a_0 - \beta_6 a_3 - \beta_7 a_2 - \beta_8 a_1 \\
\beta_6 &= 32\omega_1^3\zeta_1^3 + 24\omega_1^3\zeta_1 + 24p_1\omega_1^2\zeta_1^2 + 4p_1\omega_1^2 - a_1 - \beta_7 a_3 - \beta_8 a_2 \\
\beta_7 &= 24\omega_1^2\zeta_1^2 + 4\omega_1^2 + 8p_1\omega_1\zeta_1 - a_2 - \beta_8 a_3 \\
\beta_8 &= p_1 + 8\omega_1\zeta_1 - a_3
\end{aligned}$$

with $p_1, \omega_1, \zeta_1 > 0$.
Now, consider that the system (6) is being perturbed by the unknown input signal $\varpi(t)$ as follows

$$\begin{aligned}
\dot{\eta}_1 &= \eta_2 \\
\dot{\eta}_2 &= \eta_3 \\
\dot{\eta}_3 &= \eta_4 \\
\dot{\eta}_4 &= bu + \varpi(t) \\
y &= \eta_1 = L
\end{aligned} \tag{19}$$

with

$$\varpi(t) = -a_0\eta_1 - a_1\eta_2 - a_2\eta_3 - a_3\eta_4 + \xi(t)$$

Then, by the perfect knowledge of $\varpi(t)$, one gets the following differential flatness-based controller with feedforward of the perturbation signal $\varpi(t)$:

$$u = \frac{1}{b}(v - \varpi(t)) \tag{20}$$

with

$$v = -\alpha_3\eta_4 - \alpha_2\eta_3 - \alpha_1\eta_2 - \alpha_0\eta_1$$

Similarly as before, the perturbation input signal ϖ could be locally reconstructed by the following family of Taylor polynomial of $(r-1)$th degree:

$$\varpi(t) = \sum_{i=0}^{r-1} \gamma_i t^i \tag{21}$$

where all the coefficients γ_i are completely unknown.
Then, one can use the following extended mathematical model described in state-space form to design a robust observer for real-time estimation of the disturbance $\varpi(t)$ and time

derivatives of the flat output required for implementation of the controller (20):

$$\begin{aligned}
\dot{\eta}_1 &= \eta_2 \\
\dot{\eta}_2 &= \eta_3 \\
\dot{\eta}_3 &= \eta_4 \\
\dot{\eta}_4 &= \varpi_1 + bu \\
\dot{\varpi}_1 &= \varpi_2 \\
\dot{\varpi}_2 &= \varpi_3 \\
&\vdots \\
\dot{\varpi}_{r-1} &= \varpi_r \\
\dot{\varpi}_r &= 0 \\
y &= \eta_1 = L
\end{aligned} \tag{22}$$

where $\varpi_1 = \varpi$, $\varpi_2 = \dot{\varpi}$, $\varpi_3 = \ddot{\varpi}$, $\cdots$, $\varpi_r = \varpi^{(r-1)}$.
A Luenberger observer for the system (22) is then given by

$$\begin{aligned}
\dot{\widehat{\eta}}_1 &= \widehat{\eta}_2 + \lambda_{r+3}\left(y - \widehat{y}\right) \\
\dot{\widehat{\eta}}_2 &= \widehat{\eta}_3 + \lambda_{r+2}\left(y - \widehat{y}\right) \\
\dot{\widehat{\eta}}_3 &= \widehat{\eta}_4 + \lambda_{r+1}\left(y - \widehat{y}\right) \\
\dot{\widehat{\eta}}_4 &= \widehat{\varpi_1} + bu + \lambda_r\left(y - \widehat{y}\right) \\
\dot{\widehat{\varpi}}_1 &= \widehat{\varpi}_2 + \lambda_{r-1}\left(y - \widehat{y}\right) \\
\dot{\widehat{\varpi}}_2 &= \widehat{\varpi}_3 + \lambda_{r-2}\left(y - \widehat{y}\right) \\
&\vdots \\
\dot{\widehat{\varpi}}_{r-1} &= \widehat{\varpi}_r + \lambda_1\left(y - \widehat{y}\right) \\
\dot{\widehat{\varpi}}_r &= \lambda_0\left(y - \widehat{y}\right) \\
\widehat{y} &= \widehat{\eta}_1
\end{aligned} \tag{23}$$

The estimation error dynamics $e_1 = y - \widehat{y}$ satisfies the following dynamics:

$$\begin{aligned}
&e_1^{(r+4)} + \lambda_{r+3}e_1^{(r+3)} + \lambda_{r+2}e_1^{(r+2)} + \lambda_{r+1}e_1^{(r+1)} + \lambda_r e_1^{(r)} \\
&+\lambda_{r-1}e_1^{(r-1)} + \cdots + \lambda_2\ddot{e} + \lambda_1\dot{e}_1 + \lambda_0 e_1 = 0
\end{aligned} \tag{24}$$

Therefore, the design parameters λ_i, $i = 0, \cdots, r+3$, can be chosen so that the output estimation error e_1 exponentially asymptotically converges to zero.
On the other hand, it is assumed that the perturbation input signal $\varpi(t)$ can be locally approximated by a family of Taylor polynomials of fourth degree. Therefore, the characteristic polynomial for the dynamics of output observation error (24) is given by

$$p_{o2} = s^9 + \lambda_8 s^8 + \lambda_7 s^7 + \lambda_6 s^6 + \lambda_5 s^5 + \lambda_4 s^4 + \lambda_3 s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0 \tag{25}$$

It is then proposed the following Hurwitz polynomial to compute the proper gains for the observer:

$$p_{o2}(s) = (s + p_2)\left(s^2 + 2\zeta_2\omega_2 s + \omega_2^2\right)^4 \tag{26}$$

One then obtains that

$$\begin{aligned}
\lambda_0 &= p_2\omega_2^8 \\
\lambda_1 &= \omega_2^8 + 8p_2\zeta_2\omega_2^7 \\
\lambda_2 &= 8\omega_2^7\zeta_2 + 24p_2\omega_2^6\zeta_2^2 + 4p_2\omega_2^6 \\
\lambda_3 &= 24\omega_2^6\zeta_2^2 + 4\omega_2^6 + 32p_2\omega_2^5\zeta_2^3 + 24p_2\omega_2^5\zeta_2 \\
\lambda_4 &= 32\omega_2^5\zeta_2^3 + 24\omega_2^5\zeta_2 + 16p_2\omega_2^4\zeta_2^4 + 48p_2\omega_2^4\zeta_2^2 + 6p_2\omega_2^4 \\
\lambda_5 &= 16\omega_2^4\zeta_2^4 + 48\omega_2^4\zeta_2^2 + 6\omega_2^4 + 32p_2\omega_2^3\zeta_2^3 + 24p_2\omega_2^3\zeta_2 \\
\lambda_6 &= 32\omega_2^3\zeta_2^3 + 24\omega_2^3\zeta_2 + 24p_2\omega_2^2\zeta_2^2 + 4p_2\omega_2^2 \\
\lambda_7 &= 24\omega_2^2\zeta_2^2 + 4\omega_2^2 + 8p_2\omega_2\zeta_2 \\
\lambda_8 &= p_2 + 8\omega_2\zeta_2
\end{aligned}$$

with $p_2, \omega_2, \zeta_2 > 0$.

From the practical viewpoint, main advantage of this high-gain observer is that it could be employed for hydraulic or electromagnetic active vehicle suspension systems, requiring only information of the stiffness constant of tire k_t and the unsprung mass m_u. In addition, it can be shown that the proposed observer design methodology is quite robust with respect to parameter uncertainty and unmodeled dynamics, by considering the parameter variations into the perturbation input signal $\varpi(t)$. In fact, in (Sira-Ramirez et al., 2008a) has been presented through some experimental results that the polynomial disturbance signal-based GPI control scheme, implemented as a classical compensation network, is robust enough with respect to parameter uncertainty and unmodeled dynamics in the context of an off-line and pre-specified reference trajectory tracking tasks.

It is important to emphasize that, the proposed results are now possible thanks to the existence of commercial embedded system for automatic control tasks based on high speed FPGA/DSP boards with high computational performance operating at high sampling rates. The proposed observer could be implemented via embedded software applications without many problems.

5. Simulation results

Some numerical simulations were performed on a nonlinear quarter-vehicle suspension system characterized by the following set of realistic parameters (Tahboub, 2005) to verify the effectiveness of the proposed disturbance observer-control design methodology (see Table 1):

Parameters	Values
Sprung mass (m_s)	216.75 $[kg]$
Unsprung mass (m_u)	28.85 $[kg]$
Spring stifness (k_s)	21700 $[\frac{N}{m}]$
Damping constant (c_s)	1200 $[\frac{N\cdot s}{m}]$
Tire stifness (k_t)	184000 $[\frac{N}{m}]$
nonlinear spring stiffness (k_{ns})	2170 $[\frac{N}{m}]$
nonlinear damping constant (c_{ns})	120 $[\frac{N\cdot s}{m}]$

Table 1. Parameters of the vehicle suspension system.

Fig. 2 shows some schematic diagram for the implementation of the proposed active vibration controllers based on on-line disturbance estimation using a flatness-based controller and GPI observers.

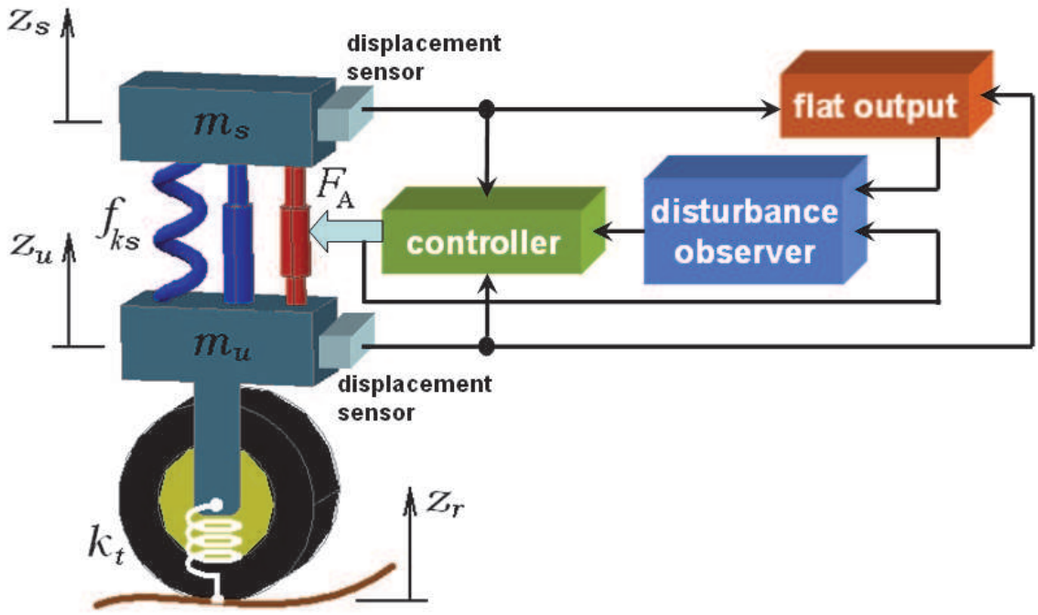

Fig. 2. Schematic diagram of the instrumentation for active vehicle suspension control implementation.

The following trajectory was utilized to simulate the unknown exogenous disturbance excitations due to irregular road surfaces (Chen & Huang, 2005):

$$z_r(t) = \begin{cases} f_1(t) + f(t) & \text{for } t \in [3.5, 5) \\ f_2(t) + f(t) & \text{for } t \in [5, 6.5) \\ f_3(t) + f(t) & \text{for } t \in [8.5, 10) \\ f_3(t) + f(t) & \text{for } t \in [10, 11.5 \\ f(t) & \text{else} \end{cases}$$

with

$$\begin{aligned} f_1(t) &= -0.0592(t-3.5)^3 + 0.1332(t-3.5)^2 \\ f_2(t) &= 0.0592(t-6.5)^3 + 0.1332(t-6.5)^2 \\ f_3(t) &= 0.0592(t-8.5)^3 - 0.1332(t-8.5)^2 \\ f_3(t) &= -0.0592(t-11.5)^3 - 0.1332(t-11.5)^2 \\ f(t) &= 0.002\sin(2\pi t) + 0.002\sin(7.5\pi t) \end{aligned}$$

Figs. 3-9 describe the robust performance of the controller (7) using the observer (14). It can be seen the high vibration attenuation level of the active vehicle suspension system compared with the passive counterpart.

Moreover, one can observe a robust and fast on-line estimation of the disturbance $\xi(t)$ as well as the corresponding time derivatives of the flat output up to third order. Similar results on the implementation of the controller (20) with disturbance observer (23) for estimation of the perturbation $\varpi(t)$ and time derivatives of the flat output are shown in Figs. 10-23.
In the computer simulations it is assumed that the perturbation input signals $\xi(t)$ and $\varpi(t)$ can be locally approximated by a family of Taylor polynomials of fourth degree.
The characteristic polynomials for the ninth order observation error dynamics were all set to be of the following form:

$$p_o(s) = (s + p_o)\left(s^2 + 2\zeta_o\omega_o s + \omega_o^2\right)^4$$

with $p_o = \omega_o = 300\text{rad/s}$ and $\zeta_o = 20$.
The characteristic polynomials associated with the closed-loop dynamics were all set to be of the form: $p_c(s) = \left(s^2 + 2\zeta_c\omega_c s + \omega_c^2\right)^2$, with $\omega_c = 10\text{rad/s}$ and $\zeta_c = 0.7071$.

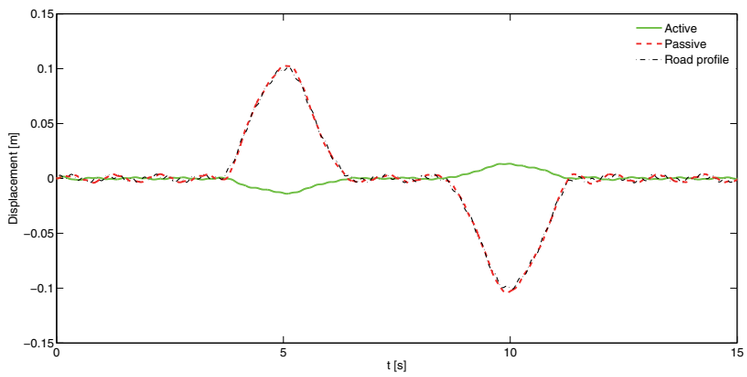

Fig. 3. Sprung mass displacement response using controller (7) and observer (14).

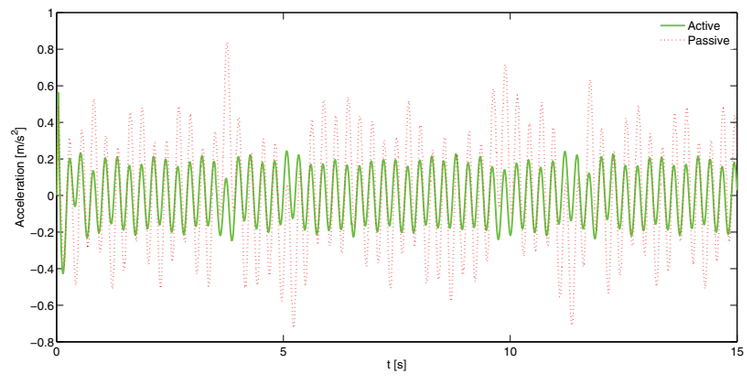

Fig. 4. Sprung mass acceleration response using controller (7) and observer (14).

In general, the proposed active vehicle suspension using a flatness-based controller and GPI observers for the estimation of unknown perturbations yields good attenuation properties and an overall robust performance.

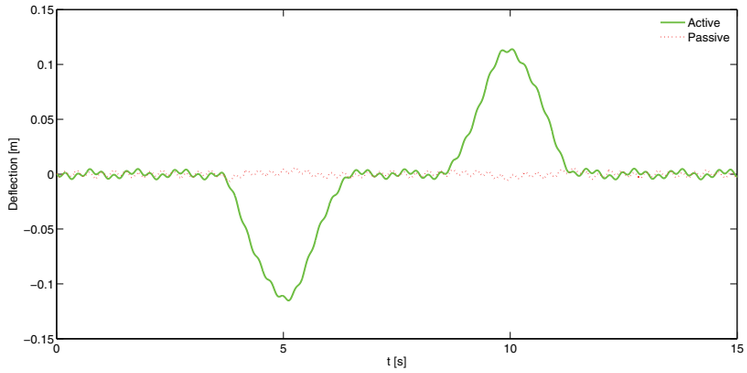

Fig. 5. Suspension deflection response ($x_1 - x_3$) using controller (7) and observer (14).

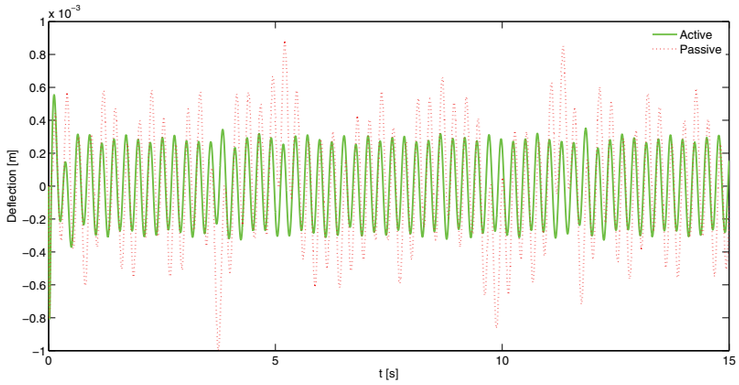

Fig. 6. Tire deflection response ($x_3 - z_r$) using controller (7) and observer (14).

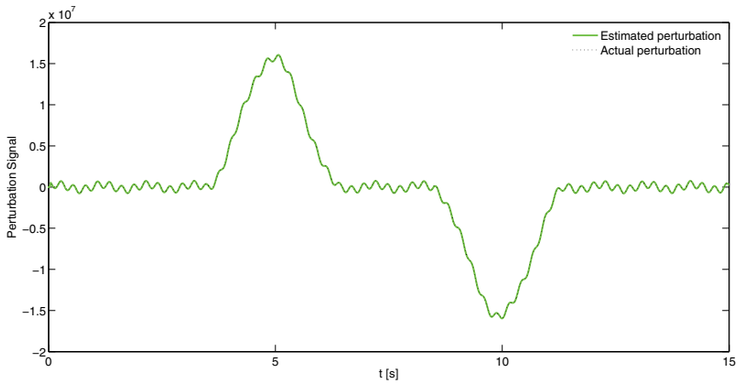

Fig. 7. Perturbation estimation $\xi(t)$ using observer (14).

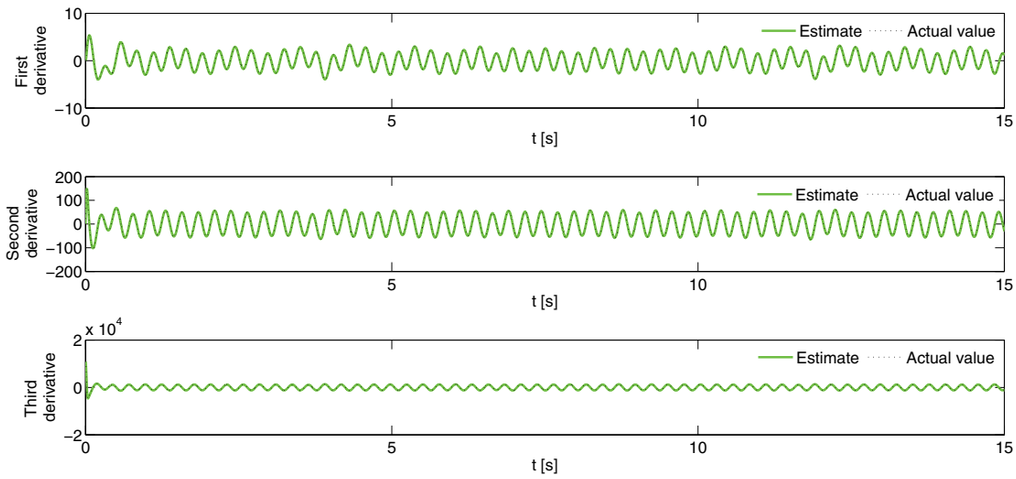

Fig. 8. Estimation of time derivatives of the flat output using the observer (14).

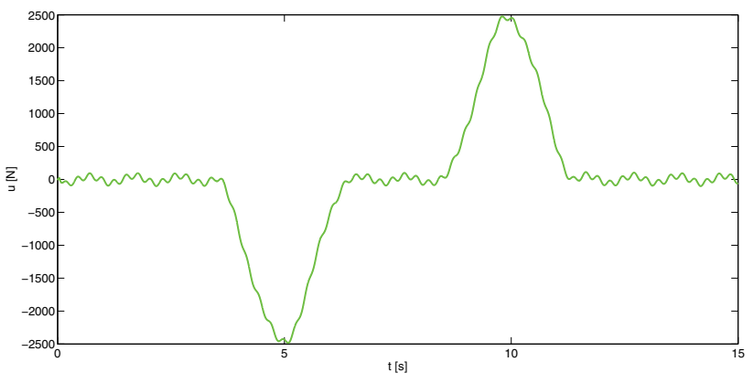

Fig. 9. Control force using the observer (14).

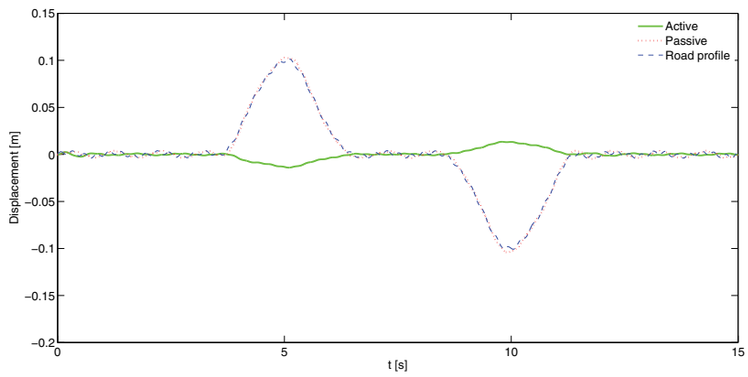

Fig. 10. Sprung mass displacement response using controller (20) and observer (23).).

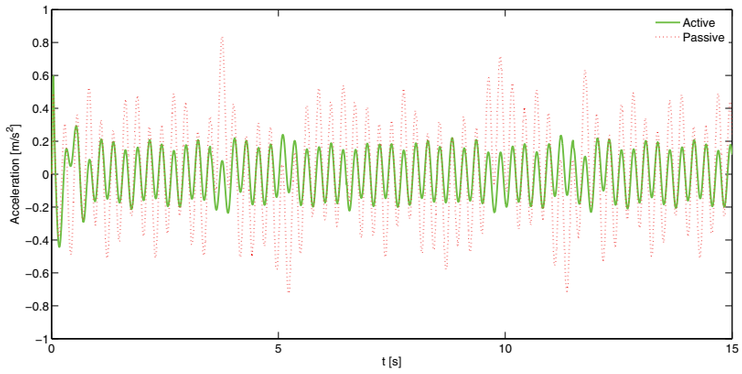

Fig. 11. Sprung mass acceleration response using controller (20) and observer (23).

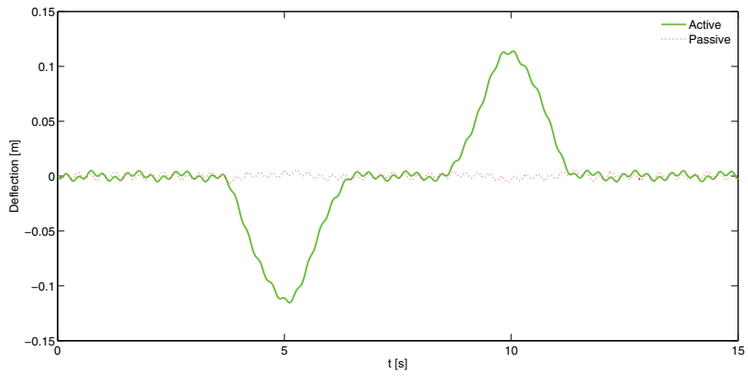

Fig. 12. Suspension deflection response ($x_1 - x_3$) using controller (20) and observer (23).

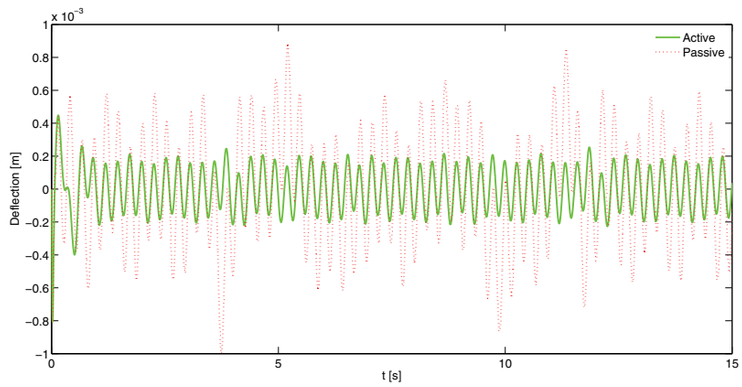

Fig. 13. Tire deflection response ($x_3 - z_r$) using controller (20) and observer (23).

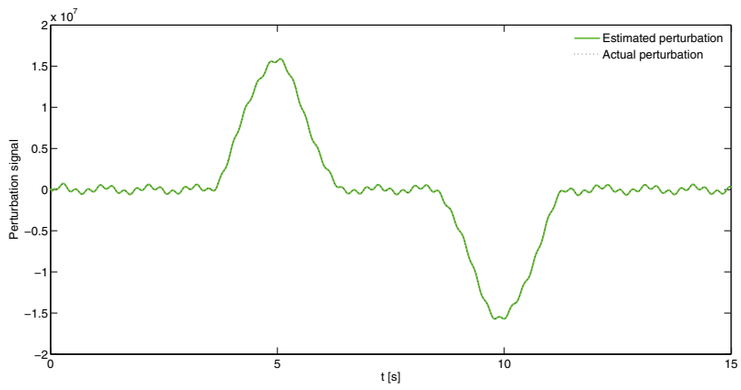

Fig. 14. Perturbation estimation $\varpi(t)$ using observer (23).

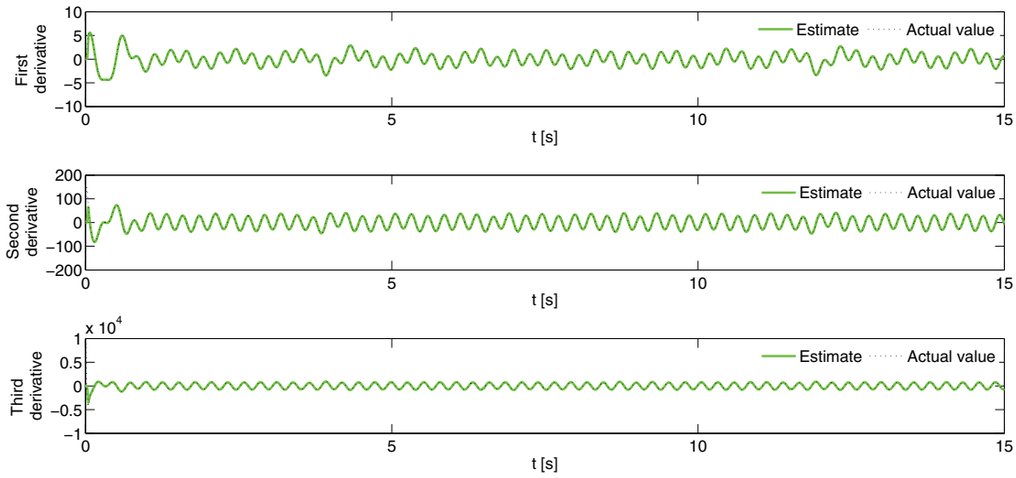

Fig. 15. Estimation of time derivatives of the flat output using the observer (23).

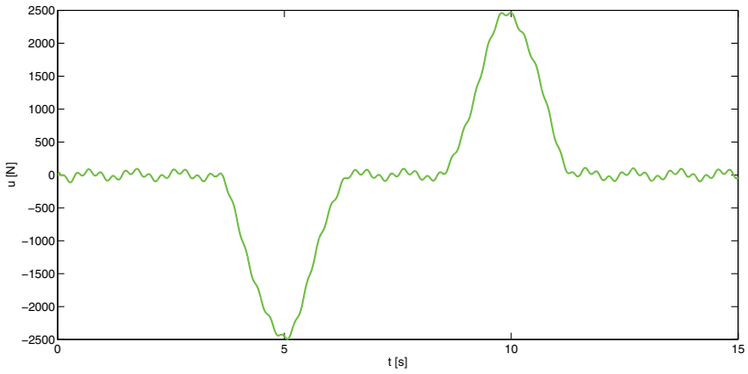

Fig. 16. Control force using the observer (23).

6. Conclusions

In this chapter a robust active vibration control scheme, based on real-time estimation and rejection of perturbation signals, of nonlinear vehicle suspension systems is described. The proposed approach exploits the structural property of differential flatness exhibited by the suspension system fot the synthesis of a flatness based controller and a robust observer. Therefore, a perturbed input-output differential equation describing the dynamics of the flat output is obtained for design purposes of the control scheme. The exogenous disturbances due to irregular road surfaces, nonlinear effects, parameter variations and unmodeled dynamics are lumped into an unknown bounded time-varying perturbation input signal affecting the differentially flat linear simplified dynamic mathematical model of the suspension system. A family of Taylor polynomials of (r-1)th degree is used to locally approximate this perturbation signal. Hence the perturbation signal is described by a rth-order mathematical model. Then, the perturbed suspension system model is expressed as a (r+4)th-order extended mathematical model.

The design of high-gain Luenberger observers, based on this kind of extended models, is proposed to estimate the perturbation signal and some time derivatives of the flat output required for implementation of differential flatness-based disturbance feedforward and feedback controllers for attenuation of vibrations in electromagnetic and hydraulic active vehicle suspension systems.

Two high-gain disturbance observer-based controllers have been proposed to attenuate the vibrations induced by unknown exogenous disturbance excitations due to irregular road surfaces, which could be employed for nonlinear quarter-vehicle active suspension models by using hydraulic or electromagnetic actuators. Computer simulations were included to show the effectiveness of the proposed controllers, as well as of the disturbance observers based on Taylor polynomials of fourth degree.

The results show a high vibration attenuation level of the active vehicle suspension system compared with the passive counterpart and, in addition, a robust and fast real-time estimation of the disturbance and time derivatives of the flat output.

7. References

Ahmadian, M. Active control of vehicle suspensions. In: *Encyclopedia of Vibration*, Edited by Braun, S.G., Ewins, D.J. & Rao, S.S. (2001), Vols. 1-3, Academic Press, San Diego, CA.

Basterretxea, K., Del Campo, I. & Echanobe, J. (2010). A semi-active suspension embedded controller in a FPGA, *2010 IEEE International Symposium on Industrial Embedded Systems*, pp. 69-78, Trento, July 7-9.

Beltran-Carbajal, F., Silva-Navarro, G., Blanco-Ortega, A. & Chavez-Conde, E. (2010a). Active Vibration Control for a Nonlinear Mechanical System using On-line Algebraic Identification, In: *Vibration Control*, M. Lallart, (Ed.), 201-214, Sciyo, Rijeka, Croatia.

Beltran-Carbajal, F., Silva-Navarro, G., Sira-Ramirez, H. & Blanco-Ortega, A. (2010b). Application of on-line algebraic identification in active vibration control, *Computación y Sistemas*, Vol. 13, No. 3, pp. 313-330.

Cao, J., Liu, H., Li, P. & Brown, D. (2008). State of the Art in Vehicle Active Suspension Adaptive Control Systems Based on Intelligent Methodologies, *IEEE Transaction on Intelligent Transportation Systems*, Vol. 9, No. 3, pp. 392-405.

Choi, S.B., Lee, H.K. & Chang, E.G. (2001). Field test results of a semi-active ER suspension system associated with skyhook controller, *Mechatronics*, Vol. 11, pp. 345-353.

Chavez-Conde, E., Beltran-Carbajal, F., Garcia-Rodriguez, C. & Blanco-Ortega, A. (2009a). Sliding Mode Based Differential Flatness Control and State Estimation of Vehicle Active Suspensions System, *IEEE International Conference on Electrical Engineering, Computing Science and Automatic Control*, pp. 544-549, Toluca, Mexico, November 10-13.

Chavez-Conde, E., Beltran-Carbajal, F., Blanco-Ortega, A. & Mendez-Azua, H. (2009b). Sliding Mode and Generalized PI Control of Vehicle Active Suspensions, *18th IEEE International Conference on Control Applications*, pp. 1726-1731, Saint Petersburg, Russia, July 8-10.

Chen, P. & Huang, A. (2005). Adaptive sliding control of non-autonomous active suspension systems with time-varying loadings, *Journal of Sound and Vibration*, Vol. 282, pp. 1119-1135.

Fliess, M., Marquez, R., Delaleau, E. & Sira-Ramirez, H. (2002). Correcteurs Proportionnels-Integraux Généralisés, *ESAIM Control, Optimisation and Calculus of Variations*, Vol. 7, pp. 23-41.

Fliess, M., Marquez, R. & Delaleau, E. (2001). State feedbacks without asymptotic observers and generalized PID regulators, Nonlinear Control in the Year 2000, *Lecture Notes in Control and Information Sciences*, Vol. 258, pp. 367-384, Springer, London.

Fliess, M., Lévine, J., Martin, Ph. & Rouchon, P. (1993). Flatness and defect of nonlinear systems: introductory theory and examples, *International Journal of Control*, Vol. 61, No. 6, pp. 1327-1361.

Gysen, B.L.J., Paulides, J.J.H., Janssen, J.L.G. & Lomonova, E. A. (2008). Active Electromagnetic Suspension System for Improved Vehicle Dynamics, *IEEE Vehicle Power and Propulsion Conference (VPPC)*, Harbin, China, September 3-5.

Isermann, R. & Munchhpf, M. (2011). *Identification of Dynamic Systems*, Springer-Verlag, Berlin.

Martins, I., Esteves, J., Marques, D.G. & Da Silva, F. P. (2006). Permanent-Magnets Linear Actuators Applicability in Automobile Active Suspensions, *IEEE Trans. on Vehicular Technology*, Vol. 55, No. 1, pp. 86-94.

Sira-Ramirez, H., Beltran-Carbajal, F. & Blanco-Ortega, A. (2008a). A Generalized Proportional Integral Output Feedback Controller for the Robust Perturbation Rejection in a Mechanical System, *e-STA Sciences et Technologies de l'Automatique*, Vol. 5, No. 4, pp. 24-32.

Sira-Ramirez, H., Feliu-Batlle, V., Beltran-Carbajal, F. & Blanco-Ortega, A. (2008b). Sigma-Delta modulation sliding mode observers for linear systems subject to locally unstable inputs, *16th Mediterranean Conference on Control and Automation*, pp. 344-349, Ajaccio, France, June 25-27.

Sira-Ramirez, H., Barrios-Cruz, E. & Marquez-Contreras, R.J. (2009). Fast adaptive trajectory tracking control for a completely uncertain DC motor via output feedback, *Computación y Sistemas*, Vol. 12, No. 4, pp. 397-408.

Sira-Ramirez, H., Silva-Navarro, G. & Beltran-Carbajal, F. (2007). On the GPI balancing control of an uncertain Jeffcot rotor model, *2007 4th International Conference on Electrical and Electronics Engineering (ICEEE)*, pp. 306-309, Mexico City, Mexico, September 5-7.

Sira-Ramirez, H. & Agrawal, S.K. (2004). *Differentially Flat Systems*, Marcel Dekker, New York.

Shoukry, Y., El-Kharashi, M. W. & Hammad, S. (2010). MPC-On-Chip: An Embedded GPC Coprocessor for Automotive Active Suspension Systems, *IEEE Embedded Systems Letters*, Vol. 2, No. 2, pp. 31-34.

Tahboub, K. A. (2005). Active Nonlinear Vehicle-Suspension Variable-Gain Control, *13th Mediterranean Conference on Control and Automation*, pp. 569-574, Limassol, Cyprus, June 27-29.

van der Schaft, A. (2000). *L_2-Gain and Passivity Techniques in Nonlinear Control*, Springer, London.

Ventura, P.J.C., Ferreira, C.D.H., Neves, C. F. C. S., Morais, R.M.P., Valente, A.L.G. & Reis, M.J.C.S. (2008). An embedded system to assess the automotive shock absorber condition under vehicle operation, *IEEE Sensor 2008 Conference*, pp. 1210-1213, Lecce, October 26-29.

Yao, G.Z., Yap, F.F., Chen, G., Li, W.H. & Yeo, S.H. (2002). MR damper and its application for semi-active control of vehicle suspension system, *Mechatronics*, Vol. 12, pp. 963-973.

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