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# Information Sharing: a Quantitative Approach to a Class of Integrated Supply Chain

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## 1. Introduction

The literature on the incorporating information on multi-echelon inventory systems is relatively recent. Milgrom & Roberts (1990) identified the information as a substitute for inventory systems from economical points of view. Lee & Whang (1998) discuss the use of information sharing in supply chains in practice, relate it to academic research and outline the challenges facing the area. Cheung & Lee (1998) examine the impact of information availability in order coordination and allocation in a Vendor Managed Inventory (VMI) environment. Cachon & Fisher (2000) consider an inventory system with one supplier and  $N$  identical retailers. Inventories are monitored periodically and the supplier has information about the inventory position of all the retailers. All locations follow an  $(R, nQ)$  ordering policy with the supplier's batch size being an integer multiple of that of the retailers. Cachon and Fisher (2000) show how the supplier can use such information to allocate the stocks to the retailers more efficiently.

Xiaobo and Minmin (2007) consider four different information sharing scenarios in a two-stage supply chain composed of a supplier and a retailer. They analyse the system costs for the various information sharing scenarios to show their impact on the supply chain performance.

Information sharing is regarded to be one of the key approaches to tame the bullwhip effect (Kelepouris et. al, 2008). Kelepouris et. al (2008) examine the operational aspect of the bullwhip effect, studying both the impact of replenishment parameters on bullwhip effect and the use of point-of-sale (POS) data sharing to tame the effect. They simulate a real situation in their model and study the impact of smoothing and safety factors on bullwhip effect and product fill rates. Also they demonstrate how the use of sharing POS data by the upper stages of a supply chain can decrease their orders' oscillations and inventory levels held.

Gavirneni (2002) illustrates how information flows in supply chains can be better utilized by appropriately changing the operating policies in the supply chain. The author considers a supply chain containing a capacitated supplier and a retailer facing independent and identically distributed demands. In his setting two models were considered. (1) the retailer is using the optimal  $(s, S)$  policy and providing the supplier information about her inventory levels; and (2) the retailer, still sharing information on her inventory levels, orders in a

period only if by the previous period the cumulative end-customer demand since she last ordered was greater than a specified value. In model 1, information sharing is used to supplement existing policies, while in model 2; operating policies were redefined to make better use of the information flows.

Hsiao & Shieh (2006) consider a two-echelon supply chain, which contains one supplier and one retailer. They investigate the quantification of the bullwhip effect and the value of information sharing between the supplier and the retailer under an autoregressive integrated moving average (ARIMA) demand of  $(0,1,q)$ . Their results show that with an increasing value of  $q$ , bullwhip effects will be more obvious, no matter whether there is information sharing or not. They show when the information sharing policy exists, the value of the bullwhip effect is greater than it is without information sharing. With an increasing value of  $q$ , the gap between the values of the bullwhip effect in the two cases will be larger.

Poisson models with one-for-one ordering policies can be solved very efficiently. Sherbrooke (1968) and Graves (1985) present different approximate methods. Seifbarghi & Akbari (2006) investigate the total cost for a two-echelon inventory system where the unfilled demands are lost and hence the demand is approximately a Poisson process. Axsäter (1990a) provides exact solutions for the Poisson models with one-for-one ordering policies. For special cases of  $(R, Q)$  policies, various approximate and exact methods have been presented in the literature. Examples of such methods are Deuermeyer & Schwarz (1981), Moinzadeh and Lee (1986), Lee & Moinzadeh (1987a), Lee and Moinzadeh (1987b), Svoronos and Zipkin (1988), (Axsäter, Forsberg, & Zhang, 1994), Axsäter (1990b), Axsäter (1993b) and Forsberg (1996). As a first step, Axsäter (1993b) expressed costs as a weighted mean of costs for one-for-one ordering policies. He exactly evaluated holding and shortage costs for a two-level inventory system with one warehouse and  $N$  different retailers. He also expressed the policy costs as a weighted mean of costs for one-for-one ordering policies. Forsberg (1995) considers a two-level inventory system with one warehouse and  $N$  retailers. In Forsberg (1995), the retailers face different compound Poisson demands. To calculate the compound Poisson cost, he uses Poisson costs from Axsäter (1990a).

Moinzadeh (2002), considered an inventory system with one supplier and  $M$  identical retailers. All the assumptions that we use in this paper are the same as the one he used in his paper, that is the retailer faces independent Poisson demands and applies continuous review  $(R, Q)$ -policy. Excess demands are backordered in the retailer. No partial shipment of the order from the supplier to the retailer is allowed. Delayed retailer orders are satisfied on a first-come, first-served basis. The supplier has online information on the inventory status and demand activities of the retailer. He starts with  $m$  initial batches (of size  $Q$ ), and places an order to an outside source immediately after the retailer's inventory position reaches  $R+s$ ,  $(0 \leq s \leq Q - 1)$ . It is also assumed that outside source has ample capacity.

To evaluate the total cost, using the results in Hadley & Whitin (1963) for one level-one retailer inventory system, Moinzadeh (2002) found the holding and backorder costs at each retailer and the holding cost at the supplier. The holding cost at each retailer is computed by the expected on hand inventory at any time (Hadley & Whitin, 1963). In the above system the lead time of the retailer is a random variable. This lead time is determined not only by the constant transportation time but also by the random delay incurred due to the availability of stock at the supplier. In his derivation Moinzadeh (2002) used the expected value of the retailer's lead time to approximate the lead time demand and pointed out that "the form of the optimal supplier policy in the context of our model is an open question and is possibly a complex function of the different combinations of inventory positions at all the

retailers in the system" (Moinzadeh, 2002). As Hadley and Whitin (1963) noted, treating the lead time as a constant equal to the mean lead time, when in actuality the lead time is a random variable, can lead to carrying a safety stock which is much too low. The amount of the error increases as the variance of the lead time distribution increases (Hadley & Whitin, 1963).

In this chapter, we, at first and in model 1, implicitly derive the exact probability distribution of this random variable and obtain the exact system costs as a weighted mean of costs for one-for-one ordering policies, using the Axsäter's (1990a) exact solutions for Poisson models with one-for-one ordering policies. Second, we, in the model 2 define a new policy for sharing information between stages of a three level serial supply chain and derive the exact value of the mean cost rate of the system. Finally, in the model 3, we define a modified ordering policy for a coverage supply chain consisting of two suppliers and one retailer to benefit from the advantage of information sharing. (Sajadifar et. al, 2008)

## 2. Model 1

In what follows we provide a detailed formulation of the basic problem explained above, and we show how to derive the total cost expression of this inventory system.

### 2.1 Problem formulation

We use the following notations:

- $S_0$  Supplier inventory position in an inventory system with a one- for-one ordering policy
- $S_1$  Retailer inventory position in an inventory system with a one-for-one ordering policy
- $L$  Transportation time from the supplier to the retailer
- $L_0$  Transportation time from the outside source to the supplier (Lead time of the supplier)
- $\lambda$  Demand intensity at the retailer
- $h$  Holding cost per unit per unit time at the retailer
- $h_0$  Holding cost per unit per unit time at the supplier
- $\beta$  Shortage cost per unit per unit time at the retailer
- $t_i$  Arrival time of the  $i$  th customer after time zero
- $c(S_0, S_1)$  Expected total holding and shortage costs for a unit demand in an inventory system with a one-for-one ordering policy
- $R$  The retailer's reorder point
- $Q$  Order quantity at both the retailer and the supplier
- $m$  Number of batches (of size  $Q$  ) initially allocated to the supplier
- $K$  Expected total holding and shortage costs for a unit demand
- $TC(R, m, s)$  Expected total holding and shortage costs of the system per time unit, when the supplier starts with  $m$  initial batches (of size  $Q$  ), and places an order to an outside source immediately after the retailer's inventory position reaches  $R + s$

Also we assume:

1. Transportation time from the outside source to the supplier is constant.
2. Transportation time from the supplier to the retailer is constant.
3. Arrival process of customer demand at the retailer is a Poisson process with a known and constant rate.
4. Each customer demands only one unit of product.

5. Supplier has online information on the inventory position and demand activities of the retailer.

To find  $K$ , the expected total holding and shortage costs for a unit demand, we express it as a weighted mean of costs for the one-for-one ordering policies. As we shall see, with this approach we do not need to consider the parameters  $L$ ,  $L_0$ ,  $h$ ,  $h_0$ , and  $\beta$  explicitly, but these parameters will, of course, affect the costs implicitly through the one-for-one ordering policy costs. To derive the one-for-one carrying and shortage costs, we suggest the recursive method in (Axsäter, 1990a and 1993b).

## 2.2 Deriving the model

To find the total cost, first, following the Axsäter's (1990a) idea, we consider an inventory system with one warehouse and one retailer with a one-for-one ordering policy. Also, as in Axsäter (1990a) let  $S_0$  and  $S_1$  indicate the supplier and the retailer inventory positions respectively in this system. When a demand occurs at the retailer, a new unit is immediately ordered from the supplier and the supplier orders a new unit at the same time. If demands occur while the warehouse is empty, shipment to the retailer will be delayed. When units are again available at the warehouse the demands at the retailer are served according to a first come first served policy. In such situation the individual unit is, in fact, already virtually assigned to a demand when it occurs, that is, before it arrives at the warehouse.

For the one-for-one ordering policy as described above, we can say that any unit ordered by the supplier or the retailer is used to fill the  $S_i^{th}$  ( $i = 0, 1$ ) demand following this order. In other words, an arbitrary customer consumes  $S_1^{th}$  ( $S_0^{th}$ ) order placed by the retailer (supplier) just before his arrival to the retailer. Axsäter (1990a) obtains the expected total holding and shortage costs for a unit demand, that is,  $c(S_0, S_1)$  for the one-for-one ordering policy.

In this paper, based on the one-for-one ordering policy as described above, we will show that the expected holding and shortage costs for the order of the  $j^{th}$  customer is exactly equal to the total costs for a unit demand in a base stock system with supplier and retailer's inventory positions  $S_0 = s + mQ$  and  $S_1 = R + j$  and so is equal to  $c(s + mQ, R + j)$  (A.12). Then, considering  $Q$  separate base stock systems in which the inventory positions of the supplier and the retailer for the  $j^{th}$  base stock system is  $s + mQ$  and  $R + j$  respectively, we obtain the exact value of  $TC(R, m, s)$ , the expected total holding and shortage costs per time unit for an inventory system with the following characteristics:

- The single retailer faces independent Poisson demand and applies continuous review  $(R, Q)$ -policy.
- The supplier starts with  $m$  initial batches (of size  $Q$ ) and places an order to an outside source immediately after the retailer's inventory position reaches  $R + s$ .
- The outside source has ample capacity.

We intend to show that

$$TC(R, m, s) = \frac{\lambda}{Q} \cdot \sum_{j=1}^Q c(s + mQ, R + j)$$

Figure 1 shows the inventory position of the retailer and the supplier between the time zero (the time the supplier places the order  $Q_0$ ) and the time the same order ( $Q_0$ ) will be sent to the retailer.

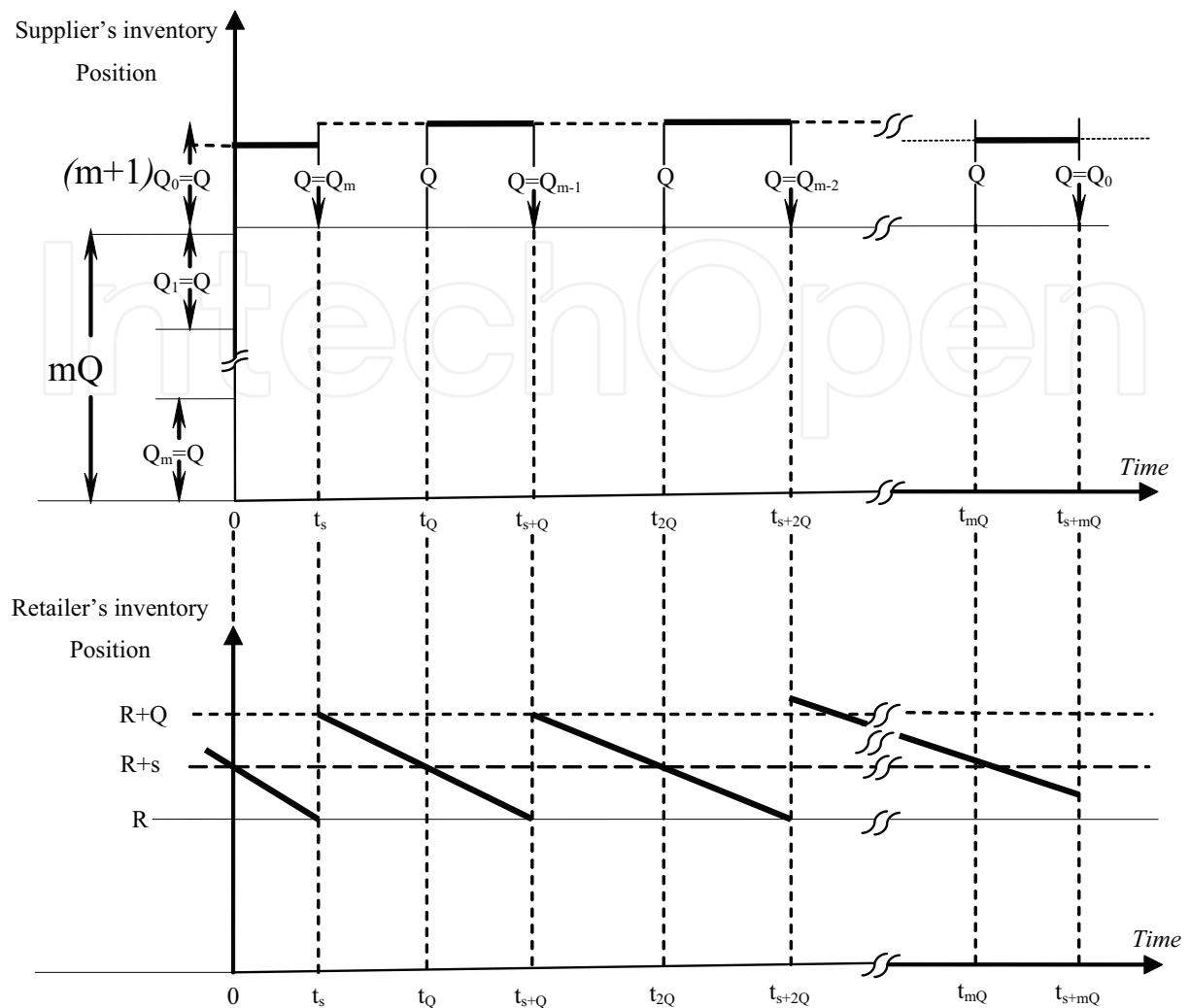


Fig. 1. Inventory position of the supplier and the retailer in  $[0, t_{s+mQ}]$

To prove this assertion, let us consider a time at which the supplier places an order to the outside source. We designate this time as time zero. We also denote the batch which the supplier orders at time zero by  $Q_0$ . At this time, the retailer's inventory position is exactly  $R + s$  and the supplier's inventory position will just reach  $(m+1)Q$ . Thus the batch  $Q_0$  will fill the  $(m+1)^{\text{th}}$  demand for the retailer batch at the warehouse. We denote the arrival times of customers who arrive after time zero by  $t_1, t_2, \dots$ . At time  $t_s$  when the  $s^{\text{th}}$  customer arrives, the retailer will order one batch of size  $Q$ , and the supplier's inventory position will drop to  $mQ$ . We note that after time zero, at the arrival time of  $(s+mQ)^{\text{th}}$  customer, i.e., at time  $t_{s+mQ}$ , the retailer will order a batch of size  $Q$ . This retailer's order will be fulfilled by the (same) batch  $Q_0$  that was ordered by the supplier at time zero. This means that the batch  $Q_0$  is released from the warehouse when  $(s+mQ)^{\text{th}}$  system demand has occurred after this order, i.e. after time zero.

The first unit in the batch  $Q_0$  will be used in the same way to fill the  $(R+1)^{\text{th}}$  retailer demand after the retailer order. Then the first unit in the batch  $Q_0$  will have the same expected retailer and warehouse costs as a unit in a base stock system with  $S_0=s+mQ$  and  $S_1=R+1$ . (the first base stock system) Therefore the corresponding expected holding and shortage costs will be equal to  $c(s+mQ, R+1)$  (A(12)).



In the same way it can be seen that the  $j^{th}$  unit in the batch  $Q_0$  will be used to fill the  $(R+j)^{th}$  retailer demand after the retailer order. Then the  $j^{th}$  unit in the batch  $Q_0$  will have the same expected retailer and warehouse costs as a unit in a base stock system with  $S_0=s+mQ$  and  $S_1=R+j$ . ( $j^{th}$  base stock system) Therefore the expected holding and shortage costs for the  $j^{th}$  unit in the batch  $Q_0$  will be equal to  $c(s+mQ, R+j)$ ,  $j=1,\dots,Q$  (A(12)).

It should be noted that each customer, demands only one unit of a batch of size  $Q$ . If we number the customers who use all  $Q$  units of this batch from 1 to  $Q$ , then the demand of any customer will be filled randomly by one of these  $Q$  units. That is, each unit of a batch of size  $Q$  will be consumed by the  $j^{th}$  ( $j=1,2,\dots,Q$ ) customer according to a discrete uniform distribution on  $1,2,\dots,Q$ . In other words, the probability that the  $i^{th}$  ( $i=1,2,\dots,Q$ ) unit of a batch of size  $Q$  is used by the  $j^{th}$  ( $j=1,2,\dots,Q$ ) customer is equal to  $1/Q$ . Therefore we can now express the expected total cost for a unit demand as:

$$K = \frac{1}{Q} \cdot \sum_{j=1}^Q c(s+mQ, R+j)$$

(1)

Since the average demand per unit of time is equal to  $\lambda$ , the total cost of the system per unit time can then be written as:

$$\begin{aligned} TC(R,m,s) &= \lambda \cdot K \\ &= \frac{\lambda}{Q} \cdot \sum_{j=1}^Q c(s+mQ, R+j) \end{aligned}$$

(2)

which proves our assertion.

3. Model 2

In this section, we consider a three-echelon inventory system with two warehouses (suppliers) and one retailer, as shown in Fig 2. This system usually called three-echelon serial inventory system. We want to find the expected total holding and shortage costs for a unit demand in three-echelon inventory system with two warehouses (suppliers) and one retailer.

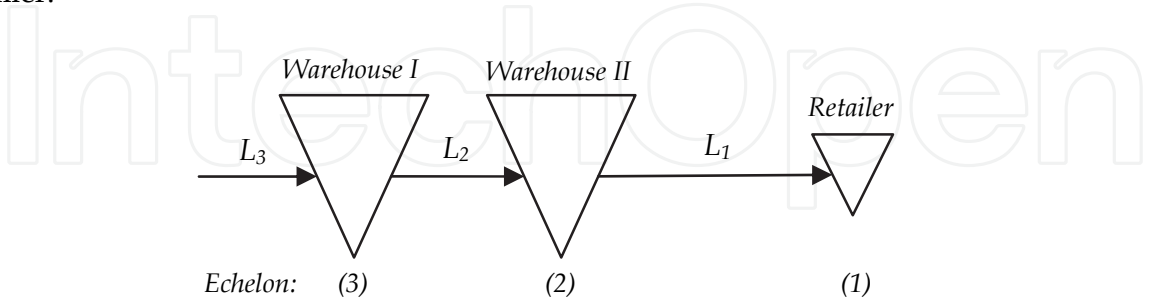


Fig. 2. Three-echelon Serial Inventory System

In this inventory system, transportation times from an outside source to the warehouse I, between warehouses, and also from the warehouse II to the retailers are constant. We assume that the retailer faces Poisson demand. Unfilled demand is backordered and the shortage cost is a linear function of time until delivery, or equivalently, a time average of the

net inventory when it is negative. Each echelon follows a base stock, or  $(S-1, S)$ , or one-for-one replenishment policies. This means essentially that we assume that ordering costs are low and can be disregarded.

The assumptions can be organized and presented as follows:

1. Transportation times between all locations are constant.
2. Arrival process of customer demand at the retailer is a Poisson process with a known and constant rate.
3. Each customer demands only one unit of product.
4. There are linear holding costs at all locations and shortage cost in the retailer.
5. Replenishment policies are one-for-one.
6. Unfilled demand is backordered and the shortage cost is a linear function of time until delivery.
7. Delayed retailer orders are satisfied on a first-come, first-served basis.
8. The outside source has ample capacity.

We fix the retailer, the warehouse II, and the warehouse I, to echelon one, two and three respectively as shown in Fig. 2. In order to derive the cost function, the following notations are used for serial inventory system:

$S_i$  Inventory position at echelon  $i$  in an inventory system with a one-for-one ordering policy

$L_1$  Transportation time from the Warehouse II to the retailer

$L_2$  Transportation time from the Warehouse I to the Warehouse II

$L_3$  Transportation time from the outside source to the Warehouse I (Lead time of the Warehouse I)

$T_i$  Random delay incurred due to the shortage of stock at the echelon  $i$  ( $i=2,3$ )

$\lambda$  Demand intensity at each echelon

$h_i$  Holding cost per unit per unit time at echelon  $i$  ( $i=1,2,3$ )

$\beta$  Shortage cost per unit per unit time at the retailer

We characterize our one-for-one replenishment policy by the  $(S_3, S_2, S_1)$  of order-up-to inventory positions which  $S_3, S_2, S_1$  are the inventory position at warehouse I (echelon 3), the inventory position at warehouse II (echelon 2), and the inventory position at retailer (echelon 1), respectively. So we consider a one-for-one replenishment rule with  $(S_3, S_2, S_1)$  as the vector of order-up-to levels.

When a demand occurs at a retailer with a demand density,  $\lambda$ , a new unit is immediately ordered from the warehouse II to warehouse I and also warehouse I immediately orders a new unit at the same time, that is, each echelon faces the same demand intensity ( $\lambda$ ). For the one-for-one ordering policy as described above, any unit ordered by the retailer is used to fill the  $S_1^{th}$  demand following this order, hereafter, referred to as its demand. It means that, an arbitrary customer consumes  $S_1^{th}$  order placed by the retailer just before his arrival to the retailer and we can also say that the customer consumes  $S_2^{th}$  ( $S_3^{th}$ ) order placed by the warehouse II (warehouse I) just before his arrival to the retailer. If the ordered unit arrives prior to its (assigned) demand, it is kept in stock and incurs carrying cost; if it arrives after its assigned demand, this customer demand is backlogged and shortage costs are incurred until the order arrives. This is an immediate consequence of the ordering policy and of our assumption that delayed demands and orders are filled on a first come, first served basis. We confine ourselves to the case where all  $S_i \geq 0$ .

To find the total cost, following the Axsäter's (1990a) idea, let  $g_i^{S_i}(\cdot)$  ( $i=1, 2, 3$ ) denote the density function of Erlang ( $\lambda, S_i$ ) distribution of the time elapsed between the placement of an order and the occurrence of its assigned demand unit:



$$g_i^{S_i}(t) = \frac{\lambda^{S_i} t^{S_i-1}}{(S_i-1)!} e^{-\lambda t} \quad (3)$$

The corresponding cumulative distribution function  $G_i^{S_i}(t)$  is:

$$G_i^{S_i}(t) = \sum_{k=S_i}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad (4)$$

An order placed by the retailer, arrives after  $L_1+T_2$  time units, and an order placed by warehouse II, arrives after  $L_2+T_3$  time units, where  $T_i$  ( $i=2,3$ ) is the random delay encountered at echelon  $i$  in case the echelon  $i$  is out of stock.

Let  $\pi_1^{S_1}(t_2)$  denotes the expected retailer carrying and shortage costs incurred to fill a unit of demand at retailer when inventory position at retailer is  $S_1$ . We evaluate this quantity by conditioning on  $T_2 = t_2$ . Note that the conditional expected cost is independent of  $S_2$  and  $S_3$ , and is given by:

$$\pi_1^{S_1}(t_2) = \beta \int_0^{L_1+t_2} (L_1+t_2-s) g_1^{S_1}(s) ds + h_1 \int_{L_1+t_2}^{\infty} (s-L_1-t_2) g_1^{S_1}(s) ds, S_1 > 0; \quad (5)$$

The conditional distribution of  $T_2$ , on condition that  $T_3=t_3$ , obtained from:

$$P\langle T_2 = 0 | T_3 = t_3 \rangle = \sum_{k=0}^{S_2-1} \frac{\lambda^k (L_2+t_3)^k}{k!} e^{-\lambda(L_2+t_3)} = 1 - G_2^{S_2}(L_2+t_3). \quad (7)$$

Also the conditional density function  $f(T_2)$  for  $0 \leq T_2 \leq L_2+t_3$  is given by:

$$f\langle T_2 = t_2 | T_3 = t_3 \rangle = g_2^{S_2}(L_2+t_3-t_2) = \frac{\lambda^{S_2} (L_2+t_3-t_2)^{S_2-1}}{(S_2-1)!} e^{-\lambda(L_2+t_3-t_2)} \quad (8)$$

The expression (6) shows the probability of time of receiving  $S_2^{th}$  demand; that is after receiving  $(S_2-1)^{th}$  demand,  $S_2^{th}$  demand occurs at the time of  $L_2+t_3-t_2$ . On the other view, we can say the time distance between receiving  $S_2^{th}$  demand and receiving the order from warehouse I ( $L_2+t_3$ ) is  $t_2$  and we call it the delay time that occurred in warehouse II. As we mentioned earlier the warehouses face a Poisson demand process with rate  $\lambda$ . Therefore we use the expression (5) in third echelon as follows:

$$P(T_3 = 0) = \sum_{k=0}^{S_3-1} \frac{\lambda^k (L_3)^k}{k!} e^{-\lambda L_3} = 1 - G_3^{S_3}(L_3) \quad (9)$$

The density function  $f(t_3)$  for  $0 \leq t_3 \leq L_3$ , because we assume that inventory positions at all facilities in this system are equal or greater than zero, is given by:

$$f(t_3) = g_3^{S_3}(L_3-t_3) = \frac{\lambda^{S_3} (L_3-t_3)^{S_3-1}}{(S_3-1)!} e^{-\lambda(L_3-t_3)} \quad (10)$$

Let  $\Pi_1^{S_1}(S_3, S_2)$  denotes the expected retailer carrying and shortage costs incurred to fill a unit of demand at retailer when  $S_3$ ,  $S_2$ , and  $S_1$  are the inventory position at warehouse I,

warehouse *II* and the retailer, respectively. Considering both states that we have delay time or have not in both warehouses, we obtain the cost that incurred to fill a unit of demand at retailer, as follows:

$$\begin{aligned} \Pi_1^{S_1}(S_3, S_2) = & (1 - G_3^{S_3}(L_3)) \left( \int_0^{L_2} g_2^{S_2}(L_2 - t_2) \pi_1^{S_1}(t_2) dt_2 + (1 - G_2^{S_2}(L_2)) \pi_1^{S_1}(0) \right) \\ & + \int_0^{L_3} g_3^{S_3}(L_3 - t_3) \left( \int_0^{L_2+t_3} g_2^{S_2}(L_2 + t_3 - t_2) \pi_1^{S_1}(t_2) dt_2 + (1 - G_2^{S_2}(L_2 + t_3)) \pi_1^{S_1}(0) \right) dt_3. \end{aligned} \quad (11)$$

The long-run average shortage and retailer carrying costs is clearly given by  $\lambda \Pi_1^{S_1}(S_3, S_2)$ . The conditional expected warehouse *II* holding cost,  $\pi_2^{S_2}(t_3)$ , on condition that  $T_3=t_3$ , is independent of  $S_3$  and given by:

$$\pi_2^{S_2}(t_3) = h_2 \int_{L_2+t_3}^{\infty} (s - L_2 - t_3) g_2^{S_2}(s) ds, S_2 > 0; \quad (12)$$

Therefore we find the average warehouse holding cost per unit for warehouse *II* when the inventory position at warehouse *I* is  $S_3$  as follows:

$$\Pi_2^{S_2}(S_3) = \int_0^{L_3} g_3^{S_3}(L_3 - t_3) \pi_2^{S_2}(t_3) dt_3 + (1 - G_3^{S_3}(L_3)) \pi_2^{S_2}(0). \quad (13)$$

Also the average warehouse *I* holding costs per unit  $\eta(S_3)$ , which depends only on the inventory position  $S_3$  is:

$$\eta(S_3) = h_3 \int_{L_3}^{\infty} g_3^{S_3}(s) (s - L_3) ds \quad (14)$$

and  $\eta(0) = 0$ .

We conclude that the long-run system-wide cost for the three-echelon serial inventory system by adding the costs which occurred in each echelon and is given by:

$$C(S_3, S_2, S_1) = \lambda (\Pi_1^{S_1}(S_3, S_2) + \Pi_2^{S_2}(S_3) + \eta(S_3)) \quad (15)$$

### 3.1 Determination the economical policy of a three-echelon inventory system with (R,Q) ordering policy and information sharing

In this section, we consider a three-echelon serial inventory system with two warehouses (suppliers) and one retailer with information exchange. The retailer applies continuous review (R,Q) policy. The warehouses have online information on the inventory position and demand activities of the retailer. The warehouse *I* and *II*, start with  $m_1$  and  $m_2$  initial batches of the same order size of the retailer, respectively. The warehouse *I* places an order to an outside source immediately after the retailer's inventory position reaches an amount equal to the retailer's order point plus a fixed value  $s_1$ , and The warehouse *II* places an order to

The warehouse *I* immediately after the retailer's inventory position reaches an amount equal to the retailer's order point plus a fixed value  $s_2$ . Transportation times are constant and the retailer faces independent Poisson demand. The lead times of the retailer and the warehouse *II*, are determined not only by the constant transportation time but also by the random delay incurred due to the availability of stock at the warehouses.

In order to find the total cost function for a unit demand in three echelon inventory system with  $(R, Q)$  ordering policy, first of all, we would present an  $(R, Q)$  ordering policy for a system with two warehouses and one retailer as showed in Fig. 2.

In this section, we want to obtain this cost function by using the cost function presented by the section 3, Hajiaghaei-keshteli and Sajadifar (2010), for the same system with one-for-one ordering policy.

We use the following notations:

- $S_i$  Echelon *i* inventory position in an inventory system with a one-for-one ordering policy
- $L_1$  Transportation time from the Warehouse *II* to the retailer
- $L_2$  Transportation time from the Warehouse *I* to the Warehouse *II*
- $L_3$  Transportation time from the outside source to the Warehouse *I* (Lead time of the Warehouse *I*)
- $\lambda$  Demand intensity at all echelons
- $h_i$  Holding cost per unit per unit time at echelon *i*
- $\beta$  Shortage cost per unit per time at the retailer
- $c(S_3, S_2, S_1)$  Expected total holding and shortage costs for a unit demand in an inventory system with a one-for-one ordering policy
- $R$  The retailer's reorder point
- $Q$  Order quantity at all locations
- $m_2$  Number of batches (of size  $Q$ ) initially allocated to the warehouse *II*
- $m_1$  Number of batches (of size  $Q$ ) initially allocated to the warehouse *I*
- $K$  Expected total holding and shortage costs for a unit demand
- $TC(R, m_1, m_2, s_1, s_2)$  Expected total holding and shortage costs of the system per time unit, when the warehouse *I* and warehouse *II*, start with  $m_1$  and  $m_2$  initial batches (of size  $Q$ ), and place an order in a batch of size  $Q$  to upper source immediately after the retailer's inventory position reaches  $R+s_1$  and  $R+s_2$  respectively.

As we shall see, with this approach we do not need to consider the parameters  $L_i$ ,  $h_i$ , and  $\beta$  explicitly, but these parameters will, of course, affect the costs implicitly through the one-for-one ordering policy costs.

When a demand occurs at the retailer, a new unit is immediately ordered from the warehouse *II* to the warehouse *I* and also the warehouse *I* immediately orders a new unit at the same time.

If demands occur while the warehouses are empty, shipments are delayed. When units are again available at the warehouses, delivered according to a first come, first served policy.

In such situation the individual unit is, in fact, already virtually assigned to a demand when it occurs, that is, before it arrives at the warehouses. For the one-for-one ordering policy, an arbitrary customer consumes  $(S_1+S_2+S_3)^{th}$ ,  $(S_1+S_2)^{th}$  and  $S_1^{th}$ , order placed by the warehouse *I*, warehouse *II*, and the retailer, respectively, just before his arrival to the retailer.

If the ordered unit arrives prior to its (assigned) demand, it is kept in stock and incurs carrying cost; if it arrives after its assigned demand, this customer demand is backlogged and shortage costs are incurred until the order arrives. This is an immediate consequence of

the ordering policy and of our assumption that delayed demands and orders are filled on a first come, first served basis.

To obtain  $TC(R, m_1, m_2, s_1, s_2)$ , we assume the warehouse *I* and *II* start with  $m_1$  and  $m_2$  initial batches (of size  $Q$ ) respectively. The warehouse *I* places an order to an outside source immediately after the retailer's inventory position reaches  $R+s_1$ , and warehouse *II* places an order to warehouse *I* immediately after the retailer's inventory position reaches  $R+s_2$ , while  $s_1$  is equal or greater than  $s_2$ .

Let us consider a time that the warehouse *I* places an order to the outside source. We set this time equal to "A". We also denote the batch which the warehouse *I* orders at time "A" by  $Q_A$ . At this time, the retailer's inventory position is just  $R+s_1$  and the warehouse *I*'s inventory position will just reach  $(m_1+1)Q$ .

After time "A", when the retailer's inventory position reaches  $R+s_2$ , warehouse *II* places an order to the warehouse *I* and her inventory position will just reach  $(m_2+1)Q$  and warehouse *I*'s inventory position will reach  $m_1Q$ . We set this time to "B".

After time "B", when  $s_2^{\text{th}}$  customer demand arrives, that is, the retailer inventory position reaches  $R$ , the retailer will order one batch (of size  $Q$ ), and the warehouse *II*'s inventory position will reach  $m_2Q$ .

We note that after time "A", at the arrival time of  $(m_1Q + s_1 - s_2)^{\text{th}}$  customer demand, the warehouse *II* will order a batch (of size  $Q$ ). This warehouse *II*'s order will be fulfilled by the (same) batch  $Q_A$ , that was ordered by the warehouse *I* at time "A", and after time "A", at the arrival time of  $(m_2Q + s_2)^{\text{th}}$  customer demand, the retailer will order a batch (of size  $Q$ ). This warehouse *II*'s order will be fulfilled by the (same) batch  $Q_B$  that was ordered by the warehouse *II* at time "B".

Besides after time "A", At the arrival time of  $(m_1Q + m_2Q + s_1)^{\text{th}}$  customer, the retailer will order a batch of size  $Q$ , this retailer's order will be fulfilled by the same batch  $Q_A$  that was ordered by the warehouse *I* at time "A". Figure 3 shows the inventory position of the retailer and the warehouses, as we detailed.

Furthermore, the  $(R+1)^{\text{th}}$  customer who arrives after this retailer's order, will use the first unit of this batch; this customer is  $(m_1Q+m_2Q+s_1+R+1)^{\text{th}}$  customer who arrives after time "A". This customer will incur a cost equal to  $c(m_1Q+s_1-s_2, m_2Q+s_2, R+1)$ , similar to  $c(S_3, S_2, S_1)$ , see equation (A.8), in which  $S_3$ ,  $S_2$ , and  $S_1$  are replaced by  $m_1Q+s_1-s_2$ ,  $m_2Q+s_2$ , and  $R+1$ , respectively.

The  $j^{\text{th}}$  unit ( $j=1, 2, \dots, Q$ ) in the batch will have to wait for the  $(R+j)^{\text{th}}$  customer who arrives after this retailer's order and it will incur a cost equal to  $c(m_1Q+s_1-s_2, m_2Q+s_2, R+j)$ , similar to (A.8), in which  $S_3$ ,  $S_2$ , and  $S_1$  are replaced by  $m_1Q+s_1-s_2$ ,  $m_2Q+s_2$ , and  $R+j$ , respectively.

It should be noted that each customer demands only one unit of a batch of size  $Q$ . if we number the customer who use all  $Q$  units of this batch from 1 to  $Q$ , then the demand of any customer will be filled randomly by one of these  $Q$  units. That is, each unit of a batch of size  $Q$  will be consumed by the  $j^{\text{th}}$  ( $j=1, 2, \dots, Q$ ) customer according to a discrete uniform distribution between  $[1, Q]$ . In other words, the probability that the  $i^{\text{th}}$  unit of a batch of size  $Q$  is used by  $j^{\text{th}}$  ( $j=1, 2, \dots, Q$ ) customer is equal to  $1/Q$ .

Therefore we can now express the expected total cost for a unit demand as:

$$K = \frac{1}{Q} \sum_{j=1}^Q c(m_1Q + s_1 - s_2, m_2Q + s_2, R + j) \quad (16)$$

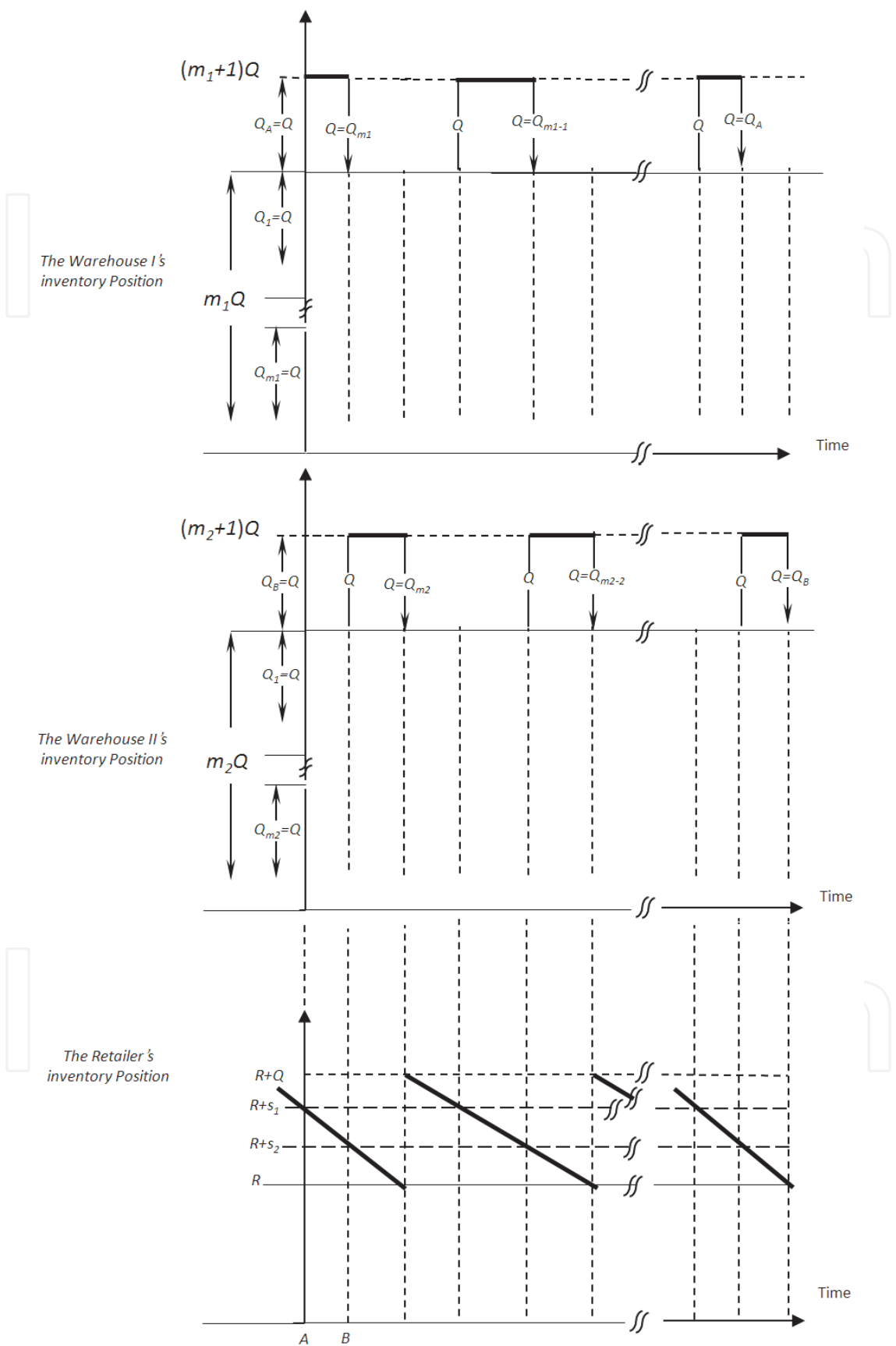


Fig. 3. Inventory position of the supplier and the warehouses

Since the average demand per unit of time is equal to  $\lambda$ , the total cost of the system per unit time can then be written as:

$$TC(R,m_1,m_2,s_1,s_2) = \lambda.K = \frac{\lambda}{Q} \sum_{j=1}^Q c(m_1Q + s_1 - s_2, m_2Q + s_2, R + j)$$

(17)

4. Model 3

In this section, we consider a single item, two-level inventory system which consisting of two suppliers and one retailer, as shown in Fig 4. Transportation times are constant. The retailer faces Poisson demands and applies continuous  $(R, Q)$  policy. Each supplier starts with  $m$  initial batches of size  $Q/2$  and places an order in a batch of size  $Q/2$  to an outside source immediately after the retailer’s inventory position reaches  $R+s$ . (Sajadifar et. al, 2008)

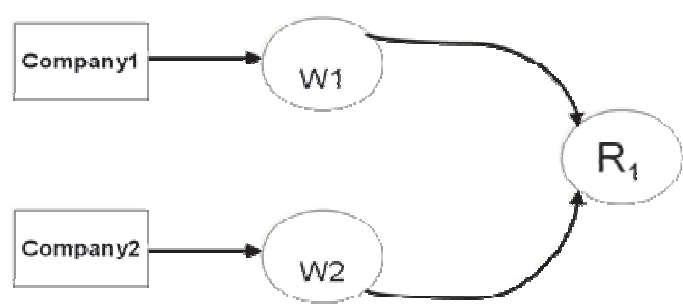


Fig. 4. A convergent two-level inventory system

4.1 Problem formulation

- The following notations are used for this system:
- $S_0$  Suppliers inventory position in an inventory system with a one- for-one ordering policy
  - $S_1$  Retailer inventory position in an inventory system with a one-for-one ordering policy
  - $L_i$  Transportation times from the supplier  $i$  to the retailer
  - $L_0^i$  Transportation times from the outside source to the supplier  $i$  (Lead time of the supplier)
  - $\lambda$  Demand intensity at the retailer
  - $h$  Holding cost per unit per unit time at the retailer
  - $h_0^i$  Holding cost per unit per unit time at the supplier  $i$
  - $\beta$  Shortage cost per unit per unit time at the retailer
  - $t_k$  Arrival time of the  $k^{th}$  customer after time zero
  - $\omega_i$  Random delay incurred due to the shortage of stock at the supplier  $i$
  - $X_i$  Lead time of the retailer when she receives a bath from the path  $i$
  - $P_{ij}$  The probability that path  $i$  is shorter than path  $j$ .
  - $g^n(t)$  Density function of the Erlang  $(\lambda, n)$
  - $G^n(t)$  Cumulative distribution function of  $g^n(t)$
  - $c_i(S_0, S_1)$  Expected total holding and shortage costs for a unit demand in an inventory system with a one-for-one ordering policy in path  $i$ .
  - $R$  The retailer’s reorder point
  - $Q$  Order quantity at the retailer
  - $m$  Number of batches (of size  $Q/2$ ) initially allocated to the suppliers
  - $K$  Expected total holding and shortage costs for a unit demand



$TC(R, m, s)$  Expected total holding and shortage costs of the system per time unit, when the suppliers starts with  $m$  initial batches (of size  $Q/2$ ), and places an order in a batch of size  $Q/2$  to outside sources immediately after the retailer's inventory position reaches  $R+s$ .

It can be seen that  $X_i = L_i + \omega_i$ . To find  $K$ , we express it as a weighted mean of costs for the one-for-one ordering policies. As we shall see, with this approach we do not need to consider the parameters  $L_i, L_0^i, h, h_0^i, \beta$  and  $\lambda$  explicitly, but these parameters will, of course, affect the costs implicitly through the one-for-one ordering policy costs. To derive the one-for-one carrying and shortage costs, we suggest the recursive method in (Axsäter, 1990a).

Also, we consider the following assumptions:

1. Orders do not cross, i.e. all orders/portions have arrived when the reorder point is reached and new orders are placed.
2. Each customer demands only one unit of product.
3. Each path that starts from outside source of the supplier  $i$  and end to the retailer is named by the path  $i$ . In other words the retailer receives each batch that shipped by the supplier  $i$  from the path  $i$  ( $i=1, 2$ ).
4. Delayed retailer orders are satisfied on a first-come, first-served basis.

#### 4.2 Deriving model

In this section, we use the method that (Haji and Sajadifar, 2008) introduced for evaluating the exact expected total costs of the inventory system, i.e., the exact expected total holding and shortage costs per time unit,  $TC(R, m, s)$ . To obtain  $TC(R, m, s)$ , using the (Axsäter, 1990a) exact solutions for Poisson models with one-for-one ordering policies they show that the expected holding and shortage costs for the order of the  $j^{th}$  customer is exactly equal to the total costs for a unit demand in a base stock system with suppliers and retailer's inventory positions  $S_0 = s + mQ$  and  $S_1 = R + j$  and so is equal to  $c(s + mQ, R + j)$ . (A.12)

Figure 5 shows the inventory position of the retailer and the each supplier between the time zero (the time the each supplier places the order  $Q_0/2$ ) and the time the same order ( $Q_0/2$ ) will be sent to the retailer.

Let us consider a time that inventory position of the retailer reaches to ' $R+s$ '. We designate this time as time zero. At this time, the suppliers immediately place an order equal to  $Q/2$  to the outside sources. We denote this batch by  $Q_0/2$ . At this time, the retailer's inventory position is exactly  $R+s$  and the suppliers' inventory positions will just reach  $(m+1)Q/2$ . Since we assume that the orders do not cross, the  $(m+1)^{th}$  order at the retailer will release the orders  $Q_0/2$  at the suppliers. It can be easily seen that the  $(s+mQ)^{th}$  customer at the retailer will be caused to an order placement at the retailer and the one which has been already assigned to this order at the suppliers are the batches  $Q_0/2$ . This means that the batch  $Q_0/2$  at each suppliers, is released from the warehouse when  $(s+mQ)^{th}$  system demand has occurred after time zero i.e. at  $t_{s+mQ}$ .

The batch  $Q_0/2$  will be received from the path  $i$  earlier than the batch  $Q_0/2$  from the path  $j$  with the probability  $P_{ij}$ . Therefore, the first unit in the batch  $Q_0/2$  (which will be received from path  $i$ ) will be used in the same way to fill the  $(R+1)^{th}$  retailer demand after the retailer order. Then the first unit in the batch  $Q_0/2$  will have the same expected retailer and warehouse costs as a unit in a base stock system with  $S_0 = s + mQ$  and  $S_1 = R + 1$  (Haji and Sajadifar 2008). Hence the corresponding expected holding and shortage costs will be equal to  $c_i(s + mQ, R + 1)$  (A(12)).

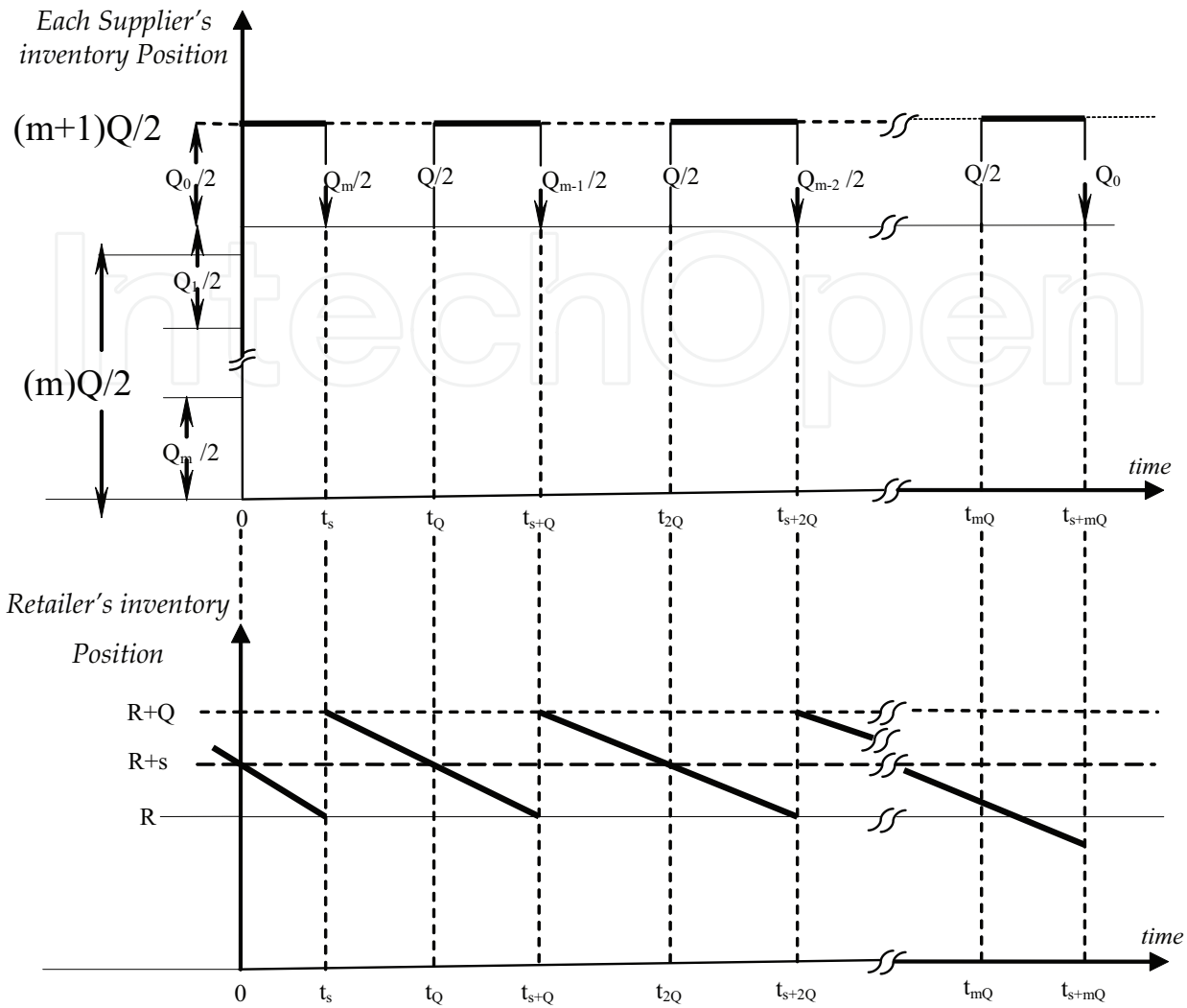


Fig. 5. Inventory position of the each supplier and the retailer in  $[0, t_s + mQ]$

In the same way it can be seen that the  $j^{\text{th}}$  unit in the batch  $Q_0/2$  (which will be received from the path  $i$ ), will be used to fill the  $(R+j)^{\text{th}}$  retailer demand after the retailer order. Then the  $j^{\text{th}}$  unit in the batch  $Q_0/2$  will have the same expected retailer and warehouse costs as a unit in a base stock system with  $S_0=s+mQ$  and  $S_1=R+j$ . Therefore the expected holding and shortage costs for the  $j^{\text{th}}$  unit in the batch  $Q_0/2$  will be equal to  $c_i(s+mQ, R+j)$ ,  $j=1, \dots, Q/2$  (A(12)). Similarly, one can easily see that the  $j^{\text{th}}$  unit in the batch  $Q_0/2$  (which will be received from the path  $j$ ), will be used to fill the  $(R+Q/2+j)^{\text{th}}$  retailer demand after the retailer order. Then this unit will have the same expected retailer and warehouse costs as a unit in a base stock system with  $S_0=s+mQ$  and  $S_1=R+Q/2+j$  and the expected holding and shortage costs for this unit will be equal to  $c_j(s+mQ, R+Q/2+j)$ ,  $j=1, \dots, Q/2$  (A(12)). It should be noted that each customer, demands only one unit of a batch. If we number the customers who use all  $Q$  units of these batches from 1 to  $Q$ , then the demand of any customer will be filled randomly by one of these  $Q$  units. That is, each unit of two batches of (total)size  $Q$  will be consumed by the  $j^{\text{th}}$  ( $j=1,2,\dots,Q$ ) customer according to a discrete uniform distribution on  $1,2,\dots,Q$ . In other words, the probability that the  $i^{\text{th}}$  ( $i=1,2,\dots,Q$ ) unit of two batches of (total)size  $Q$  is used by the  $j^{\text{th}}$  ( $j=1,2,\dots,Q$ ) customer is equal to  $1/Q$ . Therefore we can now express the expected total cost for a unit demand as:

$$\begin{aligned}
 k = & \frac{1}{Q} \left[ P_{12} \cdot \left( \sum_{i=1}^{Q/2} c_1(s + mQ, R + i) + \sum_{i=(Q/2)+1}^Q c_2(s + mQ, R + i) \right) \right. \\
 & \left. + \frac{1}{Q} \left[ P_{21} \cdot \left( \sum_{i=1}^{Q/2} c_2(s + mQ, R + i) + \sum_{i=(Q/2)+1}^Q c_1(s + mQ, R + i) \right) \right] \right]
 \end{aligned} \quad (17)$$

Since the average demand per unit of time is equal to  $\lambda$ , the total cost of the system per unit time can then be written as:

$$\begin{aligned}
 TC(R, m, s) = \lambda k = & \frac{\lambda}{Q} \left[ P_{12} \cdot \left( \sum_{i=1}^{Q/2} c_1(s + mQ, R + i) + \sum_{i=(Q/2)+1}^Q c_2(s + mQ, R + i) \right) \right. \\
 & \left. + \frac{\lambda}{Q} \left[ P_{21} \cdot \left( \sum_{i=1}^{Q/2} c_2(s + mQ, R + i) + \sum_{i=(Q/2)+1}^Q c_1(s + mQ, R + i) \right) \right] \right]
 \end{aligned} \quad (18)$$

**Corollary:** the probabilities  $P_{ij}$ , are computed as follows: ( $i, j = 1, 2$ , and  $P_{ij} + P_{ji} = 1$ )

- 1: If  $L_1 > L_2$  and  $L_0^1 > L_0^2$ , then  $P_{12} = 0$ .
- 2: If  $L_1 < L_2$  and  $L_0^1 < L_0^2$ , then  $P_{12} = 1$ .
- 3: If  $L_1 > L_2$ ,  $L_0^1 < L_0^2$ , and  $L_1 + L_0^1 < L_2 + L_0^2$ , then  $P_{12} = G^{s+mQ}(L_2 + L_0^2 - L_1)$ , (B.1).
- 4: If  $L_1 > L_2$ ,  $L_0^1 < L_0^2$ , and  $L_1 + L_0^1 > L_2 + L_0^2$ , then  $P_{12} = 0$ .
- 5: If  $L_1 < L_2$ ,  $L_0^1 > L_0^2$ , and  $L_1 + L_0^1 > L_2 + L_0^2$ , then  $P_{12} = G^{s+mQ}(L_1 + L_0^1 - L_2)$ .
- 6: If  $L_1 < L_2$ ,  $L_0^1 > L_0^2$ , and  $L_1 + L_0^1 < L_2 + L_0^2$ , then  $P_{12} = 1$ .

One can find the idea of the proofs in appendix B and more information about this section in (Sajadifar et. al, 2008).

## 5. Discussion

We, in model 1, derived the exact value of the total cost of the basic dyadic supply chain. In model 2.1 and 2.2 we, using the idea introduced in model 1, obtained the exact value of the expected total cost of the proposed inventory system. For demonstrating the effect of information sharing, we define three different types of scenarios each of which derives the benefits of sharing information between each echelon. Scenario 1: With Full information sharing, scenario 2: With semi information sharing and scenario 3: Without information sharing. For the first scenario, each echelon shares its online information to the upper echelon, that is,  $s_1$  and  $s_2$  are both positive integer. With semi information sharing, just echelon 1 shares its inventory position with echelon 2, then, only echelon 2 has online information about the retailer's inventory position, that is,  $s_1$  is a positive integer and  $s_2$  is zero. And for the last kind of relation between echelons, we assume in third scenario, that no echelon shares its online information about inventory position *that is* the both value of  $s_1$  and  $s_2$  are zero. It means that we have no  $s_i$  in this kind of relation. Numerical examples show that the total inventory system cost reduces when the information sharing is on effect. Table 1 consists of 6 pre-defined problems to show the IS effects.

Fig.6 shows the total cost of the inventory system for each problem and on each scenario. As one can easily find, the more the information would be shared between echelons, the less the total cost would be offered. Of course, from managerial point of view, the cost of

establishing information system must be considered for making any decision about sharing information. The model presented in subsection 2.2 can enhance one to derive and determine the exact value of shared information between each echelon.

| <i>Prob. No.</i> | <i>Q</i> | $\lambda$ | $\beta$ | $h_i$ | $L_i$ |
|------------------|----------|-----------|---------|-------|-------|
| 1                | 3        | 2         | 10      | 0.5   | 1     |
| 2                | 4        | 2         | 10      | 0.5   | 1     |
| 3                | 3        | 5         | 10      | 0.5   | 1     |
| 4                | 3        | 2         | 100     | 0.5   | 1     |
| 5                | 3        | 2         | 10      | 1     | 1     |
| 6                | 3        | 2         | 10      | 0.5   | 5     |

Table 1. Six Pre-Defined problems to show capability of three kinds of information sharing policy in cost reduction

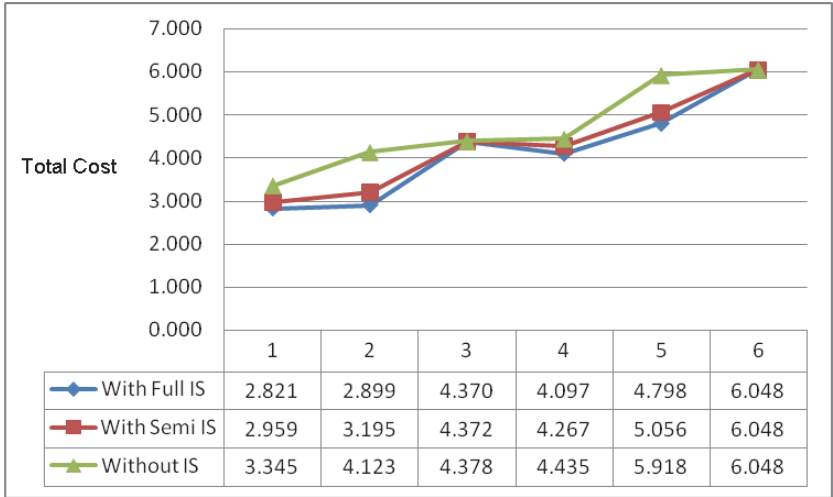


Fig. 6. Changing the  $TC^*$  in each scenario and in each problem.

In model 3, we expressed our findings as %deviation between average total cost rates between the two systems, in which:

$$\%deviation = \frac{TC_{\text{Without Information}} - TC_{\text{With Information}}}{TC_{\text{With Information}}} \times 100$$

For this purpose we fix all the parameters except  $\lambda$ ,  $L_1$ ,  $L_2$  and  $Q$ . These problems were constructed by taking all possible combinations of the following values of the parameters  $Q$ ,  $\lambda$ ,  $L_1$ , and  $L_2$ :  $Q=2,6,10, 20$ ;  $\lambda=2,5$  ;  $L_1, L_2=0, 0.5, 1, 1.5$  and  $2$ . We have assumed that the value of the parameters,  $L_0^1, L_0^2, h, h_0^1, h_0^2$  and  $\beta$  are constant and for instance are as:  $1,1,1,0.1,0.1$  and  $10$  respectively.

These numerical examples show that the savings resulting from our policy will decrease as the maximum possible lead time for an order increases. The value of information sharing will be minimal when  $Q$  is small or large. The most value of the shared information is 13% saving in total cost for  $\lambda=2$ ,  $Q=10$  and  $\sum L_i / \sum (L_i + L_0^i) = 0.2$ .

## 6. Conclusions

We, in this chapter, showed how to obtain the exact value of the total holding and shortage costs for a class of integrated two-level inventory system with information exchange. Three different models were introduced which incorporated the benefits of information sharing and we, using the idea of the one-for-one ordering policy, obtained the exact value of the expected total cost function for the inventory system in all of them. Resorting to some numerical examples generated by model 2.2, we demonstrated that increasing the information sharing between echelons of a serial supply chain can decrease the total integrated system costs. Also, analyzing the findings of model 3, we showed that the savings resulting from our policy decrease as the maximum possible lead time for an order increase, and the value of information sharing will be maximal when the order size is neither large nor small.

## 7. Appendix A

This Appendix is a summary of Axsäter, S. (1990a). For more details one can see Axsäter, S. (1990a)'s paper. We define (as in Axsäter, S. (1990a) for one retailer) the following notations:

$$g^{S_0}(t) = \text{Density function of the Erlang } (\lambda, S_0)$$

and,

$$G^{S_0}(t) = \text{Cumulative distribution function of } g^{S_0}(t).$$

Thus,

$$g^{S_0}(t) = \frac{\lambda^{S_0} t^{S_0-1}}{(S_0-1)!} e^{-\lambda t}, \quad (\text{A.1})$$

And,

$$G^{S_0}(t) = \sum_{k=S_0}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad (\text{A.2})$$

The average warehouse holding costs per unit is:

$$\gamma(S_0) = \frac{h_0^i S_0}{\lambda} (1 - G^{S_0+1}(L_0^i)) - h_0^i L_0^i (1 - G^{S_0}(L_0^i)), \quad S_0 > 0 \quad (\text{A.3})$$

And

$$\text{for } S_0 = 0, \quad \gamma(0) = 0. \quad (\text{A.4})$$

Given that the value of the random delay at the warehouse is equal to  $t$ , the conditional expected costs per unit at the retailer is:

$$\pi^{S_1}(t) = e^{-\lambda(L_i+t)} \frac{h+\beta}{\lambda} \sum_{k=0}^{S_1-1} \frac{(S_1-k)}{k!} (L_i+t)^k \lambda^k + \beta(L_i+t - \frac{S_1}{\lambda}) \quad (\text{A.5})$$

(0!=1 By definition),

The expected retailer's inventory carrying and shortage cost to fill a unit of demand is:

$$\Pi^{S_1}(S_0) = \int_0^{L_0^i} g_0^{S_0}(L_0^i - t) \pi^{S_1}(t) dt + (1 - G_0^{S_0}(L_0^i)) \pi^{S_1}(0) \quad (\text{A.6})$$

and,

$$\Pi^{S_1}(0) = \pi^{S_1}(L_0^i) \quad (\text{A.7})$$

Furthermore, for large value of  $S_0$ , we have

$$\Pi^{S_1}(S_0) \approx \pi^{S_1}(0) \quad (\text{A.8})$$

The procedure starts by determining  $\bar{S}_0$  such that

$$G^{\bar{S}_0}(L_0) < \varepsilon \quad (\text{A.9})$$

Where  $\varepsilon$  is a small positive number.

The recursive computational procedure is:

$$\Pi^{S_1}(S_0 - 1) = \Pi^{S_1-1}(S_0) + (1 - G^{S_0}(L_0^i)) \times (\pi^{S_1}(0) - \pi^{S_1-1}(0)), \quad (\text{A.10})$$

$$\Pi^0(S_0) = G^{S_0}(L_0^i) \beta L_0^i - G^{S_0+1}(L_0^i) \beta \frac{S_0}{\lambda} + \beta L_i \quad (\text{A.11})$$

and, The expected total holding and shortage costs for a unit demand in an inventory system with a one-for-one ordering policy is:

$$c_i(S_0, S_1) = \Pi^{S_1}(S_0) + \gamma(S_0) \quad (\text{A.12})$$

## 8. Appendix B

We will present the proof of the corollary 3 as follows:

$$X_i = L_i + \omega_i,$$

and

$$\omega_i = \max\{0, L_0^i - t_{s+mQ}\},$$

then

$$\begin{aligned} P_{12} &= P(X_1 < X_2) = P(t_{s+mQ} < L_0^1).P(X_1 < X_2 | t_{s+mQ} < L_0^1) \\ &+ P(L_0^1 < t_{s+mQ} < L_0^2).P(X_1 < X_2 | L_0^1 < t_{s+mQ} < L_0^2) \\ &+ P(t_{s+mQ} > L_0^2).P(X_1 < X_2 | t_{s+mQ} > L_0^2) \end{aligned}$$



$$\begin{aligned}
P_{12} &= P(X_1 < X_2) = G^{s+mQ}(L_0^1) \\
&+ G^{s+mQ}(L_2 + L_0^2 - L_1) - G^{s+mQ}(L_0^1) \\
P_{12} &= G^{s+mQ}(L_2 + L_0^2 - L_1)
\end{aligned} \tag{B.1}$$

All of the other corollaries can be proved easily in the same way.

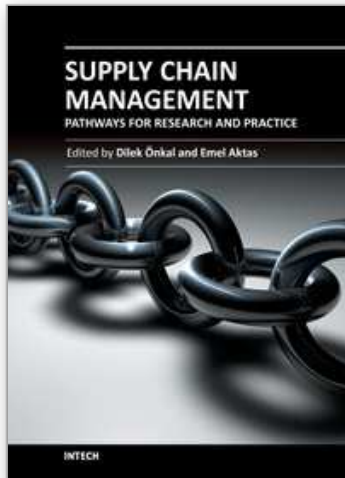
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## **Supply Chain Management - Pathways for Research and Practice**

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Challenges faced by supply chains appear to be growing exponentially under the demands of increasingly complex business environments confronting the decision makers. The world we live in now operates under interconnected economies that put extra pressure on supply chains to fulfil ever-demanding customer preferences. Relative attractiveness of manufacturing as well as consumption locations changes very rapidly, which in consequence alters the economies of large scale production. Coupled with the recent economic swings, supply chains in every country are obliged to survive with substantially squeezed margins. In this book, we tried to compile a selection of papers focusing on a wide range of problems in the supply chain domain. Each chapter offers important insights into understanding these problems as well as approaches to attaining effective solutions.

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