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A Simple Analytical Model for Remote Assessment of the Dynamics of Biomass Accumulation

Janis Abolins and Janis Gravitis
University of Latvia,
Latvian State Institute of Wood Chemistry,
Latvia

1. Introduction

Efficient means for assessment of the dynamics and the state of the stocks of renewable assets such as wood biomass are important for sustainable supplies satisfying current needs. So far attention has been paid mainly to the economic aspects of forest management while ecological problems are rising with the expected transfer from fossil to renewable resources supplies of which from forest being essential for traditional consumers of wood and for emerging biorefineries. Production of biomass is more reliant on assets other than money the land (territory) available and suitable for the purpose being the first in the number. Studies of the ecological impacts (the "footprint") of sustainable use of biomass as the source of renewable energy encounter problems associated with the productivity of forest lands assigned to provide a certain annual yield of wood required by current demand for primary energy along with other needs.

Apart from a number of factors determining the productivity of forest stands, efficiency of land-use concomitant with growing forest depends on the time and way of harvesting (Thornley & Cannell, 2000). In the case of clear-cut felling the maximum yield of biomass per unit area is reached at the time of maximum of the mean annual increment (Brack & Wood, 1998; Mason, 2008). The current annual increment (rate of biomass accumulation by a forest stand or rate of growth) culminates before the mean annual increment reaches its peak value and there is a strong correlation between the maximums of the two measures. Knowing the time of growth-rate maximum (inflection point on a logistic growth curve) allows predicting the time of maximum yield (Brack & Wood, 1998). However, the growth-rate maximum is not available from field measurements directly. Despite the progress in development of sophisticated models simulating (Cournède, P. et al., 2009; Thürig, E. et al., 2005; Welham et al., 2001) and predicting (Waring et al., 2010; Landsberg & Sands, 2010) forest growth, there still remains, as mentioned by J. K. Vanclay, a strong demand for models to explore harvesting and management options based on a few available parameters without involving large amounts of data (Vanclay, 2010).

The self-consistent analytical model described here is an attempt to determine the best age for harvesting wood biomass by providing a simple analytical growth function on the basis of a few general assumptions linking the biomass accumulation with the canopy absorbing

the radiation energy necessary to drive photosynthesis. A number of reports on employing remote sensing facilities (Baynes, 2004; Coops, et al., 1998; Lefsky et al., 2002; Richards & Brack, 2004; Tomppo E. et al., 2002 ; Waring et al.,2010) strongly support the optimism with regard to successful use of the techniques to detect the time of maximum yield of a stand well in advance by monitoring the expanding canopy.

According to the grouping of models suggested by K. Johnsen et al. in an overview of modeling approaches (Johnsen et al., 2001), the model described in this chapter belongs to simplistic traditional growth and yield models. It differs from other models of this kind by not incorporating mathematical representations of actual growth measurements over a period of time. Derived from a few essential basic assumptions the analytical representation rather provides the result that should be expected from measurements of growth under "traditional" (idealized) conditions. The chosen general approach of modeling the biomass production at the stand level allows obtaining compatible growth and yield equations (Vanclay, 1994) of a single variable – the age. Like with many other theoretical constructions the applicability of the model to reality is fairly accidental and restricted. However, since the derived equations are in good agreement with the universal growth curves obtained from measurements repeatedly confirmed and generally accepted as classic illustrations of biomass dynamics (Brack & Wood, 1998; Mason et al., 2008), it seems to offer a good approximation of the actual biomass accumulation by natural forest stands.

Equations representing the model are believed to reflect the simple assumptions made on the basis of common knowledge about photosynthesis and observations in nature: biomass is produced by biomass; the amount of produced biomass is proportional to the amount of absorbed active radiation; the absorbed radiation is proportional to effective light-absorbing area of the foliage (number and surface area of leaves) and limited by the ground area of the forest stand (the area determining the available energy flow). Projection of the canopy filling the ground area detectable by remote sensing is assumed to reflect dynamics and status (the stage) of forest growth. The height of the stand is another growth parameter accessible by remote sensing. Relationships of the latter with other measurable quantities determining the yield of accumulated biomass are well studied (Vanclay, 2009) and can be employed for remote assessment of the current annual increment and the state of forest stands (Lefsky et al., 2002; Ranson et al., 1997; Tomppo et al., 2002). The model presented hereafter has been developed to be aware of the current annual increment reaching the maximum merely from the data of remote observation of the dynamics of forest stand canopy while complemented by data of the average height would predict the yield.

2. General approach and basic equations

The analytical model offered to describe dynamics of the standing stock of wood biomass in natural forests is based on the obvious relationship between the rate of growth (rate of accumulation of biomass) y and the stock (amount of biomass) S stored in the forest stand (Garcia, 2005):

$$S(t) = \int y(t) \cdot dt \tag{1}$$

By turning to common knowledge that biomass is produced by biomass the rate of accumulation of new biomass in the first approximation can be assumed being proportional to the amount of biomass already accumulated:

$$y = \frac{dS}{dt} = aS \tag{2}$$

where a is a constant of the reciprocal time dimension and t is time. Rewriting the right-side equation of (2) in the form:

$$\frac{dS}{S} = adt\,, \tag{3}$$

and integrating it provides $\ln S = at$ and exponential growth of the stock of biomass:

$$S = const \cdot e^{at}\,, \tag{4}$$

which is unrealistic in the long run because of finite resources of nutrients and other limiting factors not taken into account in Eq. (2). The problem can be solved by setting an asymptotic limit to growth:

$$S(t) = S_{\infty} \cdot \left(1 - e^{-at}\right). \tag{5}$$

The rate of biomass accumulation y, Eq. (2), usually referred to as the current annual increment of stock measured by volume of wood mass per unit area (m^3/ha) (Brack & Wood, 1998) is not directly determined by the accumulated biomass stock. The uptake of CO_2 and photosynthesis of biomass rather depends on the total surface area of leaves determining the amount of absorbed radiation. The number of leaves and hence the light-absorbing area depend on the biomass accumulated by individual trees and the forest stand as a whole. The actual amount of the absorbed radiation that ultimately determines the rate of photosynthesis (and the annual increment) per unit area (a *hectare*) of a particular forest stand is limited regardless of the total surface area of leaves. So the concept of light-absorbing area should refer to the effective absorbing area limited by the particular area unit selected. It should be noticed here that further considerations are relevant to statistically significant numbers of individual trees and, consequently, to area units of stands comparable to hectare.

It seems to be reasonable to assume that accumulation of biomass in a forest stand occupying a large enough land area follows the same law as the rate at which the light-absorbing area (the canopy) of the growing stand expands with time. As noticed, the number and total surface area of leaves absorbing radiation is proportional to the accumulated biomass approaching some asymptotic limit L_{∞} of its own. However, the rate of expansion of the effective absorbing area also depends on the proportion of the free, unoccupied space available for expansion to intercept the radiation. Supposing the total light-absorbing area L as function of time being described by equation similar to Eq. (5):

$$L(t) = L_{\infty}\left(1 - e^{-at}\right), \tag{6}$$

the rate of expansion of the light-absorbing area expressed as:

$$\frac{dL}{dt} = const \cdot (L_{\infty} - L) \cdot L \tag{7}$$

can be written in the form:

$$\frac{dL}{dt} = const \cdot \left[L_\infty - L_\infty \cdot \left(1 - e^{-at}\right) \right] \cdot L_\infty \left(1 - e^{-at}\right) = const \cdot L_\infty^2 \cdot e^{-at} \cdot \left(1 - e^{-at}\right). \quad (8)$$

Dimension of the constant in Eq. (8) is the reciprocal of the product of area and time. Since area L_∞ also is constant it can be omitted for further convenience to focus attention on the time-dependent part of Eq. (8).
Assuming that the rate of biomass accumulation follows the rate of expansion of the light-absorbing area it can be described by equation similar to Eq. (8):

$$\frac{dS}{dt} = const \cdot \left(1 - e^{-at}\right) \cdot e^{-at}, \quad (9)$$

where the value of the constant factor (dimension of which here is the dimension of current increment) can be chosen to satisfy some selected normalizing condition, as will be done further.
The time-dependent part of Eq. (9) has a maximum at time t_m satisfying condition:

$$2e^{-at} - 1 = 0 \quad (10)$$

Wherefrom

$$at_m = \ln 2 \quad (11)$$

Exponent a determining the rate of growth in real time depends on the particular species and a number of other factors such as insolation and availability of water and nutrients at the site and has to be found from field measurements. However, existence of the maximum on the curve of the rate of growth (the curve of current annual increment often referred to as the growth curve) allows normalizing the time scale with respect to the time at which the maximum is reached. It is done by introducing dimensionless time variable

$$x = \frac{at}{\ln 2}, \quad (12)$$

or substituting at with $x \cdot ln2$ in Eq. (9), or just writing x instead of t and putting $a = ln2$. The current annual increment is normalized by choosing the constant factor to satisfy condition:

$$y_m = y(x = 1) = const \cdot \left(1 - \frac{1}{2}\right) \cdot \frac{1}{2} = const \cdot \frac{1}{4} = 1. \quad (13)$$

The normalized rate of biomass accumulation expressed by current annual increment in time scale x normalized with respect to the time when it reaches its maximum now is presented by Eq. (9) where t is substituted by variable x:

$$y(x) \equiv \frac{dS}{dx} = 4 \cdot \left(1 - e^{-ax}\right) \cdot e^{-ax} \quad (14)$$

where $a = ln2$. Function $y(x)$ is shown in Fig. 1 (a).

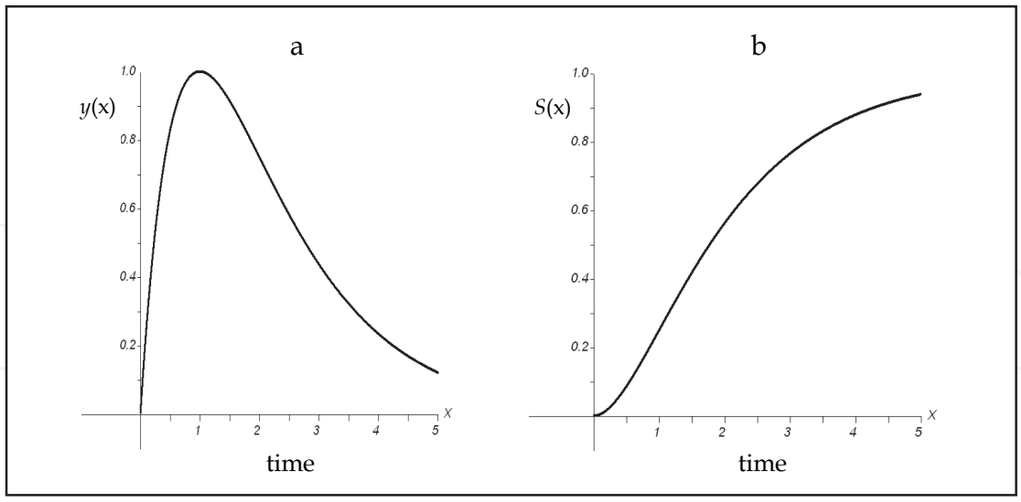

Fig. 1. a – rate of accumulation (current annual increment) of biomass $y(x)$ normalized with respect to its maximum value presented by Eq. (14) and b – stock normalized with respect to its asymptotic limit presented by Eq. (17) as functions of normalized time variable x.

Returning to Eq. (1) the biomass stored by time x = x_c is expressed by definite integral:

$$S(x_c) = \int_0^{x_c} y(x)\,dx\,. \tag{15}$$

Substituting $y(x)$ from Eq. (14) into Eq. (15) and calculating the integral the stock S is presented as function of age explicitly:

$$S(x_c) = 4\int_0^{x_c}\left(1-e^{-ax}\right)\cdot e^{-ax}dx = 4\int_0^{x_c} e^{-ax}dx - 4\int_0^{x_c} e^{-2ax}dx = 4\left[-\frac{e^{-ax}}{a}+\frac{e^{-2ax}}{2a}\right]_0^{x_c} =$$

$$= \frac{2}{2a}\cdot\left[e^{-2ax}-2e^{-ax}\right]_0^{x_c} = \frac{2}{2a}\cdot\left(e^{-2ax_c}-2e^{-ax_c}+1\right) = \frac{2}{2a}\cdot\left(1-e^{-ax_c}\right)^2 \tag{16}$$

By normalizing the stock choosing its asymptotic limit as the normalized unit $S_\infty = 1$ the result of transformations in Eq. (16) can be summarized as

$$S(x_c) = \frac{2}{2a}\cdot\left(1-e^{-ax_c}\right)^2 = S_\infty\left(1-e^{-ax_c}\right)^2 \tag{17}$$

where, as previously in Eq. (14), $a = ln2$. Function (17) in the normalized time scale is presented in Fig. 1 (b).

3. Mean annual increment and productivity

The mean annual increment of a forest stand is an essential factor illustrating the overall productivity of the stand at a given age and is expressed by the ratio of stock to age of the stand (Brack & Wood, 1998). The stock being presented by Eq. (16) the mean annual increment Z is calculated in units of the current annual increment from

$$Z(x) = \frac{2}{a} \cdot \frac{\left(1 - e^{-ax}\right)^2}{x} \tag{18}$$

where $a = ln2$. Function $Z(x)$ shown in Fig. 2 has a maximum at x satisfying condition:

$$\frac{e^{x \ln 2}}{x} = 2 \ln 2 \tag{19}$$

obtained from putting derivative of function (18) equal to zero. The value of $x \approx 1.81$ satisfying Eq.(19) is found from graphical solution of the equation (Fig. 3).

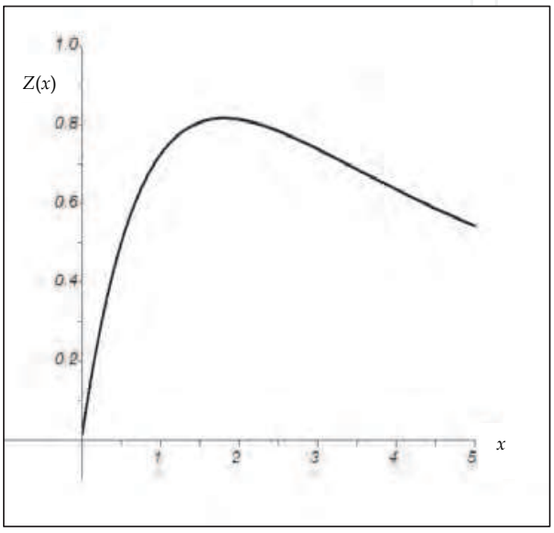

Fig. 2. Mean annual increment Eq. (18) as function of the normalized time variable x.

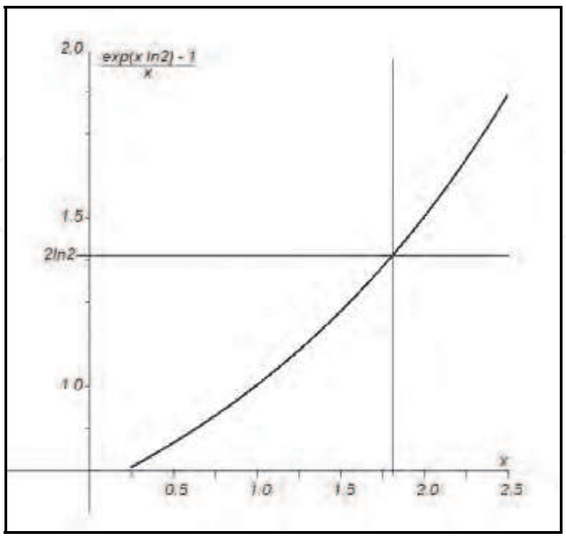

Fig. 3. Graphical solution of Eq. (19) determining position of the maximum of mean annual increment on the axis of the normalized time coordinate x.

In Fig. 4 the current annual increment (rate of biomass accumulation) and the mean annual increment are presented together wherefrom the mean annual increment is seen to reach the maximum value (equal to ≈ 0.8 of the peak value of current annual increment) at cross-point of the two curves.

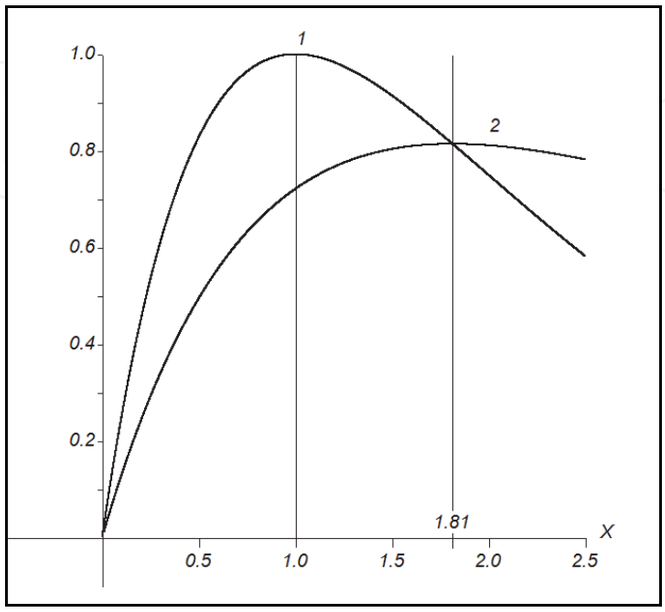

Fig. 4. Current (curve 1, Eq. 14) and mean (curve 2, Eq. 18) annual increments of biomass as functions of time x normalized with respect to the time of the growth-rate maximum chosen as the unit time interval. The mean annual increment (curve 2) is presented in the same scale as the current annual increment. The maximum of curve 2 is reached at the cross-point of the two curves at $x \approx 1.81$.

The reciprocal of the mean annual increment is a parameter characterizing the size of plantation for sustainable supply of biomass. The total area of a plantation for sustainable annual supply comprised of equal lots of stands of ages in sequence from one year to the cutting age is directly proportional to cutting age x_c and inversely proportional to the stock at cutting age $S(x_c)$:

$$A = const \cdot \frac{x_c}{S(x_c)} = const \cdot f(x_c) . \tag{20}$$

The constant is equal to the required annual yield of biomass; function $f(x_c)$ defined as

$$f(x_c) \equiv \frac{x_c}{S(x_c)} \tag{21}$$

is the reciprocal of the mean annual increment at cutting age.
At point $x \approx 1.81$ where the mean annual increment reaches maximum its reciprocal – function $f(x)$ has the minimum. If B_s is the demanded sustainable annual yield of biomass, $S(x_c)$ – the stock of biomass accumulated in the forest stand by the cutting age, and A_o – the area of the forest to be felled annually to satisfy the demand, then $B_s = S(x_c) \cdot A_o$ and the total

area of the plantation – $A = x_c \cdot A_o$. From here the yield per unit area of the whole plantation is found being proportional to the mean annual increment reaching the maximum at $x \approx 1.8$:

$$\frac{B_s}{A} = \frac{S(x_c) \cdot A_o}{x_c \cdot A_o} = \frac{S_c(x_c)}{x_c} . \tag{22}$$

As follows from Eq. (22), felling the forest at age corresponding to 1.8 units of the normalized time scale provides the maximum yield per unit area of a particular stand and hence of the whole plantation. In other words, the maximum productivity of land area under a forest is achieved when felling at the time of the mean annual increment peak.

4. Validation of the model

Neither the value of the current annual increment at maximum, nor the real time when a forest stand reaches the maximum is known *a priori*. Both parameters depend on the species and conditions represented by the quality class of the site and have to be determined by field measurements. However, the field measurements do not provide these quantities directly. They have to be found from periodic mean annual increments available from field measurements.

The growth-rate function given by Eq. (14) cannot be used directly to compare the model equation with experimental growth-rate data. For that purpose a different exponential equation can be employed containing variable parameters related to the quantities not measurable directly. The values of the variable parameters providing the best fit of the measured annual increments with the equation are chosen to evaluate the unknown quantities. A rather abundant database available for natural grey alder (*Alnus incana*) stands of up to 50 years old (Daugavietis, 2006) presents a good opportunity to test the model.

The 5-year mean annual increments available from field measurements (Daugavietis, 2006) are a good approximation for the current annual increment value at mid-time of the respective 5-year period (Fig. 5, a). By choosing a function of the type

$$y(t) = (c + kt) \cdot e^{-\frac{t}{a}} = k(b + t) \cdot e^{-\frac{t}{a}} \tag{23}$$

to describe the current annual increment it is possible to assign physical sense to variable parameters a and c and find the maximum value of the current annual increment and position of the maximum on the real-time axis by best fit of function (23) to the data from experimentally measured periodic mean increments. Under condition of taking coefficient k (of dimension y/t) equal to 1 function (23) has its maximum at time

$$t_m = a - \frac{c}{k} = a - b . \tag{24}$$

It should be noticed here that dimension of constant a in Eq. (23) is time, which is different from the constant a used in Eq. (2) with dimension of reciprocal time (frequency). The reason of choosing a different dimension of constant a in Eq. (23) is seen from Eq. (24).

By varying parameters a, b, and the maximum value of the current annual increment y_m (not available from any direct measurement) function (23) is varied for best fit to the set of experimental data normalized with respect to y_m.

The values of increments calculated from Eq. (23) coincide with the set of experimental data (Daugavietis, 2006) (Fig. 5) within standard deviation of 2.5 % of the maximum value, the correlation between the sets of calculated and experimental data being better than 0.99.
The normalized time scale is introduced by choosing variable x to satisfy condition

$$x = \frac{t}{t_m} = \frac{t}{a-b}. \tag{24}$$

By substituting the normalized time variable x for real time t in Eq. (23) the current annual increment is presented as

$$y(x) = \left[b + (a-b)\cdot x \right] \cdot e^{-\frac{a-b}{a}\cdot x}. \tag{25}$$

By defining new constant parameters $\alpha = \frac{a-b}{a}$ and $\beta = \frac{b}{a-b}$ Eq. (25) is rewritten as:

$$y(x) = (a-b)\cdot(\beta + x)\cdot e^{-\alpha x}. \tag{26}$$

Normalizing function (26) with respect to $y_m = (a-b)\cdot(\beta+1)\cdot exp(-a)$ and taking into account that $\beta + 1 = \frac{b+a-b}{a-b} = \frac{a}{a-b}$ provide

$$y(x) = \alpha \cdot e^{\alpha} \cdot (\beta + x)\cdot e^{-\alpha x}. \tag{27}$$

By substituting $y(x)$ from Eq. (27) in Eq. (15) and calculating the integral the stock normalized to $S_\infty = \frac{a\cdot(a+b)}{(a-b)^2}$ as function of cutting age is expressed by:

$$S(x_c) = 1 - \left(1 + \frac{a-b}{a+b}\cdot x_c\right)\cdot e^{-\alpha x_c}. \tag{28}$$

The mean annual increment

$$\overline{y}(x) = \frac{S(x)}{x} = \frac{1}{x}\cdot\left[1 - \left(1 + \frac{a-b}{a+b}\cdot x\right)\cdot e^{-\alpha x}\right] \tag{29}$$

reaches maximum under condition

$$\exp(\alpha x) - \alpha x \cdot \left(1 + \frac{a-b}{a+b}x\right) = 1 \tag{30}$$

providing $x_m \approx 1.77$ corresponding to optimum cutting age of $x_c = 1.8$ or 18 years in case of grey alder.
After finding the age of the maximum of current annual increment, the set of experimental points (Fig. 5, a) can be put on the normalized time scale x and compared with function (14) as shown in Fig. 5, b. The variation of the value of growth-rate maximum at this point is still available for adjustment to improve the fit between

experimental data and Eq. (14). The curves presented by Eqs. (14) and (27) with best fit parameter values are practically identical within the normalized time interval $0.5 \le x \le 2.5$. Because of a nonzero initial growth-rate Eq. (27) provides higher values on the rise while lower at later time on the decline.

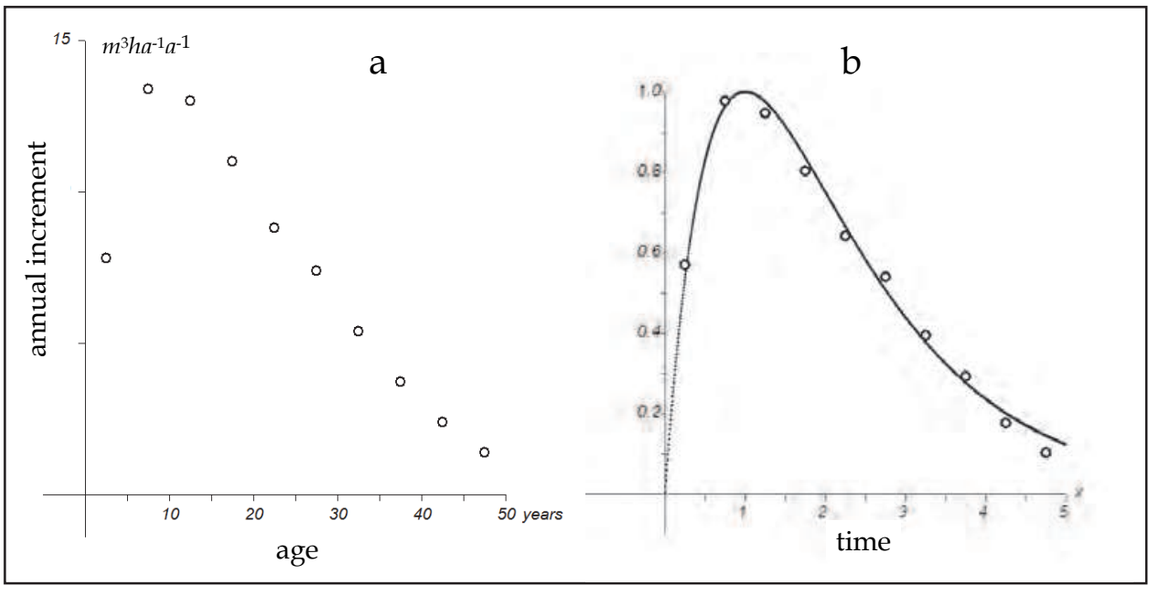

Fig. 5. a – current annual increments of grey alder stand calculated from measured 5-year periodic mean values with age (Daugavietis, 2006), in units of m^3 per *ha* per *annum*; b – best fit of Eq. (14) (solid curve) to experimental data (circles) normalized against the growth-rate maximum in the time scale of normalized age.

5. Rate of growth as function of light-absorbing area

Equation (9) describing the rate of biomass accumulation derived from Eq. (7) in section 1 is based on the assumption that dynamics of current annual increment follows dynamics of the expansion of light-absorbing area of the canopy. Returning to Eq. (7) it can be assumed to describe the relationship between the normalized rate of growth (y) and the normalized light-absorbing area (L):

$$y(L) = 4 \cdot L(1-L) \tag{31}$$

shown in Fig. 6.

It has to be noticed that the pace at which the biomass is stored is not necessarily equal to the pace at which the light-absorbing area increases. The uptake of biomass (photosynthesis) depending on the effective light-absorbing area obviously should follow with some delay, which means that the normalized (intrinsic or specific) time scale of the equation derived from Eq. (8) to describe the rate of expansion of the light-absorbing area:

$$\frac{dL}{dx} = 4\left(1 - e^{-ax}\right) \cdot e^{-ax}, \; (a = ln2) \tag{32}$$

is different from that of Eq. (14) describing the rate of biomass accumulation.

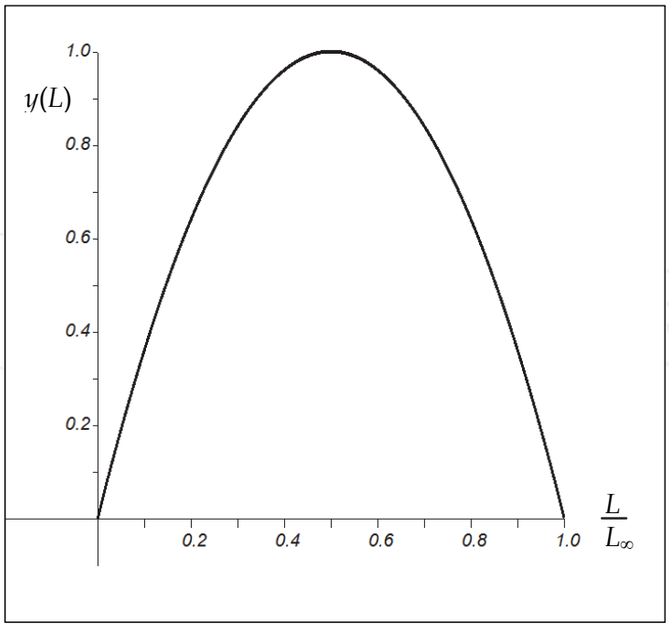

Fig. 6. Rate of biomass accumulation y as a function of the light-absorbing area L, Eq. (31).

Relationship between the units of the two normalized time variables – x_b describing the current annual increment (rate of biomass accumulation) and x_a describing the rate of expansion of the light-absorbing area can be concluded from knowing that maximum of the current annual increment is reached at $L/L_\infty = 0.5$ when $x_b = 1$. In units of time scale x_a the light-absorbing area L is expressed by integrating Eq. (32) the result of which is similar to Eq. (17):

$$L(x_a) = L_\infty \left(1 - e^{-ax_a}\right)^2 \tag{33}$$

where L is normalized in the same way as stock by taking the asymptotic limit L_∞ equal to 1. The "age" x_a at which the normalized light-absorbing area reaches the value 0.5, as follows from Eq. (33), satisfies equation:

$$1 - \frac{\sqrt{2}}{2} = e^{-ax_a} \tag{34}$$

wherefrom, remembering that $a = ln2$, the time in units of scale x_a corresponding to unit time of scale $x_b = 1$ is found being equal to

$$x_a = \frac{-\ln\left(1 - \frac{\sqrt{2}}{2}\right)}{\ln 2} \cong 1.77\ . \tag{35}$$

It means that a unit of the normalized time scale of the rate of expansion of the light-absorbing area is about 0.56 of the unit of the normalized time scale describing the rate of biomass accumulation. The units of the two normalized time scales presented in Fig. 7 are approximately equated by

$$x_b \cong 1.77 x_a \,. \tag{36}$$

As seen from Fig. 7, expansion of the light-absorbing area of the canopy (curve 1) proceeds ahead of the rate of biomass accumulation (curve 2) complying with the assumption that higher rates of the increase of the surface area (and the number) of leaves require a greater proportion of the gross product of photosynthesis lost after seasonal vegetation.
The size of the effective light-absorbing area expressed by the ratio to its asymptotic limit is presented in Fig. 7 on the lower time axis. The maximum rate of expansion dL/dx is reached at $x = x_b \approx 0.56$ ($x_a = 1$) when L = $0.25L_\infty$ while the current annual increment reaches the maximum at $x = x_b = 1$ when L = $0.5L_\infty$. By the time $x = x_b \approx 1.81$ when the mean annual increment reaches its maximum the effective light-absorbing area is equal to approximately 0.8 L_∞. The current annual increment of biomass in the stand is maintained over 0.8 of the maximum value within the range of light-absorbing area between 0.28 and 0.8 of L_∞.

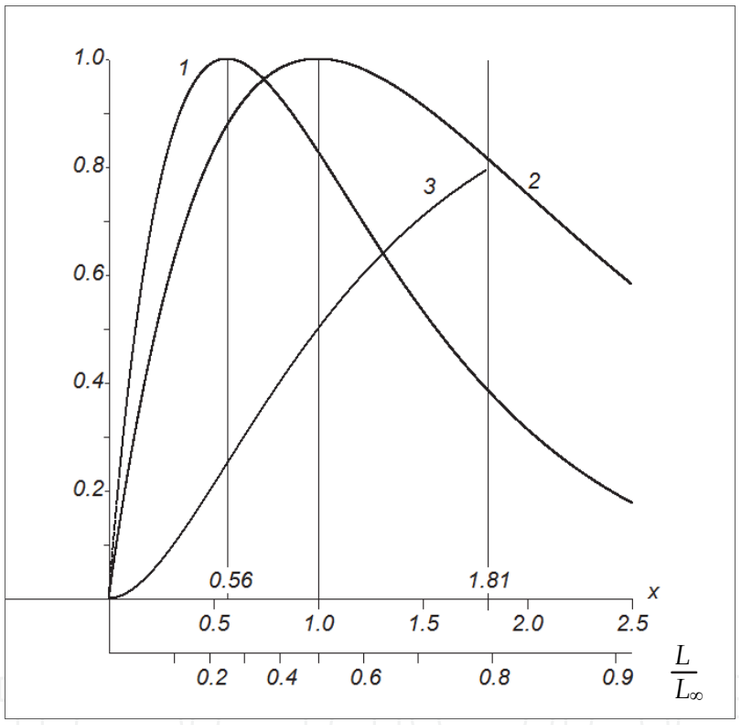

Fig. 7. Rate of expansion of the light-absorbing area (1), current annual increment (2), and the light-absorbing area (3) in time-scale $x = x_b$ normalized to the time of the current annual increment maximum. The lower axis shows the size of the light-absorbing area reached at the respective point on the time axis.

The basic components of the model – equations presenting current and mean annual increments, stock, and the rate of expansion of the light-absorbing area as functions of age expressed in the intrinsic time units are summarized in Fig. 8.

6. Conceptual remarks

The analytical expressions comprising the model are derived from rather general principles of biomass production by photosynthesis in living stands without taking into account

factors affecting forest growth other than the effective light-absorbing area of the canopy. However, since dynamics of the latter is strongly dependent on availability of nutrients, water, and some other crucial factors, the model reflects the cumulative effect of all of them through the relationship between the rate of growth and the capacity to capture the active radiation. Therefore, monitoring the canopy dynamics can provide reliable information for conclusions about that capacity and the expected end product of photosynthesis.

Determining the best time for harvesting by observing expansion of the canopy from satellites is one of attractive practical applications of the model for management of even-age stands in concert with remote sensing. Even though the canopy projection measureable by remote sensing instruments is not quite equal either to the light-absorbing area or the leaf area index, the correlation between the three is strong enough to make corrections necessary for detecting the time (age) of growth-rate maximum from remote observations of the dynamics of canopy expansion.

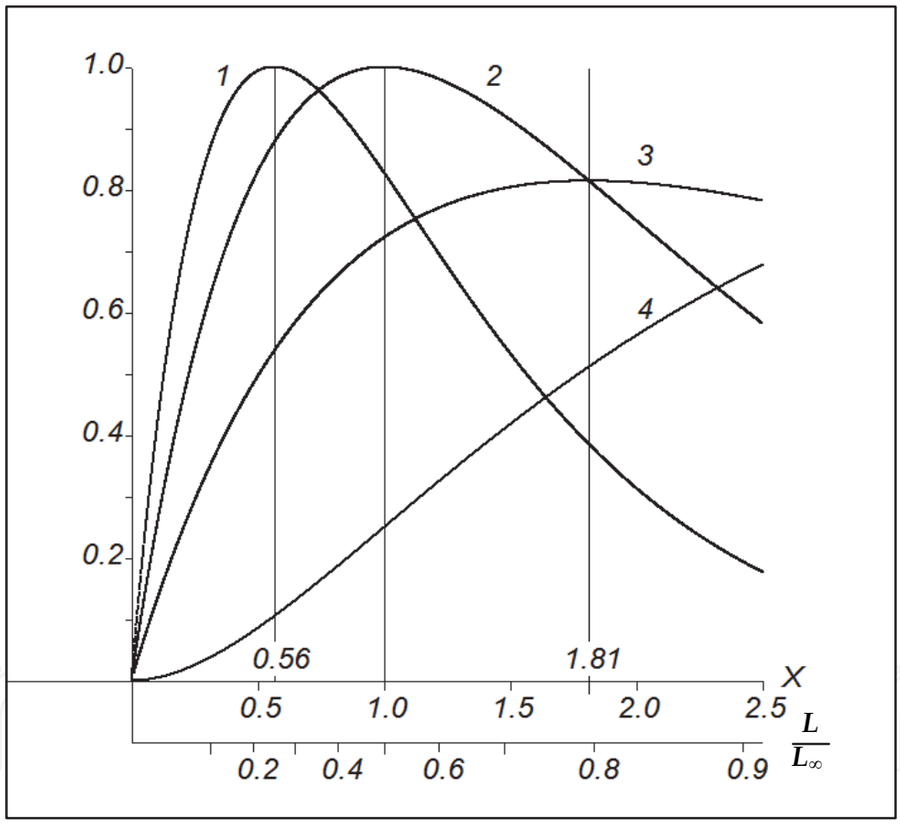

Fig. 8. Dynamics of the light-absorbing area (1), Eq. (7), the rate of production of above-ground biomass (2), Eq. (14), mean annual increment (3), Eq. (18), and the yield (4), Eq. (17), as functions of the intrinsic time provided by the rate of growth of a forest stand. The effective light-absorbing area as the ratio to its maximum value L/L_∞ , Eq. (33), is presented by the lower abscissa. Note the inflection point of curve 4 being reached before 0.25 S_∞; at the time of maximum productivity $S \cong 0.5\ S_\infty$.

The obtained analytical expression, Eq. (17), for accumulated biomass of a stand as function of age is a particular case of the well-known Richards growth equation (Zeide, 2004):

$$y(t) = const \cdot \left(1 - e^{-bt}\right)^{c} \tag{37}$$

with parameter values b = ln2 and c = 2 describing sigmoid (logistic) growth.
A generalized differential form of sigmoid growth (the growth-rate function) has been considered by C. P. D. Birch (Birch, 1999) and a detailed formalistic analysis of the family of sigmoid growth equations is given by O. Garcia (Garcia, 2005). The sigmoid shape of the yield (stock) curve Eq. (17) in the present case is predetermined by the shape (the maximum) of the obtained growth-rate function Eq. (14).
The normalized time unit introduced to provide a dimensionless common measure to match the model with experimental data is the same intrinsic time unit suggested by B. Zeide as a unit provided by organisms themselves and clarifying the meaning of parameters of growth functions (Zeide, 2004). A number of other growth factors, such as biological potential of a particular species, the site quality, changing climate, etc. are reflected in the real-time equivalent of the intrinsic time unit. For instance, comparison of best fits to available measured data of grey alder stands at sites of different quality (Daugavietis, 2006) show the stands at sites of higher quality reaching the growth-rate maximum earlier (Kosmach, 2010). Since climate change is a factor affecting forest growth (Nakawatase & Peterson, 2006), the real-time equivalents of the intrinsic time unit obtained from monitoring the growth of stands of a given species hold information for potential assessment of the changing environment accessible by remote observations and retrospective studies of forest growth.
The Richards equation (37) predicts diminishing of the current increment to zero with the age of the stand while the effective light-absorbing area given by Eq. (6) approaches a constant maximum and, therefore, should be expected to provide a constant maximum increment of biomass. However, the real growth curves (at least of natural forest stands) rather comply with Richards equation even if the underlying models do not take into account factors, such as respiration or partition, diminishing the annual above-ground biomass production. In the present case they are somehow implied in the factor (L_∞ – L) restricting the rate of expansion of the effective light-absorbing area in Eq. (7), which ultimately determining the descent of the derived growth functions, Eqs. (14) and (17), can be attributed to shading. At large, the simplified models of this kind should not be expected to hold at the very short and far ranges of the time axis their application being limited by the range of the intrinsic time units between 0.5 and 3 – the interval of interest for commercial forest management. G. E. P. Box has likely hit the point with regard to the subject by writing in 1979: "All models are wrong, but some are useful" (cited in Vanclay, 2010).

7. Conclusions

The simple logistic analytical model of biomass accumulation by forest stands derived on the basis of general assumptions about photosynthesis comprises compatible equations of growth and yield as functions of time. The function describing dynamics of the rate of growth derived as function of the effective light-absorbing area of the canopy provides a growth function representing particular case of Richards equation and is in good agreement with data obtained from experimental measurements. The model contains two related parameters: the unit of the intrinsic (normalized with respect to peak current annual increment) time scale and the effective light-absorbing area of the canopy not equal but closely related to the leaf area index or to projection of the canopy. The latter accessible by remote sensing opens the use of remote sensing data for monitoring the growth of forest

stands to predict the culmination of current annual increment the age of the stand at which being known allows predicting the optimum age for harvesting.
The model has been developed for determining the land area and the optimum harvesting age of even-aged natural stands for sustainable supply of firewood and wood biomass to satisfy the needs of paper mills and biorefineries. It can be extended to consider solutions of the same problems with regard to timber products such as boards and other construction elements of buildings.
Some further studies are necessary to find out the relationship between remote observations of canopy dynamics and dynamics of the effective light-absorbing area to realize the benefits of using the model with the opportunities provided by remote sensing to forest management.

8. Acknowledgements

The presented model has been derived on the basis of studies supported by the National Research Programs of Renewable energy resources and rational management of deciduous tree forests.

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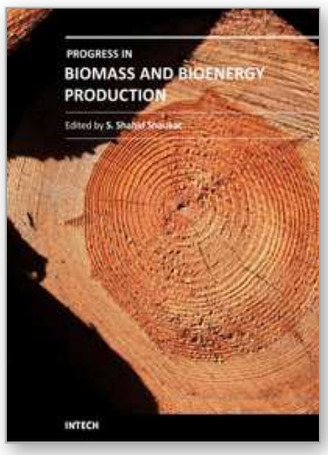

Progress in Biomass and Bioenergy Production
Edited by Dr. Shahid Shaukat

ISBN 978-953-307-491-7
Hard cover, 444 pages
Publisher InTech
Published online 27, July, 2011
Published in print edition July, 2011

Alternative energy sources have become a hot topic in recent years. The supply of fossil fuel, which provides about 95 percent of total energy demand today, will eventually run out in a few decades. By contrast, biomass and biofuel have the potential to become one of the major global primary energy source along with other alternate energy sources in the years to come. A wide variety of biomass conversion options with different performance characteristics exists. The goal of this book is to provide the readers with current state of art about biomass and bioenergy production and some other environmental technologies such as Wastewater treatment, Biosorption and Bio-economics. Organized around providing recent methodology, current state of modelling and techniques of parameter estimation in gasification process are presented at length. As such, this volume can be used by undergraduate and graduate students as a reference book and by the researchers and environmental engineers for reviewing the current state of knowledge on biomass and bioenergy production, biosorption and wastewater treatment.

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

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中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
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