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Chiral Transverse Electromagnetic Standing Waves with E||H in the Dirac Equation and the Spectra of the Hydrogen Atom

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1. Introduction

This article examines the conditions under which transverse electromagnetic (TEM) waves exist in a sourceless medium. It shows that TEM waves can be classified according to whether their Poynting vector is identically zero or nonzero. The former are non propagating TEM standing waves with $\mathbf{E} \| \mathbf{H}$ and the latter are TEM traveling and standing waves with $\mathbf{E} \perp \mathbf{H}$. The Chiral general condition under which TEM standing waves with $\mathbf{E} \| \mathbf{H}$ exist is derived. The behavior of these waves under Lorentz transformations is discussed and it is shown that these waves are lightlike and that their fields lead to well-defined Lorentz invariants. Two physical examples of these standing waves are given.

The first example is about the Dirac Equation for a free electron, which is obtained from the Maxwell Equations under the Born Fedorov approach, ($\mathbf{D} = \varepsilon(1 + T\nabla\times)\mathbf{E}$), and ($\mathbf{B} = \mu(1 + T\nabla\times)\mathbf{H}$**).** Here, it is hypothesized that an elementary particle is simply a standing enclosed electromagnetic wave with a half or whole number of wavelengths (λ). For each half number of λ the wave will twist 180° around its travel path, thereby giving rise to chirality. As for photons, the Planck constant (h) can be applied to determine the total energy (E): $E = nhc / \lambda$, where n = 1/2, 1, 3/2, 2, etc., and c is the speed of light in vacuum. The mass m can be expressed as a function of λ, since $E = mc^2$ gives $m = nh / c\lambda$, from the formula above. This result is obtained from the resulting wave equation which is reduced to a Beltrami equation $\nabla \times \mathbf{E} = -(1 / 2T)\mathbf{E}$ when the chiral factor T is given by $T = n\hbar / mc$**.** The chiral Pauli matrices are used to obtain the Dirac Equation.

The second example is on a new interpretation of the atomic spectra of the Hydrogen atom. Here we study the energy conversion laws of the macroscopic harmonic LC oscillator, the electromagnetic wave (chiral photon) and the hydrogen atom. As our analysis indicates that the energies of these apparently different systems obey exactly the same energy conversion law. Based on our results and the wave- particle duality of electron, we find that the atom in fact is a natural microscopic LC oscillator.

In the framework of classical electromagnetic field theory we analytically obtain, for the hydrogen atom, the quantized electron orbit radius $r_n = a_o n^2$, and quantized energy $E_n = -R_H hc / n^2$, (n = 1, 2, 3, · · ·), where a_0 is the Bohr radius and R_H is the Rydberg

constant. Without the adaptation of any other fundamental principles of quantum mechanics, we present a reasonable explanation of the polarization of photon,selection rules and Pauli exclusion principle. Our results also reveal an essential connection between electron spin and the intrinsic helical movement of electron and indicate that the spin itself is the effect of quantum confinement.

2. Chiral Transverse Electromagnetic standing waves with $\mathbf{E} \| \mathbf{H}$

This section provides the basis for reexamining the electromagnetic model of the electron, which is developed in others sections appearing in the present paper.

The solutions of Maxwell's equations of common interest are concerned with the propagation of electromagnetic (EM) energy in the form of transverse electromagnetic (TEM) waves in free space, material media, and transmission lines as well as transverse electric and magnetic waves in waveguides. Such solutions are characterized by the orthogonality of the electric field $\mathbf{E}$ and the magnetic flux density $\mathbf{H}$: i.e., $\mathbf{E} \perp \mathbf{H}$. An examination of many senior undergraduate and graduate electrodynamics textbooks [Jackson, 1975; Marion & Herald, 1980; Stratton, 1941; Panofsky & Phillips, 1955; Cook, 1975; Reitz & Milford, 1967; Lorrain & Corson, 1970; Portis, 1978; Smythe, 1950; Purcell, 1965] reveals that there is very little discussion of the conditions under which TEM standing waves exist. Textbooks with more of an engineering emphasis [Ramo et al., 1965.; Jordan & and Balmain, 1968.; Bekefi. & Barrett, 1977.; Shadowitz, 1975.; Rao, 1977], generally discuss standing waves in the context of the measurement of the voltage standing-wave ratio for transmission lines [Rao, 1977]. At the first time a work by Chu and Ohkawa [Chu & and Ohkawa, 1982] shows that a class of TEM waves with $\mathbf{E} \| \mathbf{H}$ exists, also, Zaghloul et al. [Zaghloul, et al.,1988] have derived the general conditions under which we have TEM waves with $\mathbf{E} \| \mathbf{H}$. From here, that it is useful to classify TEM waves according to whether their Poynting vector is identically zero or nonzero. The former are $\mathbf{E} \| \mathbf{H}$ standing waves and the latter are traveling and standing TEM waves with $\mathbf{E} \perp \mathbf{H}$ and $\mathbf{E} \not\| \mathbf{H}$. Using the Born Fedorov formalism [Torres-Silva, 2008], this article provides a discussion of the physical properties of TEM standing waves with $\mathbf{E} \| \mathbf{H}$ in connection with the interaction matter waves from its basic principles. It also suggests that $\mathbf{E} \| \mathbf{H}$ standing waves can be generated by the superposition of waves traveling in opposite directions and that the electromagnetic density of these standing waves somehow plays the same role in electrodynamics that the rest mass plays in relativistic dynamics. The background assumed in this article is that of a senior undergraduate physics or electrical student. It is hoped that this article will encourage instructors of electrodynamics courses to include a discussion of $\mathbf{E} \| \mathbf{H}$ TEM standing waves.

In this article, SI units are used. Vectors are denoted by bold letters. Other notational definitions are given as required.

2.1 Solutions of Maxwell's equations in a sourceless medium

Under the Born-Fedorov, constitutive relations between $\mathbf{D}$, $\mathbf{E}$, $\mathbf{B}$, $\mathbf{H}$ are [Torres-Silva, 2008]

$$\mathbf{D} = \varepsilon_0 (1 + T\nabla\times)\mathbf{E}\,,\ \mathbf{B} = \mu_0 (1 + T\nabla\times)\mathbf{H} \tag{1a}$$

In such a representation (BF), rotation terms are added to the basic constitutive relation whose chiral coefficients εT or μT can be either positive or negative for two stereoisomer structures. Solving the constitutive relation together with Maxwell's equations, we can easily get two eigenwaves, which are left and right circularly polarized with different wavevectors. The Maxwell's equation are

$$\nabla\times\mathbf{E}+\frac{\partial(1+T\nabla\times)\mathbf{H}}{c\partial t}=0\,,\ \nabla\times\mathbf{H}-\frac{\partial(1+T\nabla\times)\mathbf{E}}{c\partial t}=0 \tag{1b}$$

$$\nabla\bullet\mathbf{D}=0\,,\ \nabla\bullet\mathbf{B}=0 \tag{1c}$$

since $1/c^2=\varepsilon_0\mu_0$, where c is the speed of light in vacuum and ε_0, μ_0 and T (meter) are the vacuum permittivity, permeability, and the scalar chiral factor respectively.
Considering time variation as $\mathbf{E}\sim\mathbf{E}e^{i\omega_0 t}$ etc, the solution of Eqs. (1a)-(1c) can be obtained such that

$$k\nabla^2\mathbf{E}+k_0^2(1-k_0^2T^2)^{-1}\mathbf{E}+2\omega_0\mu_0\varepsilon_0T((1-k_0^2T^2)^{-1}\nabla\times\mathbf{E}=0\,, \tag{2a}$$

$$k\nabla^2\mathbf{H}+k_0^2(1-k_0^2T^2)^{-1}\mathbf{H}+2\omega_0\mu_0\varepsilon_0T((1-k_0^2T^2)^{-1}\nabla\times\mathbf{H}=0 \tag{2b}$$

Due to the chiral scalar $T\geq 0$ or $T\leq 0$, a general TEM solution of Eq. (2a) can be expressed in the form due to d'Alembert [Smythe, 1950],

$$\boldsymbol{E}(\mathrm{r},t)=\boldsymbol{E}_+(\eta)+\boldsymbol{E}_-(\xi)\,, \tag{3}$$

where $\boldsymbol{E}_+(\eta)=\sum_{i=1}^{3}E_+(\eta)_i\mathbf{l}_i$, $\boldsymbol{E}_-(\xi)=\sum_{i=1}^{3}E_-(\xi)_i\mathbf{l}_i$, $\eta\equiv\mathrm{k}\bullet\mathrm{r}-\omega_0 t$, $\xi\equiv\mathrm{k}\bullet\mathrm{r}+\omega_0 t$, and l_i are mutually orthogonal unit vectors in an arbitrary orthogonal coordinate system. Notice that the most general TEM solution of Eq. (2a or 2b) is a sum of solutions of the form of Eq. (3a) over all possible propagation vectors, k. Here, k and ω, the angular frequency, are related by $k_0^2=\omega_0^2/c^2$. The two solutions of Eq. (2a or 2b) given by Eq. (3a) can be considered as the vector electromagnetic field of two TEM waves traveling in opposite directions, i.e., energy propagation takes place in two opposite directions. In general, there is more energy flow in one direction than in the other so there is net energy flow in one direction. Such a TEM wave is called a traveling wave. In the special case where the energy propagated in one direction is equal to that propagated in the opposite direction, there is no net energy flow in the medium and the sum of the two TEM waves form what is generally known as a standing wave. Mathematically, the amount of energy density propagated is proportional to the magnitude of the Poynting vector [Marion & Herald, 1980] $\mathbf{S}$, where

$$\mathbf{S}\propto\mathbf{E}\times\mathbf{H}\,. \tag{4}$$

The condition for a standing wave is that the time average of $\mathbf{S}$ vanishes . This can be achieved if (i) $\mathbf{S}$ is zero all the time everywhere in the region of space under consideration, i.e., $\mathbf{S}(\mathrm{r},t)=0$, or (ii) $\mathbf{S}$ changes sign with time and has a zero time average, i.e., $\mathbf{S}(\mathrm{r},t)_{\mathrm{av}}=0$. Examination of Eq. (4) shows that $\mathbf{S}=0$ if either $\mathbf{E}$ or $\mathbf{H}$ is zero (the cases of electro- and magnetostatics) or if $\mathbf{E}\,||\,\mathbf{H}$. In this last case, a particular solution of Eq. (2a, 2b)

is when $k_0^2T^2=1$, where we have the condition $\mathbf{E}\|\mathbf{H}$, and $\mathbf{E}=i\eta\mathbf{H}$, so we find the Beltrami force free equation $\mathbf{E}+2T\nabla\times\mathbf{E}=0$ and the vector Poynting vanishes.
This means that for time-varying fields (i) leads to TEM standing waves with $\mathbf{E}\|\mathbf{H}$ whereas (ii) leads to the more familiar TEM standing waves with $\mathbf{E}\perp\mathbf{H}$.The fact that $\boldsymbol{S}(\mathrm{r},t)=0$, for time-varying fields leads to $\mathbf{E}\|\mathbf{H}$ and TEM standing waves will be used to derive the general conditions and connection to derive the Dirac equation.
The derivation of the general conditions for the existence of TEM standing waves with $\mathbf{E}\|\mathbf{H}$ was given first in a condensed form by [Zaghloul, et al.,1988]. Our derivation can be simplified by assuming that $\nabla\bullet(1+k_0^2T^2)\boldsymbol{A}+\partial\phi/\partial t=0$,. If $k_0^2T^2=1$, $\partial\phi/\partial t=0$, and $\nabla\phi=0$. This is equivalent and consistent with the choice of the Lorentz gauge.
In our approach if only if $kT=\pm1$, then we have TEM standing waves with $\mathbf{E}\|\mathbf{H}$. Here, T is a chiral scalar factor. This approach defines a class of $\mathbf{E}\|\mathbf{H}$ TEM standing waves that may be characterized by the fact that the chiral vector potential $\mathbf{F}$ satisfies the equation $\nabla\times\mathbf{F}=k\mathbf{F}$ as has been shown by [Torres-Silva, 2008]. An example of a vector potential satisfying $\nabla\times\mathbf{F}=k\mathbf{F}$ with $\mathbf{E}\|\mathbf{H}$ fields

$$\boldsymbol{F}(\mathrm{r},t)=F_0(\sin kz,\cos kz,0)\cos\omega t\,, \tag{5}$$

$$\mathbf{E}(\mathrm{r},t)=\omega F_0(\sin kz,\cos kz,0)\sin\omega t\,, \tag{6}$$

$$\mathbf{H}(\mathrm{r},t)=\eta F_0(\sin kz,\cos kz,0)\cos\omega t\,. \tag{7}$$

This solution corresponds to two circularly polarized waves [Chu & and Ohkawa, 1982] propagating opposite to each other in such a way that their Poynting vectors are cancelled out, so $\mathbf{E}=\mathrm{i}\eta\mathbf{H}$. Therefore, a single helical photon with energy $\hbar\omega$ carries a magnetic helicity of $\hbar c$.

2.2 Physical properties of $\mathbf{E}\|\mathbf{H}$ TEM standing waves

The TEM waves with $\mathbf{E}\|\mathbf{H}$ do not propagate, and hence are standing waves. It has been shown earlier that they are characterized by $\mathbf{S}(\mathrm{r},t)=0$. This means that there is no energy transfer between any two points in space at any time. How such waves can be generated and maintained in some region of space is an interesting question. One answer is that they are due to the superposition os waves traveling in opposite directions. It should be noted that for TEM waves with $\mathbf{E}\|\mathbf{H}$, investigation of the direction of $\mathbf{E}$ and $\mathbf{H}$ at any one point in space cannot uniquely determine the direction of the propagation vector $\mathbf{k}$. The direction of k can be inferred from a consideration of the directions of the fields at different points. The two fields $\mathbf{E}$ and $\mathbf{H}$ remain parallel to each other at each point but their direction will change from point topoint such that all of these directions lie in parallel planes, the transverse planes. This provides a unique definition of the direction of the propagation vector. It might be more appropriate to use the term wave vector in place of propagation vector for these standing waves since they do not propagate.
If $\mathbf{E}\|\mathbf{H}$ in one Lorentz frame K, then in any other frame K', $\mathbf{E}'$ may not be parallel to $\mathbf{H}'$. Moreover, no frame exists for which $\mathbf{E}'\perp\mathbf{H}'$. This is a direct consequence of the invariance

of $\mathbf{E} \cdot \mathbf{H}$, which will be discussed in next Sec. The TEM standing waves with $\mathbf{E} \| \mathbf{H}$, like all other TEM waves with $\mathbf{E} \perp \mathbf{H}$, keep $k_1 = \mathbf{E}^2 - \eta^2 \mathbf{H}^2$ and $k_2 = 2\mathbf{E} \cdot \eta \mathbf{H}$ invariant in all Lorentz frames ($\eta = \sqrt{\mu / \varepsilon}$). The TEM waves with $k_1 = k_2 = 0$ are known as null or pure radiation fields .

Notice that the invariants k_1 and k_2 are also gauge invariant. The EM invariants k_1 and k_2 can be combined to form another invariant [Zaghloul, et al; Torres-Silva, 2008],

$$\begin{aligned} k_1^2 + k_2^2 &= \left(\mathbf{E}^2 - \eta^2 \mathbf{H}^2\right)^2 + 4\left(\mathbf{E} \cdot \eta \mathbf{H}\right)^2 \\ &= \left(\mathbf{E}^2 + \eta^2 \mathbf{H}^2\right)^2 - 4\left|\mathbf{E} \times \eta \mathbf{H}\right|^2 , \\ &= 4\left(\varepsilon^2 - S^2 / c^2\right) \end{aligned} \tag{8}$$

where ε is the EM energy density and $\mathbf{S}$ is the Poynting vector. In a reference frame with $\mathbf{E} \| \mathbf{H}$ and $\mathbf{S} = 0$,

$$k_1^2 + k_2^2 = 4\varepsilon_{\mathrm{sw}}^2 , \tag{9}$$

where $\varepsilon_{\mathrm{sw}}$ is the EM standing-wave energy density. In any other frame, if $\mathbf{S}' \neq 0$, and $\varepsilon' \neq \varepsilon_{\mathrm{sw}}$, then

$$k_1'^2 + k_2'^2 = 4\left(\varepsilon'^2 - S'^2 / c^2\right) = k_1^2 + k_2^2 = 4\varepsilon_{\mathrm{sw}}^2 \tag{10}$$

or

$$\varepsilon'^2 - S'^2 / c^2 = \varepsilon_{\mathrm{sw}}^2 . \tag{11}$$

Equation (11) states that for any TEM wave, the difference between the square of the EM energy density (ε') and the square of the magnitude of the Poynting vector (S') in any frame is equal to the square of the EM energy density ($\varepsilon_{\mathrm{sw}}$) in the frame for which $\mathbf{E} \| \mathbf{H}$. This is true of all EM waves since $\varepsilon_{\mathrm{sw}} = 0$ for classical EM traveling waves with $\mathbf{E} \perp \mathbf{H}$ (null fields) whilst $\varepsilon_{\mathrm{sw}} \neq 0$ for parallelizable fields (generic fields) and for classical standing waves with $\mathbf{E} \perp \mathbf{H}$. It should be emphasized that $\varepsilon_{\mathrm{sw}}$ is an invariant and that ε is not an invariant [Zaghloul, et al.,1988].

Equation (11) can be compared with the expression for the invariant squared magnitude of the energy-momentum four-vector p_μ of a particle that is given by [Lorrain & Corson, 1970; Torres-Silva, 2008]

$$c^2 p_\mu p^\mu = E^2 - c^2 p^2 = m_0^2 c^4 , \tag{12}$$

where $E = mc^2$ is the self-energy, p is the magnitude of the momentum, m_0 is the rest mass, and m is the mass of the particle. This similarity suggests that somehow $\varepsilon_{\mathrm{sw}}$ plays the same role in electrodynamics that the rest energy of a particle plays in relativistic dynamics. In other words, the generation of $\mathbf{E} \| \mathbf{H}$ TEM standing waves allows the localization of electromagnetic energy. This result for $\mathbf{E} \| \mathbf{H}$ standing waves differs from that of $\mathbf{E} \perp \mathbf{H}$ standing waves since the energy oscillates back and forth, for the latter waves,

and there exists no Lorentz frame where the energy is static, which is the situation for the former waves. This localization of EM energy could open the door for some very interesting possibilities such as the storage ("bottling") of EM energy using, for example, a laser beam to make what could be thought of as a laser battery as well as verifying experimentally if the storing of EM energy E in a medium can be associated with an increase of the mass of the medium by m_0, where $m_0 = E/2c^2 = \hbar / 2cT$. "Electromagnetic mass" where gravitational mass and other physical quantities originate from the electromagnetic field alone. We can say that the electromagnetic mass models which are the sources of purely electromagnetic origin have not only connection associated with the conjecture of Lorentz but even a physics having novel features.

3. Chiral Dirac equation from electromagnetic standing waves with $\mathbf{E} \| \mathbf{H}$

The purpose of this section is to (a) derive a two-component force-free Dirac particle equation that incorporates all of the physically meaningful information about the particle contained in the force-free, four-component Dirac equation, (b) solve that two-component equation and, from the resulting two-component wave function, obtain the internal spin of the particle, (c) show that the spin and rest mass of the particle are the result of the phenomenon described by Schrodinger [Schrodinger, 1930] as *Zitterbewegung*, and (d) demonstrate that the relativistic increase in mass with velocity of the force-free particle is due to an increase in the rate of *Zitterbewegung* with velocity.

We introduce a new unitary transformation under which, for force-free motion, we obtain *uncoupled two-component* equations *which we can identify separately as a particle equation and an antiparticle equation*. In uncoupling the two equations and solving each equation as separate from and unrelated to the other, although that information is still there, hidden in the Hamiltonian, since the two equations can be put back together and transformed once again into the usual four-component Dirac equation, with a particle solution and an antiparticle solution. Our particle and antiparticle spinors are not Weyl or Majorana spinors; nevertheless and state that two of our spinors equal one pair of particle and antiparticle Dirac spinors.

The Foldy-Wouthuysen transformation [Foldy & Wouthuysen, 1950] decomposes the Dirac equation into two-component states of positive and negative energy. However, as the Foldy-Wouthuysen transformation does not provide a pair of particle and antiparticle equations. For the two-component equation produced by our separation process to be an equation for a particle (or for the companion equation to be an equation for an antiparticle), we must be able of demonstrating that, like the four-component equation, each of the two-component equations obtained leads to a relativistically correct four-vector, the fourth component of which is a scalar probability density, with a three-vector describing a probability current. For definiteness, throughout most of this section we will refer to the particle under consideration as an electron.

Based on the results of Fues and Hellmann [Fues & Hellmann, 1930], Schrödinger had come to the conc1usion that it was not practical to obtain general information about the electron by solving the four-component Dirac equation directly, and set out in [Schrodinger, 1930] to see what general information could be obtained without direct solutions. Schrodinger focused on that aspect of the Dirac equation and introduced the concept of Zitterbewegung in an attempt to deal with this problem when obtaining general information about solutions

to the Dirac equation. In addition to being discussed by Schrödinger in incisive detail in his original article [Schrodinger, 1930], *Zitterbewegung* in the four-component Dirac basis has been extensively addressed in books devoted to the the Dirac equation, [Rose, 1961.] and [Thaller, 1992] (who summarize Schrödinger's approach), as well as by Hanl and Papapetrou [Hanl & Papapetrou, 1940] in a paper that first analyzes Zitterbewegung in detail and then attempts to mimic the effects of *Zitterbewegung* with a mechanical model based on a classical point charge.

The relativistic particle equation in our paper presents an interpretation of Zitterbewegung similar to, yet quite different from, the interpretation for the force-free, four-component Dirac equation. For the Dirac Hamiltonian *Zitterbewegung* results from an interference between two positive and two negative energy components of the Dirac spinor [Lock, 1979]. Solutions of our two-component, force-free equation will be shown to have one positive energy component interfering with one negative energy component. In contrast to the solutions in the Dirac basis, the concept of the motion of a point has disappeared in our two-component case, and has been replaced by a density function and an internal motion (rotation) of that density function.

The difference between the interpretation of *Zitterbewegung* for our two-component particle Hamiltonian and for the Dirac Hamiltonian is to be expected, since it has been demonstrated that *Zitterbewegung* has different interpretations after different unitary transformations of the Dirac equation [Lock, 1979]. From another perspective, the different interpretation in our paper is also to be expected from comparing our result to equation-reduction techniques due to Feshbach [Feshbach, 1958]. In particular, whereas the Hamiltonian for the Dirac equation is linear in the components of the momentum operator, the Hamiltonian of our two-component particle equation is nonlocal.

The central difference between the two-component relativistic particle equation developed in the present section and the four-component Dirac equation is that for the Dirac equation the *Zitterbewegung* is explicit, as demonstrated by Schrödinger. In the case of the two-component particle equation obtained here, the effect of *Zitterbewegung* is implicit. Also, as Schrödinger noted before demonstrating the utility and importance of the concept of *Zitterbewegung*, information about a Dirac particle described in the usual Dirac basis can be difficult to extract. As an example consider Huang's insight (using the Dirac basis) [Huang, 2006] into the fact that internal rotation of the Dirac electron is associated with *Zitterbewegung*.

In many respects, the present section is a direct extension of papers written by Weyl in 1929 [Weyl, 1929 a;. Weyl, 1929 b; Weyl, 1929 c]. The importance of Weyl in the early interpretation of the Dirac equation has been succinctly described by Miller [Miller,1994]. In the first of the three papers Weyl asserts that, since the wave function of the electron "can only involve two components", two components of the solution of the Dirac equation shou1d be ascribed to the electron and two to the proton [Weyl, 1952]. In light of Weyl's later proof [Dirac, 1931] that negative and positive energy electrons must have the same mass, Dirac was led to the concept of the second solution of the Dirac equation being not a proton but antimatter, "having the same mass and opposite charge to an electron" [Dirac,1928]. However, in our case we have the condition +m, -e, T>0 to the electron and -m, +e, T<0 to the positron because for a particle in a rest frame $T = \hbar / 2mc$.

As Dirac well understood [Dirac, 1928], a two-component theory linear in the derivatives with respect to both the time and spatial coordinates is not possible. However, as we will demonstrate, by relaxing the restriction of linearity with respect to derivatives of the spatial

coordinates, a two-component equation for the force-free electron is possible, and the solutions we obtain to that equation provide the insights that Schrödinger was seeking [Torres-silva, 2011]. Moreover, when we obtain the complete solution of the two-component equation for the electron, we find significant additional characteristics not apparent in solutions of the four-component equation of the Dirac theory of the electron.

Finally, because in the Dirac basis the chirality operator does not commute with the Hamiltonian, it might appear that chirality (handedness, as opposed to the kinematic concept of helicity, associated with the labels ψ_L and ψ_R) cannot be a characteristic of a Dirac particle with mass. We will use our two-component solutions to demonstrate that this would be a false conclusion; for a Dirac particle with mass the usual definition of chirality can be replaced by a concept specific to a particle with mass, a concept that we will refer to a intrinsic electromagnetic chirality, $T = \hbar / 2mc$ with $\psi_L = \psi_{\vec{E}} = \vec{\sigma} \cdot \mathbf{E}$ and $\psi_R = \psi_{\vec{H}}. = \vec{\sigma} \cdot \boldsymbol{H}^{\bullet}$ explained below.

3.1 Two-component equations for the electron and anti-electron

The usual choice of an orthogonal set of four plane-wave solutions of the free-particle Dirac equation does not lend itself readily to direct and complete physical interpretation except in low energy approximation. A different choice of solutions can be made which yields a direct physical interpretation at all energies. Besides the separation of positive and negative energy states there is a further separation of states for which the spin is respectively parallel or antiparallel to the direction of the momentum vector. This can be obtained from the Maxwell's equation without charges and current in the $\mathbf{E} \| \boldsymbol{H}$ configuration. Dirac's four-component equation for the relativistic electron is.

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\psi = \hat{H}^{D}\psi \,, \tag{13}$$

where:

$$\hat{H}^{D} = c(\vec{\alpha} \cdot \hat{\vec{p}}) + mc^2\beta \,, \tag{14}$$

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad k = 1,2,3, \tag{15}$$

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{16}$$

and I is the two-by-two identity matrix. We now reduce the force-free Dirac equation.

This Hamiltonian commutes with the momentum vector $\hat{\mathrm{p}}$, and the usual procedure is to seek simultaneous eigenfunctions of H and p. These eigenfunctions are, however, not uniquely determined, and for given eigenvalues of H and p, there remains a twofold degeneracy. In order to resolve this degeneracy we seek a dynamical variable which commutes with both H and $\hat{\mathrm{p}}$. Such a variable is $\hat{\sigma} \cdot \hat{\mathrm{p}}$, where $\hat{\sigma}$ is the matrix Pauli. It is obvious that this variable commutes with $\hat{\mathrm{p}}$. To verify that it also commutes with H, we write $\hat{\alpha} = \rho_1 \hat{\sigma} = \hat{\sigma} \rho_1$ and recalling that β commutes with operator $\hat{\sigma}$, we have

$$\hat{\sigma}\cdot\hat{\mathrm{p}}H - H\hat{\sigma}\cdot\hat{\mathrm{p}} = \left(c\rho_1\hat{\sigma}\cdot\hat{\mathrm{p}}\right)\left(\hat{\sigma}\cdot\hat{\mathrm{p}}\right) - \left(\hat{\sigma}\cdot\hat{\mathrm{p}}\right)\left(c\rho_1\hat{\sigma}\cdot\hat{\mathrm{p}}\right) = 0\,, \tag{17}$$

since ρ_1 commutes with $\hat{\sigma}$.
We now proceed to find simultaneous eigenfunctions of the commuting variables *H*, p and $\hat{\sigma}\cdot\hat{\mathrm{p}}$. We have, since the components of p commute,

$$\left(\hat{\sigma}\cdot\hat{\mathrm{p}}\right)^2 = p^2\,, \tag{18}$$

where *p* is the magnitude of the momentum vector. Thus for a simultaneous eigenstate of $\hat{\mathrm{p}}$ and $\hat{\sigma}\cdot\hat{\mathrm{p}}$, the value of $\hat{\sigma}\cdot\hat{\mathrm{p}}$ will be +*p* or –*p*, corresponding to states for which the spin is parallel or antiparallel, respectively, to the momentum vector.
A simultaneous eigenfunction of *H* and *p* will have the form of a plane wave

$$\psi_j = u_j \exp\left[i\left(\mathrm{p}\cdot\mathrm{r} - Wt\right)/h\right], \qquad j = 1,2,3,4\,, \tag{19}$$

where the ψ_j are the four components of the state function and u_j four numbers to be determined. In the argument of the exponential function, p represents the eigenvalues of the components of the momentum for this state and *E* the corresponding eigenvalue of *H*. Then *E* can have either of the two values.

$$W = \pm\varepsilon = \pm\left(m^2c^4 + c^2p^2\right)^{\frac{1}{2}}. \tag{20}$$

We now demand that ψ_j be also an eigenfunction of $\hat{\sigma}\cdot\hat{\mathrm{p}}$ belonging to one of the eigenvalues $p_{\mathbf{E}}$, say, where $p_{\mathbf{E}} = \pm p$. Employing the usual matrix representation for $\hat{\sigma}$, we have

$$\hat{\sigma}\cdot\hat{p} = \begin{pmatrix} p_z & p_x - ip_y & 0 & 0 \\ p_x + ip_y & -p_z & 0 & 0 \\ 0 & 0 & p_z & p_x - ip_y \\ 0 & 0 & p_x + ip_y & -p_z \end{pmatrix}. \tag{21}$$

In the above matrix, p_x, p_y, and p_z are operators, but since this matrix is to operate on an eigenfunction of p, the operators can be replaced by their eigenvalues. We shall, without risk of confusion, use the same symbols for the eigenvalues as for the corresponding operators.
The eigenvalue equation is

$$\hat{\sigma}\cdot\hat{\mathrm{p}}\psi = p_{\mathbf{E}}\psi\,, \tag{22}$$

Since *W* can be given either of the two values $\pm\varepsilon$ and $p_{\mathbf{E}}$, the two values $\pm p$, we have found for given p four linearly independent plane wave solutions. It is easily verified that they are mutually orthogonal.
The physical interpretation of the solutions is now clear. Each solution represents a homogeneous beam of particles of definite momentum p, of definite energy, either $\pm\varepsilon$, and with the spin polarized either parallel or antiparallel to the direction of propagation.

3.2 Dirac equation deduced from Maxwell's equations with $\mathbf{E} \| \mathbf{H}$

Following our first section Transverse electromagnetic standing waves with $\mathbf{E} \| \boldsymbol{H}$ the vector potential is

$$\boldsymbol{F}(\mathrm{r},t) = \mathrm{Re}\left(e^{i\omega t}\right)\left[\mathbf{A}_k(\mathrm{r}) + k^{-1}\nabla \times \boldsymbol{A}_k(r)\right] \tag{23}$$

will lead to TEM standing waves with $\mathbf{E} \| \boldsymbol{H}$. This approach defines a class of $\mathbf{E} \| \boldsymbol{H}$ TEM standing waves that may be characterized by the fact that the vector potential A satisfies the equation $\nabla \times \mathbf{A} = k\mathbf{A}$, with $k = \omega / c$. Thus we have $\nabla \times \boldsymbol{E} = k\boldsymbol{E}$ and $\nabla \times \boldsymbol{H}^{\bullet} = k\boldsymbol{H}^{\bullet}$, where $\boldsymbol{H}^{\bullet}$ is the complex conjugate of $\boldsymbol{H}$. If we make the conjecture that the chiral factor T considered as a hidden scalar satisfies $Tp_{\mathbf{E}} = \pm\hbar/2$, then we can transform $\nabla \times \boldsymbol{E} = k\boldsymbol{E}$ as

$$i\hat{\sigma} \cdot \nabla \times \boldsymbol{E} = k\hat{\sigma} \cdot i\boldsymbol{E} \tag{24}$$

which can be put as

$$\hat{\sigma} \cdot p(\hat{\sigma} \cdot \boldsymbol{E}) = p_{\mathbf{E}}(\hat{\sigma} \cdot \boldsymbol{E}) \tag{25}$$

where we are considered $\nabla \cdot \boldsymbol{E} = 0$ (no charges), and

$$\hat{\sigma} \cdot \boldsymbol{E} = \begin{pmatrix} E_z & E_x - iE_y & 0 & 0 \\ E_x + iE_y & -E_z & 0 & 0 \\ 0 & 0 & E_z & E_x - iE_y \\ 0 & 0 & E_x + ie_y & -E_z \end{pmatrix} \tag{26}$$

The vector function ψ is given by

$$\psi_{\mathbf{E}} = \begin{pmatrix} E_x - E_y \\ |\boldsymbol{E}| - E_z \\ -\dfrac{|\boldsymbol{E}|c}{W - mc^2}(E_x - E_y) \\ -\dfrac{|\boldsymbol{E}|c}{W - mc^2}(|E| - E_z) \end{pmatrix} \tag{27}$$

with $|\boldsymbol{E}| = \dfrac{Wmc}{eh}$, $E = \hbar\omega$ and $p_{\mathbf{E}}T = \pm n\dfrac{\hbar}{2}$, where n = 1/2, 1, 3/2, 2 In this case of an electron n=1/2, the electric and magnetic fields are 90º out of phase, the energy density $\varepsilon_{\mathrm{sw}}$ is constant and proportional to $4\pi G(\boldsymbol{E}^2 + i\eta\boldsymbol{H}^2) \sim 4\pi k^2 F_0^2$, which numerically, we find in SI units as $4\pi k^2 F_0^2 = (1.3\times10^{-9} E E_c^{-1} m^{-1})^2$. Here, E_c is the critical field for electron-positron pair production $E_c = m_e{}^2 e^{-1} \sim 1.3\times10^{18} Vm^{-1}$ (see equations (5,6,7)of this paper) and [Torres-Silva, 2011]. Here, it is hypothesized that an elementary particle is simply a standing enclosed electromagnetic wave with a half or whole number of wavelengths (λ). For each half number of λ the wave will twist 180° around its travel path, thereby giving rise to chirality. As for photons, the Planck constant (h) can be applied to determine the total energy (E):

$E = nhc / \lambda$, where n = 1/2, 1, 3/2, 2, etc., and c is the speed of light in vacuum. The mass (m) can be expressed as a function of λ , since $E = mc^2$ gives $m = nh / c\lambda$, from the formula above.

In this form we have a close connection between the Dirac Equation and the Maxwell' equations, with a direct and complete physical interpretation in the $\mathbf{E} \| \boldsymbol{H}$ configuration. Here we are obtained a clear connection between the Planck constant $\hbar$, the quiral factor T, and the electromagnetic mass m.

To find the two uncoupled, two-component equations, one for the electron and one for the anti-electron (positron), we begin this reduction with the Cini-Touschek transformation [Cini & Touschek, 1958].

$$\hat{H}^T = M\hat{H}^D M^{-1}, \tag{28}$$

where:

$$M = \exp[iS], \tag{29}$$

$$S = \frac{\mathrm{i}}{2\hat{p}}\beta\left[\vec{\alpha}\cdot\hat{\vec{p}}\right]\mathrm{arc\ cot}\left(\frac{\hat{p}}{mc}\right), \tag{30}$$

and $\hat{\vec{p}}$ and $\hat{p}$ are defined by

$$\hat{\vec{p}} = -\mathrm{i}\hbar\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right);\ \hat{p}^2 = \hat{\vec{p}}\cdot\hat{\vec{p}}\,. \tag{31}$$

Under this transformation

$$\hat{H}^D \to \hat{H}^T = \sqrt{\left(\frac{m^2c^4}{\hat{p}^2} + c^2\right)}\vec{\alpha}\cdot\hat{\vec{p}}\,. \tag{32}$$

Instead of the transformation M in Eq. (29) we introduce the transformation

$$U = N_{chiral}M, \tag{33}$$

where

$$N_{chiral} = \frac{1}{\sqrt{2}}\begin{pmatrix} I & I \\ -I & I \end{pmatrix} \tag{34}$$

in which each component I is the two-by-two identity matrix.

Now we operate Eq. (25) with $\mathrm{i}\hbar\partial / \partial t$. This reduces the force-free Dirac equation to the pair of uncoupled two-spinor equations

$$\mathrm{i}\hbar\frac{\partial}{\partial t}(\vec{\sigma}\cdot\vec{E}) = \sqrt{\left(\frac{m^2c^4}{\hat{p}^2} + c^2\right)}\vec{\sigma}\cdot\hat{\vec{p}}(\vec{\sigma}\cdot\vec{E}) \tag{35}$$

and

$$-i\hbar\frac{\partial}{\partial t}(\vec{\sigma}\cdot\vec{H}^{\bullet}) = \sqrt{\left(\frac{m^2c^4}{\hat{p}^2}+c^2\right)}\vec{\sigma}\cdot\hat{\vec{p}}(\vec{\sigma}\cdot\vec{H}^{\bullet})\,. \tag{36}$$

In addition to Eqs. (35) and (36), it is instructive to factor the operator $\hat{p}^2$ out of the denominator and write Eqs. (35) and (36), respectively, as

$$i\hbar\frac{\partial}{\partial t}(\vec{\sigma}\cdot\vec{E}) = \sqrt{(m^2c^4+c^2\hat{p}^2)}\,\frac{1}{\sqrt{\hat{p}^2}}\vec{\sigma}\cdot\hat{\vec{p}}(\vec{\sigma}\cdot\vec{E})\,,\ (\omega_0 T/c\le 1) \tag{37}$$

and

$$-i\hbar\frac{\partial}{\partial t}(\vec{\sigma}\cdot\vec{H}^{\bullet}) = \sqrt{(m^2c^4+c^2\hat{p}^2)}\,\frac{1}{\sqrt{\hat{p}^2}}\vec{\sigma}\cdot\hat{\vec{p}}(\vec{\sigma}\cdot\vec{H}^{\bullet})\,,\ (\omega_0 T/c\le 1) \tag{38}$$

Where, $\psi_L = \psi_{\mathbf{E}} = \vec{\sigma}\cdot\vec{E}$, and $\psi_R = \psi_{\mathbf{H}^{\bullet}} = \vec{\sigma}\cdot\vec{H}^{\bullet}$. In this form it is easy to see that one can obtain Eq. (38) from Eq. (37) either by replacing *t* in Eq. (37) by -*t*, or by replacing $\sqrt{\left(m^2c^4+c^2\hat{p}^2\right)}$, the energy operator for a particle of momentum $\vec{p}$, by the operator $-\sqrt{\left(m^2c^4+c^2\hat{p}^2\right)}$, the operator for the negative of the energy in Eq. (37).

For the equation as written by Dirac, the pair of solutions are states of positive and negative energy, interpreted by Dirac and others as the electron and, eventually, the anti-e1ectron (the positron). A later interpretation [Mulligan, 2008], [Feynman, 1949] treats the solutions of the Dirac equation as a pair of different types of solutions-one type moving forward in time and the other type moving backward in time;

the solutions which move forward in time are interpreted as electrons, while those that move backwards in time are interpreted as anti-electrons. Although a study of the literature shows that the physical implications of the two different interpretations are profound, Feynman demonstrated that, *mathematically*, organizing the types of solutions according to his interpretation is equivalent to organizing the solutions according to the Dirac interpretation. From the way in which we rewrite the Dirac equation here as the combination of Eqs. (37) and (38), the difference between the two interpretations can easily be seen as simply a matter of whether to associate a minus sign with the energy *E* or with the time *t* compared with the sign of the chiral scalar T.

That Eq. (38) be a time-reversed or negative energy solution of Eq. (37) is clearly a necessary condition for recognizing the two-component spinor $\vec{\sigma}\cdot\vec{E}$ as a force-free electron.

As required by Dirac, the four-component operator $\hat{H}^D$ in Eq. (13) depends only linearly on the operator $\hat{\vec{p}}$. In contrast, the operator on the right-hand side of Eqs. (35) and (36) also depends on $\hat{\vec{p}}^2$, which appears in the denominator. A series expansion of the right-hand side of Eqs. (37) and (38) involves $\hat{\vec{p}}^2$ to all orders. Thus, whereas the Hamiltonian in Eq. (1) is local, the operator on the right-hand side of Eqs. (37) and (38) is nonlocal. It is well-known that nonlocality can result [13,14] when eliminating explicit reference to channels-in this case, the reference to the coupling of anti-electrons to electrons, which is explicit in Eq. (19) but no longer explicit in Eqs. (37) and (38), since each equation can be solved independently of the other but with the connection $\boldsymbol{E} = i\eta\boldsymbol{H}$.

One of the conditions imposed by Dirac in writing down a relativistic equation for the electron was that the "square" of that equation be the Klein-Gordon equation. Imposing the

additional condition of linearity of $\hat{H}^D$ in the components $\hat{p}_k$ led Dirac to Eqs. (13) and (14). It is trivial to see that squaring either Eqs. (37) and (38) also leads to the Klein-Gordon equation.

3.3 Internal rotation and electron mass

As was the case with the four-component force-free Dirac equation, our two-component force-free equation exhibits Zitterbewegung as an interference between components. In our case, each solution has one positive energy component and one negative energy component. Also, our solutions immediately exhibit a frequency

$$\Omega' = W / \hbar \tag{39}$$

and relate that frequency to an internal rotation of the electron. This frequency agrees with the frequency of rotation determined by Huang from consideration of the characteristics of the *Zitterbewegung* associated with the Dirac equation, but not obtained there so readily; as Huang points out, *Zitterbewegung* as rotation is not a feature which is immediately apparent in the four-component solutions of the Dirac equation.

Here, we consider in more detail the mass of the electron as a property which can be attributed to its internal rotation.

The angular frequency Ω' given by Eq. (39) is the accepted value of the rotational frequency associated with the intrinsic spin of an electron in motion with momentum $\vec{p}$, as discussed by Huang [Huang, 2006]. An additional important aspect of the internal rotation of the electron is the realization and identification that the rest-mass energy of a Dirac particle "appears simply as the energy of the internal motion in the rest frame."

The condition $\omega_0 T / c = 1$, in eq. allow to write the angular frequency Ω for an electron at rest, Eq. (39) becomes

$$\omega_0 T / c = 1 \rightarrow \omega_o = \Omega = \frac{2mc^2}{\hbar} \tag{40}$$

Eq. (40) demonstrates that if *Zitterbewegung* is the mechanism that gives rise to the rest mass of the electron, it also must be the mechanism that gives rise to the relativistic mass as a function of the velocity of the electron.

Moreover, we can attribute the increase in *Zitterbewegung* for an electron in motion directly to the time dilation due to motion of the electron relative to the observer. This can be demonstrated by repeating the well-known fact that if we define a velocity υ_z for the electron by

$$\upsilon_z = \frac{p_z}{M}, \tag{41}$$

where M the relativistic mass

$$M = \frac{m}{\sqrt{1 - \frac{\upsilon_z^2}{c^2}}}, \tag{42}$$

the expression $\sqrt{m^2c^4 + p_z^2c^2}$ for the energy E becomes

$$E = \frac{mc^2}{\sqrt{1-\frac{\upsilon_z^2}{c^2}}} \tag{43}$$

The time dependence of the angle of the intrinsic rotation of the electron is given by $\frac{2}{\hbar}Et$, or

$$\frac{2}{\hbar}Et = \frac{2}{\hbar}\frac{mc^2}{\sqrt{1-\frac{\upsilon_z^2}{c^2}}}t \tag{44}$$

We can write Eq. (75) in the alternate form

$$\frac{2}{\hbar}Et = \frac{2}{\hbar}mc^2\tau\,, \tag{45}$$

where

$$\tau = \frac{1}{\sqrt{1-\frac{\upsilon_z^2}{c^2}}}t\,. \tag{46}$$

Consequently, the angular frequency Ω' of the rotation measured by an observer who sees the electron moving with velocity υ_z is related to the rest frame angular frequency Ω by

$$\Omega' = \frac{\Omega}{\sqrt{1-\frac{\upsilon_z^2}{c^2}}}\,. \tag{47}$$

The above discussion is presented in detail because Hönl and Papapetrou, in their mechanical model of of the electron, accidentally apply time dilation to *decrease* the rotational frequency, rather than to dilate the time. Accordingly, they conclude that the rate of rotation decreases rather than increases as the velocity of the particle increases. In light of Torres-Silva's [18] insight that the internal motion of the electron is the source of its mass, the relation between increased internal motion and increased mass becomes a critical aspect of the source of mass in a Dirac particle.

Dorling [Dorling, 1970] recognized that Eq. (47) represents an adjustment to a frequency associated with the electron at rest and attempts to relate this frequency to *Zitterbewegung*. We can now readily make the necessary connection based on our results for the force-free electron. To effect the decrease in mass required by the binding process, according to our results the rotational frequency associated with the *Zitterbewegung* would have to decrease by an amount appropriate in energy of the electron. That is exactly what Eq. (47) describes. Therefore, in the case of Eq. (47) the role of time as measured by a clock attached to the electron and a dock held by an external observer is the reverse of that for the free electron. For the bound electron the external observer sees the rate of rotation due to Zitterbewegung correspondingly reduced.

In his paper on *Zitterbewegung* [Schrodinger, 1930], Schrödinger demonstrates that *Zitterbewegung* introduces motion perpendicular to the direction of net motion of the electron.

The need to introduce components p_x and p_y was first recognized by Neamtan [Neamtan, 1952], who did not associate these components with *Zitterbewegung*. Moreover, Neamtan was working with the four-component Dirac equation, and his solutions do not lend themselves to the interpretation possible here as a result of our use of two-component wave functions, to which we can apply Eqs. (37)-(38).

Our results allow us to write a solution for a particle which is intrinsically right-handed, a particle which is intrinsically left-handed, or, by taking a linear combination of solutions, a particle such as the electron that has no intrinsic chirality. In obtaining the solution of Eq. (35) we have effectively solved the Dirac equation for the particle moving in the direction of the positive *z*-axis and projected out that part of the particle wave function with positive helicity and positive intrinsic chirality; in obtaining one solution, we have effectively solved the Dirac equation for the particle moving in the direction of the positive *z*-axis and projected out that part of the particle wave function with positive helicity and negative intrinsic chirality. This is of substantial practical importance, since in electron β-decay only that part of the wave function with negative intrinsic chirality takes part.

The general consensus of papers on *Zitterbewegung* is that in the rest frame of the electron *Zitterbewegung* has a characteristic amplitude of $\hbar / 2mc$. This information can be used to obtain an order of magnitude estimate of el. in the rest frame. We set

$$\frac{\int_0^\infty \mathrm{e}^{(-2/\hbar)\alpha x} x\,\mathrm{d}x}{\int_0^\infty \mathrm{e}^{(-2/\hbar)\alpha x}\,\mathrm{d}x} = \frac{\hbar}{2mc} \tag{48}$$

which yields

$$\alpha = mc\ . \tag{49}$$

Before leaving the subject of the relation of *Zitterbewegung* to mass, we return to discussion of Zitterbewegung in the context of the Weyl equations. he treats the mass term in each of the two coupled spinors of the Weyl basis as a source for the other, a coupling proportional to the electron mass. Accordingly, while we already have related the size of the electron perpendicular to its motion to the electron mass. Our analysis allows us also to relate the length of the electron to its mass. Penrose describes the motion of the Dirac point position of the stationary electron along the axis of rotation of the electron (which we have taken to be the *z*-axis) as motion first with velocity e and negative helicity in the positive *z*-direction and then with positive helicity in the negative *z*-direction. The Dirac point performs this oscillatory motion with the de Broglie frequency

$$\upsilon = \frac{mc^2}{2\pi\,\hbar}\ . \tag{50}$$

Multiplying the time for one motion in each direction by the velocity *c* gives for the length *L* of the electron

$$L = \frac{\pi \hbar}{mc} . \tag{51}$$

While this calculation results in an order of magnitude estimate of the length of the electron, even when at rest the motion of the electron distribution along its axis of rotation.

4. New interpretation of the atomic spectra of the hydrogen atom: a mixed mechanism of classical LC circuits and quantum wave-particle duality

Following Huang, [Huang, 2006], we study the energy conversion laws of the macroscopic harmonic LC oscillator, the electromagnetic wave (photon) and the hydrogen atom. As our analysis indicates that the energies of these apparently different systems obey exactly the same energy conversion law. Based on our results and the wave- particle duality of electron, we find that the atom of Hydrogen in fact is a natural chiral microscopic LC oscillator.

In the framework of classical electromagnetic field theory we analytically obtain, for the hydrogen atom, the quantized electron orbit radius $r_n = a_o n^2$, and quantized energy $W_n = -R_H hc / n^2$, (n = 1, 2, 3, ·), where a_o is the Bohr radius and R_H is the Rydberg constant. Without the adaptation of any other fundamental principles of quantum mechanics, we present a reasonable explanation of the polarization of photon, selection rules and Pauli exclusion principle. Our results also reveal an essential connection between electron spin and the intrinsic helical movement of electron and indicate that the spin itself is the effect of quantum confinement.

In fact, at the heart of quantum mechanics lies only the Schrödinger equation, which is the fundamental equation governing the electron. According to quantum theory, it is the electromagnetic interaction (by the exchange of photons) which hold electrons and nuclei together in the atoms [Planck, 1900; Einstein, 1905; Bohr, 1913; Stern,1920; de Broglie, 1924 ; Pauli, 1924 ; Heisenberg, 1927 ; Yang, 1950]. But, up to now, quantum theory never provides a practical scheme that electron and nuclei can absorb and emit photons.

In this section, we investigate the energy relationship of electron in the hydrogen atom. Significantly, we find a process of perfect transformation of two forms of energy (kinetic and field energy) inside the atom and the conservation of energy in the system. By applying the principle of wave-particle duality and comparing to known results of the macroscopic harmonic LC oscillator and microscopic photon, we are assured that electron kinetic energy in fact is a kind of magnetic energy and the atom is a natural microscopic LC oscillator. Moreover, the mixed mechanism (classical LC circuits / quantum wave particle duality) turns out to have remarkably rich and physical properties which can used to describe some important quantum principles and phenomena, for instance, polarization of photon, Zeeman effect, Selection rules, the electron's mass and spin, zero point energy (ZPE) , the Pauli exclusion principle.

4.1 Energy transformation and conversion in hydrogen atom

Classically, as shown in Fig. 1, the hydrogen atom consists of one electron in orbit around one proton with the electron being held in place via the electric Coulomb force. Equation of motion is

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m_e u^2}{r} , \tag{52}$$

where m_e is mass of electron. Eq. (52) can be rewritten in the form of kinetic energy W_k and field energy W_f (stored in the capacitor of hydrogen atom) as follows:

$$\frac{e^2}{2C_r} = \frac{1}{2} m_e u^2 , \tag{53}$$

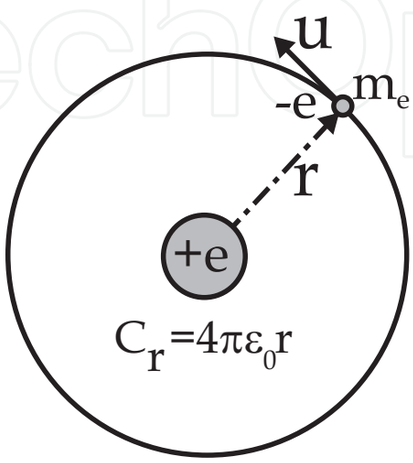


Fig. 1. The diagram illustrating the hydrogen atom

where $C_r = 4\pi\varepsilon_0 r$ is the capacitance of the hydrogen system. Thus the total energy of the hydrogen system is given by

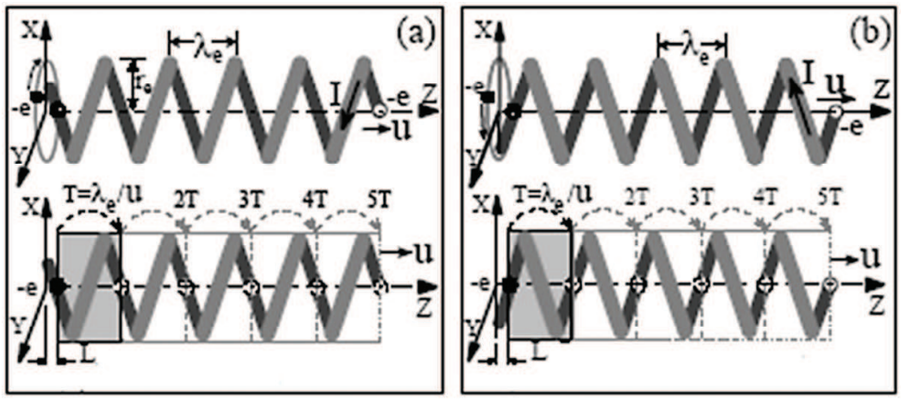
$$W_{total} = \left| \frac{1}{2} m_e u^2 - \frac{e^2}{4\pi\varepsilon_0 r} \right| = \frac{e^2}{2C_r} . \tag{54}$$

It should be pointed out that Eq. (53) and (54) are the foundation of our study. These two equations together indicate a process of perfect periodically transformation of two forms of energy (kinetic energy $W_k = m_e u^2 / 2$ and field energy $W_f = \frac{e^2}{2C_r}$) inside the atom and the conservation of energy in the system

$$W_{total} = W_f = W_k \tag{55}$$

Recall the macroscopic harmonic LC oscillator where two forms of energy, the maximum field energy $W_f = \frac{Q_0^2}{2C}$ of the capacitor C (carrying a charge Q_0) and the maximum magnetic energy $W_m = \frac{L_0^2}{2L}$ of the inductor L, are mutually interchangeable ($W_{total} = W_f = W_k$) with a exchange periodic $T = 2\pi\sqrt{LC}$. And for a microscopic photon (electromagnetic wave), the maximum field energy $W_f = \frac{1}{2}\varepsilon_0 \boldsymbol{E}_0^2$ and the maximum magnetic energy $W_m = \frac{1}{2}\mu_0 \boldsymbol{H}_0^2$ also satisfy $W_{total} = W_f = W_k$.

Based on the above energy relationship for three totally different systems and the requirement of the electromagnetic interaction (by exchanging photon) between electron and nuclei, we assure that the kinetic energy of electron, Eq. (53) is a kind of magnetic energy and the hydrogen atom is a natural microscopic LC oscillator.
Recently, a multinational team of physicists had observed for the first time a process of internal conversion between bound atomic states when the binding energy of the converted electron becomes larger than the nuclear transition energy [Carreyre et al. 2000; Kishimoto et al. 2000]. This observation indicate that energy can pass resonantly between the nuclear and electronic parts of the atom by a resonant process similar to that which operates between an inductor and a capacitor in an LC circuit. These experimental results can be considered a conclusive evidence of reliability of our LC mechanism.
Here raise an important question: how can the electron function as an excellent microscopic inductor?



a) Left hand electron (S=1), b) Right hand electron (S=-1)

Fig. 2. A free electron moving along a helical orbit with a helical pitch of de Broglie wavelength λ_e

The answer lies in the intrinsic wave-particle duality nature of electron. In our opinion, the wave-particle nature of electron is only a macroscopic behavior of the intrinsic helical motion of electron within its world.

4.2 Chirality and "inducton" of free electron

De Broglie suggested that all particles, not just photons, have both wave and particle properties [Reines & Sobel, 1974]. The momentum wavelength relationship for any material particles was given by

$$\lambda = h / p\,, \tag{56}$$

where λ is called de Broglie wavelength, h is Planck's constant and p the momentum of the particle. The subsequent experiments established the wave nature of the electron [9, 10]. Eq. (56) implies that, for a particle moving at high speed, the momentum is large and the wavelength is small. In other words, the faster a particle moves, the shorter is its wavelength. Furthermore, it should be noted that any confinement of the studied particle will shorten than λ and help to enhance the so-called quantum confinement effects.

As shown in Fig. 2 (a) and (b), based on Eq. (56) and the demanding that the electron would be a microscopic inductor, we propose that a free electron can move along a helical orbit (the helical pitch is de Broglie wavelength λ_e) of left-handed or right-handed. In this paper, the corresponding electrons are called "Left-hand" and "Right-hand" electron which are denoted by Chirality Indexes S = 1 and S = −1, respectively. Hence, the electron can now be considered as a periodic-motion quantized inductive particle which is called "inducton" (see Fig. 2). Moreover, the particle-like kinetic energy of electron can be replaced with a dual magnetic energy carried by a "inducton". Therefore, we have

$$W_k = \frac{1}{2} m_e u^2 = \frac{1}{2} L_e I^2 , \tag{57}$$

where u is the axial velocity of the helical moving electron and L_e is the inductance of the quantized "inducton".

The above relation indicates that the mass of electron is associated with an amount of magnetic energy. From Fig. 2, the electric current, for one de Broglie wavelength, is given by

$$I = \frac{eu}{\lambda_e} . \tag{58}$$

From Eq. (58), it is important to note that the electric current should be defined within an integral number of de Broglie wavelength. Hence, the electric current $I = \frac{eu}{2\pi r}$ $I = \frac{eu}{2\pi r}$ (where r is the electronic orbital radius in the hydrogen atom), which was widely used in the semiclassical Bohr model, may be physically invalid. Collecting Eq. (57) and (58) together, we have the inductance of single "inducton"

$$L_e = \frac{m_e \lambda_e^2}{e^2} . \tag{59}$$

Then the dual nature of electron can be uniquely determined by L_e, the periodic T (or frequency $f = \frac{1}{T} = \frac{u}{\lambda_e}$), the initial phase φ_0 and the chirality (S = 1 or S = −1).

4.3 Atomic spectra of hydrogen atom

A. Quantized radius and energy by the application of helical electron orbit to the hydrogen atom (Fig. 2), we can explain the stability of the atom but also give a theoretical interpretation of the atomic spectra. Fig. 3 shows four possible kinds of stable helical electron orbits in hydrogen atom, and each subgraph corresponds to a electron of different motion manner within the atom. The electrons can be distinguished by the following two aspects. First consider the chirality of electron orbits, as shown in Fig. 3, the electrons of Fig. 3(a) and (c) are "Left-hand" labelled by S = 1, while electrons of Fig. 3(b) and (d) are "Right-hand" labelled by S = −1. Secondly consider the direction of electron orbital magnetic moment μL, Fig. 3(a) and (b) show that the μL are in the Z direction (Up) while (c) and (d) in the −Z direction (Down), the corresponding electrons are labelled by J = 1 and J = −1, respectively, here J is called Magnetic Index. Hence, the electrons of different physical

properties become distinguishable, they are Up "Left-hand" (ULH) electron (J = 1, S = 1), Up "Right-hand" (URH) electron (J = 1, S = −1), Down "Left-hand" (DLH) electron (J = −1, S = 1) and Down "Right-hand" (DRH) electron (J = −1, S = −1).

As shown in Fig. 3(a), the helical moving electron around the orbit mean radius r can now be regarded as a quantized "inducton" of λ_r, thus the hydrogen atom is a natural microscopic LC oscillator.

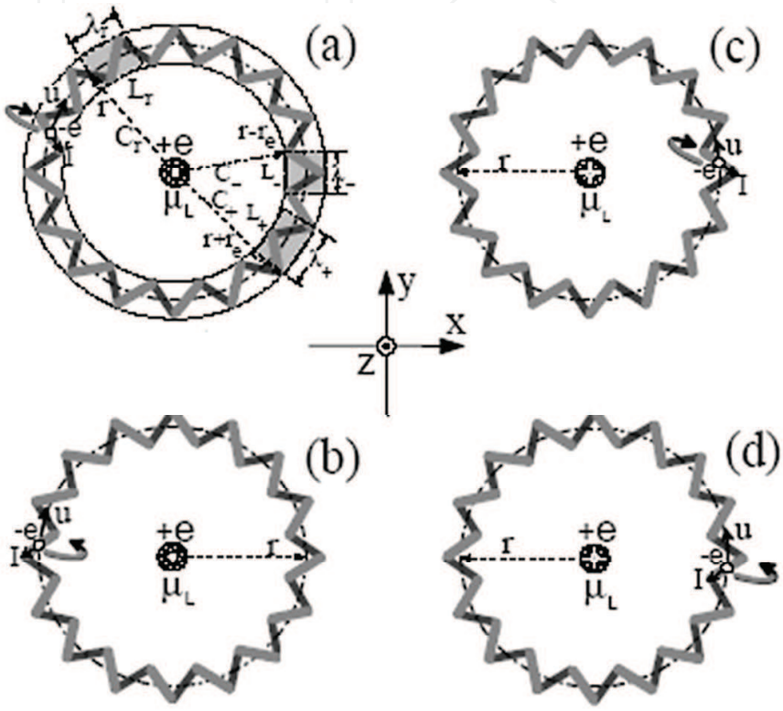


Fig. 3. The quadruple degenerate stable helical electron or-bits in hydrogen atom. a) Up Left-hand electron ULH electron (J=1, S=1); b) Up Right-hand electron URH electron (J=1, S=-1); c) Down Left-hand electron DLH electron (J=-1, S=1); d) Down Right-hand electron DRH electron (J=-1, S=-1)

We consider that the physical properties of the hydrogen atom can be uniquely determined by these natural LC parameters. To prove that our theory is valid in explaining the structure of atomic spectra, we study the quantized orbit radius and the quantized energy of hydrogen atom and make a comparison between our results of LC mechanism and the known results of quantum theory. For the system of λ_r, the LC parameters of the hydrogen atom is illustrated in Fig. 3. Then the LC resonant frequency is

$$\nu_r = \frac{1}{2\pi\sqrt{L_r C_r}}. \tag{60}$$

Recall the well-known relationship $E = h\nu_r$, we have

$$W = E = h\nu_r = \frac{e^2}{8\pi\varepsilon_0 r}. \tag{61}$$

Combining Eq. 60 and Eq. 61 gives

$$\lambda_r = \frac{2h}{e}\sqrt{\pi\varepsilon_0 r / m_e} \ . \tag{62}$$

Then the stable electron orbits are determined by

$$\frac{2\pi r}{\lambda_r} = n \ , \text{(n = 1, 2, 3 } \cdots), \tag{63}$$

where n is called Principal oscillator number. The integer n shows that the orbital allow integer number of "induction" of the de Broglie wavelength λ_r . From Eq. 52 and Eq. 53, the quantized electron orbit mean radius is given by

$$r_n = \frac{\varepsilon_0 h^2}{\pi m_e e^2} n^2 = a_0 n^2 \ , \tag{64}$$

where a_0 is the Bohr radius. And the quantized energy is

$$W_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{m_e e^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -R_H \frac{hc}{n^2} \ , \tag{65}$$

where R_H is the Rydberg constant. Surprisingly, the results of Eq. (64) and (65) are in excellent agreement with Bohr model. Besides, taking Fig. 3 into account, we can conclude that the quantized energies of Eq. (65) are quadruple degenerate.

5. Concluding remarks

In this article we have examined the conditions under which transverse electromagnetic (TEM) waves according to whether their Poynting vector is identically zero or nonzero. We have studied the non propagating TEM standing waves with $\mathbf{E} \| \mathbf{H}$. The Chiral general condition under which TEM standing waves with $\mathbf{E} \| \mathbf{H}$ exist is derived. Two physical examples of these standing waves are given.

The first example is about the Dirac Equation for a free electron, which is obtained from the Maxwell Equations under the Born Fedorov approach, ($\mathbf{D} = \varepsilon_0(1 + T\nabla\times)\mathbf{E}$), and ($\mathbf{B} = \mu_0(1 + T\nabla\times)\mathbf{H}$ **).** Here, it is hypothesized that an elementary particle is simply a standing enclosed electromagnetic wave with a half or whole number of wavelengths (λ). For each half number of λ the wave will twist 180° around its travel path, thereby giving rise to chirality. As for photons, the Planck constant (h) can be applied to determine the total energy (E): $E = nhc / \lambda$, where n = 1/2, 1, 3/2, 2, etc., and c is the speed of light in vacuum. The mass m can be expressed as a function of λ , since $E = mc^2$ gives $m = nh / c\lambda$, from the formula above. This result is obtained from the resulting wave equation which is reduced to a Beltrami equation $\nabla\times\mathbf{E} = -(1/2T)\mathbf{E}$ when the chiral factor T is given by $T = n\hbar / mc$. The chiral Pauli matrices are used to obtain the Dirac Equation.

In the second example, we have found a perfect transformation of two forms of energy (kinetic and field energy) inside the hydrogen atom and the conservation of energy in the

system. Then, we have shown that the helical moving electron can be regarded as an inductive particle ("inducton") while atom as a microscopic LC oscillator, then the indeterministic quantum phenomena can be well explained by the deterministic classical theory. For a microscopic photon (electromagnetic wave), the maximum field energy $W_f = \frac{1}{2}\varepsilon_0 \boldsymbol{E}_0^2$ and the maximum magnetic energy $W_m = \frac{1}{2}\mu_0 \boldsymbol{H}_0^2$ are connected iff $\boldsymbol{E} = i\eta \boldsymbol{H}$.

The Poynting vector vanishes and the Hydrogen atom does not radiate and it is stable. In particular, with this approach we can show another phenomena such how a pairing Pauli electron can move closely and steadily in a DNA-like double helical electron orbit. Moreover, we can have pointed out that the mass of electron, the intrinsic "electron spin", the Pauli exclusion principle and the Dirac equation are all really the quantum confinement effects of the intrinsic chirality of particles of helical motion produced by electromagnetic fields..

We have shown that the quantum mechanism is nothing but an electromagnetic theory (with the radius of the helical orbit $r_e \to 0$) of the LC/wave-particle duality mixed mechanism. Our mixed mechanics force us to rethink the nature and the nature of physical world. We believe all elementary particles, similar to photon and electron, are only some different types of energy representation.

From our study, it has been shown that the electron follows a perfectly defined trajectory in its motion, which confirms the de Broglie-Bohm's prediction (Bohm, 1952). Also in our work, it is found that the known wave-particle duality can be best manifested by showing that the wave motion associated with a electron is just the phenomenon of its complex helical motion in real space.

It is hoped that this article will encourage electrodynamics course instructors to include a discussion of $\mathbf{E} \| \mathbf{H}$ TEM standing waves in atomic systems.

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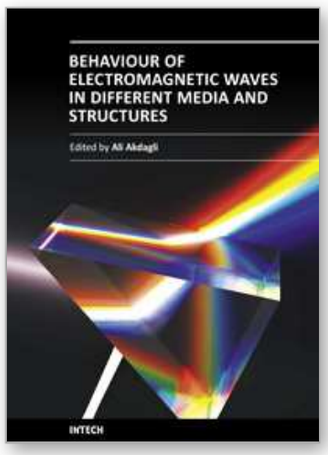

Behaviour of Electromagnetic Waves in Different Media and Structures

Edited by Prof. Ali Akdagli

ISBN 978-953-307-302-6
Hard cover, 440 pages
Publisher InTech
Published online 09, June, 2011
Published in print edition June, 2011

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