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Coherent Current States in Two-Band Superconductors

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1. Introduction

To present day overwhelming majority works on theory of superconductivity were devoted to single gap superconductors. More than 50 years ago the possibility of superconductors with two superconducting order parameters were considered by V. Moskalenko

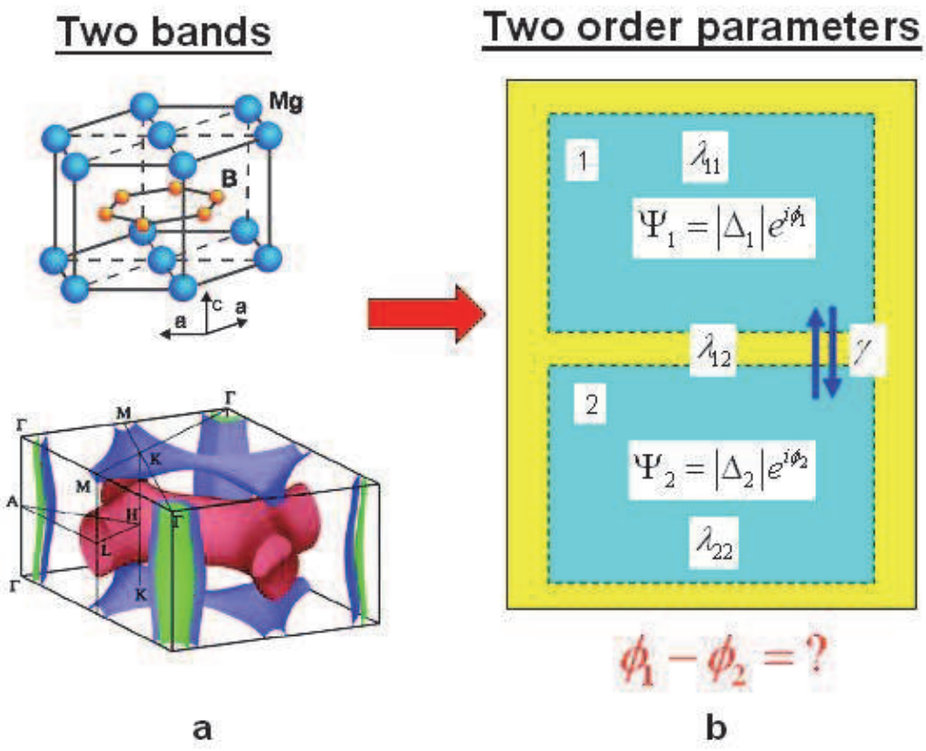


Fig. 1. a. The structure of MgB_2 and the Fermi surface of MgB_2 calculated by Kortus et al. (Kortus et al., 2001).
b. The coexistence of two complex order parameters (in momentum space).

(Moskalenko, 1959) and H. Suhl, B.Matthias and L.Walker (Suhl et al., 1959). In the model of superconductor with the overlapping energy bands on Fermi surface V.Moskalenko has theoretically investigated the thermodynamic and electromagnetic properties of two-band superconductors. The real boom in investigation of multi-gap superconductivity started after the discovery of two gaps in MgB_2 (Nagamatsu et al., 2001) by the scanning tunneling (Giubileo et al., 2001; Iavarone et al., 2002) and point contact spectroscopy (Szabo et al., 2001; Schmidt et al., 2001; Yanson & Naidyuk, 2004). The structure of MgB_2 and the Fermi surface of MgB_2 calculated by Kortus et al. (Kortus et al, 2001) are presented at Fig.1.a. The compound MgB_2 has the highest critical temperature $T_c = 39$ K among superconductors with phonon mechanism of the pairing and two energy gaps $\Delta_1 \approx 7meV$ and $\Delta_2 \approx 2,5meV$ at $T = 0$. At this time two-band superconductivity is studied also in another systems, *e.g.* in heavy fermion compounds (Jourdan et al., 2004; Seyfarth et al., 2005), high-T_c cuprates (Kresin & Wolf, 1990), borocarbides (Shulga et al., 1998), liquid metallic hydrogen (Ashcroft, 2000; Babaev, 2002; Babaev et. al, 2004). Recent discovery of high-temperature superconductivity in iron-based compounds (Kamihara et al., 2008) have expanded a range of multiband superconductors. Various thermodynamic and transport properties of MgB_2 and iron-based superconductors were studied in the framework of two-band BCS model (Golubov et al., 2002; Brinkman et al., 2002; Mazin et al., 2002; Nakai et al., 2002; Miranovic et al., 2003; Dahm & Schopohl, 2004; Dahm et al., 2004; Gurevich, 2003; Golubov & Koshelev, 2003). Ginzburg-Landau functional for two-gap superconductors was derived within the weak-coupling BCS theory in dirty (Koshelev & Golubov, 2003) and clean (Zhitomirsky & Dao, 2004) superconductors. Within the Ginzurg-Landau scheme the magnetic properties (Askerzade, 2003a; Askerzade, 2003b; Doh et al., 1999) and peculiar vortices (Mints et al., 2002; Babaev et al., 2002; Gurevich & Vinokur, 2003) were studied.

Two-band superconductivity proposes new interesting physics. The coexistence of two distinctive order parameters $\Psi_1 = |\Psi_1|\exp(i\phi_1)$ and $\Psi_2 = |\Psi_2|\exp(i\phi_2)$ (Fig.1.b.) renewed interest in phase coherent effects in superconductors. In the case of two order parameters we have the additional degree of freedom, and the question arises, what is the phase shift $\delta\phi = \phi_1 - \phi_2$ between Ψ_1 and Ψ_2 ? How this phase shift manifested in the observable effects? From the minimization of the free energy it follows that in homogeneous equilibrium state this phase shift is fixed at 0 or π , depending on the sign of interband coupling. It does not exclude the possibility of soliton-like states $\delta\phi(x)$ in the ring geometry (Tanaka, 2002). In nonequilibrium state the phases ϕ_1 and ϕ_2 can be decoupled as small plasmon oscillations (Leggett mode) (Legett, 1966) or due to formation of phase slips textures in strong electric field (Gurevich & Vinokur, 2006).

In this chapter we are focusing on the implication of the $\delta\phi$-shift in the coherent superconducting current states in two-band superconductors. We use a simple (and, at the same time, quite general) approach of the Ginsburg–Landau theory, generalized on the case of two superconducting order parameters (Sec.2). In Sec.3 the coherent current states and depairing curves have been studied. It is shown the possibility of phase shift switching in homogeneous current state with increasing of the superfluid velocity v_s. Such switching manifests itself in the dependence $j(v_s)$ and also in the Little-Parks effect (Sec.3). The Josephson effect in superconducting junctions is the probe for research of phase coherent effects. The stationary Josephson effect in tunnel S_1-I-S_2 junctions (I - dielectric) between

two- and one- band superconductors have been studied recently in a number of articles (Agterberg et al., 2002; Ota et al., 2009; Ng & Nagaosa, 2009). Another basic type of Josephson junctions are the junctions with direct conductivity, S-C-S contacts (C – constriction). As was shown in (Kulik & Omelyanchouk, 1975; Kulik & Omelyanchouk, 1978; Artemenko et al., 1979) the Josephson behavior of S-C-S structures qualitatively differs from the properties of tunnel junctions. A simple theory (analog of Aslamazov-Larkin theory(Aslamazov & Larkin, 1968)) of stationary Josephson effect in S-C-S point contacts for the case of two-band superconductors is described in Sec.4).

2. Ginzburg-Landau equations for two-band superconductivity.

The phenomenological Ginzburg-Landau (GL) free energy density functional for two coupled superconducting order parameters ψ_1 and ψ_2 can be written as

$$F_{GL} = F_1 + F_2 + F_{12} + \frac{H^2}{8\pi},$$

Where

$$F_1 = \alpha_1 |\psi_1|^2 + \frac{1}{2}\beta_1 |\psi_1|^4 + \frac{1}{2m_1}\left| \left(-i\hbar\nabla - \frac{2e}{c}\vec{A} \right)\psi_1 \right|^2 \tag{1}$$

$$F_2 = \alpha_2 |\psi_2|^2 + \frac{1}{2}\beta_2 |\psi_2|^4 + \frac{1}{2m_2}\left| \left(-i\hbar\nabla - \frac{2e}{c}\vec{A} \right)\psi_2 \right|^2 \tag{2}$$

and

$$F_{12} = -\gamma\left(\psi_1^*\psi_2 + \psi_1\psi_2^*\right) + \eta \begin{pmatrix} \left(-i\hbar\nabla - \frac{2e}{c}\vec{A} \right)\psi_1 \left(i\hbar\nabla - \frac{2e}{c}\vec{A} \right)\psi_2^* + \\ + \left(i\hbar\nabla - \frac{2e}{c}\vec{A} \right)\psi_1^* \left(-i\hbar\nabla - \frac{2e}{c}\vec{A} \right)\psi_2 \end{pmatrix} \tag{3}$$

The terms F_1 and F_2 are conventional contributions from ψ_1 and ψ_2, term F_{12} describes without the loss of generality the interband coupling of order parameters. The coefficients γ and η describe the coupling of two order parameters (proximity effect) and their gradients (drag effect) (Askerzade, 2003a; Askerzade, 2003b; Doh et al., 1999), respectively. The microscopic theory for two-band superconductors (Koshelev & Golubov, 2003; Zhitomirsky & Dao, 2004; Gurevich, 2007) relates the phenomenological parameters to microscopic characteristics of superconducting state. Thus, in clean multiband systems the drag coupling term (η) is vanished. Also, on phenomenological level there is an important condition , that absolute minimum of free GL energy exist: $|\eta| < \frac{1}{2\sqrt{m_1 m_2}}$ (see Yerin et al., 2008).

By minimization the free energy $F=\int(F_1+F_2+F_{12}+\frac{H^2}{8\pi})d^3r$ with respect to ψ_1, ψ_2 and $\vec{A}$ we obtain the differential GL equations for two-band superconductor

$$\begin{cases}\frac{1}{2m_1}\left(-i\hbar\nabla-\frac{2e}{c}\vec{A}\right)^2\psi_1+\alpha_1\psi_1+\beta_1|\psi_1|^2\psi_1-\gamma\psi_2+\eta\left(-i\hbar\nabla-\frac{2e}{c}\vec{A}\right)^2\psi_2=0\\ \frac{1}{2m_2}\left(-i\hbar\nabla-\frac{2e}{c}\vec{A}\right)^2\psi_2+\alpha_2\psi_2+\beta_2|\psi_2|^2\psi_2-\gamma\psi_1+\eta\left(-i\hbar\nabla-\frac{2e}{c}\vec{A}\right)^2\psi_1=0\end{cases} \tag{4}$$

and expression for the supercurrent

$$\begin{aligned}&j=-\frac{ie\hbar}{m_1}\left(\psi_1^*\nabla\psi_1-\psi_1\nabla\psi_1^*\right)-\frac{ie\hbar}{m_2}\left(\psi_2^*\nabla\psi_2-\psi_2\nabla\psi_2^*\right)\\ &-2ie\hbar\eta\left(\psi_1^*\nabla\psi_2-\psi_2\nabla\psi_1^*-\psi_1\nabla\psi_2^*+\psi_2^*\nabla\psi_1\right)-\\ &-\left(\frac{4e^2}{m_1c}|\psi_1|^2+\frac{4e^2}{m_2c}|\psi_2|^2+\frac{8\eta e^2}{c}\left(\psi_1^*\psi_2+\psi_2^*\psi_1\right)\right)\vec{A}.\end{aligned} \tag{5}$$

In the absence of currents and gradients the equilibrium values of order parameters $\psi_{1,2}=\psi_{1,2}^{(0)}e^{i\chi_{1,2}}$ are determined by the set of coupled equations

$$\begin{aligned}&\alpha_1\psi_1^{(0)}+\beta_1\psi_1^{(0)3}-\gamma\psi_2^{(0)}e^{i(\chi_2-\chi_1)}=0,\\ &\alpha_2\psi_2^{(0)}+\beta_2\psi_2^{(0)3}-\gamma\psi_1^{(0)}e^{i(\chi_1-\chi_2)}=0.\end{aligned} \tag{6}$$

For the case of two order parameters the question arises about the phase difference $\phi=\chi_1-\chi_2$ between ψ_1 and ψ_2. In homogeneous zero current state, by analyzing the free energy term F_{12} (3), one can obtain that for $\gamma>0$ phase shift $\phi=0$ and for $\gamma<0$ $\phi=\pi$. The statement, that ϕ can have only values 0 or π takes place also in a current carrying state, but for coefficient $\eta\neq0$ the criterion for ϕ equals 0 or π depends now on the value of the current (see below).

If the interband interaction is ignored, the equations (6) are decoupled into two ordinary GL equations with two different critical temperatures T_{c_1} and T_{c_2}. In general, independently of the sign of γ, the superconducting phase transition results at a well-defined temperature exceeding both T_{c_1} and T_{c_2}, which is determined from the equation:

$$\alpha_1(T_c)\alpha_2(T_c)=\gamma^2. \tag{7}$$

Let the first order parameter is stronger then second one, i.e. $T_{c_1}>T_{c_2}$. Following (Zhitomirsky & Dao, 2004) we represent temperature dependent coefficients as

$$\begin{aligned}&\alpha_1(T)=-a_1(1-T/T_{c1}),\\ &\alpha_2(T)=a_{20}-a_2(1-T/T_{c1}).\end{aligned} \tag{8}$$

Phenomenological constants $a_{1,2}, a_{20}$ and $\beta_{1,2}, \gamma$ can be related to microscopic parameters in two-band BCS model. From (7) and (8) we obtain for the critical temperature T_c:

$$T_c = T_{c1}\left(1+\sqrt{\left(\frac{a_{20}}{2a_2}\right)^2+\frac{\gamma^2}{a_1 a_2}}-\frac{a_{20}}{2a_2}\right). \tag{9}$$

For arbitrary value of the interband coupling γ Eq.(6) can be solved numerically. For $\gamma = 0$, $T_c = T_{c1}$ and for temperature close to T_c (hence for $T_{c2} < T \le T_c$) equilibrium values of the order parameters are $\psi_2^{(0)}(T) = 0$, $\psi_1^{(0)}(T) = \sqrt{a_1(1-T/T_c)/\beta_1}$. Considering in the following weak interband coupling, we have from Eqs. (6-9) corrections $\sim \gamma^2$ to these values:

$$\begin{aligned}
\psi_1^{(0)}(T)^2 &= \frac{a_1}{\beta_1}(1-\frac{T}{T_c})+\frac{\gamma^2}{\beta_1}\left(\frac{1}{a_{20}-a_2(1-\frac{T}{T_c})}-\frac{T}{T_c}\frac{1}{a_{20}}\right), \\
\psi_2^{(0)}(T)^2 &= \frac{a_1}{\beta_1}(1-\frac{T}{T_c})\frac{\gamma^2}{(a_{20}-a_2(1-\frac{T}{T_c}))^2}.
\end{aligned} \tag{10}$$

Expanding expressions (10) over $(1-\frac{T}{T_c}) \ll 1$ we have conventional temperature dependence of equilibrium order parameters in weak interband coupling limit

$$\begin{aligned}
\psi_1^{(0)}(T) &\approx \sqrt{\frac{a_1}{\beta_1}}\left(1+\frac{1}{2}\frac{a_{20}+a_2}{{a_{20}}^2 a_1}\gamma^2\right)\sqrt{1-\frac{T}{T_c}}, \\
\psi_2^{(0)}(T) &\approx \sqrt{\frac{a_1}{\beta_1}}\frac{\gamma}{a_{20}}\sqrt{1-\frac{T}{T_c}}.
\end{aligned} \tag{11}$$

Considered above case (expressions (9)-(11)) corresponds to different critical temperatures $T_{c_1} > T_{c_2}$ in the absence of interband coupling γ. Order parameter in the second band $\psi_2^{(0)}$ arises from the "proximity effect" of stronger $\psi_1^{(0)}$ and is proportional to the value of γ.
Consider now another situation, which we will use in the following as the model case. Suppose for simplicity that two condensates in current zero state are identical. In this case for arbitrary value of γ we have

$$\alpha_1(T) = \alpha_2(T) \equiv \alpha(T) = -a\left(1-\frac{T}{T_c}\right), \beta_1 = \beta_2 \equiv \beta. \tag{12}$$

$$\psi_1^{(0)} = \psi_2^{(0)} = \sqrt{\frac{|\gamma|-\alpha}{\beta}}. \tag{13}$$

2. Homogeneous current states and GL depairing current

In this section we will consider the homogeneous current states in thin wire or film with transverse dimensions $d \ll \xi_{1,2}(T), \lambda_{1,2}(T)$, where $\xi_{1,2}(T)$ and $\lambda_{1,2}(T)$ are coherence lengths

and London penetration depths for each order parameter, respectively. This condition leads to a one-dimensional problem and permits us to neglect the self-magnetic field of the system. (see Fig.2) . In the absence of external magnetic field we use the calibration $\vec{A}=0$.

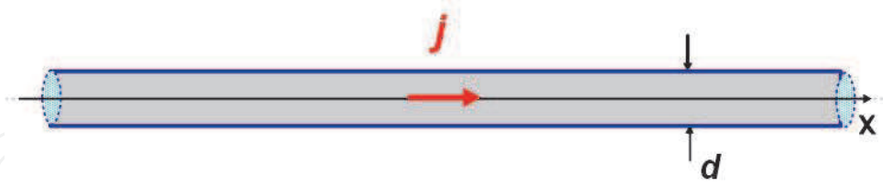


Fig. 2. Geometry of the system.

The current density j and modules of the order parameters do not depend on the longitudinal direction x. Writing $\psi_{1,2}(x)$ as $\psi_{1,2}=|\psi_{1,2}|\exp\left(i\chi_{1,2}(x)\right)$ and introducing the difference and weighted sum phases:

$$\begin{cases} \phi=\chi_1-\chi_2, \\ \theta=c_1\chi_1+c_2\chi_2, \end{cases} \tag{14}$$

for the free energy density (1)-(3) we obtain

$$\begin{aligned} F=&\alpha_1|\psi_1|^2+\frac{1}{2}\beta_1|\psi_1|^4+\alpha_2|\psi_2|^2+\frac{1}{2}\beta_2|\psi_2|^4+\\ &+\hbar^2\left(\frac{|\psi_1|^2}{2m_1}+\frac{|\psi_2|^2}{2m_2}+2\eta|\psi_1||\psi_2|\cos\phi\right)\left(\frac{d\theta}{dx}\right)^2+\\ &+\hbar^2\left(c_2^2\frac{|\psi_1|^2}{2m_1}+c_1^2\frac{|\psi_2|^2}{2m_2}+2c_1c_2\eta|\psi_1||\psi_2|\cos\phi\right)\left(\frac{d\phi}{dx}\right)^2-\\ &-2\gamma|\psi_1||\psi_2|\cos\phi. \end{aligned} \tag{15}$$

Where

$$c_1=\frac{\frac{|\psi_1|^2}{m_1}+2\eta|\psi_1||\psi_2|\cos\phi}{\frac{|\psi_1|^2}{m_1}+\frac{|\psi_2|^2}{m_2}+4\eta|\psi_1||\psi_2|\cos\phi},\ c_2=\frac{\frac{|\psi_2|^2}{m_2}+2\eta|\psi_1||\psi_2|\cos\phi}{\frac{|\psi_1|^2}{m_1}+\frac{|\psi_2|^2}{m_2}+4\eta|\psi_1||\psi_2|\cos\phi}. \tag{16}$$

The current density j in terms of phases θ and ϕ has the following form

$$j=2e\hbar\left(\frac{|\psi_1|^2}{m_1}+\frac{|\psi_2|^2}{m_2}+4\eta|\psi_1||\psi_2|\cos\phi\right)\frac{d\theta}{dx}. \tag{17}$$

Total current j includes the partial inputs $j_{1,2}$ and proportional to η the drag current j_{12}. In contrast to the case of single order parameter (De Gennes, 1966), the condition $div\mathrm{j}=0$ does not fix the constancy of superfluid velocity. The Euler – Lagrange equations for

$\theta(x)$ and $\phi(x)$ are complicated coupled nonlinear equations, which generally permit the soliton like solutions (in the case $\eta = 0$ they were considered in (Tanaka, 2002)). The possibility of states with inhomogeneous phase $\phi(x)$ is needed in separate investigation. Here, we restrict our consideration by the homogeneous phase difference between order parameters $\phi = const$. For $\phi = const$ from equations it follows that $\theta(x) = qx$ (q is total superfluid momentum) and $\cos\phi = 0$, i.e. ϕ equals 0 or π. Minimization of free energy for ϕ gives

$$\cos\phi = sign\left(\gamma - \eta\hbar^2 q^2\right). \tag{18}$$

Note, that now the value of ϕ, in principle, depends on q, thus, on current density j. Finally, the expressions (15), (17) take the form:

$$\begin{aligned} F = {} & \alpha_1|\psi_1|^2 + \frac{1}{2}\beta_1|\psi_1|^4 + \frac{\hbar^2}{2m_1}|\psi_1|^2 q^2 + \alpha_2|\psi_2|^2 + \frac{1}{2}\beta_2|\psi_2|^4 + \frac{\hbar^2}{2m_2}|\psi_2|^2 q^2 - \\ & -2\left(\gamma - \eta\hbar^2 q^2\right)|\psi_1||\psi_2| sign\left(\gamma - \eta\hbar^2 q^2\right), \end{aligned} \tag{19}$$

$$j = 2e\hbar\left(\frac{|\psi_1|^2}{m_1} + \frac{|\psi_2|^2}{m_2} + 4\eta|\psi_1||\psi_2| sign\left(\gamma - \eta\hbar^2 q^2\right)\right)q. \tag{20}$$

We will parameterize the current states by the value of superfluid momentum q, which for given value of j is determined by equation (20). The dependence of the order parameter modules on q determines by GL equations:

$$\alpha_1|\psi_1| + \beta_1|\psi_1|^3 + \frac{\hbar^2}{2m_1}|\psi_1| q^2 - |\psi_2|\left(\gamma - \eta\hbar^2 q^2\right) sign\left(\gamma - \eta\hbar^2 q^2\right) = 0, \tag{21}$$

$$\alpha_2|\psi_2| + \beta_2|\psi_2|^3 + \frac{\hbar^2}{2m_2}|\psi_2| q^2 - |\psi_1|\left(\gamma - \eta\hbar^2 q^2\right) sign\left(\gamma - \eta\hbar^2 q^2\right) = 0. \tag{22}$$

The system of equations (20-22) describes the depairing curve $j(q,T)$ and the dependences $|\psi_1|$ and $|\psi_2|$ on the current j and the temperature T. It can be solved numerically for given superconductor with concrete values of phenomenological parameters.

In order to study the specific effects produced by the interband coupling and dragging consider now the model case when order parameters coincide at $j = 0$ (Eqs. (12), (13)) but gradient terms in equations (4) are different. Eqs. (20)-(22) in this case take the form

$$\begin{aligned} & f_1\left(1 - \left(1 + |\tilde{\gamma}|\right) f_1^2\right) - f_1 q^2 + f_2\left(\tilde{\gamma} - \tilde{\eta} q^2\right) sign\left(\tilde{\gamma} - \tilde{\eta} q^2\right) = 0 \\ & f_2\left(1 - \left(1 + |\tilde{\gamma}|\right) f_2^2\right) - k f_2 q^2 + f_1\left(\tilde{\gamma} - \tilde{\eta} q^2\right) sign\left(\tilde{\gamma} - \tilde{\eta} q^2\right) = 0 \end{aligned} \tag{23}$$

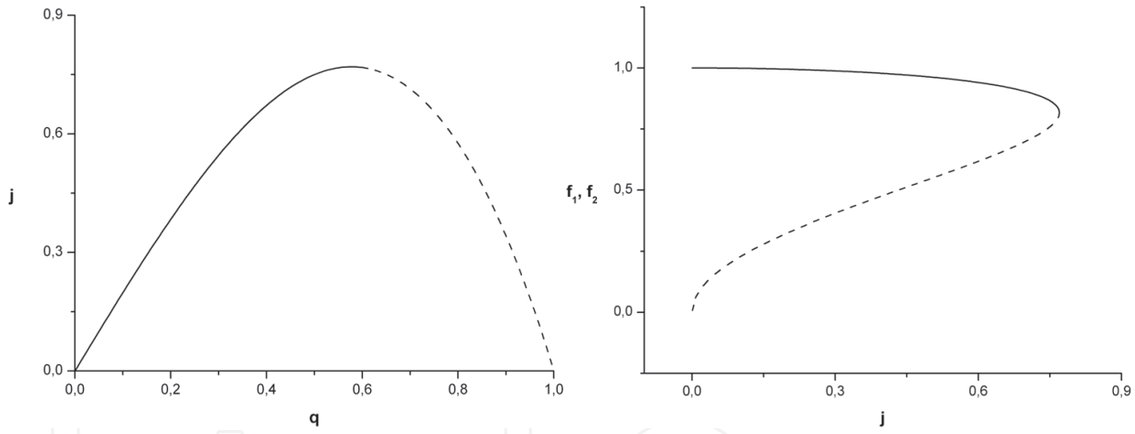
$$j = f_1^2 q + k f_2^2 q + 2\tilde{\eta} f_1 f_2 q \, sign\left(\tilde{\gamma} - \tilde{\eta} q^2\right) \tag{24}$$

Here we normalize $\psi_{1,2}$ on the value of the order parameters at $j = 0$ (13), j is measured in units of $2\sqrt{2}e\frac{|\gamma| + |\alpha|}{\beta}\sqrt{\frac{|\alpha|}{m_1}}$, q is measured in units of $\sqrt{\frac{\hbar^2}{2m_1|\alpha|}}$, $\tilde{\gamma} = \frac{\gamma}{|\alpha|}$, $\tilde{\eta} = 2\eta m_1$, $k = \frac{m_1}{m_2}$.
If $k = 1$ order parameters coincides also in current-carrying state $f_1 = f_2 = f$ and from eqs. (23), 24) we have the expressions

$$f^2(q) = \frac{1 - q^2 + \left|\tilde{\gamma} - \tilde{\eta} q^2\right|}{1 + |\tilde{\gamma}|} \tag{25}$$

$$j(q) = 2 f^2 \left(1 + \tilde{\eta} sign\left(\tilde{\gamma} - \tilde{\eta} q^2\right)\right) q, \tag{26}$$

which for $\tilde{\gamma} = \tilde{\eta} = 0$ are conventional dependences for one-band superconductor (De Gennes, 1966) (see Fig. 3 a,b).



(a) (b)

Fig. 3. Depairing current curve (a) and the graph of the order parameter modules versus current (b) for coincident order parameters. The unstable branches are shown as dashed lines.

For $k \neq 1$ depairing curve $j(q)$ can contain two increasing with q stable branches, which corresponds to possibility of bistable state. In Fig. 4 the numerically calculated from equations (23,24) curve $j(q)$ is shown for $k = 5$ and $\tilde{\gamma} = \tilde{\eta} = 0$.
The interband scattering ($\tilde{\gamma} \neq 0$) smears the second peak in $j(q)$, see Fig.5.
If dragging effect ($\tilde{\eta} \neq 0$) is taking into account the depairing curve $j(q)$ can contain the jump at definite value of q (for $k = 1$ see eq. 34), see Fig.6. This jump corresponds to the switching of relative phase difference from 0 to π.

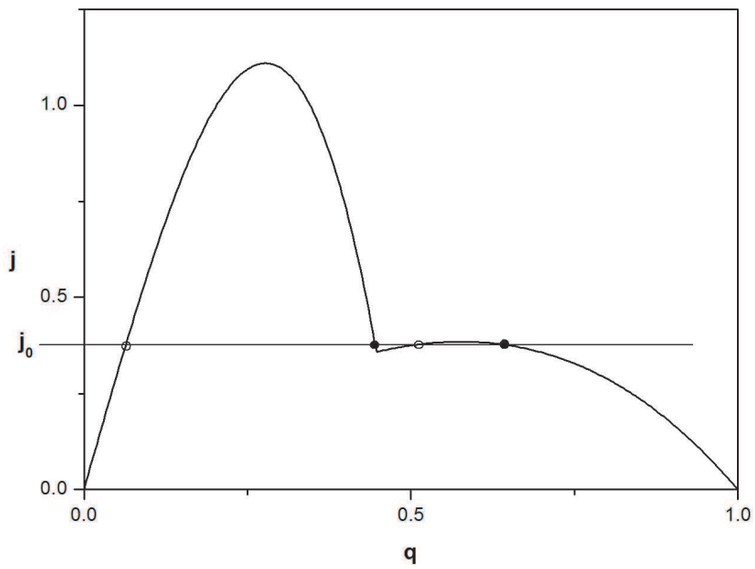


Fig. 4. Dependence of the current j on the superfluid momentum q for two band superconductor. For the value of the current $j = j_0$ the stable ($\circ$) and unstable ($\bullet$) states are depicted. The ratio of effective masses $k = 5$, and $\tilde{\gamma} = \tilde{\eta} = 0$.

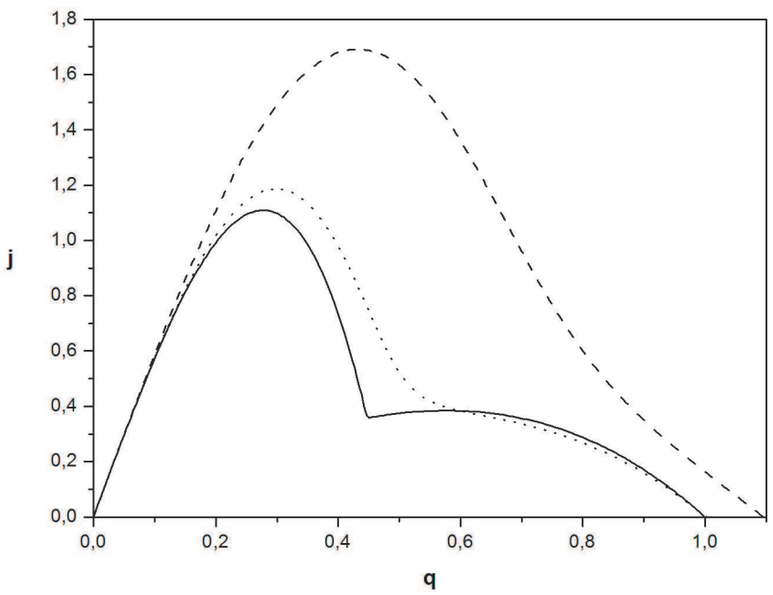


Fig. 5. Depairing current curves for different values of interband interaction: $\tilde{\gamma} = 0$ (solid line), $\tilde{\gamma} = 0.1$ (dotted line) and $\tilde{\gamma} = 1$ (dashed line). The ratio of effective masses $k = 5$, and $\tilde{\eta} = 0$.

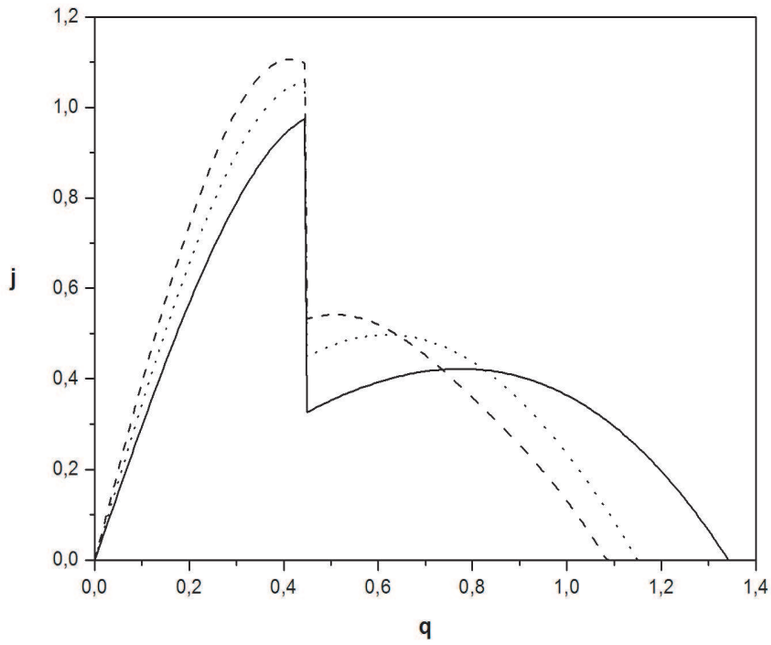


Fig. 6. Depairing current curves for different values of the effective masses ratio $k = 1$ (solid line), $k = 1.5$ (dotted line) and $k = 5$ (dashed line). The interband interaction coefficient $\tilde{\gamma} = 0.1$ and drag effect coefficient $\tilde{\eta} = 0.5$.

4. Little-Parks effect for two-band superconductors

In the present section we briefly consider the Little–Parks effect for two-band superconductors. The detailed rigorous theory can be found in the article (Yerin et al., 2008). It is pertinent to recall that the classical Little–Parks effect for single-band superconductors is well-known as one of the most striking demonstrations of the macroscopic phase coherence of the superconducting order parameter (De Gennes, 1966; Tinkham, 1996). It is observed in open thin-wall superconducting cylinders in the presence of a constant external magnetic field oriented along the axis of the cylinder. Under conditions where the field is essentially unscreened the superconducting transition temperature $T_{c\Phi}$ (Φ is the magnetic flux through the cylinder) undergoes strictly periodic oscillations (Little–Parks oscillations)

$$\frac{T_c - T_{c\Phi}}{T_c} \propto \min\left(\frac{\Phi}{\Phi_0} - n\right)^2, (n = 0, \pm 1, \pm 2, \ldots), \tag{27}$$

where $T_{c\Phi} \equiv T_{c\Phi}\big|_{\Phi=0}$ and $\Phi_0 = \pi\hbar c / e$ is the quantum of magnetic flux.

How the Little–Parks oscillations (27) will be modified in the case of two order parameters with taking into account the proximity (γ) and dragging (η) coupling? Let us consider a superconducting film in the form of a hollow thin cylinder in an external magnetic field **H** (see Fig.6).

We proceed with free energy density (19), but now the momentum q is not determined by the fixed current density **j** as in Sec.3. At given magnetic flux $\oint \vec{A} \cdot d\vec{l} = \int \vec{H} \cdot d\vec{\sigma} = \Phi$ the superfluid momentum q is related to the applied magnetic field

$$q = \frac{1}{R}\left(n - \frac{\Phi}{\Phi_0} \right). \tag{28}$$

At fixed flux Φ the value of q take on the infinite discrete set of values for $n = 0, \pm 1, \pm 2, \ldots$. The possible values of n are determined from the minimization of free energy $F[|\psi_1|, |\psi_2|, q]$. As a result the critical temperature of superconducting film depends on the magnetic field. The dependencies of the relative shift of the critical temperature $\Delta t_c \equiv (T_c - T_{c\Phi}) / T_c$ for different values of parameters γ, β, R were calculated in (Yerin et al., 2008). The dependence of $\Delta t_c(\Phi)$ as in the conventional case is strict periodic function of Φ with the period Φ_0 (contrary to the assertions made in Askerzade, 2006). The main qualitative difference from the classical case is the nonparabolic character of the flux dependence $\Delta t_c(\Phi)$ in regions with the fixed optimal value of n . More than that, the term $\left(\gamma - \eta\hbar^2 q^2\right) sign\left(\gamma - \eta\hbar^2 q^2\right)$ in the free energy (19) engenders possibility of observable singularities in the function $\Delta t_c(\Phi)$, which are completely absent in the classical case (see Fig.8.).

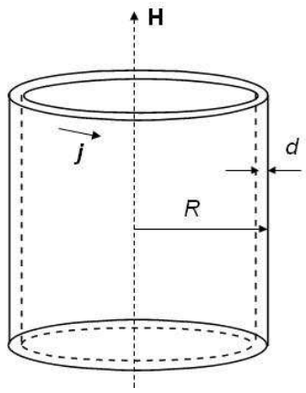


Fig. 7. Geometry of the problem.

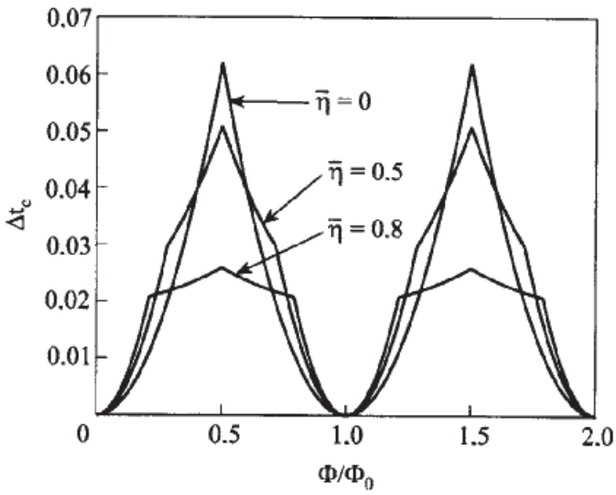


Fig. 8. $\Delta t_c(\Phi)$ for the case where the bands 1 and 2 have identical parameters and values of $\tilde{\eta}$ are indicated.

5. Josephson effect in two-band superconducting microconstriction

In the Sec.3 GL-theory of two-band superconductors was applied for filament's length $L \to \infty$. Opposite case of the strongly inhomogeneous current state is the Josephson microbridge or point contact geometry (Superconductor-Constriction-Superconductor contact), which we model as narrow channel connecting two massive superconductors (banks). The length L and the diameter d of the channel (see Fig. 9) are assumed to be small as compared to the order parameters coherence lengths ξ_1, ξ_2.

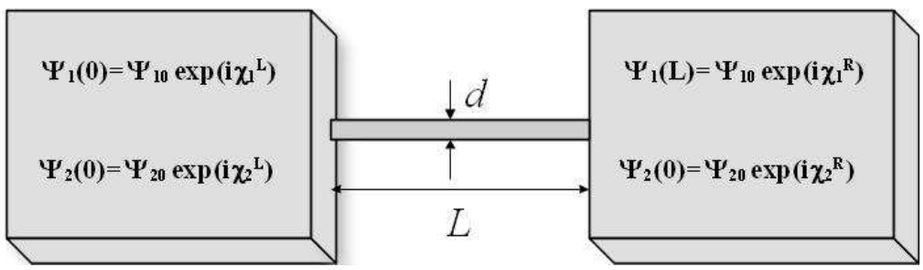


Fig. 9. Geometry of S-C-S contact as narrow superconducting channel in contact with bulk two-band superconductors. The values of the order parameters at the left (L) and right (R) banks are indicated

For $d \ll L$ we can solve one-dimensional GL equations (4) inside the channel with the rigid boundary conditions for order parameters at the ends of the channel.
In the case $L \ll \xi_1, \xi_2$ we can neglect in equations (4) all terms except the gradient ones and solve equations:

$$\begin{cases} \dfrac{d^2\psi_1}{dx^2}=0, \\ \dfrac{d^2\psi_2}{dx^2}=0 \end{cases} \tag{29}$$

with the boundary conditions:

$$\psi_1(0)=\psi_{01}\exp\left(i\chi_1{}^L\right),\ \psi_2(0)=\psi_{02}\exp\left(i\chi_1{}^R\right), \tag{30}$$
$$\psi_1(L)=\psi_{01}\exp\left(i\chi_2{}^L\right),\ \psi_2(L)=\psi_{02}\exp\left(i\chi_1{}^R\right).$$

Calculating the current density j in the channel we obtain:

$$j = j_{11} + j_{22} + j_{12} + j_{21}, \tag{31}$$

$$j_{11}=\frac{2e\hbar}{Lm_1}\psi_{01}^2\sin\left(\chi_1{}^R-\chi_1{}^L\right),$$

$$j_{22}=\frac{2e\hbar}{Lm_1}\psi_{01}^2\sin\left(\chi_2{}^R-\chi_2{}^L\right),$$

$$j_{12} = \eta \frac{4e\hbar}{L} \psi_{01}\psi_{02} \sin\left(\chi_1{}^R - \chi_2{}^L\right),$$

$$j_{21} = \eta \frac{4e\hbar}{L} \psi_{02}\psi_{01} \sin\left(\chi_2{}^R - \chi_1{}^L\right).$$

The current density j (31) consists of four partials inputs produced by transitions from left bank to right bank between different bands. The relative directions of components j_{ik} depend on the intrinsic phase shifts in the banks $\delta\phi^L = \chi_1^L - \chi_2^L$ and $\delta\phi^R = \chi_1^R - \chi_2^R$ (Fig.10).

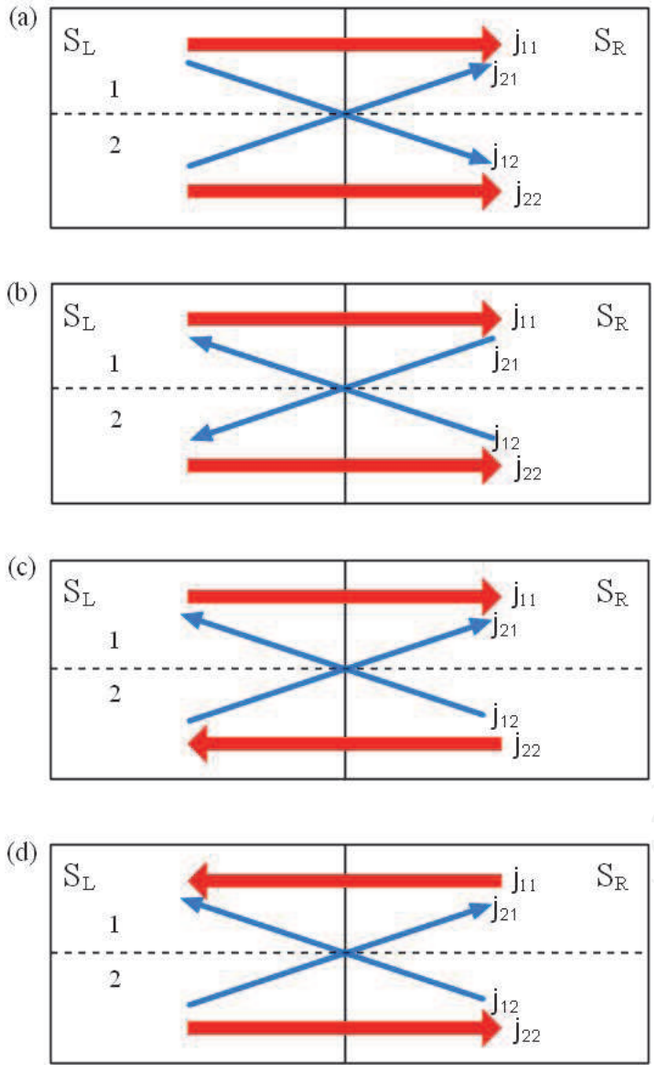


Fig. 10. Current directions in S-C-S contact between two-band superconductors. (a) – there is no shift between phases of order parameters in the left and right superconductors; (b) - there is the π -shift of order parameters phases at the both banks ; (c) – π -shift is present in the right superconductor and is absent in the left superconductor; (d) – π -shift is present in the left superconductor and is absent in the right superconductor .

Introducing the phase difference on the contact $\varphi = \chi_1^R - \chi_1^L$ we have the current-phase relation $j(\phi)$ for different cases of phase shifts $\delta\phi^{R,L}$ in the banks:

a. $\delta\phi^R = \delta\phi^L = 0$

$$j \equiv j_c \sin\phi = \frac{2e\hbar}{L}\left(\frac{\psi_{01}^2}{m_1} + \frac{\psi_{02}^2}{m_2} + 4\eta\psi_{01}\psi_{02}\right)\sin\phi$$

b. $\delta\phi^R = \delta\phi^L = \pi$

$$j \equiv j_c \sin\phi = \frac{2e\hbar}{L}\left(\frac{\psi_{01}^2}{m_1} + \frac{\psi_{02}^2}{m_2} - 4\eta\psi_{01}\psi_{02}\right)\sin\phi$$

c. $\delta\phi^R = \pi, \delta\phi^L = 0$

$$j \equiv j_c \sin\phi = \frac{2e\hbar}{L}\left(\frac{\psi_{01}^2}{m_1} - \frac{\psi_{02}^2}{m_2}\right)\sin(\phi)$$

d. $\delta\phi^R = 0, \delta\phi^L = \pi$

$$j \equiv j_c \sin\phi = \frac{2e\hbar}{L}\left(-\frac{\psi_{01}^2}{m_1} + \frac{\psi_{02}^2}{m_2}\right)\sin(\phi)$$

The critical current j_c in cases a) and b) is positively defined quadratic form of ψ_{01} and ψ_{02} for $|\eta| < \frac{1}{2\sqrt{m_1 m_2}}$. In cases c) and d) the value of j_c can be negative. It corresponds to the so-called π – junction (see *e.g.* (Golubov et. al, 2004)) (see illustration at Fig.11).

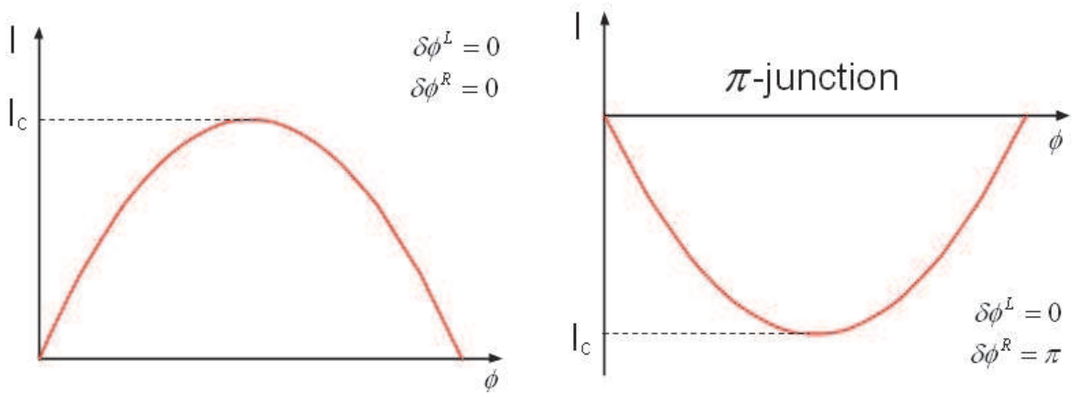


Fig. 11. Current-phase relations for different phase shifts in the banks.

This phenomenological theory, which is valid for temperatures near critical temperature T_c, is the generalization of Aslamazov-Larkin theory (Aslamazov & Larkin, 1968) for the case of two superconducting order parameters. The microscopic theory of Josephson effect in S-C-S junctions (KO theory) was developed in (Kulik & Omelyanchouk, 1975; Kulik &

Omelyanchouk, 1978;) by solving the Usadel and Eilenberger equations (for dirty and clean limits). In papers (Omelyanchouk & Yerin, 2009; Yerin & Omelyanchouk, 2010) we generalized KO theory for the contact of two-band superconductors. Within the microscopic Usadel equations we calculate the Josephson current and study its dependence on the mixing of order parameters due to interband scattering and phase shifts in the contacting two-band superconductors. These results extend the phenomenological theory presented in this Section on the range of all temperatures $0 < T < T_c$. The qualitative feature is the possibility of intermediate between $\sin\phi$ and $-\sin\phi$ behavior $j(\phi)$ at low temperatures (Fig.12).

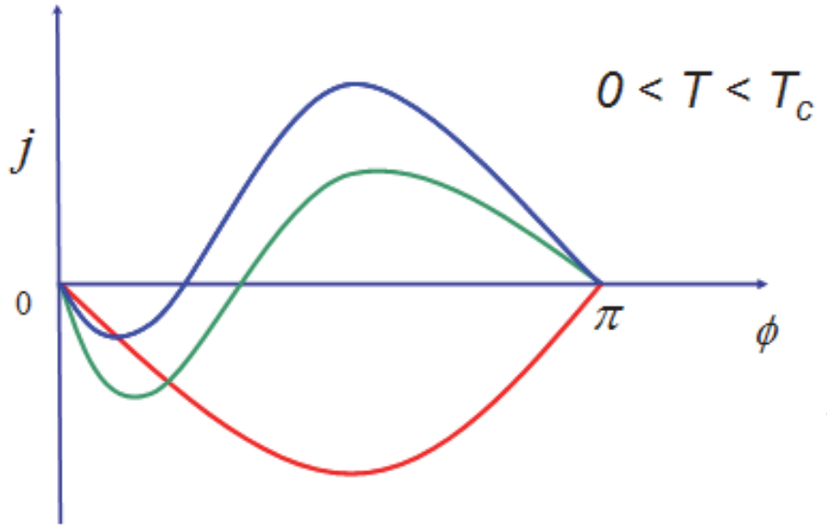


Fig. 12. The possible current-phase relations $j(\phi)$ for hetero-contact with $\delta\phi^R = 0, \delta\phi^L = \pi$.

6. Conclusion

In this chapter the current carrying states in two-band superconductors are described in the frame of phenomenological Ginzburg-Landau theory. The qualitative new feature, as compared with conventional superconductors, consists in coexistence of two distinct complex order parameters Ψ_1 and Ψ_2. It means the appearing of an additional internal degree of freedom, the phase shift between order parameters. We have studied the implications of the $\delta\phi$-shift in homogeneous current state in long films or channels, Little-Parks oscillations in magnetic field, Josephson effect in microconstrictions. The observable effects are predicted. Along with fundamental problems, the application of two band superconductors with internal phase shift in SQUIDS represents significant interest (see review (Brinkman & Rowell, 2007).

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8. References

Agterberg D. F., Demler E., & Janko B. (2002). Josephson effects between multigap and single-gap superconductors, *Phys. Rev. B*, V.66, Iss.21, p.214507.

Artemenko S. N., Volkov A. F. & Zaitsev A. V. (1979). Theory of nonstationary Josephson effect in short superconducting junctions, *Zh. Eksp. Teor. Fiz.*, V. 76, No.5, p.1816-1833.

Ashcroft N. W. (2000). The Hydrogen Liquids. *J. Phys.: Condens. Matter*, V.12, No.8A, p. A129-A137.

Askerzade I. N. (2003). Temperature dependence of the London penetration depth of YNi2B2C borocarbids using two-band Ginzburg-Landau theory. *Acta Physica Slovaca*, V.53, No. 4, p.321-327.

Askerzade I. N. (2003). Ginzburg–Landau theory for two-band s-wave superconductors: application to non-magnetic borocarbides LuNi2B2C, YNi2B2C and magnesium diboride MgB2. *Physica C*, V.397, Iss.3-4, p.99-111.

Askerzade I. N. (2006). Ginzburg-Landau theory: the case of two-band superconductors. *Usp. Fiz. Nauk*, V.49, Iss.10, p.1003-1016.

Aslamazov L.G. & Larkin A.I. (1969). The Josephson effect in point superconducting junctions. *Pis'ma Zh. Eksp. Teor. Fiz.*, V.9, No.2, p.150-154.

Babaev E. (2002). Vortices with Fractional Flux in Two-Gap Superconductors and in Extended Faddeev Model. *Phys. Rev. Lett.*, V.89, Iss. 89, p.067001.

Babaev E., Faddeev L. D. & Niemi A. J. (2002). Hidden symmetry and knot solitons in a charged two-condensate Bose system. *Phys. Rev. B*, V.65, Iss.10, p.100512(R).

Babaev E., Sudbo A. & Ashcroft N. W. (2004). A superconductor to superfluid phase transition in liquid metallic hydrogen. *Nature*, V.431, No. 7009, p.666-668.

Brinkman A., Golubov A. A., Rogalla H., Dolgov O. V., Kortus J., Kong Y., Jepsen O. & Andersen O. K. (2002). Multiband model for tunneling in MgB2 junctions. *Phys. Rev.B*, V.65, Iss.18, p.180517(R).

Brinkman A. & Rowell J. (2007). MgB2 tunnel junctions and SQUIDs. *Physica C,* 456, p.188-195.

Dahm T., Graser S. & Schopohl N. (2004). Fermi surface topology and vortex state in MgB2. *Physica C*, V.408-410, p.336-337.

Dahm T. & Schopohl N. (2003). Fermi Surface Topology and the Upper Critical Field in Two-Band Superconductors: Application to MgB2. *Phys. Rev. Lett.*, V.91, Iss. 1, p.017001.

De Gennes P. G. (1966). *Superconductivity of Metals and Alloys*, Benjamin, ISBN 0738201014, New York.

Doh H., Sigrist M., Cho B.K. & Lee S.I. (1999). Phenomenological Theory of Superconductivity and Magnetism in Ho1-xDyxNi2B2C. *Phys. Rev. Lett.*, V.83, Iss.25, p.5350-5353.

Giubileo F., Roditchev D., Sacks W., Lamy R., Thanh D.X., Klein J., Miraglia S., Fruchart D., Marcus J. & Monod P. (2001). Two-Gap State Density in MgB2: A True Bulk Property Or A Proximity Effect? *Phys. Rev. Lett.*, V.87, Iss.17, p.177008.

Golubov A. A., Kortus J., Dolgov O. V., Jepsen O., Kong Y., Andersen O. K., Gibson B. J., Ahn K. & Kremer R. K. (2002). Specific heat of MgB2 in a one- and a two-band model from first-principles calculations. *J. Phys.: Condens. Matter*, V.14, No.6, p.1353-1361.

Golubov A. A. & Koshelev A. E. (2003). Upper critical field in dirty two-band superconductors: Breakdown of the anisotropic Ginzburg-Landau theory. *Phys. Rev. B*, V.68, Iss.10, p.104503.

Golubov A. A., Kupriyanov M. Yu. & Il'ichev E. (2004). The current-phase relation in Josephson junctions. *Rev. Mod. Phys.*, V.76, Iss.2, p. 411–469.

Golubov A. A. & Mazin I. I. (1995). Sign reversal of the order parameter in s wave superconductors. *Physica C*, V.243, Iss.1-2, p.153-159.

Gurevich A. (2003). Enhancement of the upper critical field by nonmagnetic impurities in dirty two-gap superconductors, *Phys. Rev. B*, V.67, Iss.18, p.184515.

Gurevich A. (2007). Limits of the upper critical field in dirty two-gap superconductors. *Physica C*, V. 456, Iss. 1-2, p.160-169.

Gurevich A. & Vinokur V.M. (2003). Interband Phase Modes and Nonequilibrium Soliton Structures in Two-Gap Superconductors. *Phys. Rev. Lett.*, V.90, Iss.4, p.047004.

Gurevich A. & Vinokur V. M. (2006). Phase textures induced by dc-current pair breaking in weakly coupled multilayer structures and two-gap superconductors, *Phys. Rev. Lett.*, V.97, Iss.13, p.137003.

Iavarone M., Karapetrov G., Koshelev A.E., Kwok W.K., Crabtree G.W., Hinks D.G., Kang W.N., Choi E.-M., Kim H.J. & Lee S.I. (2002). Two-Band Superconductivity in MgB2. *Phys. Rev. Lett.*, V.89, Iss.18, p.187002.

Jourdan M., Zakharov A., Foerster M. & Adrian H. (2004). Evidence for Multiband Superconductivity in the Heavy Fermion Compound UNi2Al3. *Phys. Rev. Lett.*, V.93, Iss.9, p.097001.

Kamihara Y., Watanabe T., Hirano M. & Hosono H. (2008). Iron-based layered superconductor La[O(1-x)F(x)]FeAs (x = 0.05-0.12) with T(c) = 26 K. *J. Am. Chem. Soc.*, V.130, Iss.11, p.3296-3297.

Kortus J., Mazin I. I., Belashchenko K. D., Antropov V. P. & L. L. Boyer (2001). Superconductivity of Metallic Boron in MgB2. *Phys. Rev. Lett.*, V.86, Iss.20, p.4656.

Koshelev A. E. & Golubov A. A. (2003). Mixed state of a dirty two-band superconductor: pplication to MgB2, *Phys. Rev. Lett.*, V.90, Iss.17, p.177002.

Kresin V. Z. & Wolf S. A (1990). Multigap structure in cuprates. *Physica C*, V.169, Iss.5-6, p.476–484.

Kulik I. O. & Omelyanchouk A. N. (1975). Microscopic theory of Josephson effect in superconducting microbridges, *Pis'ma Zh. Eksp. Teor. Fiz.*, V.21, No.4, p.216-219.Kulik I. O. & Omelyanchouk A. N. (1978). Josephson effect in superconducting bridges:

microscopic theory, *Fiz. Nizk. Temp.*, V.4, No.3, p.296-311.

Leggett J. (1966). Number-phase fluctuations in two-band superconductors. *Progr. Theor. Phys.*, V.36, No. 5, pp. 901-930.

Mazin I. I., Andersen O. K., Jepsen O., Dolgov O. V., Kortus J., Golubov A. A., Kuz'menko A. B. & van der Marel D. (2002). Superconductivity in MgB2: Clean or Dirty? *Phys. Rev. Lett.*, V.89, Iss.10, p.107002.

Mints R. G., Papiashvili I., Kirtley J. R., Hilgenkamp H., Hammerl G. & Mannhart J. (2002). Observation of Splintered Josephson Vortices at Grain Boundaries in YBa2Cu3O7-δ. *Phys. Rev. Lett.*, V.89, Iss.6, p.067004.

Miranovic P., Machida K. & Kogan V. G. (2003). Anisotropy of the Upper Critical Field in Superconductors with Anisotropic Gaps: Anisotropy Parameters of MgB2. *J. Phys. Soc. Jpn.*, V.72, No.2, p.221-224.

Moskalenko V.A. (1959). Superconductivity of metals within overlapping energy bands. *Fiz. Met. Metallov.*, V.8, p.503-513.

Nagamatsu J., Nakagawa N., Muranaka T., Zenitani Y. & Akimitsu J. (2001), Superconductivity at 39 K in magnesium diboride. *Nature*, V.410, No.6824, p.63-64.

Nakai A., Ichioka M. & Machida K. (2002). Field Dependence of Electronic Specific Heat in Two-Band Superconductors. *J. Phys. Soc. Jpn.*, V.71, No.1, p.23-26.

Ng T. K. & Nagaosa N. (2009). Broken time-reversal symmetry in Josephson junction involving two-band superconductors, *EPL* V.87, No.1, p.17003.

Omelyanchouk A.N. & Yerin Y.S. (2010). Josephson effect in point contacts between two-band superconductors. In: *Physical Properties of Nanosystems*, Bonca J. & Kruchinin S., pp. 111-119, Springer, ISBN: 978-94-007-0043-7, Berlin.

Ota Y., Machida M., Koyama T. & Matsumoto H. (2009). Theory of heterotic superconductor-insulator-superconductor Josephson junctions between single- and multiple-gap superconductors, *Phys. Rev. Lett.*, V.102, Iss.23, p.237003.

Schmidt H., Zasadzinski J.F., Gray K.E. & Hinks D.G. (2001). Evidence for Two-Band Superconductivity from Break-Junction Tunneling on MgB2. *Phys. Rev. Lett.*, V.88, Iss.12, p.127002.

Seyfarth G., Brison J. P., Méasson M.-A., Flouquet J., Izawa K., Matsuda Y., Sugawara H. & Sato H. (2005). Multiband Superconductivity in the Heavy Fermion Compound PrOs4Sb12. *Phys. Rev. Lett.*, V.95, Iss. 10, p.107004.

Shulga S. V., Drechsler S.-L., Fuchs G., Müller K.-H., Winzer K., Heinecke M. & Krug K. (1998). Upper Critical Field Peculiarities of Superconducting YNi2B2C and LuNi2B2C. *Phys. Rev. Lett.*, V.80, Iss.8, p.1730-1733.

Suhl H., Matthias B.T. & Walker L.R. (1959), Bardeen-Cooper-Schrieffer Theory of Superconductivity in the Case of Overlapping Bands. *Phys. Rev. Lett.*, V.3, Iss.12, p. 552-554.

Szabo P., Samuely P., Kacmarcik J., Klein T., Marcus J., Fruchart D., Miraglia S., Mercenat C., & Jansen A.G.M. (2001). Evidence for Two Superconducting Energy Gaps in MgB2 by Point-Contact Spectroscopy. *Phys. Rev. Lett.*, V.87, Iss.13, p.137005.

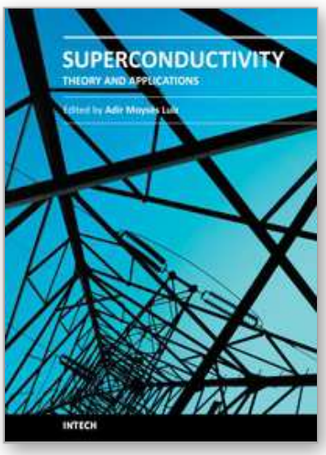
Tanaka Y. (2002). Soliton in Two-Band Superconductor. *Phys. Rev. Lett.*, V.88, Iss.1, p.017002.Tinkham M. (1996). *Introduction to Superconductivity*, McGraw-Hill, ISBN 0-07-064878-6, New York.

Yanson I. K. & Naidyuk Yu. G. (2004), Advances in point-contact spectroscopy: two-band superconductor MgB2. *Low Temp.Phys.*, V.30, Iss.4, p.261-275.

Yerin Y. S., Kuplevakhskii S. V. & Omelyanchuk A. N. (2008). Little–Parks effect for two-band superconductors. *Low Temp. Phys.*, V.34, Iss.11, p.891-898.

Yerin Y. S. & Omelyanchouk A. N. (2010). Josephson currents in point contacts between dirty two-band superconductors. *Low Temp. Phys.*, V.36, Iss.10, p.969-974.

Zhitomirsky M. E. & Dao V.-H. (2004). Ginzburg-Landau theory of vortices in a multigap superconductor, *Phys. Rev. B*, V.69, Iss.5, p.054508.



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Superconductivity was discovered in 1911 by Kamerlingh Onnes. Since the discovery of an oxide superconductor with critical temperature (Tc) approximately equal to 35 K (by Bednorz and Müller 1986), there are a great number of laboratories all over the world involved in research of superconductors with high Tc values, the so-called â€œHigh-Tc superconductorsâ€. This book contains 15 chapters reporting about interesting research about theoretical and experimental aspects of superconductivity. You will find here a great number of works about theories and properties of High-Tc superconductors (materials with Tc > 30 K). In a few chapters there are also discussions concerning low-Tc superconductors (Tc < 30 K). This book will certainly encourage further experimental and theoretical research in new theories and new superconducting materials.

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