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Mathematically Fuzzy Approach to Quality Control

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1. Introduction

Romulus: Did she lay, this noble bird?
Achilles: Almost, my Emperor.
Romulus: Almost? What does that mean? A hen either lays or she doesn't.
Achilles: Not yet, my Emperor.
(The Emperor makes a decisive gesture).
Romulus: Not at all.

(F. Dürrenmatt, "Romulus the Great", 1949)

2. Quality as a mathematical function

Quality control, which initially emerged and developed in the sphere of material production and consumption, now is rapidly spreading also in nonmaterial spheres, such as development and implementation of various projects, economic programs and managerial solutions [1], which in essence are the processes which quality is considered from the point of view of operations research theory [2].

The fact that quality is functional is known since the last quarter of the twentieth century. At that time Russian researchers V. Solodovnikov and N. Filimonov have formulated the concept of quality functional [3]. Curiously enough, they came to the conclusion about necessity of mathematically fuzzy (hereinafter MF) approach to evaluation of acceptability of performance of complicated control systems [4]. This fully complies with the methodology of mathematical fuzziness which foundations were laid by the American scientist Lotfi Zadeh [5], who discovered that the more complicated is a system, so much the less we are capable to provide precise and at the same time practically valuable judgment about its behavior. Therefore, for the systems which complexity exceeds certain threshold level, "precision" and "significance" become almost mutually exclusive characteristics [6].

Besides, the functionality of quality is known since the times, when Japanese scientist G. Taguchi proposed "quality loss function" [7,8] for radio-electronic components and products, for which the operating performance was evaluated using binary scale (e.g., "bad" or "good"). At the same time, QFD ("Quality function deployment") [8,9] method has been developed in the USA; it is the procedure for transforming user demands into quality parameters of the processes of product planning, manufacturing, installation and enhancement (quality improvement).

MF approach to quality control is possible only in case if it is considered, figuratively speaking, from mathematically «rigorous» point of view. Therefore, it is necessary to clarify, to which class of mathematical representations the concept «quality» may be ascribed. For this purpose we must briefly deepen into terminological aspects of quality as a mathematical phenomenon. With this objective in mind, process approach [2] on the basis of standard [10] is already used.

All this has laid the groundwork of modern quality control, both for material and nonmaterial objects, which in the context of above said we will name simply "objects". As prominent Austrian-British philosopher of the 20-th century L. Wittgenstein notes in his treatise [11], "objects contain the possibility of all states of affairs".

Quality is judged by presence with the object of one (one-dimensional) or several parallel (multidimensional or vectorial) properties [2,12], hereinafter denoted as P where necessary.

They represent different aspects of the object and are determined by values which in qualitative sense are common for many objects, but in quantitative sense are individual for each object [13]. Therefore, the value is a denominated property [14] and serves as the quality index (hereinafter – QI) of the object.

The properties may be either point-wise, i.e. occurring singly in full scope which is denoted by 1 (while their absence is denoted by 0) [14], or permanent, i.e. persisting continuously during prescribed time interval. Occurrence of point-wise property or loss of permanent property must be confirmed by the result of appropriate QI check. In contrast to single occurrence of point-wise property, loss of permanency (incurring loss of quality of the object) may take place either at a point or within a finite interval.

In case of a point-wise property in nonmaterial sphere of activities, specifically, a quality management process or quality management system (hereinafter – QMS), the fact of its certification at an industrial enterprise, suppose, in year 2011, is subject to confirmation by certificate granted by a certification body.

An example of a point-wise property in the sphere of material production may be the JIT (Just-in-time) logistics concept, created in late 1950-ies in the Japanese company Toyota Motors, later accepted by other Japanese automobile manufacturers [15] and now having worldwide recognition. It suggests supply of a resource exactly at the time when it is needed, allowing reduction of stock reserves and related expenses. Examples in non-material sphere of activity may be the just-in-time fulfillment of contractual obligations and payments; an analog of this in social-behavioral aspect is just-in-time coming of a subordinate functionary to his chief, as well as paying visits.

In case of permanent property, the interval of loss of permanence represents the period (or time) of property recovery, e.g., recovery of system performance or device operability, respectively.

An example of interval loss of permanence, e.g., in the sphere of management activity, may be a short-term stoppage of work of an institution or enterprise administration for a few hours during working day for reasons which might be foreseen and timely eliminated. Exception here might be only force majeure circumstances. An example of such loss in material sphere is a temporary exceedance of allowable limits of mains voltage.

An example of point-wise loss of permanent property may be occurrence of defective products at an industrial enterprise in conditions of prescribed zero defects of production, e.g., violation of requirements to the appearance of the items (in particular, items made of plastics [16,17]).

QI may be either quantitative, if it has metric scale, or qualitative, if its values are established according to ranking scale or to classificatory scale of quality gradations [18, 19]. Point-wise property may have any of the above mentioned scales. Permanent property may have only metric scale, and only in anticipation of eventual interval loss of quality.
However, if possibility of point-wise loss of property is envisioned, then only non-graded (free) axis is conceivable.
The above said allows making some conclusions and generalizations:

- Quality is inherent to any object; in contrast to property, quality cannot appear and disappear,
- quality is functional; it may be either compliant or incompliant with the requirements raised to it by reason of presence or absence of the property with the object,
- verbal evaluations «1» and «0» correspond to presence or absence of the property,
- if the object has several properties, quality becomes multidimensional and exists only when its presence is supported concurrently by all QI,
- property may be either point-wise (occurring one-time) or permanent, i.e. persisting continuously,
- existence or absence of a property is evidenced by the values of QI, either measured or evaluated by other methods,
- these values must either lie within required (prescribed) limits of appropriate scales or be point-wise.

Now it is permissible to ask: if quality is functional [3 – 9], to which kind of mathematical functions it should be categorized, and what should be the appearance of mathematical function of quality? To get the answer, let us turn to the theory of functions [20], which allows, in case of single QI, taking it as independent variable x, to search for quality function (hereinafter – QF) in the generalized form as

$$Q = \rbrack Q(x) \qquad x \in E, \tag{1}$$

where: Q is the symbolic value «1» in case of compliance, or «0» in case of incompliance of quality with the requirements raised to it,
Q(x) is the quality function (QF),
] is the symbol of a mathematical operation, decision rule or method for determining compliance or incompliance of quality with the requirements raised to it,
E means the definition domain of QF.
As is known, the concept of "function" is characterized by the following attributes:

- representation method,
- range of values,
- range of definition.

If these concepts will provide substance to the formal expression (1), then QF will be actually found.
Let us start from representation method. Four such methods are known [20]:

- analytic,
- graphical,
- tabular,
- verbal (in the form of verbal expression).

As Russian scientist N. Nazarov has rigorously proved, quality is not a quantitative value [18, 21]; hence, two methods remain applicable – graphical and verbal. Graphical representation method for function assignment is not enough; its mathematical formulation is required. Only verbal representation of a function remains available for this purpose. Addressing to [20], we find precedents of verbal representation of functions. These include:

Dirichlet function – equaling to 1, when x is rational number, and 0, when x is irrational number,

Boolean function – which has the same range of values as the previous function, though the range of its definition is different.

This gives ground to assume that QF may also be set verbally, i.e., following [2,3], to assign value 1 to quality presence, and value 0 – to quality absence, what, apropos, represents the range of values of QF sought for.

It should be noted here that by no means these values are the measure or, horrible ictum[1]), the quantity of quality. 1 and 0 here lose their numerical value [22] and quantitative essence in favor of verbal essence, accepting the sense of propositions «is present» and «is absent», respectively, i.e. become nothing more than the symbols of verbal evaluations.

Further on we will denote the verbal evaluations of quality range of values as ⌋Q (quality is compliant with the requirements raised to it) and ⌉Q (quality is incompliant with the requirements raised to it). The symbols ⌋ and ⌉ represent the right parts of notations «floor» – ⌊ ⌋ and «ceiling» – ⌈ ⌉ [23 – 25], located to the left of Q. The symbols ⌋ and ⌉ are generalized into symbolic operator ⁆ appearing in the formula (1). This mnemonics corresponds to the widely known manual gestures, and the symbols are ⌋ and ⌉contained in character sets of standard computer software.

Taking the above said into consideration, QF may be set on the basis of decision rule

$$Q =]Q(x) = \begin{cases} 1, \text{ if } \rfloor Q; \\ 0, \text{ if } \rceil Q. \end{cases} \quad (2)$$

The meaning of the equality sign «=» in formulas (1) and (2), as well as the essence of values 1 and 0 in formula (2), is solely symbolic, linguistic and, therefore, is not numerical, because the question is only about presence or absence of the quality P with the object being checked.

As can be seen from (2), the range of values as well as representation method of QF searched for is the same as for Boolean function. It remains to clarify the range of definition E of the function searched for. This range, which we will denote as E_Q, represents the set of values taken by QIs. The appearance of this set of values depends on the applied method of QI gradation according to the principle of preference of various ranking scales [19]. It may be single as well, i.e. may consist of one element, e.g., for already mentioned single (one-time) measurements or evaluations.

From the point of view of mathematical inequalities, functional requirements to QI may be presented either by semi-open intervals

$$x \geq x_l \qquad [x_l, \infty), \quad (3)$$

$$x \leq x_r \qquad (0, x_r] \quad (4)$$

or by segment

[1] Note: horrible to say (lat.)

$$x_l \leq x \leq x_r \qquad [x_l, x_r] \tag{5}$$

where x_l and x_r are the so-called functional thresholds [16,26], left and right, respectively.
The range of definition E_Q of quality function is represented by the set

$$E_Q = \{x_l,\ldots,x_r\},$$

while the range of definition E_B of Boolean function is represented by the set

$$E_B = \{1,0\}.$$

If cardinalities of E_Q and E_B sets are compared, it is evident that, except for point-wise properties, when $E_Q = \{1\}$,

$$\text{card } E_Q \geqslant \text{card } E_B, \tag{6}$$

where card means cardinality of a set.
Naturally, for point-wise properties

$$\text{card } E_Q < \text{card } E_B \tag{7}$$

From the point of view of theory of functions, relationships (6) and (7) allow to make conclusion that QF is an extension of Boolean function in regard of range of definition.
The results of fulfillment of functional requirements (3) – (5) are presented in table 1.

Quality index, QI	Relationships of QI and functional thresholds	Functional thresholds	
		left, x_l	right, x_r
x	≥	⌋Q	⌉Q
	≤	⌉Q	⌋Q

Table 1. Dependence of quality Q (⌋Q or ⌉Q) on fulfillment of functional requirements

If the object possesses not one but several properties, then a certain quality function Q_j will correspond to each j-th property. In this case, assertion about the quality Q_0 of the object in whole may be made only on the basis of logical proposition

$$Q_0 = Q_1 \& Q_2 \& \ldots \& Q_j \& \ldots \& Q_n, \qquad j = \left(\overline{1,n}\right), \tag{8}$$

where Q_j means quality of j–th property of the object,
& means logical conjunction sign.

Expression (8) testifies that ⌋Q_0 is achievable only on condition of ⌋Qj for each QI. Figuratively speaking, it may be collated (both in form and fact) with a chain which breaks if only one of its links is broken.
Thus, by means of QF, mathematical-logical formulation (8) was obtained, corresponding to the known assertion that quality is represented by the set of object properties [12, 27].

3. Mathematically fuzzy characteristic functions of quality

The graphical method of setting QF (2) may give visual presentation of the mutual relationship of the only two possible values $Q(x)$ of quality, 0 and 1, with QI values lying

along x axis; for this purpose fig. 1a and fig. 1b show graphical presentation of QF for the half-intervals (3) and (4), respectively, of its domain of definition.
Reviewing of fig.1 from the point of view of mathematical analysis leads to a conclusion that these are the plots of discontinuous piecewise-linear functions with discontinuities at points x_l and x_r. Numerical 0 of QI x at each plot coincides at the origin of coordinates with non-numerical zero relating to absence of the property with the object. Discontinuity of the function is explained by the fact that 0 and 1 along vertical axis are non-numerical. There is no mathematical fuzziness here, and cannot be in principle. However, if one looks at these plots from, so to say, the point of view of mathematical fuzziness, then, pursuant to [28, 29], it is obvious that the plot at fig. 1a resembles the rectangular mathematically fuzzy (MF) S-number (sigmoid), while the plot at fig. 1b resembles the rectangular MF Z-number (zetoid).
In order to pass from purely outward resemblance to exact matching with these MF numbers, "eine grosse Kleinichkeit"[2] is needed – vertical segments, the so-called terms[3], connecting 0 and 1 of functional thresholds – x_l and x_r.
This fact is taken into consideration at fig. 2, which, as may be ascertained, is the MF analog of fig.1 and presents the graphical method of setting QF as MF number for these thresholds [28,29]. We will denote such MF function as characteristic function (hereinafter - CF), it structurally repeats the decision rule (2) for QF and the MF characteristic of functional thresholds (3, 4), or of the combination thereof (5):

$$Q = \rceil Q(x) = \alpha_Q(x) = \left\{ \begin{array}{l} 1, \text{ if } \rfloor Q; \\ 0, \text{ if } \rceil Q. \end{array} \right\} \tag{9}$$

where $\alpha_Q(x)$ is the membership function [30], in this case - of QI x to the property $Q(x)$.
Now let us find out, which MF number corresponds to segment (5) of QF domain of definition. As shown at fig.3a, it will correspond to intersection of sigmoid 1 and zetoid 2 mentioned above. Using the terminology of the theory of sets, this intersection represents the set

$$\Pi = (S \cap Z) = \{x \mid x \in S \text{ и } x \in Z\}, \tag{10}$$

where: S, Z – the sets corresponding to S- and Z- numbers,
$\cap$ – operator of intersection of sets,
| – Sheffer stroke,
$\in$ – logical inclusion sign,
x – QI.
Fig. 3b shows the result of this intersection – the rectangular tolerant (closed) MF number, let us name it Π-number or Π-oid by analogy with S- and Z-numbers. The characteristic property of Π-number is fulfillment of conditions

$$\rfloor Q \in \Pi \quad \text{and} \quad \rceil Q \notin \Pi, \tag{11}$$

where $\notin$ means logical exclusion sign.
Relations (11) mean that quality is compliant with the raised requirements ($\rfloor Q$) only within Π number, and is incompliant with the raised requirements outside it, i.e. ($\rceil Q$).

[2] Note: a big trifle (German);
[3] Note: from Latin word "termin" – borde

As these three MF numbers correspond to three possible domains of definition (3) – (5) of QF, they may be considered as the main (basic) MF characteristic numbers of quality. As far as the domains (3) – (5) are concerned, they may be named the domains of S-, Z- and Π-quality, or briefly S-, Z- and Π- quality, designated as Q_S, Q_Z and Q_Π, respectively.
Thus, not only traditional mathematical methods, but also the methodology of mathematical fuzziness [5], may be applied to quality as the subject of applied mathematical research.
Basing on the main MF characteristic numbers of quality, auxiliary (or second-rank) MF numbers may be derived, which are met in course of quality control of point-wise values or permanent values.
In case of point-wise property of the object, when $x_l = x_r$, Π-number degenerates into the so-called [30] singleton[4]. We will denote it as MF I-number, due to its appearance, fig. 4, and will regard it as additional characteristic number of quality.
In case of permanent property of the object, its point-wise absence may be characterized by anti-singleton, let us denote it as Ī. Geometric appearance of this singleton is shown at fig.5 as drooping vertical straight segment symbolizing point-wise absence of property (⌉P). As may be seen at fig.5, its x – x axis has neither arrow orientation nor scale, and incompliance of quality (⌉**Q)** with the raised requirements occurs here once and at single point.
Examples from material and non-material spheres characterized by singleton I and by anti-singleton Ī were presented above, where point-wise properties and their absence were discussed. A propos, another characteristic number related to loss (⌉**P**) and recovery (⌋P) of property P may be seen there.
According to the stated above, loss of property (⌉**P)** leads to incompliance of quality with the requirements raised to it (⌉**Q**), while recovery of property (⌋P) restores such compliance to (⌋Q). It is shown at fig.6 and characterizes ⌉ and Q at the interval ($x_{⌉Q}$, $x_{⌋Q}$), where $x_{⌉Q}$ and $x_{⌋Q}$ denote the points of incompliance (⌉) and compliance (⌋Q) of quality of permanence by the property P(x) at QI axis x. Its width

$$b = x_{⌋Q} - x_{⌉Q}$$

in terms of MF language is the base or carrier of time (or period) of quality recovery. Appearance of this number at fig. 6 resembles the letter U, therefore we will name it MF auxiliary characteristic number U of quality absence.
For reasons of simplicity, later we will denote MF characteristic numbers S, Z, Π, I in general case by common symbol Ǫ (the so-called "candle apple"[5]).
Similarly, MF characteristic numbers Ī and U in general will be represented by common symbol Ǭ, but they are not discussed here in detail because they are subjects for separate study as they refer to cases difficult for prediction and to unforeseen situations.
Therefore, further we will basically discuss the generalized MF quality characteristic numbers Ǫ or, what is the same, quality Ǫ-numbers. By the way, all the symbolic notations used above were taken from the corresponding Unicode tables of Microsoft Office Word; in particular, hexagonal numbers of symbols Ǫ и Ǭ are 01EA and 01EC, respectively.
Classification of MF characteristic numbers of quality and of quality absence is presented in table 2.

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[4] Note: individual item existing in singular (eng.)
[5] Note: "growing up from pedicle" (contemporary folklore)

Rank	Quality presence	Name	Designation	Position at x axis of quality index
Main	Q̣	Sigmoid	S	Local
		Zetoid	Z	
		Π-oid	Π	
Auxiliary		Singleton	I	
	Q̣̄	Anti-singleton	Ī	Free
		U-oid	U	

Table 2. Mathematically fuzzy characteristic numbers of quality

Examples from the field of production quality control characterized by MF characteristic numbers of quality are given in table 3.

MF characteristic numbers of quality	Objects and/or facilities of quality control
1	2
S	Check of dimensions of male parts by go gauges "GO". Check of dimensions of female parts by no-go gauges "NO GO". Internal diameter of male thread Functional characteristics of a micro switch: forward travel (until direct actuation) of the driving element; overtravel (after direct actuation); force of reverse actuation at the driving element; operational life (limit number of changeovers).
Z	Check of dimensions of female parts by go gauges "GO". Check of dimensions of male parts by no-go gauges "NO GO". Check of defects of geometric shape using universal measuring instruments or by amplitude sensor. Internal diameter of male thread. Functional imperfections of elastic members (e.g., nonlinearity and hysteresis of their elastic response). Functional characteristics of a micro switch: force of direct actuation; voltage drop at normally closed contacts.
Π	Check of limiting dimensions of machine parts confined by tolerance range, using universal measuring instruments and/or by double-limit contact sensor.
I	Just-in-time delivery of component parts, subassemblies, units etc. to the assembly line.
Ī	Hang-up of contacts of micro switches in the zone of expected direct and/or reverse actuation
U	Single or intermittent failures of measuring or automatic checking facilities

Table 3. Examples of production quality control characterized by MF characteristic numbers of quality

Vectorial nature of properties of objects and of corresponding QIs [12], especially in regard of complicated systems [31], allows speaking about composite MF characteristic numbers of quality which we will denote as Ǫ^{n}-numbers, where n means the number of dimensions of object properties vector. The corresponding Ǭ^{n}-numbers will be considered here only fragmentary, because they are more suitable for analysis of catastrophic situations (so to say, "post-flight analysis"), rather than to quality control. In this connection, QF also becomes vectorial, i.e. multi-component.

For simplicity reasons, let us consider the elementary case of a two-dimensional vector of properties which we will denote as vectorial MF characteristic two-component number Ǫ^{2} of quality. Indices 1 and 2 in the latter notation correspond to the set of two numbers, the main and auxiliary, from Ǫ- and/or Ǭ-range. If such two-component number is composed only from Ǫ-numbers, then, naturally, it will represent a Ǫ^{2}-number. If even one of the numbers of the two-component set happens to be Ǭ-number, then, by virtue of relation (9), the resulting Ǫ^{2}-number also will be Ǭ-number. This rule applies to similar composite numbers of any vectorial dimensionality. As the saying is, a scabbed sheep will mar the flock.

From the point of view of theory of sets, two-component MF numbers of quality represent a union of two heterogeneous sets, what is presented in general form as

$$Ǫ^2 = Ǫ_{12} = (Ǫ_1 \cup Ǫ_2) = \{x \mid x_1 \in Ǫ_1 \text{ and } x_2 \in Ǫ_2\}, \qquad (12),$$

where Ǫ$_1$ and Ǫ$_2$ mean the main and auxiliary MF numbers, respectively,

$\cup$ means the sign of logical uniting of sets.

As $x_1 \neq x_2$ in the area under consideration (where $\neq$ is the symbol of absence of equivalence), then numbers Ǫ$_1$ and Ǫ$_2$ cannot lie in the same plane. Therefore, following [32], let us create from the two planes corresponding to them the dimensionless non-numerical (verbal) axis containing the verbal segment [0,1] corresponding to its non-numerical property.

Such a coordinate system for the two-component MF characteristic number of quality Ǫ^{2} (Ǫ$_1$ – main number, Ǫ$_2$ – auxiliary number conjugated with Ǫ$_1$) is shown at fig. 7. The main plane x_1-0-1, corresponding to Ǫ$_1$ number, lies in the plane of drawing, and the auxiliary plane x_2-0-1 forms a spatial right angle with it.

Classification of the main two-component characteristic numbers of quality is given in table 4.

Type	Logical set structure	Geometric image
1	S∨Z∨Π ∪ S∨Z∨Π	Piecewise-planar surface
2	S∨Z∨Π ∪ I	Planar piecewise linear form
3	I ∪ S∨Z∨Π	-*-
4	I ∪ I	Vertical linear segment

Table 4. Classification of two-component characteristic numbers of quality

Geometric images of Ǫ²-numbers of the 1st kind from table 4 are systemized in table 5. Their coordinate axes correspond to the axes at fig. 7. In a similar way, one can systemize and represent graphically other kinds of MF numbers of this table, as well as various combinations of Ǫ- and $\bar{Ǫ}$-numbers, because it is easy to show that formula (12) also covers the case when $Q_2 = \bar{Ǫ}$.

Investigation of geometric images of Ǫ²-numbers in table 5 shows that all of them, except ΠΠ-number, consist of only two geometric forms resembling street shelters: a shed, e.g., ZS-number, or a booth, e.g., ΠS-number. It is not too difficult to note that the entire variety of images of Ǫ²-numbers obtained using the coordinate method considered above may be reproduced by sequential rotations around the vertical axis at the angles being multiple of π/2, namely:

for the "shed"–

$$SZ \looparrowleft ZZ \looparrowleft ZS \looparrowleft SS \looparrowleft SZ: \quad (13)$$

for the " booth" –

$$\Pi S \looparrowleft S\Pi \looparrowleft \Pi Z \looparrowleft Z\Pi \looparrowleft \Pi S, \quad (14)$$

where ↰ means the mathematical symbol of rotation at the angle π/2, in this case – counterclockwise.

Presence of rotations (13) and (14) indicates eventual group-theoretical properties of the structure formed by Ǫ²-numbers.

MF characteristic numbers of quality allow identifying and classifying multi-component (vectorial) double goals and goals of larger dimensionality stipulated by a joint action, e.g., by implementation of QMS into manufacturing activity of an enterprise. The table of kinds of QMS measurable goals [14] is convenient for this purpose, where only the due dates of achieving the goals are changed for later dates. After adding Ǫ- and $\bar{Ǫ}$-numbers to this table, we get table 6.

As one may see, the goals of this table are characterized by single-component Ǫ-, two-component Ǫ², while the latter of the goals in the list – by ternary (three-component) MF Ǫ³ –number IZS. In this connection, it seems that we deal with the structure of notations of MF numbers which may be regarded as MF symbolic language of quality control. A word in this language is a sequence of Ǫ-numbers, ordered by the priority of actions corresponding to them, namely, to organizing and implementing the measures related to quality assurance and quality control, as shown, in particular, in table 6. From the semantic point of view, this corresponds to the known statement: "the meaning of a certain word is its usage in the language" [33].

For practical application of the stated above MF approach to quality control it is necessary to know, at least in first approximation, the influence of measurement errors on the Ǫ-numbers which characterize this quality. Basing on the investigations [34, 35], it may be shown, that if one turns a priori to the probability function of MF appurtenance of this error, then for the case of most commonly encountered Gaussian normal probability law, this MF function is a symmetrical MF R-L-number. Upon completion of additive MF operations with the main

Kinds of goals			Examples of goals	Ǫ and Ǭ numbers of quality	
				main	auxiliary
Absolute	Index value is presented in explicit form.		To increase the number of contracts with customers to 50. To achieve time between failures of one thousand hours for the product under design.	S	
	Index value is not presented in explicit form.		To exclude cases of usage of unverified measuring instruments during acceptance tests (0 cases). To exclude cases of failures of performance (termination) of contracts with customers through the organization's fault (0 cases).	Ī	
	Range of index values (from – to, not larger than, not less than)		Availability coefficient of the product under design shall be not less than 0,95.	S	
			Number of claims lodged to the enterprise shall be not larger than 5.	Z	
			Duration of analysis of a contract with the consumers – 3-5 days	Π	
Relative	Relative value of index		To certify 100% of test equipment.	I	
			To revise 20% of enterprise standards.	S	
	Absolute increment of index value		To increase the profit per one employee of the enterprise by 5 thousand rubles. To reduce average duration of contract issuance by one day.		
	Relative increment of index value показателя		To increase the number of won tenders by 10%. To increase the average wage at the enterprise by 20%.		
	Increment of index value is not expressed in numeric form.		To increase the assortment of manufactured types (kinds) of products. To decrease the number of occupational injuries.	I	
Temporary goals (goals with prescribed deadline)	Short-term (period of achievement – not over 1 year)	Goals of the first two kinds	*All the goals stated below, set for the term not over one year*		
		The goal is not expressed in numeric form	To certify QMS in regard of compliance with GOST R ISO 9001 – 2001 during 2011 To perform training of quality service personnel during 2010. To purchase in 2011 the new climate chamber for tests.		
	Long-term (period of achievement – over 1 year)	Goals of the first two kinds (for term over 1 year)	To rewrite before 2012 all enterprise standards in the framework of preparation of QMS to certification in regard of compliance with GOST R.	Z	
			At that, to rewrite before 2011 not less than 50% of enterprise standards.		S
		Short-term for every year.	To replace, by year 2012, 100% of measurement tools used during state tests of the products by measurement tools included in the state register of measurement tools	I	Z
			At that, by year 2011, to replace not less than 70% of measurement tools.		S

Table 6. Kinds of QMS measurable goals

(S-, Z- and Π-) characteristic numbers of quality, it leads to changing of location of the latter on the scale of QI x which is measured. For the simplest three-term R-L-number which is appropriate here, S-number shifts by two inter-term intervals to the right, Z-number shifts by same distance to the left, while Π-number symmetrically narrows from left and from right by one inter-term interval without shifting.

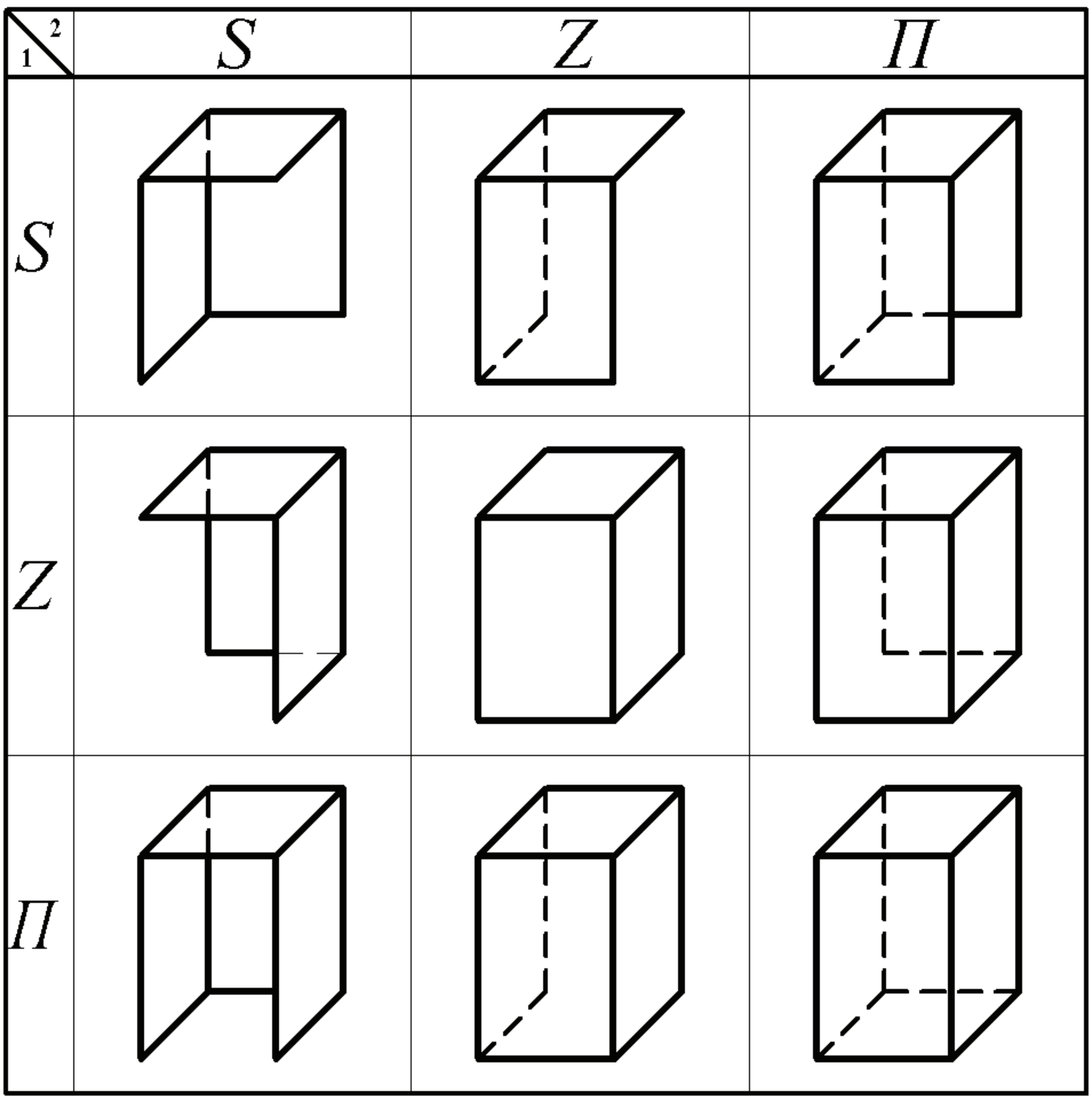

Table 5. Geometric image of mathematically fuzzy two-component Q̧-numbers of quality

In conclusion, I express my gratitude to Doctor of physical and mathematical sciences, Professor of Moscow State Technical University named after N.E. Bauman – M.I. Kiselev for every kind of assistance, valuable comments and proposals during work over this chapter; to the students of the chair headed by him who assisted me; and especially – to my daughter Nadezhda who undertook the labor of computer-aided making-up of the manuscript.

4. Figures

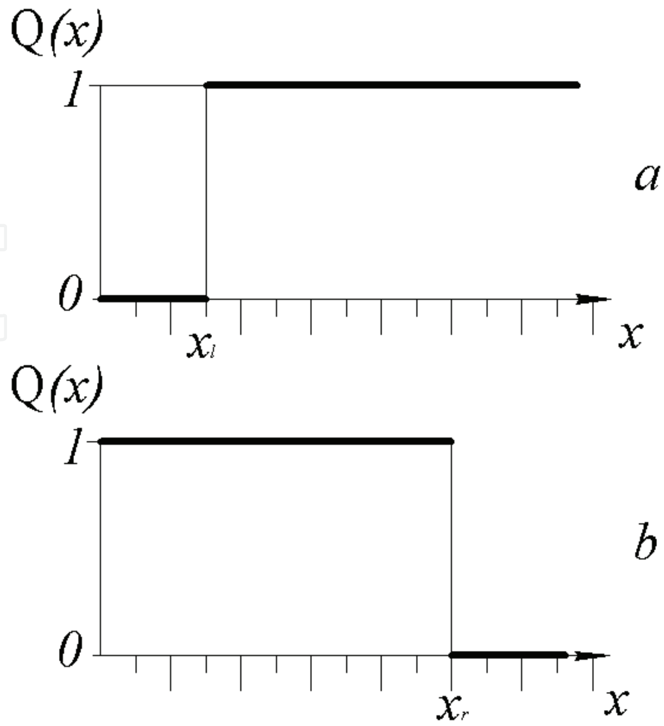

Fig. 1. Analytic quality function $Q_a = Q_a(x)$ for functional thresholds:
a) left, $x \geqslant x_l$, b) right, $x \leqslant x_r$,
x – quality index

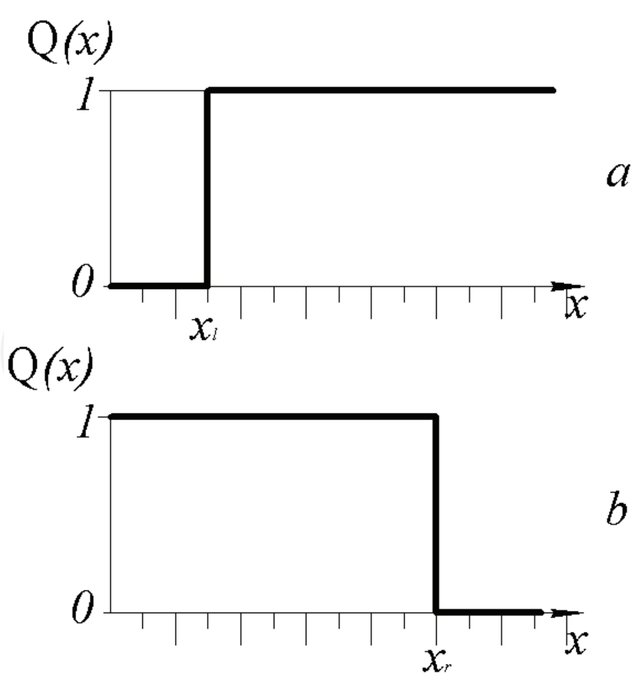

Fig. 2. Mathematically fuzzy (MF) quality function $Q = Q(x)$ for functional thresholds:
a) left (sigmoid S) $x \geqslant x_l$, b) right (zetoid Z) $x \leqslant x_r$,
x – quality index

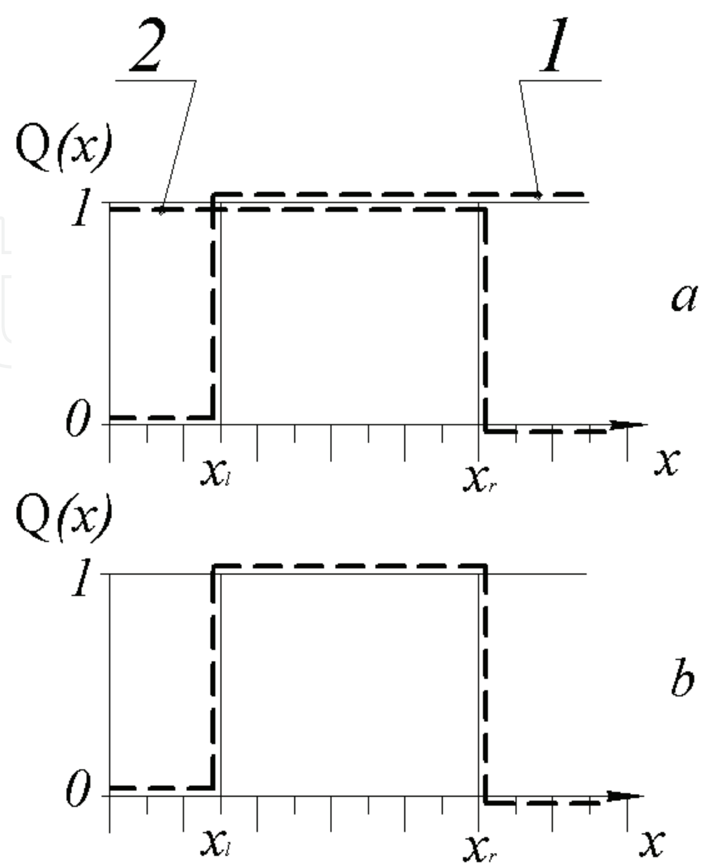

Fig. 3. Formation of MF rectangular tolerant Π-number from sigmoid S and zetoid Z.
a) intersection of sigmoid (1) and zetoid (Z),
b) characteristic Π-number of quality, or Π-oid,
x – quality index,Q (*x*) – MF quality function

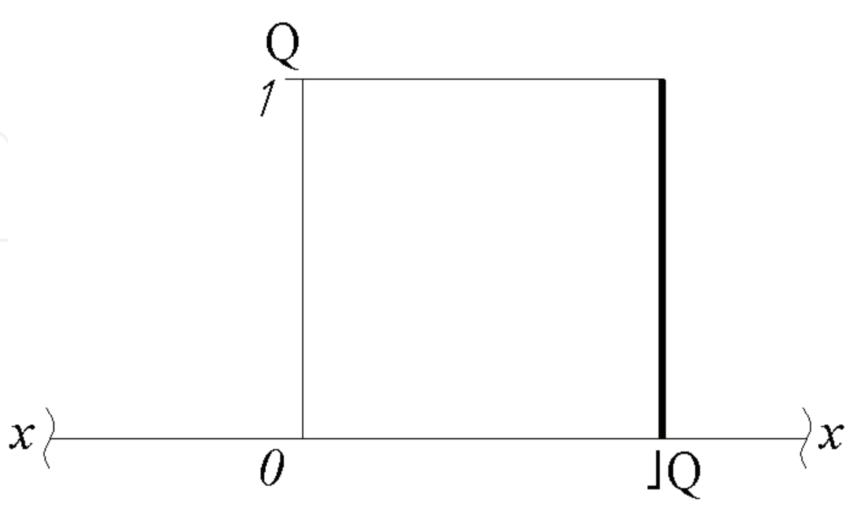

Fig. 4. Degenerate Π-number or singleton I corresponding to point-wise MF quality function
x – *x* – free axis of singleton location,
Q – axis of verbal values of quality (0 and 1)

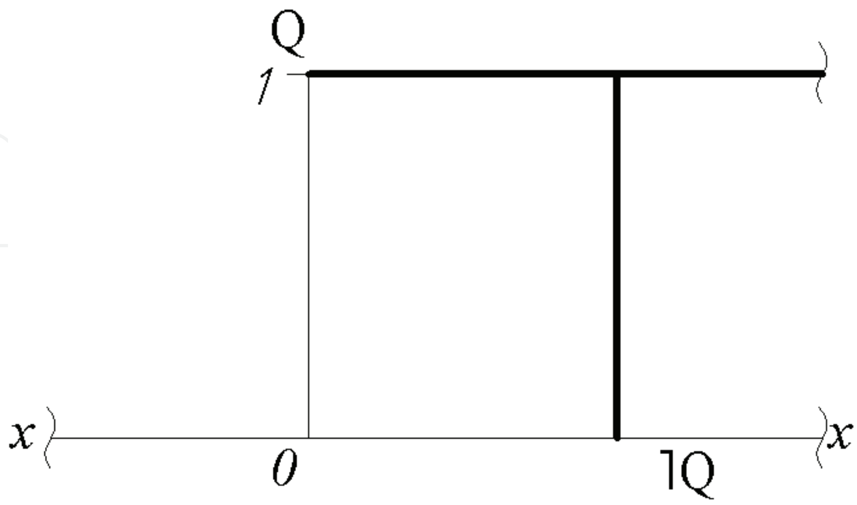

Fig. 5. Anti-singleton $\bar{I}$ of permanent quality function Q (x)
x – x – free axis of anti-singleton location,
Q – axis of verbal values of quality (0 and 1)

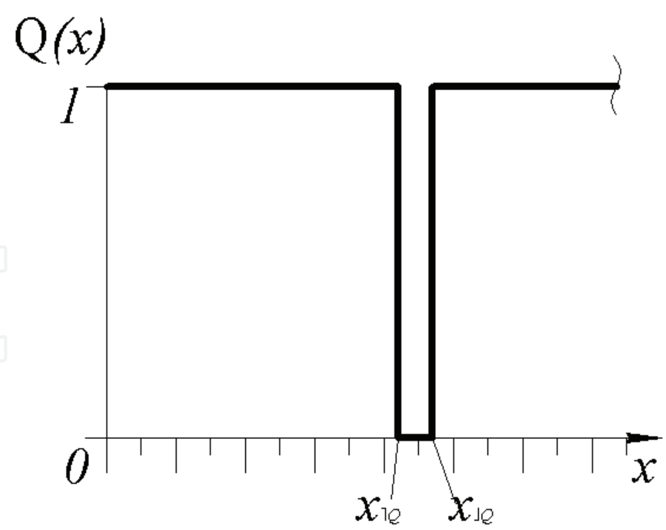

Fig. 6. MF U-number of interval absence of permanency of MF quality function Q (x)
x – quality index,
$x_{\rceil Q}$, $x_{\rfloor Q}$ - points of loss and recovery of permanency of the property characterized by MF quality function at the interval ($x_{\rceil Q}$, $x_{\rfloor Q}$)

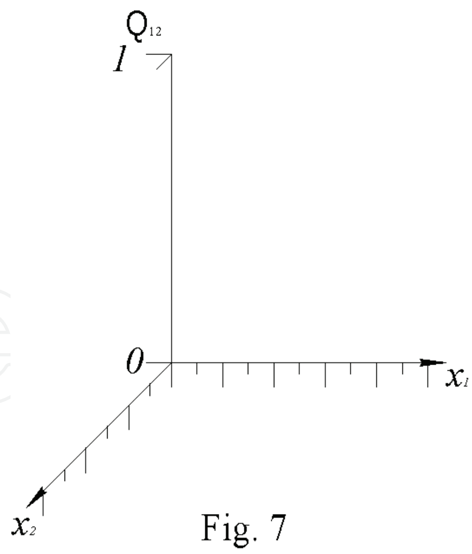

Fig. 7. Coordinate system of the composite MF characteristic quality number Q^2 .
a - main coordinate plane x_1 - 0 - 1, corresponding to the main number Q_1 ;
b - auxiliary coordinate plane x_2 - 0 - 1, corresponding to the auxiliary number Q_2 ;
c – plane of quality absence;
$Q_{1,2}$ - axis of verbal quality values (0 and 1) common for numbers Q_1 and Q_2

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Applications and Experiences of Quality Control

Edited by Prof. Ognyan Ivanov

ISBN 978-953-307-236-4
Hard cover, 704 pages
Publisher InTech
Published online 09, February, 2011
Published in print edition February, 2011

The rich palette of topics set out in this book provides a sufficiently broad overview of the developments in the field of quality control. By providing detailed information on various aspects of quality control, this book can serve as a basis for starting interdisciplinary cooperation, which has increasingly become an integral part of scientific and applied research.

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