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A Collaborative Vendor – Buyer Deteriorating Inventory Model for Optimal Pricing, Shipment and Payment Policy with Two – Part Trade Credit

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1. Introduction

The classical economic order quantity model of Wilson's was developed with the assumption that the buyer must pay off immediately on arrival of the goods in the inventory system. In fact, offering buyers to delays payment for goods received is considered as a sales promotional tool in the business world. With offer of trade credit, vendor increases sales, attracts more buyers and reduces on – hand stock level. Under this marketing strategy, the time of the buyer's capital tied up in stock reduced which eventually reduces the buyer's holding cost of finance. In addition, during this allowable credit period, the buyer can earn interest on the generated revenue. For the small – scale industries having a limited finance, the trade credit acts as a source of short – term funds. Goyal (1985) developed an economic order quantity model with a constant demand rate under the condition of permissible delay in payments. After that numbers of variants of the trade credit problem have been analyzed. For example Shah (1993a, 1993b), Aggarwal and Jaggi (1995), Kim et al. (1995), Jamal et al. (1997), Shinn (1997), Chu et al. (1998), Chen and Chung (1999), Chang and Dye (2001), Teng (2002), Chung and Huang (2003), Shinn and Hwang (2003), Chung and Liao (2004, 2006), Chung et al. (2005), Teng et al. (2005), Ouyang et al. (2005) and their cited references. For up – to day available literature on permissible delay period, refer to the article by Shah et al. (2010).

The above cited references assume that the vendor offer the buyer a "one – part" trade credit, i.e. the vendor offers a permissible delay period. If the account is settled within this period, no interest is charged to the buyer. As a result, with no incentive for making early payments, and earning interest through generated revenue during the credit period, the buyer postpones payment up to the last day of the permissible period offered by the vendor. As an outset, from the vendor's end, offering trade credit leads to delayed cash inflow and increases the risk of cash flow shortage and bad debt. To increase cash inflow and reduce the risk of a cash crisis and bad debt, the vendor may offer a cash discount to attract the buyer to pay for goods earlier. i.e. the vendor offers a "two – part" trade credit to the buyer to balance the trade off between delayed payment and cash discount. For example, under an agreement, the vendor agrees to a 2% discount to the buyer's purchase price if payment is made within 10 days. Otherwise, full payment is to be settled within 30 days after the

delivery. In financial management, this credit is denoted as "2|10 net 30". If the vendor only offers the buyer a 30 days credit period, i.e. "one – part" trade credit, then this is denoted as "net 30" (Brigham, 1995). The papers related to this credit policy are by Lieber and Orgler (1975), Hill and Riener (1979), Kim and Chung (1990), Arcelus and Srinivasan (1993), Arcelus et al. (2001, 2003). Ouyang et al. (2002), Chang (2002) and Huang and Chung (2003) developed inventory models when two – credit policy is offered by the vendor to the buyer.
The above cited model's are derived either from the vendor's or the buyer's end. However, the two players may have their own goals. The decision taken from the buyer's end may not be agreeable to vendor and vice versa. Lee et al. (1997) argued that without coordinated inventory management in the supply chain may result in excessive inventory investment, revenue reduction and delays in response to customer satisfaction. Therefore, the joint discussion is more beneficial as compared to the individual decision. Goyal (1976) first developed a single vendor – single buyer integrated inventory model. Banerjee (1986) extended Goyal's (1976) model under assumption of a lot – for – lot production for the vendor. Later, Goyal (1988) established that if vendor produces an integer multiple of the buyer's purchase quantity then the inventory cost can be reduced. Lu (1995) generalized Goyal's (1988) model by relaxing the assumption that the vendor can supply to the buyer only after finishing the entire lot size. Bhatnagar et al. (1993), Goyal (1995), Viswanathan (1998), Hill (1997, 1999), Kim and Ha (2003), Kalle et al. (2003), Li and Liu (2006) developed more batching and shipping policies for an integrated inventory model. However, these articles did not incorporate the effect of trade credit on the integrated optimal decision. Abad and Jaggi (2003) developed a vendor – buyer integrated model assuming lot – for – lot production under a permissible delay in payments. Later, Shah (2009) extended Abad and Jaggi's(2003) model for deteriorating items. In both the articles, the vendor offered a "one – part" trade credit to the buyer.
Ho et al. (2008) studied impact of a "two – part" trade credit policy in the integrated inventory model. This model assumed that units in inventory remain of 100% utility during the cycle time. However, the products like medicines and drugs, food products, vegetables and fruits, fashion goods, x – ray films etc loose its 100% utility in due course of time. In this chapter, we analyze effect of a "two – part" trade credit policy in the integrated inventory model when units are subject to constant deterioration and demand is retail price sensitive. The supplier offers the buyer a cash discount if payment is made before an allowable period, and if the buyer does not pay within the allowable period, the full account against purchases made before the delay payment due date. The joint profit is maximized with respect to the optimal payment policy, selling price, lot – size and the number of shipments from vendor to buyer in one production run. An algorithm is developed to determine the optimal policy. Numerical examples are given to validate the theoretical results. The sensitivity analysis of the optimal solutions with respect to model parameters is also carried out.

2. Assumptions and notations

The proposed model is formulated using the following assumptions and notations.

1. The integrated inventory system comprises of a single – vendor and single buyer for a single item.
2. Shortages are not allowed.
3. The inventory holding cost rates excluding interest charges for the vendor is I_v and for the buyer is I_b.
4. To accelerate the cash inflow and reduce the risk of bad debt, the vendor offers a discount β ($0 < \beta < 1$) off the purchase price, if the buyer settles the account within time M_1. Otherwise, the full account is due within time M_2, where $M_2 > M_1 \geq 0$.

5. The vendor's unit production cost is \$ C_v and unit sale price is \$ C_b. The buyer's unit retail price is \$ P. Here $P > C_b > (1-\beta)C_b > C_v$.
6. During the allowable credit period to the buyer, the vendor opts to give up an immediate cash inflow until a later date. Thus, the vendor endures a capital opportunity cost at a rate I_{vo} during the time between delivery and payment of the item.
7. During period [M_1, M_2], a cash flexibility rate f_{vc} is available to quantize the advantage of early cash income for the vendor.
8. During the credit period (i.e. M_1 or M_2), the buyer earns interest at a rate of I_{be} on the revenue generated by selling the product.
9. The demand rate for the item is a decreasing function of the sale price and is given by $R(P) = aP^{-\eta}$, where a > 0 is scaling demand, and η > 1 is a price – elasticity coefficient.
10. The capacity utilization "ρ" is defined as the ratio of the demand rate, R(P) to the production rate p(P), i.e. ρ = R(P)/p(P) where ρ < 1 and is fixed.
11. The buyer's cycle time is T, order quantity is Q per order.
12. The buyer's ordering cost per order is A_b.
13. During the production period, the vendor produces in batches of size nQ (where n is a positive integer) and incurs a batch set up cost A_v. After the production of first Q units, the vendor ships them to the buyer and then makes continuous shipping at every T-units of time until the vendor's inventory level depletes to zero.
14. The units in inventory deteriorate at a constant rate, θ ($0 < \theta < 1$). The deteriorated can neither be repaired nor replaced during the cycle time T.

3. Mathematical model

The inventory on hand depletes due to price – sensitive demand and deterioration of units. The rate of change of inventory at any instant of time 't' is governed by the differential equation,

$$\frac{dI(t)}{dt} = -R(P) - \theta I(t); \qquad 0 \le t \le T$$

with initial condition I(0) = Q and boundary condition I(T) = 0. The solution of the differential equation is

$$I(t) = \frac{R(P)}{\theta}\left\{e^{\theta(T-t)} - 1\right\}; \qquad 0 \le t \le T$$

and procurement quantity, Q is

$$Q = I(0) = \frac{R(P)}{\theta}\left\{e^{\theta T} - 1\right\}$$

3.1 Vendor's total profit per unit time

During each production run, the vendor produces in batches of the size nQ with a batch set up cost A_v. The cycle length of the vendor is nT- units. Therefore, the vendor's set up cost per unit time is (A_v/nT). Using method given by Joglekar (1988), with the unit production

cost C_v, the inventory holding cost rate excluding interest charges I_v and capital opportunity cost per $ per unit time I_{vo}, the vendor's carrying cost per unit time is

$$\frac{C_v(I_v + I_{vo})}{T}\left[(n-1)(1-\rho)+\rho\right]\int_0^T I(t)dt$$
$$= \frac{C_v(I_v + I_{vo})}{T}R(P)\left[(n-1)(1-\rho)+\rho\right]\left[e^{\theta T} - \theta T - 1\right]$$

For each unit of item, the vendor charges $\left(\left(1-K_j\beta\right)C_b\right)$ if the buyer pays at time M_j,j=1,2, $K_1 = 1$ and $K_2 = 0$. The opportunity cost at the finance rate I_{vo} per unit time for offering trade credit is $\left(\left(1-K_j\beta\right)C_b \cdot I_{vo} \cdot M_j \cdot Q/T\right)$. However, if the buyer pays at M_1 - time, during $M_2 - M_1$ the vendor can use the revenue ((1 – β)C_b) to avoid a cash flow crisis. The advantage gain per unit time from early payment at a cash flexibility rate f_{vc} is $\left(K_j \cdot (1-\beta)C_b \cdot f_{vc} \cdot (M_2 - M_1)Q/T\right)$.

Thus, the vendor's total profit per unit time is the revenue generated plus the advantage from early payment minus production cost, set up cost, inventory holding cost and opportunity cost for offering trade credit.

$$\begin{aligned} TVP_j(n) = {} & \left(1-K_j\beta\right)C_b\frac{Q}{T} - C_v\frac{Q}{T} - \frac{A_v}{nT} \\ & - \frac{C_v(I_v + I_{vo})R(P)}{\theta^2 T}\left[(n-1)(1-\rho)+\rho\right]\left[e^{\theta T} - \theta T - 1\right] \\ & - (1-K_j\beta)vI_{vo}M_j\frac{Q}{T} + K_j(1-\beta)C_b f_{vc}(M_2 - M_1)\frac{Q}{T}, \end{aligned} \tag{1}$$

$$j = 1,2; \quad K_1 = 1, \quad K_2 = 0$$

3.2 Buyer's total profit per unit time

The buyer's ordering cost is A_b for each order of Q – units, so the ordering cost per unit time is (A_b/T). The inventory holding cost excluding interest charges per unit time is

$$\left(\frac{\left(1-K_j\beta\right)C_b I_b R(P)}{\theta^2 T}\left[e^{\theta T} - \theta T - 1\right]\right)$$

On the basis of length of the payment time, two cases arise: (i) T< M_j and (ii) T ≥ M_j ; j=1,2 These two cases are shown in Figure 1.

Case: (i) T < M_j; j = 1, 2.

Here, the buyer's cycle time ends before the payment time. So buyer does not pay opportunity cost for the items kept in stock. The buyer earns interest at the rate of I_{be} on the revenue generated; hence, the interest earned per unit time is,

$$\frac{1}{T}\left[PI_{be}\int_0^T R(P)tdt + PI_{be}Q(M_j - T)\right] = \frac{PI_{be}R(P)}{T}\left[\frac{T^2}{2} + \frac{1}{\theta}\left(e^{\theta T} - 1\right)\left(M_j - T\right)\right]$$

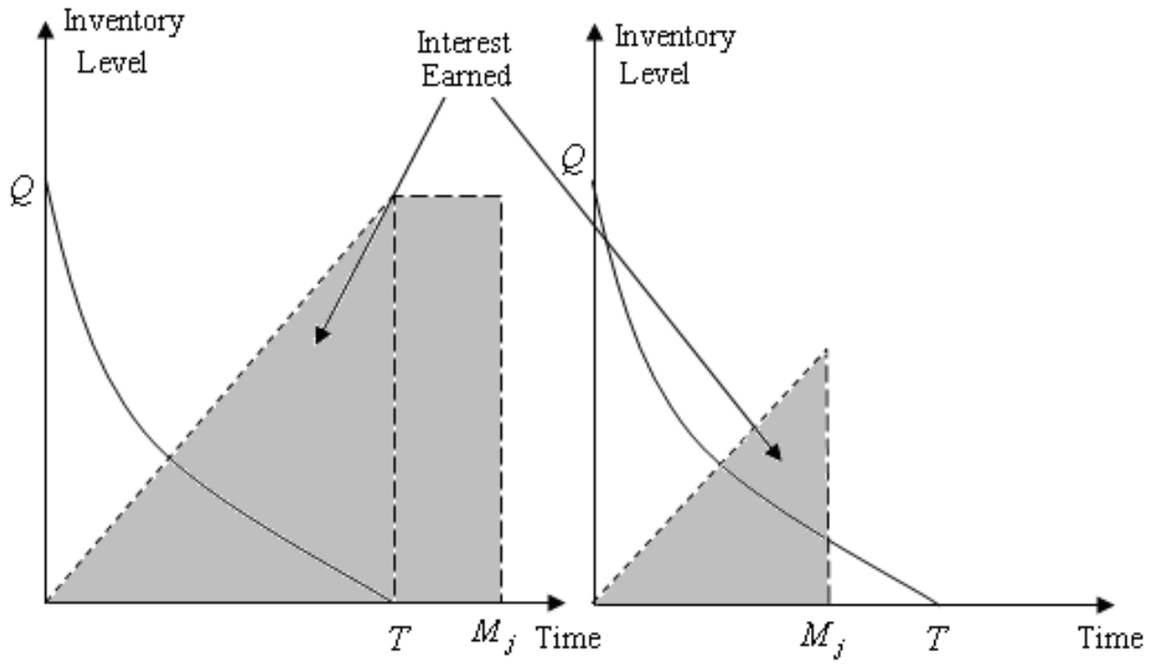

Fig. 1. Inventory and interest earned for the buyer under trade credit

Case: (ii) $T \geq Mj$; $j = 1, 2$

In this case, the buyer's allowable payment time ends on or before the inventory is depleted to zero. The interest earned per unit time is

$$\frac{PI_{be}}{T}\int_0^{M_j} R(P)tdt = \frac{PI_{be}R(P)M_j^2}{2T}.$$

After the due date M_j, the buyer pays interest charges at the rate of I_{bc}. Therefore, the interest charges payable per unit time is,

$$\frac{\left(1-K_j\beta\right)C_bI_{bc}}{T}\int_{M_j}^{T} I(t)dt = \frac{\left(1-K_j\beta\right)C_bI_{bc}R(P)}{\theta^2T}\left[e^{\theta(T-M_j)} - \theta(T-M_j) - 1\right]$$

The buyer purchase cost per unit time is $((1 - K_j \beta)\, C_b\, Q/T)$ and revenue generated per unit time is (PQ/T). Therefore, the buyer's total profit per unit time is revenue generated plus interest earned minus the total cost comprises of the purchase cost, ordering cost, inventory holding cost excluding interest charges and interest charges payable, i.e.

$$TBP_j(P,T) = \begin{cases} TBP_{j1}(P,T) & T < M_j \\ TBP_{j2}(P,T) & T \geq M_j \end{cases}; j = 1,2 \qquad (2)$$

Where

$$TBP_{j1}(P,T)=\frac{PQ}{T}-\left(1-K_j\beta\right)\frac{C_bQ}{T}-\frac{A_b}{T}$$
$$-\frac{\left(1-K_j\beta\right)C_bI_bR(P)}{\theta^2T}\left[e^{\theta T}-\theta T-1\right] \quad (3)$$
$$+\frac{PI_{be}R(P)}{T}\left[\frac{T^2}{2}+\frac{1}{\theta}\left(e^{\theta T}-1\right)\left(M_j-T\right)\right]$$

And

$$TBP_{j2}(P,T)=\frac{PQ}{T}-\left(1-K_j\beta\right)\frac{C_bQ}{T}-\frac{A_b}{T}$$
$$-\frac{\left(1-K_j\beta\right)C_bI_bR(P)}{\theta^2T}\left[e^{\theta T}-\theta T-1\right]+\frac{PI_{be}R(P)M_j^2}{2T} \quad (4)$$
$$-\frac{\left(1-K_j\beta\right)C_bI_{bc}R(P)}{\theta^2T}\left[e^{\theta(T-M_j)}-\theta(T-M_j)-1\right]$$

3.3 The joint total profit per unit time

When the buyer and vendor opt for the joint decision, the joint total profit per unit time is,

$$TP_j(n,P,T)=\begin{cases}TP_{j1}(n,P,T) & T<M_j\\ TP_{j2}(n,P,T) & T\geq M_j\end{cases};j=1,2 \quad (5)$$

Where

$$TP_{j1}(n,P,T)=TVP_j(n)+TBP_{j1}(P,T)$$
$$=\left(P-C_v\right)\frac{Q}{T}-\frac{1}{T}\left(\frac{A_v}{n}+A_b\right)-\frac{\left(1-K_j\beta\right)C_bI_{vo}M_jQ}{T}$$
$$+\frac{C_v(I_v+I_{vo})R(P)}{\theta^2T}\left[(n-1)(1-\rho)+\rho\right]\left[e^{\theta T}-\theta T-1\right]$$
$$+\frac{K_j(1-\beta)C_bf_{vc}(M_2-M_1)Q}{T}$$
$$+\frac{PI_{be}R(P)}{T}\left[\frac{T^2}{2}+\frac{1}{\theta}\left(e^{\theta T}-1\right)\left(M_j-T\right)\right] \quad (6)$$
$$-\frac{\left(1-K_j\beta\right)C_bI_bR(P)}{\theta^2T}\left[e^{\theta T}-\theta T-1\right]$$
$$+\frac{PI_{be}R(P)}{T}\left[\frac{T^2}{2}+\frac{1}{\theta}\left(e^{\theta T}-1\right)\left(M_j-T\right)\right]$$

And

$$
\begin{aligned}
TP_{j2}(n,P,T) &= TVP_j(n) + TBP_{j2}(P,T) \\
&= (P-C_v)\frac{Q}{T} - \frac{1}{T}\left(\frac{A_v}{n} + A_b\right) - \frac{\left(1-K_j\beta\right)C_b I_{vo} M_j Q}{T} \\
&- \frac{C_v(I_v + I_{vo})R(P)}{\theta^2 T}\left[(n-1)(1-\rho)+\rho\right]\left[e^{\theta T} - \theta T - 1\right] \\
&+ \frac{K_j(1-\beta)C_b f_{vc}(M_2 - M_1)Q}{T} + \frac{PI_{be}R(P)M_j^2}{2T} \\
&- \frac{\left(1-K_j\beta\right)C_b I_b R(P)}{\theta^2 T}\left[e^{\theta T} - \theta T - 1\right] \\
&- \frac{\left(1-K_j\beta\right)vI_{bc}R(P)}{\theta^2 T}\left[e^{\theta(T-M_j)} - \theta(T-M_j) - 1\right]
\end{aligned}
\tag{7}
$$

Assuming θ to be very small, ignoring θ^2 and its higher powers, we get

$$
\begin{aligned}
TP_{j1} &= (P-c_v)R(P)\left(1+\frac{\theta T}{2}\right) - \frac{\left(1-K_j\beta\right)C_b I_b R(P)T}{2} \\
&- \frac{C_v(I_v + I_{vo})R(P)\left[(n-1)(1-\rho)+\rho\right]T}{2} + \frac{PI_{be}R(P)}{T}\left[M_j\left(1+\frac{\theta T}{2}\right) - \frac{T}{2}\right] \\
&- \left(1-K_j\beta\right)C_b I_{vo} M_j R(P)\left(1+\frac{\theta T}{2}\right) - \frac{1}{T}\left(\frac{A_v}{n} + A_b\right) \\
&+ K_j(1-\beta)C_b f_{vc}(M_2 - M_1)R(P)\left(1+\frac{\theta T}{2}\right)
\end{aligned}
\tag{6a}
$$

Also

$$
\begin{aligned}
TP_{j2} &= (P-C_v)R(P)\left(1+\frac{\theta T}{2}\right) - \left(1-K_j\beta\right)C_b I_{vo} M_j R(P)\left(1+\frac{\theta T}{2}\right) \\
&- \frac{C_v(I_v + I_{vo})R(P)\left[(n-1)(1-\rho)+\rho\right]T}{2} - \frac{1}{T}\left(\frac{A_v}{n} + A_b\right) \\
&+ K_j(1-\beta)C_b f_{vc}(M_2 - M_1)R(P)\left(1+\frac{\theta T}{2}\right) + \frac{PI_{be}R(P)M_j^2}{2T} \\
&- \frac{\left(1-K_j\beta\right)C_b(I_b + I_{bc})R(P)T}{2} - \left(1-K_j\beta\right)C_b I_{bc} R(P)M_j \\
&+ \frac{\left(1-K_j\beta\right)C_b I_{bc} R(P)M_j^2}{2T}
\end{aligned}
\tag{7a}
$$

The problem now is to compute the optimal values of n, P and T such that TP_j (n, P, T); j=1, 2 in equation (5) is maximized.

4. Solution methodology

For fixed P and T, the second order partial derivative of equation (5) with respect to 'n' is, $\frac{\partial^2 TP_j(n,P,T)}{\partial n^2} = \frac{-2A_v}{n^3 T} < 0$ for j = 1, 2 suggest that TP_j (n, P, T) is a concave function in 'n'. This guarantees that the search for the optimal shipment number n* is reduced to find a local optimal solution.

4.1 Determination of the optimal cycle time 'T' for any given 'n' and 'P'

For given n and P, the partial derivative of TP_{j1} (n, P, T) in (6 – a) with respect to T, $\frac{\partial^2 TP_{j1}(n,P,T)}{\partial T^2} = -\frac{2}{T^3}\left(\frac{A_v}{n} + A_b\right) < 0$ suggests that TP_{j1} (n, P, T) is a concave function in T. Hence, there exists unique value of $T = T_{j1}(n, P)$ (say) which maximizes $TP_{j1}(n, P, T)$. $T_{j1}(n, P)$ can be obtained by setting $\frac{\partial TP_{j1}(n,P,T)}{\partial T} = 0$ and is given by,

$$T_{j1} = \sqrt{\frac{2\left(\frac{A_v}{n} + A_b\right)}{R(P)\left\{\begin{array}{l} C_v\left(I_v + I_{vo}\right)\left[(n-1)(\rho-1)+\rho\right] - (P - C_v)\theta \\ +\left(1 - K_j\beta\right)C_b I_{vo} M_j \theta - K_j(1-\beta)C_b f_{vc}(M_2 - M_1)\theta \\ +\left(1 - K_j\beta\right)C_b I_b + PI_{be}\left(1 - \theta M_j\right) \end{array}\right\}}} \tag{8}$$

To ensure T_{j1} (n, P) < M_j, we substitute (8) into inequality $T_{j1}(n, P) < M_j$ and obtain

$$\frac{A_v}{n} + A_b < \frac{R(P)M_j^2}{2}\left[\begin{array}{l} C_v\left(I_v + I_{vo}\right)\left[(n-1)(\rho-1)+\rho\right] - (P - C_v)\theta \\ -K_j(1-\beta)C_b f_{vc}(M_2 - M_1)\theta \\ +\left(1 - K_j\beta\right)C_b\left(I_b + I_{vo}M_j\theta\right) + PI_{be}\left(1 - \theta M_j\right) \end{array}\right] \tag{9}$$

Substituting (8) into (6), the joint total profit for case 1 is,

$$TP_{j1}(n, P) = TP_j(n, P, T_{j1}(n, P)) \tag{10}$$

Furthermore, from (9), we have T_{j2} (n, P) ≥ M_j if and only if

$$\frac{A_v}{n} + A_b \geq \frac{R(P)M_j^2}{2}\left[\begin{array}{l} C_v\left(I_v + I_{vo}\right)\left[(n-1)(\rho-1)+\rho\right] - (P - C_v)\theta \\ -K_j(1-\beta)C_b f_{vc}(M_2 - M_1)\theta \\ +\left(1 - K_j\beta\right)C_b\left(I_b + I_{vo}M_j\theta\right) + PI_{be}\left(1 - \theta M_j\right) \end{array}\right] \tag{11}$$

The second order partial derivative of TP_{j2} (n, P, T) in (7 – a) is,

$$\frac{\partial^2 TP_{j2}(n,P,T)}{\partial T^2} = -\frac{1}{T^3}\left\{R(P)M_j^2\left[\left(1-K_j\beta\right)C_bI_{bc} - PI_{be}\right] + 2\left(\frac{A_v}{n} + A_b\right)\right\} < 0 \tag{12}$$

which suggests that for fixed n and P, TP_{j2} (n, P, T) is a concave function in T.

By solving the equation $\frac{\partial TP_{j2}(n,P,T)}{\partial T} = 0$, we obtain the value of T= T_{j2} (n, P) (say) which maximizes TP_{j2} (n, P, T) and is given by

$$T_{j2} = \sqrt{\frac{2\left(\frac{A_v}{n} + A_b\right) - PI_{be}R(P)M_j^2 + \left(1-K_j\beta\right)C_bI_{bc}R(P)M_j^2}{R(P)\left\{\begin{matrix} C_v(I_v + I_{vo})\left[(n-1)(\rho-1)+\rho\right] - K_j(1-\beta)C_bf_{vc}(M_2-M_1)\theta \\ -(P-C_v)\theta + \left(1-K_j\beta\right)C_b\left(I_b + I_{bc} + I_{vo}\theta M_j\right)\end{matrix}\right\}}} \tag{13}$$

Substituting (13) into (7 – a), the joint total profit for case 2 is

$$TP_{j2}(n, P) = TP_j(n, P, T_{j2}(n, P)) \tag{14}$$

For simplicity, define

$$\Delta_j = \frac{R(P)M_j^2}{2}\left[\begin{matrix} C_v(I_v + I_{vo})\left[(n-1)(1-\rho)+\rho\right] + (P-C_v)\theta \\ +\left(1-K_j\beta\right)C_b\left(I_b + I_{bc} + I_{vo}\theta M_j\right) \\ +K_j(1-\beta)C_bf_{vc}(M_2-M_1)\theta + PI_{be}\left(1-\theta M_j\right)\end{matrix}\right], \text{ j =1,2} \tag{15}$$

Since, $M_2 > M_1 \geq 0$, $K_1 = 1$ and $K_2 = 0$, we have $\Delta_2 > \Delta_1$.

Theorem 1: For given n and P,

a. When $\frac{A_v}{n} + A_b < \Delta_1$, if max {$TP_{11}$ (n, P), TP_{21} (n, P)} = TP_{11} (n, P) then the optimal payment time is M_1 and optimum cycle time is TP_{11} (n, P). Otherwise, the optimal payment time is M_2 and optimum cycle time is TP_{21} (n, P).

b. When $\Delta_1 \leq \frac{A_v}{n} + A_b < \Delta_2$, if max {$TP_{21}$ (n, P), TP_{12} (n, P)} = TP_{21} (n, P) then the optimal payment time is M_2 and optimum cycle time is TP_{21} (n, P). Otherwise the optimal payment time is M_1 and optimum cycle time is TP_{12} (n, P)

c. When $\frac{A_v}{n} + A_b \geq \Delta_2$, if max {$TP_{12}$ (n, P), TP_{22} (n, P)} = TP_{12} (n, P) then the optimal payment time is M_1 and optimum cycle time is TP_{12} (n, P) Otherwise the optimal payment time is M_2 and optimum cycle time is TP_{22} (n, P)

Proof: It immediately follows from (9), (11) and (15).

4.2 Determination of the buyer's optimal retail price for any given n

For computing optimal value of retail price; P we follows methodology given by Teng et al. (2005).

Define

$$f_j(P) = \Delta_j,\ j = 1, 2 \tag{16}$$

It is easy to check that $f_j(P)$ is strictly decreasing function of P for given n. Also $\lim_{P \to 0} f_j(P) = \infty$ and $\lim_{P \to \infty} f_j(P) = 0$ for fixed n, guarantees that there exist a unique value of P_{jo} such that

$$f_j(P_{jo}) = \frac{A_v}{n} + A_b \tag{17}$$

Then, (9) and (11) reduce to

$$\text{if and only if} \quad P < P_{jo}, \text{ then } T_{j1}(n, P) < M_j \tag{18}$$

and

$$\text{if and only if} \quad P \geq P_{jo}, \text{ then } T_{j2}(n, P) \geq M_j \tag{19}$$

respectively.
Now our problem is to find the optimal value of retail price; P which maximize the joint total profit

$$TP_j(n,P) = \begin{cases} TP_{j1}(n,P), & if \quad P < P_{jo} \\ TP_{j2}(n,P), & if \quad P \geq P_{jo} \end{cases} \qquad j = 1,2 \tag{20}$$

For fixed n, the optimal value of P which maximizes $TP_{ji}(n, P)$, j = 1, 2 and i =1,2, can be obtained by first order necessary condition $\frac{\partial TP_{ji}(n,P)}{\partial P} = 0$ and examining the second order sufficient condition $\frac{\partial^2 TP_{ji}(n,P)}{\partial P^2} < 0$ for concavity.

From the above arguments, we outline the computational algorithm to find the optimal solution (n*, P*, T*).

Computational algorithm

Step 1 Set n =1.
Step 2 For j = 1,2.

i. Determine P_{jo} by solving (17).

ii. If there exists a P_{j1} such that $P_{j1} < P_{j0}$, $\frac{\partial TP_{j1}(n,P)}{\partial P} = 0$ and $\frac{\partial^2 TP_{j1}(n,P)}{\partial P^2} < 0$, then compute $T_{j1}(n, P_{j1})$ using (8) and $T_{j1}(n, P_{j1})$ using (10).
Otherwise, set $TP_{j1}(n, P_{j1}, T_{j1}(n, P_{j1})) = 0$.

iii. If there exists a P_{j2} such that $P_{j2} \geq P_{j0}$, $\frac{\partial TP_{j2}(n,P)}{\partial P} = 0$ and $\frac{\partial^2 TP_{j2}(n,P)}{\partial P^2} < 0$, then compute $T_{j2}(n, P_{j2})$ using (13) and $T_{j2}(n, P_{j2})$ using (14).
Otherwise, set $TP_{j2}(n, P_{j2}, T_{j2}(n, P_{j2})) = 0$.

Step 3 Set $TP\left(n, P^{(n)}, T^{(n)}\right) = \max_{\substack{j=1,2 \\ i=1,2}} TP_{ij}\left(n, P_{ji}, T_{ji}\left(n, P_{ji}\right)\right)$ then ($P^{(n)}$, $T^{(n)}$) is the optimal solution for given n.

Step 4 If TP (n, $P^{(n)}$, $T^{(n)}$) ≥ TP (n-1, $P^{(n-1)}$, $T^{(n-1)}$), then go to step 5. Otherwise, go to step 6.

Step 5 Set n = n + 1, go to step 2.

Step 6 Set TP (n, P^*, T^*) ≥ TP (n-1, $P^{(n-1)}$, $T^{(n-1)}$), then (n*, P*, T*)is the optimal solution.

Knowing the optimal solution (n*, P*, T*), the optimal order quantity per order for the buyer Q* can be obtained using $Q^* = \frac{R(P^*)}{\theta}\left\{e^{\theta T^*} - 1\right\}$.

5. Numerical illustration

Example 1 In order to validate the solution procedure, consider an integrated inventory system with following parametric values: a=250,000, ρ=0.9, η=1.25, C_v=$2/unit, C_b=$4.5/unit, A_v=$1000/Set up, A_b=$300/Order, I_v=0.08/$/annum, I_b=0.08/$/annum, I_{vo}=0.09/$/annum, I_{bc}=0.16/$/annum, I_{be}=0.12/$/annum and f_{vc}=0.17/$/annum. Consider, a trade credit term "2|10 net 30", i.e. M_1=10 days, M_2=30 days and β=2% is offered by the vendor to the buyer. The deterioration rate of units in inventory is 5%.

Using the computational procedure, the maximum total joint profit of the integrated system is TP(n*, P*, T*) = $ 109628.38. The buyer makes the payment within 10 days and avails of 2% discount in purchase cost, the retail price is P*=$ 10.6616/unit, the replenishment cycle time T* = T_{12} = 0.2330 year = 85.04 days and the ordering quantity Q*=3041.09units/order.

The optimal shipment from the vendor to the buyer us n*= 10.

Example 2 In Table 1, we study the effects of credit terms M_1 and M_2. The no trade credit is taken as a bench mark. The relationship between credit terms and profits of buyer, vendor and total are calculated.

It is observed that the profit gain in percentage is positive for the integrated decision. i.e. total profit for the integrated decision under the two – part trade credit policy is beneficial than the total profit when no credit is offered. It is also observed that the profit gain in percentage is not always positive for the vendor. Under credit terms "2|10, net 30" or if vendor extends the due date to M_2=30 days after the delivery, the vendor's profit gains in percentage are negative.

Table 1 also suggests that if the vendor offers the payment due date at 30 days then offering a 2% discount can encourage the buyer to settle the payment earlier. However, if the vendor extends the due date to 60 days or 90 days, the integrated profit will be maximized as the buyer pays at the end of the net period. The offer of due date at 60 days or 90 days after delivery by the vendor will not accelerate cash inflows. Hence, in an integrated supply chain, the vendor needs to decide the credit policy very carefully to get mutual benefit from a two – part trade credit scenario.

Example 3 Using the same data as in Example 1, we compare the impact of trade credit for independent and coordinated decision in Table 2. The optimal solutions of "cash on delivery" (i.e.M_1 = M_2 = 0 and β = 0) and "2|10 net 30" are computed.

In independent decision, buyer is dominant decision maker and then the vendor defines his policy.

M_1	M_2	Optimum Payment time	n	P	T	R(P)	Q	Profit			Profit gain (%)		
								Buyer	Vendor	Integrated	Buyer	Vendor	Integrated
0	0	-	10	10.75	87.47	12843	3096	78100	30961	109061	-	-	-
0	30	30	12	10.59	T_2=84.95	13076	3061	78292	31312	109604	0.25	1.13	0.49
10		10	10	10.66	T_{12}=85.04	12976	3041	78706	30922	109628	0.78	-0.13	0.52
20		20	10	10.73	T_{12}=84.52	12876	2999	78947	30560	109507	1.084	-1.23	0.41
0	60	60	13	10.66	T_2=65.80	12975	2350	79996	30074	110070	2.43	-2.86	0.93
10		60	13	10.66	T_{12}=65.81	12975	2350	79996	30075	110071	2.43	-2.86	0.93
20		60	13	10.66	T_{12}=65.81	12975	2350	79997	30077	110074	2.43	-2.86	0.93
0	90	90	13	10.72	T_2=66.03	12882	2341	78870	32131	111001	0.98	3.78	1.78
10		90	13	10.72	T_{12}=66.03	12882	2341	78870	32133	111003	0.98	3.78	1.78
20		90	13	10.72	T_{12}=66.03	12882	2341	78870	32134	111005	0.98	3.78	1.78

Table 1. Optimal solution under different payment time

Table 2 suggests that under both an independent and coordinated policy, offer of trade credit to the buyer fallout in a lower retail price and hence, pushes up market demand and total joint profit. However, when the vendor and buyer work independently, irrespective of whether or not the vendor offers trade credit to the buyer, the retail price which maximizes the buyer's profit is much higher than that in a coordinated policy. This in turn reduces demand and hence the buyer's order quantity decreases for each subsequent order. This lowers profit of the vendor as well as the channel significantly. Therefore, the joint decision

Decision making	Credit terms	Payment time	*n*	*P*	*T* (days)	*R(P)*	*Q*	*nQ*	Profit		
									Buyer	Vendor	Integrated
Independent	Cash on Delivery	0	11	24.05	114.72	4696	1487	16357	89348	10539	99887
	Trade Credit (2|10 net 30)	10	11	23.89	112.47	4733	1460	16060	91296	10754	102050
Coordinated	Cash on Delivery	0	10	10.75	87.47	12843	3096	30960	78100	30961	109061
	Trade Credit (2|10 net 30)	10	10	10.66	85.04	12976	3041	30410	78706	30922	109628
Allocated									98075	11553	109628

Table 2. Optimal solution under different payment scenario

opted by the players of the supply chain can significantly improve the profit of the entire supply chain. From the vendor's end a joint decision is more advantageous than the independent decision. This is not true for the buyer. Therefore, to make the joint decision beneficial to the vendor and buyer both, Goyal (1976)'s method is implemented to enjoy long term partnership which benefits both the vendor and buyer.
We reallocate TP(n*, P*, T*) and obtained

$$\text{Buyer's profit} = TP\left(n^*, P^*, T^*\right) \times \frac{TBP\left(P_B^*, T_B^*\right)}{\left[TBP\left(P_B^*, T_B^*\right) + TVP\left(n_v^*\right)\right]}$$

$$= 109628 \times \frac{91296}{102050}$$

$$= 98075$$

and

$$\text{Vendor's profit} = TP\left(n^*, P^*, T^*\right) \times \frac{TVP\left(n_v^*\right)}{\left[TBP\left(P_B^*, T_B^*\right) + TVP\left(n_j^*\right)\right]}$$

$$= 109628 \times \frac{10754}{102050}$$

$$= 11553$$

The allocated results are listed at the bottom of Table 2.
Table 3 exhibits the benefits of a collaborative lot size credit policy. This shows that the profit increase of a joint decision is $ 9174 (= 10906 – 9987) for the "cash on delivery scenario and $ 7578(= 109628 – 102050) for the "2|10 net 30" scenario respectively. Under independent decision, offer of trade credit improves profit by 2.17% as compared to cash on delivery. The joint decision improves profit by 0.52%. The surplus capital generated for the supply chain by joint decision and trade credit policy is $ 9741 which is 8.93% increase in the profit. This concludes that the player can expect larger channel profit from the coordination and trade credit policy.

	Independent	Coordinated	Improvement
Cash on delivery	99887	109061	9174 (9.18%)
Trade credit (2|10, net 30)	102050	109628	7578 (7.43 %)
Improvement	2163 (2.17 %)	567 (0.52 %)	9741 (8.93 %)

Table 3. Improvement solution for coordinated system

Example 4 In this example, we compute the relative performances for various values of the model parameters. The values of ρ, (A_b/A_v) and (I_b/I_v) are varied. The other model parameters take values as given in Example 1. The offer of "2|10 net 30" by the vendor is consider. The optimal solutions and the integrated profit are exhibited in Table 4.
It is observed that increase in ρ, lowers the buyer cycle time and tempted to take advantage of a trade credit more frequently. The buyer's retail price decreases and integrated profit increases significantly. The number of shipments increases significantly.

A_b/A_v	I_b/I_v	n			P			T			Integrated Profit		
		ρ=0.1	ρ=0.5	ρ=0.9	ρ=0.1	ρ=0.5	ρ=0.9	ρ=0.1	ρ=0.5	ρ=0.9	ρ=0.1	ρ=0.5	ρ=0.9
0.01	1.0	31	48	133	10.64	10.59	10.43	10.00	8.89	8.15	110166	110758	111773
	1.5	31	49	139	10.66	10.60	10.43	9.93	8.69	7.82	110144	110735	111747
	2.0	32	49	141	10.66	10.60	10.43	9.61	8.62	7.65	110119	110718	111728
	2.5	33	50	143	10.66	10.61	10.44	9.29	8.43	7.51	110095	110696	111709
	3.0	34	51	144	10.66	10.61	10.46	9.00	8.23	7.40	110071	110676	111690
0.1	1.0	10	16	45	10.80	10.68	10.52	38.47	36.69	34.88	109257	109674	110576
	1.5	11	15	46	10.83	10.72	10.57	35.41	35.43	33.78	109112	109647	110492
	2.0	11	15	46	10.85	10.76	10.61	34.85	34.82	33.02	109043	109576	110424
	2.5	11	15	47	10.87	10.79	10.64	34.31	34.22	32.04	108976	109507	110346
	3.0	12	15	47	10.88	10.82	10.67	33.96	33.36	31.39	108851	109439	110282
0.5	1.0	4	6	16	10.99	10.92	10.81	112.49	103.20	100.57	108085	108262	108931
	1.5	5	6	17	11.04	10.96	10.87	96.10	95.84	95.17	107621	108071	108710
	2.0	5	6	17	11.07	10.99	10.90	93.19	92.02	91.55	107444	107887	108532
	2.5	5	6	17	11.12	11.04	10.94	90.67	89.08	88.41	107274	107710	108361
	3.0	5	6	18	11.15	11.07	10.96	88.24	86.29	84.49	107108	107539	108169

Table 4. Sensitivity analysis of optimal solution for changes in model parameters

Furthermore, the increase in the value of (A_b/A_v) (i.e. the relative ordering cost for the buyer increases), the number of shipment decreases, cycle time and retail price increases but integrated profit lowers down. When the relating holding cost rate (excluding interest charge) for the buyer increases, the buyer's cycle time will decrease and hence number of shipment increases marginally. The integrated profit decreases. See figures 2 – 19.

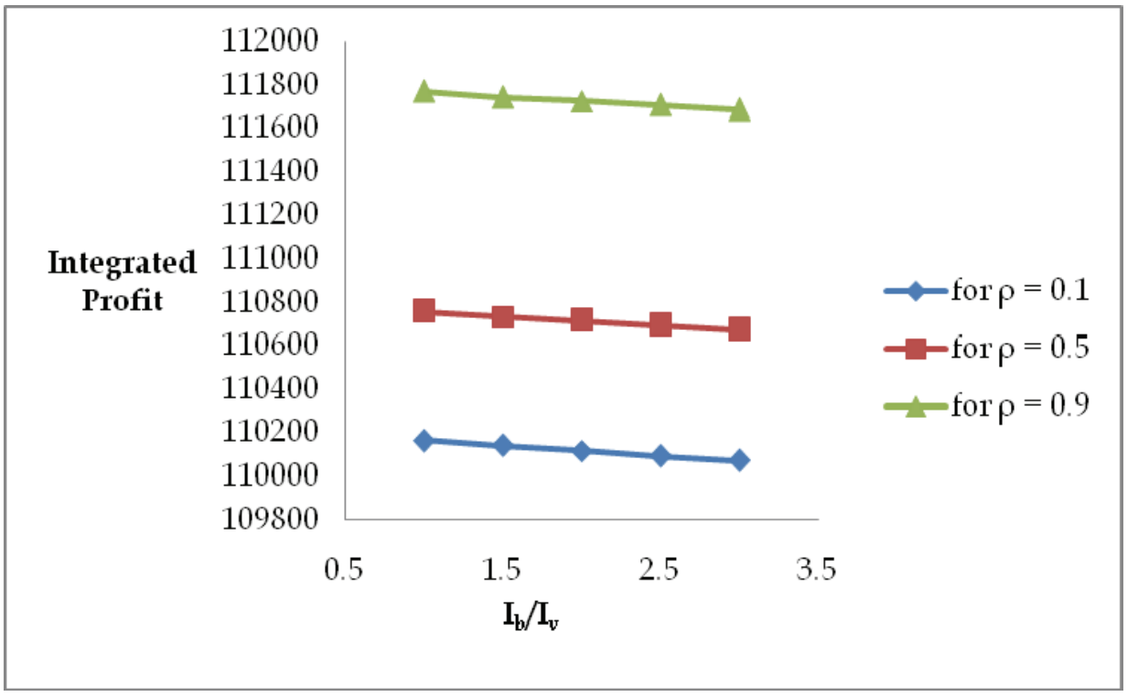

Fig. 2. Sensitivity analysis of Integrated Profit with respect to I_b/I_v for ρ.

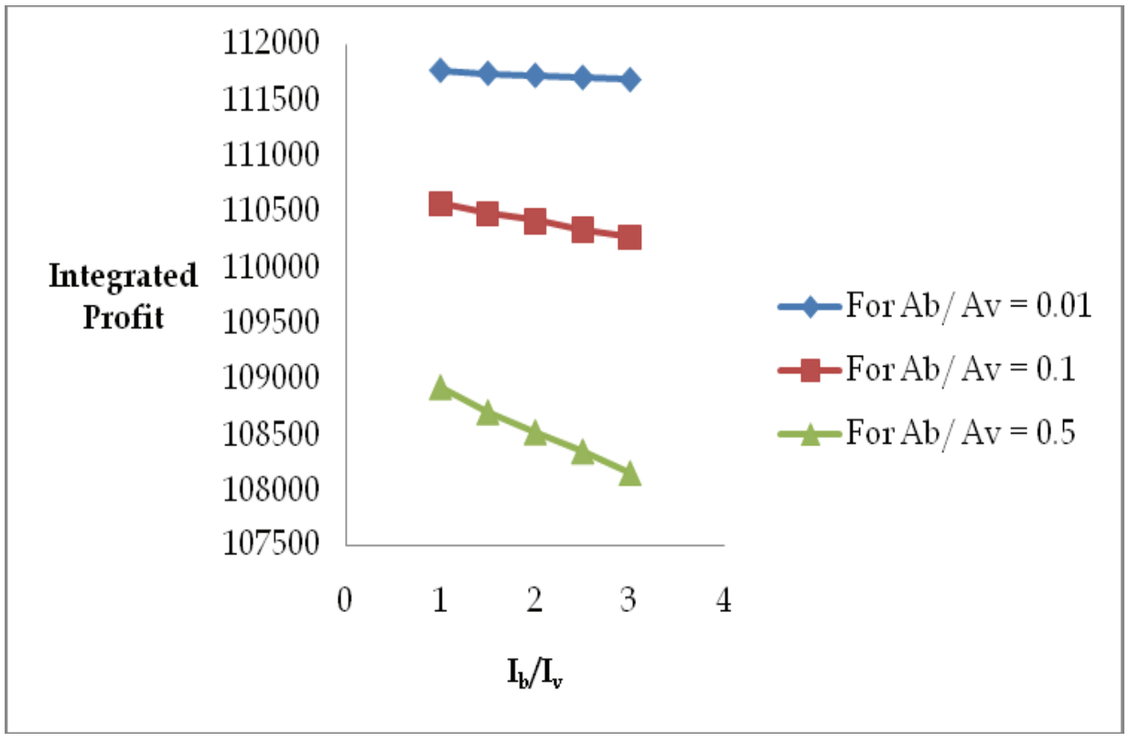

Fig. 3. Sensitivity analysis of Integrated Profit with respect to I_b/I_v for A_b/A_v.

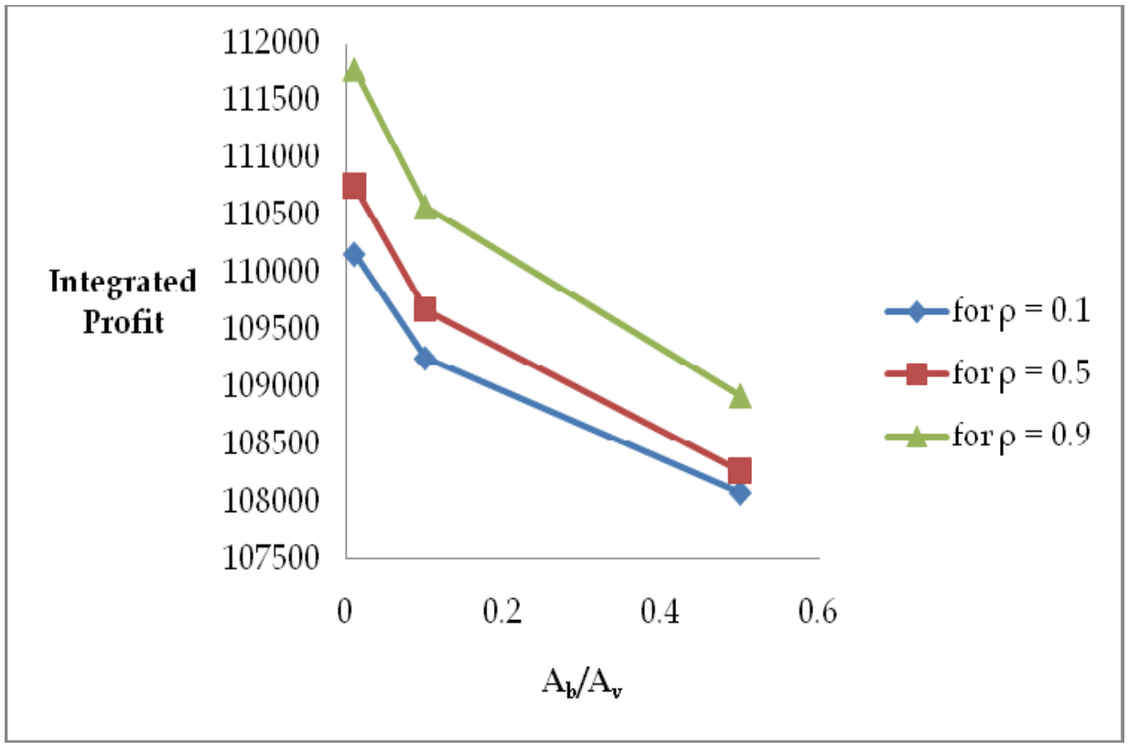

Fig. 4. Sensitivity analysis of Integrated Profit with respect to A_b/A_v for ρ.

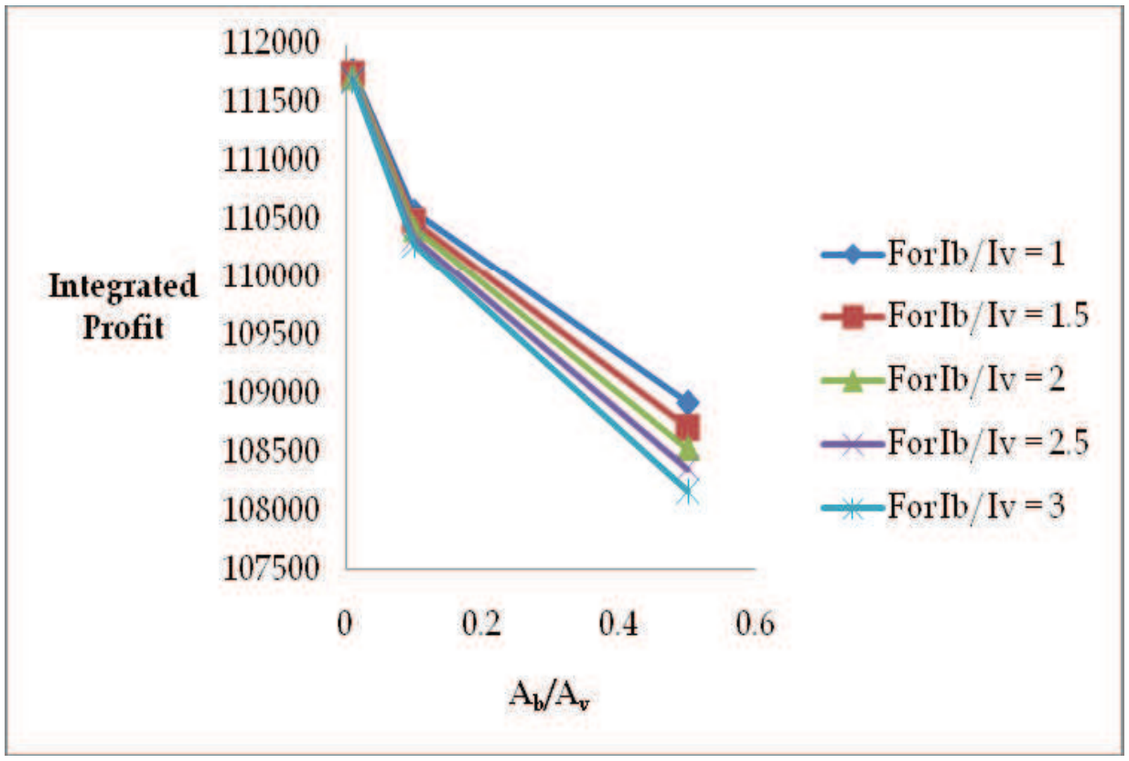

Fig. 5. Sensitivity analysis of Integrated Profit with respect to A_b/A_v for I_b/I_v.

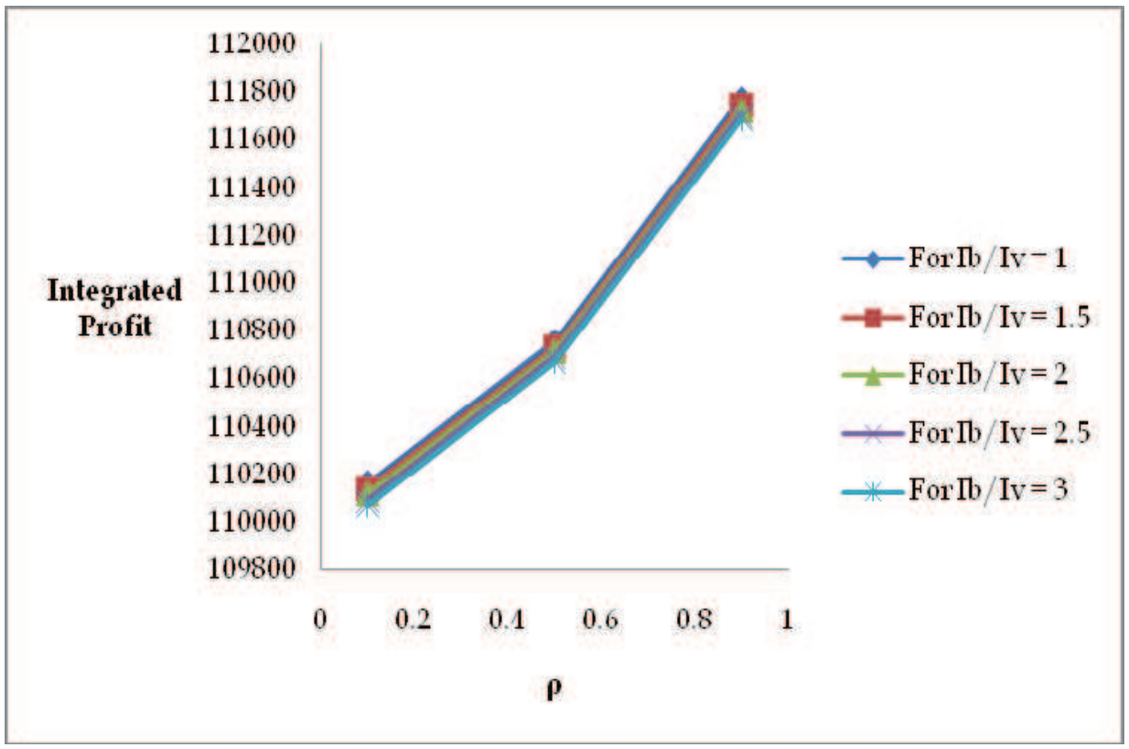

Fig. 6. Sensitivity analysis of Integrated Profit with respect to ρ for I_b/I_v.

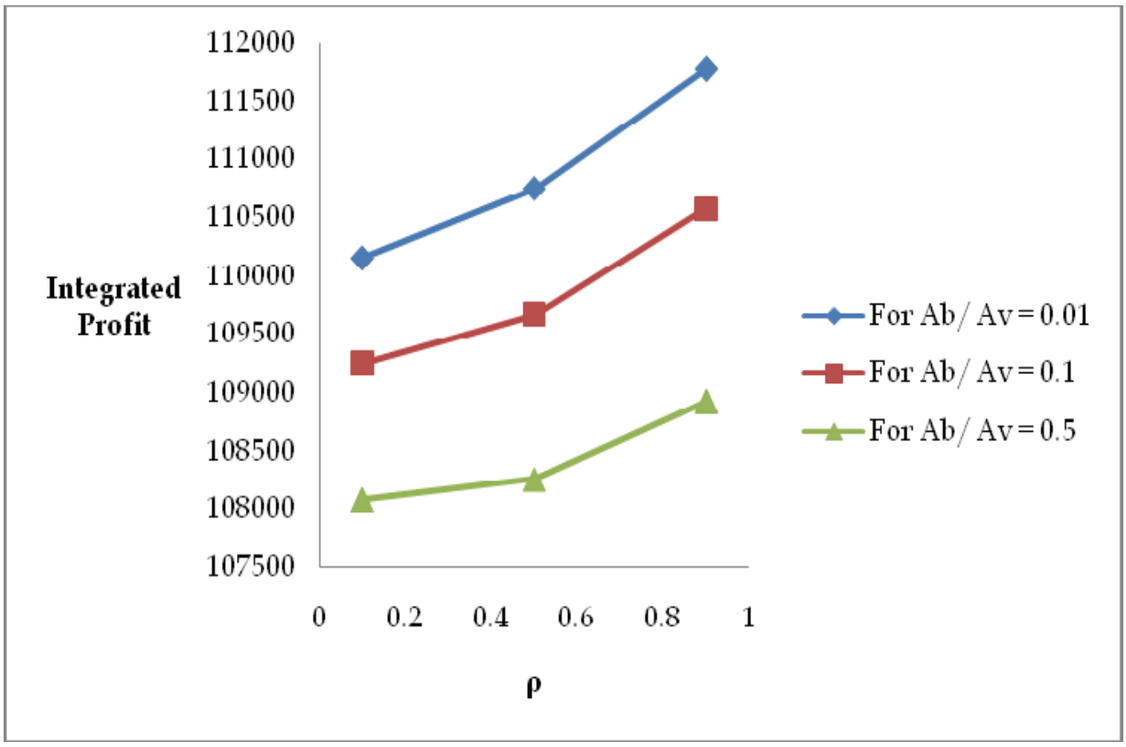

Fig. 7. Sensitivity analysis of Integrated Profit with respect to ρ for A_b/A_v.

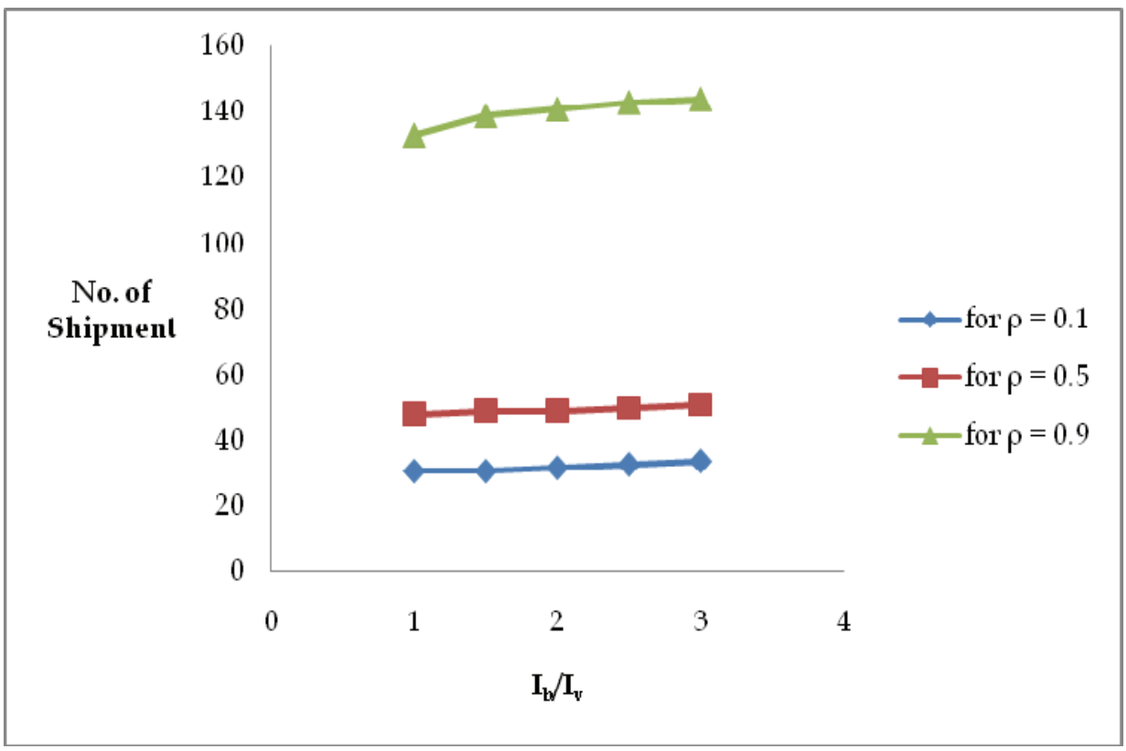

Fig. 8. Sensitivity analysis of Number of shipment with respect to I_b/I_v for ρ.

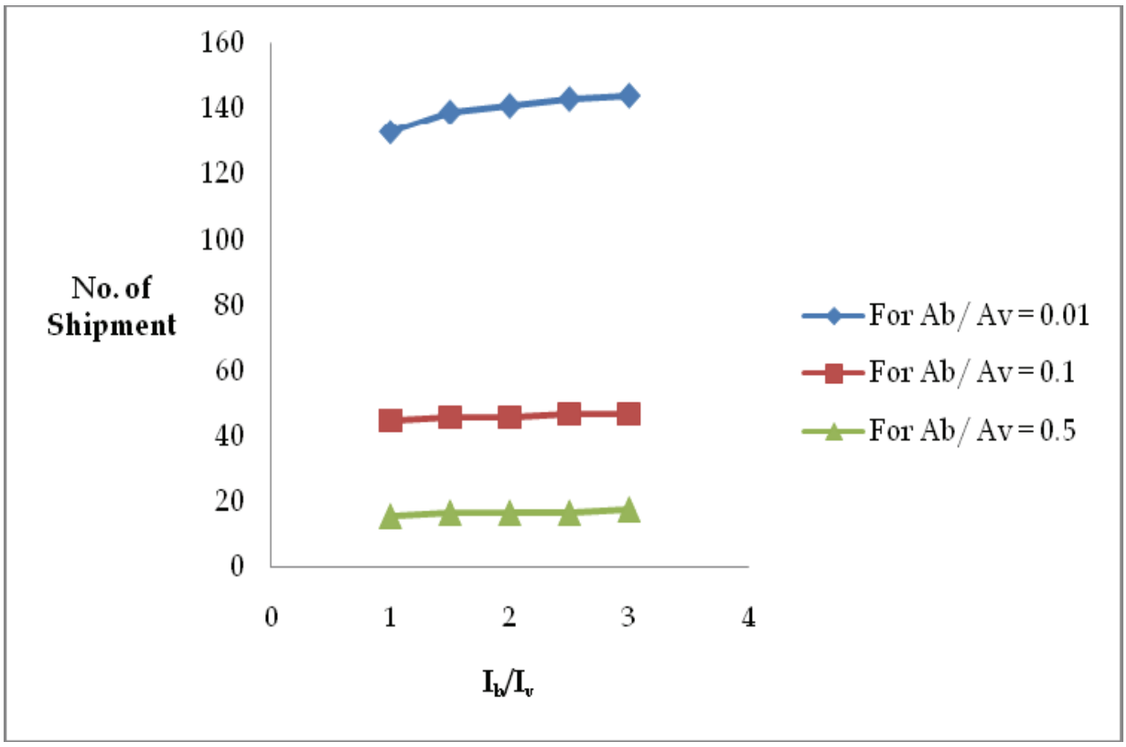

Fig. 9. Sensitivity analysis of Number of shipment with respect to I_b/I_v for A_b/A_v.

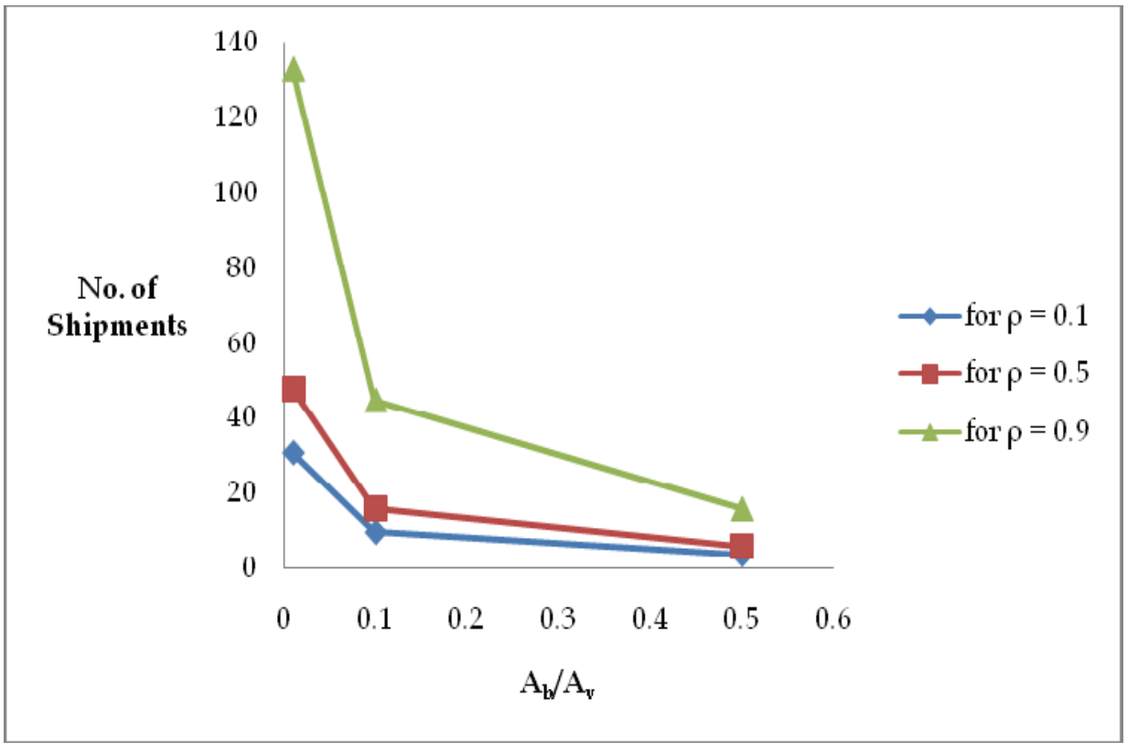

Fig. 10. Sensitivity analysis of Number of shipment with respect to A_b/A_v for ρ.

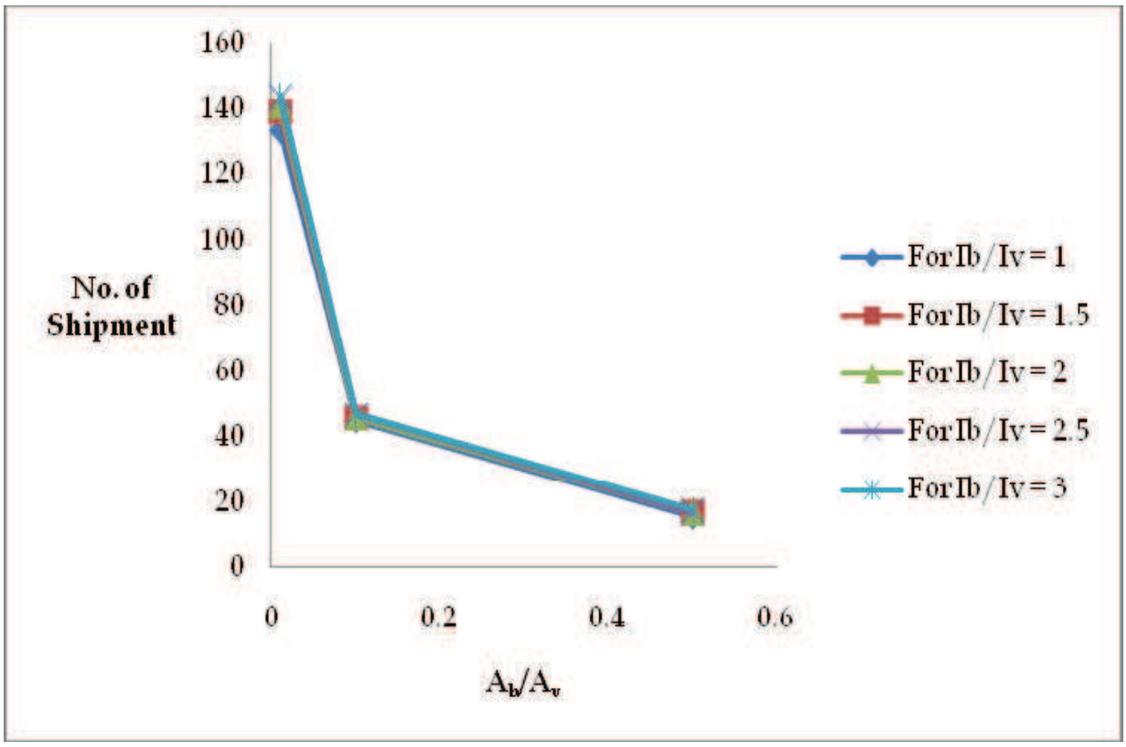

Fig. 11. Sensitivity analysis of Number of shipment with respect to A_b/A_v for I_b/I_v.

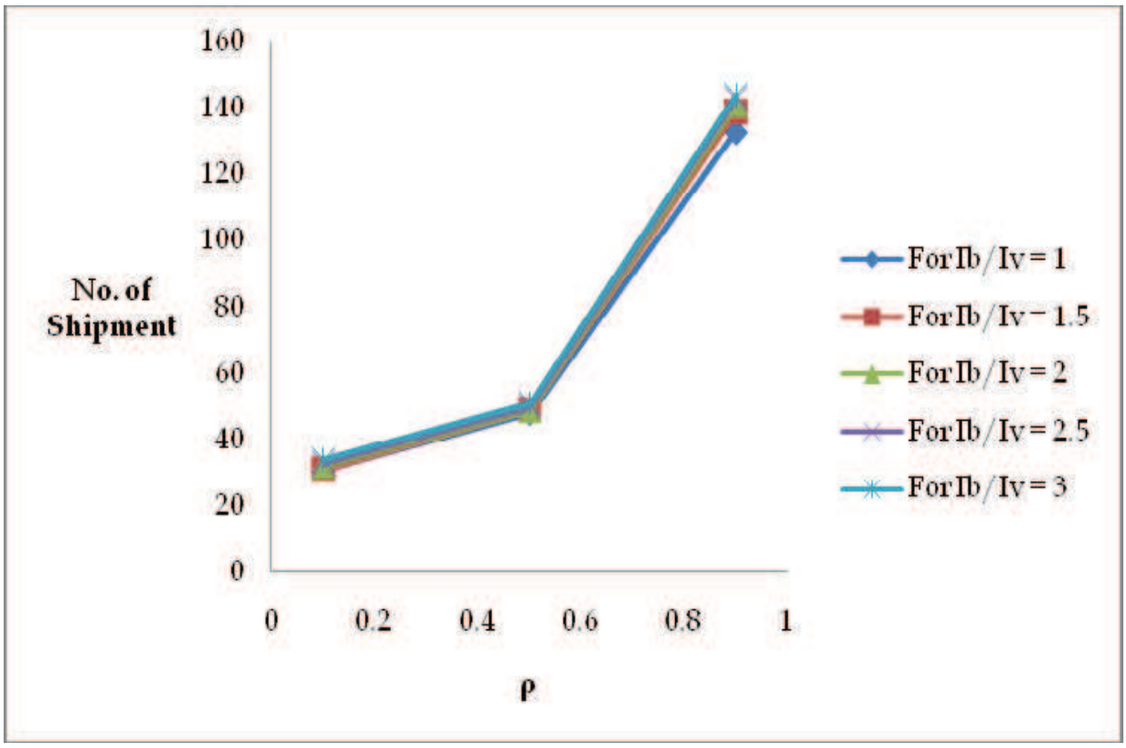

Fig. 12. Sensitivity analysis of Number of shipment with respect to ρ for I_b/I_v.

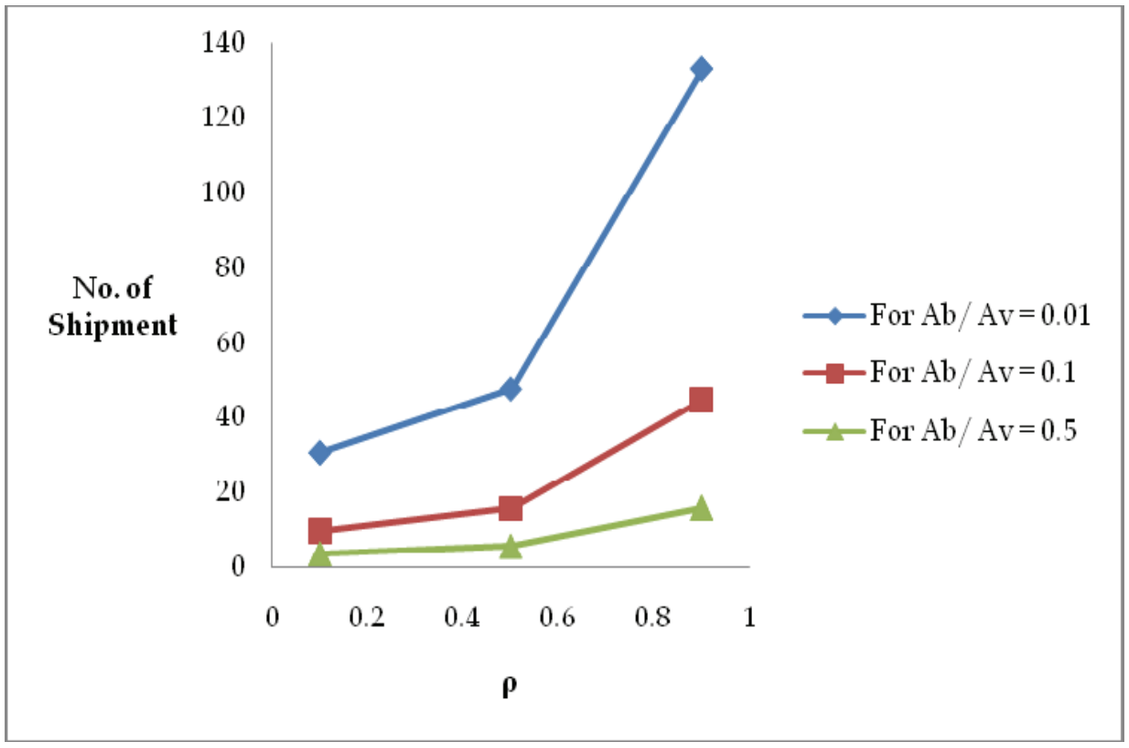

Fig. 13. Sensitivity analysis of Number of shipment with respect to ρ for A_b/A_v.

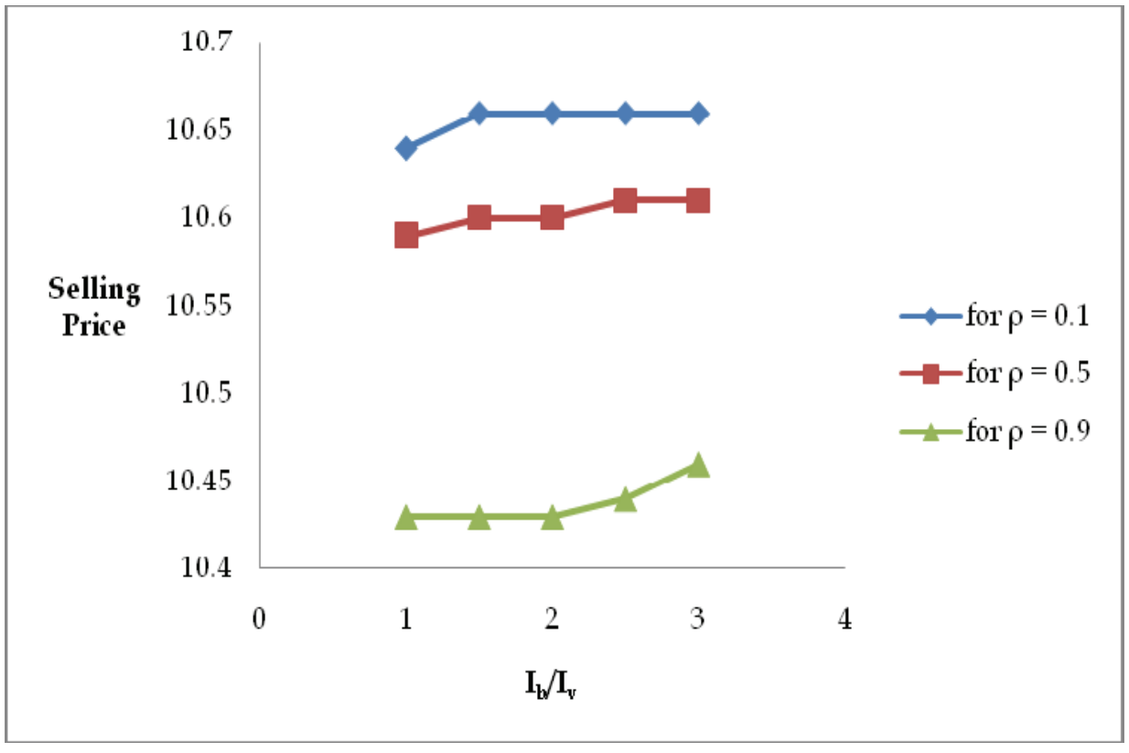

Fig. 14. Sensitivity analysis of Selling Price with respect to I_b/I_v for ρ.

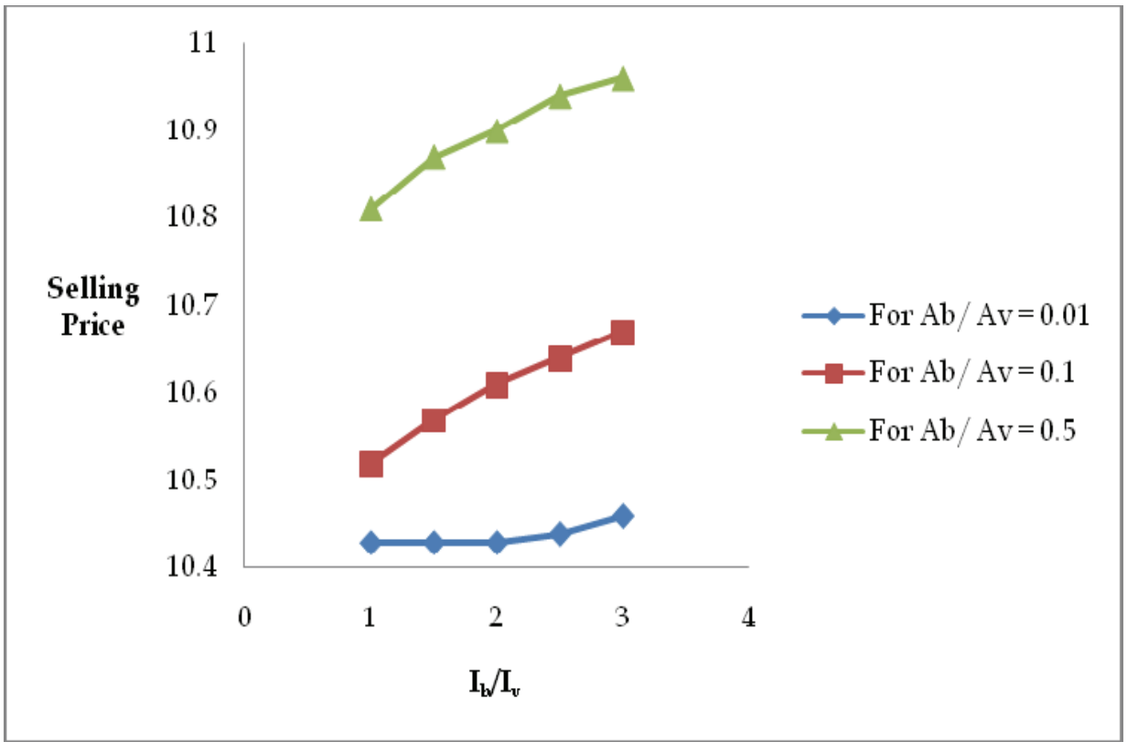

Fig. 15. Sensitivity analysis of Selling Price with respect to I_b/I_v for A_b/A_v.

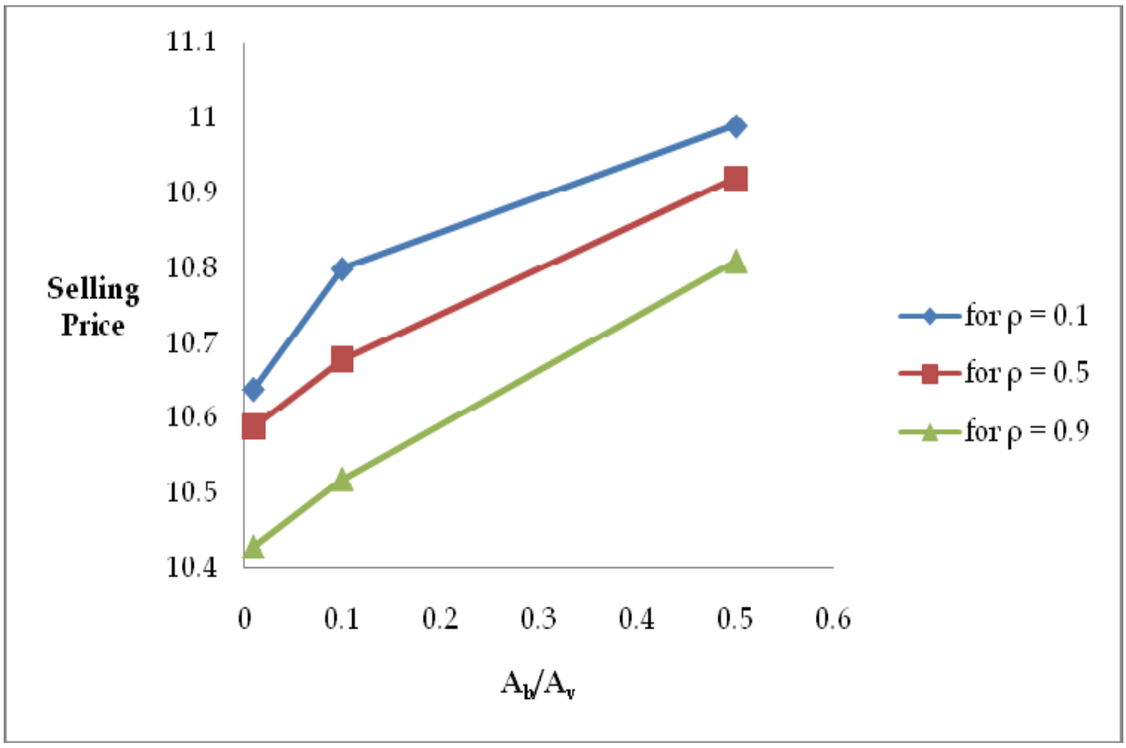

Fig. 16. Sensitivity analysis of Selling Price with respect to A_b/A_v for ρ.

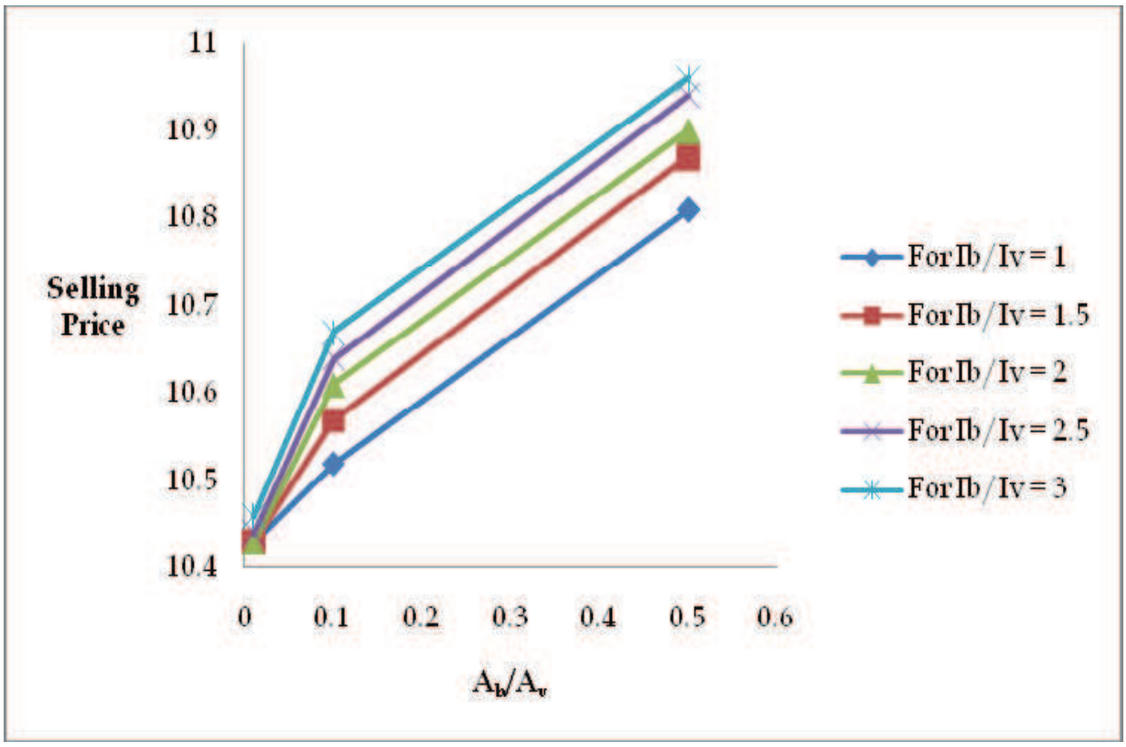

Fig. 17. Sensitivity analysis of Selling Price with respect to A_b/A_v for I_b/I_v.

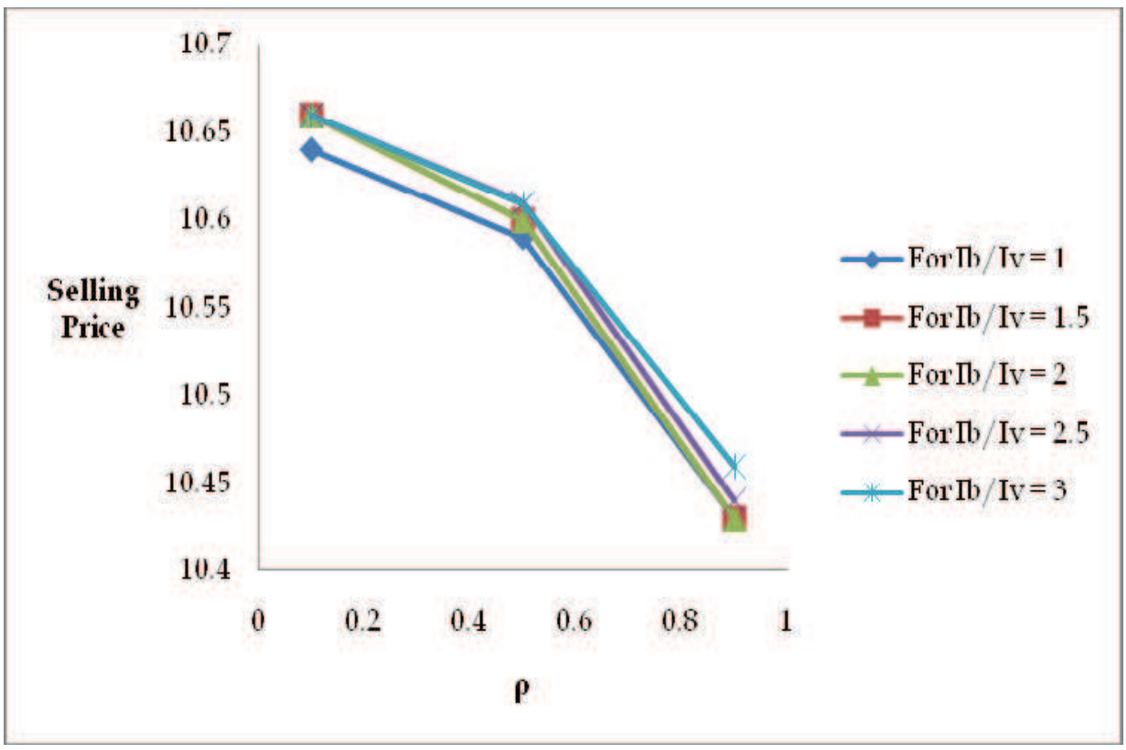

Fig. 18. Sensitivity analysis of Selling Price with respect to ρ for I_b/I_v.

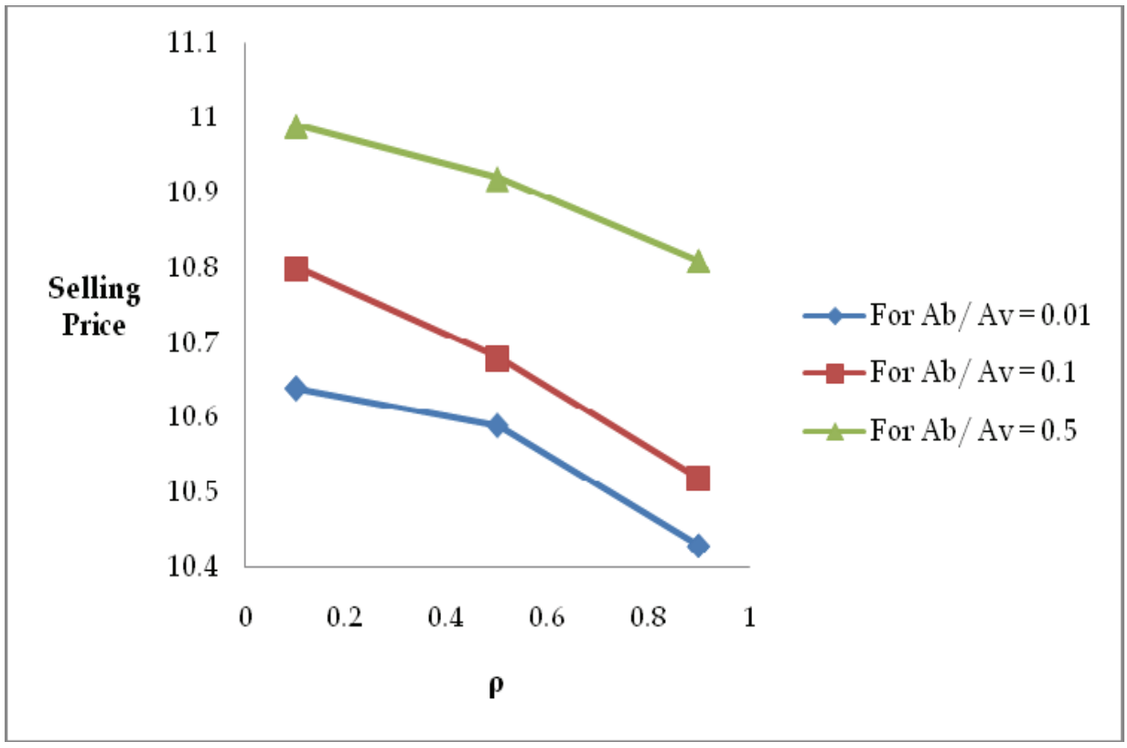

Fig. 19. Sensitivity analysis of Selling Price with respect to ρ for A_b/A_v.

6. Conclusions

In this chapter, a collaborative vendor – buyer inventory model is analyzed when the market demand is sensitive to the retail price, units in inventory deteriorate at a constant rate and the vendor offers two payment options namely trade credit and early – payments with discount in purchase price to the buyer. A solution procedure is constructed to compute the best payment option, the optimal retail price, cycle time, order quantity and the numbers of shipments per production run from the vendor to the buyer which maximizes the integrated profit. Numerical examples are given to validate the proposed model.

It is concluded that a two – part trade credit offer can increase profits of the buyer, vendor and the entire supply chain. It is observed that as the vendor and buyer take joint decision, the channel profit will increase significantly. Supply chain integration is useful in the vendor's profit gain and buyer's cash flow management. To entire buyer to opt for joint decision, the vendor should share additional profits.

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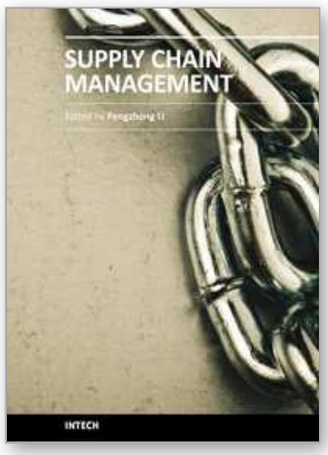

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The purpose of supply chain management is to make production system manage production process, improve customer satisfaction and reduce total work cost. With indubitable significance, supply chain management attracts extensive attention from businesses and academic scholars. Many important research findings and results had been achieved. Research work of supply chain management involves all activities and processes including planning, coordination, operation, control and optimization of the whole supply chain system. This book presents a collection of recent contributions of new methods and innovative ideas from the worldwide researchers. It is aimed at providing a helpful reference of new ideas, original results and practical experiences regarding this highly up-to-date field for researchers, scientists, engineers and students interested in supply chain management.

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