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# Distributed Fusion Prediction for Mixed Continuous-Discrete Linear Systems

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## 1. Introduction

The integration of information from a combination of different types of observed instruments (sensors) are often used in the design of high-accuracy control systems. Typical applications that benefit from this use of multiple sensors include industrial tasks, military commands, mobile robot navigation, multi-target tracking, and aircraft navigation (see (hall, 1992, Bar-Shalom, 1990, Bar-Shalom & Li, 1995, Zhu, 2002, Ren & Key, 1989) and references therein). One problem that arises from the use of multiple sensors is that if all local sensors observe the same target, the question then becomes how to effectively combine the corresponding local estimates. Several distributed fusion architectures have been discussed in (Alouani, 2005, Bar-Shalom & Campo, 1986, Bar-Shalom, 2006, Li et al., 2003, Berg & Durrant-Whyte, 1994, Hamshemipour et al., 1998) and algorithms for distributed estimation fusion have been developed in (Bar-Shalom & Campo, 1986, Chang et al., 1997, Chang et al, 2002, Deng et al., 2005, Sun, 2004, Zhou et al., 2006, Zhu et al., 1999, Zhu et al., 2001, Roecker & McGillem, 1998, Shin et al, 2006). To this end, the Bar-Shalom and Campo fusion formula (Bar-Shalom & Campo, 1986) for two-sensor systems has been generalized for an arbitrary number of sensors in (Deng et al., 2005, Sun, 2004, Shin et al., 2007) The formula represents an optimal mean-square linear combination of the local estimates with matrix weights. The analogous formula for weighting an arbitrary number of local estimates using scalar weights has been proposed in (Shin et al., 2007, Sun & Deng, 2005, Lee & Shin 2007). However, because of lack of prior information, in general, the distributed filtering using the fusion formula is globally suboptimal compared with optimal centralized filtering (Chang et al., 1997). Nevertheless, in this case it has advantages of lower computational requirements, efficient communication costs, parallel implementation, and fault-tolerance (Chang et al., 1997, Chang et al, 2002, Roecker & McGillem, 1998). Therefore, in spite of its limitations, the fusion formula has been widely used and is superior to the centralized filtering in real applications.

The aforementioned papers have not focused on prediction problem, but most of them have considered only distributed filtering in multisensory continuous and discrete dynamic models. Direct generalization of the distributed fusion filtering algorithms to the prediction problem is impossible. The distributed prediction requires special algorithms one of which for discrete-time systems was presented in (Song et al. 2009). In this paper, we generalize the results of (Song et al. 2009) on mixed continuous-discrete systems. The continuous-discrete

approach allows system to avoid discretization by propagating the estimate and error covariance between observations in continuous time using an integration routine such as Runge-Kutta. This approach yields the optimal or suboptimal estimate continuously at all times, including times between the data arrival instants. One advantage of the continuous-discrete filter over the alternative approach using system discretization is that in the former, it is not necessary for the sample times to be equally spaced. This means that the cases of irregular and intermittent measurements are easy to handle. In the absence of data the optimal prediction is given by performing only the time update portion of the algorithm.

Thus, the primary aim of this paper is to propose two distributed fusion predictors using fusion formula with matrix weights, and analysis their statistical properties and relationship between them. Then, through a comparison with an optimal centralized predictor, performance of the novel predictors is evaluated.

This chapter is organized as follows. In Section 2, we present the statement of the continuous-discrete prediction problem in a multisensor environment and give its optimal solution. In Section 3, we propose two fusion predictors, derived by using the fusion formula and establish the equivalence between them. Unbiased property of the fusion predictors is also proved. The performance of the proposed predictors is studied on examples in Section 4. Finally, concluding remarks are presented in Section 5.

## 2. Statement of problem – centralized predictor

We consider a linear system described by the stochastic differential equation

$$\dot{x}_t = F_t x_t + G_t v_t, \quad t \geq 0, \quad (1)$$

where  $x_t \in \mathbb{R}^n$  is the state,  $v_t \in \mathbb{R}^q$  is a zero-mean Gaussian white noise with covariance  $E(v_t v_s^T) = Q_t \delta(t-s)$ , and  $F_t \in \mathbb{R}^{n \times n}$ ,  $G_t \in \mathbb{R}^{n \times q}$ , and  $Q_t \in \mathbb{R}^{q \times q}$ .

Suppose that overall discrete observations  $Y_{t_k} \in \mathbb{R}^m$  at time instants  $t_1, t_2, \dots$  are composed of  $N$  observation subvectors (local sensors)  $y_{t_k}^{(1)}, \dots, y_{t_k}^{(N)}$ , i.e.,

$$Y_{t_k} = [y_{t_k}^{(1)T} \dots y_{t_k}^{(N)T}]^T, \quad (2)$$

where  $y_{t_k}^{(i)}, i=1, \dots, N$  are determined by the equations

$$\begin{aligned} y_{t_k}^{(1)} &= H_{t_k}^{(1)} x_{t_k} + w_{t_k}^{(1)}, \quad y_{t_k}^{(1)} \in \mathbb{R}^{m_1}, \\ &\vdots \\ y_{t_k}^{(N)} &= H_{t_k}^{(N)} x_{t_k} + w_{t_k}^{(N)}, \quad y_{t_k}^{(N)} \in \mathbb{R}^{m_N}, \\ k &= 1, 2, \dots; \quad t_{k+1} > t_k \geq t_0 = 0; \quad m = m_1 + \dots + m_N, \end{aligned} \quad (3)$$

where  $y_{t_k}^{(i)} \in \mathbb{R}^{m_i}$  is the local sensor observation,  $H_{t_k}^{(i)} \in \mathbb{R}^{n \times m_i}$ , and  $\{w_{t_k}^{(i)} \in \mathbb{R}^{m_i}, k = 1, 2, \dots\}$  are zero-mean white Gaussian sequences,  $w_{t_k}^{(i)} \sim \mathcal{N}(0, R_{t_k}^{(i)})$ ,  $i=1, \dots, N$ . The distribution of the initial state  $x_0$  is Gaussian,  $x_0 \sim \mathcal{N}(\bar{x}_0, P_0)$ , and  $x_0, v_t$ , and  $\{w_{t_k}^{(i)}\}, i = 1, \dots, N$  are assumed mutually uncorrelated.

A problem associated with such systems is to find the distributed weighted fusion predictor  $\hat{x}_{t+\Delta}$ ,  $\Delta \geq 0$  of the state  $x_{t+\Delta}$  based on overall current sensor observations

$$Y_{t_1}^{t_k} = \{Y_{t_1}, \dots, Y_{t_k}\}, \quad t_1 < \dots < t_k \leq t \leq t + \Delta, \quad \Delta \geq 0. \quad (4)$$

## 2.1 The optimal centralized predictor

The optimal centralized predictor is constructed by analogy with the continuous-discrete Kalman filter (Lewis, 1986, Gelb, 1974). In this case the prediction estimate  $\hat{x}_{t+\Delta}^{\text{opt}}$  and its error covariance  $P_{t+\Delta}^{\text{opt}}$  are determined by the combining of *time update* and *observation update*,

$$\begin{cases} \dot{\hat{x}}_s^{\text{opt}} = F_s \hat{x}_s^{\text{opt}}, & t_k \leq s \leq t + \Delta, \quad \hat{x}_{s=t_k}^{\text{opt}} = \hat{x}_{t_k}^{\text{opt}}, \\ \dot{P}_s^{\text{opt}} = F_s P_s^{\text{opt}} + P_s^{\text{opt}} F_s^T + \tilde{Q}_s, & P_{s=t_k}^{\text{opt}} = P_{t_k}^{\text{opt}}, \end{cases} \quad (5)$$

where the initial conditions represent filtering estimate of the state  $\hat{x}_{t_k}^{\text{opt}}$  and its error covariance  $P_{t_k}^{\text{opt}}$  which are given by the continuous-discrete Kalman filter equations (Lewis, 1986, Gelb, 1974):

*Time update between observations:*

$$\begin{cases} \dot{\hat{x}}_\tau^{\text{opt}} = F_\tau \hat{x}_\tau^{\text{opt}}, & t_{k-1} \leq \tau \leq t_k, \quad \hat{x}_{\tau=t_{k-1}}^{\text{opt}} = \hat{x}_{t_{k-1}}^{\text{opt}}, \\ \dot{P}_\tau^{\text{opt}} = F_\tau P_\tau^{\text{opt}} + P_\tau^{\text{opt}} F_\tau^T + \tilde{Q}_\tau, & P_{\tau=t_{k-1}}^{\text{opt}} = P_{t_{k-1}}^{\text{opt}}, \end{cases} \quad (6a)$$

*Observation update at time  $t_k$ :*

$$\begin{cases} \hat{x}_{t_k}^{\text{opt}} = \hat{x}_{t_k}^{\text{opt}} + L_{t_k}^{\text{opt}} (Y_{t_k} - H_{t_k} \hat{x}_{t_k}^{\text{opt}}), \\ L_{t_k}^{\text{opt}} = P_{t_k}^{\text{opt}} H_{t_k}^T (H_{t_k} P_{t_k}^{\text{opt}} H_{t_k}^T + R_{t_k})^{-1}, \\ P_{t_k}^{\text{opt}} = (I_n - L_{t_k}^{\text{opt}} H_{t_k}) P_{t_k}^{\text{opt}}. \end{cases} \quad (6b)$$

Here  $I_n$  is the  $n \times n$  identity matrix,  $\tilde{Q}_t = G_t Q_t G_t^T$ ,  $Y_{t_k}^T = [y_{t_k}^{(1)T} \dots y_{t_k}^{(N)T}]$ ,  $H_{t_k}^T = [H_{t_k}^{(1)T} \dots H_{t_k}^{(N)T}]$ ,  $R_{t_k} = \text{diag}[R_{t_k}^{(1)} \dots R_{t_k}^{(N)}]$ , and the matrices  $F_t$ ,  $G_t$ ,  $Q_t$  and  $R_{t_k}^{(i)}$  are defined in (1)-(3). Note that in the absence of observation  $Y_{t_k}$ , the centralized predictor includes two *time update* equations (5) and (6a), and in case of presence at time  $t=t_k$  the initial conditions  $\hat{x}_{t_k}^{\text{opt}}$  and  $P_{t_k}^{\text{opt}}$  for (5) computed by the *observation update* equations (6b).

Many advanced systems now make use of a large number of sensors in practical applications ranging from aerospace and defence, robotics automation systems, to the monitoring and control of process generation plants. Recent developments in integrated sensor network systems have further motivated the search for decentralized signal processing algorithms. An important practical problem in the above systems is to find a fusion estimate to combine the information from various local estimates to produce a global

(fusion) estimate. Moreover, there are several limitations for the centralized estimators in practical implementation, such as computational cost and capacity of data transmission. Also numerical errors of the centralized estimator design are drastically increased with dimension of the state  $x_t \in \mathbb{R}^n$  and overall observations  $Y_{t_k} \in \mathbb{R}^m$ . In these cases the centralized estimators may be impractical. In next Section, we propose two new fusion predictors for multisensor mixed continuous-discrete linear systems (1), (3).

### 3. Two distributed fusion predictors

The derivation of the fusion predictors is based on the assumption that the overall observation vector  $Y_{t_k}$  combines the local subvectors (individual sensors)  $y_{t_k}^{(1)}, \dots, y_{t_k}^{(N)}$ , which can be processed separately. According to (1) and (3), we have  $N$  unconnected dynamic subsystems ( $i = 1, \dots, N$ ) with the common state  $x_t$  and local sensor  $y_{t_k}^{(i)}$ :

$$\begin{aligned}\dot{x}_t &= F_t x_t + G_t v_t, \quad t \geq t_0, \\ y_{t_k}^{(i)} &= H_{t_k}^{(i)} x_{t_k} + w_{t_k}^{(i)}, \\ k &= 1, 2, \dots; \quad t_{k+1} > t_k \geq t_0 = 0,\end{aligned}\tag{7}$$

where  $i$  is the index of subsystem. Then by the analogy with the centralized prediction equations (5), (6) the optimal local predictor  $\hat{x}_{t+\Delta}^{(i)}$  based on the overall local observations  $\{y_{t_1}^{(i)}, \dots, y_{t_k}^{(i)}\}$ ,  $t_k \leq t \leq t + \Delta$  satisfies the following *time update and observation update* equations:

$$\begin{cases} \dot{\hat{x}}_s^{(i)} = F_s \hat{x}_s^{(i)}, \quad t_k \leq s \leq t + \Delta, \quad \hat{x}_{s=t_k}^{(i)} = \hat{x}_{t_k}^{(i)}, \\ \dot{P}_s^{(ii)} = F_s P_s^{(ii)} + P_s^{(ii)} F_s^T + \tilde{Q}_s, \quad P_{s=t_k}^{(ii)} = P_{t_k}^{(ii)}, \end{cases}\tag{8}$$

where the initial conditions  $\hat{x}_{t_k}^{(i)}$  and its error covariance  $P_{t_k}^{(ii)}$  are given by the continuous-discrete Kalman filter equations

*Time update between observations:*

$$\begin{cases} \dot{\hat{x}}_\tau^{(i)} = F_\tau \hat{x}_\tau^{(i)}, \quad t_{k-1} \leq \tau \leq t_k, \quad \hat{x}_{\tau=t_{k-1}}^{(i)} = \hat{x}_{t_{k-1}}^{(i)}, \\ \dot{P}_\tau^{(ii)} = F_\tau P_\tau^{(ii)} + P_\tau^{(ii)} F_\tau^T + \tilde{Q}_\tau, \quad P_{\tau=t_{k-1}}^{(ii)} = P_{t_{k-1}}^{(ii)}, \end{cases}\tag{9a}$$

*Observation update at time  $t_k$ :*

$$\begin{cases} \hat{x}_{t_k}^{(i)} = \hat{x}_{t_k}^{(i)} + L_{t_k}^{(i)} \left( y_{t_k}^{(i)} - H_{t_k}^{(i)} \hat{x}_{t_k}^{(i)} \right), \\ L_{t_k}^{(i)} = P_{t_k}^{(ii)} H_{t_k}^{(i)T} \left( H_{t_k}^{(i)} P_{t_k}^{(ii)} H_{t_k}^{(i)T} + R_{t_k}^{(i)} \right)^{-1}, \\ P_{t_k}^{(ii)} = \left( I_n - L_{t_k}^{(i)} H_{t_k}^{(i)} \right) P_{t_k}^{(ii)}. \end{cases}\tag{9b}$$

Thus from (8) we have  $N$  local filtering  $\hat{x}_t^{(i)} = \hat{x}_{s=t}^{(i)}$  and prediction  $\hat{x}_{t+\Delta}^{(i)} = \hat{x}_{s=t+\Delta}^{(i)}$  estimates, and corresponding error covariances  $P_t^{(ii)}$  and  $P_{t+\Delta}^{(ii)}$  for  $i=1, \dots, N$  and  $t \geq t_k$ . Using these values we propose two fusion prediction algorithms.

### 3.1 The fusion of local predictors (FLP Algorithm)

The fusion predictor  $\hat{x}_{t+\Delta}^{\text{FLP}}$  of the state  $x_{t+\Delta}$  based on the overall sensors (2), (3) is constructed from the local predictors  $\hat{x}_{t+\Delta}^{(i)}$ ,  $i = 1, \dots, N$  by using the fusion formula (Zhou et al., 2006, Shin et al., 2006):

$$x_{t+\Delta}^{\text{FLP}} = \sum_{i=1}^N a_{t+\Delta}^{(i)} \hat{x}_{t+\Delta}^{(i)}, \quad \sum_{i=1}^N a_{t+\Delta}^{(i)} = I_n, \quad (10)$$

where  $a_{t+\Delta}^{(1)}, \dots, a_{t+\Delta}^{(N)}$  are  $n \times n$  time-varying matrix weights determined from the mean-square criterion,

$$J_{t+\Delta}^{\text{FLP}} = E \left[ \left\| x_{t+\Delta} - \sum_{i=1}^N a_{t+\Delta}^{(i)} \hat{x}_{t+\Delta}^{(i)} \right\|^2 \right]. \quad (11)$$

The Theorems 1 and 2 completely define the fusion predictor  $\hat{x}_{t+\Delta}^{\text{FLP}}$  and its overall error covariance  $P_{t+\Delta}^{\text{FLP}} = \text{cov}(\tilde{x}_{t+\Delta}^{\text{FLP}}, \tilde{x}_{t+\Delta}^{\text{FLP}})$ ,  $\tilde{x}_{t+\Delta}^{\text{FLP}} = x_{t+\Delta} - \hat{x}_{t+\Delta}^{\text{FLP}}$ .

**Theorem 1:** Let  $\hat{x}_{t+\Delta}^{(1)}, \dots, \hat{x}_{t+\Delta}^{(N)}$  are the local predictors of an unknown state  $x_{t+\Delta}$ . Then

a. The weights  $a_{t+\Delta}^{(1)}, \dots, a_{t+\Delta}^{(N)}$  satisfy the linear algebraic equations

$$\sum_{i=1}^N a_{t+\Delta}^{(i)} [P_{t+\Delta}^{(ij)} - P_{t+\Delta}^{(iN)}] = 0, \quad \sum_{i=1}^N a_{t+\Delta}^{(i)} = I_n, \quad j=1, \dots, N-1; \quad (12)$$

b. The local covariance  $P_{t+\Delta}^{(ii)} = \text{cov}(\tilde{x}_{t+\Delta}^{(i)}, \tilde{x}_{t+\Delta}^{(i)})$ ,  $\tilde{x}_{t+\Delta}^{(i)} = x_{t+\Delta} - \hat{x}_{t+\Delta}^{(i)}$  satisfies (8) and local cross-covariance  $P_{t+\Delta}^{(ij)} = \text{cov}(\tilde{x}_{t+\Delta}^{(i)}, \tilde{x}_{t+\Delta}^{(j)})$ ,  $i \neq j$  describes the time update and observation update equations:

$$\begin{cases} \dot{P}_{\tau}^{(ij)} = F_{\tau} P_{\tau}^{(ij)} + P_{\tau}^{(ij)} F_{\tau}^T + \tilde{Q}_{\tau}, & P_{\tau=t_{k-1}}^{(ij)} = P_{t_{k-1}}^{(ij)}, \quad t_{k-1} \leq \tau \leq t_k, \\ P_{t_k}^{(ij)} = (I_n + L_{t_k}^{(i)} H_{t_k}^{(i)}) P_{t_k}^{(ij)} (I_n + L_{t_k}^{(j)} H_{t_k}^{(j)})^T, & t=t_k, \\ \dot{P}_s^{(ij)} = F_s P_s^{(ij)} + P_s^{(ij)} F_s^T + \tilde{Q}_s, & P_{s=t_k}^{(ij)} = P_{t_k}^{(ij)}, \quad t_k \leq s \leq t+\Delta; \end{cases} \quad (13)$$

c. The fusion error covariance  $P_{t+\Delta}^{\text{FLP}}$  is given by

$$P_{t+\Delta}^{\text{FLP}} = \sum_{i,j=1}^N a_{t+\Delta}^{(i)} P_{t+\Delta}^{(ij)} a_{t+\Delta}^{(j)T}. \quad (14)$$

**Theorem 2:** The local predictors  $\hat{x}_{t+\Delta}^{(1)}, \dots, \hat{x}_{t+\Delta}^{(N)}$  and fusion predictor  $\hat{x}_{t+\Delta}^{\text{FLP}}$  are unbiased, i.e.,  $E(\hat{x}_{\tau}^{(i)}) = E(x_{\tau})$  and  $E(\hat{x}_{t+\Delta}^{\text{FLP}}) = E(x_{t+\Delta})$  for  $0 \leq \tau \leq t+\Delta$ .

The proofs of Theorems 1 and 2 are given in Appendix.

Thus the local predictors (8) and fusion equations (10)-(14) completely define the FLP algorithm. In particular case at  $N=2$ , formulas (10)-(12) reduce to the Bar-Shalom and Campo formulas (Bar-Shalom & Campo, 1986):

$$\begin{aligned}
\hat{x}_{t+\Delta}^{\text{FLP}} &= a_{t+\Delta}^{(1)} \hat{x}_{t+\Delta}^{(1)} + a_{t+\Delta}^{(2)} \hat{x}_{t+\Delta}^{(2)}, \\
a_{t+\Delta}^{(1)} &= \left[ P_{t+\Delta}^{(22)} - P_{t+\Delta}^{(21)} \right] \left[ P_{t+\Delta}^{(11)} + P_{t+\Delta}^{(22)} - P_{t+\Delta}^{(12)} - P_{t+\Delta}^{(21)} \right]^{-1}, \\
a_{t+\Delta}^{(2)} &= \left[ P_{t+\Delta}^{(11)} - P_{t+\Delta}^{(12)} \right] \left[ P_{t+\Delta}^{(11)} + P_{t+\Delta}^{(22)} - P_{t+\Delta}^{(12)} - P_{t+\Delta}^{(21)} \right]^{-1}.
\end{aligned} \tag{15}$$

Further, in parallel with the FLP we offer the other algorithm for fusion prediction.

### 3.2 The prediction of fusion filter (PFF Algorithm)

This algorithm consists of two parts. The first part fuses the local filtering estimates  $\hat{x}_{t_k}^{(1)}, \dots, \hat{x}_{t_k}^{(N)}$ . Using the fusion formula, we obtain the fusion filtering (FF) estimate

$$\hat{x}_{t_k}^{\text{FF}} = \sum_{i=1}^N b_{t_k}^{(i)} \hat{x}_{t_k}^{(i)}, \quad \sum_{i=1}^N b_{t_k}^{(i)} = I_n, \tag{16}$$

where the weights  $b_{t_k}^{(1)}, \dots, b_{t_k}^{(N)}$  do not depend on lead  $\Delta$ .

In the second part we predict the fusion filtering estimate  $\hat{x}_{t_k}^{\text{FF}}$  using the *time update* prediction equations. Then the fusion predictor  $\hat{x}_{t+\Delta}^{\text{PFF}}$  and its error covariance  $P_{t+\Delta}^{\text{PFF}} = \text{cov}(\tilde{x}_{t+\Delta}^{\text{PFF}}, \tilde{x}_{t+\Delta}^{\text{PFF}})$ ,  $\tilde{x}_{t+\Delta}^{\text{PFF}} = x_{t+\Delta} - \hat{x}_{t+\Delta}^{\text{PFF}}$  satisfy the following equations:

$$\begin{cases} \dot{\hat{x}}_s^{\text{PFF}} = F_s \hat{x}_s^{\text{PFF}}, & t_k \leq s \leq t+\Delta, \quad \hat{x}_{s=t_k}^{\text{PFF}} = \hat{x}_{t_k}^{\text{FF}}, \\ \dot{P}_s^{\text{PFF}} = F_s P_s^{\text{PFF}} + P_s^{\text{PFF}} F_s^T + \tilde{Q}_s, & P_{s=t_k}^{\text{PFF}} = P_{t_k}^{\text{FF}}. \end{cases} \tag{17}$$

Next Theorem completely defines the PFF algorithm.

**Theorem 3:** Let  $\hat{x}_{t_k}^{(1)}, \dots, \hat{x}_{t_k}^{(N)}$  are the local filtering estimates of an unknown state  $x_t$ . Then

a. The weights  $b_{t_k}^{(1)}, \dots, b_{t_k}^{(N)}$  satisfy the linear algebraic equations

$$\sum_{i=1}^N b_{t_k}^{(i)} \left[ P_{t_k}^{(ij)} - P_{t_k}^{(iN)} \right] = 0, \quad \sum_{i=1}^N b_{t_k}^{(i)} = I_n, \quad j = 1, \dots, N-1; \tag{18}$$

b. The local covariance  $P_{t_k}^{(ii)}$  and cross-covariance  $P_{t_k}^{(ij)}$  in (18) are determined by equations (9) and (13), respectively;

c. The initial conditions  $\hat{x}_{t_k}^{\text{FF}}$  and  $P_{t_k}^{\text{FF}}$  in (17) are determined by (16) and formula

$$P_{t_k}^{\text{FF}} = \sum_{i,j=1}^N b_{t_k}^{(i)} P_{t_k}^{(ij)} b_{t_k}^{(j)T}, \tag{19}$$

respectively;

d. The fusion predictor  $\hat{x}_{t+\Delta}^{\text{PFF}}$  in (17) is unbiased, i.e.,  $E(\hat{x}_{t+\Delta}^{\text{PFF}}) = E(x_{t+\Delta})$ .

The proof of Theorem 3 is given in Appendix.

### 3.3 The relationship between FLP and PFF

Here we establish the relationship between the prediction fusion estimates  $\hat{x}_{t+\Delta}^{\text{FLP}}$  and  $\hat{x}_{t+\Delta}^{\text{PFF}}$  determined by (10) and (16), respectively.



**Theorem 4:** Let  $\hat{x}_{t+\Delta}^{\text{FLP}}$  and  $\hat{x}_{t+\Delta}^{\text{PFF}}$  be the fusion prediction estimates determined by (10) and (16), respectively, and the local error covariances  $P_s^{(ij)}$ ,  $t_k \leq s \leq t+\Delta$ ,  $i, j = 1, \dots, N$  are nonsingular. Then

$$\hat{x}_{t+\Delta}^{\text{FLP}} = \hat{x}_{t+\Delta}^{\text{PFF}} \quad \text{for } \Delta > 0. \quad (20)$$

The proof of Theorem 4 is given in Appendix.

**Remark 1 (Uniqueness solution):** When the local prediction covariances  $P_{t+\Delta}^{(ij)}$ ,  $i, j = 1, \dots, N$  are nonsingular, the quadratic optimization problem (11) has a unique solution, and the weights  $a_{t+\Delta}^{(1)}, \dots, a_{t+\Delta}^{(N)}$  are defined by the expressions (11). The same result is true for the covariance  $P_{t_k}^{(ij)}$  and the weights  $b_{t_k}^{(1)}, \dots, b_{t_k}^{(N)}$  (Zhu et al., 1999, Zhu, 2002).

**Remark 2 (Computational complexity):** According to Theorem 4, both the predictors FLP and PFF are equivalent; however, from a computational point of view they are different. To predict the state  $x_{t+\Delta}$  using FLP we need to compute the matrix weights  $a_{t+\Delta}^{(1)}, \dots, a_{t+\Delta}^{(N)}$  for each lead  $\Delta > 0$ . This contrasts with PFF, wherein the weights  $b_{t_k}^{(1)}, \dots, b_{t_k}^{(N)}$  are computed only once, since they do not depend on the leads  $\Delta$ . Therefore, FLP is deemed more complex than PFF, especially for large leads.

**Remark 3 (Real-time implementation):** We may note that the local filter gains  $L_t^{(i)}$ , the error cross-covariances  $P_t^{(ij)}$ ,  $P_{t+\Delta}^{(ij)}$ , and the weights  $a_{t+\Delta}^{(i)}$ ,  $b_{t_k}^{(i)}$  may be pre-computed, since they do not depend on the current observations  $y_{t_k}^{(i)}$ ,  $i = 1, \dots, N$ , but only on the noises statistics  $Q_t$  and  $R_t^{(i)}$ , and system matrices  $F_t$ ,  $G_t$ ,  $H_t^{(i)}$ , which are part of the system model (1), (3). Thus, once the observation schedule has been settled, the real-time implementation of the fusion predictors FLP and PFF requires only the computation of the local estimates  $\hat{x}_t^{(i)}$ ,  $\hat{x}_{t+\Delta}^{(i)}$ ,  $i = 1, \dots, N$  and final fusion predictors  $\hat{x}_{t+\Delta}^{\text{FLP}}$  and  $\hat{x}_{t+\Delta}^{\text{PFF}}$ .

**Remark 4 (Parallel implementation):** The local estimates  $\hat{x}_t^{(i)}$ ,  $\hat{x}_{t+\Delta}^{(i)}$ ,  $i = 1, \dots, N$  are separated for different sensors. Therefore, they can be implemented in parallel for various types of observations  $y_t^{(i)}$ ,  $i = 1, \dots, N$ .

## 4. Examples

### 4.1 The damper harmonic oscillator motion

System model of the harmonic oscillator is considered in (Lewis, 1986). We have

$$\dot{x}_t = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\alpha \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t, \quad 0 \leq t \leq t^*, \quad (21)$$

where  $x_t = [x_{1,t} \ x_{2,t}]^T$ , and  $x_{1,t}$  is position,  $x_{2,t}$  is velocity, and  $v_t$  is zero-mean white Gaussian noise with intensity  $q$ ,  $E(v_t v_s) = q \delta_{t-s}$ ,  $x_0 \sim N(\bar{x}_0, P_0)$ . Assume that the observation system contains  $N$  sensors which are observing the position  $x_{1,t}$ . Then we have

$$\begin{aligned} y_{t_k}^{(1)} &= H_{t_k}^{(1)} x_{t_k} + w_{t_k}^{(1)}, \\ &\vdots \\ y_{t_k}^{(N)} &= H_{t_k}^{(N)} x_{t_k} + w_{t_k}^{(N)}, \quad 0 = t_0 < t_1 < t_2 < \dots < t^*, \end{aligned} \quad (22)$$

where  $H_{t_k}^{(j)} = [1 \ 0]$ , and  $w_{t_k}^{(j)}$ ,  $j = 1, \dots, N$  are uncorrelated zero-mean white Gaussian noises with constant variances  $r^{(j)}$ , respectively.



For model (21), (22), three predictors are applied: centralized predictor (CP) in (5), (6), FLP in (10) and PFF in (16), (17). The performance comparison of the fusion predictors for  $N=2,3$  was expressed in the terms of computation load (CPU time  $T_{CPU}$ ) and MSEs,  $P_{i,t+\Delta} = E(x_{i,t+\Delta} - \hat{x}_{i,t+\Delta})^2$ , where  $\hat{x}_{i,t+\Delta} = \hat{x}_{i,t+\Delta}^{CP}, \hat{x}_{i,t+\Delta}^{FLP}$  or  $\hat{x}_{i,t+\Delta}^{PFF}$ ,  $i=1,2$ . The model parameters, noise statistics, initial conditions, and lead are taken to

$$\begin{aligned} \omega_n^2 &= 3, \quad \alpha = 2.5, \quad t^* = 3, \quad q = 5, \quad r^{(1)} = 3.0, \quad r^{(2)} = 2.0, \quad r^{(3)} = 1.0, \\ \bar{x}_0 &= [10.0 \quad 0.0]^T, \quad P_0 = \text{diag}[0.5 \quad 0.5], \\ \Delta &= 0.1 \sim 0.5 \text{ (sec)}, \quad t_k - t_{k-1} = 0.1. \end{aligned} \quad (23)$$

Figs. 1 and 2 illustrate the MSEs for position ( $x_1$ ),  $P_{1,t+\Delta}^{CP}, P_{1,t+\Delta}^{FLP}, P_{1,t+\Delta}^{PFF}$ , and analogously for velocity ( $x_2$ ),  $P_{2,t+\Delta}^{CP}, P_{2,t+\Delta}^{FLP}, P_{2,t+\Delta}^{PFF}$  at  $N=2,3$  and lead  $\Delta=0.2$ . The analysis of results in Figs. 1 and 2 show that the fusion predictors FLP and PFF have the same accuracy, i.e.,  $P_{i,t+\Delta}^{FLP} = P_{i,t+\Delta}^{PFF}$ , and the MSEs of each predictor are reduced from  $N=2$  to  $N=3$ . The usage of three sensors allows to increase the accuracy of fusion predictors compared with the optimal CP for two sensors, i.e.,  $P_{i,t+\Delta}^{FLP}(N=3) = P_{i,t+\Delta}^{PFF}(N=3) < P_{i,t+\Delta}^{CP}(N=2)$ . Moreover the differences between optimal  $P_{i,t+\Delta}^{CP}$  and fusion MSEs  $P_{i,t+\Delta}^{FLP}, P_{i,t+\Delta}^{PFF}$  are small, especially for steady-state regime. The results of numerical experiments on an Intel® Core 2 Duo with 2.6GHz CPU and 3G RAM are reported. The CPU time for CP, FLP, and PFF are represented in Table 1. We find that although  $P_{i,t+\Delta}^{FLP}$  and  $P_{i,t+\Delta}^{PFF}$  are equal (see Theorem 4), the CPU time  $T_{CPU}^{PFF}$  for evaluation of the prediction  $\hat{x}_{i,t+\Delta}^{PFF}$  is 4~5 times less than  $T_{CPU}^{FLP}$  for  $\hat{x}_{i,t+\Delta}^{FLP}$  ( $T_{CPU}^{PFF} < T_{CPU}^{FLP}$ ) and this difference tends to increase with increasing the dimension of the state  $n$  or the number of sensors  $N$ . This is due to the fact that the PFF's weights  $b_{t_k}^{(i)}$  do not depend on the leads  $\Delta$  in contrast to the FLP's weights  $a_{t+\Delta}^{(i)}$ . Also, since CPU time difference between CP and PFF is negligible, PFF algorithm prefer to implement in real application rather than CP, especially for distributed system or sensor network.

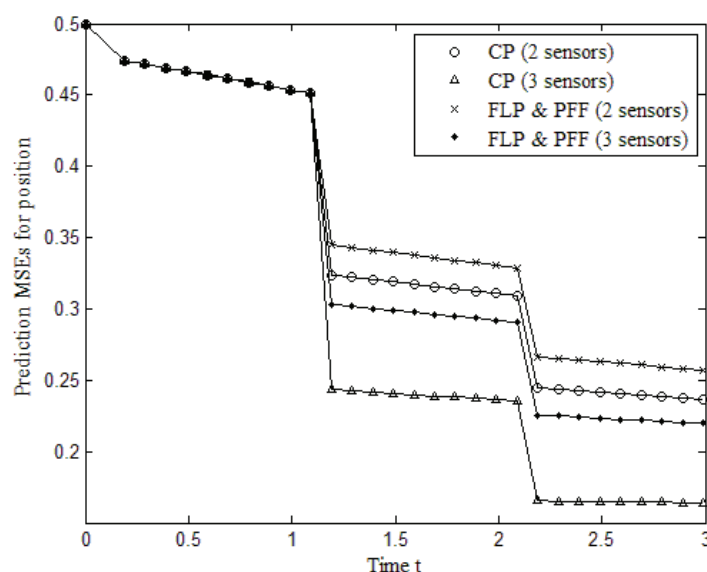


Fig. 1. Position MSE comparison of three predictors at  $N=2,3$  and lead  $\Delta=0.2$ .

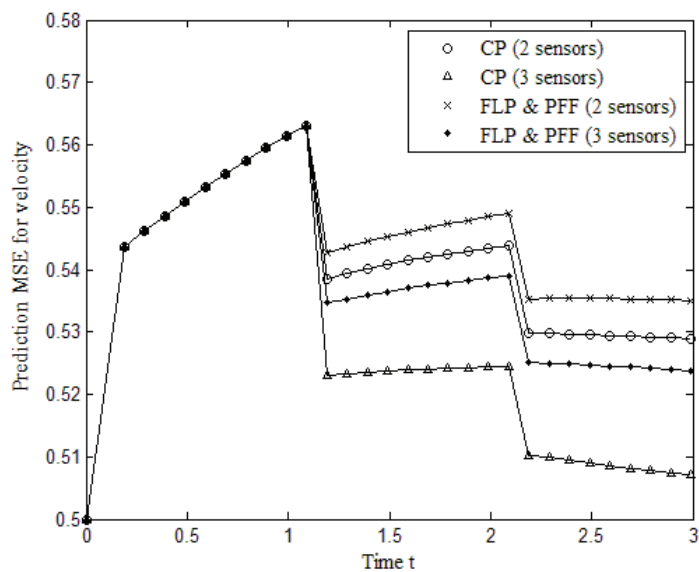


Fig. 2. Velocity MSE comparison of three predictors at N = 2,3 and lead Δ =0.2.

Number of sensors	Lead Δ (sec)	CPU time (sec)		
		$T_{CPU}^{CP}$	$T_{CPU}^{FLP}$	$T_{CPU}^{PFF}$
N = 2	0.1	0.172	0.826	0.185
	0.2	0.298	1.475	0.310
	0.3	0.384	1.863	0.405
	0.4	0.500	2.552	0.550
	0.5	0.656	3.137	0.691
N = 3	0.1	0.187	1.024	0.200
	0.2	0.305	1.743	0.340
	0.3	0.452	2.454	0.471
	0.4	0.602	3.306	0.621
	0.5	0.754	4.203	0.776

Table 1. Comparison of CPU time at N = 2,3 and Δ = 0.1 ~ 0.5

4.2 The water tank mixing system

Consider the water tank system which accepts two types of different temperature of the water and throw off the mixed water simultaneously (Jannerup & Hendricks, 2006). This system is described by

$$\dot{x}_t = \begin{bmatrix} -0.0139 & 0 & 0 \\ 0 & -0.0277 & 0 \\ 0 & 0.1667 & -0.1667 \end{bmatrix} x_t + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} v_t, \quad t \geq 0,$$

(24)

where  $x_t = [x_{1,t} \ x_{2,t} \ x_{3,t}]^T$  and  $x_{1,t}$  is water level,  $x_{2,t}$  is water temperature,  $x_{3,t}$  is sensor temperature, and  $v_t$  is a white Gaussian noise with intensity  $q$ ,  $E(v_t v_s) = q \delta_{t-s}$ ,  $x_0 \sim N(\bar{x}_0, P_0)$ . The measurement model contains two sensors (N = 2) which sense water level. Then we have

$$y_{t_k}^{(i)} = [1 \ 0 \ 0]x_{t_k} + w_{t_k}^{(i)}, \quad 0=t_0 < t_1 < t_2 < \dots < t^*, \quad i=1,2, \quad (25)$$

where  $w_{t_k}^{(1)}$  and  $w_{t_k}^{(2)}$  are uncorrelated white Gaussian sequences with zero-mean and constant intensities  $r^{(1)}$  and  $r^{(2)}$ , respectively.

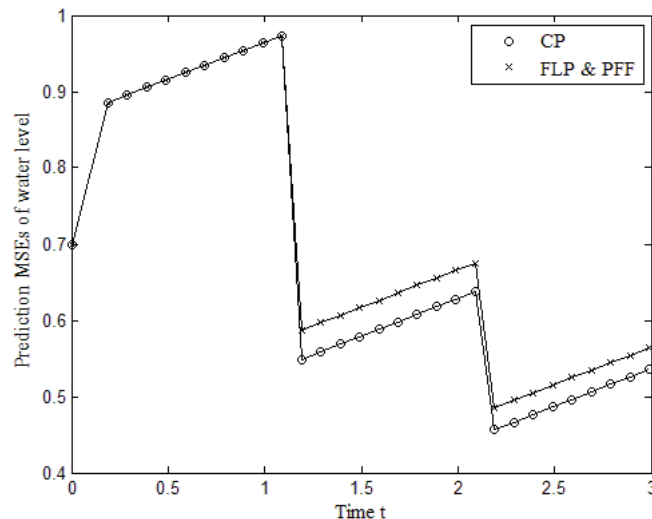


Fig. 3. CP, FLP and PFF MSEs for water level at lead  $\Delta = 0.2s$

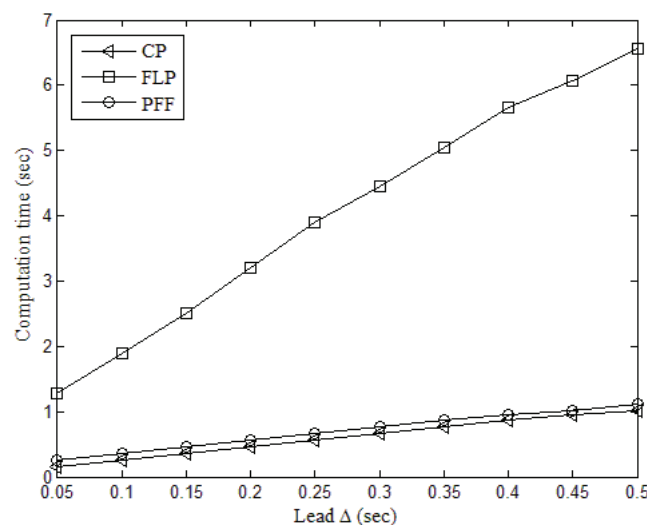


Fig. 4. Computational time of water tank mixing system using 3 predictors at leads  $\Delta = 0.05, 0.1, \dots, 0.5s$ .

The parameters are subjected to  $q=1$ ,  $r^{(1)}=2$ ,  $r^{(2)}=1$ ,  $t^*=3$ ,  $\bar{x}_0=[1 \ 1 \ 0]^T$ ,  $P_0 = \text{diag}[0.7 \ 0.7 \ 0.1]$ ,  $t_k - t_{k-1} = 0.1$ ,  $\Delta = 0.05 \sim 0.5$ . Fig. 3 illustrates the MSEs of the water level  $P_{1,t+\Delta}^{CP}$ ,  $P_{1,t+\Delta}^{FLP}$  and  $P_{1,t+\Delta}^{PFF}$  at lead  $\Delta=0.2$ . As we can see in Fig. 3 the CP is better than the fusion predictors and the fusion MSEs for water level ( $x_1$ ) of FLP and PFF are equal, i.e.,  $P_{1,t+\Delta}^{CP} < P_{1,t+\Delta}^{FLP} = P_{1,t+\Delta}^{PFF}$ . The CPU times for CP, FLP and PFF are represented in Fig. 4, where it is shown that FLP requires considerably more CPU time than PFF, but CPU time of PFF is similar to CP.

Thus, from Examples 4.1 and 4.2 we can confirm that PFF is preferable to FLP in terms of computation efficiency.

## 5. Conclusions

In this chapter, two fusion predictors (FLP and PFF) for mixed continuous-discrete linear systems in a multisensor environment are proposed. Both of these predictors are derived by using the optimal local Kalman estimators (filters and predictors) and fusion formula. The fusion predictors represent the optimal linear combination of an arbitrary number of local Kalman estimators and each is fused by the MSE criterion. Equivalence between the two fusion predictors is established. However, the PFF algorithm is found to more significantly reduce the computational complexity, due to the fact that the PFF's weights  $b_{t_k}^{(i)}$  do not depend on the leads  $\Delta > 0$  in contrast to the FLP's weights  $a_{t+\Delta}^{(i)}$ .

## Appendix

### Proof of Theorem 1

(a), (c) Equation (12) and formula (14) immediately follow as a result of application of the general fusion formula [20] to the optimization problem (10), (11).

(b) In the absence of observations differential equation for the local prediction error  $\tilde{x}_\tau^{(i)} = x_\tau - \hat{x}_\tau^{(i)}$  takes the form

$$\dot{\tilde{x}}_\tau^{(i)} = \dot{x}_\tau - \dot{\hat{x}}_\tau^{(i)} = F_\tau \tilde{x}_\tau^{(i)} + G_\tau v_\tau. \quad (\text{A.1})$$

Then the prediction cross-covariance  $P_\tau^{(ij)} = E(\tilde{x}_\tau^{(i)} \tilde{x}_\tau^{(j)T})$  associated with the  $\tilde{x}_\tau^{(i)}$  and  $\tilde{x}_\tau^{(j)}$  satisfies the time update Lyapunov equation (see the first and third equations in (13)). At  $t=t_k$  the local error  $\tilde{x}_{t_k}^{(i)}$  can be written as

$$\tilde{x}_{t_k}^{(i)} = x_{t_k} - \hat{x}_{t_k}^{(i)} = x_{t_k} - \hat{x}_{t_k}^{(i)} - L_{t_k}^{(i)} \left[ y_{t_k}^{(i)} - H_{t_k}^{(i)} \hat{x}_{t_k}^{(i)} \right] = \tilde{x}_{t_k}^{(i)} - L_{t_k}^{(i)} \left[ H_{t_k}^{(i)} x_{t_k} + w_{t_k}^{(i)} - H_{t_k}^{(i)} \hat{x}_{t_k}^{(i)} \right] = (I_n - L_{t_k}^{(i)} H_{t_k}^{(i)}) \tilde{x}_{t_k}^{(i)} - L_{t_k}^{(i)} w_{t_k}^{(i)}. \quad (\text{A.2})$$

Given that random vectors  $\tilde{x}_{t_k}^{(i)}$ ,  $w_{t_k}^{(i)}$  and  $w_{t_k}^{(j)}$  are mutually uncorrelated at  $i \neq j$ , we obtain observation update equation (13) for  $P_{t_k}^{(ij)} = E(\tilde{x}_{t_k}^{(i)} \tilde{x}_{t_k}^{(j)T})$ .

This completes the proof of Theorem 1.

### Proof of Theorem 2

It is well known that the local Kalman filtering estimates  $\hat{x}_\tau^{(i)}$  are unbiased, i.e.,  $E(\hat{x}_\tau^{(i)}) = E(x_\tau)$  or  $E(\tilde{x}_\tau^{(i)}) = E(x_\tau - \hat{x}_\tau^{(i)}) = 0$  at  $0 \leq \tau \leq t_k$ . With this result we can prove unbiased property at  $t_k < \tau \leq t+\Delta$ . Using (8) we obtain

$$\dot{\tilde{x}}_\tau^{(i)} = \dot{x}_\tau - \dot{\hat{x}}_\tau^{(i)} = F_\tau \tilde{x}_\tau^{(i)} + G_\tau v_\tau, \quad \tilde{x}_{t_k}^{(i)} = \tilde{x}_{t_k}^{(i)}, \quad t_k \leq \tau \leq t+\Delta, \quad (\text{A.3})$$

or

$$\frac{d}{d\tau} E(\tilde{x}_\tau^{(i)}) = F_\tau E(\tilde{x}_\tau^{(i)}), \quad E(\tilde{x}_{t_k}^{(i)}) = E(\tilde{x}_{t_k}^{(i)}) = 0, \quad t_k \leq \tau \leq t+\Delta. \quad (\text{A.4})$$

Differential equation (A.4) is homogeneous with zero initial condition therefore it has zero solution  $E(\tilde{x}_\tau^{(i)}) \equiv 0$  or  $E(\hat{x}_\tau^{(i)}) = E(x_\tau)$ ,  $t_k \leq \tau \leq t+\Delta$ .

Since the local predictors  $\hat{x}_{t+\Delta}^{(i)}$ ,  $i = 1, \dots, N$  are unbiased, then we have

$$E(\hat{x}_{t+\Delta}^{\text{FLP}}) = \sum_{i=1}^N a_{t+\Delta}^{(i)} E(\hat{x}_{t+\Delta}^{(i)}) = \left[ \sum_{i=1}^N a_{t+\Delta}^{(i)} \right] E(x_{t+\Delta}) = E(x_{t+\Delta}). \quad (\text{A.5})$$

This completes the proof of Theorem 2.

### Proof of Theorem 3

*a., c.* Equations (18) and (19) immediately follow from the general fusion formula for the filtering problem (Shin et al., 2006)

*b.* Derivation of observation update equation (13) is given in Theorem 1.

*d.* Unbiased property of the fusion estimate  $\hat{x}_{t+\Delta}^{PFF}$  is proved by using the same method as in Theorem 2.

This completes the proof of Theorem 3.

### Proof of Theorem 4

By integrating (8) and (17), we get

$$\hat{x}_{t+\Delta}^{(i)} = \Phi(t+\Delta, t_k) \hat{x}_{t_k}^{(i)}, \quad i = 1, \dots, N, \quad \hat{x}_{t+\Delta}^{PFF} = \Phi(t+\Delta, t_k) \hat{x}_{t_k}^{FF}, \quad (\text{A.6})$$

where  $\Phi(t, s)$  is the transition matrix of (8) or (17). From (10) and (16), we obtain

$$\begin{aligned} \hat{x}_{t+\Delta}^{FLP} &= \sum_{i=1}^N a_{t+\Delta}^{(i)} \hat{x}_{t+\Delta}^{(i)} = \sum_{i=1}^N a_{t+\Delta}^{(i)} \Phi(t+\Delta, t_k) \hat{x}_{t_k}^{(i)} = \sum_{i=1}^N A_{t, t_k, \Delta}^{(i)} \hat{x}_{t_k}^{(i)}, \\ \hat{x}_{t+\Delta}^{PFF} &= \Phi(t+\Delta, t_k) \hat{x}_{t_k}^{FF} = \sum_{i=1}^N \Phi(t+\Delta, t_k) b_{t_k}^{(i)} \hat{x}_{t_k}^{(i)} = \sum_{i=1}^N B_{t, t_k, \Delta}^{(i)} \hat{x}_{t_k}^{(i)}, \end{aligned} \quad (\text{A.7})$$

where the new weights take the form:

$$A_{t, t_k, \Delta}^{(i)} = a_{t+\Delta}^{(i)} \Phi(t+\Delta, t_k), \quad B_{t, t_k, \Delta}^{(i)} = \Phi(t+\Delta, t_k) b_{t_k}^{(i)}. \quad (\text{A.8})$$

Next using (12) and (18) we will derive equations for the new weights (A.8). Multiplying the first (N-1) homogeneous equations (18) on the left hand side and right hand side by the nonsingular matrices  $\Phi(t+\Delta, t_k)$  and  $\Phi(t+\Delta, t_k)^T$ , respectively, and multiplying the last non-homogeneous equation (18) by  $\Phi(t+\Delta, t_k)$  we obtain

$$\begin{aligned} \sum_{i=1}^N \Phi(t+\Delta, t_k) b_{t_k}^{(i)} \left[ P_{t_k}^{(ij)} - P_{t_k}^{(iN)} \right] \Phi(t+\Delta, t_k)^T &= 0, \quad j=1, \dots, N-1; \\ \sum_{i=1}^N \Phi(t+\Delta, t_k) b_{t_k}^{(i)} &= \Phi(t+\Delta, t_k). \end{aligned} \quad (\text{A.9})$$

Using notation for the difference  $\delta P_s^{(ijN)} = P_s^{(ij)} - P_s^{(iN)}$  we obtain equations for  $B_{t, t_k, \Delta}^{(i)}$ ,  $i = 1, \dots, N$  such that

$$\sum_{i=1}^N B_{t, t_k, \Delta}^{(i)} \delta P_{t_k}^{(ijN)} \Phi(t+\Delta, t_k)^T = 0, \quad j=1, \dots, N-1; \quad \sum_{i=1}^N B_{t, t_k, \Delta}^{(i)} = \Phi(t+\Delta, t_k). \quad (\text{A.10})$$

Analogously after simple manipulations equation (12) takes the form

$$\begin{aligned} \sum_{i=1}^N a_{t+\Delta}^{(i)} \Phi(t+\Delta, t_k) \Phi(t+\Delta, t_k)^{-1} \left[ P_{t+\Delta}^{(ij)} - P_{t+\Delta}^{(iN)} \right] &= \sum_{i=1}^N A_{t, t_k, \Delta}^{(i)} \Phi(t+\Delta, t_k)^{-1} \delta P_{t+\Delta}^{(ijN)} = 0, \\ \sum_{i=1}^N a_{t+\Delta}^{(i)} \Phi(t+\Delta, t_k) &= \sum_{i=1}^N A_{t, t_k, \Delta}^{(i)} = \Phi(t+\Delta, t_k). \end{aligned} \quad (\text{A.11})$$

or

$$\sum_{i=1}^N A_{t_k, \Delta}^{(i)} \Phi(t+\Delta, t_k)^{-1} \delta P_{t+\Delta}^{(ijN)} = 0, \quad j = 1, \dots, N-1; \quad \sum_{i=1}^N A_{t_k, \Delta}^{(i)} = \Phi(t+\Delta, t_k). \quad (\text{A.12})$$

As we can see from (A.10) and (A.12) if the equality

$$\delta P_{t_k}^{(ijN)} \Phi(t+\Delta, t_k)^T = \Phi(t+\Delta, t_k)^{-1} \delta P_{t+\Delta}^{(ijN)} \quad (\text{A.13})$$

will be hold then the new weights  $A_{t_k, \Delta}^{(i)}$  and  $B_{t_k, \Delta}^{(i)}$  satisfy the identical equations. To show that let consider differential equation for the difference  $\delta P_s^{(ijN)} = P_s^{(ij)} - P_s^{(iN)}$ . Using (13) we obtain the Lyapunov homogeneous matrix differential equation

$$\delta \dot{P}_s^{(ijN)} = \dot{P}_s^{(ij)} - \dot{P}_s^{(iN)} = F_s (P_s^{(ij)} - P_s^{(iN)}) + (P_s^{(ij)} - P_s^{(iN)}) F_s^T = F_s \delta P_s^{(ijN)} + \delta P_s^{(ijN)} F_s^T, \quad t_k \leq s \leq t+\Delta, \quad (\text{A.14})$$

which has the solution

$$\delta P_{t+\Delta}^{(ijN)} = \Phi(t+\Delta, t_k) \delta P_{t_k}^{(ijN)} \Phi(t+\Delta, t_k)^T. \quad (\text{A.15})$$

By the nonsingular property of the transition matrix  $\Phi(t+\Delta, t_k)$  the equality (A.13) holds, then  $A_{t_k, \Delta}^{(i)} = B_{t_k, \Delta}^{(i)}$ , and finally using (A.7) we get

$$\hat{x}_{t+\Delta}^{\text{FLP}} = \sum_{i=1}^N A_{t_k, \Delta}^{(i)} \hat{x}_{t_k}^{(i)} = \sum_{i=1}^N B_{t_k, \Delta}^{(i)} \hat{x}_{t_k}^{(i)} = \hat{x}_{t+\Delta}^{\text{PFF}}. \quad (\text{A.16})$$

**This completes the proof of Theorem 4.**

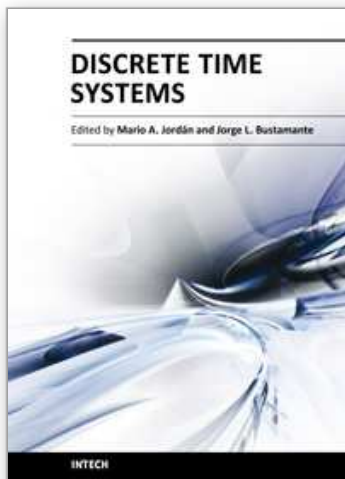
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## **Discrete Time Systems**

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Discrete-Time Systems comprehend an important and broad research field. The consolidation of digital-based computational means in the present, pushes a technological tool into the field with a tremendous impact in areas like Control, Signal Processing, Communications, System Modelling and related Applications. This book attempts to give a scope in the wide area of Discrete-Time Systems. Their contents are grouped conveniently in sections according to significant areas, namely Filtering, Fixed and Adaptive Control Systems, Stability Problems and Miscellaneous Applications. We think that the contribution of the book enlarges the field of the Discrete-Time Systems with signification in the present state-of-the-art. Despite the vertiginous advance in the field, we also believe that the topics described here allow us also to look through some main tendencies in the next years in the research area.

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