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The Linear Method for Acoustical Source Localization (Constant Speed Localization Method) - A Discussion of Receptor Geometries and Time Delay Accuracy for Robust Localization

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1. Introduction

One of the most widely used methodology for the passive localization of acoustic sources is based on the measurement of the time delay of arrival (TDOA) of the source signal to receptors pairs. In 2D, two pairs of receptors are necessary, implying the need of 3 receptors. In 3D, three pairs are needed, and a minimum of 4 receptors. The only data available to solve for the source spatial coordinates are the receptors spatial position and the best possible computation of TDOA between receptors pairs. In a 2D problem if we have two receptors and we compute a TDOA between them, it is a well known fact that the source capable to produce that delay must be placed over one of two symmetric hyperbolas, Figure 1. Because this is true for each pair, becomes clear that the source must be placed in the intersection of the hyperbolas of two different pairs. That is why this method is known as hyperbolic localization. HL for short.

The resulting system of equations is non linear. In 3D the hyperbolas become hyperboloids, a third coordinate appears as unknown, and one more pair of receptors is needed. This reasoning justifies the minimum number of receptors mentioned above. Of course, although the mathematical minimum is correct, in finite computations the pairs available can provide a numerically inadequate set of equations. To provide more pairs, and receptors, than necessary made available an ample set of equations from where to choose the adequate ones. Nevertheless, non linearity and equation redundancy are different issues that should not be confused.

For the sake of self consistency the equations of the HL problem are developed.

Be $s = \{x, y, z\}$ the unknown spatial position of the source. For each receptor m_i we have its position $\{xi, yi, zi\}$ and the vector $\vec{\mathbf{r_i}} = \vec{\mathbf{s}} - \vec{\mathbf{m_i}}$ that points from the receptor to the source. Assuming spherical sound propagation the following relationship is satisfied by each receptor pair:

$$r_i - r_j = d_{ij} = v\tau_{ij} \tag{1}$$

where d_{ij}, a signed quantity, is the difference between the distances of each receptor to the source, v is the sound propagation speed in the medium and τ_{ij} is the TDOA computed from the receptors registers. The τ_{ij} s are signed quantities too. Working over Equation 1, the

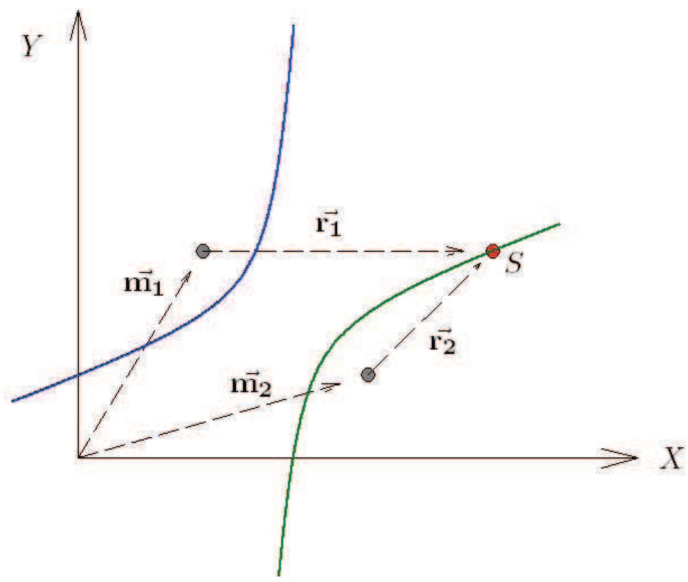

Fig. 1. A source positioned over the hyperbolas, irrespective of the distance, will produce the same TDOA absolute value. Which one is the involved hyperbola is determined by the TDOA sign.

following expression is obtained:

$$(x_i - x_j)x + (y_i - y_j)y + (z_i - z_j)z + d_{ij}r_j = \frac{m_i^2 - m_j^2 - d_{ij}^2}{2} \tag{2}$$

The same equation can be written for other two pairs. Assuming that the three pairs are constructed from three receptors the resulting system of equations is:

$$\begin{aligned} (x_i - x_j)x + (y_i - y_j)y + (z_i - z_j)z + d_{ij}r_j &= 0.5(m_i^2 - m_j^2 - d_{ij}^2) \\ (x_k - x_l)x + (y_k - y_l)y + (z_k - z_l)z + d_{kl}r_l &= 0.5(m_k^2 - m_l^2 - d_{kl}^2) \\ (x_i - x_k)x + (y_i - y_k)y + (z_i - z_k)z + d_{ik}r_k &= 0.5(m_i^2 - m_k^2 - d_{ik}^2) \end{aligned} \tag{3}$$

where

$$\begin{aligned} r_q &= \sqrt{(x_q - x)^2 + (y_q - y)^2 + (z_q - z)^2} \\ m_q &= \sqrt{x_q^2 + y_q^2 + z_q^2} \ ; \text{ for } q = j, k, l \end{aligned} \tag{4}$$

Equations 3 constitute a nonlinear system of equations and can be solved, iteratively, by traditional numerical methods. In 1987 many authors, in closely sequenced papers, presented a different way to obtain Equation 3 (Abel & Smith, 1987; Friedlander, 1987; H.C.Schau & Robinson, 1987). First they choose one of the receptors, for example receptor j, as a master receptor. This allows computing all the receptor-source distances as a function of the distance of the master receptor to the source. The values of d_{ij} are computed from the τ_{ij} and the medium propagation speed.

$$d_{jl} = r_j - r_l \Longrightarrow r_l = r_j - d_{jl} \tag{5}$$

Second, receptor m_j is renamed m_0 and r_j as r_0, obtaining

$$(x_i - x_j)x + (y_i - y_j)y + (z_i - z_j)z + d_{ij}r_0 = \frac{m_i^2 - m_j^2 - d_{ij}^2}{2} + d_{ij}d_{0j} \quad (6)$$

where r_0 is now the distance between the master receptor and the source, the so called range, computed as

$$r_0 = \sqrt{(x - x_0)^2 + (y - y_0)^2) + (z - z_0)^2} \quad (7)$$

In Equation 6, the unknowns still are $\{x, y, z\}$. One way to overcome the non linearity of the system was to introduce r_0 as a new unknown or parameter (Friedlander, 1987). The new unknown required the introduction of one more equation, expanding the original equations system. At that time nobody believed that the values of r_0 and $\{x, y, z\}$ obtained from the expanded system would satisfy Equation 7. It seems that nobody checked it either in the last 20 years. Because the clear non linear nature of Equation 7 many authors developed ways to solve the new expanded system by iterative methods (Chan & Ho, 1994).
The use of redundant pairs made it necessary to combine iterative methods with least square procedures, increasing the difficulty. In 2000, (Huang et al., 2000) found that the redundant system can be solved correctly in only one iteration. It was not noticed that it only can happen if the system is linear or if the initial guess in the nonlinear system is always coincident with the right solution.

2. The constant speed localization method, CSLM

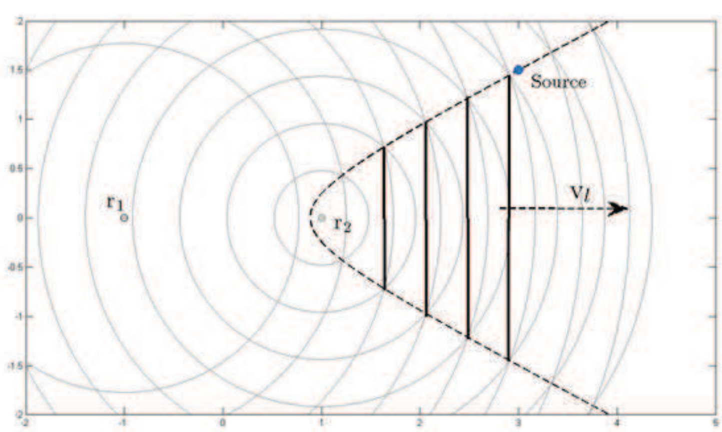

Fig. 2. Straight front propagation

In 2007 the authors (Militello & Buenafuente, 2007) presented a new way of interpreting the source localization problem, from now on CSLM (Constant Speed Localization Method). This allowed demonstrating that the problem could be transformed into a linear one by the mere fact of adding an additional receiver to the minimum required in the hyperbolic localization method. It was also shown that the work of Friedlander *et al.* and methods derived from it are special cases of the general case presented, making clear the linearity of the method. To explain the CSLM the receptors are considered to act as sources, each one emitting sound. But each one starts emitting in the inverse order they capture the sound from the source. In this way, all the wave fronts emitted will intersect the source at the same time.

Two receptors at a distance $2c$ from each other received the signal with a time delay t_a. For a sound speed v a spatial delay is defined as $2a = t_a v$. Now the two receptors start emitting with a time delay t_a. Both circles will intersect, and the successive intersections will describe a hyperbola. The hyperbola is symmetric with respect to the line joining the receptors and one of the branches will contain the source. But, if we join the successive intersection points with a straight line, as in Figure 2, a straight front can be identified. In (Militello & Buenafuente, 2007) it was proved that this front propagates with a constant speed $v_l = va/c$. Because of the straight front speed property the method is called Constant Speed Localization.
Each receptor pairs will produce one straight front propagating at a constant speed, and all the fronts will reach the source at the same time, i.e. all the constant speed traveling straight lines will intersect at the source position. In this way, a linear system of equations having as unknowns the source coordinates and the time of arrival can be constructed. The unknowns are clearly independent, and there is neither preferred coordinate system nor time origin. If one receptor position is considered as the coordinate centre, and the distance from this point to the source is called the range, the values of vt appearing in the equation can be substituted by r_0 and Friedlandert's equations are recovered. This is the only case where $R = vt = \sqrt{x^2 + y^2}$.
A detailed development of CSLM for 2D and 3D problems in its general form is presented in (Militello & Buenafuente, 2007). Here, for the sake of comparison, the equations are developed taking into account Friedlander's methodology and the following particular form is obtained:

$$(x_i - x_j)x + (y_i - y_j)y + (z_i - z_j)z + d_{ij}vt = \frac{m_i^2 - m_j^2 - d_{ij}^2}{2} + d_{ij}d_{0j} \tag{8}$$

To reach (8) the time origin is established as the time when receptor m_0 starts emitting. In the original CSLM method the time origin is the time when the furthest receptor starts emitting. Because the problem is linear in time and space, a time or a coordinate shift do not introduces changes in the solution nature.
Equations 6 and 8 are almost identical. The difference is that r_0 is replaced by vt. This replacement is consistent with the meaning of r_0 in Friedlandert's formulation and the meaning of the independent variable t in the CSLM formulation. Then r_0 is an independent variable because it can be obtained as the product of the independent variable t by the sound speed in the medium.
Now the linear nature of both methods and their equivalence has been established. Because a new independent variable appears, r_0 or t, one more equation is needed. The linear system can be solved by using a minimum of four sensors instead of three in a 2D problem and five sensors instead of four in a 3D problem. But the use of the correct number of sensors does not preclude the appearance of numerical errors when solving the system.
Something worth noting: in the CSLM method it is necessary to create a common time axis. It can only be done if the TDOA are not only computed between the active receptor pairs but also among one receptor, lets say a master one, and one of the receptors of each active pair. This is totally equivalent to Friedlandert's method when all the receptors positions are computed as a function of the position of the master receptor. Then, the computational work load involved in both methods is the same.

3. The design of the reception system

There are many variables and uncertainties in the design of a receptor system. To mention some of them the following list is proposed:
Uncertainties:

1. The error in TDOA estimations. This error depends on the ability to identify a specific perturbation introduced by the source in each sensor register and to assign a time to it. Or in the ability to compute the TDOA for a receptor pair.
2. The geometrical position of the receptor. Nowadays receptors are small in size and the pressure centre of a microphone can be determined with an error of the order of millimetres.

Design variables:

1. The spatial distribution of receptors.
2. The receptors chosen to constitute active pairs.

As it will be shown, the design variables will be responsible of the system performance. It will govern the way the effects of uncertainties are amplified in some detection scenarios and the quality of detection when the relative position of the source changes respect to our detection system.

3.1 Selecting the active pairs and the master receptor (time origin)

The study is focused in the way the design variables affects the source localization through the inevitable TDOA uncertainties. The superscript $^\circ$ is used to indicate the correct or exact values. They will be affected by an uncertainty value so that $\tau_{ij} = \tau_{ij}^\circ \pm e_{ij}$. By replacing it in (8) and rearranging terms:

$$\begin{aligned} (x_i - x_j)x^\circ + (y_i - y_j)y^\circ + (z_i - z_j)z^\circ + v\tau_{ij}^\circ vt^\circ - 0.5(m_i^2 - m_j^2 - v^2(d_{ij}^\circ)^2) &= 0 \quad (9) \\ \pm v^2 e_{ij} t^\circ - 0.5 e_{ij}^2 \pm v\tau_{ij}^\circ e_{ij} + v^2(\tau_{ij}^\circ \tau_{0j}^\circ \pm \tau_{ij}^\circ e_{0j} \pm \tau_{0j}^\circ e_{ij} \pm e_{ij} e_{oj}) &= \epsilon_{ij} \quad (10) \end{aligned}$$

Equation 9 recasts Equation 8. Equation 10 is an error and can be seen as a contribution to the uncertainty value of the left hand side of the original equation system. Neglecting second order terms and adding up uncertainties an upper bound can be computed.

$$\epsilon_{ij} = v^2 \left(e_{ij}(t^\circ + \tau_{ij}^\circ + \tau_{0j}^\circ) + \tau_{ij}^\circ \tau_{0j}^\circ + \tau_{ij}^\circ e_{0j} \right) \quad (11)$$

This upper bound can be reduced if all the active pairs include the master receptor. In doing so $\tau_{00}^\circ = 0$. In this case Equation 11 can be further simplified to:

$$\epsilon_{i0} = v e_{i0}(vt^\circ + d_{i0}^\circ) \quad (12)$$

From this equation many conclusions can be drawn about the amplification of the TDOA inaccuracies. The main factors are:

1. The speed of sound in the medium.
2. The distance from the source.
3. The TDOA uncertainty.

In other words, for a given medium, the further the source the higher is the error. And, for a given set of receptors, it seems that the active pairs should be chosen so that one of the receptors appears in all the pairs and the distance between receptors is kept to a minimum.

4. Error propagation

Although the rules extracted in the preceding sections seems logical, they are not conclusive. This is due to the fact that in a linear problem the quality of the solution depends on the conditioning of the system of equations. In 3D the number of unknowns is four so that four pairs are needed. The system of equations gets the form $\mathbf{Mx} = \mathbf{b}$, where

$$\mathbf{M} = \begin{bmatrix} x_i - x_j & y_i - y_j & z_i - z_j & d_{ij} \\ x_k - x_l & y_k - y_l & z_k - z_l & d_{kl} \\ x_m - x_n & y_m - y_n & z_m - z_n & d_{mn} \\ x_p - x_q & y_p - y_q & z_p - z_q & d_{pq} \end{bmatrix} \tag{13}$$

$$\mathbf{x} = \begin{bmatrix} x & y & z & vt \end{bmatrix}^{\mathrm{T}} \tag{14}$$

$$\mathbf{b} = \frac{1}{2} \begin{bmatrix} m_i^2 - m_j^2 - d_{ij}^2 + 2d_{ij}d_{0j} \\ m_k^2 - m_l^2 - d_{kl}^2 + 2d_{kl}d_{0l} \\ m_m^2 - m_n^2 - d_{mn}^2 + 2d_{mn}d_{0n} \\ m_p^2 - m_q^2 - d_{pq}^2 + 2d_{pq}d_{0q} \end{bmatrix} \tag{15}$$

and the solution is

$$\mathbf{x} = \mathbf{M}^{-1}\mathbf{b} \tag{16}$$

provided that the inverse of $\mathbf{M}$ exists. Notice the use of eight different sensors, which is the most general case to construct the system. But, as one sensor can be part of many pairs, this number can be reduced to five. Because of the uncertainties pointed up before matrices $\mathbf{M}$ and $\mathbf{b}$ are perturbed. As before only TDOA uncertainties are considered. The real equation system becomes

$$(\mathbf{M} + \boldsymbol{\delta}\mathbf{M})\,\hat{\mathbf{x}} = (\mathbf{b} + \boldsymbol{\delta}\mathbf{b}) \tag{17}$$

being $\hat{\mathbf{x}}$ an approximation to the exact solution.

$$\hat{\mathbf{x}} = \mathbf{x}^{\circ} + \boldsymbol{\delta}\mathbf{x} \tag{18}$$

Because the system is linear, perturbation theory can be applied in order to obtain a bound to the expected error in the system solution. The relative solution error will satisfy:

$$\frac{\|\boldsymbol{\delta}\mathbf{x}\|}{\|\mathbf{x}^{\circ}\|} \leq \frac{\operatorname{cond}(\mathbf{M})}{1 - \operatorname{cond}(\mathbf{M})\frac{\|\boldsymbol{\delta}\mathbf{M}\|}{\|\mathbf{M}\|}} \left(\frac{\|\boldsymbol{\delta}\mathbf{M}\|}{\|\mathbf{M}\|} + \frac{\|\boldsymbol{\delta}\mathbf{b}\|}{\|\mathbf{b}\|} \right) \tag{19}$$

where $\operatorname{cond}(\mathbf{M})$ is the matrix condition number defined as:

$$\operatorname{cond}(\mathbf{M}) = \|\mathbf{M}\| \left\|\mathbf{M}^{-1}\right\| \geq 1 \tag{20}$$

where $\|\cdot\|$ is a matrix norm, usually the l_2 norm. In a badly conditioned system the $\operatorname{cond}(\mathbf{M})$ is bigger than 1. If it is assumed that the perturbed matrices have a small norm and $\operatorname{cond}(\mathbf{M})$ is not a big number, (Moon & Stirling, 2000), the relative error in system solution can be approximated by

$$\frac{\|\boldsymbol{\delta}\mathbf{x}\|}{\|\mathbf{x}^{\circ}\|} \leq \operatorname{cond}(\mathbf{M}) \left(\frac{\|\boldsymbol{\delta}\mathbf{M}\|}{\|\mathbf{M}\|} + \frac{\|\boldsymbol{\delta}\mathbf{b}\|}{\|\mathbf{b}\|} \right) + O(e^2) \tag{21}$$

Being e the order of magnitude of the TDOA uncertainty. From Equation 21 it can be seen that the relative error in the system solution can be approximated as the sum of the relative error in the matrix plus the relative error in the independent term, amplified by the condition number. In order to clarify the effect of this equation in the results two examples are presented.

4.1 Directivity of a given sensor configuration

In this context the term "directivity" is defined as $1/\text{cond}(\mathbf{M})$, having a maximum value of 1, and is used to point how a given sensor configuration will amplify the uncertainties from a source placed over a circle around the designed master receptor. Matrix **M** has three columns that can be evaluated from the receptors coordinates, but the fourth one depends on the relative positions of source and receptors pairs, the TDOA. Matrix **M** can be easily constructed from any expected source position and its condition evaluated. Following Equation 21 the value $1/\text{cond}(\mathbf{M})$ can be seen as a directivity property. A high value in a given direction indicates that direction as a preferred one with small uncertainty amplification.

Simulation A.

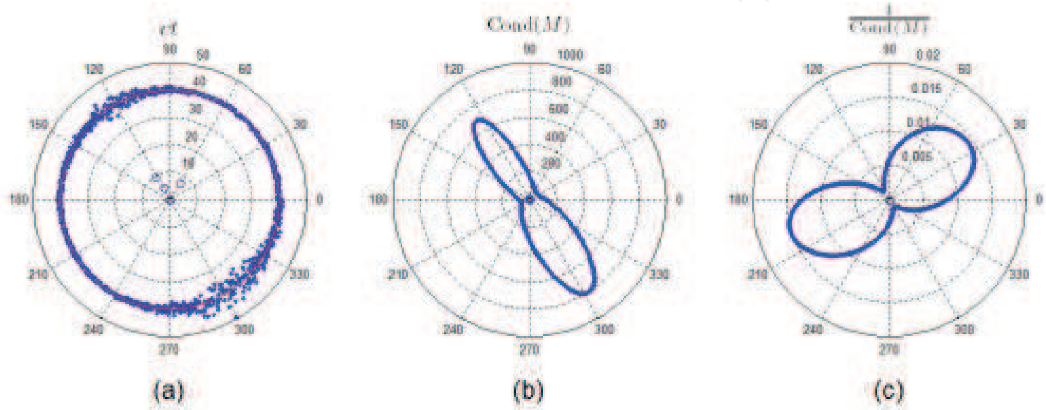

Fig. 3. **Simulación A**. (a) A starting receptors configuration and range computation with CSLM. (b) Matrix **M** condition showing the lobes responsible of error amplification. (c) Receptors array directivity, minimum directivity in the maximum error propagation direction.

A set of receptors are positioned: $m_0\{0,0\}$, $m_1\{-5,8\}$, $m_2\{4,6\}$, and $m_3\{-2,4\}$. The receptors pairs are $\{m_0, m_1\}$, $\{m_0, m_3\}$ and $\{m_0, m_2\}$. It must be noticed that receptors m_0, m_1 and m_3 seems to be over a straight line at 120° from the X axis but they are not. If they are over the same line the system is singular and can not be inverted. A circle of radius 40 m centered at m_0 is drawn and 1000 sources uniformly distributed over it. For each source exact, within machine precision, quantities are computed. The exact TDOAs are computed and perturbed with a random Gaussian error distribution. The error standard deviation is set to 10us. The values of vt computed for each source are plotted in Figure 3(a). Figure 3(b) plots the computed matrix condition and clearly shows the coincidence of big condition values with high source localization error. An amplification factor of 800 can be seen at 300°. Figure 3(c) is the directivity, showing a big value in the directions where the computed error will be low. From the traveling straight front point of view a wrong selection of receptors pairs will produce almost parallel lines, making it difficult to compute their intersection. Why the 120° direction produces less dispersion than the 300° one? It will be explained latter.

Simulation B

A robust configuration is defined as the one with not pronounced directivity lobes. Under this point of view the best one will be the one with no lobes and a directivity value near one. In order to achieve this receptors are placed in the vertex of an equilateral triangle and the master receptor is placed at the triangle centre of gravity, Figure 4. The triangle side is $4\sqrt{3}$

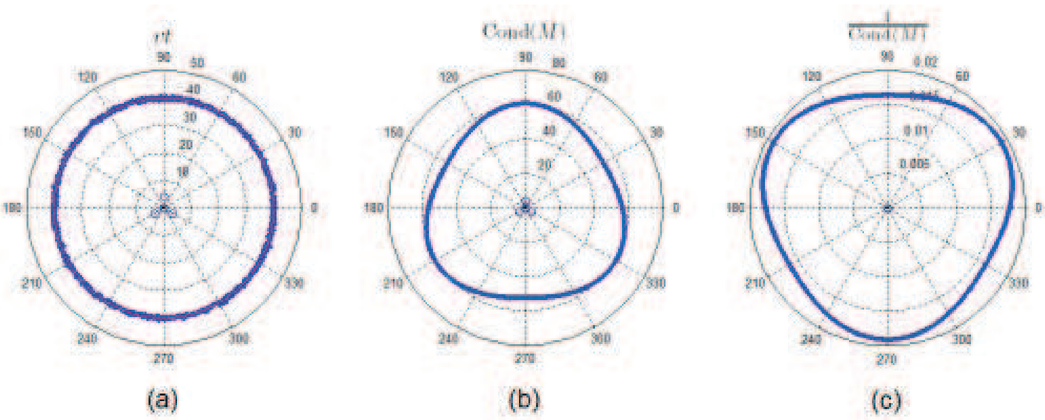

Fig. 4. **Simulación B**. A centred triangle configuration. a)Computed range with CSLM. b)Matrix M condition. c) Directivity.

m. The TDOA uncertainties are computed in exactly the same manner as in Simulation A. It can be seen that three lobes appear with a very uniform shape. The directivity is uniform too. It should be noticed that a directivity number better than 0.02 is not achieved for this configuration. Simulation B shows how with the same computational and hardware costs a better system can be constructed. The matrix condition number increases as the distance to the source increases. The ideal number of 1 is hard to get. For the triangular configuration of SIMULATION B a condition number of 1.4 is obtained for a source placed at the triangle centre, in top of the master receptor.

5. An upper bound for the solution error

When designing a reception system the effect of TDOA error in system performance is capital. All the electronics and computational effort used in reducing this uncertainty will have a direct impact in localization. Equation 21 provides an easy way to predict the value of uncertainty necessary for a desired performance. Assuming no error in receptors positions the perturbed matrix can be written as

$$\boldsymbol{\delta}\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 & e_{ij}v \\ 0 & 0 & 0 & e_{kl}v \\ 0 & 0 & 0 & e_{mn}v \\ 0 & 0 & 0 & e_{pq}v \end{bmatrix} \tag{22}$$

where e_{ij} is the error in computing the TDOA for each receptors pair. The maximum value for e_{ij} is set to $e_{\max}$. The l_1 norm is computed for this matrix obtaining a bound for the perturbed matrix:

$$\|\boldsymbol{\delta}\mathbf{M}\| < nve_{\max} \tag{23}$$

In Equation 23 n is the number of receptor pairs.
To compute an upper bound to $\|\boldsymbol{\delta}\mathbf{b}\|$ it must be recalled that $d_{ij} = d_{ij}^{\circ} + ve_{ij}$. The perturbed $\mathbf{b}$ can be written as:

$$\boldsymbol{\delta}\mathbf{b} = -\frac{v^2}{2}\begin{bmatrix} e_{ij}^2 + 2\tau_{ij}e_{ij} \\ e_{kl}^2 + 2\tau_{kl}e_{kl} \\ e_{mn}^2 + 2\tau_{mn}e_{mn} \\ e_{pq}^2 + 2\tau_{pq}e_{pq} \end{bmatrix} \tag{24}$$

Now, if e_{ij} is neglected with respect to τ_{ij} (remember that τ_{ij} is the TDOA and e_{ij} the error in computing it. It is assumed that $e_{ij} << \tau_{ij}$), e_{ij} is bounded by $e_{\max}$ and d_{ij} is bounded by $D = d_{ij}^{\max}$, :

$$\begin{aligned} v^2 e_{ij}(e_{ij+2\tau_{ij}}) &\approx v^2 e_{ij} 2\tau_{ij} \\ &< 2ve_{\max} d_{ij} \\ &< 2ve_{\max} D \end{aligned} \tag{25}$$

Then, an upper bound for the perturbation is

$$\|\boldsymbol{\delta}\mathbf{b}\|_1 < nve_{\max} D \tag{26}$$

From (25) and (26) the relative error in source positioning can be bounded:

$$\frac{\|\boldsymbol{\delta}\mathbf{x}\|}{\|\mathbf{x}^{\circ}\|} < nve_{\max}\left(\frac{1}{\|\mathbf{M}\|} + \frac{D}{\|\mathbf{b}\|}\right)\mathrm{cond}(\mathbf{M}) \tag{27}$$

Finally the value of $e_{\max}$ can be computed from it:

$$e_{\max} = \frac{\Delta R}{Rnv\left(\frac{1}{\|\mathbf{M}\|} + \frac{D}{\|\mathbf{b}\|}\right)\mathrm{cond}(\mathbf{M})} \tag{28}$$

The values of the range R and its allowed uncertainty ΔR must be introduced and matrices $\mathbf{M}$ and $\mathbf{b}$ must be computed. The following examples will show how Equation 28 can be used.

Simulation C

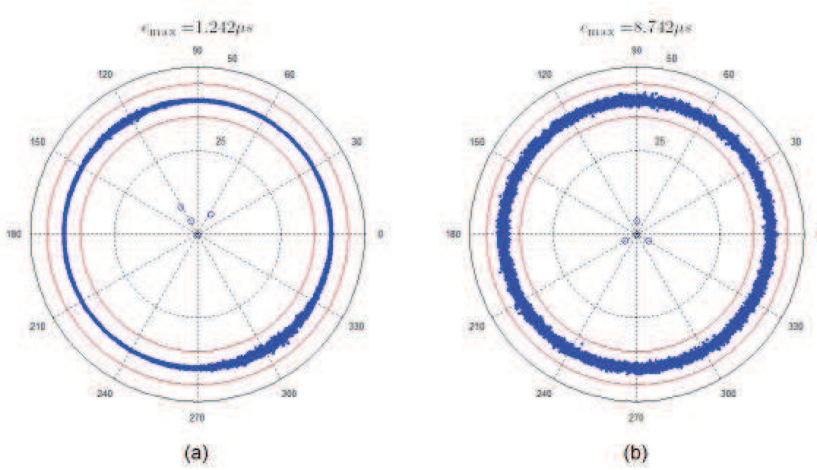

Fig. 5. **Simulation C**. 1000 sources are localized around the receptors. The red circles show the allowed error bound of ± 5 m.

For the examples the two configurations studied in simulations A and B are used. The range was 40 m and an uncertainty of $\pm$ 5 m ($\Delta R = 5$) is introduced. From (28) the values of $e_{\max}$ are computed for the 1000 sources equally spaced. The smallest value, $e_{\max}^{\min}$, imposes the hardware and software quality. Now the TDOAs are perturbed with a random Gaussian error with a standard deviation equal to $e_{\max}^{\min}$. The source position is computed. The results for both configurations are depicted in Figure 5. Configuration A needs an $e_{\max}^{\min}$ equal to 1.242 μs to keep positioning for the worst conditions within bounds. Configuration B can do the job with $e_{\max}^{\min}$ equal to 8.742 μs. All the sources are localized within bounds. It should be noticed the low values of $e_{\max}^{\min}$ needed to ensure an error of $\pm$ 5m in a 40 m range. To the best of the authors' knowledge is the first time this kind of quantification can be done a priori.

Simulation D

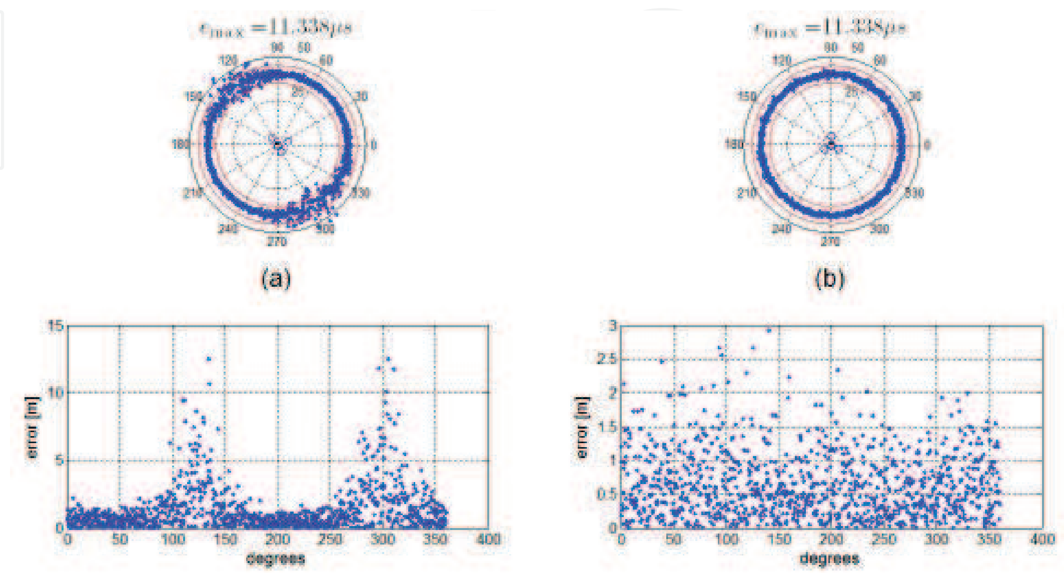

Fig. 6. Simulación D. Ascertaining whether the acquisition frequency is adequate or not. a) The acquisition frequency of 44100 Hz is not enough. b) 44100 Hz is enough.

One of the methods used for TDOA computation is the *Generalized Cross Correlation*, GCC (Knapp & Carter, 1976). In this method the uncertainty is bounded by the acquisition frequency used for the signal. For a given acquisition frequency a lower uncertainty can be achieved by the use of interpolation techniques (Tervo & Lokki, 2008), or regressive techniques (Brandstein & Silverman, 1997). With Equation 28 it can be established if interpolation is needed or not. For SIMULATION C the signal sampling frequency is 44100 Hz. A Gaussian noise with standard deviation equal to $\frac{1}{2}\Delta t = \frac{1}{2*44100}$ is added to the TDOA exact values. The resulting system is solved for each source position. The results can be seen in Figure 6. For configuration A, in order to keep the error within bounds it is necessary to use interpolation algorithms. Configuration B will do the job using the GCC algorithm alone.
It should be noticed that configuration A do not present noticeable differences at 120° and 300° as it did before. The only change is that receptor m_3 is used as coordinate centre instead of m_0.

6. 3D examples

If uniform directivity is considered a desirable property for a detection system the goal must be to achieve it with the minimum hardware and computational work, i.e: receptors and receptors pairs involved. A tetrahedron with a receptor at the geometrical centre is proposed as a guess, Figure 7(a). It is not a blind guess because of the properties shown by the centred equilateral triangle array (see Figure 4). The tetrahedron is a five receptors array, the minimum required. The receptor at the centre is designed as the master. All pairs include the master receptor. The distance from the centre to the corner is 1m. Figure 7(b) shows the directivity

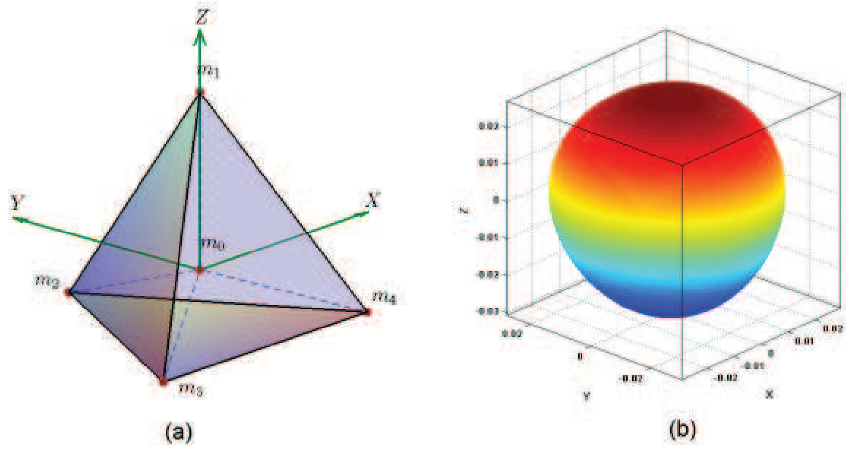

Fig. 7. 3D examples. (a) Receptors spatial configuration. All receptors pairs include the master receptor m0 (b) System directivity

pattern for sources located in a surrounding sphere of radius 10m. The maximum directivity value is 0.017.
The system is perturbed by changing the receptors pairs. The centre receptor is still used as the master. The receptors pairs are depicted by the solid lines in Figure 8(a–c). The corresponding computed directivity pattern is shown in Figure 9(a–c). The results are astonishing.

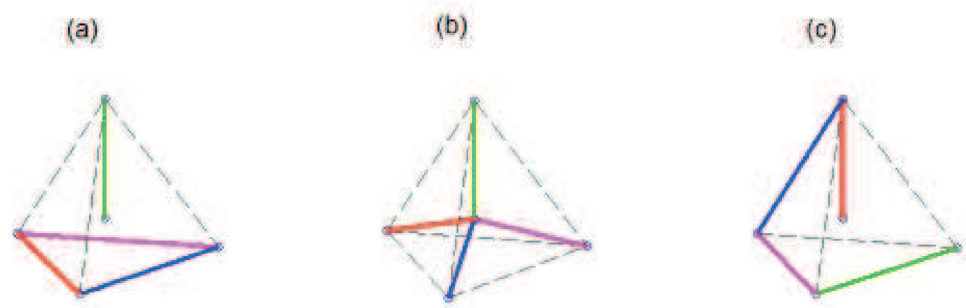

Fig. 8. Three detection systems made from the same receptors but choosing different pairs. The master receptor is always the one in the tetrahedron centre.

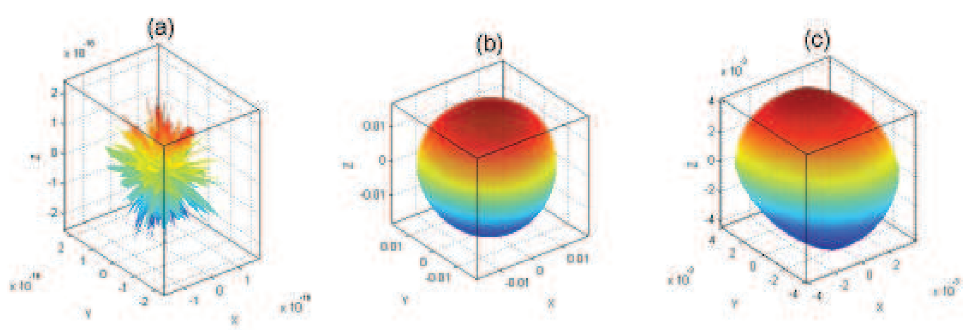

Fig. 9. Directivities computed for the three receptor systems from Figure 8.

The conclusion is that for the same hardware configuration receptors pairs are paramount to determine system directivity.

An alternative is the six receptors arrays of Figure 10(a). The distance of each receptor to the centre is 1 m. The master receptor is the one on top. Three pairs are constructed from the obvious on axis locations. A fourth pair is constructed with two corners from different axes. One more receptor is used. Eight directivity lobes in the axis direction can be seen, Figure 10(b). The maximum directivity is 0.01. Although the fourth receptor pair selection breaks symmetry the directivity pattern is symmetric.

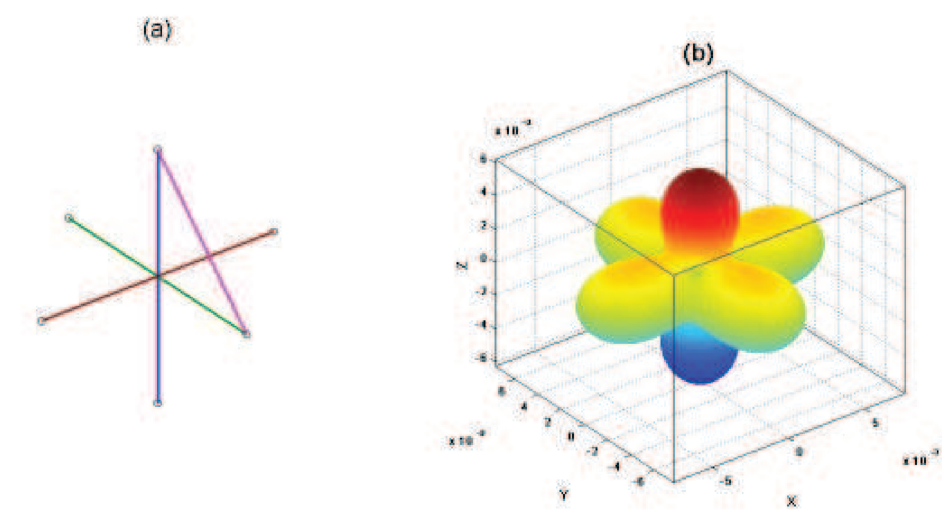

Fig. 10. Six receptors arrangement. a) Selected pairs, b) Computed directivities.

7. Experiment dimensions and effectiveness forecast

7.1 Localization errors as a function of TDOA uncertainties

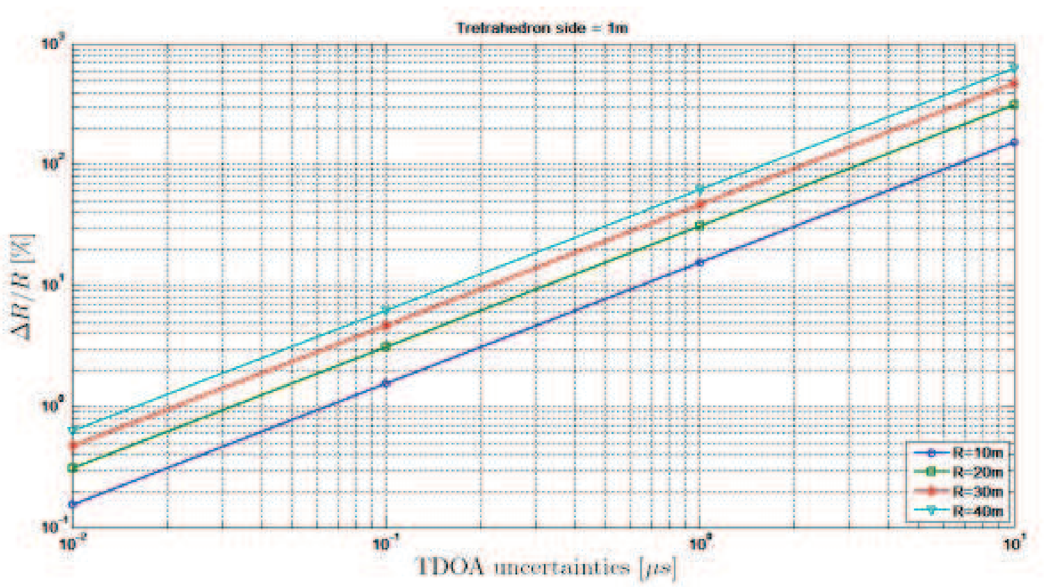

Fig. 11. Relative localization error for a 1 meter tetrahedron array side. TDOA uncertainties of 0.01, 0.1, 1 and 10 microseconds are considered for source distances of 10, 20, 30 and 40 m.

The starting experimental setting is a tetrahedron array with 1m side. For a source position at $r[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]$ with $r = 10$, 20, 30 and 40 meters and TDOA uncertainties of 0.01, 0.1, 1 and 10 microseconds, by using Equation 28 the relative positioning error is computed. The results are plotted in Figure 11. For $1\mu s$ uncertainty the relative error in localizing a source at 20 m is 30%. That is 6 m. It can be seen that in order to reduce the localization uncertainty one order of magnitude, the TDOA uncertainty must be reduced one order of magnitude too.

7.2 The effect of receptors arrangement size

Assuming that the arrangement dimensions can be chosen freely Equation 28 is now computed for the same values of TDOA uncertainties but changing the tetrahedron sides to 0.01, 0.1, 1 and 10 meters. The source is placed at 20 metres. Results are plotted in Figure 12. For the same value of TDOA uncertainties, increasing the side one order of magnitude reduces the localization error in two orders of magnitude.

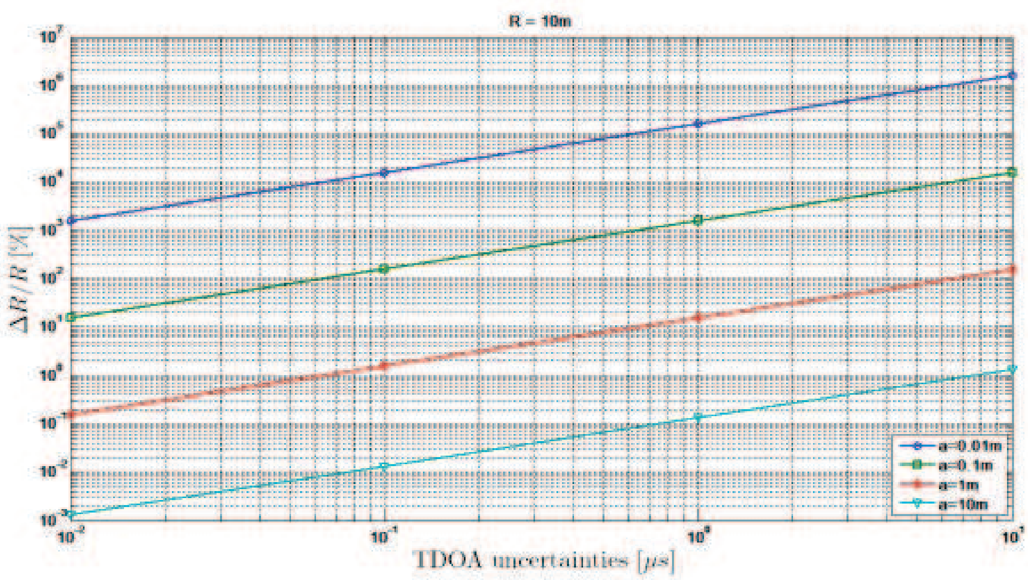

Fig. 12. Relative localization error for a 0.01, 0.1, 1 and 10 meters tetrahedron array side. TDOA uncertainties of 0.01, 0.1, 1 and 10 microseconds are considered for source distance of 10 m.

7.3 The experiment

For the experiments a tetrahedron of 4m side has been constructed. The microphones used are of the ICP type. The signal conditioning is a PCB with a low pass filter set at 10 Khz. A KHEITLEY USB ®, 16 bits, card attached to a portable PC is used as A/D converter.
An acoustic gun shot is used as the source. Acquisition frequency is set to 100 KHz. TDOA are computed by using the GCC algorithm.
From Figure 11, to localize a source with an upper bound of 17% relative error, a TDOA uncertainty of $10\mu s$, approximately, is needed. A GCC method will produce an uncertainty determined by the acquisition frequency, i.e., 10 us.
The complete experiment is mounted in a football stadium, Figure 13. The sources are placed over a 20m circle around the receptors arrangement. To install the receptors the following procedure is followed. One long vertical stick carries the central and the top microphones. Three short sticks with a microphone at the end are placed around the long one on a circle

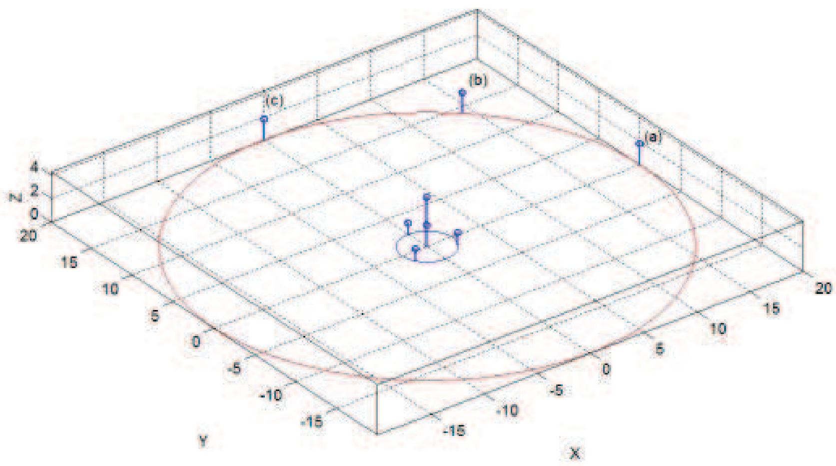

Fig. 13. Receptors array and source position for the experimental setting.

of radius $\frac{4}{\sqrt{3}}$. They position the lower microphones at 1m over the floor. The source height is coincident with the height of the centre receptor. Positioning of the source, end of gun barrel, with respect to the receptors centre is made with an estimated error of $\pm 5cm$, which is less than 0.6% of the radius. Positioning of the receptors is checked with a theodolite Model Wild-Leica-T2 of 1 second precision.

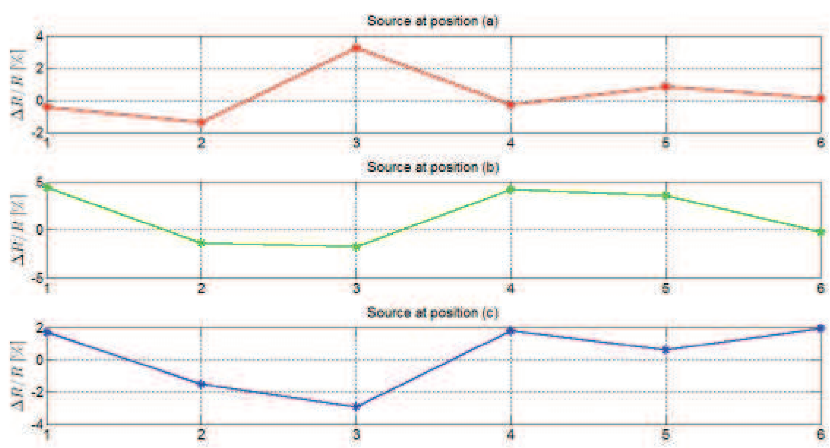

Fig. 14. Error computed for three gun shots at each source location. Notice that all of them are within the prescribed bound of 8%.

For each position the experiment is repeated six times. Figure 14 shows the results. It can be seen that the relative error remains within the 5% level, which is one third of the upper bound forecasted of 17%. The safety margin is in agreement with the ones that can be seen in Figure 5 (b).

8. Discussion, conclusions and future research

Nowadays a DSP can carry on thousands of operation per second. At first glance to solve the localization problem in one step or ten could be considered irrelevant. To add one more receptor does not seem a big deal because redundancy is a common practice. To solve the HL non-linear original problem or the linear expanded one can be a matter of taste. But it is not. A linear system allows using well known, well established error propagation methods. Equation 28 is an invaluable tool for the one in charge to design a source localization system. For a given array a directivity pattern can be computed and observed easily with software like MATLAB®. Plots like the ones in Figures 9 and 10 will help in designing the acquisition system. The experiment shows that the upper bound computed is reliable.

Three points, among others, have not been reviewed in this work: the uncertainty in receptors position, the effect of using redundant pairs and adequate receptor pair selection for a given receptor geometry.

It is clear from this work that matrix **M** condition is important. It can be computed if the receptors pattern, receptors pairs and source position are known. The condition does not depend on geometrical or TDOA uncertainties. Geometrical uncertainties will add or will establish the upper bound for the $\boldsymbol{\delta}\mathbf{M}$ matrix norm. A rule of thumb is that receptors position uncertainties must be in the order of $v \cdot e_{max}$. For a time uncertainty of $10\mu s$ in air, the number is 0.35 cm. For a high frequency acquisition and very low errors in TDOA the ability to correctly position the receptors centre will impose the limits.

The use of redundant pairs seems plausible. At first glance it can be imagined that many of the selected pairs will produce a better problem conditioning or a more robust pseudoinverse for a given source location. But a meaningful error reduction can be obtained only if the condition is improved.

It has been shown that pair selection for a given receptors array is paramount. Recently (Gillette & Silverman, 2008) produced a redundant system by introducing more equations. The equations do not came from the introduction of more receptors but for arranging new pairs with the same existing receptors. In the author's opinion the same reasoning of the previous paragraph can be done.

This work does not give a guideline on receptors orientation and preferred receptors pairs. Research is carried on in order to develop a rationale to reduce the conditioning and is a matter of future research. It is worth noting the work of (Yang & Scheuing, 2005) as an effort to find a good receptors distribution geometry.

This work do shows a 2D and a 3D robust detection system and a simple way to validate them or any other configuration.

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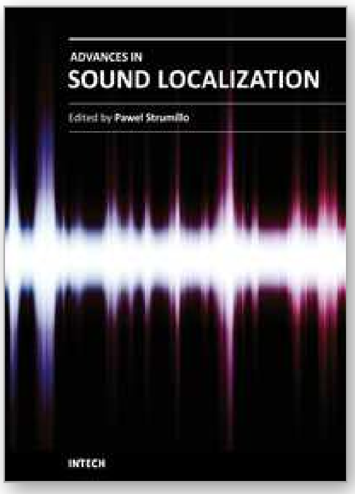

Advances in Sound Localization

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Sound source localization is an important research field that has attracted researchers' efforts from many technical and biomedical sciences. Sound source localization (SSL) is defined as the determination of the direction from a receiver, but also includes the distance from it. Because of the wave nature of sound propagation, phenomena such as refraction, diffraction, diffusion, reflection, reverberation and interference occur. The wide spectrum of sound frequencies that range from infrasounds through acoustic sounds to ultrasounds, also introduces difficulties, as different spectrum components have different penetration properties through the medium. Consequently, SSL is a complex computation problem and development of robust sound localization techniques calls for different approaches, including multisensor schemes, null-steering beamforming and time-difference arrival techniques. The book offers a rich source of valuable material on advances on SSL techniques and their applications that should appeal to researches representing diverse engineering and scientific disciplines.

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