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CA in Urban Systems and Ecology: From Individual Behaviour to Transport Equations and Population Dynamics

José Luis Puliafito
Universidad de Mendoza
Argentina

1. Introduction

One way of seeing Cellular Automata (CA) is as cell- based computational models for describing the evolution of spatially distributed systems. Each cell represents a "local" state of the system that can vary according to its past states and to the present states of a "vicinity" of cells, following some set of relations known as "transitions rules".
More important than how these transition rules are (i.e linear, non linear, discrete, etc,), is that distant parts of such system can interact one to another only through its neighbours; in other words, what we are actually considering in CA models, is that the system obeys *the principle of locality*. For this seems to be the case of most systems in nature, CA models have found potential applicability in a wide variety of phenomena, ranging from macroscopic scales, like urban systems, down to microscopic scales like in solid state physics. J.F Nystrom (2001) has even argued in favour of the idea that fundamental laws of physics should arise from simple transitions rules of some Universal CA, in a structured space following R. Buckminster Fuller's synergetic geometry.
This brings us to a central point, which is that, in nature, space is as essential as time for describing any process; disregarding if we are more interested in watching at the temporal behaviour of certain group of state variables or if we are more interested in taking static pictures of some distributed properties in space, there will be always a spatiotemporal evolution process taking place behind.
A good example are urban and environmental systems; social scientists have been discussing since long ago how population and economy of regions interact and evolve through the years, while geographers and urban analysts have been doing it looking at its spatial structures. Both have contributed in equal parts to our present understanding of sustainable development. However, ¿can economists explain development without considering where was located the infrastructure support? or ¿can urbanists explain the structure of a city without considering the historical circumstances? Both views tend to describe one aspect of the evolution looking at the other as frame constrains, usually given in terms of literal stories. The same happens in many other fields of science treating with complexity.
A more modern view stands on the growing availability of informatics tools, and pushes towards constructing spatiotemporal models. But this is not a simple task; most attempts

flow amongst "top-down" approaches, based on continuity equations in partial derivates - including fractional order diffusion equations for explaining behaviours with long-range dependency (Angulo J.M, et al. 2001)- and "bottom-up" approaches, mostly given by discrete rule-based CAs (Park S. and Wagner D. F., 1997). While the former go more for the classic type, which puts emphasis in an extensive view of the system (that is in its behaviour and consistency as a whole), the latter are more of the evolutionary type, giving more emphasis to a detailed view, trying to describe self-organization and innovation proper of complexity.

However, as discrete space-time models have become more attractive, due to the intensive use of dynamical raster GIS (Geographic Information Systems) (Batty, M, 1996; Mitas L. 1997 et al.; Park S. and Wagner D. F. , 1997), different kinds of CAs , as well as seamless discrete-continuous approaches, are opening new theoretical avenues.

For instance, as regards mobile agent-based CAs, the number of agents (population) grows initially from implanted "seeds" reproducing and spreading on the back cells, in accordance with transition rules and information on their development capability held in different GIS layers (Batty M. and Torrens P., 2001).

Not far from these, some seamless continuous/discrete approaches face the modelling problem in terms of a particle-field duality, just as in the path sampling method used in physics for solving continuity equations (Mitasova, H. and Mitas L., 2000). Multidimensional complexity can be treated herein by means of particles and fields in different scales. Likewise, some approaches use spatiotemporal convolution equations with kernels limited in space (i.e gaussian or similar), or even space-variant kernels (heterogeneous) (Wikle C., 2001), in a way that complex spatiotemporal processes are described as the propagation of dispersive or non-dispersive wave packets.

The distinctive feature of the seamless and mobile agent-based CAs models is that they use particles - or rather pseudo-particles - as an attempt to match the continuum response of an extensive view with the discrete and evolutionary behaviour of a detailed view; these can be considered as descriptions halfway between classical physics kinetics and unstable system dynamics. Issues to be primarily considered herewith are: a reduction in the amount of information involved, the interlacing of layers or embedding of contributing models, as well as the setting of scales for representative particles of the included processes.

The modelling of ecological systems offers also good examples; the emergency of complex spatiotemporal patterns in the population dynamics of certain species has been since long time of great interest in Ecology. Random walking and diffusion equations are used to describe the movement of animals in their own environment, and to forecast their spatial distribution under the influence of the diverse territorial heterogeneities (Jeanson R. et al., 2003). Such models are found on a regular basis but there is still a long conceptual way to go.

Complex spatiotemporal patterns in the activities carried out by some social insects, such as ants and termites, reveal that individuals can collectively do better at performing tasks than isolated. This is not only observed in the typical pattern scales, usually far larger than the size of individuals, but also in their shape, featuring arrangements in various delicate and regular structures. Despite individual randomness and limitations, collective structures arise effectively in response to several functional and adaptive requirements (protection against predators, the substrate of social life and reproduction, thermal regulation, etc.) (Theraulaz G. et al., 2003).

Twenty years of research have revealed that the origin of hierarchical complexity is more a consequence of the multiplicity of individual responses to stimuli, derived of relatively

simple behaviours, than of the ability of each insect to process a large amount of information. Hence, the resulting patterns seem to emerge from non-linear interactions among individuals and between individuals and their environment, all this through mechanisms like templates, stigmergy and self-organization (Theraulaz G. et al., 2003; Ball P., 1998).

These features in particular have pushed traditional temporal dynamic analysis towards incorporating more explicitly space (Spatial Ecology), through metapopulation and transition rules models like the Cellular Automata. The interest is in the link between the spatial structure of the environment and of the occupying population with the species features, their development, survival and even their diversity [Pascala, S. and Levin, S. 1997; Tilman,D and Kareiva, P. 1997].

The latter also points at phenomenological models with differential equations in partial derivatives, such as the reaction-diffusion equations based on the Alan Turing model (1952). This was originally applied to the morphogenesis of skin spots in animals like zebras, jaguars and leopards, and later extended, by several authors, to nearly all the range of biological and ecological patterns, being cellular morphogenesis and the spatial segregation of species included. Basically, it describes the non-linear interaction of two- species concentrations: one is an "activator" (rather of local action) and the other an "inhibitor" (of a longer reach), so periodical structures rise as a consequence of different diffusion speeds (Meinhardt, H., 1982). An outstanding example in the biological level is the chemotaxonomic spatiotemporal behaviour of two bacterial species, which can be externally controlled and shapes propagating waves and patterns (Lebiedz D. and Brandt-Pollmann U., 2003).

The variety of approaches is not as much a consequence of the type of system under consideration, as of the need to integrate multidimensional interactions at various levels, where a spatiotemporal model rises from any of the following (Popov V.L. and Psakhie S.G., 2001):

a. the macroscopic dynamics of the system and by finding solutions to partial integro-differential equations (if known);
b. the microscopic dynamics of the real system and by finding interaction laws through molecular dynamics methods or first-principle methods;
c. the replacement of the real system by a certain medium model (having rougher microscopic behaviour but the same macroscopic dynamics as the former), while formulating proper transition laws.

The third type of approach is where CA and seamless models are actually placed; in particular the use of Cellular Automata has widely spread because of its intrinsic capacity to simulate complexity, specifically self-organization and innovation. However, and going back to the beginning, it should be bared in mind that such models are eventually tensorial computational methods based on finite spatial cells, thus defining an *excitable elastoplastic medium* that represents the species-space/environment system in question. In any case, Cellular Automata can successfully model several types of excitable media, not only due to some insensibility of "macroscopic" dynamics in relation to the structure and nature of interactions in their "microscopic" order, but also to the fact that most systems and even hypothetical mathematical objects, are described by some kind of *transport equations* (Popov V.L. and Psakhie S.G., 2001).

It can be surprising that despite the obvious conceptual division between the animate and the inanimate worlds certain population phenomena are described similarly. In fact,

reaction-diffusion systems are frequently found and rather important in many areas of physics. For instance, through the *band theory*, crystalline solids such as semimetals and semiconductors can be described on an electrical basis by means of two charge transport equations, one on electrons (negative charge) and other on holes (positive charge) in specific energy bands. The concentration of each carrier can be described in a similar way to Turing's, since both transport equations are coupled through the generation/recombination of carriers, similar to predator-prey interactions in Ecology.

As the active and inhibitor species rise naturally herewith, leading to stigmergy-like based mechanisms, *¿is it possible that some of the methods and principles used in solid state physics be also applicable to ecological and urban systems?*; if so, and even though living systems would have more plasticity, a crystallographic metaphor would be useful to model certain aspects relevant to spatiotemporal evolution in social species, at least under stationary or quasi-stationary conditions. This approximation has been studied and applied to spatiotemporal modelling of urban areas, thus showing its viability and potentiality at explaining several heterogeneously distributed urban phenomena (Puliafito, J.L. 2006).

We must bear in mind herewith that at describing the spatiotemporal dynamic evolution of populations of real individuals through transport equations, one is not only implicitly considering the existence of definite interactions of the species with its space/environment, but also stability regions in the associated state space that are similar to the energy bands in solid materials (multistability; Theraulaz G. et al., 2003). Therefore, either systems can stray away slightly from the previous dynamic relations so that restoring forces will tend to preserve evolution within a states region (linearity, elasticity), or can stray away largely with transitions among regions (non-linearity, plasticity). In this sense, experience proves that social behaviour and complex and regular spatiotemporal structures usually emerge under conditions where species reach some critical spatial density.

In such train of thoughts, herein an ecosystem is not the mere association of interactions in terms of the whole, or the sum of strongly-interacting independent elements, but a rather coherent sum of elementary units made up of living individuals and their immediate surrounding space-environment; the latter being regarded as a multidimensional representation of the resources needed for its survival, physical space in itself included.

A review of some of the investigations done by the author dealing with the above questions is presented in this chapter.

2. A bridge from the stochastic behaviour at the individual level to the associated behaviour at the collective level

A research program dealing with all these features should start at modelling generic individuals as automata exploring the environment, capturing and feeding from discrete units of matter and energy, thus developing some sort of random-walk mostly confined to a certain territory. This leads, as it will be briefly shown here, to spatiotemporal behaviours that resemble quantum stochastic systems. Apart from the theoretical interest, it yields the possible application of simplifying analogies to the population dynamics of dense concentrations –even in a restrictive manner -, as it lays a bridge towards a collective description similar to the band theory of solids (Puliafito, José L and Puliafito, S. E. 2006).

2.1 First class Bioautomaton – Langevin equation

Let us consider some type of autonomous homeostatic device, which, for our purposes, we can call *bioautomaton*. In an elementary class (first class bioautomaton), such an ideal device

is an open biophysical system moving step-wise and randomly in space, aiming at capturing, storing and processing discreet units of matter/energy (resources), thus assuring its "survival". It can be seen as a black box excited by Poisson impulses and responding with limited spatial displacements through an appropriate transfer function. Due to these mechanical displacements, the resources that are available in the near-by space–environment can be captured. The latter can also be considered as a black box excited by a random step function, giving such energy impulses to the bioautomaton. (Fig 1)

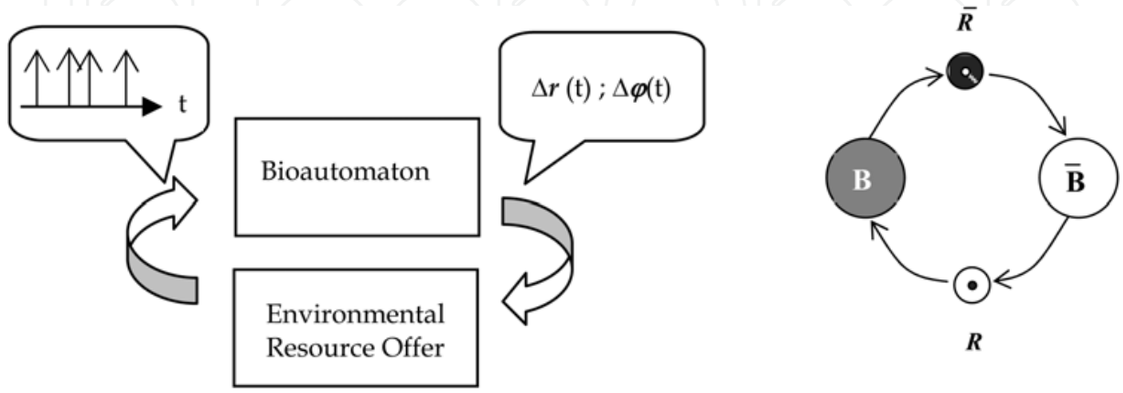

Fig. 1. Left: Scheme of the bioautomaton/environment interaction; the bioautomaton has an internal reserve of energy . Heat exchange fluxes are not displayed for simplicity. Right: The bioautomaton-medium interaction seen as a closed system; B particle is the bioautomaton and $\bar{B}$ antiparticle represents the near-by medium; the exchange is given by a discrete flux of resources R and of residues $\bar{R}$.

The evolution in the state space of the whole system (bioautomaton / environment) is a stochastic process, depending, on one side, on the efficiency of the bioautomaton to collect resources and to adequately use its internal energy reserve and, on the other side, on the environmental offer and its renewal capacity. There will be stationary or quasi-stationary random solutions, as long as the expected value of the rate of energy consumption per period between impulses is higher or equal to the average minimum consumption rate:

$$< \boldsymbol{\varepsilon_{\delta i} / T_{\delta i}} >= \varepsilon_0 / T_0 \geq (\delta\varepsilon/\delta\tau)_{min} \quad (1)$$

Eq. (1) can be considered as the *first-class functional* of a bioautomaton or "survival" functional, where $(\delta\varepsilon/\delta\tau)_{min}$ plays a similar role to basal metabolism in living organisms. Here, the device's "survival" consists of a set of conditions resulting in the sustenance in time of its internal reactions, within a relatively steady range, balancing dynamically the energetic exchange with the environment.

Since under stationary conditions the bioautomaton's movement and survival are limited to an optimal use of its internal energy reservoir, a certain potential function can be associated to this storage, as a measure of the probability to capture new resources. This can be defined as a spatiotemporal convolution between a certain window *S(r)*, representative of the perception and capture radio of the bioautomaton, and the spatial density of resources ρ(*r*) (ζ it's a process constant):

$$U_{br}(\boldsymbol{r}, t) = -(1/2\pi\zeta) \int_{\tau} S(\boldsymbol{r}(t) - r') \rho(r') \, d\tau \quad (2)$$

When taking a gaussian window and a localized distribution of resources (eg. disc type) a "well" spatial function is obtained, which recognizes approximately the regions given by the three degrees of homogeneity in classical mechanics (K_h= 2 parabolic for $r \leq 1{,}5\ r_0$, K_h=1 linear for $1{,}5\ r_0 < r < 2r_0$ and K_h= -1 newtonian for $r \geq 2\ r_0$, with r_0 as a characteristic radius). Unlike a classical potential, which is determined by the medium, U_{br} depends on what the environment can offer as regards means, as well as the degree of utilisation (or efficiency) the bioautomaton can get out of them; that is, it represents *the expected interaction bioautomaton-environment*. Thus, its interpretation as a potential function is conditioned to the resulting movement being a stationary or quasi-stationary process, or, in other words, being an efficient estimator of the spatial distribution of resources.
A generalized *Langevin stochastic differential equation* derives from the previous definitions for the bioautomaton, which can be analysed from partial solutions for the homogeneity regions above given (3).

$$m.\frac{d^2}{dt^2}\,\bar{\mathbf{r}} + f\,\frac{d}{dt}\,\bar{\mathbf{r}} - \frac{\partial}{\partial \boldsymbol{r}} U_{br}\,(\,\bar{\mathbf{r}}, t\,) = \bar{\mathbf{F}}_{ex}(t) = m.\,\bar{\boldsymbol{n}}(t) \tag{3}$$

Note that equation (3) has reduced the quite complex interactions to the dissipative stochastic movement of a m mass and f friction punctual particle, subjected to certain excitation and restitution forces dependent on the $U_{br}(\,\boldsymbol{r}, t\,)$ virtual potential. Formally, it can be interpreted as a generalized type of Brownian movement, where $\bar{\boldsymbol{n}}(t)$ represents white shot noise.

2.2 Behaviour in K_h = 2 zones

Near the distribution centre of $\rho(r)$, $U_{br}(r)$ takes the shape of a second degree parabola (K_h = 2), in a way that the potential gradient (the potential reactive force) is approximately proportional to displacements:

$$\frac{d^2}{dt^2}\,\bar{\boldsymbol{r}} + \beta\,\frac{d}{dt}\,\bar{\boldsymbol{r}} + \omega_0^2\,\bar{\boldsymbol{r}} = \bar{\boldsymbol{n}}(t) \tag{4}$$

with $\beta = f/m$ and $\omega_0^2 = k/m$, which corresponds to the movement of a particle in a viscose medium under the action of a central field. In the case of the bioautomaton, β must be understood more generically as the relation between the total dissipative forces (outer and inner) and the total equivalent mass that includes the inert mass and the associated biomass. The bidimensional problem can be described in terms of an analytical process with a complex random variable $\boldsymbol{r}(t) = \boldsymbol{x}(t) + j\,\boldsymbol{y}(t)$. The stochastic processes $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ are also described through independent differential equations of the type given in (4), coupled through proper coefficients $\omega_{0x} = \omega_{0y} = \omega_0$ and $\beta_x = \beta_y = \beta$, which presupposes spatial isotropy.
The essential properties of the movement originate in the characteristic equation for the autocorrelation of any of the two components. The weakly dumped harmonic case is of particular interest $(\beta/2\omega_0 < 1)$, as it describes a range of solutions corresponding to a *stochastic oscillator* in which trajectories are stochastic "orbits":

$$\boldsymbol{r}\,(t) = \mathbf{r}\,.\,\exp\,[-0{,}5\,\beta t + j\,(\,\omega_d\,t + \boldsymbol{\varphi}\,)] \tag{5}$$

with $\omega_d = \omega_0\,(1 - \beta^2/4\omega_0^2)^{1/2}$; $\mathbf{r} = (\mathbf{x}^2 + \mathbf{y}^2)^{1/2}$; $\boldsymbol{\varphi}$=atan $(\mathbf{y}/\mathbf{x})$. This case gives the longer possible *average life times* of the bioautomaton ($\tau \sim 2/\beta$), thus becoming the most appropriate for the definitions above given.

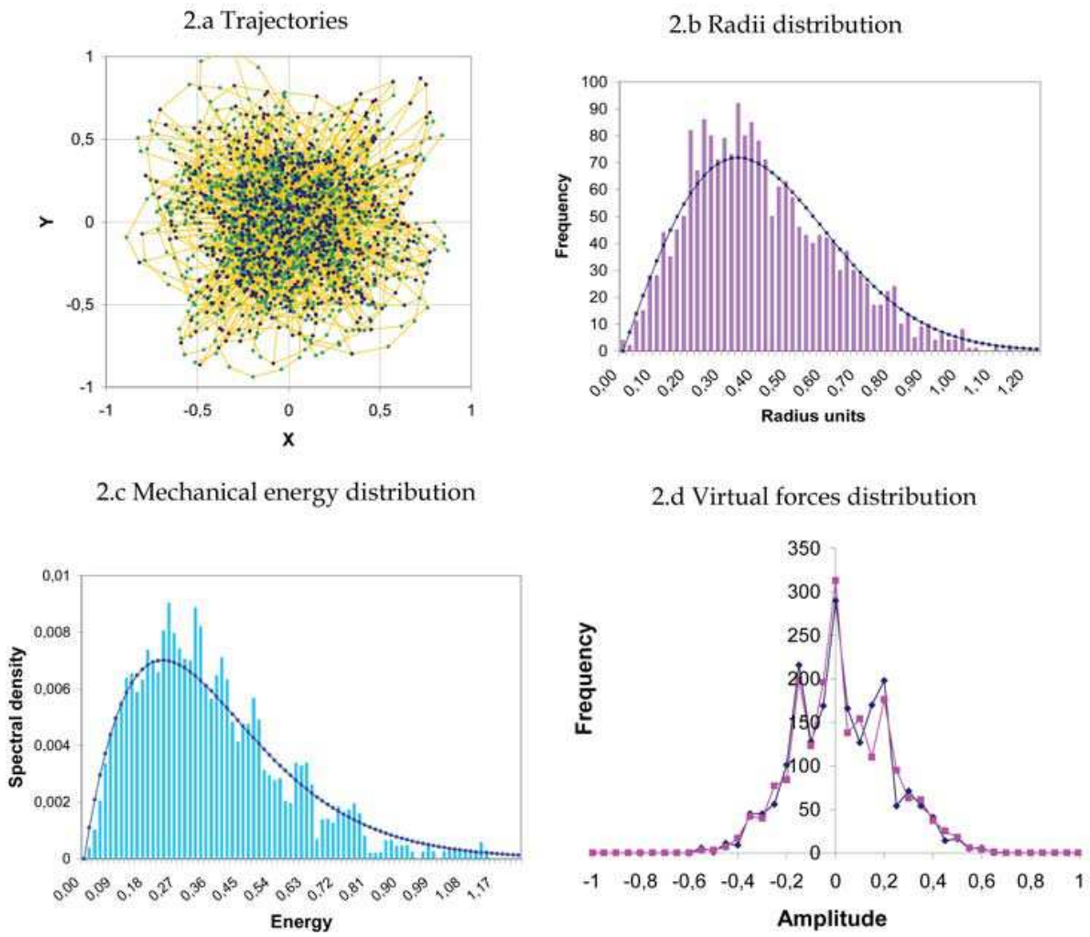

Fig. 2. Numerical simulation of bioautomaton in K_h= 2 zone, with Poisson impulses excitation and random directions; simulation parameters are: Mass m=10; Friction f=3.145; Elastic constant k= 0.987 (ω_0 = 0,31416; β = 0,3145; ξ= 0,500)

Considering in particular when **n**(t) is shot noise, representing discrete supply events, the resultant of the apparent excitation forces $\bar{\mathbf{F}}_{ex}$, can be expressed as follows:

$$\bar{\mathbf{F}}_{\mathbf{ex}}(\boldsymbol{s}) \approx \sum_i \gamma.\Delta s.\delta(s-\boldsymbol{s_i})\ \ \bar{\boldsymbol{s}}^{\circ} \rightarrow \bar{\mathbf{F}}_{\mathbf{ex}}(t) = \sum_i (\boldsymbol{\varepsilon_i} / \bar{v_i})\ \delta(t - \mathbf{t_i}) \qquad (6)$$

In the first expression of eq. (6) γ is the apparent density of energy per longitude unit, Δs the mean step and $\delta(s-\boldsymbol{s_i})$ the delta Dirac function for $s=\boldsymbol{s_i}$, thus describing an impulse train with events located in (i) random positions over the $\boldsymbol{s}$ trajectory. Random positions $\boldsymbol{s_i}$ can be considered as independent events, resulting in a Poisson process with a density parameter α_s =N/<$\boldsymbol{s}$> of points, determined by the distribution of resources in space, and a expected value <$\mathbf{F_{ex}}$> = γ δs α_s. The second expression of eq. (6) is in explicit function of time, where $\bar{v}_i$ is the instantaneous vector velocity of the bioautomaton over trajectory $\boldsymbol{s}$ and $\boldsymbol{\varepsilon_i}$ the specific energy of the resources captured in s = $\boldsymbol{s_i}$ random positions. In this way average trajectories in stationary processes will *depend directly on the number N of captured resources* . In fig.2 a numerical simulation of the stochastic differential equation (4) is shown. Figure (2.a) shows the trajectories and the encounter positions with resources corresponding to the

input impulse sequence. Hence, *a cloud of excitation points* is associated to the trajectories, which is denser in the centre and dilutes to the outside. As expected, the position radii follow a Rayleigh distribution (fig 2.b); accordingly, the distributions of x-y distances, as well (as of x-y velocity components), follow a gaussian form. The power spectrum of radii has also a concordant distribution in frequency, given by a second-order transfer function shifted by the natural angular frequency of the system.

Some of the peculiar properties of shot noise excitation arise already when the distributions of the mechanical energy of the system and of the virtual forces are considered. The distribution of the mechanical energy (fig 2.c) (as in addition to its transitions) has a spectral density that nearly follows a Planck type distribution with two degrees of freedom (the correlation factor is approximately 0.97):

$$F(\varepsilon) = A.\ \varepsilon^2 / \exp\left(\varepsilon/\varepsilon_T - 1\right) \tag{7}$$

where ε represents energy, ε_T a "thermal" equilibrium energy , and A a proper characteristic constant. The third factor is the Bose-Einstein distribution, but here ε is squared instead of at a cubic power as in the Planck radiation law. In this sense, while the distribution of the x-y instant components of the input virtual forces follow a sine law (vector decomposition in uniformly distributed random phases), their mobile media (within a window as long as the feeding period T*) show the expected tendency to gaussian distributions, but with a clear fine reticular structure (fig. 2.d).

The former results show the need to consider the *quantification* of the bioautomaton's behaviour as regards the *number of capture events associated to the noise density* (N). In fact, it is possible to analyse the device behaviour by decomposing its excitation in terms of *a random sum of k- input modes*, where k=1 always represents one input impulse per feeding period T*, k=2 represents a random sequence of just two impulses per period, k=3 represents three impulses, and so on. Denoting $\check{z}(t)$ as the mobile average of excitation forces $z(t)$, one gets the distribution $f_{\check{z}}(\mathbf{z})$, shown in figure (2.d), by doing

$$f_{\check{z}}(\mathbf{z}) = (1/\pi)\ \sum_k\ b_k\, f_{zk}(\mathbf{z}) \tag{8.a}$$

where: $$f_{zk}(\mathbf{z}) = \int_0^{\Omega_{max}} J_0{}^k(a\,\Omega).\cos \Omega z\ d\Omega\ ;\ b_k\ =\ (\lambda.T^*)^k\,.\,\exp\left(-\lambda.T^*\right) / k! \tag{8.b}$$

In eq. (8.a) and (8.b) $f_{zk}(\mathbf{z})$ is the *k-modal component* of the distribution density, expressed as the cosine Fourier transform of the k power of the first kind Bessel function of zero order, in the sthocastic frequency domain Ω; Ω_{max} is a cut-off stochastic frequency which rises from the forces reticular structure in stationary conditions. At the same time b_k is a weight factor of mode k, expressed as a *Poisson probability coefficient*.

A k=0 mode can be also defined, which means that no impulse is arriving (no resource is captured), so that the movement is carried out just by the use of the internal storage of energy. According to the *virial theorem* (case K_h= 2), the total average mechanical energy is *half of the system total energy in the k=1 mode*.

Taking into account the previous considerations it is also possible to decompose in k-modes the average value of the angular moment, as well as of the mechanical energy. In fact, the expected angular moment when excited by N capture events per feeding period is:

$$\langle \mathbf{M}_\phi \rangle^2 = N^2\, \varepsilon_0{}^2 / \omega_0{}^2 = m_0{}^2 \omega_0{}^2 r_0{}^4 \tag{9}$$

In eq. (8) one can immediately define an *action constant* $\varepsilon_0/\omega_0 = a/2\pi = \check{a}$, which represents the average rate of the energy consumed per capture event, associated to *a certain consumption capacity* of the bioautomaton. In this way, considering k=0 as the *basal mode*, k-modal components in K_h=2 zones for the angular moment and the energy can be written as:

$$M_{\phi 0k} = (k + ½) \check{a} \tag{10}$$

$$E_{0k} = (k + ½) \check{a}\, \omega_0 \tag{11}$$

which have the same form as in the *quantum harmonic oscillator*. However, one must consider that these are not pure states but the *average values* of *associated state groups*, in a way that their composition, through the Poisson coefficients given in (8.b), define the general *mixed state* of the Bioautomaton in zones K_h=2.

As the internal energy of the bioautomaton, given by kinetic energy added to the storage energy, is $E_i = m_0 \upsilon^2$, where $\upsilon = v_0$ can be taken as a *typical velocity of bioautomaton-medium interaction*, other virial relationships can be drawn from here in terms of *associated wavelengths* as well as stating the relativity of the average exploration radio and the effective mass of the bioautomaton, respect of its actual average velocity *v*. In fact, alterations can occur while the bioautomaton still keeps the stationary regime; if ω is the apparent frequency of the average forced excitation regime, produced by the search movement of the bioautomaton, the relative frequency ω/ω_0 results from the variation of the average relative velocity bioautomaton v/υ. These relationships express the average spatial response of the bioautomaton- medium feedback system when trying to keep its stationary regime under alterations of interaction parameters.

Equations (10) and (11) and their associated virial relationships establish as a whole, an *allometric relation* between the resource flux and the device effective mass, of the type $(\delta\varepsilon/\delta\tau)_{min} \sim a\, m_0{}^b$ similar to the ones observed in the real biological world, according to the *theory of biological similitude* of Max Klieber (1932) and to research works carried out more recently like Hemmingsen (1960) and Günther et al. (1992). [1]

2.3 Behaviour in distant zones

If the device is deployed far from the resource centre, that is, within an interval of distances of the virtual potential corresponding to regions K_h=-1, the generalized Langevin equation has no lineal term on distance but one of the Newtonian type (-1/r). In this case two kinds of behaviours can be basically considered, one that is stationary and another that is a transition from K_h=-1 to K_h=2.

In the first case (K_h=-1) the average trajectories are longer and with less chances of capturing resources. A group of stationary solutions here demands lower frictions or higher energy per resource. Besides, the selective character of the non-lineal form of the differential equation makes stationary solutions critically dependent on the set of values chosen for the device parameters and its excitation. In this case trajectories are also mostly confined into certain average radio, but with spatial distributions that are compounded of various

[1] The magnitude $(\delta\varepsilon/\delta\tau)_{min}$ represents the basal metabolism, m_0 is the mass expressed in kg weight and *a* and *b* are proper allometric parameters. Max Klieber first proposed the allometric relation for most mammals adopting b=0,738, and Hemmingsen and other authors extended such relation even for different homeothermic, poikilothermic and unicellular species with b=0.75.

anisotropies. This is partly due to an additional degree of freedom (stochastic rotation), and partly to the group composition of k-modal solutions similar to the ones seen before.
The second case refers to a bioautomaton having inadequate parameters for keeping a stationary dynamic behaviour in region K_h=-1, but instead they are adequate for region K_h=2. Sinulation tests show that there is a quite fast transition from the first to the second region, passing through region K_h=1. Once the device reaches the parabolic region, its behaviour becomes stationary again, as described before.

2.4 Comparison with quantum-stochastic systems

The former points suggest certain similarity to quantum stochastic systems, mainly due to the discrete character of the resource absorption and that the movement takes the form of a random step sequence, confined more or less to a certain exploration area.
In order to go deeper into this similarity, it is necessary to focus on the dynamics of the bioautomaton-environment system from the possible transitions of states. In this sense, apart from the stationary movements seen above, there can exist forced displacements that would result from the *virtual movement of the resource centre.* This would occur, for instance, when the resource flux diminishes in an originally dense zone. A *drift* or a *migration* of the bioautomaton can be conceived here. In fact, if diminishing the potential storage turns into an estimation of the distance to the resource centre equivalent to a K_h=1 region, slight state changes would force the bioautomaton to "accompany" the virtual displacement of the centre (drift). If diminishing the potential storage becomes so large that the estimated resource centre occurs at a virtual distance equivalent to a K_h=-1 zone instead, a transition would take place (migration).
This can be alternatively appreciated from the *Chapman-Kolmogorov* equation, which is a property of the transition functions in Markov processes. Due to Kolmogorov, *progressive and regressive diffusion equations* can be derived from it, being the regressive the *Fokker-Planck* diffusion equation. As a Markov process (increasing times) is also so in an inverted manner (decreasing times), the progressive equation can be understood as an *antidifussion,* or as the diffusion of trajectories of an *antiparticle*, which would represent the virtual motion of the resource centre. Hence, interaction must be seen as a rather symmetric exchange between two poles; if the position is fixed in the bioautomaton, an incident flux of resources is seen, while if the position is fixed in the resource centre an incident flux of "voids" (or residues) is seen (fig. 1 right).
In the strict stationary case, the progressive and regressive diffusion equations present a *closed symmetry*, thus implying that the consumed resources and the residues produced by the automaton are equalled to the resources produced and residues processed by the environment; in a drift (the bioautomaton follows closely the resource centre) there is a *practically closed symmetry* (quasi-stationary regime), and it is possible to refer such equations to a system of mobile coordinates leading back to the previous case. Finally, symmetry breaks down definitely during a migration and the said equations express two rather independent trajectory fluxes, one for the particle and the other for the antiparticle.
As for what was stated above, the pair of equations generalized for stationary or quasi-stationary bidimensional movements (with means and variances not depending on the absolute position) show somehow the *expected flux of resources and residues* for growing times ($t > t_0$) from the point of view of particle B:

$$\begin{cases} \partial p/\partial t + \bar{\nabla} \bullet (\bar{v}_F\, p) + D\, \nabla^2 p = 0 \\ \partial p/\partial t + \bar{\nabla} \bullet (\bar{v}_R\, p) - D\, \nabla^2 p = 0 \end{cases} \quad (12)$$

with $p = p(x,y,t;x_0,y_0,t_0)$ the bidimensional transition probability, v_R a "regressive" velocity, v_F a "progressive" velocity and D a diffusion coefficient. Following for instance Smolin L., 2007 (inspired in Nelson E., 1966) a wave equation similar *to Schrödinger's equation* can be derived, with $ă^2 = 2m_0^2 D^2$:

$$j\, ă\, \partial\psi / \partial t = - (ă^2/2\, m_0)\, \nabla^2\psi + U_b.\, \psi \quad (13)$$

Two fields are here defined, ψ and its conjugated ψ^*, associated to *the normalized average flux density* of resources and residues, in a way that their product is the transition probability $\psi\psi^* = p\ (x,y,t;x_0,y_0,t_0)$, consequently establishing the quantum similarity of the system bioautomaton-medium[2].

Finally, the k-modal decomposition of ψ and ψ^* can be incorporated, introducing sets of orthogonal wave functions or *associated wave function groups* $\psi = \sum_k B_k\, \psi_k\, (x,y,t;x_0,y_0,t_0)$ and $\psi^* = \sum_k B_k\, \psi_k^*(x,y,t;x_0,y_0,t_0)$; where $B_k = b_k^{1/2}$ with b_k the Poisson coefficients. They describe the expected configuration of resources and residues by means of its k-modal wave functions: the ψ_k ones associated to the incoming or incident flux and the ψ_k^* associated to the outgoing or reflected flux, thus producing a *mixed general state* of the bioautomaton, as compared to a *quantum stochastic system.*

However, it should be emphasized here that the bioautomaton is not a quantum system but a *classical system with quantum similarity,* which eventually falls near the treatment of quantum dissipative systems given in modern ontological interpretations of Quamtum Mechanics, such as those of *consistent histories,* according to which the purpose of a quantum theory is to predict instances of probabilities of various alternative histories[3]. The consistency criterion states that a system's history can be described on the basis of classical probabilities for each alternative history, compatible (consistent) with Schrödinger equation.

2.5 Transition to collective systems

The bioautomaton can only be considered as a very vague and simplified representation of a biological organism. Notwithstanding, taken as a basic component of a relatively stationary population, and far from describing the life cycle and reproductive function of living beings, still can be used for studying some aspects of real collective behaviours. For that, it is not

[2] The average Hamiltonian of the bioautomaton-medium system is H (x ;y) = kinetic energy + resource energy + storage = $½\, m_0 v_0^2 + \xi' m_0 v_0^2 + U_b$; the resource component $H_{res} = \xi' m_0 v_0^2 = m_0 u_0^2$ plays here a similar role to Bhom's *quantum potential* (Bohm, D. , 1952; Smolin L., 2007)

[3] This is confirmed in various elements as in the generalised Langevin equation (3), which is equivalent to the one proposed by Magalinski in 1942, later continued by other authors as a general method to analyse quantum dissipative systems (Hänggi P., Ingold G-L., 2005). Or in the conclusions reached by Wang Q. A. (2005), which states that there exist commuting and uncertainty relations in the classical stochastic processes similar to the ones predicted by Heisenberg, and also by Faigle U., Schoenhuth A. (2006), which establish a general type of stochastic models with quantum prediction (Quantum Predictor Models), out of which the bioautomaton would be a subclass. Likewise, it is sustained in the very derivation of the Schrödinger's associated equation, which although following a formalism similar to Fényes and E. Nelson's (Smolin L. 2005), here it is relatively direct and gives rise naturally to the equivalent of Bohm's quantum potential.

hard to imagine bioautomata that are being subjected to second class functionals, by which mutual interactions can have complementary or exclusion symmetries, or even subject to third class functionals by which group survival can be optimised. Beyond the interest in drawing or not quantum analogies, its importance resides, mainly, in the effects the extension of the previous outcomes have over the behaviour of an ensemble of devices, and eventually over the population dynamics of biological species.

In this sense, the fact that the statistical behaviour of a bioautomaton can be represented in average by means of wave functions, allows to glimpse also stationary or quasi-stationary solutions regarding group behaviour, resulting from the superposition of individual wave functions. For such reason it is quite possible that a bioautomata ensemble tends to a sort of *periodical spatial structure* in *fluctuating cells*. Hence, dynamics can be described in terms of transport equations (arising from eq. 12) and framed within some *appropriate band theory.*

Accordingly, a *basic equivalent model* could be outlined in terms of a virtual substrate with two energy bands: a population band and a resource band where their associated pseudo-particles – *the inhabitant and the recurson* – represent (in principle in an anti-symmetrical way) the interacting population and their space- environment structure correspondingly. Within this context, a high-density ecosystem, can be compared in some sense to an elastoplastic network and thence treated in a similar way to the solid state of matter; that is, as state transitions in a *pseudo-crystalline virtual substrate* subdued to a general exclusion principle.

3. Modelling urban dynamics based on an analogy to solid state physics

Urban regions in particular, being high density human ecosystems, tend to present statistical virial relations and highly structured territorial occupations. The crystallographic picture can bring a new insight to the modelling problem, providing additionally some of the powerful tools used in the solid state theory.

As for the pseudo-particles, here the inhabitant can be mostly conceived as an average individual, while the recurson more like a hamper of resources, depending on the present population needs and the cultural trends; as in quasi stationary frames, changes in the hamper composition and its weights are limited, the total composition can be roughly replaced by a single representative resource, which in most cities can be accomplished by using the available statistics on the real estate values. Complicated interactions can be treated hereby as transitions of the process agents within groups of states or energy bands, ruled by the well known Fermi- Dirac statistics based on the exclusion principle. Hence, the bands structure will represent the organization and hierarchies of the system, and the Fermi energy level a measure of its density.

Beyond this standing point, the analogy also provides a way to introduce a proper potential, by defining an equivalent population charge on the basis of the spatial properties of the interaction between complementary agents. It is possible then to build a static representation of the urban region in terms of the field theory and therefore to represent the spatiotemporal processes by means of coupled transport equations of opposite charge carriers (Puliafito, José L. 2006).

3.1 Characterization of an urban region

A band model and the characterization of pseudo particles can be derived, for a given urban region, observing the statistical properties of the spatial distributions of population and of

real estate values. A case study was developed for Great Mendoza, an urban region of about 850,000 inhabitants (1995) located at the foot of the Andes Mountains, 32.8º South latitude and 68.8º West longitude in the Province of Mendoza, western Argentina.

A statistical assessment developed from a GIS raster representation of Great Mendoza's distributions -using 350 x 350 m2 grid elements and official census data collected between 1990 and 1992-, reveals the existence of two subsystems: one dense and central and the other rather diluted and peripheral (fig 3). The former contributes to the characterization of the main urban agglomerate whereas the latter to its interphases of expansion. The dense subsystem can be represented appropriately by statistics of the Fermi-Dirac type (FD) and its respective spectral densities:

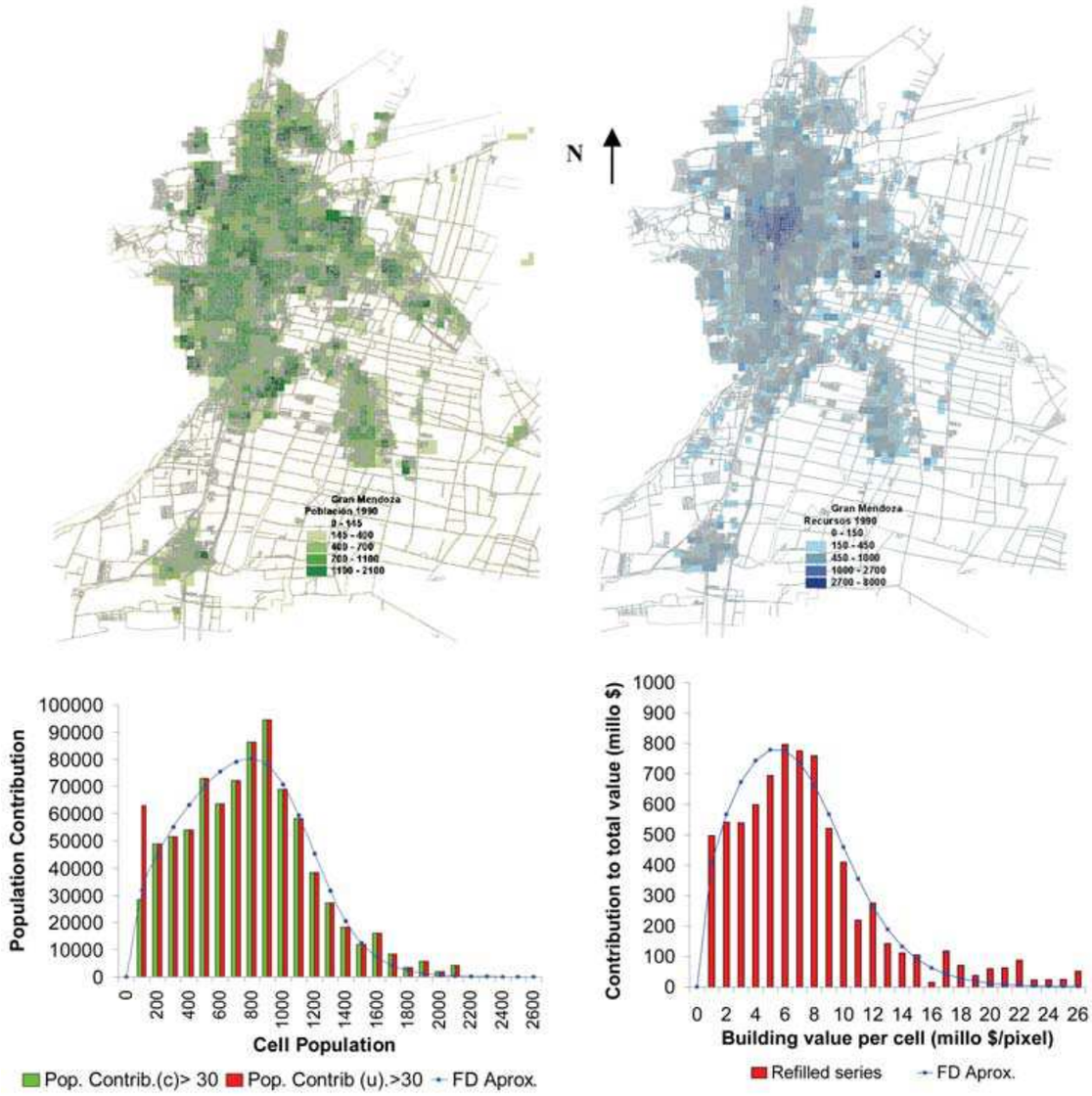

Fig. 3. Characterization of Great Mendoza– Above: GIS raster images of the population distribution (left) and of real estat values (right.)-. Below: Spectral densities of population and real estate and their approximations by Fermi-Dirac density distributions

$$F(N_p) = 1/\{1- \exp[(N_p-N_F)/N_T]\} \rightarrow \Delta N_P = A_p N_p^{1/2} F(N_p) \quad (14.a)$$

$$F(R_p) = 1/\{1- \exp[(R_p-R_F)/R_T]\} \rightarrow \Delta R_p = B_r R_p^{1/2} F(R_p) \quad (14.b)$$

where eq. (14.a) stand for inhabitants and (14.b) for recursons. The FD parameters are obtained within correlation factors of 0.98999 and 0.978 respectively (table 1).

Inhabitant (N_P)			Recurson (R_P)		
N_F: Fermi density [inhab/pixel]	N_T: Thermal density [inhab/pixel]	A_P: Density of states [inhab/pixel]	R_F: Fermi density [millon$/pixel]	R_T: "Thermal" density [millon$/pixel]	B_R: Density of states [millon$/pixel]
1140	165	3198	8.5	2.3	424.6

Table 1. Fermi-Dirac parameters for Great Mendoza 1990/2

The *crystallographic equivalent model* (a virtual substrate) is constructed over the properties of FD approximations (Kittel C., 1995):

$$\delta N_p/\Delta\varepsilon = 2/3\, N_p/\varepsilon \quad (15)$$

$$g_V(\varepsilon) = (1/2\pi^2)((2m_p)^{3/2} / \check{a}_p^3)\, \varepsilon^{1/2} \quad (16)$$

Equation (15) represents an associated energy state space (ε) with an Δε uncertainty, and eq. (16) the population volumetric density of states $g_V(\varepsilon)$ in this space (directly related to A_p in eq.14.a); here m_p represents an effective mass of the pseudo particle inhabitant, while $\check{a}_p = a_p/2\pi$ is an equivalent action constant proper of the urban process scale.

Fitting energy uncertainty to half of the excess of biokinetic energy over the daily rest metabolism of an inhabitant and the effective mass to the biokinetic proportion of its average mass, an effective mass of 12.83 [kg] and an action constant of 153.9 [J.seg] are obtained. Scaling can be completed assuming that in eq. (14.a) and eq. (14.b) a "thermal" equilibrium is fulfilled (stationary conditions) so that k. $N_T = R_T$, with k = 13939.39 [$/inhab] representing one recurson per inhabitant. In such case Br results in 30460 [rec/pixel], which permits to estimate the effective mass of the recurson as $m_r = 4.48657 m_p \cong 4.5\, m_p$

The band model can be finally obtained, taking into account that the recurson is the inhabitant's anti-particle, satisfying a representation analogous to a semimetal.

$$N_F / N_T = (E_F - E_{BP}) / E_T\,;\; R_F / R_T = (E_{BR} - E_F) / E_T \quad (17)$$

Considereing the bottom of the population band is the zero energy level, the ceiling of the resources band is at 32 J and the Fermi energy level at 20,05 J .

3.2 Growth and circulation model

When representing the urban virtual substrate with a band structure analogous to a semimetal, it is possible to anticipate equivalent processes of conduction, as much of the free type as by movement of vacancies. The former can be associated mainly to fast dynamics on a daily basis, whereas the latter to the medium term transport that arises from expansion. Naturally, population dynamics adopts thence a similar transport model (circulation and growth):

$$\frac{\partial}{\partial t} p(x,y,t) = g_p\,(x,y,t,T) + 1/q_p\ \nabla.\boldsymbol{J}_p\,(x,y,t) \tag{18}$$

$$\frac{\partial}{\partial t} r(x,y,t) = g_r\,(x,y,t,T) - 1/q_p\ \nabla.\boldsymbol{J}_r\,(x,y,t)$$

where $[p(x,y,t);\ r(x,y,t)]$ represents the surface concentrations of both pseudo particles, inhabitants and recursons, $[g_p(x,y,t,T);\ g_r(x,y,t,T)]$ their speeds of growth and $[\boldsymbol{J}_p(x,y,t);\ \boldsymbol{J}_r(x,y,t)]$ the corresponding current densities, for which a *population equivalent charge* q_p will be defined afterwards. Besides, the growth part of the transport model specified in the pair of equations (18) follows the form:

$$\delta p = \eta_{0p}\, p_0 - \gamma\, p.r\ /\ p_0 \tag{19}$$

$$\delta r = \eta_{0r}\, r_0 - \beta\, \gamma\, p.r\ /\ r_0$$

where $\delta p = g_p.\Delta t$ and $\delta r = g_r.\Delta t$ represent the variations of concentrations of pseudo particles in a Δt interval, η_{0p} and η_{0r} their free growth rates, γ a factor of mutual control of population and $\beta = r_0/p_0$ an urban quality factor, with p_0 and r_0 the respective local stationary concentrations at statistical "temperature" T_0. Thus, the growth for each pseudo particle adopts the form of a balance between generation (production) and recombination (loss), as it could happen in the case of doped solid materials because of extrinsic excitation, a form that in addition can be linked to a prey-predator model typical of population dynamics in ecology (see for example: Bossel H., 1986; Pacala S. and Levin S., 1997). This requires the definition of population and resources growth rates, and of a recombination rate, that here is to be interpreted as a cross limitation to the free rates of growth.
As for the circulatory part of the transport model, this one follows the form:

$$1/q_p)\ \boldsymbol{J}_p\,(x,y,z,t) = -\mu_p\,.\,p.\nabla V + D_p\,.\nabla p \tag{20}$$

$$(1/q_p\,)\ \boldsymbol{J}_r\,(x,y,z,t) = -\mu_r\,.r.\nabla V - D_r\,.\nabla r$$

Currents adopt in each case the form of a dynamic balance between a drift current, mobilized by the gradient of an *urban potential*, and a diffusion current, mobilized by the gradient of the corresponding concentration. Currents demand the definition of an appropriate urban potential (in which the population charge mentioned above takes part), and a spatial tensor of mobility and diffusion $[\mu_p, D_p; \mu_r, D_r]$.
This type of transport and growth model is naturally attainable by means of bidimensional cellular automata of mobile agents, characterized by a set of parameters that are a function of space and of the statistical temperature of the system. Hereby, nevertheless, the additional advantage lays in that the analogy with the solid state of matter allows a more conceptual bottom-up interpretation, diminishing therefore the necessities of model parameterization to an indispensable minimum.

3.3 Urban potential and the population equivalent charge

From the point of view of the individual contribution of an inhabitant, the urban potential represents a measurement of its energy reserve, as a result of the capacity of the individual to collect resources from the environment, as seen already for the bioautomaton. Appart from what is stated in ec. (2), it can also be defined by means of a bottom-up approach analogous to the Thomas-Fermi model, used in solid state physics (Kittel C., 1995), taking

advantage of the scaling of characteristic constants already made in the band model; therefore, it is also possible to define the value of population charge.[4]
Assuming a monostructure of bands, a model of Thomas-Fermi adapted to the case can be specified as follows:

$$V_p(x,y) \cong (\check{a}_p^2/2m_p)\{(3\pi^2 p_{0m})^{2/3} - sgz \cdot (3\pi^2 \left| z(x,y) \right|)^{2/3}\} \quad (21)$$

$$z(x,y) = p(x,y) - r(x,y)m_p/m_r$$

The energy term given by $\varepsilon_{F0} = (\check{a}_p^2/2m_p)(3\pi^2 p_{0m})^{2/3}$ is the Fermi level for zero statistical temperature, where p_{0m} corresponds to the average concentration of pseudo particles inhabitants in their basal state (rest state), and $z(x,y)$ is the associated net concentration to the distribution of population charge ($r(x,y)$ is given in [rec.]) with its corresponding sign (sgz). From this theory and ec (2) , it is possible to find a suitable value for q_p . For the case study, a population equivalent charge $q_p = 5.832$ [(J.m)$^{1/2}$] can be found.

Fig. 4. Urban Potential for Great Mendoza (1990/92) in equipotential contour lines format.

Urban Potential in fig. 4, resulting from ec. (21), shows the centre of the city as a positive (dark) peak due to a bigger concentration of resources (Capital Department). Using this urban potential it is easy to distinguish metropolitan residential areas, resources injection areas, as well as the variation of urban quality and the relation between poles . The single

[4] For urban evolutionary situations being governed mainly by spatial nuclei of activity concentration, the Thomas - Fermi model allows the description of inhomogeneities in the distribution of population and resources, by means of smooth variations of the Fermi level, within a unique structure of bands (impoverished or enriched by resources and/or population). In evolutionary situations governed by fragmentation, the description must be made by zones with interphases that can present very steep transitions and even different band structures, altogether implying nonlinear local behaviours

reading of this potential map already gives substantial information of the city, thenceforth constituting a valuable synthetic way of representation in itself, even to the extent of a qualitative evaluation of future evolution.

3.4 Growth parameters

From the pair of equations (19) and since $p = p_0 + \delta p$ and $r = r_0 + \delta r$, for situations of normal growth in which $\delta p/p_0 << 1$ and $\delta r/r_0 << 1$, one gets (despising quadratic powers) the following relative variations of concentrations:

$$\delta p/p_0 \cong [\eta_{0p}(1-\gamma) - \beta\gamma\,(1+\eta_{0r})] \,/\, [1+\gamma\,(1-\eta_{0p})+\beta\gamma\,(1+\eta_{0r})] \qquad (22)$$

$$\delta r/r_0 = \eta_{0r} - (1/\beta)\,(\eta_{0p} - \delta p/p_0)$$

where the factor of urban quality $\beta = r_0/p_0$ is a function of space $\beta(x,y)$. Using this last property it is possible to fit the growth model to the case study, computing the average factor of urban quality by department β_{md} and associating them to the expected rates of annual population and GGP growth in the early 90's (Fig. 5)

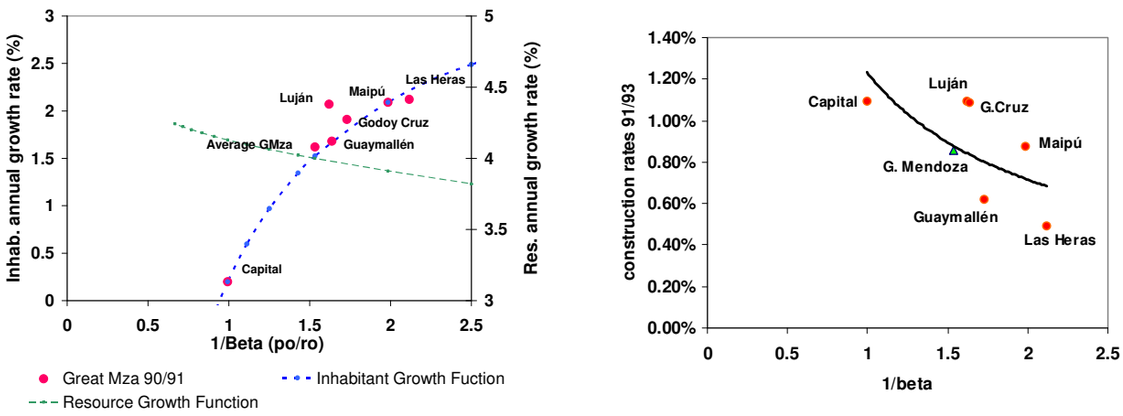

Fig. 5. Left: Demographic growth rates for Great Mendoza (90/91) as a function of the inverse quality factor $1/\beta$ and adjustment of the theoretical growth model . Right: Construction growth rates for Great Mendoza at the beginning of the 90s

The resulting growth parameters for the case study are η_{0p} = 4,2 % , η_{0r}= 8,1 % , and γ = 3,66%. The free demographic growth is an annual net birth rate η_{0p} (birth of inhabitants minus their mortality), representative of the effective procreative capacity. Similarly, the free economical growth rate η_{0r} represents a maximum average annual net growth of resources (here taken as real estate rates) since it results from the adjustment to the expected GGP growth.

It should be bared in mind that these growth parameters are representative of a stationary behaviour; other behaviours can arise out of the cultural substrate, which can modulate the balancing of population as much as through the parametric variation of η_{0p} as of γ . Likewise, the economical substrate influences on the balancing of resources through own parametric variations of η_{0r} and γ , but hereby depending more on macro economic conditions given at a national or a regional scale, rather than on a metropolitan scale. This justifies the need to represent them as functions of statistical temperature.

3.5 Mobility and diffusion factors

The circulation part of the transport model is written in the pair of equations (20). For deriving the mobility and diffusion factors one can consider that the system has reached a stationary situation where $J_p(t=0) \cong 0$ and $J_r(t=0) \cong 0$, then:

$$D_p / \mu_p \cong p_0 \left| dV/dp_0 \right| ; \quad D_r / \mu_r \cong r_0 \left| dV/dr_0 \right| \tag{23}$$

As the urban potential V is approximated by Thomas-Fermi, one gets a generalized expression of D_p/μ_p as a function of space [5]:

$$D_p/\mu_p \cong \frac{2\,\varepsilon_P(x,y) \left| 1-(dr/dp)_0\, m_p/m_r) \right|}{3\, q_p \left| 1-(r_0/p_0)\, m_p/m_r) \right|^{1/3}} \tag{24}$$

where $\varepsilon_P(x,y) = (\check{a}_p^2/2m_p)(3\pi^2 p_0(x,y))^{2/3}$ can be considered the isolated contribution of $p_0(x,y)$ to Fermi's level. The former applies for genuine stationary conditions, but for a quasi stationary frame there should be a limiting trend as follows:

$$D_p/\mu_p = D_r/\mu_r \cong 2/3\, \varepsilon_F(x,y)/q_p \tag{25}$$

Once the D/μ quotient has been specified for each pixel, a numerical value of each parameter can be found by estimating mobility factors, as in solid state theory:

$$\mu_p = q_p\, \tau_p / m_p ; \; \mu_n = q_p\, \tau_r / m_r \tag{26}$$

The characteristic time parameters τ_p and τ_r, can be interpreted as the *average free time periods between relocations* of inhabitant and recursons. In the case of τ_p, its value is representative of the average time invested daily per inhabitant in terms of displacements (in one direction) for different activities, which for the case study was about 25 min in 1990 [6].

An average measure of D/μ quotient is the given by Einstein's equation $D_{p0}/\mu_{p0} = D_{r0}/\mu_{r0} = KT/q_p$ $[(J/m)^{1/2}]$, being for the case study $\mu_{p0} = 1{,}15\ 10^{-04}$, $\mu_{r0} = 2{,}59\ 10^{-05}$ for the effective mobility factors $[(J.m)^{1/2}$ sec/kg] and $D_{p0} = 5{,}91\ 10^{-5}$, $D_{r0} = 1{,}51\ 10^{-5}$ for the effective diffusion factors [m2 /seg]. Factor D_{r0} in particular, can be interpreted as the city average "thermal" expansion, gives a relative surface expansion of 1.65. Since this is practically $(1+\beta_0)$, where $\beta_0 \cong 0.65$ is the city average quality factor, it gives a net relative expansion of β_0 per inhabitant in excess, which is in accordance to the fact that excess concentration will be rearranged trying to conserve the former average quality factor.

The mobility and diffusion factors link the daily commutation regime to the expansion regime, in a way that the city structure depends directly on the average relocation time period and vice versa.

3.6 Implementation and testing of the model

The general scheme of calculation associated to the model, consists of an iterative process of n periods (annual periods have been used for the case study), in which, as from an initial

[5] Mobility and diffusion factors vary in space and hence do not follow the analogous relationship to Einstein's equation $D/\mu = kT/q$. For an explanation on Einstein's equation see for example Kittel C., 1995.

[6] Weighted sum of invested time in bus and car journeys not including Luján de Cuyo.

state, the urban potential and the growth and transport of population and resources are computed for each cell (64x 86 elements of 350x 350 m2), thus filling in an evolutionary gridded data base. The parametric inputs of the model are given by the growth free rates and mutual control of population and resources, as well as by their respective mobilities.

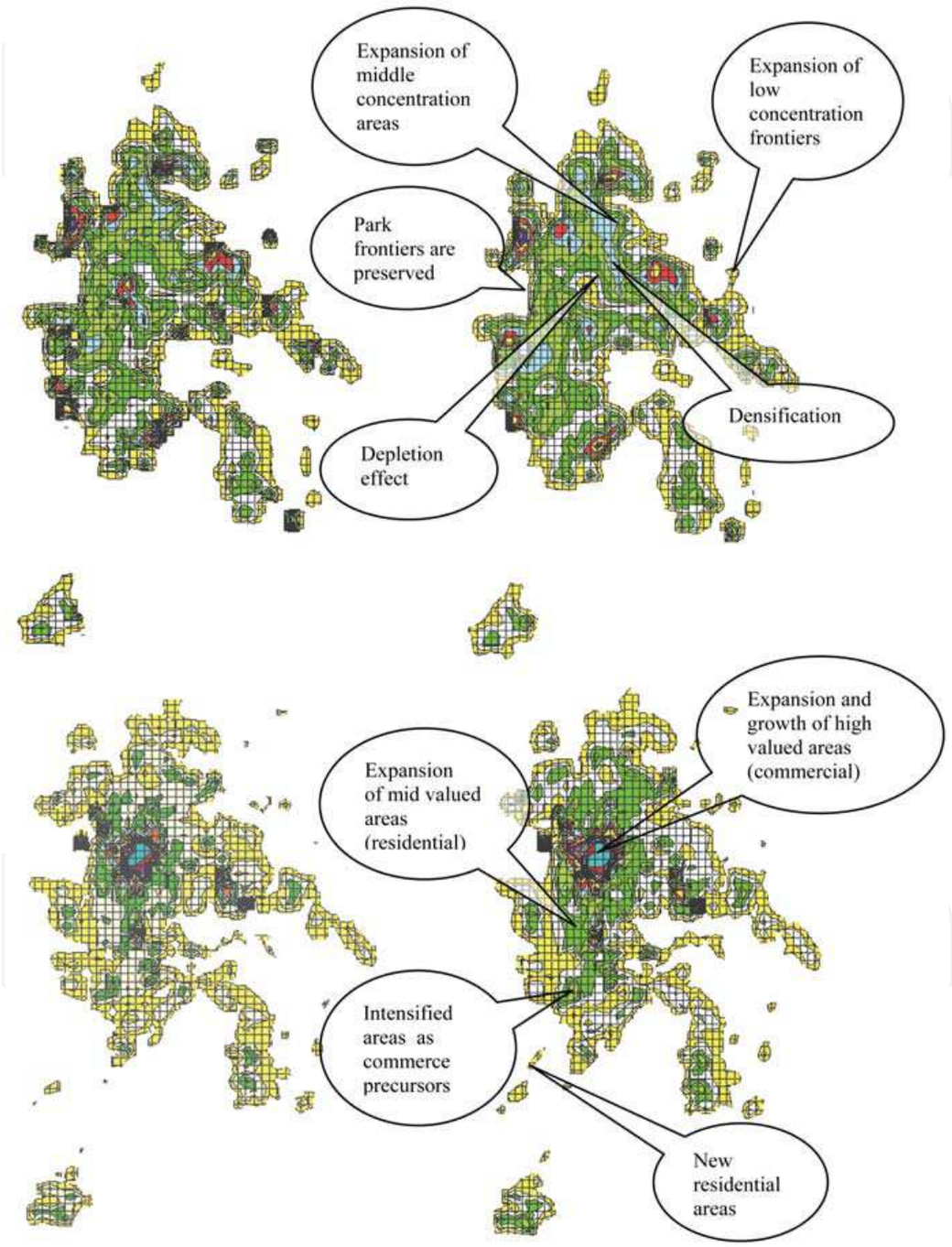

Fig. 6. Comparison between 1990/1 (left figure) and final state (right figure) for the distribution of inhabitants (top figures) and of recursons (bottom figures).

Spatiotemporal uncertainty, associated to the initial state, limits the model space and time resolution and might even cause its instability. It includes the combined effect of errors due to the gathering, sampling and conditioning of demographic and cadastral data, which on the other hand are not strictly co-temporal. For the case study the time uncertainty was lower than 1.4 year and a space uncertainty not larger than one pixel (350 x 350 m^2).
This model has been tested for the case study in quasi stationary conditions. Initial data correspond to Great Mendoza in 1990/1, with the associated characteristic constants previously discussed. Parameters have been kept constant throughout all periods.
The results of a simulation for five years show a good correspondence with growth and distribution trends seen in such decade. From the maps one can clearly distinguish how equidensity areas evolve (fig. 6). Only considering the spatial aspects observed here, one can already recognize the principal types of effects that could be expected in mid-term evolutions in any city, as for instance the one conveyed by the seven transition rules of the Batty-Torrens model (Batty M., Torrens P., 2001). It is particularly interesting the depletion effect (sometimes called "donut" effect) seen in the main centre of the city, which here arises naturally as a consequence of resources and inhabitants competitive growth and diffusion. This qualitative correspondence to the principal trends of evolution of Great Mendoza during the last decade, acquires more importance when considering it together with the reasonable overall temporal behaviour of state variables.

4. Some reflections on population growth and economy

Since the beginning of the last century the world is experiencing an important demographic transition, which will probably impact on economic growth. Many demographers and social scientists are trying to understand the key drivers of such transition as well as its profound implications. A correct understanding can help to predict other important trends at global scale, as the primary energy demand and the carbon emission to the atmosphere, which may be leading to an important climate change.
Inspired on the former works, a set of coupled differential equations has been proposed in Puliafito S. Enrique et al. (2007) to describe the changes of population and gross domestic product, modelled as competing-species as in Lokta-Volterra relations. In fact, if the development and population dynamics of cities could be explained in terms of the above given model, it would be natural then to expect that global population growth and economy follow also a predator-prey type model (eq. 19). Based on that, changes of primary energy consumption and carbon emissions would be then modelled similarly. The estimated results for the temporal evolution of world population, gross domestic product, primary energy consumption and carbon emissions were calculated from year 1850 to year 2150. The calculated scenarios are in good agreement with common world data and projections for the next 100 years.
Economic growth models give population growth a major role, but some show population as detrimental to economic growth and others show population as a major contributor. In fact, population growth has two effects: it increases the number of consumers, and it increases the number of workers devoted to productive activity and research. However, population growth increases the scale of the economy, therefore permitting industries, enterprises, and the entire economy to exploit economies of scale. Models based on technological progress, or on generation of new ideas generally conclude that population growth and the size of the population have a positive effect on growth of per capita output

by specifying technological progress as a function of the number of people engaged in R&D activity. But models based on congestion, come to the conclusion that increasing population produces a slowing economy, since more investment is needed to maintain same per capita output. The debate on whether population growth is detrimental or beneficial to the welfare of humanity essentially comes down to the opposing conclusions of the Solow and Malthusian models vs. the exogenous growth models (Galor, O., Weil, D. , 2000).
The definition of economic growth as an increase in output per capita implies an inverse relationship between output (GDP) and population, but this is not necessarily a cause-effect relationship; if population causes total output to increase faster than population does, only then it will produce an increase in per capita output. Although in many countries population growth seems to be negatively related to economical growth, empirical evidence does not unambiguously support either view of population growth.
For a closer look on this, consider population p when changes are taken as continuous and are unregulated by external factors; then it can be expressed in differential form as:

$$1/p\ (dp/dt) = \eta \tag{27}$$

which gives as solution a growing exponential function of the type $p(t) = P_0 exp\ (\eta t)$, where η is the growth rate. However, many demographic and ecological studies recognize that, for long periods of time, the growth rate η is not constant, but decreases as population increases. So the actual population presents apparently a (auto-) limiting factor. In fact, this limitation can be expressed as in differential form as:

$$1/p\ (dp/dt) = \eta - \alpha\ p \tag{28}$$

where the crude growth rate η is limited by the product of $\alpha.P_t$, being $\alpha = \eta/P_m$, and P_m the maximum supporting population for a given environment, which produces the "S-shaped" curve, known as logistic curve. Also the economic output (GDP) sometimes is modeled in a logistic form. Although population and gross domestic product may be fitted to logistic type curves, there is no clear indication on which may be the value of the maximum carrying capacity, nor a clear explanation for this limitation process. One possible feedback mechanism, which may explain this limitation processes is linked to the availability of resources, as it can be seen from ecological and biological studies and the discussion given in the former points. Consequently, a pair of nonlinear-coupled differential equation, similar to the Lokta-Volterra relations for two species interaction, is proposed:

$$\begin{cases} 1/p\ (dp/dt) = a - g\ m \\ 1/g\ (dg/dt) = \kappa - b\ p \end{cases} \tag{29}$$

where the left members represent the relative changes in the population p and available resources g, $b.p$ is the annual resource consumption by the population p, k is the annual resource renovation, m is the annual death rate, a is the per capita consumption and regulates the birth rate n. Interesting to note is that depending on the chosen parameters, these coupled no linear relationships may show a chaotic behavior. Eq (29) shows that for low values of g population will increase rapidly regulated only by mortality rate m, but as p growths the GDP growths is slowed down by increasing p, which in turn will slow down the population growth.

If p and g have similar temporal variation, which corresponds to a stationary frame where the ratio g/p (per capita output) is approximately constant, it is possible to foresee that p and g will also produce a logistic type equation. However, as for non stationary frames, the ratio of g/p is not constant, the logistic type curve can only be achieved if also a and b are not constant but they have the proper variations. To represent these types of frames adequately (particularly the transitory in short terms), an additional function $f(t)$ can be included to the set of Eq. (29), which might be interpreted as an external excitation function comprising all other causes of variation not included in the predator-prey solely mechanism; in fact, the Lokta-Volterra model is a closed one because the eventual changes in the carrying capacity of the substrate are not explicit. To make them explicit, considering now an open model, the substrate has to be taken as varying along the time, for example due to the changing culture and technology. To generalize this open model, disregarding if it is expressed in terms of the rates of production or consumption of the species, and at same time to capture the influence of the variation of the substrate as rates over the populations of the considered species, we can write:

$$\begin{cases} 1/p\ (dp/dt) = \alpha_1.f/p + \alpha_2.g + \alpha_3 \\ 1/g\ (dg/dt) = \beta_1.f/g + \beta_2.p + \beta_3 \end{cases} \tag{30}$$

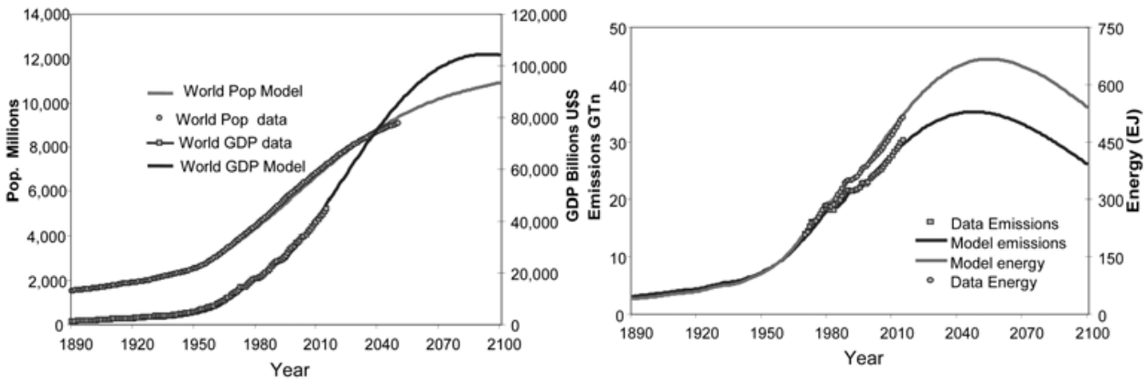

Fig. 7. (Left) Exogenous model for world population (millions inhabitants) and GDP (Billions U$S); (Right) Exogenous model for world primary energy consumption (EJ) and carbon emissions (GTn) (from 1890 to 2004 measured or estimated values; from 2005 to 2100 projected values). Coefficient values used in Eq. (30) are p_1=0.0004, p_2= -7.4×10^{-8}, p_3= 0.64%, P_0=1522; g_1= 0.0014, g_2= -2.5×10^{-6}, g_3= 1.68%, G_0=1234); coefficient values used in Eq. (31): ε_1=0.0009, ε_2= -2.8×10^{-6}, ε_3= 1.64%, E_0=40; σ_1= 0.0009, σ_2= -2.8×10^{-6}, σ_3= 1.45%, C_0=3). The external function $f=A.exp(\tau.t)$ plus short impulses is used to represent big international crisis with A=2, τ=0.04 from 1890 to 1963, and τ=-0.04 thereafter. Sources of data EIA (2005).

The experience shows that most positive culture and technology changes arise from scenarios with an increasing g/p rate; therefore, a first approximation is to set f equal to g/p. The figure 7 shows satisfactory results for Eq (30) in such condition; the coefficients α_1, α_2, α_3 and β_1, β_2, β_3 are obtained from the annual changes applying a multi-linear regression.
The annual changes in energy consumption and carbon emission show similar behaviour as changes in GDP and population. Despite that there is not enough certain information of carbon emissions and energy consumption from 1890 to 1970, the energy demand e and

carbon emission c are strongly coupled to g and p, so that a similar set of differential equations as (30) can be suitable to estimate the annual changes in both variables:

$$\begin{cases} 1/e\ (de/dt) = \varepsilon_1.f_1/e + \varepsilon_2.p + \varepsilon_3 \\ 1/c\ (dc/dt) = \sigma_1.f_2/c + \sigma_2.p + \sigma_3 \end{cases} \quad (31)$$

where $f_1=f.e/g$; and $f_2=f.\ c/g$, and f is the same function used for the external excitation of g and p in Eq. (30), for the exogenous model; $\varepsilon_1.\ e/g$ (%) is the efficiency improvement through more technological investment, $\varepsilon_2.\ p$ (%), is the per capita energy consumption, and ε_3 is the residual increase in energy consumption not explained by the other two coefficients, or the natural increase without an external excitation. Same can be said for the natural rate of changes in the carbon emissions. Some results are also shown in Fig. 7.

5. Conclusions

Throughout this chapter we have been exploring some of the fundaments of CA models and the reasons of why these are being so widely applied nowadays, particularly to urban systems and ecology, all of which seems to be connected directly with the fact that the transport equations are common as much to the socioeconomic phenomena as to physics. However, it is not immediate that population dynamics can be described similarly by means of reaction-diffusion equations; on the contrary, perhaps on this outstanding fact rests one of the clues to explain how individual behaviour, usually seen at the "microscopic" scale as mostly stochastic or eventually moved by free-will, can fit into the associated collective behaviour seen at the "macroscopic" scale.

In this sense, by means of the bioautomaton theory we have seen that the discrete character of the device-environment interaction, leads to describe stationary individual behaviour in a similar way to what is done in quantum stochastic systems. The most important aspect of this similarity is that the statistical behaviour of a bioautomaton can be represented in average by means of wave functions, in a way that stationary or quasi-stationary solutions regarding group behaviour can result from the superposition of individual wave functions. As a wave function is a measure of the probability of a stationary exchange between each device and its immediate surrounding, periodical spatial structures can emerge in certain conditions; hence, stationary dynamics would be described in terms of transport equations framed within some appropriate band theory.

This theoretical speculation is justified for real ecological systems when we consider that social behaviour and complex and regular spatiotemporal structures emerge under conditions where species reach some critical spatial density, thus giving place to outstanding interaction mechanisms as templates, stigmergy and self-organization. This suggest that an ecosystem is not the mere association of interactions as a whole or a collection of highly interactive independent elements, but a rather coherent sum of elementary units composed of living individuals and their near-by space-environment, the latter being regarded as a multidimensional representation of the necessary resources for survival, physical space in itself included.

Within this context, a high-density ecosystem can be compared in some sense to an elastoplastic network and thence treated in a similar way to the solid state of matter; that is, as state transitions in a pseudo-crystalline virtual substrate subdued to a general exclusion

principle. In fact, at describing the spatiotemporal dynamic evolution of populations of real individuals through transport equations, one is not only considering the interactions of the species with its space/environment, but also stability regions in the associated state space that are similar to the energy bands in solid materials.

Standing on these principles we have reviewed a feasible model for urban evolution, which is outlined in terms of a virtual substrate with two energy bands: a population band and a resource band where their associated pseudo-particles – the inhabitant and the recurson – represent (in principle in an anti-symmetrical way) the interacting population and their space- environment structure correspondingly.

The characterization of the energy band model for Great Mendoza, starts of the statistical properties of spatial distribution of inhabitants and of real estate values, which have been assimilated to Fermi-Dirac statistics; after determining the characteristic parameters of associated pseudo particles and of the band structure in itself, the case study can be represented in an analogous form to a semimetal.

Taking advantage of the solid state picture, the net concentration of pseudo particles can be linked to a proper urban potential function, through the use of a Thomas –Fermi approximation and an equivalent population charge. Thereafter, a static combined representation of the urban region is feasible in terms of the field theory.

With these elements, the dynamics of urban systems can be constructed over cellular automata with mobile agents, by using similar transport equations as in solid state. The circulatory part of the model adopts the balance form between two components (diffusion and drift), describing the concentration and sprawl of population and resources present in the cities. The model production part , described in terms of generation-recombination of pseudo particles, is comparable to a predator-prey model as well, typical of population dynamics in Ecology. Using the characterization of pseudo particles it is possible to adjust the diffusion and mobility coefficients, and growth to the well-known urban behaviour, with a "bottom-up" approach that diminishes the need of parameterization to an indispensable minimum.

A test in stationary conditions along a five-year period, shows that the principal state variables of the case study evolve in time as it would be expected from the application of classical methods based on statistical progression, and with a spatial response compatible with the principal effects expected in a mid-term evolution in a city. In this sense, this analogy plausibly explains the varied growth rates of the political departments, as well as the principal urban trends for spatial occupation for Great Mendoza in the last decade.

The methodology and model here discussed open new possible ways of approaching urban evolution. Although it has been presented as a stand-alone tool, it can be combined through its parametric inputs with other CA models (i.e. in successive embedded scales or lower structural bands) or even with non spatial social-economical models, thus orienting it more to long-term simulation, where innovation and changing scenarios are required. It also provides a way for describing fragmented urban development by means of zones which may have very different band structures, implying non-linear local behaviours in the resulting interphases.

Finally, a global perspective of the former ideas has been presented in the context of a research of the projection of the energy demand, the carbon emissions and the link to possible climate changes. Several authors have proposed that world population, the primary energy demand and the gross domestic product are the main drivers (or state variables) for the carbon emission problem, while per capita consumption, energy intensity and emission efficiency, among others, are taken as indicators of the system.

As the development and population dynamics of urban regions is represented by transport equations that include a production part , described in terms of generation-recombination of pseudo particles representing population and resources, it seems natural to expect that global population growth and economy follow also a predator-prey type model. Based on that, changes of primary energy consumption and carbon emissions can be then modelled.
Here we have seen that a set of coupled differential equations of this type can describe the changes in the main state variables in a plausible way. Indeed, some studies have observed both positive and inverse relation between population growth and GDP, depending on the time frame and the group of countries involved in the studies; with the coupled model here shown is possible to represent well the three different scenarios or transitional phases from "Malthusian, post Malthusian and modern growth", proposed by some scholars. Other researches propose logistic variation of the population as a way to describe the demographic transitions. Here, the interrelation between these variables, the growth rate and their expected logistic type shape curve arises naturally as the interaction of population and economic output as described in the coupled differential equations. The results of the model were compared to several agencies projection, showing comparable results, but most importantly is the ability to capture conceptually and mathematically the range of current thoughts and models used by the international agencies.
Cellular Automata have shown a great potential for modelling a wide range of types and scales of phenomena, but it is still an open question why this is so. A research on the foundation of this capability, as the one intended here, might contribute not only to a better understanding of the principles involved but also to a better and wider use of the tool.

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Cellular Automata - Simplicity Behind Complexity

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Cellular automata make up a class of completely discrete dynamical systems, which have became a core subject in the sciences of complexity due to their conceptual simplicity, easiness of implementation for computer simulation, and their ability to exhibit a wide variety of amazingly complex behavior. The feature of simplicity behind complexity of cellular automata has attracted the researchers' attention from a wide range of divergent fields of study of science, which extend from the exact disciplines of mathematical physics up to the social ones, and beyond. Numerous complex systems containing many discrete elements with local interactions have been and are being conveniently modelled as cellular automata. In this book, the versatility of cellular automata as models for a wide diversity of complex systems is underlined through the study of a number of outstanding problems using these innovative techniques for modelling and simulation.

InTech Europe
University Campus STeP Ri
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Phone: +385 (51) 770 447
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