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ECG Signal Compression Using Discrete Wavelet Transform

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1. Introduction

Transmission techniques of biomedical signals through communication channels are currently an important issue in many applications related to clinical practice. These techniques can allow experts to make a remote assessment of the information carried by the signals, in a very cost-effective way. However, in many situations this process leads to a large volume of information. The necessity of efficient data compression methods for biomedical signals is currently widely recognized. This chapter introduces the compression of ElectroCardioGram (ECG or EKG) signals using Discrete Wavelet Transform (DWT). It is well known that modern clinical systems require the storage, processing and transmission of large quantities of ECG signals. ECG signals are collected both over long periods of time and at high resolution. This creates substantial volumes of data for storage and transmission. Data compression seeks to reduce the number of bits of information required to store or transmit digitized ECG signals without significant loss of signal quality. Although storage space is currently relatively cheap, electronic ECG archives could easily become extremely large and expensive. Moreover, sending ECG recordings through mobile networks would benefit from low bandwidth demands. ECG signal compression attracted considerable attention over the last decade. Several examples of ECG compression algorithms have been described in the literature with compression ratios ranging approximately from 2:1 up to 50:1 (Jalaleddine et al., 1990; Addison, 2005). The main goal here is to provide an up-to-date introduction to this fascinating field; through presenting some of the latest algorithmic innovations and to stimulate readers to investigate the subject in greater depth using the extensive set of references provided (Addison, 2005; Padma et al., 2009). Section 2 introduces the production of the ECG signal and its main time- and frequency-domain parameters. Different ECG signal compression techniques including direct, transformed and optimization methods are presented in section 3. Section 4 discusses the fundamentals of DWTs and their filter bank realizations. Subjective and objective performance measures of compression algorithms are explained in section 5. In section 6, DWT based ECG signal compression algorithms are presented. This includes optimization-based, SPIHT, 2-D, hybrid, and linear prediction based algorithms. Thresholding, and coding of DWT coefficients considering energy packing efficiency and binary significant map are discussed in sections 7 and 8 respectively.

2. ElectroCardioGraphy

ECG signal is a recording of the electrical activity of the heart over time produced by an electrocardiograph and is a well-established diagnostic tool for cardiac diseases. ECG signal is monitored by placing sensors at defined positions on chest and limb extremities of the subject. Each heart beat is caused by a section of the heart generating an electrical signal which then conducts through specialized pathway to all parts of the heart. These electrical signals also get transmitted through the chest to the skin where they can be recorded. The following four steps in the generation of ECG signal can be monitored:

1. The S-A node (natural pacemaker) creates an electrical signal.
2. The electrical signal follows natural electrical pathways through both atria. The movement of electricity causes the atria to contract, which helps push blood into the ventricles.
3. The electrical signal reaches the A-V node (electrical bridge). There, the signal pauses to give the ventricles time to fill with blood.
4. The electrical signal spreads through the His-Purkinje system. The movement of electricity causes the ventricles to contract and push blood out to lungs and body.

ECG signal is obtained from a machine known as an Electrocardiograph, which captures the signal through an array of electrode sensors placed at standard locations on the skin of the human body. Modern electrocardiographs record ECG signals by digitizing and then storing the signal in magnetic or optical discs. An automated diagnostic system is required to speed up the diagnostic process and assist the cardiologists in examining patients using non-invasive techniques. Electrical impulses in the heart originate in the sinoatrial node and travel through the heart muscle where they impart electrical initiation of systole or contraction of the heart. The electrical waves can be measured at selectively placed electrodes (electrical contacts) on the skin. Electrodes on different sides of the heart measure the activity of different parts of the heart muscle. An ECG displays the voltage between pairs of these electrodes, and the muscle activity that they measure, from different directions, also understood as vectors. The ECG signal is composed from five waves labeled using five capital letters from the alphabet: P, Q, R, S, and T. The width of a wave on the horizontal axis represents a measure of time. The height and depth of a wave represent a measure of voltage. An upward deflection of a wave is called positive deflection and a downward deflection is called negative deflection. A typical representation of the ECG waves is presented in Figure (1) (Moody, (1992).

The electrocardiogram essentially reads the electrical impulses that stimulate the heart to contract. It is probably the most useful tool to determine whether the heart has been injured or how it is functioning. The ECG signal is made up of a number of segments or waves of different durations, amplitudes, and forms: 'slow', low-frequency P and T waves and short and high-frequency Q, R, and S waves, forming the QRS complex. P wave, QRS wave, and T wave, they are diagnostic critical waves. The P wave represents the atrial depolarization where the blood is squeezed from the atria to the ventricles. The QRS segment is when the ventricles depolarize and squeeze the blood from the right ventricle to the aorta. The T wave represents the period of time when the ventricles repolarize (get ready for the next heart beat). Most of the ECG signal energy is concentrated in the QRS complex, but there are diagnostically important changes in the low amplitude PQ and ST intervals, the P and T waves.

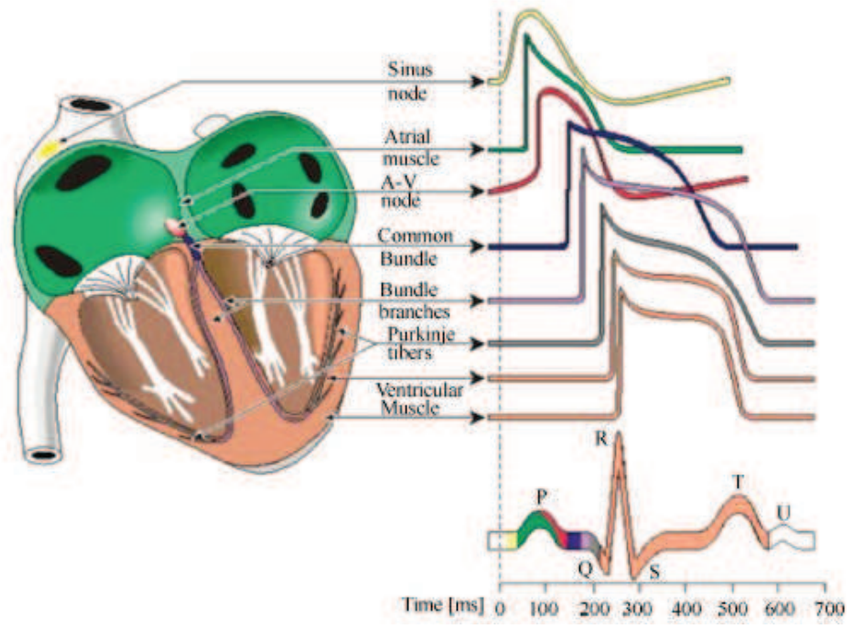


Fig. 1. A typical representation of the ECG waves.

Figure (2) illustrates the ECG signal in time and frequency domains. Compressing the ECG signal while preserving the original shape of the reconstructed signal and especially the amplitudes of Q, R and S peaks, without introducing distortions in the low amplitude ST segment, P and T waves are the main objectives of this chapter. In fact, most ECG compression algorithms produce ripple effects around QRS complexes and could also reduce the sharp waves' amplitudes.

3. ECG signal compression

Data reduction of ECG signal is achieved by discarding digitized samples that are not important for subsequent pattern analysis and rhythm interpretation. The data reduction algorithms are empirically designed to achieve good reduction without causing significant distortion error. ECG compression techniques can be categorized into: direct time-domain techniques; transformed frequency-domain techniques and parameters optimization techniques.

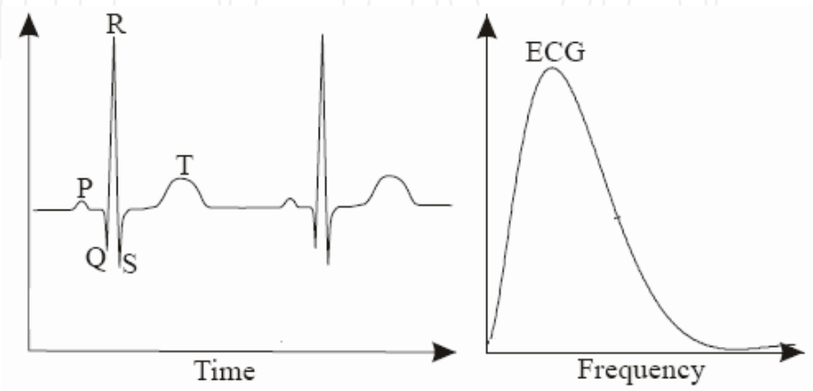


Fig. 2. ECG signal in time and frequency domains.

1. **Direct Signal Compression Techniques:** Direct methods involve the compression performed directly on the ECG signal. These are also known as time domain techniques dedicated to compression of ECG signal through the extraction of a subset of significant samples from the original sample set. Which signal samples are significant, depends on the underlying criterion for the sample selection process. To get a high performance time-domain compression algorithm, much effort should be put in designing intelligent sample selection criteria. The original signal is reconstructed by an inverse process, most often by drawing straight lines between the extracted samples. This category includes the FAN (Dipersio & Barr, 1985), CORTES (Abenstein & Tompkins, 1982), AZTEC (Cox et al., 1968), Turning Point (Mueller W., 1978) and TRIM (Moody et al., 1989) algorithms. The more recent cardinality constrained shortest path technique (Haugland et al., 1997) also fits into this category. Many of the time domain techniques for ECG signal compression are based on the idea of extracting a subset of significant signal samples to represent the original signal. The key to a successful algorithm is the development of a good rule for determining the most significant samples. Decoding is based on interpolating this subset of samples. The traditional ECG time domain compression algorithms all have in common that they are based on heuristics in the sample selection process. This generally makes them fast, but they all suffer from sub-optimality.
2. **Transformed ECG Compression Methods:** Transform domain methods, as their name implies, operate by first transforming the ECG signal into another domain. These methods mainly utilize the spectral and energy distributions of the signal by means of some transform, and properly encoding the transformed output. Signal reconstruction is achieved by an inverse transformation process. This category includes traditional transform coding techniques applied to ECG signals such as the Karhunen–Loève transform (Olmos et al., 1996), Fourier transform (Reddy & Murthy, 1986), Cosine transform (Ahmed et al., 1975), subband-techniques (Husøy & Gjerde, (1996), vector quantization (VQ) (Mammen & Ramamurthi, 1990), and more recently the wavelet transform (WT) (Chen et al., 1993; (Miaou et al.., 2002). Wavelet technique is the obvious choice for ECG signal compression because of its localized and non-stationary property and the well-proven ability of wavelets to see through signals at different resolutions. Wavelets are mathematical functions that cut up data into different scale-shift components. The wavelet decomposition splits the analyzing signal into average and detail coefficients, using finite impulse response digital filters. The main task in wavelet analysis (decomposition and reconstruction) is to find a good analyzing function (mother wavelet) to perform an optimal decomposition. Wavelet-based ECG compression methods have been proved to perform well. The ability of DWT to separate out pertinent signal components has led to a number of wavelet-based techniques which supersede those based on traditional Fourier methods. The discrete wavelet transform has interesting mathematics and fits in with standard signal filtering and encoding methodologies. It produces few coefficients, and the user does not have to worry about losing energy during the transform process or its inverse. While the DWT is faster and maps quickly to the sub-band coding of signals, the Continuous Wavelet Transform (CWT) allows the user to analyze the signal at various scales and translations according to the problem.
3. **Optimization Methods For ECG Compression:** More recently, many interesting optimization based ECG compression methods, the third category, have been developed. The goal of most of these methods is to minimize the reconstruction error

given a bound on the number of samples to be extracted or the quality of the reconstructed signal to be achieved. In (Haugland et al., 1997), the goal is to minimize the reconstruction error given a bound on the number of samples to be extracted. The ECG signal is compressed by extracting the signal samples that, after interpolation, will best represent the original signal given an upper bound on their number. After the samples are extracted they are Huffman encoded. This leads to the best possible representation in terms of the number of extracted signal samples, but not necessarily in terms of bits used to encode such samples. In (Nygaard et al., 1999), the bit rate has been taken into consideration in the optimization process.

The vast majority of the above mentioned methods do not permit perfect reconstruction of the original signals. In fact; there is no automatic way to assure that the distortion in the reconstructed signal will not affect clinically important features of the ECG. To preserve the clinical diagnostic features of the reconstructed ECG signals both the wavelet filters' parameters and the threshold levels in all subbands should be selected carefully. Thus, the aim is to present ECG compression technique that achieves maximum data volume reduction while preserving the significant signal morphology features upon reconstruction. This has been achieved through the minimization of both the bit rate and the distortion of the reconstructed ECG signal through parameterization of the wavelet filters and the selection of optimum threshold levels of the wavelet coefficients in different subbands.

4. Discrete wavelet transform

In technical literature, a number of time–frequency methods are currently available for the high resolution signal decomposition. This includes the short time Fourier transform (STFT), Wigner–Ville transform (WVT), Choi–Williams distribution (CWD) and the WT. Of these, the wavelet transform has emerged as the most favored tool by researchers as it does not contain the cross terms inherent in the WVT and CWD methods while possessing frequency-dependent windowing which allows for arbitrarily high resolution of the high frequency signal components. The DWT is the appropriate tool for the analysis of ECG signals as it removes the main shortcomings of the STFT; namely it uses a single analysis window which is of fixed length in both time and frequency domains. This is a major drawback of the STFT, since what are really needed are a window of short length (in time domain) for the high frequency content of a signal and a window of longer length for the low frequency content of the signal. The WT improves upon the STFT by varying the window length depending on the frequency range of analysis. This effect is obtained by scaling (contractions and dilations) as well as shifting the basis wavelet. The continuous wavelet transform (CWT) transforms a continuous signal into highly redundant signal of two continuous variables — translation and scale. The resulting transformed signal is easy to interpret and valuable for time-frequency analysis. The continuous wavelet transform of continuous function, $f(x)$ relative to real-valued wavelet, $\psi(x)$ is described by:

$$W_{\psi}(s,\tau) = \int_{-\infty}^{\infty} f(x)\psi_{s,\tau}(x)\,dx \tag{1}$$

where,

$$\psi_{s,\tau}(x) = \frac{1}{\sqrt{s}}\psi\left(\frac{x-\tau}{s}\right) \tag{2}$$

s and τ are called scale and translation parameters, respectively. $W_\psi(s,\tau)$ denotes the wavelet transform coefficients and ψ is the fundamental mother wavelet. If $W_\psi(s,\tau)$ is given, $f(x)$ can be obtained using the inverse continuous wavelet transform (ICWT) that is described by:

$$f(x) = \frac{1}{C_\psi} \int_0^\infty \int_{-\infty}^{\infty} W_\psi(s,\tau) \frac{\psi_{s,\tau}(x)}{s^2} d\tau\, ds \tag{3}$$

where, $\Psi(u)$ is the Fourier transform of $\psi(x)$ and

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\Psi(u)|^2}{|u|} du \tag{4}$$

The discrete wavelet transform can be written on the same form as Equation (1), which emphasizes the close relationship between CWT and DWT. The most obvious difference is that the DWT uses scale and position values based on powers of two. The values of s and τ are: $s = 2^j, \tau = k * 2^j$ and $(j,k) \in Z^2$ as shown in Equation (5).

$$\psi_{j,k}(x) = \frac{1}{\sqrt{s_o^j}} \psi\left(\frac{x - k\tau_o s_o^j}{s_o^j}\right) \tag{5}$$

The key issues in DWT and inverse DWT are signal decomposition and reconstruction, respectively. The basic idea behind decomposition and reconstruction is low-pass and high-pass filtering with the use of down sampling and up sampling respectively. The result of wavelet decomposition is hierarchically organized decompositions. One can choose the level of decomposition j based on a desired cutoff frequency. Figure (3-a) shows an implementation of a three-level forward DWT based on a two-channel recursive filter bank, where $h_0(n)$ and $h_1(n)$ are low-pass and high-pass analysis filters, respectively, and the block $\downarrow 2$ represents the down sampling operator by a factor of 2. The input signal $x(n)$ is recursively decomposed into a total of four subband signals: a coarse signal $C_3(n)$, and three detail signals, $D_3(n)$, $D_2(n)$, and $D_1(n)$, of three resolutions. Figure (3-b) shows an implementation of a three-level inverse DWT based on a two-channel recursive filter bank, where $\tilde{h}_0(n)$ and $\tilde{h}_1(n)$ are low-pass and high-pass synthesis filters, respectively, and the block $\uparrow 2$ represents the up sampling operator by a factor of 2. The four subband signals $C_3(n), D_3(n), D_2(n)$ and $D_1(n)$, are recursively combined to reconstruct the output signal $\tilde{x}(n)$. The four finite impulse response filters satisfy the following relationships:

$$h_1(n) = (-1)^n h_0(n) \tag{6}$$

$$\tilde{h}_0(n) = h_0(1 - n) \tag{7}$$

$$\tilde{h}_1(n) = (-1)^n h_0(1 - n) \tag{8}$$

so that the output of the inverse DWT is identical to the input of the forward DWT.

5. Compression algorithms performance measures

5.1 Subjective judgment

The most obvious way to determine the preservation of diagnostic information is to subject the reconstructed data for evaluation by a cardiologist. This approach might be accurate in some cases but suffers from many disadvantages. One drawback is that it is a subjective measure of the quality of reconstructed data and depends on the cardiologist being consulted, thus different results may be presented. Another shortcoming of the approach is that it is highly inefficient. Moreover, the subjective judgment solution is expensive and can generally be applied only for research purposes (Zigel et al., 2000).

5.2 Objective judgment

Compression algorithms all aim at removing redundancy within data, thereby discarding irrelevant information. In the case of ECG compression, data that does not contain diagnostic information can be removed without any loss to the physician. To be able to compare different compression algorithms, it is imperative that an error criterion is defined such that it will measure the ability of the reconstructed signal to preserve the relevant diagnostic information. The criteria for testing the performance of the compression algorithms consist of three components: compression measure, reconstruction error and computational complexity. The compression measure and the reconstruction error depend usually on each other and determine the rate-distortion function of the algorithm. The computational complexity component is related to practical implementation consideration and is desired to be as low as possible.

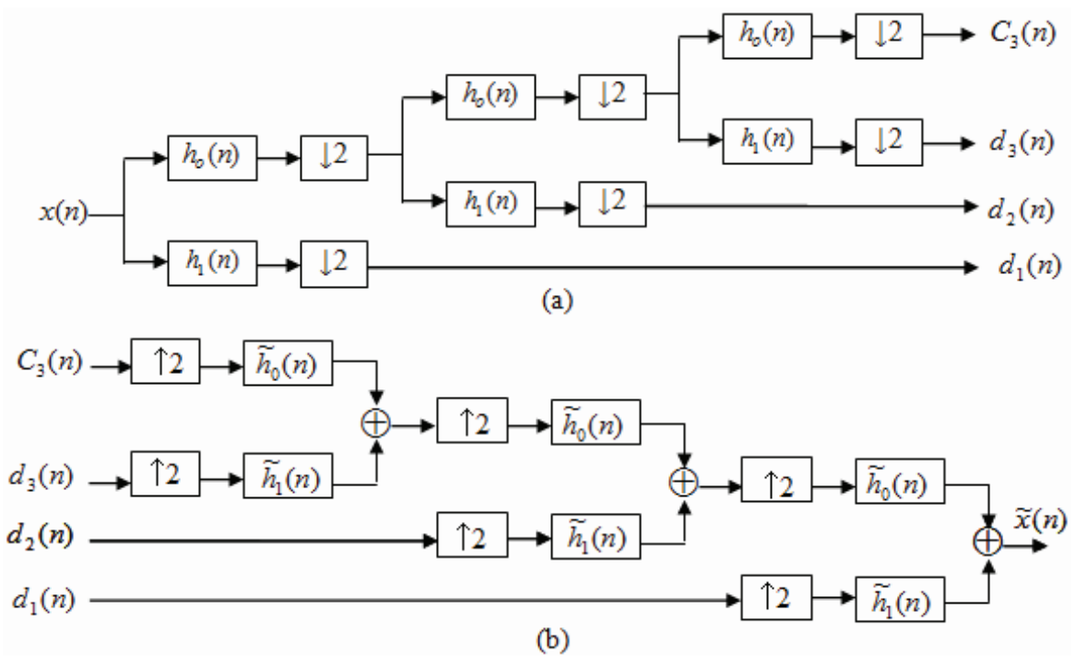


Fig. 3. A three-level two-channel iterative filter bank (a) forward DWT (b) inverse DWT

The compression ratio (*CR*) is defined as the ratio of the number of bits representing the original signal to the number required for representing the compressed signal. So, it can be calculated from:

$$CR = \frac{N\, b_C}{(N_S + M)(b_S + 1)} \tag{9}$$

Where, b_C is the number of bits representing each original ECG sample. One of the most difficult problems in ECG compression applications and reconstruction is defining the error criterion. Several techniques exist for evaluating the quality of compression algorithms**.** In some literature, the root mean square error (*RMS*) is used as an error estimate. The *RMS* is defined as

$$RMS = \sqrt{\frac{\sum_{n=1}^{N} (x(n) - \hat{x}(n))^2}{N}} \tag{10}$$

where $x(n)$ is the original signal, $\hat{x}(n)$ is the reconstructed signal and N is the length of the window over which the *RMS* is calculated(Zou & Tewfik, 1993). This is a purely mathematical error estimate without any diagnostic considerations.

The distortion resulting from the ECG processing is frequently measured by the percent root-mean-square difference (*PRD*) (Ahmed et al., 2000). However, in previous trials focus has been on how much compression a specific algorithm can achieve without loosing too much diagnostic information. In most ECG compression algorithms, the *PRD* measure is employed. Other error measures such as the *PRD* with various normalized root mean square error and signal to noise ratio (*SNR*) are used as well (Javaid et al., 2008). However, the clinical acceptability of the reconstructed signal is desired to be as low as possible. To enable comparison between signals with different amplitudes, a modification of the RMS error estimate has been devised. The *PRD* is defined as:

$$PRD = \sqrt{\frac{\sum_{n=1}^{N} (x(n) - \hat{x}(n))^2}{\sum_{n=1}^{N} x^2(n)}} \tag{11}$$

This error estimate is the one most commonly used in all scientific literature concerned with ECG compression techniques. The main drawbacks are the inability to cope with baseline fluctuations and the inability to discriminate between the diagnostic portions of an ECG curve. However, its simplicity and relative accuracy make it a popular error estimate among researchers (Benzid et al., 2003; Blanco-Velasco et al., 2004).

As the PRD is heavily dependent on the mean value, it is more appropriate to use the modified criteria:

$$PRD_1 = \sqrt{\frac{\sum_{n=1}^{N} (x(n) - \hat{x}(n))^2}{\sum_{n=1}^{N} (x(n) - \bar{x})^2}} \tag{12}$$

where $\bar{x}$ is the mean value of the signal. Furthermore, it is established in (Zigel et al., 2000), that if the PRD_1 value is between 0 and 9%, the quality of the reconstructed signal is either

'very good' or 'good', whereas if the value is greater than 9% its quality group cannot be determined. As we are strictly interested in very good and good reconstructions, it is taken that the PRD value, as measured with (11), must be less than 9%.

In (Zigel et al., 2000), a new error measure for ECG compression techniques, called the weighted diagnostic distortion measure (*WDD*), was presented. It can be described as a combination of mathematical and diagnostic subjective measures. The estimate is based on comparing the PQRST complex features of the original and reconstructed ECG signals. The *WDD* measures the relative preservation of the diagnostic information in the reconstructed signal. The features investigated include the location, duration, amplitudes and shapes of the waves and complexes that exist in every heartbeat. Although, the *WDD* is believed to be a diagnostically accurate error estimate, it has been designed for surface ECG recordings. More recently (Al-Fahoum, 2006), quality assessment of ECG compression techniques using a wavelet-based diagnostic measure has been developed. This approach is based on decomposing the segment of interest into frequency bands where a weighted score is given to the band depending on its dynamic range and its diagnostic significance.

6. DWT based ECG signal compression algorithms

As described above, the process of decomposing a signal x into approximation and detail parts can be realized as a filter bank followed by down-sampling (by a factor of 2) as shown in Figure (4). The impulse responses $h[n]$ (low-pass filter) and $g[n]$ (high-pass filter) are derived from the scaling function and the mother wavelet. This gives a new interpretation of the wavelet decomposition as splitting the signal x into frequency bands. In hierarchical decomposition, the output from the low-pass filter h constitutes the input to a new pair of filters. This results in a multilevel decomposition. The maximum number of such decomposition levels depends on the signal length. For a signal of size N, the maximum decomposition level is $\log_2(N)$.

The process of decomposing the signal x can be reversed, that is given the approximation and detail information it is possible to reconstruct x. This process can be realized as up-sampling (by a factor of 2) followed by filtering the resulting signals and adding the result of the filters. The impulse responses h' and g' can be derived from h and g. If more than two bands are used in the decomposition we need to cascade the structure.

In (Chen et al., 1993), the wavelet transform as a method for compressing both ECG and heart rate variability data sets has been developed. In (Thakor et al., 1993), two methods of data reduction on a dyadic scale for normal and abnormal cardiac rhythms, detailing the errors associated with increasing data reduction ratios have been compared. Using discrete orthonormal wavelet transforms and Daubechies D_{10} wavelets, Chen et al., compressed ECG data sets resulting in high compression ratios while retaining clinically acceptable signal quality (Chen & Itoh, 1998). In (Miaou & Lin, 2000), D_{10} wavelets have been used, with the incorporating of adaptive quantization strategy which allows a predetermined desired signal quality to be achieved. Another quality driven compression methodology based on Daubechies wavelets and later on biorthogonal wavelets has been proposed (Miaou & Lin, 2002). The latter algorithm adopts the set partitioning of hierarchical tree (SPIHT) coding strategy. In (Bradie, 1996), the use of a wavelet-packet-based algorithm for the compression of the ECG signal has been suggested. By first normalizing beat periods using multi rate processing and normalizing beat amplitudes the ECG signal is converted into a near cyclostationary sequence (Ramakrishnan & Saha, 1997). Then Ramakrishnan and Saha

employed a uniform choice of significant Daubechies D_4 wavelet transform coefficients within each beat thus reducing the data storage required. Their method encodes the QRS complexes with an error equal to that obtained in the other regions of the cardiac cycle. More recent DWT data compression schemes for the ECG include the method using non-orthogonal wavelet transforms (Ahmed et al., 2000), and SPIHT algorithm (Lu et al., 2000).

6.1 Optimization-based compression algorithm

As it has been mentioned before, many of the resulting wavelet coefficients are either zero or close to zero. These coefficients are divided into two classes according to their energy content; namely: high energy coefficients and low energy coefficients. By coding only the larger coefficients, many bits are already discarded. The high energy coefficients should be compressed very accurately because they contain more information. So, they are threshold with low threshold levels. However, the low energy coefficients that represent the details are threshold with high threshold levels. The success of this scheme is based on the fact that only a fraction of nonzero value wavelet coefficients may be encoded using a small number of bits.
In (Zou & Tewfik, 1993), the problem of finding a wavelet that best matches the wave shape of the ECG signal has been addressed. The main idea behind this approach is to find the minimum distortion representation of a signal, subject to a given bit budget or to find the minimum bit rate representation of a signal, subject to a target PRD. If, for a given wavelet, the error associated with the compressed signal is minimal, then its wavelet coefficients are considered to best represent the original signal. Therefore, the selected wavelet would more effectively match the signal under analysis when compared to standard wavelets (Daubechies, 1998). The DWT of the discrete type signal x[n] of length N is computed in a recursive cascade structure consisting of decimators $\downarrow 2$ and complementing low-pass (h) and high-pass (g) filters which are uniquely associated with a wavelet. The signal is iteratively decomposed through a filter bank to obtain its discrete wavelet transform. This gives a new interpretation of the wavelet decomposition as splitting the signal into frequency bands. Figure (4) depicts a diagram of the filter bank structure. In hierarchical decomposition, the output from the low-pass filter constitutes the input to a new pair of filters. The filters coefficients corresponding to scaling and wavelet functions are related by

$$g[n] = (-1)^n h[L - n], \quad n = 0, 1, \ldots, L-1 \tag{13}$$

where L is the filter length. To adapt the mother wavelet to the signals for the purpose of compression, it is necessary to define a family of wavelets that depend on a set of parameters and a quality criterion for wavelet selection (i.e. wavelet parameter optimization). These concepts have been adopted to derive a new approach for ECG signal compression based on dyadic discrete orthogonal wavelet bases, with selection of the mother wavelet leading to minimum reconstruction error. An orthogonal wavelet transform decomposes a signal into dilated and translated versions of the wavelet function $\psi(t)$. The wavelet function $\psi(t)$ is based on a scaling function $\varphi(t)$ and both can be represented by dilated and translated versions of this scaling function.

$$\varphi(t) = \sum_{n=0}^{L-1} h(n)\varphi(2t-n) \quad \text{and} \quad \psi(t) = \sum_{n=0}^{L-1} g(n)\varphi(2t-n) \tag{14}$$

With these coefficients *h(n)* and *g(n)*, the transfer functions of the filter bank that are used to implement the discrete orthogonal wavelet transform, can be formulated.

$$H(z) = \sum_{b=0}^{L-1} h(n) z^{-n} \quad and \quad G(z) = \sum_{b=0}^{L-1} g(n) z^{-n} \tag{15}$$

For a finite impulse response (FIR) filter of length *L*, there are $L/2+1$ sufficient conditions to ensure the existence and orthogonality of the scaling function and wavelets (Donoho & Johnstone, 1998). Thus $L/2-1$ degrees of freedom (free parameters) remain to design the filter *h*.

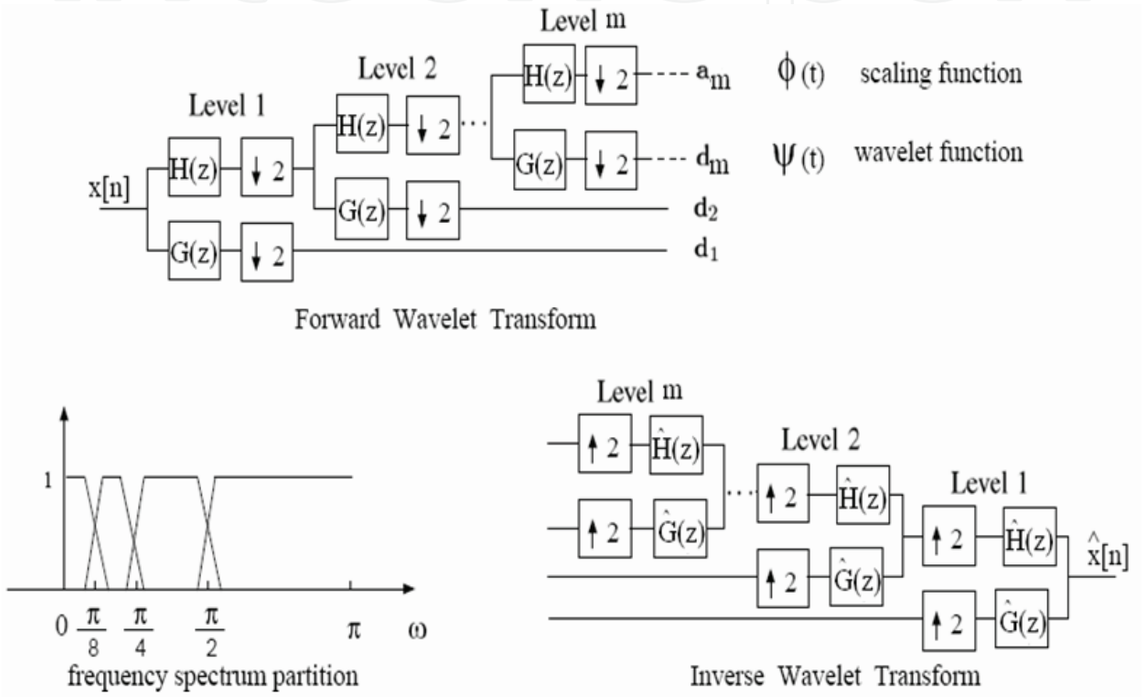


Fig. 4. The DWT implementation using a filter bank structure.

The lattice parameterization described in (Vaidyanathan, 1993) offers the opportunity to design *h* via unconstrained optimization: the *L* coefficients of *h* can be expressed in term of $L/2-1$ new free parameters. These parameters can be used to choose the wavelets which results in a good coding performance. The Daubechies wavelet family was constructed by using all the free parameters to maximize the number of vanishing moments. Coiflet wavelets were designed by imposing vanishing moments on both the scaling and wavelet functions. In (Zou & Tewfik, 1993) wavelet parameterizations have been used to systematically generate *L*-tap orthogonal wavelets using the $L/2-1$ free parameters for *L* = 4, 6 and 8. The order of a wavelet filter is important in achieving good coding performance. A higher order filter can be designed to have good frequency localization which in turn increases the energy compaction. Consequently, by restriction to the orthogonal case, *h* defines ψ. For this purpose consider, the orthogonal 2x2 rotational angles, realized by the lattice section shown in Figure (5), and defined by the matrix:

$$R(\beta_i) = \begin{bmatrix} \cos\beta_i & -\sin\beta_i \\ \sin\beta_i & \cos\beta_i \end{bmatrix} \tag{16}$$

The polyphase matrix $H_p(z)$ can be defined in terms of the rotational angles as

$$H_p(z) = \begin{bmatrix} H_e(z^2) & H_o(z^2) \\ G_e(z^2) & G_o(z^2) \end{bmatrix} = \prod_{i=o}^{L/2-1} \left(\begin{bmatrix} \cos\beta_i & -\sin\beta_i \\ \sin\beta_i & \cos\beta_i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \right) \tag{17}$$

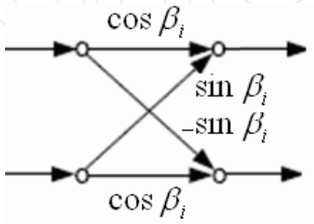


Fig. 5. Lattice Implementation

where, $H_e(z)$, $H_o(z)$, $G_e(z)$ and $G_o(z)$ are defined, respectively, from the decomposition of $H(z)$ and $G(z)$ as

$$H(z) = H_e(z^2) + z^{-1} H_o(z^2) \tag{18a}$$

and

$$G(z) = G_e(z^2) + z^{-1} G_o(z^2) \tag{18b}$$

To obtain the expressions for the coefficients of *H(z)* in terms of the rotational angles, it is necessary to multiply out the above matrix product. In order to parameterize all orthogonal wavelet transforms leading to a simple implementation, the following facts should be considered.

1. Orthogonality is structurally imposed by using lattice filters consisting of orthogonal rotations.
2. The sufficient condition for constructing a wavelet transform, namely one vanishing moment of the wavelet, is guaranteed, by assuring the sum of all rotation angles of the filters to be exactly -45^o .

A suitable architecture for the implementation of the orthogonal wavelet transforms are lattice filters. However, the wavelet function should be of zero mean, which is equivalent to the wavelet having at least one vanishing moment and the transfer functions *H(z)* and *G(z)* have at least one zero at z =-1 and z=1 respectively. These conditions are fulfilled if the sum of all rotation angles is 45^o (Xie & Morris, 1994), i.e.,

$$\sum_{i=1}^{L/2} \beta_i = 45^o \tag{19}$$

Therefore, a lattice filter whose sum of all rotation angles is 45^o performs an orthogonal WT independent of the angles of each rotation. For a lattice filter of length *L*, *L/2* orthogonal

rotations are required. Denote the rotation angles by β_i , $i = 1, 2, \ldots, L/2$, and considering the constraint given in (19), the number of design angles θ s is *L/2-1.* The following is the relation between the rotation angles and the design angles.

$$\left.\begin{aligned} \beta_1 &= 45^o - \theta_1 \,, \\ \beta_i &= (-1)^i (\theta_{i-1} + \theta_i) \;\; for \;\; i = 2, 3, \ldots, L/2 - 1, \\ \beta_{L/2} &= (-1)^{L/2} \; \theta_{L/2-1} \end{aligned}\right\} \tag{20}$$

At the end of the decomposition process, a set of vectors representing the wavelet coefficients is obtained

$$C = \left\{ d_1 \;,\; d_2 \;,\; d_3 \;,\; \ldots \;,\; d_j \;,\; \ldots \;,\; d_m \;,\; a_m \right\} \tag{21}$$

where, *m* is the number of decomposition levels of the DWT. This set of approximation and detail vectors represents the DWT coefficients of the original signal. Vectors d_j contain the detail coefficients of the signal in each scale *j*. As *j* varies from 1 to *m*, a finer or coarser detail coefficients vector is obtained. On the other hand, the vector a_m contains the approximation wavelet coefficients of the signal at scale *m*. It should be noted that this recursive procedure can be iterated $\left(m \le \log_2 N\right)$ times at most. Depending on the choice of *m*, a different set of coefficients can be obtained. The inverse transform can be performed using a similar recursive approach. Thus, the process of decomposing the signal *x* can be reversed, that is given the approximation and detail information it is possible to reconstruct *x*. This process can be realized as up-sampling (by a factor of *2*) followed by filtering the resulting signals and adding the result of the filters. The impulse responses *h'* and *g'* can be derived from *h* and *g*. However, to generate an orthogonal wavelet, *h* must satisfy some constraints. The basic condition is $\sum_{n=1}^{L} h(n) = \sqrt{2}$, to ensure the existence of ϕ . Moreover, for orthogonality, *h* must be of norm one and must satisfy the quadratic condition

$$\sum_{n=1}^{L} h(n) \sum_{n=1}^{L} h(n - 2k) = 0, \;\; for \;\; k = 1, \ldots, L/2 - 1 \tag{22}$$

The lattice parameterization described in (Vaidyanathan, 1993) offers the opportunity to design *h* using unconstrained optimization by expressing the $L/2 - 1$ free parameters in terms of the design parameter vector θ . For instance, if *L = 6*, two-component design vector, $\theta = [\theta_1 \;,\; \theta_2]$ is needed, and *h* is given by (Vaidyanathan, 1993):

$$i = 0 \;,\; 1 \qquad h(i) = \left[\left(1 + (-1)^i \cos\theta_1 + \sin\theta_1\right)\left(1 - (-1)^i \cos\theta_2 - \sin\theta_2\right) + (-1)^i \; 2 \sin\theta_2 \cos\theta_1 \right] / 4\sqrt{2}$$

$$i = 2 \;,\; 3 \qquad h(i) = \left[1 + \cos(\theta_1 - \theta_2) + (-1)^i \; \sin(\theta_1 - \theta_2) \right] / 2\sqrt{2}$$

$$i = 4 \;,\; 5 \qquad h(i) = 1/\sqrt{2} - h(i-4) - h(i-2) \tag{23}$$

For other values of L, expressions of h are given in (Maitrot et al., 2005). With this wavelet parameterization there are infinite available wavelets which depend on the design parameter vector θ to represent the ECG signal at hand. Different values of θ may lead to different quality in the reconstructed signal. In order to choose the optimal θ values, and thus the optimal wavelet, a blind criterion of performance is needed. Figure (6) illustrates the block diagram of the proposed compression algorithm. In order to establish an efficient solution scheme, the following precise problem formulation is developed. For this purpose, consider the one-dimensional vector $x(i)$, $i=1, 2, 3, \ldots, N$ represents the frame of the ECG signal to be compressed; where N is the number of its samples. The initial threshold values are computed separately for each subband by finding the mean (μ) and standard deviation (σ) of the magnitude of the non-zero wavelet coefficients in the corresponding subband. If the σ is greater than μ then the threshold value in that subband is set to (2*μ), otherwise, it is set to (μ-σ). Also, define the targeted performance measures PRD_{target} and CR_{target} and start with an initial wavelet design parameter vector $\theta = [\theta_{10}, \theta_{20}, \ldots, \theta_{L-10}]$ to construct the wavelet filters $H(z)$ and $G(z)$. Figure (7) illustrates the compression algorithm for satisfying predefined PRD (PRD_1) with minimum bit rate representation of the signal. The same algorithm with little modifications is used for satisfying predefined bit rate with minimum signal distortion measured by PRD (PRD_1); case 2. In this case, the shaded two blocks are replaced by: CR calculation and predefined CR is reached?, respectively.

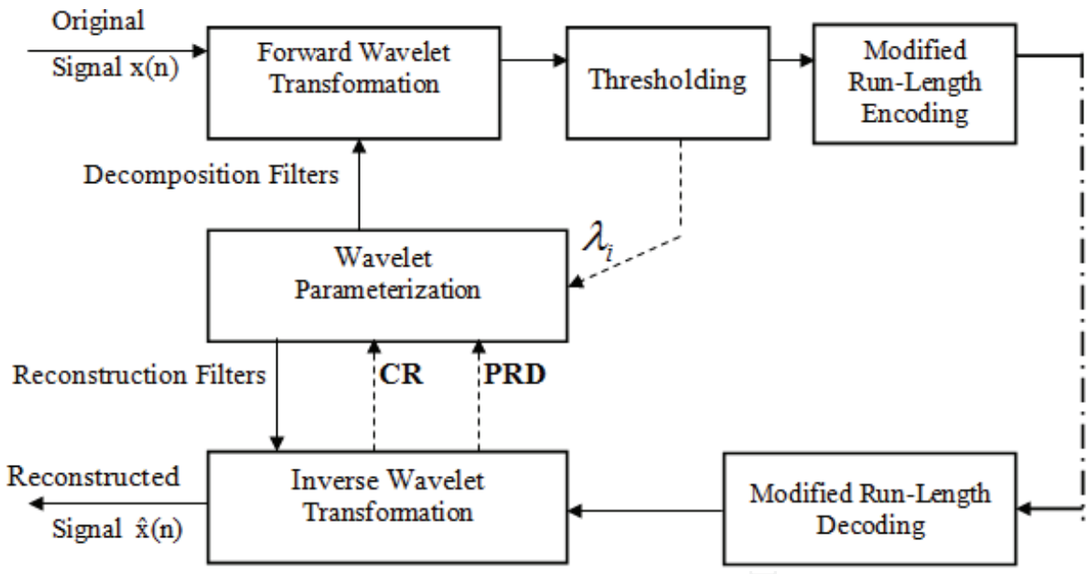


Fig. 6. Block diagram for the proposed compression algorithm.

6.2 Compression of ECG signals using SPIHT algorithm

SPIHT is an embedded coding technique; where all encodings of the same signal at lower bit rates are embedded at the beginning of the bit stream for the target bit rate. Effectively, bits are ordered in importance. This type of coding is especially useful for progressive transmission and transmission over a noisy channel. Using an embedded code, an encoder can terminate the encoding process at any point, thereby allowing a target rate or distortion parameter to be met exactly. Typically, some target parameters, such as bit count, is monitored in the encoding

process and when the target is met, the encoding simply stops. Similarly, given a bit stream, the decoder can cease decoding at any point and can produce reconstruction corresponding to all lower-rate encodings. EZW, introduced in (Shapiro, 1993) is a very effective and computationally simple embedded coding algorithm based on discrete wavelet transform, for image compression. SPIHT algorithm introduced for image compression in (Said & Pearlman, 1996) is a refinement to EZW and uses its principles of operation.

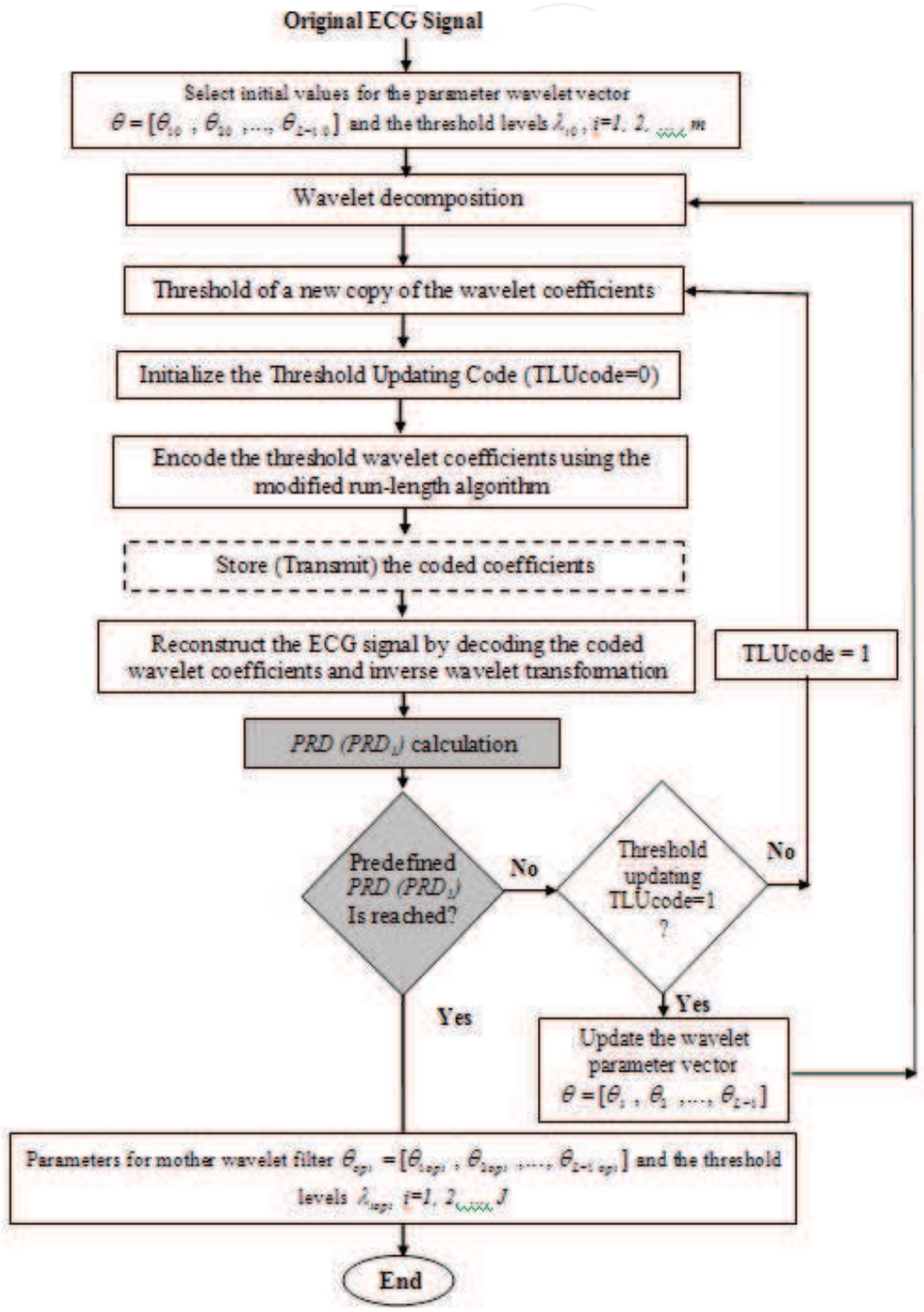


Fig. 7. Compression Algorithm for Satisfying Predefined *PRD* with Minimum Bit Rate.

These principles are partial ordering of transform coefficients by magnitude with a set partitioning sorting algorithm, ordered bit plane transmission and exploitation of self-similarity across different scales of an image wavelet transform. The partial ordering is done by comparing the transform coefficients magnitudes with a set of octavely decreasing thresholds. In this algorithm, a transmission priority is assigned to each coefficient to be transmitted. Using these rules, the encoder always transmits the most significant bit to the decoder. In (Lu et al., 2000), SPIHT algorithm is modified for 1-D signals and used for ECG compression. For faster computations SPIHT algorithm can be described as follows:

1. ECG signal is divided to contiguous non-overlapping frames each of N samples and each frame is encoded separately.
2. DWT is applied to the ECG frames up to L decomposition levels.
3. Each wavelet coefficient is represented by a fixed-point binary format, so it can be treated as an integer.
4. SPIHT algorithm is applied to these integers (produced from wavelet coefficients) for encoding them.
5. The termination of encoding algorithm is specified by a threshold value determined in advance; changing this threshold, gives different compression ratios.
6. The output of the algorithm is a bit stream (0 and 1). This bit stream is used for reconstructing signal after compression. From it and by going through inverse of SPIHT algorithm, we compute a vector of N wavelet coefficients and using inverse wavelet transform, we make the reconstructed N sample frame of ECG signal.

In (Pooyan et al., 2005), the above algorithm is tested with N=1024 samples, L=6 levels and the DWT used is biorthogonal 9/7 (with symmetric filters h(n) with length 9 and g(n) with length 7). The filters' coefficients are given in Table (1).

n	0	±1	±2	±3	±4
h(n)	0,852699	0,377403	-0.11062	-0.023849	0.037829
g(n)	0.788485	0.418092	-0.04069	-0.064539	

Table 1. Coefficients of the Biorthogonal 9/7 Tap Filters.

6.3 2-D ECG compression methods based on DWT

By observing the ECG waveforms, a fact can be concluded that the heartbeat signals generally show considerable similarity between adjacent heartbeats, along with short-term correlation between adjacent samples. However, most existing ECG compression techniques did not utilize such correlation between adjacent heartbeats. A compression scheme using two-dimensional DWT transform is an option to employ the correlation between adjacent heartbeats and can thus further improve the compression efficiency. In (Reza et al., 2001; Ali et al., 2003) a 2-D wavelet packet ECG compression approach and a 2-D wavelet based ECG compression method using the JPEG2000 image compression standard have been presented respectively. These 2-D ECG compression methods consist of: 1) QRS detection, 2) preprocessing (cut and align beats, period normalization, amplitude normalization, mean removal), 3) transformation, and 4) coefficient encoding. Period normalization helps utilizing the interbeat correlation but incurs some quantization errors. Mean removal helps maximizing the interbeat correlation since dc value of each beat is different due to baseline change. Recently (Tai et al., 2005), a 2-D approach for ECG compression that utilizes the

redundancy between adjacent heartbeats has been presented. The QRS complex in each heartbeat is detected for slicing and aligning a 1-D ECG signal to a 2-D data array, and then 2-D wavelet transform is applied to the constructed 2-D data array. Consequently, a modified SPIHT algorithm is applied to the resulting wavelet coefficients for further compression. The way that the 2-D ECG algorithm presented in (Tai et al., 2005) differs from other 2-D algorithms, (Reza et al., 2001; Ali et al., 2003), is that it not only utilizes the interbeat correlation but also employs the correlation among coefficients in relative subbands. More recently (Wang & Meng, 2008), a new 2-D wavelet-based ECG data compression algorithm has been presented. In this algorithm a 1-D ECG data is first segmented and aligned to a 2-D data array, thus the two kinds of correlation of heartbeat signals can be fully utilized. And then 2-D wavelet transform is applied to the constructed 2-D ECG data array. The resulting wavelet coefficients are quantized using a modified vector quantization (VQ). This modified VQ algorithm constructs a new tree vector which well utilizing the characteristics of the wavelet coefficients. Experimental results show that this method is suitable for various morphologies of ECG data, and that it achieves higher compression ratio with the characteristic features well preserved.

6.4 Hybrid ECG signal compression methods

Hybrids ECG signal compression methods are constructed from more than time and/or frequency domain techniques (Ahmed et al., 2007). These include Modified Discrete Cosine Transform (MDCT) and DWT; linear prediction coding and DWT. By studying the ECG waveforms, it can be concluded that the ECG signals generally show two types of correlation, namely correlation between adjacent samples within each ECG cycles (intrabeat correlation) and correlation between adjacent heartbeats (interbeat correlation) (Xingyuan & Juan, 2009). However, most existing ECG compression techniques did not utilize such correlation between adjacent heartbeats. Hybrid compression methods of ECG signals are discussed in this section, which fully utilizes the interbeat correlation and thus can further improve the compression efficiency.

6.4.1 ECG signal compression based on combined MDCT and DWT

In (Ahmed et al., 2008), a hybrid two-stage electrocardiogram (ECG) signals compression method based on the MDCT and DWT has been proposed. The ECG signal is partitioned into blocks and the MDCT is applied to each block to decorrelate the spectral information. Then, the DWT is applied to the resulting MDCT coefficients. The resulting wavelet coefficients are then threshold and compressed using energy packing and binary-significant map coding technique for storage space saving. MDCT is a linear orthogonal lapped transform, based on the idea of time domain aliasing cancellation (TDAC). It is designed to be performed on consecutive blocks of a larger dataset, where subsequent blocks are overlapped so that the last half of one block coincides with the first half of the next block. This overlapping, in addition to the energy-compaction qualities of the DCT, makes the MDCT especially attractive for signal compression applications. Thus, it helps to avoid artifacts stemming from the block boundaries (Britanak & Rao, 2002; Nikolajevic & Fettweis, 2003). MDCT is critically sampled, which means that though it is 50% overlapped, a sequence data after MDCT has the same number of coefficients as samples before the transform (after overlap-and-add). This means that, a single block of IMDCT data does not correspond to the original block on which the MDCT was performed. When subsequent

blocks of inverse transformed data are added, the errors introduced by the transform cancel out TDAC. The MDCT is defined as (Nikolajevic & Fettweis, 2003):

$$X_C(k) = \sum_{n=0}^{N-1} x(n) \cos\left[\left(n + \frac{M+1}{2}\right)\left(k + \frac{1}{2}\right)\frac{\pi}{M}\right], \quad k = 0, 1, \ldots M-1 \tag{24}$$

where, *x(n), n=0, 1, 2, …,* N-1 is the sequence to be transformed, N=2M is the window length and M is the number of transform coefficients. The computation burden can be reduced if the transform coefficients given by equation (24) are rewritten in the following recursive form

$$X_C(k) = x(0) \cos\left[(M+1)\frac{\theta_k}{2}\right] + V_1 \cos\left[(M+3)\frac{\theta_k}{2}\right] - V_2 \cos\left[(M+1)\frac{\theta_k}{2}\right] \tag{25}$$

Where,

$$V_m = x(m) + 2\cos\theta_k V_{m+1} - V_{m+2}, \; m = N-1, N-2, \ldots, 1, 0 \tag{26}$$

and

$$\theta_k = \left(k + \frac{1}{2}\right)\frac{\pi}{M} \tag{27}$$

The MDCT computation algorithm of a data sequence *x(n)* can be summarized in the following:

1. Partition the data sequence in N_b consecutive blocks, each one with *N=64* samples.
2. Recursively generate the V_m from the input sequence *x(n)* according to (26) and (27).
3. Calculate the MDCT coefficients for each block by evaluating the k-th MDCT coefficient using (25) at the N-th step.

In the decompression stage, the inverse MDCT, that is termed IMDCT, is adopted. Because there are different numbers of inputs and outputs, at first glance it might seem that the MDCT should not be invertible. However, perfect invertability is achieved by adding the overlapped IMDCTs of subsequent overlapping blocks, causing the errors to cancel and the original data to be retrieved. The IMDCT transforms the *M* real coefficients, $X_C(0), X_C(1), \ldots, X_C(M-1)$, into *N=2M* real numbers, *x(0), x(1), ….. , x(N-1)*, according to the formula:

$$x(n) = \sum_{k=0}^{M-1} X_C(k) \cos\left[\left(n + \frac{M+1}{2}\right)\left(k + \frac{1}{2}\right)\frac{\pi}{M}\right], \quad n = 0, 1, \ldots N-1 \tag{28}$$

Again, the computation burden of *x(n)* can be reduced considerably if equation (28) is rewritten in the following recursive form

$$x(n) = X_C(0) \cos\left[\frac{\theta_n}{2}\right] + V_1 \cos\left[\frac{3\theta_n}{2}\right] - V_2 \cos\left[\frac{\theta_n}{2}\right] \tag{29}$$

Where, $V_m = X_C(m) + 2\cos\theta_n V_{m+1} - V_{m+2}$ and $\theta_n = \left(n + \frac{M+1}{2}\right)\frac{\pi}{M}$ (30)

6.4.2 ECG signal compression based on the linear prediction of DWT coefficients

In (Abo-Zahhad et al., 2000; Ahmed & Abo-Zahhad, 2001), a new hybrid algorithm for ECG compression based on the compression of the linearly predicted residuals of the wavelet coefficients is presented. The main goal of the algorithm is to reduce the bit rate while keeping the reconstructed signal distortion at a clinically acceptable level. In this algorithm, the input signal is divided into blocks and each block goes through a discrete wavelet transform; then the resulting wavelet coefficients are linearly predicted. In this way, a set of uncorrelated transform domain signals is obtained. These signals are compressed using various coding methods, including modified run-length and Huffman coding techniques. The error corresponding to the difference between the wavelet coefficients and the predicted coefficients is minimized in order to get the best predictor.

7. Thresholding and coding of DWT coefficients

Thresholding DWT coefficients are very similar to the method that our ears take to de-noise a music signal. We concentrate on the high peaks and try to ignore the low crackling of the white noise. Because DWT coefficients are based on amplitude and location of the signal, we can separate much of the noise from the signal relatively easily. The technique of thresholding takes the DWT coefficients, and throws out (makes them zero) coefficients below a certain threshold, leaving the peaks of the signal. Then each coefficient after thresholded is quantized. A non-uniform quantization method is commonly used to increase the compression and decrease the distortion in the reconstructed signal. The quantized coefficients are then encoded. The wavelet domain representation itself does not introduce any compression. Compression is obtained by encoding the thresholded wavelet coefficients using optimal thresholding levels. Given that most of the energy in the signal is in the lower subbands, it is reasonable to assume that after thresholding a substantial number of higher band wavelet coefficients will be set to zeros. Since these zeros tend to occur in clusters, as a direct consequence of the way in which the data are organized in vectors, run-length coding of these zeros makes sense. The basic idea of this technique is to encode a sequence of equal symbols with a certain codeword depending on the length of that sequence. Thus, two types of codewords may be used: the counter-words and the value-words. For example, the string “aaabbbbd” is encoded as: (a, 3), (b, 4), and (d, 1). In case of ECG compression, the run-length coding is done by representing the thresholded wavelet coefficients vectors in the forum of (Run, Level), where Run is the number of zeros before each nonzero coefficients, and Level is the amplitude of the coefficient following a number of zeros given by Run. The event that the last coefficient are all zeros is represented by the special code (0 , 0). For example, the set of wavelet coefficients given by Wbefore = {0 1 0 0 0 4 5 0 0 0 0 0 0 0 0 0 } is run-length coded as Wafter = { (1 , 1) (3 , 4) (0 , 5) (0 , 0) }. As it has been mentioned in section 3, the compression is based on representing the thereshold wavelet coefficients with a small number of bits. This has been carried out by discarding the WT-coefficients, which are less than a given threshold. These coefficients are considered insignificant with their values set to zero. The remaining N_S coefficients are the significant coefficients. The number of the discarded coefficients is $N_I=N-N_S$. Most of these coefficients are concentrated at the end of the coefficients’ vector. In technical literature, many algorithms are suggested to deal with signals that have repeated samples' values such as run-length coding and Huffman coding. The need of at least one bit for the mostly repeated sample is the main limitation of the Huffman coding. The disadvantage of the run-

length algorithm is the need of two words for the representation of each group of repeated samples: one for the repeated value and the other for the number of repetitions. In this section a more efficient coding algorithm, a modified run-length algorithm, is presented for dealing with this situation. The algorithm is based on representing each significant coefficient by b_S+1 bits. The insignificant coefficients (of value zero) are manipulated in a different manner. First, the repeated groups of zeros are counted and the resulting count is represented by b_S+1 bits. Then the train of coefficients representing the ECG signal is transformed to another train of numbers. Some of these numbers represent the significant coefficients and the rest are the numbers representing the repeated group of zeros (K_1, K_2, …., K_M). Here, M denotes the number of these groups. The problem here is how to differentiate between the coefficients and the numbers representing the group of zeros. For example, the number 18 may be found twice in the new train of numbers, where the first 18 may be a significant coefficient and the second one may indicate 18 repeated zeros. To overcome this problem, the first bit in the representation of each number is used as a control bit. In case of the significant coefficient this bit is set to one and in case of repeated zeros it is reset to zero.

- representation of significant residual coefficient

1	b_S – bits

- representation of a group of repeated zeros

0	b_S – bits

8. Quantization and coding of DWT coefficients

A quantizer simply reduces the number of bits needed to store the transformed coefficients by reducing the precision of those values. A quantization scheme maps a large number of input values into a smaller set of output values. This implies that some information is lost during the quantization process. The original wavelet coefficients $c(n)$ cannot be recovered exactly after quantization. An encoder further compresses the quantized values losslessly to give better overall compression. The most commonly used encoders are the Huffman encoder and the arithmetic encoder, although for applications requiring fast execution, simple run-length encoding (RLE) has proven very effective (Ahmed & Abo-Zahhad, 2001). In the following, wavelet coefficients quantization and coding algorithms are described.

8.1 Energy packing efficiency strategy

In this section, the quantization strategy adopted is based on the energy packing efficiency (EPE). It guarantees the balance between the compression achievement and information loss. Here, quantization process is performed by selecting an appropriate threshold level λ to control the compression ratio. Due to the careful representation of the ECG signal performed by DWT, it is reasonable to assume that only a few coefficients contain information about the real signal while others appear as less important details. The goal is to extract these significant coefficients and to ignore others smaller than specified threshold level λ. The optimal value of λ is determined such that the reconstructed signal is as close to the original one as possible. Usually the selection of optimal threshold level is not an easy task, because some of the coefficients that represent the actual signal details may be also killed, and as a result, signal distortion is the side effect. In (Abo-Zahhad & Rajoub, 2001, 2002) Energy Packing Efficiency (EPE) strategy has been utilized for decreasing the

distortion of the reconstructed signal. This has been performed by thresholding the wavelet coefficients of the approximation and details subbands with different threshold levels.
As it can be deduced from the above discussion, the approximation band is the smallest band in size and it includes high amplitude approximation coefficients. The wavelet coefficients other than these included in the approximation band, detail coefficients, have small magnitudes. Most of the energy is captured by these coefficients of the lowest resolution band. This can be seen from the decomposition of 4096-sample ECG signal up to the fifth level. The total energy of the signal is 94393.5. About 99.73% of this energy is concentrated in the 136 approximation coefficients and only 0.27% of the energy is concentrated in the remaining 3960 detail coefficients. Here, threshold levels are defined according to the energy packing efficiencies of the signal for all subbands. EPE for a set of coefficients in the ith subband is defined as the ratio of the energy captured by the subband coefficients and the energy captured by the whole number of coefficients.

$$EPE_i = \frac{\sum_{n=1}^{L_i} (c(n))^2}{\sum_{n=1}^{L} (c(n))^2} x100 \tag{31}$$

Where L_i and L are the number of coefficients in the ith subband and the whole number of coefficients respectively. A large threshold could attain high data reduction but poor signal fidelity and a small threshold would produce low data reduction but high signal fidelity. To explore the effect of threshold level (λ) selection and the coefficients representation on the compression ratio and PRD, the following thresholding rule is set:

Keep all the wavelet coefficients in the approximation subband without thresholding and calculate the threshold value for each details subband separately by preserving the higher amplitude wavelet coefficients in the ith details subband that contribute to α_i % of the energy in that subband.

One important feature of this rule is that the integer part of the wavelet coefficients in each subband is represented by different number of bits.

8.2 Binary significant map coding algorithm

The coding algorithm adopted here is based on grouping the significant coefficients in one vector and the locations of the insignificant coefficients in another vector. The significant coefficients are arranged from high scale coefficients to low scale coefficients. Each significant coefficient is decomposed into integer part and fractional part, where M-bits are assigned to represent the integer part (signed representation) and N-bits represent the fractional part; i.e. each coefficient is represented by N+M bits. A binary significant map is used as flags to indicate if the coefficient is significant or not. This binary stream is compressed further as will be shown in the following:

1. Threshold the wavelet coefficients, $c(n)$, to produce the threshold coefficients $\overline{c}(n)$. The threshold level (λ) is determined by using the above-mentioned rule such that the distortion in the reconstructed signal $\overline{x}_i$ is acceptable. The distortion is measured using PRD and/or visual inspection. The optimal non-orthogonal wavelet transform developed in (Ahmed et al., 2000) may be used to minimize the PRD in least mean square sense. Here, the threshold λ is determined such that the PRD is less or equal to a prescribed acceptable value defined by a cardiologist.

2. Search the vector $\bar{c}(n)$ to isolate the significant coefficient in another vector $\hat{C}_S(m)$.
3. Use finite word length representation to represent the integer and fractional parts of the coefficients, $\hat{C}_S(m)$. The number of bits used to represent these coefficients is determined as follows:
   3.1 Search the vector $\hat{C}_S(m)$ to find the maximum coefficient (in absolute value) and determine the number of bits that represents this coefficient. This can be done by finding $k = \text{Int}\left\langle \max|\hat{C}_S(m)| \right\rangle$ where $\text{Int}\langle . \rangle$ denotes the integer part. Then convert k to a binary number and count the number of bits, M.
   3.2 Similarly, find the number of bits, N, that represent the minimum value of the fractional part of each significant coefficient in such a way to keep the distortion within acceptable limits.
4. Generate a binary stream, b(n), of 1's and 0's that encodes the zero-locations in $\bar{c}(n)$. This is done by coding each significant coefficient in $\bar{c}(n)$ by a binary 1. The length of the binary stream equals n_1, where n_1 designates the index value of the last significant coefficients in $\bar{c}(n)$. Hence, there is no need to encode the zeros for $n > n_1$. The value of n_1 need not be stored because it can be determined as the length of the vector b(n) in the decoding process.
5. Compress the binary stream using run length encoding of 0's and 1's as follows:
   5.1 Set i = 1, Run-type= b(i), and set the run length Z to 1;
   If b(i) ≠ b(i+1) increment i by Z. Else, while b(i+1) = b(i), increment i by 1 and Z by 1 end; end.
   5.2 From Table (2), find the inequality that Z satisfies. Then output the symbol that specifies the run type followed by the number Z. i.e., code = [code χ Z] , where χ designates concatenation operator.
   5.3 If index < n_1 set Z=1 and go to step (5.1).
6. Represent the obtained run length code in binary format. There are 16 different symbols that can be generated from step 5. These are the digits 0-9 and the letters A-F. Hence, 4 bits can be used to represent each symbol.

Symbol	Run Type	Range
A	0	$100 \le Z \le 999$
B	0	$10 \le Z \le 99$
C	0	$2 \le Z \le 9$

Symbol	Run Type	Range
D	1	$100 \le Z \le 999$
E	1	$10 \le Z \le 99$
F	1	$2 \le Z \le 9$

Table 2. Run Length Encoding of 0's and 1.

9. Conclusion

In literature, numerous ECG compression methods have been developed. They may be defined either as reversible methods (offering low compression ratios but guaranteeing an exact or near-lossless signal reconstruction), irreversible methods (designed for higher compression ratios at the cost of a quality loss that must be controlled and characterized), or scalable methods (fully adapted to data transmission purposes and enabling lossy reconstruction). Choosing one method mainly depends on the use of the ECG signal. In the case of the needs of a first diagnosis, a reversible compression would be most suitable.

However, if compressed data has to be stored on low-capacity data supports, an irreversible compression would be necessary. Finally, scalable techniques clearly suit data transmission. All compression solutions presented in this chapter adopt DWT as a reversible compression tool. As a consequence, the following question remains: why should they all be compressed using the same algorithm? Unsurprisingly, this discussion still remains open.

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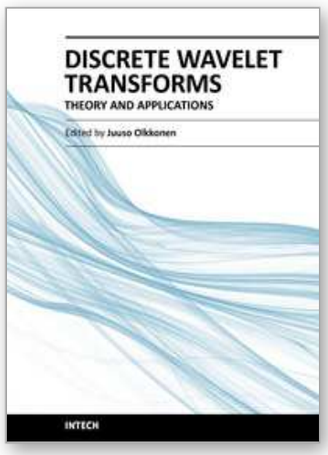
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Discrete wavelet transform (DWT) algorithms have become standard tools for discrete-time signal and image processing in several areas in research and industry. As DWT provides both frequency and location information of the analyzed signal, it is constantly used to solve and treat more and more advanced problems. The present book: Discrete Wavelet Transforms: Theory and Applications describes the latest progress in DWT analysis in non-stationary signal processing, multi-scale image enhancement as well as in biomedical and industrial applications. Each book chapter is a separate entity providing examples both the theory and applications. The book comprises of tutorial and advanced material. It is intended to be a reference text for graduate students and researchers to obtain in-depth knowledge in specific applications.

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