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Agent-based Simulation Analysis for Effectiveness of Financing Public Goods with Lotteries

Ichiro Nishizaki, Tomohiko Sasaki and Tomohiro Hayashida
Hiroshima University
Japan

1. Introduction

In this chapter, we present the result of agent-based simulations to examine the effectiveness of financing public goods. Morgan (2000) develops a mathematical equilibrium model of lotteries for financing public goods. Moreover, Morgan and Sefton (2000) conduct experiments with human subjects and focus on the following three points. First, when it is efficient to provide a positive amount of public goods, the provision of the public goods through the lottery mechanism is more than the provision through the voluntary contribution mechanism. Second, the provision of the public goods increases with the size of the lottery prize. Third, wagers of the lottery mechanism vary with the return from the public goods. On the whole the results of the experiments support the above three predictions from the mathematical equilibrium model.

Simulation analysis is advantageous for implementing a model of a certain social system and examining the effectiveness of the social system, and from this viewpoint we employ simulation analysis in order to show the effectiveness of lotteries for financing and to examine the validity of the mathematical equilibrium model and the experiments with human subjects. While mathematical models are based on optimization such as maximization of the individual payoff or utility, our agent-based simulation model is based on adaptive behaviors of agents, that is, agents evaluate results of their decisions and revise policies to select one among multiple alternatives as actual decision makers do so. From this sense we can expect a reasonable interpretation of the gaps between the results of the mathematical equilibrium model and the experiments with human subjects.

As concerns approaches based on adaptive behavior models, Holland and Miller (1991) interpret most of economic systems as complex adaptive systems, and point out that simulations using artificial societies with adaptive agents is effective for analysis of such economic systems. Axelrod (1997) insists on the need for simulation analysis in social sciences, and states that purposes of the simulation analysis include prediction, performance, training, entertainment, education, proof and discovery.

In the literature, some successful attempts of agent-based simulations are reported. For the iterated prisoner's dilemma game, Axelrod (1987) examine the effectiveness of strategies generated in an artificial society system, in which agents endowed with strategies are adaptively evolved by using a genetic algorithm.

Dorsey et al. (1994) employ an artificial decision mechanism using neural networks to imitate decision making of auctioneers, and compare the behavior of artificial agents with that of real auctioneers which often deviates from Nash equilibria. To estimate bid functions of bidders, i.e., to establish appropriate weights of a neural network for determining bids, they employ a genetic adaptive neural network algorithm based on genetic algorithms instead of the error back propagation algorithm which is a commonly used method. The objective of their study is to identify the optimal bid function given the bids of the experimental subjects, and then they explore neural networks as an aid in evaluating economic data.

Andreoni and Miller (1995) use genetic algorithms to model decision making in auctions. As in Dorsey et al. (1994), they also compare the decisions of artificial adaptive agents with the experimental data by human subjects, and find that the two types of decisions by the artificial agents and the human subjects resemble each other. Erev and Rapoport (1998) investigate a market entry game by using an adaptive learning model based on reinforcement learning proposed by Roth and Erev (1995). Rapoport et al. (2002) also deal with market entry games, and compare the decisions observed in experiments with human subjects with the actions of artificial adaptive agents with a learning mechanism using reinforcement learning, and analyze behavioral patterns in the aggregate level of the experimental data.

Leshno et al. (2002) consider equilibrium problems in market entry games through agent-based simulations with agents' decision mechanism based on neural networks, and the neural networks are trained not by some teacher signals but by outcomes of games. They compare the results of the simulations with the experimental data by human subjects shown in Sundali et a. (1995), and find some similarities between phenomena of the simulations and the experiments. Nishizaki et al. (2005) investigate the effectiveness of a socio-economic system for preserving global commons by simulation analysis. Moreover, using an agent-based simulation model, Nishizaki et al. (2009) examine the behavior of agents with respect to the social norm by varying values of some parameters. A number of attempts have been made for performing multi-agent based simulations and developing the related techniques underlying ideas from distributed artificial intelligence and multi-agent systems (BanerjeSen, 2002; Chellapilla and Fogel, 1999; Conte et al., 1997; Downing et al., 2001; Epstein and Axtell, 1996; Moss and Davidsson, 2001; Niv et al., 2002; Parsons et al., 2002; Sichman et al., 2003).

By the above mentioned researches and the related articles, the effectiveness of simulation analysis with artificial adaptive agents has been recognized. In this chapter, to examine the effectiveness of lotteries for financing public goods, we give results of the agent-based simulations with decision and learning mechanism based on neural networks and genetic algorithms by extensively varying values of the parameters of the mathematical equilibrium model (Morgan, 2000) which is also the basis of the experiments with human subjects by Morgan and Sefton (2000). In particular, it should be noted that, in our system, a genetic algorithm is used not to establish appropriate weights of a neural network such as Leshno et al. (2002) but to develop agents with good performance, as used in Nishizaki et al. (2009).

In the simulations, we deal with three parameters: the exogenous contribution which becomes the prize in the lottery game and utilizes for funding the public goods directly in the voluntary contribution game, the marginal per capita return of the public good provision, and the group size which is the number of players in the games. Furthermore, providing a simple learning mechanism and a more elaborate one, we examine which of agent behaviors with those two learning mechanisms approaches closer to the prediction of the mathematical equilibrium model.

In section 2, we briefly review the mathematical equilibrium model by Morgan (2000) and the experiment with human subjects by Morgan and Sefton (2000). In section 3, we describe agent-based simulation model with learning mechanisms based on neural networks and genetic algorithms. We give the results of the simulations in section 4, and analyze sensitivity with respect to parameters of our model in section 5. Finally, in section 6 we make some concluding remarks.

2. A mathematical equilibrium model and experiments for financing public goods by lotteries

A mathematical equilibrium model for financing public goods by lotteries is developed by Morgan (2000). Let the set of players be denoted by $N = \{1, \ldots, n\}$, where a player is a contributor in a voluntary contribution game or a bettor in a lottery game. In general, player i has the following payoff function:

$$U_i = w_i + h_i(G), \tag{1}$$

where w_i is the wealth of player i and $G \in \mathbb{R}_+$ denotes the level of the public goods provided; $\mathbb{R}_+$ is the set of non-negative real numbers; player i has a diminishing marginal payoff from the provision of the public goods, i.e., the payoff $h_i(G)$ from the provision G of the public goods is characterized by $h_i'(\cdot) > 0$ and $h_i''(\cdot) < 0$; and U_i is assumed to be quasi-linear.

For a voluntary contribution game, player i chooses $x_i \in [0, w_i]$ so as to maximize the payoff

$$U_i = w_i - x_i + h_i(x(N)), \tag{2}$$

where x_i is the amount of wealth contributed by player i, and $x(N) \equiv \sum_{i \in N} x_i$ denotes the total contribution of all the players.

For a lottery game, player i chooses a wager $x_i \in [0, w_i]$ so as to maximize the expected payoff

$$U_i = w_i - x_i + \frac{x_i}{x(N)} R + h_i(x(N) - R), \tag{3}$$

where R is a prize of some fixed amount.

The results of the mathematical equilibrium model by Morgan (2000) are summarized as follows.

1. Voluntary contributions underprovide the public goods relative to first-best levels.

2. The lottery with a fixed prize has a unique equilibrium.

3. The lottery with a fixed prize provides more of the public goods than the voluntary contributions.

In the experiment by Morgan and Sefton (2000), a linear-homogeneous version of the above-mentioned model by Morgan (2000) is dealt with. For the voluntary contribution game, each player has the same endowment e, and an exogenous contribution R is used to fund the public goods together with the total contribution of all the players. Thus, the payoff of player i is represented by

$$U_i = e - x_i + \beta(x(N) + R), \tag{4}$$

where β is the constant marginal per capita return of the provision of the public goods, and player i chooses a contribution $x_i \in [0, e]$ so as to maximize the payoff (4).

Assuming $\beta < 1$, for all i, the predicted equilibrium contribution of the voluntary contribution game is

$$x_i^{VC} = 0. \tag{5}$$

For the lottery game, the whole sum of wagers is assigned to the public good provision, and the exogenous contribution R is used to fund a prize. Therefore, the expected payoff of player i is represented by

$$U_i = e - x_i + R\frac{x_i}{x(N)} + \beta x(N). \tag{6}$$

Player i chooses a wager $x_i \in [0, e]$ so as to maximize the payoff (6). Then, the predicted equilibrium contribution of the lottery game is

$$x_i^L = \frac{R(n-1)}{n^2(1-\beta)}. \tag{7}$$

In the experiment, the payoff (4) is given to a subject in the voluntary contribution game or the payoff (6) in the lottery game. The primary parameters of the experiment are given as: the number of players $n = 4$, the initial endowment $e = 20$, the exogenous contribution $R = 8$, and the marginal per capita return $\beta = 0.75$. The game is iterated 20 times each treatment. The results are summarized as follows.

1. In the voluntary contribution game, the average contribution in the initial round was about 10.5, it decreased as rounds proceeded, and finally it became 8.075 in the final 20th round. The final average contribution 8.075 was considerably larger than the equilibrium contribution $x_i^{VC} = 0$.

2. In the lottery game, the average wager in the initial round was about 10, it was almost changeless as rounds proceeded, and finally it became 10.35 in the final 20th round. The final average wager 10.35 was larger than the equilibrium wager $x_i^L = \frac{8(4-1)}{4^2(1-0.75)} = 6$ and the final average contribution of 8.075 in the voluntary contribution game of the experiment.

3. In the treatment of the lottery game with the exogenous contribution $R = 16$ which was twice as large as that of the baseline treatment, the average wager in the initial round was about 13, it was almost changeless as rounds proceeded, and finally it became 13.825 in the final 20th round. This result implies that large prize lotteries will be more successful fund-raising devices than smaller scale endeavors.

4. In the treatment of the lottery game without the marginal per capita return, i.e., $\beta = 0$, the average wager in the initial round was about 8, it extremely decreased as rounds proceeded, and finally it became 2.425 in the final 20th round. This result implies that wagers are substantially reduced when the link between the public good provision and the lottery proceeds is broken.

3. A simulation model

In most of mathematical models, it is assumed that players are rational and then they maximize their payoffs. Such optimization approaches are not always appropriate for

analyzing human behaviors and social phenomena. Models based on adaptive behavior can be alternatives to the optimization models, and it is natural that behaviors of agents in simulation models are described by using adaptive behavioral rules. In particular, we employ a learning model of agents taking account of not only a payoff of self but also those of the others from a viewpoint that observation of other players' actions and payoffs may affect learning of agents (Duffy and Feltovich, 1999).

To examine the influence of learning mechanisms on the performance of agents, we employ an agent model with a decision and learning mechanism based on neural networks (e.g. Hassoun (1995)) and genetic algorithms (e.g. Goldberg (1989)), and this choice enables us to provide two grades of learning mechanisms: one is a simple learning mechanism based only on genetic algorithms, and the other is a more elaborate one based on both genetic algorithms and the error back propagation algorithm.

An agent corresponds to a neural network which is characterized by synaptic weights between two nodes in the neural network, a threshold which is a parameter for an output of a node, and a learning rate concerned with learning by error-correction. Because a structure of neural networks is determined by the number of layers and the number of nodes in each layer, an agent is prescribed by the fixed number of parameters of the neural network. By forming a chromosome consisting of these parameters characterizing an agent, each of the artificial agents is evaluated through the fitness computed from the payoff obtained by playing the voluntary contribution or the lottery game, and they evolve in our artificial genetic system. From this sense, in our simulation model, a player of the game is referred to as an artificial autonomous agent. The structures of a neural network and a chromosome in the genetic algorithm are depicted in Fig. 1.
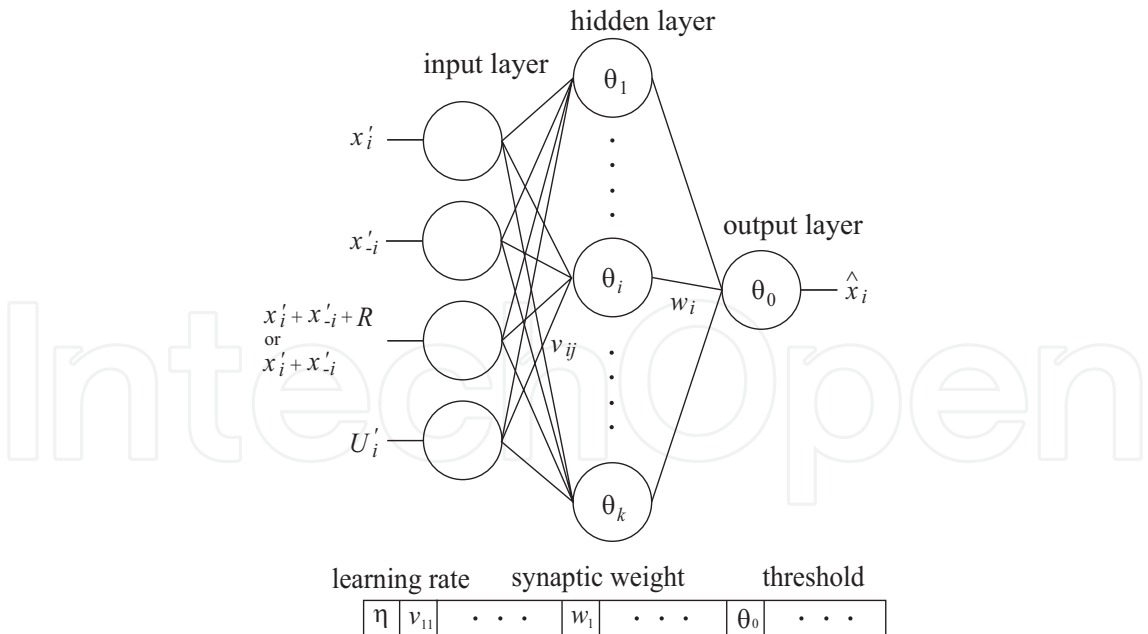

Fig. 1. Structures of a neural network and a chromosome in the genetic algorithm

We design a simulation model related to the experiment by Morgan and Sefton (2000) to compare with their results. In our simulation model, the population of agents is divided into a certain number of groups, and each group has the same number of agents. Each agent in a group determines the amount of contribution or wager by an output of the neural network

and plays the voluntary contribution or the lottery games. In the voluntary contribution game, an agent obtains a payoff defined by (4). In the lottery game, an agent obtains the following payoff:

$$U_i = \begin{cases} e - x_i + R + \beta x(N) & \text{if winning} \\ e - x_i + \beta x(N) & \text{otherwise.} \end{cases} \tag{8}$$

The payoff (8) differs from (6) of the mathematical model in the third term, which is an expected payoff $R\frac{x_i}{x(N)}$.

We provide two learning mechanisms for artificial autonomous agents in our simulation system. One is a simple learning mechanism based only on genetic algorithms, and in this mechanism agents evolve, that is, the synaptic weights and the thresholds are revised through the fitness which is computed by payoffs obtained in the games. The other is a learning mechanism based on both the genetic algorithm and the error back propagation algorithm, and in addition to learning by the genetic algorithm, after finishing games, the parameters of the neural network for the agent are revised by the error back propagation algorithm with teacher signals (target outputs) obtained by computing optimal contributions or wagers for the given contributions or wagers of the other agents. For convenience of reference, let the simple learning mechanism and the more complicated one be denoted by GA and GABP, respectively. By providing the two learning mechanisms, we can verify whether actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

The flows of simulations with GA and GABP are shown in Fig. 2, and they are summarized as follows.

Step 1 (Generating the initial population) Let the number of agents in a group and the number of groups in the population of the simulations be n and 10, respectively. Then, the initial population of $10n$ agents is formed.

Step 2 (Dividing the population into groups) From the population, n agents are randomly chosen and then one group is formed, and this procedure is repeated 10 times. Eventually, 10 groups are made in all.

Step 3 (Playing games) For each group, the voluntary contribution game or the lottery game is played by n agents.

The voluntary contribution game.

Step 3-1-VC (Determining the amount of a contribution) The contribution x_i' of agent i, the sum x_{-i}' of the contributions of the other agents, the total fund $x_i' + x_{-i}' + R$ for the public goods, and the payoff U_i' of agent i in the previous generation are input values to a neural network for agent i, and they are normalized into $[0,1]$. Let $\hat{x}_i$ be an output of the neural network. In particular, for the first generation, the input values are randomly determined from $[0,1]$. The contribution of agent i of the present generation is determined as $x_i = \lfloor (e+1)\hat{x}_i \rfloor$, where $\lfloor \cdot \rfloor$ denotes rounding off fractions.

Step 3-2-VC (Informing about the contributions of the others) Agent i is informed about the sum x_{-i} of the contributions of the other agents in the present generation.

Step 3-3-VC (Computing the payoff) The payoff U_i of agent i is calculated by (4).
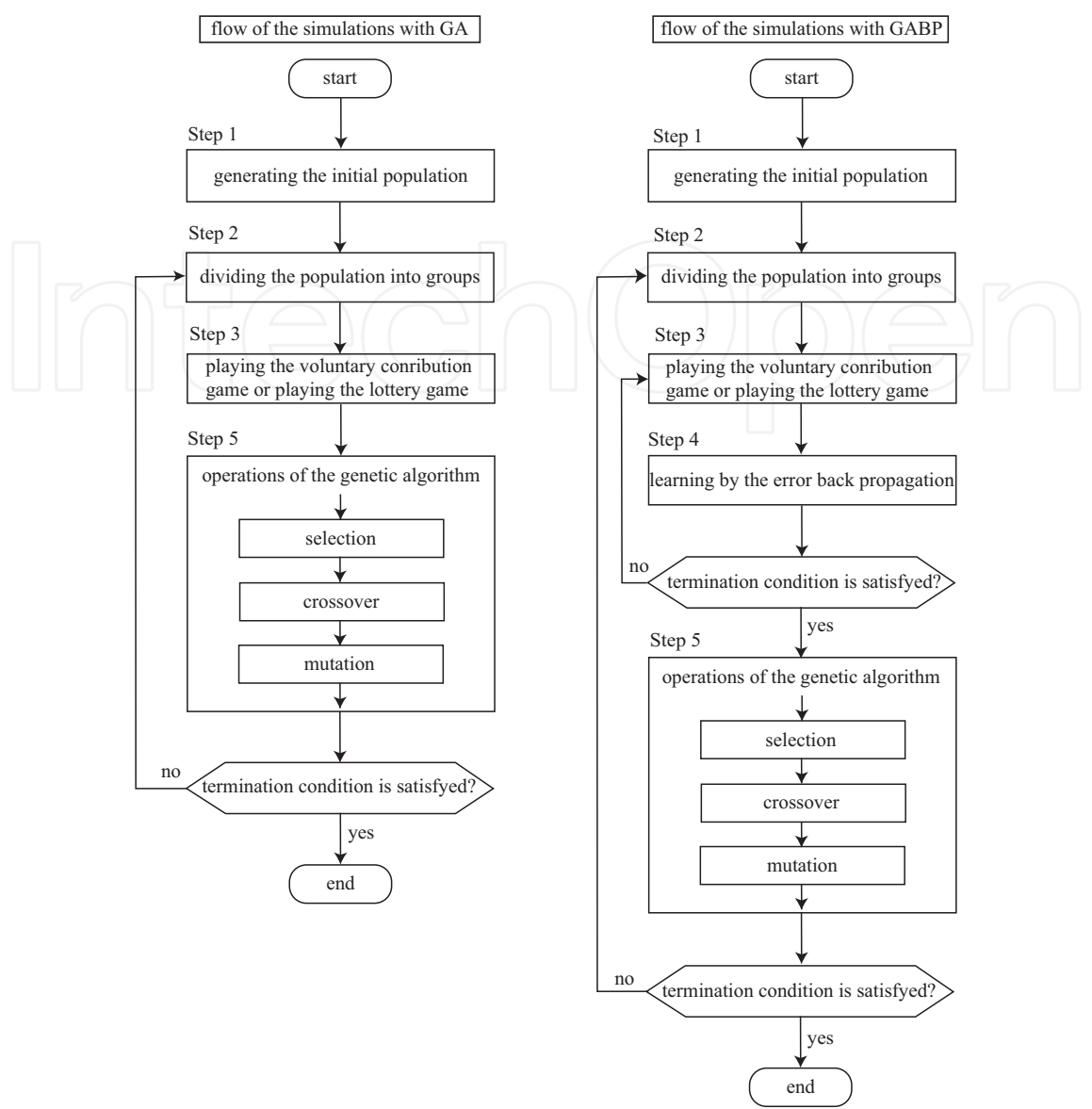
The lottery game.

Fig. 2. Flowcharts of simulations with GA and GABP

Step 3-1-L (Determining the amount of a wager) The wager x'_i of agent i, the sum x'_{-i} of the wagers of the other agents, the total fund $x'_i + x'_{-i}$ for the public goods, and the payoff U'_i of agent i in the previous generation are input values to the neural network for agent i, and they are normalized into $[0, 1]$. Let $\hat{x}_i$ be an output of the neural network. In particular, for the first generation, input values are randomly determined from $[0, 1]$. The wager of agent i is determined as $x_i = \lfloor (e + 1)\hat{x}_i \rfloor$.

Step 3-2-L (Drawing lotteries) After wagers of all the agents are determined, winners are selected by a roulette wheel in which agent i draws a winning with the probability $p_i = x_i / \sum_{j=1}^{n} x_j$.

Step 3-3-L (Informing about the contributions of the others) Agent i is informed about the result of the lottery and the sum x_{-i} of the wagers of the other agents in the present generation.

Step 3-4-L (Computing the payoff) The payoff U_i of agent i is calculated by (8).

Step 4 (Learning by the error back propagation algorithm) *This step is executed only for GABP.* The synaptic weights of the neural network for agent i are revised by teacher signals obtained by computing the optimal wagers for the given wagers of the other agents in the error back propagation algorithm. For agent i, the wagers of self and the sums of the wagers of the other agents for the last k games are recorded and they are used as training data. If the round number does not reach the given maximal round, return to Step 3.

Step 5 (Performing genetic operations) The following genetic operations are performed to each of the chromosomes for all the agents, and then the population of the next generation is formed.

Step 5-1 (Reproduction) The fitness f_i of each agent is obtained by appropriately scaling the payoff x_i obtained in the present generation. As a reproduction operator, the elitist roulette wheel selection is adopted. The elitist roulette wheel selection is a combination of the elitism and the roulette wheel selection. The elitist means that a chromosome with the largest fitness is preserved into the next generation. By a roulette wheel with slots sized by the probability $p_i^{selection} = f_i / \{\sum_{i=1}^{10n} f_i\}$, each chromosome is selected into the next generation.

Step 5-2 (Crossover) A single-point crossover operator is applied to any pair of chromosomes with the probability of crossover p^c. Namely, a point of crossing over on the chromosomes is randomly selected and then two new chromosomes are created by swapping subchromosomes which are the right-hand side parts of the selected point of crossing over on the original chromosomes. The operation of crossover is depicted in Fig. 3.
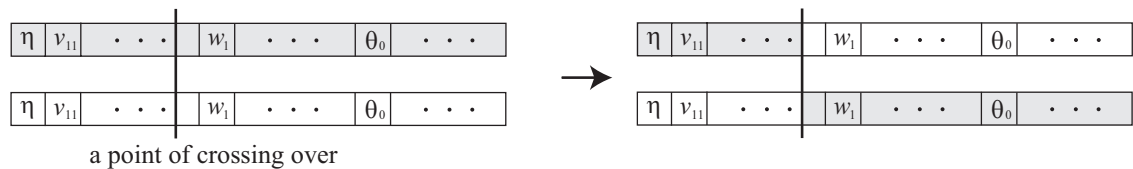

a point of crossing over

Fig. 3. The operation of crossover

Step 5-2 (Mutation) With the given small probability of mutation p^m, each gene, which represents a synaptic weight, a threshold or a learning rate, in a chromosome is randomly changed. The selected gene is replaced by a random number in $[-1, 1]$ for a synaptic weight, or in $[0, 1]$ for a threshold and a learning rate.

If the number of generations does not reach the given final generation, return to Step 2.

4. The results of the simulations

4.1 Treatments of the simulations

In the simulations, the voluntary contribution and the lottery games are played by agents, and there are three important parameters in the model: the exogenous contribution R which is used to fund the public goods or a prize, the marginal per capita return β of the public good provision, and the group size n which is the number of agents in a group. Then, we conduct the three simulations: the exogenous contribution simulation, the marginal per capita return simulation, and the group size simulation. Furthermore, providing two learning mechanisms, GA and GABP, we verify whether actions of agents with more elaborate learning mechanism

are closer to the predictions of the mathematical equilibrium model. Each simulation consists of four treatments, and all of the simulations are summarized in Table 1.

simulations	voluntary contribution		lottery	
	GA	GABP	GA	GABP
exogenous contribution R	R-VC-GA	R-VC-GABP	R-L-GA	R-L-GABP
marginal per capita return β	β-VC-GA	β-VC-GABP	β-L-GA	β-L-GABP
group size n	n-VC-GA	n-VC-GABP	n-L-GA	n-L-GABP

Table 1. Treatments of the simulations

The general settings of the simulations and the parameters of the neural networks and the genetic algorithm are given as follows.

1. The initial endowment is usually set at $e = 20$, and only for the case where the equilibrium wager is larger than 20, it is set at $e = 40$.

2. Let n denote a group size, and because 10 groups are provided for each treatment, the population size of each generation becomes $10n$.

3. Each treatment of the simulations shown in Table 1 is performed 10 runs.

4. There are 6 units in the hidden layer of the neural networks.

5. Each of the output functions of units in the hidden and the output layers is a logistic function $f(x) = 1/\{1 + \exp(-x)\}$.

6. For the GABP treatments, the game is repeated 10 rounds in each generation.

7. After finishing the game in each round, the error back propagation algorithm is performed using 10 sets of the training data. To do so, each agent records the results of the games, i.e., x_i and x_{-i}, for the last 10 games.

8. Each of the initial values of synaptic weights and thresholds is set at 1 so that a contribution or a wager in the first generation becomes the maximal values, i.e., 20 or 40, and the initial value of the learning rate is set at a random number in $[0, 1]$.

9. For simulations with GABP, the fitness is computed by using the payoff only at the first round in each generation in order to exclude the effect of learning by the error back propagation algorithm.

10. The probabilities of crossover and mutation are specified at $p_c = 0.6$ and $p_m = 0.01$, respectively.

11. When a certain gene is selected for mutation, the gene is replaced by a random number in $[-1, 1]$ if it is for a synaptic weight, and the gene is replaced by a random number in $[0, 1]$ if it is for a threshold or a learning rate.

12. The simulations last until generation 1000 which is the final generation for treatments with the group size $n = 2, 4, 10$, or until generation 2000 which is the final generation for treatments with $n = 50, 100$.

4.2 The exogenous contribution simulation

In the exogenous contribution simulation, the group size and the marginal per capita return are fixed at $n = 4$ and $\beta = 0.75$, respectively, related to the experiment by Morgan and Sefton, and each treatment consists of four cases with $R = 2, 4, 8, 16$. Each of the treatments is repeated 10 times, and then numerical data given in the tables and the figures of this section are averages of the 10 runs.

4.2.1 The voluntary contribution games: R-VC-GA and R-VC-GABP

The result of the voluntary contribution games is summarized in Table 2 where the average contributions of the last 100 generations in the GA treatments are shown in the fourth column of GA, the average contributions of the 10 rounds in the final generation in the GABP treatment are shown in the third column of GABP, and the result of the experiment by Morgan and Sefton is also given in the rightmost column. As seen in the table, the average contributions of both the GA and the GABP treatments are close to the equilibrium of zero, and the contributions of the GABP treatments are closer to the equilibrium than those of the GA treatments. Thus, the result supports the predictions of the mathematical equilibrium model that the equilibria are zero regardless of the value of R, and it is found that the actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

R	equilibrium	GABP	GA	experiment with human subjects
2	0	0.033	0.220	—
4	0	0.022	0.199	—
8	0	0.023	0.175	$10.5 \rightarrow 8.075$
16	0	0.031	0.228	—

Table 2. The voluntary contribution games: treatments R-VC-GA and R-VC-GABP

Transitions of contributions of the GA treatments are depicted in Fig. 4. The left graph with the full length of the 1000 generations shows the whole transitions of the GA treatments, and the right graph with the initial 150 generations is given to see changes in the early generations. Moreover, the equilibrium of contribution given by (5) is shown at the both sides of the vertical axis. As seen in Fig. 4 and Table 2, the average contributions of all the treatments with $R = 2, 4, 8, 16$ approach 0.2 up to around generation 200, and for the pace of convergence of the average contributions, an obvious difference among the four treatments is not found.
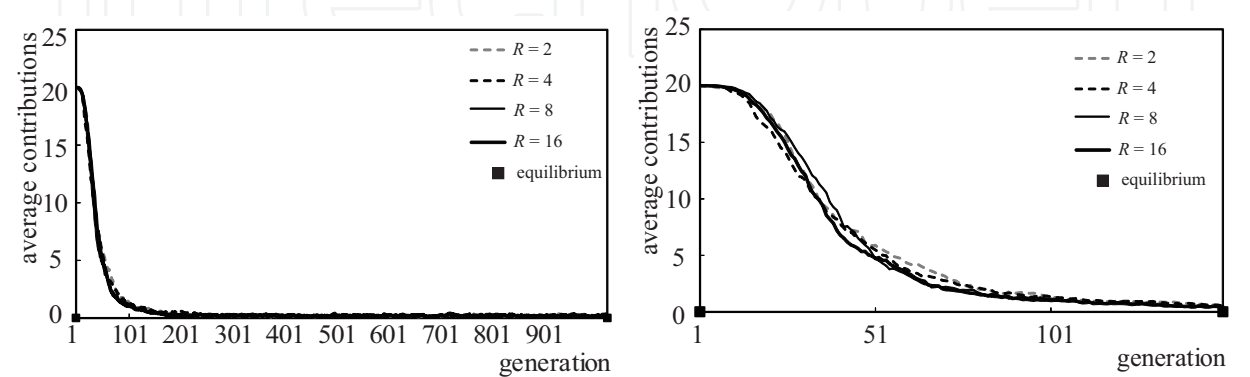

Fig. 4. Transitions of treatments R-VC-GA

Transitions of contributions of the GABP treatments are depicted in Figure 5. The left graph with the full length of the 1000 generations also shows the whole transitions of the GABP treatments, and the right graph tracing transitions of the 10 rounds in the final generation is given to show the effect of learning by the error back propagation algorithm. As seen in Fig. 5 and Table 2, the average contributions of all the treatments with $R = 2, 4, 8, 16$ approach zero up to about generation 200, and for convergence of the sequence, an obvious difference among the four treatments is not also found. By the learning by the error back propagation algorithm, average contributions approach almost zero at the fourth round in all the four treatments.
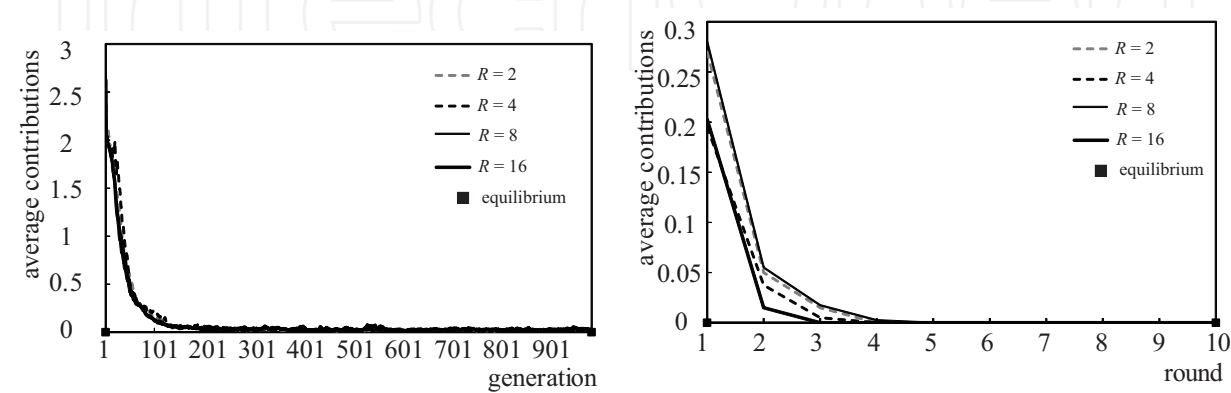
Fig. 5. Transitions of treatments R-VC-GABP

We compare the result of the voluntary contribution games in the exogenous contribution simulation with the corresponding result of the experiment by Morgan and Sefton. In the experiment, the voluntary contribution game with $R = 8$ is played. The average contribution at the initial round of the game is about 10.5, it decreases as the round proceeds, and it finally becomes 8.075 at the final 20th round of the game. This contribution is considerably larger than the equilibrium of zero, but the contribution slightly decreases as subjects gain experiences. For the result of the simulation, the average contributions decrease from 20 to almost zero until around generation 200 in the both GA and GABP treatments.

We summarize the result of the voluntary contribution games as follows. The contributions of both the simulation and the experiment decrease as the learning develops. While the contribution of the experiment is larger than the equilibrium, that of the simulation approaches the values of the equilibrium. Because the repetition of the game in the simulation is vary large compared with that of the experiment, it suggests that human subjects with rich experience of the game may make contributions close to the values of the equilibrium. The contributions of the experiment correspond to those of the simulation from generation 39 to generation 43. Although this correspondence depends on the initial arrangement of the simulation, in general it would be expected to exist some correspondence between the result of the experiment and a part of the whole transition of the simulation with a larger process of the learning.

4.2.2 The lottery games: R-L-GA and R-L-GABP

The result of the lottery games is summarized in Table 3. The equilibria of wagers shown in the second column of the table is calculated by (7), and they increase with growing the exogenous contribution R, i.e., the size of the prize. As seen in the table, the average wagers of both the GA and the GABP treatments are close to the values of the equilibria, and the

wagers of the GABP treatments are closer to the values of the equilibria than those of the GA treatments. Thus, the result supports the predictions of the mathematical equilibrium model that the equilibria of wagers increase as the value of R becomes larger, and it is found that actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

R	equilibrium	GABP	GA	experiment with human subjects
2	1.5	1.471	2.222	—
4	3	2.975	3.735	—
8	6	5.954	5.868	$10 \rightarrow 10.35$
16	12	11.835	10.560	$13 \rightarrow 13.825$

Table 3. The lottery games: treatments R-L-GA and R-L-GABP

Transitions of wagers of the GA treatments are depicted in Fig. 6, which is in a form similar to Fig. 4 of the voluntary contribution games. As seen in Fig. 6 and Table 3, the differences among the average wagers of the treatments can be seen from around generation 40, and the average contributions of the treatments with $R = 2, 4, 8, 16$ converge at about 2.2, 3.7, 5.9, 10.6, respectively, after around generation 150. Compared with the equilibria of wagers, the average wagers of the treatments with $R = 2, 4$ are slightly larger than the values of the equilibria, and those of the treatments with $R = 8, 16$ are slightly smaller than the values of the equilibria. For convergence of the sequences, an obvious difference among the four treatments is not found.
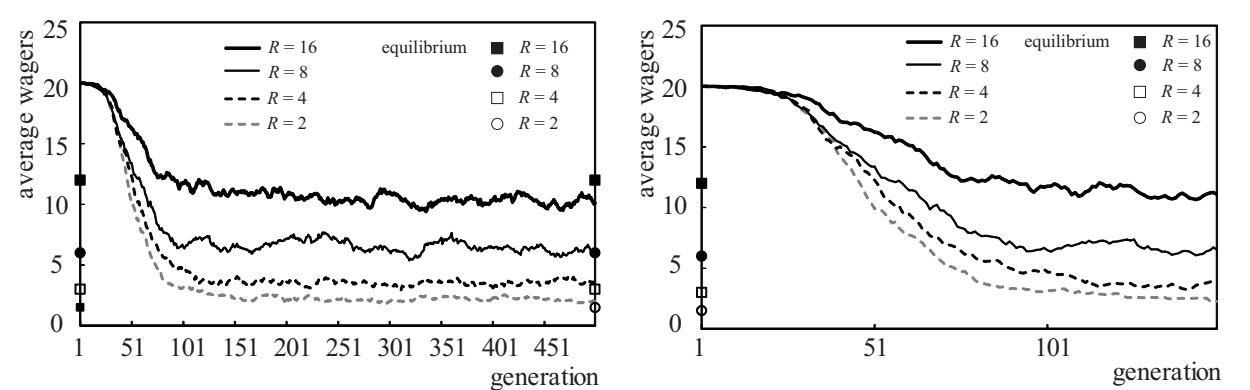

Fig. 6. Transitions of treatments R-L-GA

Transitions of wagers of the GABP treatments are depicted in Fig. 7, which is also in a form similar to Fig. 5 of the voluntary contribution games. After around generation 70, each of the average wagers clearly converges at the corresponding equilibrium. Compared with the GA treatments, the transitions of the GABP treatments converge at the equilibria more exactly and earlier, and variances of the wagers are obviously smaller than those of the GA treatments. By the learning of the error back propagation algorithm, the average wagers of the treatments with $R = 2, 4, 8$ converge almost at the equilibria after the third round, and even for the treatment with $R = 16$, although there exists an oscillation around the equilibrium, the average wagers after the sixth round stably converge at the equilibrium.

To examine the relation between the average wagers of the simulations and the equilibria of the mathematical model, we perform additional treatments with $R = 1.4, 2.7, 5.4, 6.7, 9.4, 10.7, 13.4$ in addition to the original treatments with $R = 2, 4, 8, 16$. In Fig. 8, given the
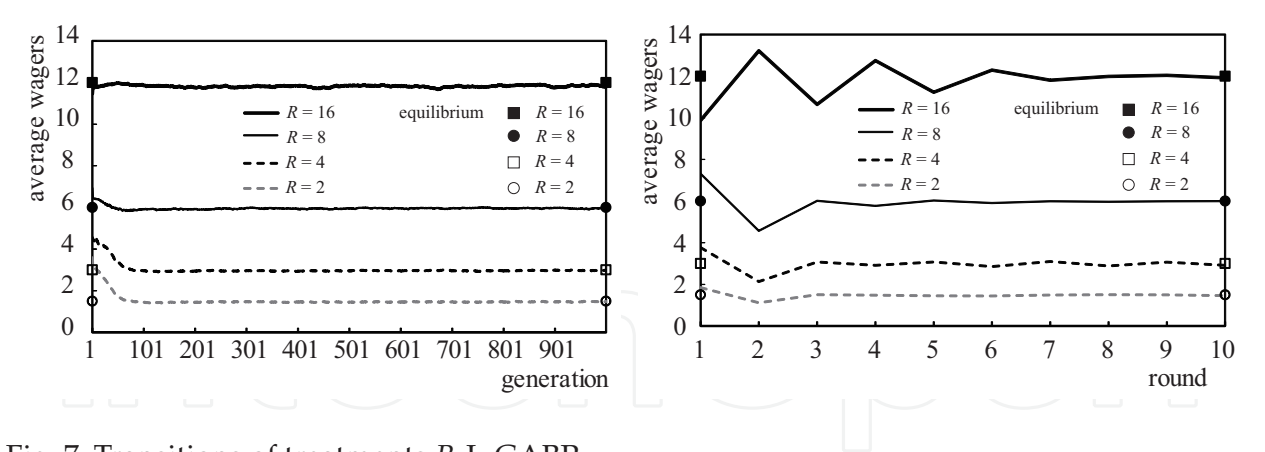
Fig. 7. Transitions of treatments R-L-GABP

equilibria in the horizontal axis, we show the differences between the average wagers of the simulations and the equilibria in the vertical axis. An seen in the left graph of Fig. 8 for the GA treatments, the average wagers of the simulations are higher than the values of the corresponding equilibria in the games whose equilibrium values are smaller than 6, and the average wagers of the simulations are lower than the values of the equilibria in the games whose equilibrium values are larger than 8. In contrast, for the GABP treatments, the average wagers of the simulation are close to the equilibria regardless of the sizes of the equilibria. As we described in the previous section, the learning mechanism of GABP is complicated and requires a heavy load of computation. Because in the learning of the experimental subjects or ordinary people, complicated computations are not performed generally from the viewpoint of bounded rationality, it is supposed that the learning of the experimental subjects might be closer to the learning mechanism of GA rather than that of GABP. This suggestion might give some reason for the fact that the average wagers by human subjects shown in Table 3 from the experiments by Morgan and Sefton (2000) are larger than the value of the equilibria.
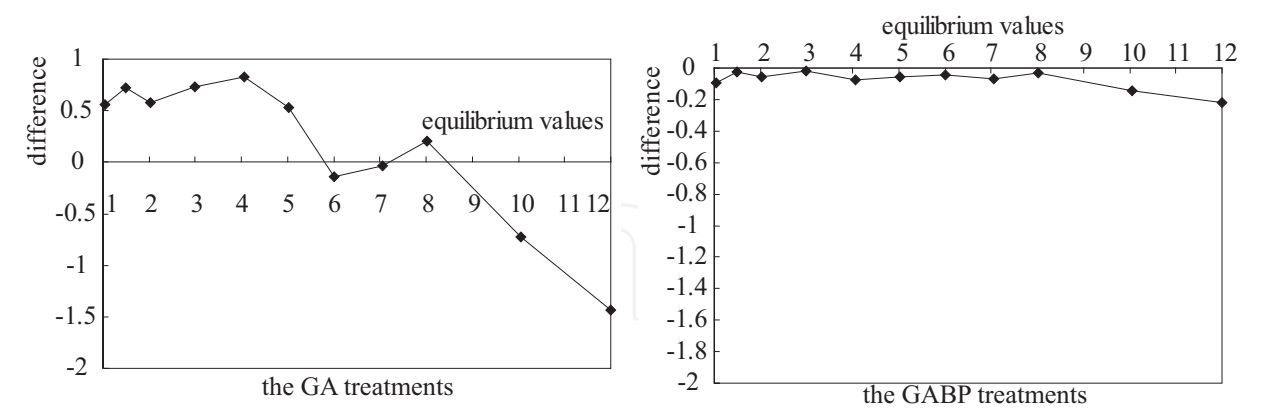

Fig. 8. Differences between the wagers of the simulation and the equilibria

We compare the result of the lottery games in the exogenous contribution simulation with the corresponding result of the experiment by Morgan and Sefton. In the experiment, by comparing two lottery games with $R = 8, 16$, they examine how the size of the prize influences the wagers of the experimental subjects. The average wager at the initial round of the game in the treatment with $R = 8$ is about 10, the round goes on but it rarely changes, and it finally becomes 10.35 at the 20th round of the game. The average wager at the initial round of the

game in the treatment with $R = 16$ is about 13, it is almost changeless even though the round proceeds, and it finally becomes 13.825 at the 20th round of the game. Although the change by acquiring experience is not found in each of the treatments with $R = 8, 16$, the experiment supports the equilibrium prediction that the wagers increase as the value of R becomes larger. For the corresponding results of the simulation, in the GA treatment with $R = 8$, the average wager starts at 20, it decreases as the generation goes on, and after around generation 150 it converges at almost 6. In the GA treatment with $R = 16$, after around generation 150, the average wager finally oscillates in the interval between 10 and 11. In the GABP treatments, the average wagers converge sooner and closer to the equilibria than those in the GA treatments. In particular, the wagers of the human subjects in the experiments $R = 8$ and $R = 16$ correspond to parts of the transition of the wagers of the simulation. Namely, for the treatment with $R = 8$, the transition of wagers from 10 to 10.35 in the experiment corresponds to the transition from a wager at generation 68 to a wager at generation 70 in the simulation, and for the treatment with $R = 16$, the transition from 13 to 13.825 in the experiment corresponds to the transition from a wager at generation 69 to a wager at generation 73 in the simulation. Finally, as seen in Table 3, Figs. 6 and 7, the results of the simulation including the results of the treatments not only with $R = 8, 16$ but also with $R = 2, 4$ more clearly support the equilibrium prediction that the wagers increase as the value of R grows larger.

4.2.3 Summary of the exogenous contribution simulation

To conclude this subsection, we summarize the results of the simulation for the exogenous contribution R.

- Although it can be found that there exists a clear difference between the equilibrium values of the mathematical model and the average contributions of the experiment with human subjects in the voluntary contribution games, in the simulation, we observe that the average contributions of the simulation are sufficiently close to the value of the equilibria with the passage of time or with enough learning of agents.

- While the result of the experiment by Morgan and Sefton supports the equilibrium prediction that the wagers increase as the value of R grows larger, the result of the simulation supports it more obviously.

- In both of the voluntary contribution games and the lottery games, the contributions and the wagers of the GABP treatments are close to the equilibrium values of the mathematical model, compared with the GA treatments. Thus, it is found that the actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

- From comparing Tables 2 and 3, we observe that the lottery mechanism provides more of the public goods than the voluntary contributions mechanism.

4.3 The marginal per capita return simulation

In the marginal per capita return simulation, the exogenous contribution and the group size are fixed at $R = 8$ and $n = 4$, respectively, as in the experiment by Morgan and Sefton, and each treatment consists of five cases with $\beta = 0.00, 0.25, 0.50, 0.75, 0.95$.

4.3.1 The voluntary contribution games: β-VC-GA and β-VC-GABP

The result of the voluntary contribution games is summarized in Table 4 which is similar to that of the exogenous contribution simulation. As seen in the table, although the average contributions of the GA and the GABP treatments with $\beta = 0.95$ slightly deviate from the equilibrium contribution of zero, the average contributions of the other treatments are very close to the equilibrium of zero. Because a gap between a marginal payoff from the private good and a marginal return from the public good provision decreases as the value of β approaches one, particularly in the GA treatment with $\beta = 0.95$, it becomes difficult for agents to discriminate between a payoff from the private goods and a return from the public good provision, and therefore it seems that the average contribution slightly deviates from the equilibrium of zero. Thus, the result of the simulation supports the predictions of the mathematical equilibrium model that the equilibrium contributions are zero if $\beta < 1$ and β is not close to 1. Moreover, the contributions of the GABP treatments are closer to zero than those of the GA treatments, and then it follows that the actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

β	equilibrium	GABP	GA	experiment with human subjects
0.00	0	0.009	0.061	—
0.25	0	0.010	0.065	—
0.50	0	0.011	0.091	—
0.75	0	0.023	0.175	$10.5 \to 8.075$
0.95	0	0.237	1.751	—

Table 4. The voluntary contribution games: treatments β-VC-GA and β-VC-GABP

Transitions of contributions of the GA treatments are shown in Fig. 9. Because the result of the treatment with $\beta = 0$ almost overlaps with that of $\beta = 0.25$, the transition of the treatment with $\beta = 0$ is omitted from the figure. As seen in Fig. 9 and Table 4, the average contributions of all the treatments with $\beta = 0.00, 0.25, 0.50, 0.75$ approach about zero up to around generation 100, and for convergence of the sequences, the smaller the value of β is, the sooner the average contribution approaches zero. In the treatment with $\beta = 0.95$, although the average contribution slightly deviates from the equilibrium of zero, it becomes below 2.0 after around generation 400.
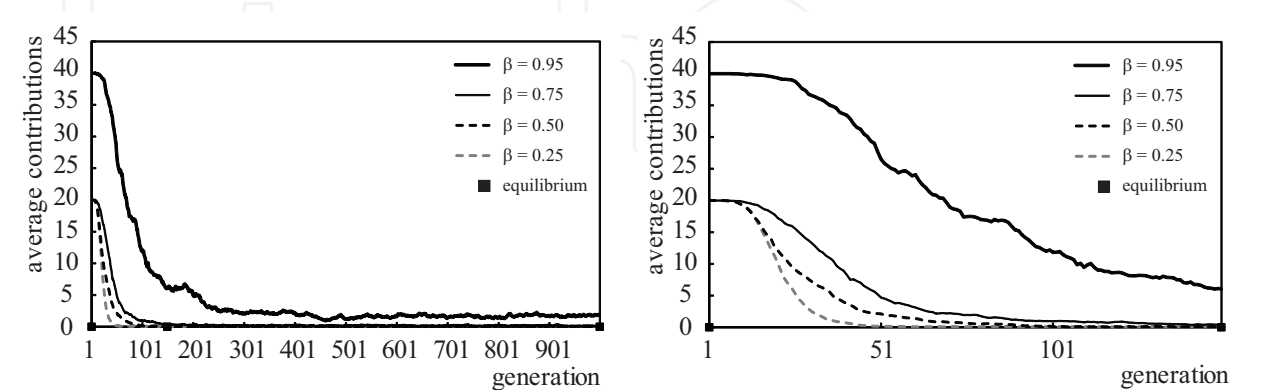

Fig. 9. Transitions of treatments β-VC-GA

Transitions of contributions of the GABP treatments are shown in Fig. 10. While the transitions of the average contributions shown in Fig. 10 is similar to those of Figure 9, the finally

converging contributions of the GABP treatments are smaller than those of the GA treatments as seen Table 4. As seen in the right graph of Fig. 10, the average contributions of the first round in the final generation are already close to zero for the treatments with $\beta = 0, 0.25, 0.50, 0.75$, and even for the treatment with $\beta = 0.95$, after the third round the average contribution almost converges at zero by the learning by the error back propagation algorithm.
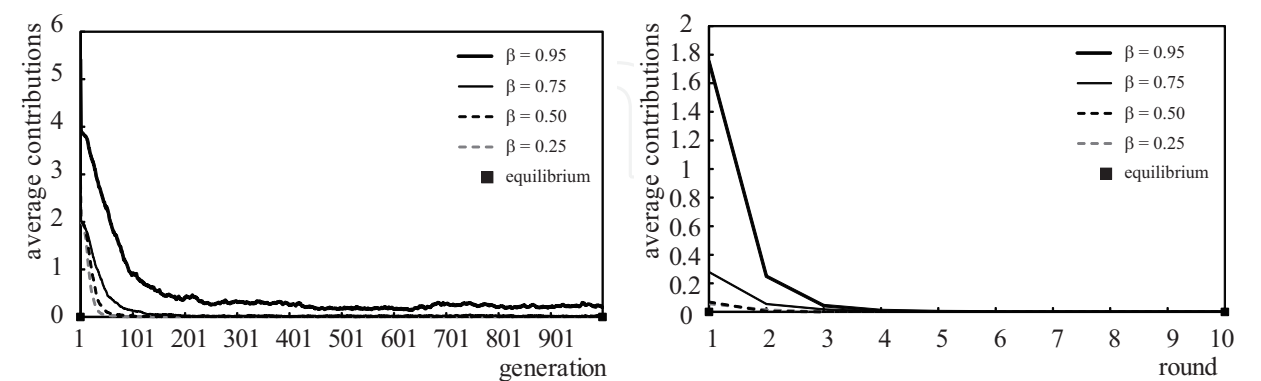

Fig. 10. Transitions of treatments β-VC-GABP

For the treatments with respect to the marginal per capita return in the experiment by Morgan and Sefton, they conduct only the treatment with $\beta = 0.75$. As for the case with $\beta = 0.75$, in the exogenous contribution simulation we already gave the description about the comparison between the results of the simulation and the experiment for this case.

4.3.2 The lottery games: β-L-GA and β-L-GABP

The result of the lottery games is summarized in Table 5. The equilibria of wagers shown in the second column of the table is calculated by (7), and they increase with the marginal per capita return β. As seen in the table, almost all the average wagers of the GA and the GABP treatments are close to the equilibrium wagers except for the GA treatment with $\beta = 0.95$, and the average wagers of the GABP treatments are closer to the equilibria than those of the GA treatments. For the GA treatment with $\beta = 0.95$, from the same reason as that of the voluntary contribution games, it becomes difficult for agents to discriminate between a payoff from the private goods and a return from the public good provision, and therefore it seems that the average wager deviates from the equilibrium of 30. Thus, the result of the simulation almost supports the predictions of the mathematical equilibrium model that the equilibrium wagers increase as the value of β becomes larger, and it is also found that the actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

β	equilibrium	GABP	GA	experiment with human subjects
0.00	1.5	1.46	1.58	$8 \to 2.425$
0.25	2	1.95	2.09	—
0.50	3	2.97	4.01	—
0.75	6	5.95	5.87	$10 \to 10.35$
0.95	30	28.46	13.00	—

Table 5. The lottery games: treatments β-L-GA and β-L-GABP

Transitions of wagers of the GA treatments are depicted in Fig. 11. As seen in Fig. 11 and Table 5, the differences among the average wagers of the treatments can be seen from around generation 20, each of the average wagers of all the treatments converges after around generation 100, and the average wagers increase as the value of β becomes larger. Only for the treatment with $\beta = 0.95$, the average wager deviates from the equilibrium, and for the other treatments, the average wagers converge almost at the equilibria. The average wagers of the treatments with $\beta = 0.00, 0.25, 0.50, 0.75$ are almost the same as or slightly larger than the values of the equilibria, and that of the treatment with $\beta = 0.95$ is smaller than the value of the equilibrium. For speed of the convergence of the sequences, an obvious difference among them is not found.
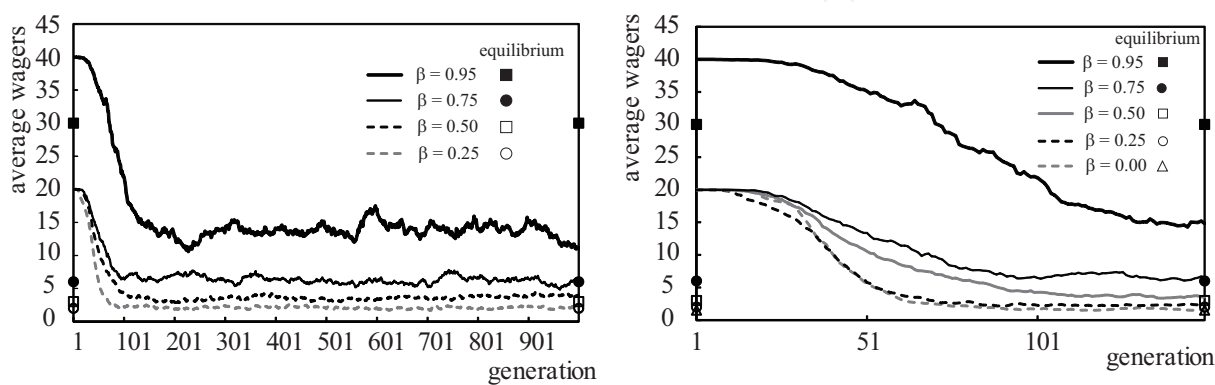
Fig. 11. Transitions of treatments β-L-GA

Transitions of wagers of the GABP treatments are depicted in Fig. 12. After around generation 70, each of the average wagers clearly converges at the corresponding equilibrium except for the treatment with $\beta = 0.95$. Compared with the GA treatments, the transitions of the GABP treatments converge at the equilibria more exactly and earlier, and variances of the wagers are obviously smaller than those of the GA treatments. Although the average wager of the treatment with $\beta = 0.95$ slightly deviates from the equilibrium, it is fairly close to the equilibrium, compared with that of the GA treatment. By the learning of the error back propagation algorithm, the wagers of the treatments with $\beta = 0, 0.25, 0.50, 0.75$ converge almost at the corresponding equilibria after the third round, and even for the treatment with $\beta = 0.95$, although there exists an oscillation around the equilibrium, the wagers after the seventh round stably converge at the equilibrium.

The relation between the average wagers of the simulations and the equilibria from the mathematical model are shown in Fig. 13. To examine the relation, we perform the another nine treatments with $\beta = 0.6250, 0.7000, 0.7300, 0.7860, 0.8125, 0.8500, 0.9000, 0.9250, 0.9400$ in addition to the original treatments with $\beta = 0.00, 0.25, 0.50, 0.75, 0.95$. An seen in the left graph of Fig. 13 for the GA treatments, the average wagers of the simulations are close to the values of the equilibria in the games such that the equilibria are smaller than 10. When larger than 10, the difference between the average wagers of the simulation and the equilibria of the mathematical model increases with the value of the equilibrium. In contrast, for the GABP treatments, the wagers of the simulation are close to the equilibria regardless of the sizes of the equilibria. The result of the GA treatments whose learning mechanism is simpler than that of GABP is consistent with the result of the experiment by Morgan and Sefton (2000) that the wager of 2.425 in the treatment with $\beta = 0.00$ is relatively close to the equilibrium of 1.5 and
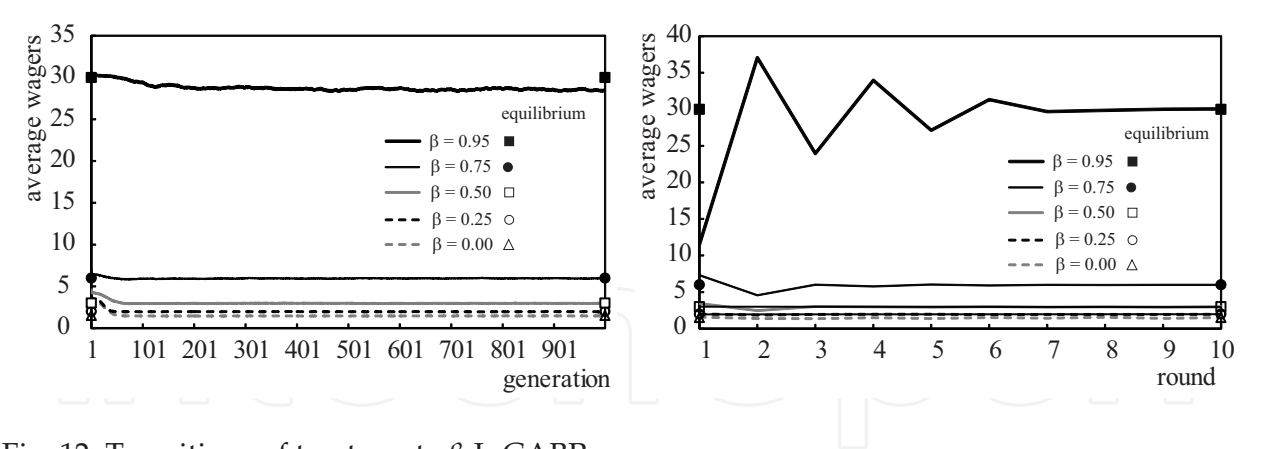
Fig. 12. Transitions of treatments β-L-GABP

the wager of 10.35 in the treatment with $\beta = 0.75$ is considerably larger than the equilibrium of 6, as seen Table 5.
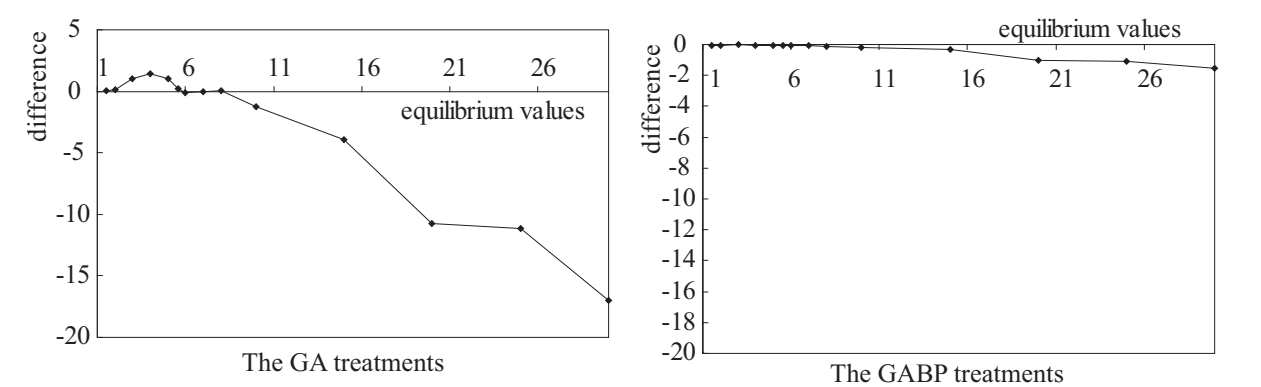

Fig. 13. Differences between the wagers of the simulation and the equilibria

We compare the result of the lottery games in the marginal per capita return simulation with the corresponding result of the experiment by Morgan and Sefton. In the experiment, by comparing two lottery games with $\beta = 0.0$ and $\beta = 0.75$, they examine how the size of the marginal per capita return influences the wagers of the experimental subjects. The average wager at the initial round of the game in the treatment with $\beta = 0.0$ is about 8, it decreases as the round goes on, and it finally becomes 2.425 at the 20th round of the game. The average wager at the initial round of the game in the treatment with $\beta = 0.75$ is about 10, it remains almost the same even though the round goes on, and it finally becomes 10.35. The change of wagers by acquiring experience can be found only in the treatments with $\beta = 0.0$, and the experiment supports the equilibrium prediction that the wagers increase with the marginal per capita return β.

For the corresponding results of the simulation, in the GA treatment with $\beta = 0.0$, the average wager starts at 20, it decreases as the generation goes on, and after around generation 50 it converges at about 1.5. In the GA treatment with $\beta = 0.75$, after around generation 100, the average wager finally converges at almost 6. In the GABP treatments, the average wagers converge sooner and closer to the equilibria than those of the GA treatments. In particular, the transitions of wagers of the human subjects in the experiments $\beta = 0.0$ and $\beta = 0.75$ correspond to parts of the transitions of wagers in the simulation. Namely, for the treatment

with $\beta = 0.0$, the transition of wagers from 8 to 2.425 in the experiment corresponds to the transition from a wager at generation 44 to a wager at generation 73 in the simulation, and for the treatment with $\beta = 0.75$, the transition from 10 to 10.35 in the experiment corresponds to the transition from a wager at generation 66 to a wager at generation 71 in the simulation. Finally, as seen in Table 5, Figs. 11 and 12, the results of the simulation including the results of the treatments not only with $\beta = 0.00, 0.75$ but also with $\beta = 0.25, 0.50, 0.95$ more clearly support the equilibrium prediction that the wagers increase as the value of β grows larger.

4.3.3 Summary of the marginal per capita return simulation
To conclude this subsection, we summarize the results of the simulation for the marginal per capita return β.

- Although the average contribution and wager of the treatments with $\beta = 0.75$ in the experiment with human subjects differ from the equilibria of the mathematical model, in the simulation, we observe that the average contribution and wager of the simulation are sufficiently close to the values of the equilibria with the passage of time or with enough learning of agents.

- While the result of the experiment by Morgan and Sefton supports the equilibrium prediction that the wagers increase with the marginal per capita return β, the result of the simulation supports it more obviously.

- In both of the voluntary contribution games and the lottery games, the contributions and the wagers of the GABP treatments are closer to the values of the equilibria in the mathematical model than those of the GA treatments. Thus, it is found that the actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

- From comparing Tables 4 and 5, we observe that the lottery mechanism provides more of the public goods than the voluntary contributions mechanism does.

- As the marginal per capita return β approaches one, i.e., $\beta \to 1$, in particular for the lottery game, the average wager in the simulation deviates from the equilibria because it becomes difficult for agents to discriminate between a payoff from the private goods and a return from the public good provision.

4.4 The group size simulation
In the group size simulation, the exogenous contribution and the marginal per capita return are fixed at $R = 8$ and $\beta = 0.75$, respectively, and each treatment consists of five cases with $n = 2, 4, 10, 50, 100$.

4.4.1 The voluntary contribution games: n-VC-GA and n-VC-GABP
The result of the voluntary contribution games is summarized in Table 6. Because any treatment with respect to the group size n is not conducted in the experiment by Morgan and Sefton, the result from the experiment is not shown in the table.
As seen in the table, for all of the GABP treatments and the GA treatments with $n = 2, 4, 10$, the average contributions are very close to the equilibrium of zero, and the contributions of the GABP treatments are closer to zero than those of the GA treatments. The average contributions of the GA treatments with large group sizes such as $n = 50, 100$ slightly deviate from the

equilibrium contribution. It is supposed that the reason is because the amount of the public good provision is extremely enlarged as the group size n becomes larger. Namely, the increase of the public good provision makes the payoff from the private goods relatively smaller, and then because the effect of increasing the payoff by maximizing the private goods is reduced, it seems that the average contributions slightly deviate from the equilibrium.

n	equilibrium	GABP	GA
2	0	0.01	0.33
4	0	0.02	0.18
10	0	0.04	0.36
50	0	0.16	1.64
100	0	0.28	2.20

Table 6. The voluntary contribution games: treatments n-VC-GA and n-VC-GABP

Transitions of contributions of the GA treatments are shown in Fig. 14, where the results of the treatments with $n = 2, 4, 10$ and $n = 50, 100$ are separately depicted in the left and the right graphs, respectively, because the maximal generations of them are not the same, and to see changes in the early generations the lower graph is also provided. As seen in Fig. 14 and Table 6, the average contributions of the treatments with $n = 2, 4, 10$ approach almost zero up to around generation 200. For the treatments with $n = 50, 100$, the average contributions become below 1.8 and 2.6, respectively, after around generation 300, and they finally converge not at zero which is the values of the equilibria but at about 1.6 and 2.2, respectively. By convergence of the sequences, we do not observe an obvious difference among the treatments with $n = 2, 4, 10$, but the convergences of the treatments with $n = 2, 4, 10$ are earlier than those of the treatments with $n = 50, 100$.

Transitions of contributions of the GABP treatments are shown in Fig. 15, where the left and the right graphs are for the treatments with $n = 2, 4, 10$ and with $n = 50, 100$, respectively, and the lower graph is provided for seeing the effect of learning by the error back propagation algorithm. The average contributions of the treatments with $n = 2, 4, 10$ approach almost zero up to around generation 200, and even for the treatments with $n = 50, 100$, the average contributions converge below 0.3 after around generation 200. By the learning by the error back propagation algorithm, the average contributions approach zero for all of the treatments at the third round. In particular, for the treatments with $n = 50, 100$, although the average contributions in the early rounds are larger than 1.0, it is observed that they quickly converge at zero by the learning by the error back propagation algorithm.

4.4.2 The lottery games: n-L-GA and n-L-GABP

The result of the lottery games is summarized in Table 7. The equilibria of wagers shown in the second column of the table is calculated by (7), and they decrease as the group size n becomes larger. As seen in the table, for all of the GABP treatments and the GA treatments with $n = 2, 4, 10$, the average wagers of the simulation are close to the values of the equilibria in the mathematical model, and the wagers of the GABP treatments are closer to the values of the equilibria than those of the GA treatments. The results of the GA and GABP treatments where the group size n does not exceed 50 support the predictions of the mathematical equilibrium model that the equilibrium wagers decrease as the value of n becomes larger. Conversely, however, the average wagers of the treatments with $n = 100$ are larger than those of the
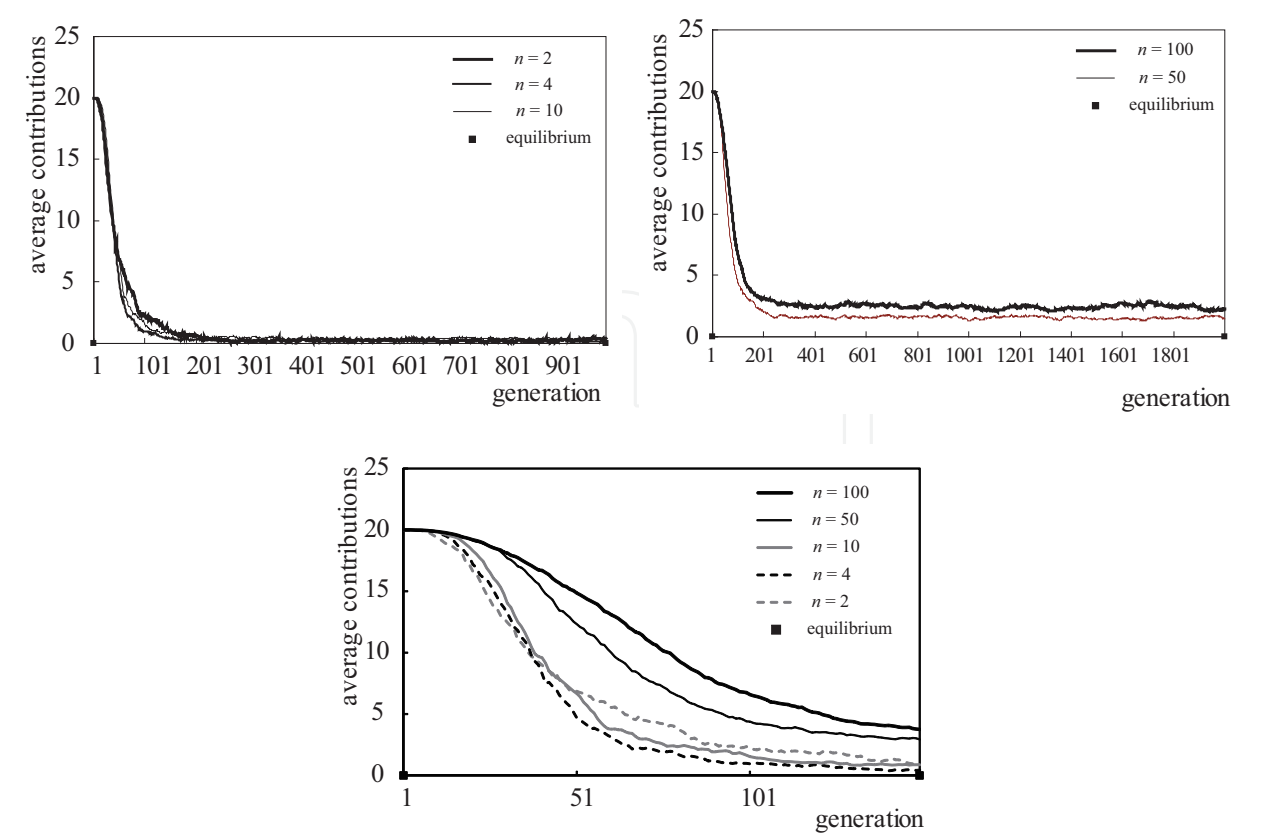
Fig. 14. Transitions of treatments n-VC-GA

treatments with $n = 50$; and besides, the average wagers of the treatments with $n = 50, 100$ deviate from the equilibria. We think that the reason of the deviation is the same as that of the voluntary contribution games in the simulation with respect to the group size as mentioned in the previous subsection.

n	equilibrium	GABP	GA
2	8	7.86	6.70
4	6	5.95	5.87
10	2.88	3.12	3.80
50	0.63	2.27	2.96
100	0.32	2.69	3.24

Table 7. The lottery games: treatments n-L-GA and n-L-GABP

Transitions of wagers of the GA treatments are depicted in Fig. 16. As seen in Fig. 16 and Table 7, although the transitions of the treatments with $n = 2, 4, 10$ slightly oscillate after around generation 200, they are not way from the corresponding equilibria. For the treatments with $n = 50, 100$, however, the average wagers converge at about 3 and they do not approach to the corresponding equilibria of 0.63 and 0.32 anymore.

Transitions of wagers of the GABP treatments are depicted in Fig. 17. It is found that after around generation 100, each of the average wagers of the treatments with $n = 2, 4, 10$ exactly converges at the corresponding equilibrium. Contrastively, the average wagers of the treatments with $n = 50, 100$ converge at about 2.5 which is larger than the values of
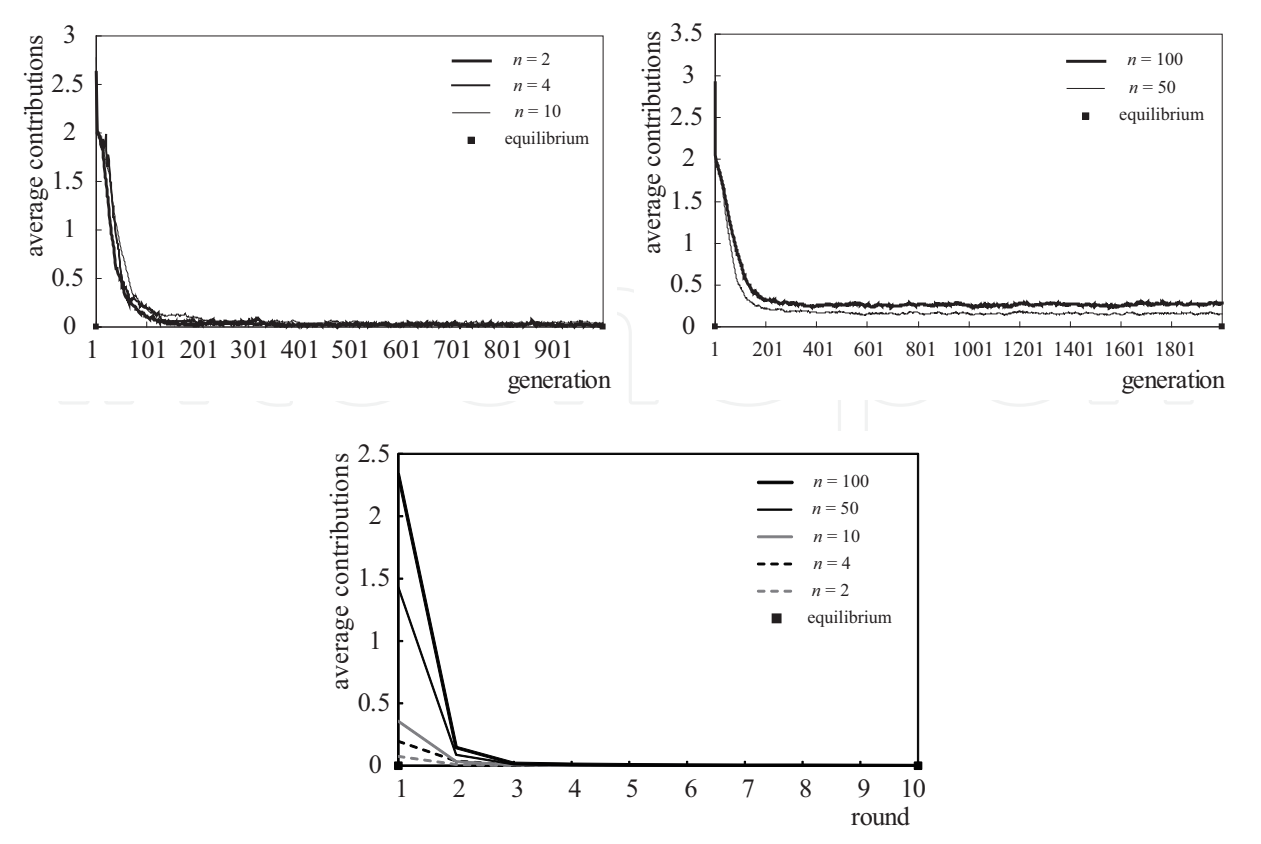
Fig. 15. Transitions of treatments n-VC-GABP

the corresponding equilibria. Comparing with the results of the GA treatments shown in Fig. 16, transitions of the GABP treatments converge at the equilibria more exactly and earlier, and variances of the wagers are obviously smaller than those of the GA treatments. As seen in the lower graph in Fig. 17, by the learning of the error back propagation algorithm, the average wagers of the treatments with $n = 2, 4$ converge almost at the equilibria after the third round, and for the treatment with $n = 10$, although there exists an oscillation around the equilibrium, the wagers after the ninth round converge at the equilibrium. For the treatments with $n = 50, 100$, however, the average wagers violently oscillate around about 3, which is larger than the values of the equilibria.

When the utility functions (4) and (6) are employed, one finds that the enlargement of the public good provision incurred by increase in the group size n makes the payoff from the private goods relatively smaller than the return of the public good provision. Thus, it is difficult for us to analyze the influence of the group size by using the simulation model with utility functions of agents (4) and (6). To relax this difficulty, we provide the following utility function diminishing return of the public good provision

$$U_i = e - x_i + R \frac{x_i}{x(N)} + \beta \alpha \log(x(N)), \tag{9}$$

where a parameter α is specified as follows. The parameter α is determined so that, at equilibrium, in case of $n = 4$, $R = 8$, and $\beta = 0.75$, the third term of (6) representing the
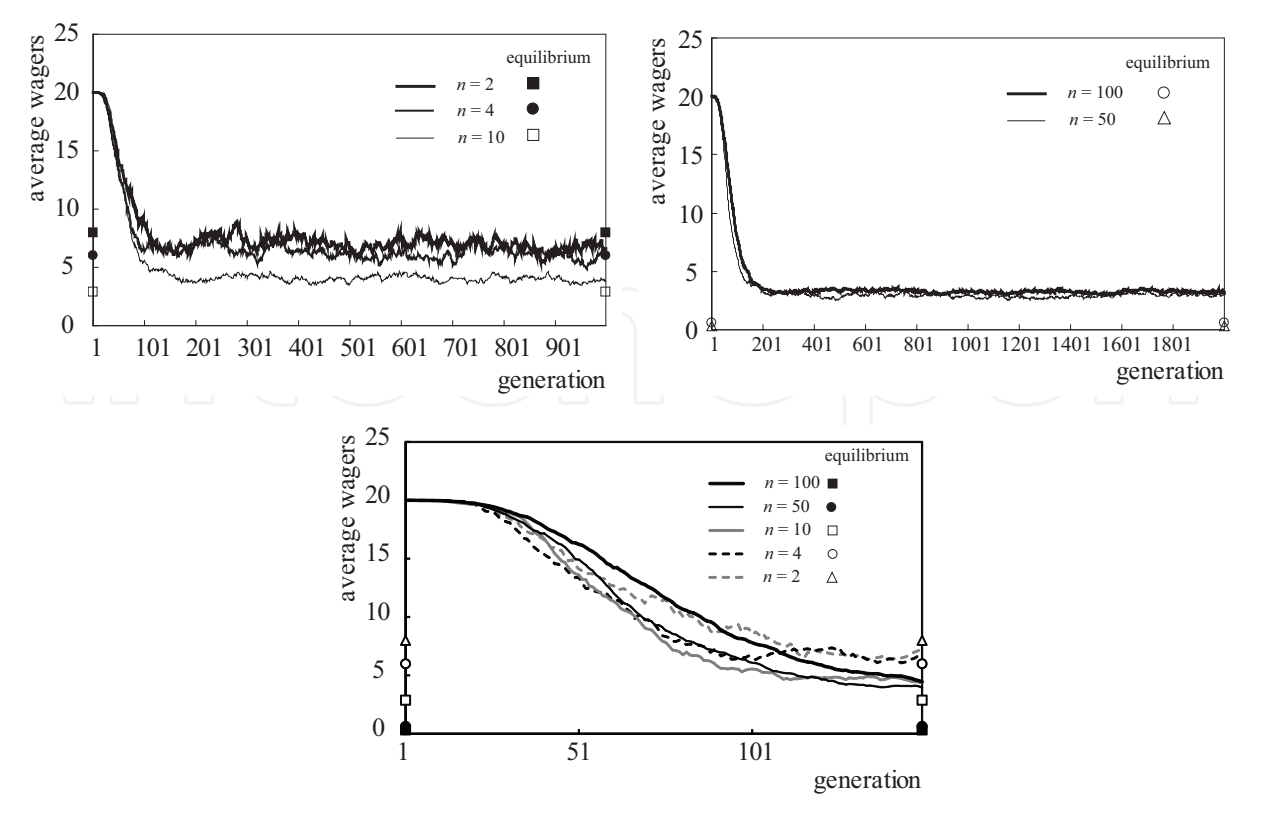
Fig. 16. Transitions of treatments n-L-GA

public good provision is equal to that of (9). To be more precise, assuming

$$0.75 \cdot 4 \cdot 6 \approx 0.75\alpha \log(4 \cdot 6),$$

we set $\alpha = 4$. We perform another simulation with the GABP treatments with $n = 50, 100$. The result of the simulation is given in Table 8, and it should be noted that the equilibria are not the same as those of Table 7 because of the different utility function (9).

n	equilibrium	GABP
50	0.374	1.02
100	0.109	1.01

Table 8. The lottery games: treatment n-L-GABP with the revised utility function

Comparing the results shown in Tables 7 and 8, the average wagers of the treatment with the revised utility function (9) are closer to the equilibria than those of the treatments with the original utility function (6). Although the average wagers in the original simulation shown in Table 7 increase when the group size increases from $n = 50$ to $n = 100$ and this phenomenon is against the predictions of the mathematical equilibrium model that the wagers decrease as the value of n becomes larger, the average wagers of the simulation with the revised utility function shown in Table 8 slightly decrease. Thus, it is found that if a utility function can be specified appropriately, it is possible to obtain results of the simulation supporting the predictions of the mathematical equilibrium model that the wagers decrease as the group size n becomes larger.
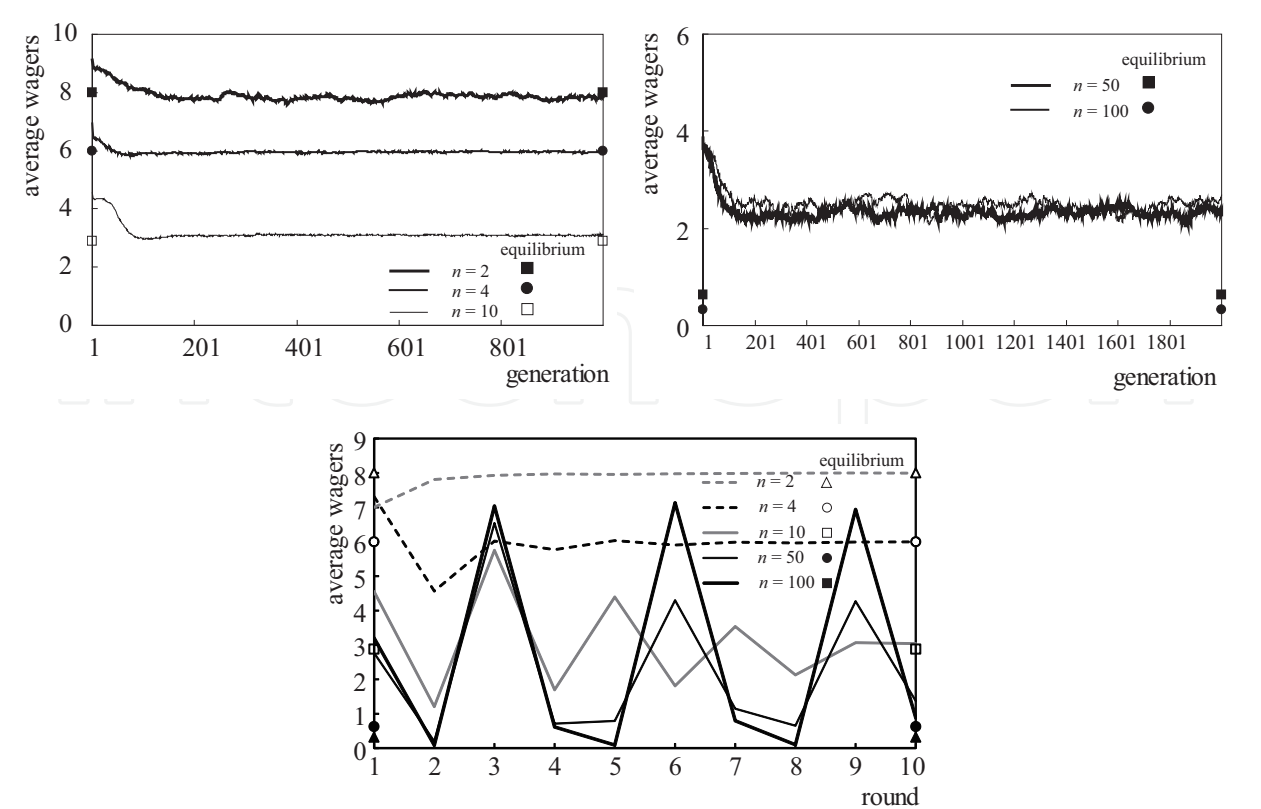
Fig. 17. Transitions of treatments n-L-GABP

4.4.3 Summary of the group size simulation

To conclude this subsection, we summarize the results of the simulation for the group size n.

- We obtain the results that the average contributions and wagers of the simulations are close to the corresponding equilibria expect for the treatments of the lottery games with $n = 50, 100$. For the lottery games, the result of the treatments with group sizes smaller than 50 supports the equilibrium prediction that the wagers decrease as the group size n becomes larger.

- For the lottery games, the result of the treatments with large group sizes such as $n = 100$ in the simulation with the original setting is not consistent with the predictions of the mathematical equilibrium model. However, if a utility function can be specified appropriately for the treatments with large group sizes, we show that it is possible to obtain results of the simulation which support the predictions of the mathematical equilibrium model.

- In both of the voluntary contribution games and the lottery games, the contributions and the wagers of the GABP treatments are closer to the values of the equilibria in the mathematical model than those of the GA treatments. Thus, it is found that the actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

- Comparing Tables 6 and 7, we observe that the lottery mechanism provides more of the public goods than the voluntary contribution mechanism.

5. Sensitivity analysis

In our simulation model, we employ the agent-based simulation model in which artificial adaptive agents have a mechanism of decisions and learning based on neural networks and genetic algorithms. Because the structure and the performance of the simulation model depend on the parameters of neural networks and genetic algorithms, it is important to verify whether or not the specified parameters are appropriate. In this section, we analyze the sensitivity on the parameters of the number of units in the hidden layer of the neural networks, the number of learning data in the error back propagation algorithm, the probabilities of crossover and mutation in the genetic algorithm. In practice, after we have evaluated the results of the sensitivity analysis shown in the following, we determine the values of these parameters which are used in the simulations shown in the previous section. For the sensitivity analysis, the treatments described in the following subsections are performed. Each treatment of the simulations is repeated 10 times, and numerical data given in the tables in the following subsections are averages of the 10 runs in the same way as the simulations shown in the previous section.

5.1 The number of units in the hidden layer of the neural networks

The number of units in the hidden layer of the neural networks is set at 6 in the simulations shown in the previous section. In this setting we show the effectiveness of lotteries for financing, and examine the validity of the mathematical equilibrium model and the experiments with human subjects. In this subsection, by varying the number of units in the hidden layer, we examine appropriate values of the parameter. To do so, we carry out simulations where the number of units is 2, 4, 6, or 8, and values of the other parameters of the neural networks and the genetic algorithms are the same as those of the simulations in the previous section. We provide two treatments where the exogenous contribution is set at $R = 2, 8, 16$, fixing the marginal per capita return and the group size at $\beta = 0.75$ and $n = 4$; and the marginal per capita return is set at $\beta = 0.00, 0.50, 0.75$, fixing the exogenous contribution and the group size at $R = 8$ and $n = 4$. Because the number of units in the hidden layer is a parameter for the neural networks, we perform the treatments with the learning mechanism of GABP, and both the voluntary contribution game and the lottery game are played. The average contributions and wagers of the simulation for sensitivity analysis is given in Tables 9 and 10.

the number of units	voluntary contribution			lottery		
	$R = 2$	$R = 8$	$R = 16$	$R = 2$	$R = 8$	$R = 16$
2	0.20	0.19	0.18	**1.48**	**5.97**	11.73
4	0.05	0.05	0.04	1.46	5.92	**11.85**
6	**0.03**	**0.04**	**0.02**	**1.48**	**5.95**	**11.78**
8	**0.02**	**0.01**	**0.02**	1.46	5.90	11.77
equilibrium	0.00	0.00	0.00	1.50	6.00	12.00

Table 9. Sensitivity analysis for the number of units in the hidden layer: the exogenous contribution R

In the tables, the contribution or the wager which is the closest to the values of the corresponding equilibrium is highlighted by underlined and boldfaced numbers, and the contribution or the wager which is the second closest is highlighted by boldfaced numbers.

the number of units	voluntary contribution			lottery		
	$\beta = 0.00$	$\beta = 0.50$	$\beta = 0.75$	$\beta = 0.00$	$\beta = 0.50$	$\beta = 0.75$
2	0.15	0.13	0.19	1.45	**2.99**	**5.96**
4	**0.01**	0.02	**0.04**	1.44	2.62	5.09
6	**0.01**	**0.01**	**0.04**	**1.46**	**2.97**	**5.95**
8	**0.01**	**0.01**	**0.04**	**1.46**	2.78	4.86
equilibrium	0.00	0.00	0.00	1.50	3.00	6.00

Table 10. Sensitivity analysis for the number of units in the hidden layer: the marginal per capita return β

As seen in Tables 9 and 10, the contributions or the wagers of the case where the number of units is 6 are the first or the second closest to the values of the equilibria, and therefore we conclude that it is appropriate to provide 6 units in the hidden layer of the neural networks.

5.2 The number of learning data in the error back propagation algorithm

The number of learning data in the error back propagation algorithm is set at 10 in the simulations shown in the previous section. In this subsection, by varying the number of learning data, we examine appropriate values of the parameter. To do so, we carry out simulations where the number of learning data is 3, 5, or 10, and values of the other parameters of the neural networks and the genetic algorithms are the same as those of the simulations in the previous section.

We provide two treatments which are the same as the treatments of the simulation for the sensitivity analysis withe respect to the number of units in the hidden layer of the neural networks. Because the number of learning data is a parameter for the neural networks, we perform the treatments with the learning mechanism of GABP. The result of the simulation for sensitivity analysis is shown in Tables 11 and 12.

the number of learning data	voluntary contribution			lottery		
	$R = 2$	$R = 8$	$R = 16$	$R = 2$	$R = 8$	$R = 16$
3	0.04	**0.02**	0.03	**1.45**	5.87	11.75
5	**0.02**	**0.02**	0.03	1.44	**5.93**	**11.78**
10	**0.03**	0.04	**0.02**	**1.48**	**5.95**	**11.78**
equilibrium	0.00	0.00	0.00	1.50	6.00	12.00

Table 11. Sensitivity analysis for the number of learning data: the exogenous contribution R

the number of learning data	voluntary contribution			lottery		
	$\beta = 0.00$	$\beta = 0.50$	$\beta = 0.75$	$\beta = 0.00$	$\beta = 0.50$	$\beta = 0.75$
3	**0.01**	0.02	**0.04**	1.47	**2.97**	5.92
5	**0.01**	**0.01**	**0.04**	**1.46**	**2.98**	5.93
10	**0.01**	**0.01**	**0.04**	**1.46**	2.97	**5.95**
equilibrium	0.00	0.00	0.00	1.50	3.00	6.00

Table 12. Sensitivity analysis for the number of learning data: the marginal per capita return β

As seen in Tables 11 and 12, the contributions or the wagers of the case where the number of learning data is 10 are the first or the second closest to the values of the equilibria as well as

the case with learning data of 5, and therefore we think that it is appropriate to provide 10 sets of learning data for the error back propagation algorithm in the simulation.

5.3 The probability of crossover in the genetic algorithm

The probability of crossover in the genetic algorithm is set at 0.6 in the simulations shown in the previous section. In this subsection, by varying the probability of crossover, we examine appropriate values of the parameter. To do so, we carry out simulations where the probability of crossover is 0.5, 0.6, or 0.7, and values of the other parameters of the neural networks and the genetic algorithms are the same as those of the simulations in the previous section.

We provide two treatments similarly, and because the probability of crossover is a parameter for the genetic algorithm, we perform the treatments with the learning mechanism of GA. The result of the simulation for sensitivity analysis is shown in Tables 13 and 14.

the probability of crossover	voluntary contribution			lottery		
	$R = 2$	$R = 8$	$R = 16$	$R = 2$	$R = 8$	$R = 16$
0.5	0.24	0.25	0.26	<u>**1.94**</u>	**6.39**	**10.28**
0.6	<u>**0.22**</u>	<u>**0.18**</u>	**0.23**	2.22	<u>**5.87**</u>	<u>**10.56**</u>
0.7	<u>**0.22**</u>	**0.23**	<u>**0.22**</u>	**2.18**	7.36	9.42
equilibrium	0.00	0.00	0.00	1.50	6.00	12.00

Table 13. Sensitivity analysis for the probability of crossover in the genetic algorithm: the exogenous contribution R

the probability of crossover	voluntary contribution			lottery		
	$\beta = 0.00$	$\beta = 0.50$	$\beta = 0.75$	$\beta = 0.00$	$\beta = 0.50$	$\beta = 0.75$
0.5	0.07	<u>**0.09**</u>	0.22	1.74	<u>**3.28**</u>	**6.60**
0.6	<u>**0.06**</u>	<u>**0.09**</u>	**0.18**	<u>**1.58**</u>	4.01	<u>**5.87**</u>
0.7	<u>**0.06**</u>	<u>**0.09**</u>	<u>**0.17**</u>	**1.69**	3.44	7.39
equilibrium	0.00	0.00	0.00	1.50	3.00	6.00

Table 14. Sensitivity analysis for the probability of crossover in the genetic algorithm: the marginal per capita return β

As seen in Tables 13 and 14, the contributions or the wagers of the case where the probability of crossover is 0.6 are the first or the second closest to the values of the equilibria, and therefore the probability 0.6 is appropriate as the crossover probability in the genetic algorithm.

5.4 The probability of mutation in the genetic algorithm

The probability of mutation in the genetic algorithm is set at 0.01 in the simulations shown in the previous section. In this subsection, by varying the probability of mutation, we examine appropriate values of the parameter. To do so, we carry out simulations where the probability of mutation is 0.01, 0.03, or 0.05, and values of the other parameters of the neural networks and the genetic algorithms are the same as those of the simulations in the previous section.

We provide two treatments similarly, and because the probability of mutation is a parameter for the genetic algorithm, we perform the treatments with the learning mechanism of GA. The result of the simulation for sensitivity analysis is shown in Tables 15 and 16.

the probability of mutation	voluntary contribution			lottery		
	$R = 2$	$R = 8$	$R = 16$	$R = 2$	$R = 8$	$R = 16$
0.01	**<u>0.22</u>**	**<u>0.18</u>**	**<u>0.23</u>**	**<u>2.22</u>**	**<u>5.87</u>**	**10.56**
0.03	**1.89**	**1.47**	**1.55**	**2.93**	**7.52**	**<u>10.63</u>**
0.05	3.06	3.09	3.05	4.18	7.65	10.50
equilibrium	0.00	0.00	0.00	1.50	6.00	12.00

Table 15. Sensitivity analysis for the probability of mutation in the genetic algorithm: the exogenous contribution R

the probability of mutation	voluntary contribution			lottery		
	$\beta = 0.00$	$\beta = 0.50$	$\beta = 0.75$	$\beta = 0.00$	$\beta = 0.50$	$\beta = 0.75$
0.01	**<u>0.06</u>**	**<u>0.09</u>**	**<u>0.18</u>**	**<u>1.58</u>**	**<u>4.01</u>**	**<u>5.87</u>**
0.03	**0.26**	**0.48**	**1.75**	**2.13**	**4.62**	**7.33**
0.05	0.62	1.33	3.03	2.75	5.31	7.54
equilibrium	0.00	0.00	0.00	1.50	3.00	6.00

Table 16. Sensitivity analysis for the probability of mutation in the genetic algorithm: the marginal per capita return β

As seen in Tables 15 and 16, the contributions or the wagers of the case where the probability of mutation is 0.01 are almost the closest to the equilibria, and therefore the probability 0.01 is appropriate as the mutation probability in the genetic algorithm.

6. Conclusions

In this chapter, we have presented an agent-based simulation model in which artificial adaptive agents have a mechanism of decisions and learning based on neural networks and genetic algorithms. By the simulations, we have shown the effectiveness of lotteries for financing, and have examined the validity of the mathematical equilibrium model and the experiments with human subjects in detail.

Dealing with three parameters: the exogenous contribution, the marginal per capita return, and the group size, we have performed simulations and examined the effectiveness of the lottery mechanism compared with the voluntary contribution mechanism. From the result of the simulations, we have observed that the transitions of the average contributions and wagers approach almost the corresponding the predictions of the mathematical equilibrium model, and the actions of agents with more elaborate learning mechanism are closer to the values of the equilibria. Moreover, from the simulation, it is also found that the lottery mechanism provides more of the public goods than the voluntary contribution mechanism does. Thus, the results of the simulation support the equilibrium prediction more obviously compared with the experiments with human subjects, and with the results of the simulations, we have given some interpretation on the differences between the equilibrium of the mathematical model and the result of the experiments with human subjects.

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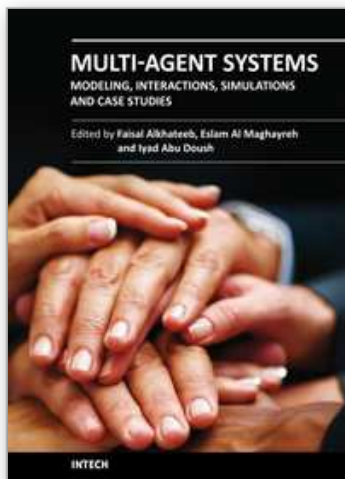
Multi-Agent Systems - Modeling, Interactions, Simulations and Case Studies

Edited by Dr. Faisal Alkhateeb

A multi-agent system (MAS) is a system composed of multiple interacting intelligent agents. Multi-agent systems can be used to solve problems which are difficult or impossible for an individual agent or monolithic system to solve. Agent systems are open and extensible systems that allow for the deployment of autonomous and proactive software components. Multi-agent systems have been brought up and used in several application domains.

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