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# A Model of Adding Relations in Two Levels of a Linking Pin Organization Structure with Two Subordinates 

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## 1. Introduction

A pyramid organization is a hierarchical structure based on the principle of unity of command (Koontz, 1980) that every member except the top in the organization should have a single immediate superior. On the other hand an organization characterized by System 4 (Likert, 1976) has a structure in which relations between members of the same section are added to the pyramid organization structure. Members of middle layers of System 4 which are both members of the upper units and chiefs of the lower units are called linking pins, and this type of organization is called a linking pin organization. In the linking pin organization there exist relations between each superior and his direct subordinates and those between members which have the same immediate superior.
The linking pin organization structure can be expressed as a structure where every pair of siblings which are nodes which have the same parent in a rooted tree is adjacent, if we let nodes and edges in the structure correspond to members and relations between members in the organization respectively. Then the linking pin organization structure is characterized by the number of subordinates of each member, that is, the number of children of each node and the number of levels in the organization, that is, the height of the rooted tree, and so on (Robbins, 2003; Takahara \& Mesarovic, 2003). Moreover, the path between a pair of nodes in the structure is equivalent to the route of communication of information between a pair of members in the organization, and adding edges to the structure is equivalent to forming additional relations other than those between each superior and his direct subordinates and between members which have the same direct subordinate.
The purpose of our study is to obtain an optimal set of additional relations to the linking pin organization such that the communication of information between every member in the organization becomes the most efficient. This means that we obtain a set of additional edges to the structure minimizing the sum of lengths of shortest paths between every pair of all nodes.
We have obtained an optimal depth for a model of adding edges between every pair of nodes with the same depth to a complete K-ary linking pin structure of height $H(H=2,3$, ...) where every pair of siblings in a complete K-ary tree of height $H$ is adjacent (Sawada, 2008). A complete $K$-ary tree is a rooted tree in which all leaves have the same depth and all internal nodes have $K(K=2,3, \ldots)$ children (Cormen et al., 2001). Figure 1 shows an example
of a complete $K$-ary linking pin structure of $K=2$ and $H=5$. In Fig. 1 the value of $N$ expresses the depth of each node.


Fig. 1. An example of a complete $K$-ary linking pin structure of $K=2$ and $H=5$
This model gives us an optimal level when we add relations in one level to the organization structure which is a complete $K$-ary linking pin structure of height $H$, but this model cannot be applied to adding relations in two or more levels. This chapter expands the above model into the model of adding relations in two levels to the organization structure, which is that of adding edges between every pair of nodes at each depth of two depths to a complete binary $(K=2)$ linking pin structure of height $H(H=3,4, \ldots)$.
If $l_{i, j}\left(=l_{j, i}\right)$ denotes the distance, which is the number of edges in the shortest path from a node $v_{i}$ to a node $v_{j}\left(i, j=1,2, \ldots, 2^{H+1}-1\right)$ in the complete binary linking pin structure of height $H$, then $\Sigma_{i<j} l_{i, j}$ is the total distance. Furthermore, if $l_{i, j}^{\prime}$ denotes the distance from $v_{i}$ to $v_{j}$ after adding edges in this model, $l_{i, j}-l_{i, j}^{\prime}$ is called the shortening distance between $v_{i}$ and $v_{j}$, and $\Sigma_{i<j}\left(l_{i, j}-l_{i, j}^{\prime}\right)$ is called the total shortening distance.
In Section 2 for the model of adding relations in one level we show formulation of a total shortening distance and an optimal depth which maximizes the total shortening distance. In Section 3 for the model of adding relations in two levels we formulate a total shortening distance and obtain an optimal pair of depths which maximizes the total shortening distance.

## 2. A model of adding relations in one level

This section shows an optimal depth $L^{*}$ by maximizing the total shortening distance, when edges between every pair of nodes at one depth $L(L=2,3, \ldots, H)$ are added to a complete binary linking pin structure of height $H(H=2,3, \ldots)$ (Sawada, 2008).

### 2.1 Formulation of total shortening distance

Let $\sigma_{H}(L)$ denote the total shortening distance, when we add edges between every pair of nodes with a depth of $L$.
The total shortening distance $\sigma_{H}(L)$ can be formulated by adding up the following three sums of shortening distances: (i) the sum of shortening distances between every pair of nodes whose depths are equal to or greater than $L$, (ii) the sum of shortening distances between every pair of nodes whose depths are less than $L$ and those whose depths are equal
to or greater than $L$ and (iii) the sum of shortening distances between every pair of nodes whose depths are less than $L$.
The sum of shortening distances between every pair of nodes whose depths are equal to or greater than $L$ is given by

$$
\begin{equation*}
\alpha_{H}(L)=\{W(H-L)\}^{2} 2^{L} \sum_{i=1}^{L-1} i 2^{i} \tag{1}
\end{equation*}
$$

where $W(h)$ denotes the number of nodes of a complete binary tree of height $h(h=0,1,2, \ldots)$. The sum of shortening distances between every pair of nodes whose depths are less than $L$ and those whose depths are equal to or greater than $L$ is given by

$$
\begin{equation*}
\beta_{H}(L)=W(H-L) 2^{L+1} \sum_{i=1}^{L-2} \sum_{j=1}^{i} j 2^{j} \tag{2}
\end{equation*}
$$

and the sum of shortening distances between every pair of nodes whose depths are less than $L$ is given by

$$
\begin{equation*}
\gamma(L)=2^{L} \sum_{i=1}^{L-3} \sum_{j=1}^{i} j(i-j+1) 2^{j}, \tag{3}
\end{equation*}
$$

where we define

$$
\begin{equation*}
\sum_{i=1}^{0} \cdot=0 \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\sum_{i=1}^{-1} \cdot=0 . \tag{5}
\end{equation*}
$$

From these equations, the total shortening distance $\sigma_{H}(L)$ is given by

$$
\begin{align*}
\sigma_{H}(L) & =\alpha_{H}(L)+\beta_{H}(L)+\gamma(L) \\
& =\{W(H-L)\}^{2} 2^{L} \sum_{i=1}^{L-1} i 2^{i}+W(H-L) 2^{L+1} \sum_{i=1}^{L-2} \sum_{j=1}^{i} j 2^{j}+2^{L} \sum_{i=1}^{L-3} \sum_{j=1}^{i} j(i-j+1) 2^{j} . \tag{6}
\end{align*}
$$

Since the number of nodes of a complete binary tree of height $h$ is

$$
\begin{equation*}
W(h)=2^{h+1}-1, \tag{7}
\end{equation*}
$$

$\sigma_{H}(L)$ of Eq.(6) becomes

$$
\begin{equation*}
\sigma_{H}(L)=(L-2) 2^{2 H+2}+2^{2 H-L+3}-2^{H+L+3}+(L+1) 2^{H+3}+L(L-1) 2^{L} . \tag{8}
\end{equation*}
$$

### 2.2 An optimal depth $L^{*}$

In this subsection, we seek $L=L^{*}$ which maximizes $\sigma_{H}(L)$ of Eq.(8).
Let $\Delta \sigma_{H}(L) \equiv \sigma_{H}(L+1)-\sigma_{H}(L)$, so that we have

$$
\begin{equation*}
\Delta \sigma_{H}(L)=4\left(1-2^{-L}\right) 2^{2 H}+8\left(1-2^{L}\right) 2^{H}+L(L+3) 2^{L} \tag{9}
\end{equation*}
$$

for $L=2,3, \ldots, H-1$. Let us define $x$ as

$$
\begin{equation*}
x=2^{H}, \tag{10}
\end{equation*}
$$

then $\Delta \sigma_{H}(L)$ in Eq.(9) becomes

$$
\begin{equation*}
\tau_{L}(x)=4\left(1-2^{-L}\right) x^{2}+8\left(1-2^{L}\right) x+L(L+3) 2^{L} \tag{11}
\end{equation*}
$$

which is a quadratic function of the continuous variable $x$. By differentiating $\tau_{L}(x)$ in Eq.(11) with respect to $x$, we obtain

$$
\begin{equation*}
\tau_{L}^{\prime}(x)=8\left(1-2^{-L}\right) x+8\left(1-2^{L}\right) \tag{12}
\end{equation*}
$$

Since $\tau_{L}(x)$ is convex downward from

$$
\begin{equation*}
4\left(1-2^{-L}\right)>0 \tag{13}
\end{equation*}
$$

and

$$
\begin{equation*}
\tau_{L}\left(2^{L+1}\right)=L(L+3) 2^{L}>0 \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
\tau_{L}^{\prime}\left(2^{L+1}\right)=8\left(2^{L}-1\right)>0 \tag{15}
\end{equation*}
$$

we have $\tau_{L}(x)>0$ for $x \geq 2^{L+1}$. Hence, we have $\Delta \sigma_{H}(L)>0$ for $H \geq L+1$; that is, $L=2,3$, ..., H-1.
From the above results, the optimal depth of this model is $L^{*}=H$.

### 2.3 Numerical examples

Table 1 shows the optimal depths $L^{*}$ and the total shortening distances $\sigma_{H}\left(L^{*}\right)$ in the case of $H=2,3, \ldots, 20$.

## 3. A model of adding relations in two levels

This section obtains an optimal pair of depths $(M, N)$ * by maximizing the total shortening distance, when edges between every pair of nodes with depth $M(M=2,3, \ldots, H-1)$ and those between every pair of nodes with depth $N(N=M+1, M+2, \ldots, H)$ which is greater than $M$ are added to a complete binary linking pin structure of height $H(H=3,4, \ldots \ldots)$.

### 3.1 Formulation of total shortening distance

Using formulation of the model of adding relations in one level shown in Subsection 2.1, we formulate the total shortening distance of the model of adding relations in two levels $S_{H}(M$, $N)$.

| $H$ | $L^{*}$ | $\sigma_{H}\left(L^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 2 | 8 |
| 3 | 3 | 112 |
| 4 | 4 | 960 |
| 5 | 5 | 6528 |
| 6 | 6 | 211200 |
| 7 | 7 | 1083392 |
| 8 | 8 | 5324800 |
| 9 | 9 | 25356288 |
| 10 | 10 | 117878784 |
| 11 | 11 | 537870336 |
| 12 | 12 | 2418180096 |
| 13 | 13 | 10742497280 |
| 14 | 14 | 47255977984 |
| 15 | 15 | 206183596032 |
| 16 | 16 | 893408772096 |
| 17 | 17 | 3848412856320 |
| 18 | 18 | 16492941803520 |
| 19 | 19 | 70369327185920 |
| 20 | 20 |  |

Table 1. Optimal depths $L^{*}$ and total shortening distances $\sigma_{H}\left(L^{*}\right)$
Let $V_{1}$ denote the set of nodes whose depths are less than $M$. Let $V_{2}$ denote the set of nodes whose depths are equal to or greater than $M$ and are less than $N$. Let $V_{3}$ denote the set of nodes whose depths are equal to or greater than $N$.
The sum of shortening distances between every pair of nodes in $V_{3}$ is given by

$$
\begin{equation*}
A_{H}(N)=\alpha_{H}(N) \tag{16}
\end{equation*}
$$

from Eq.(1). The sum of shortening distances between every pair of nodes in $V_{3}$ and nodes in $V_{1}$ and $V_{2}$ is given by

$$
\begin{equation*}
B_{H}(N)=\beta_{H}(N) \tag{17}
\end{equation*}
$$

from Eq.(2). The sum of shortening distances between every pair of nodes in $V_{1}$ is given by

$$
\begin{equation*}
C(M)=\gamma(M) \tag{18}
\end{equation*}
$$

from Eq.(3), and the sum of shortening distances between every pair of nodes in $V_{1}$ and nodes in $V_{2}$ is given by

$$
\begin{equation*}
D(M, N)=\beta_{N-1}(M) \tag{19}
\end{equation*}
$$

from Eq.(2). The sum of shortening distances between every pair of nodes in $V_{2}$ is formulated as follows.
The sum of shortening distances between every pair of nodes in each linking pin structure whose root is a node with depth $M$ is given by

$$
\begin{equation*}
E(M, N)=\gamma(N-M) 2^{M} \tag{20}
\end{equation*}
$$

from Eq.(3). The sum of shortening distances between every pair of nodes in two different linking pin structures whose roots are nodes with depth $M$ is given by summing up $F(M, N)$ and $G(M, N) . F(M, N)$ which is the sum of shortening distances by adding edges only between nodes with depth $M$ is given by

$$
\begin{equation*}
F(M, N)=\alpha_{N-1}(M) \tag{21}
\end{equation*}
$$

from Eq.(1). $G(M, N)$ which is the sum of additional shortening distances by adding edges between nodes with depth $N$ after adding edges between nodes with depth $M$ is expressed by

$$
\begin{equation*}
G(M, N)=\left(2^{M}-1\right) \sum_{i=1}^{N-M-2} 2^{N-i} \sum_{j=1}^{N-M-i-1} 2^{N-M-j}(N-M-i-j) \tag{22}
\end{equation*}
$$

where we define

$$
\begin{equation*}
\sum_{i=1}^{0} \cdot=0 \tag{23}
\end{equation*}
$$

and

$$
\begin{equation*}
\sum_{i=1}^{-1} \cdot=0 \tag{24}
\end{equation*}
$$

From these equations, the total shortening distances $S_{H}(M, N)$ is given by

$$
\begin{align*}
S_{H}(M, N) & =A_{H}(N)+B_{H}(N)+C(M)+D(M, N)+E(M, N)+F(M, N)+G(M, N) \\
& =\{W(H-N)\}^{2} 2^{N} \sum_{i=1}^{N-1} i 2^{i}+W(H-N) 2^{N+1} \sum_{i=1}^{N-2} \sum_{j=1}^{i} j 2^{j} \\
& +2^{M} \sum_{i=1}^{M-3} \sum_{j=1}^{i} j(i-j+1) 2^{j}+W(N-M-1) 2^{M+1} \sum_{i=1}^{M-2} \sum_{j=1}^{i} j 2^{j}  \tag{25}\\
& +2^{N} \sum_{i=1}^{N-M-3} \sum_{j=1}^{i} j(i-j+1) 2^{j}+\{W(N-M-1)\}^{2} 2^{M} \sum_{i=1}^{M-1} i 2^{i} \\
& +\left(2^{M}-1\right) \sum_{i=1}^{N-M-2} 2^{N-i} \sum_{j=1}^{N-M-i-1} 2^{N-M-j}(N-M-i-j)
\end{align*}
$$

Since the number of nodes of a complete binary tree of height $h$ is

$$
\begin{equation*}
W(h)=2^{h+1}-1 \tag{26}
\end{equation*}
$$

$S_{H}(M, N)$ of Eq.(25) becomes

$$
\begin{align*}
S_{H}(M, N) & =(N-2) 2^{2 H+2}+2^{2 H-N+3}-2^{H+N+3}+(N+1) 2^{H+3}+(N-M) 2^{N+M+1}  \tag{27}\\
& +(N-M)(N-M-3) 2^{N}+M(M-1) 2^{M}
\end{align*}
$$

### 3.2 An optimal depth $\boldsymbol{N}^{*}$ for a fixed value of $\boldsymbol{M}$

In this subsection, we seek $N=N^{*}$ which maximizes $R_{H, M}(N)=S_{H}(M, N)$ for a fixed value of $M(M=2,3, \ldots, H-1)$.
Let $\Delta R_{H, M}(N) \equiv R_{H, M}(N+1)-R_{H, M}(N)$, so that we have

$$
\begin{align*}
\Delta R_{H, M}(N) & =4\left(1-2^{-N}\right) 2^{2 H}+8\left(1-2^{N}\right) 2^{H}+(N-M+2) 2^{N+M+1}  \tag{28}\\
& +\{(N-M)(N-M+1)-4\} 2^{N}
\end{align*}
$$

for $N=M+1, M+2, \ldots, H-1$. Let us define $x$ as

$$
\begin{equation*}
x=2^{H}, \tag{29}
\end{equation*}
$$

then $\Delta R_{H, M}(N)$ in Eq.(28) becomes

$$
\begin{align*}
T_{M, N}(x)= & 4\left(1-2^{-N}\right) x^{2}+8\left(1-2^{N}\right) x+(N-M+2) 2^{N+M+1}  \tag{30}\\
& +\{(N-M)(N-M+1)-4\} 2^{N}
\end{align*}
$$

which is a quadratic function of the continuous variable $x$. By differentiating $T_{M, N}(x)$ in Eq.(30) with respect to $x$, we obtain

$$
\begin{equation*}
T_{M, N}^{\prime}(x)=8\left(1-2^{-N}\right) x+8\left(1-2^{N}\right) \tag{31}
\end{equation*}
$$

Since $T_{M, N}(x)$ is convex downward from

$$
\begin{equation*}
4\left(1-2^{-N}\right)>0 \tag{32}
\end{equation*}
$$

and

$$
\begin{equation*}
T_{M, N}\left(2^{N+1}\right)=(N-M) 2^{N+M+1}+(N-M)(N-M+1) 2^{N}+\left(2^{M}-1\right) 2^{N+2}>0 \tag{33}
\end{equation*}
$$

and

$$
\begin{equation*}
T_{M, N}^{\prime}\left(2^{N+1}\right)=8\left(2^{N}-1\right)>0 \tag{34}
\end{equation*}
$$

we have $T_{M, N}(x)>0$ for $x \geq 2^{N+1}$. Hence, we have $\Delta R_{H, M}(N)>0$ for $H \geq N+1$; that is, $N=M+1, M+2, \ldots, H-1$.
From the above results, the optimal depth $N^{*}$ for a fixed value of $M(M=2,3, \ldots, H-1)$ is $N^{*}=H$.

### 3.3 An optimal pair of depths ( $M, N$ )*

In this subsection, we seek $(M, N)=(M, N)^{*}$ which maximizes $S_{H}(M, N)$ in Eq.(27).
Let $Q_{H}(M)$ denote the total shortening distance when $N=H$, so that we have

$$
\begin{align*}
Q_{H}(M) & \equiv S_{H}(M, H) \\
& =(H-4) 2^{2 H+2}+(H-M) 2^{H+M+1}+(H+2) 2^{H+3}+(H-M)(H-M-3) 2^{H}  \tag{35}\\
& +M(M-1) 2^{M} .
\end{align*}
$$

Let $\Delta Q_{H}(M) \equiv Q_{H}(M+1)-Q_{H}(M)$, so that we have

$$
\begin{equation*}
\Delta Q_{H}(M)=(H-M-2)\left(2^{M}-1\right) 2^{H+1}+M(M+3) 2^{M}>0 \tag{36}
\end{equation*}
$$

for $M=2,3, \ldots, H-2$.
From the results in Subsection 3.2 and 3.3, the optimal pair of depths is $(M, N)^{*}=(H-1, H)$.

### 3.4 Numerical examples

Tables 2-19 show the optimal depths $N^{*}$ for a fixed value of $M(M=2,3, \ldots, H-1)$ and the total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=3,4, \ldots, 20$.

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | :---: |
| 2 | 3 | 120 |

Table 2. Optimal depth $N^{*}$ and total shortening distance $S_{H}\left(M, N^{*}\right)$ in the case of $H=3$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 4 | 1000 |
| 3 | 4 | 1040 |

Table 3. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=4$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 5 | 6664 |
| 3 | 5 | 6896 |
| 4 | 5 | 7040 |

Table 4. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=5$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 6 | 39176 |
| 3 | 6 | 39984 |
| 4 | 6 | 41024 |
| 5 | 6 | 41472 |

Table 5. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=6$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | :---: |
| 2 | 7 | 212232 |
| 3 | 7 | 214576 |
| 4 | 7 | 218304 |
| 5 | 7 | 222592 |
| 6 | 7 | 223872 |

Table 6. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=7$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 8 | 1085960 |
| 3 | 8 | 1092144 |
| 4 | 8 | 1103040 |
| 5 | 8 | 1118848 |
| 6 | 8 | 1136000 |
| 7 | 8 | 1139456 |

Table 7. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=8$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 9 | 5330952 |
| 3 | 9 | 5346352 |
| 4 | 9 | 5375168 |
| 5 | 9 | 5421696 |
| 6 | 9 | 5486464 |
| 7 | 9 | 5554432 |
| 8 | 9 | 5563392 |

Table 8. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=9$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 10 | 25370632 |
| 3 | 10 | 25407536 |
| 4 | 10 | 25479360 |
| 5 | 10 | 25602688 |
| 6 | 10 | 25794432 |
| 7 | 10 | 26055936 |
| 8 | 10 | 26324992 |
| 9 | 10 | 26347520 |

Table 9. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=10$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 11 | 117911560 |
| 3 | 11 | 117997616 |
| 4 | 11 | 118169792 |
| 5 | 11 | 118477440 |
| 6 | 11 | 118986624 |
| 7 | 11 | 119764224 |
| 8 | 11 | 120813568 |
| 9 | 11 | 121880576 |
| 10 | 11 | 121935872 |

Table 10. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=11$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 12 | 537944072 |
| 3 | 12 | 538140720 |
| 4 | 12 | 538542272 |
| 5 | 12 | 539280000 |
| 6 | 12 | 540551040 |
| 7 | 12 | 542618880 |
| 8 | 12 | 545748992 |
| 9 | 12 | 549949440 |
| 10 | 12 | 554190848 |
| 11 | 12 | 554323968 |

Table 11. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=12$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 13 | 2418343944 |
| 3 | 13 | 2418786352 |
| 4 | 13 | 2419704000 |
| 5 | 13 | 2421424768 |
| 6 | 13 | 2424473472 |
| 7 | 13 | 2429637888 |
| 8 | 13 | 2437969920 |
| 9 | 13 | 2450526208 |
| 10 | 13 | 2467325952 |
| 11 | 13 | 2484219904 |
| 12 | 13 | 2484535296 |

Table 12. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=13$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 14 | 10742857736 |
| 3 | 14 | 10743840816 |
| 4 | 14 | 10745905344 |
| 5 | 14 | 10749837952 |
| 6 | 14 | 10756949888 |
| 7 | 14 | 10769339648 |
| 8 | 14 | 10790156288 |
| 9 | 14 | 10823602176 |
| 10 | 14 | 10873890816 |
| 11 | 14 | 10941067264 |
| 12 | 14 | 11008458752 |
| 13 | 14 | 11009196032 |

Table 13. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=14$

| M | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 15 | 47256764424 |
| 3 | 15 | 47258927152 |
| 4 | 15 | 47263514816 |
| 5 | 15 | 47272362624 |
| 6 | 15 | 47288616832 |
| 7 | 15 | 47317521664 |
| 8 | 15 | 47367469056 |
| 9 | 15 | 47451049984 |
| 10 | 15 | 47585060864 |
| 11 | 15 | 47786323968 |
| 12 | 15 | 48054943744 |
| 13 | 15 | 48324050944 |
| 14 | 15 | 48325754880 |

Table 14. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=15$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | :---: |
| 2 | 16 | 206185299976 |
| 3 | 16 | 206190018608 |
| 4 | 16 | 206200111296 |
| 5 | 16 | 206219772544 |
| 6 | 16 | 206256342912 |
| 7 | 16 | 206322406656 |
| 8 | 16 | 206438938624 |
| 9 | 16 | 206639501312 |
| 10 | 16 | 206974445568 |
| 11 | 16 | 207510925312 |
| 12 | 16 | 208316153856 |
| 13 | 16 | 209390370816 |
| 14 | 16 | 210465685504 |
| 15 | 16 | 210469584896 |

Table 15. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=16$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | ---: |
| 2 | 17 | 893412442120 |
| 3 | 17 | 893422665776 |
| 4 | 17 | 893444686016 |
| 5 | 17 | 893487940224 |
| 6 | 17 | 893569206144 |
| 7 | 17 | 893717845248 |
| 8 | 17 | 893984192512 |
| 9 | 17 | 894452142080 |
| 10 | 17 | 895255930880 |
| 11 | 17 | 896596930560 |
| 12 | 17 | 898743681024 |
| 13 | 17 | 901964857344 |
| 14 | 17 | 906261004288 |
| 15 | 17 | 910559608832 |
| 16 | 17 | 910568456192 |

Table 16. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=17$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | :---: |
| 2 | 18 | 3848420720648 |
| 3 | 18 | 3848442740784 |
| 4 | 18 | 3848490451136 |
| 5 | 18 | 3848584823424 |
| 6 | 18 | 3848763606912 |
| 7 | 18 | 3849093911808 |
| 8 | 18 | 3849693181952 |
| 9 | 18 | 3850762752000 |
| 10 | 18 | 3852638185472 |
| 11 | 18 | 3855856398336 |
| 12 | 18 | 3861222801408 |
| 13 | 18 | 3869811376128 |
| 14 | 18 | 3882696409088 |
| 15 | 18 | 3899879129088 |
| 16 | 18 | 3917067321344 |
| 17 | 18 | 3917087244288 |

Table 17. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=18$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | :---: |
| 2 | 19 | 16492958580744 |
| 3 | 19 | 16493005766704 |
| 4 | 19 | 16493108527296 |
| 5 | 19 | 16493313000064 |
| 6 | 19 | 16493703071616 |
| 7 | 19 | 16494429738240 |
| 8 | 19 | 16495761438720 |
| 9 | 19 | 16498167943168 |
| 10 | 19 | 16502454577152 |
| 11 | 19 | 16509963563008 |
| 12 | 19 | 16522842488832 |
| 13 | 19 | 16544312819712 |
| 14 | 19 | 16578670067712 |
| 15 | 19 | 16630210428928 |
| 16 | 19 | 16698936655872 |
| 17 | 19 | 16767675006976 |
| 18 | 19 | 16767719571456 |

Table 18. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=19$

| $M$ | $N^{*}$ | $S_{H}\left(M, N^{*}\right)$ |
| :---: | :---: | :---: |
| 2 | 20 | 70369362837512 |
| 3 | 20 | 70369463500848 |
| 4 | 20 | 70369683701952 |
| 5 | 20 | 70370124104320 |
| 6 | 20 | 70370969257856 |
| 7 | 20 | 70372554708224 |
| 8 | 20 | 70375484438528 |
| 9 | 20 | 70380832198656 |
| 10 | 20 | 70390477056000 |
| 11 | 20 | 70407640281088 |
| 12 | 20 | 70437690687488 |
| 13 | 20 | 70489218449408 |
| 14 | 20 | 70575109013504 |
| 15 | 20 | 70712543477760 |
| 16 | 20 | 70918704463872 |
| 17 | 20 | 71193598099456 |
| 18 | 20 | 71468518473728 |
| 19 | 20 | 71468617564160 |

Table 19. Optimal depths $N^{*}$ and total shortening distances $S_{H}\left(M, N^{*}\right)$ in the case of $H=20$

Table 20 shows the optimal pairs of depths $(M, N)^{*}$ and the total shortening distances $S_{H}$ $(M, N)^{*}$ in the case of $H=3,4, \ldots, 20$.

| $H$ | $(M, N)^{*}$ | $S_{H}(M, N)^{*}$ |
| :---: | :---: | ---: |
| 3 | $(2,3)$ | 120 |
| 4 | $(3,4)$ | 1040 |
| 5 | $(4,5)$ | 7040 |
| 6 | $(5,6)$ | 223872 |
| 7 | $(6,7)$ | 1139456 |
| 8 | $(7,8)$ | 5563392 |
| 9 | $(8,9)$ | 26347520 |
| 10 | $(9,10)$ | 121935872 |
| 11 | $(10,11)$ | 554323968 |
| 12 | $(11,12)$ | 2484535296 |
| 13 | $(12,13)$ | 11009196032 |
| 14 | $(13,14)$ | 48325754880 |
| 15 | $(14,15)$ | 210469584896 |
| 16 | $(15,16)$ | 910568456192 |
| 17 | $(16,17)$ | 3917087244288 |
| 18 | $(17,18)$ | 16767719571456 |
| 19 | $(18,19)$ | 71468617564160 |
| 20 | $(19,20)$ |  |

Table 20. Optimal pairs of depths $(M, N)$ * and total shortening distances $S_{H}(M, N)^{*}$

## 4. Conclusions

This study considered obtaining optimal depths of adding edges to a complete binary linking pin structure where every pair of siblings in a complete binary tree is adjacent maximizing the total shortening distance which is the sum of shortening lengths of shortest paths between every pair of all nodes in the complete binary linking pin structure. This means to obtain optimal levels of adding relations to a linking pin organization structure in which relations between members of the same section are added to the pyramid organization structure such that the communication of information between every member in the organization becomes the most efficient.
For the model of adding edges between every pair of nodes at one depth $L$ to a complete binary linking pin structure of height $H$, we had already obtained an optimal depth $L^{*}=H$ in our paper (Sawada, 2008). This result shows that the most efficient way of adding relations between all members in one level is to add relations at the lowest level, irrespective of the number of levels in the organization structure.
This chapter expanded the above model into the model of adding relations in two levels of the organization structure, which is that of adding edges between every pair of nodes with depth $M$ and those between every pair of nodes with depth $N$ which is greater than $M$ to a complete binary linking pin structure of height $H$. We obtained an optimal pair of depth ( $M$, $N)^{*}=(H-1, H)$ which maximizes the total shortening distances. In the case of $H=5$ illustrated with the example in Fig. 1 an optimal pair of depths is $(M, N)^{*}=(4,5)$. This result means that the most efficient manner of adding relations between all members in each level of two levels is to add relations at the lowest level and the second lowest level, irrespective of the number of levels in the organization structure.

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