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On the Electrodinamics of Space-Time Periodic Mediums in a Waveguide of Arbitrary Cross Section

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1. Introduction

Theoretical investigations of parametric interaction between the electrodynamics waves and space-time periodic filling of the waveguide of arbitrary cross section are reviewed. The cases of dielectric, anisotropic and magnetodielectric periodically modulated filling are considered. The analytical method of solution of the problems of electrodynamics of space-time periodic mediums in a waveguide is given. The propagation of transverse-electric (TE) and transverse-magnetic (TM) waves in the waveguide mentioned above are investigated. Physical phenomena of electrodynamics of space-time periodic mediums in the region of "weak" and "strong" interactions between the travelling wave in the waveguide and the modulation wave are studied.

Propagation of electromagnetic waves in the medium whose permittivity and permeability are modulated in space and time with help of pump waves of various nature (electromagnetic wave, ultrasonic wave, etc.) under the harmonic law, represents one of the basic problem of the electromagnetic theory. In the scientific literature the most part of such researches concerns to electrodynamics of periodically non-stationary and non-uniform mediums in the unlimited space [1-15], while the same problems in the limited modulated mediums, for example, in the waveguides of arbitrary cross section remain still insufficiently studied and there is no strict analytical theory of the propagation of electromagnetic waves in similar systems (although in the scientific literature already appeared the articles on the problems, mentioned above [16-25].

Meanwhile the investigation of the propagation of electromagnetic waves in the waveguides with space-time periodically modulated filling represents a great interest not only from point of view of development of theory but also from point of view of possibility of practice application of similar waveguides in the ultrahigh frequency electronics. For example, the waveguides with periodically non-stationary and non-uniform filling can be applied for designing of multifrequency distributing back-coupling lasers (DBS lasers), Bragg reflection lasers (DBR lasers), mode transformers, parametric amplifiers, multifrequency generators, transformers of low and higher frequency, Bragg resonators and filters, prismatic polarizer, diffraction lattices, oscillators, mode converters, wave-channeling devices with a fine periodic structure, etc [14], [26-30].

2. Electromagnetic waves in a waveguide with space-time periodic filling

Let us consider the regular ideal waveguide of arbitrary cross section which axis coincides with the OZ axis of certain Cartesian frame. Let the permittivity and permeability of the filling of the waveguide with help of pump wave are modulated in space and time under the periodic law (Fig.1.1) [23, 25]

$$\varepsilon = \varepsilon_0\left[1 + m_\varepsilon \cos k_0 (z - ut)\right],\ \mu = \mu_0\left[1 + m_\mu \cos k_0 (z - ut)\right] \tag{1.1}$$

where m_ε и m_μ are the modulation indexes, u is the modulation wave velocity , k_0 is the modulation wave number, $k_0 u$ is the modulation wave frequency, ε_0 и μ_0 are the permittivity and permeability of the filling in the absence of modulation. The signal wave with frequency ω_0

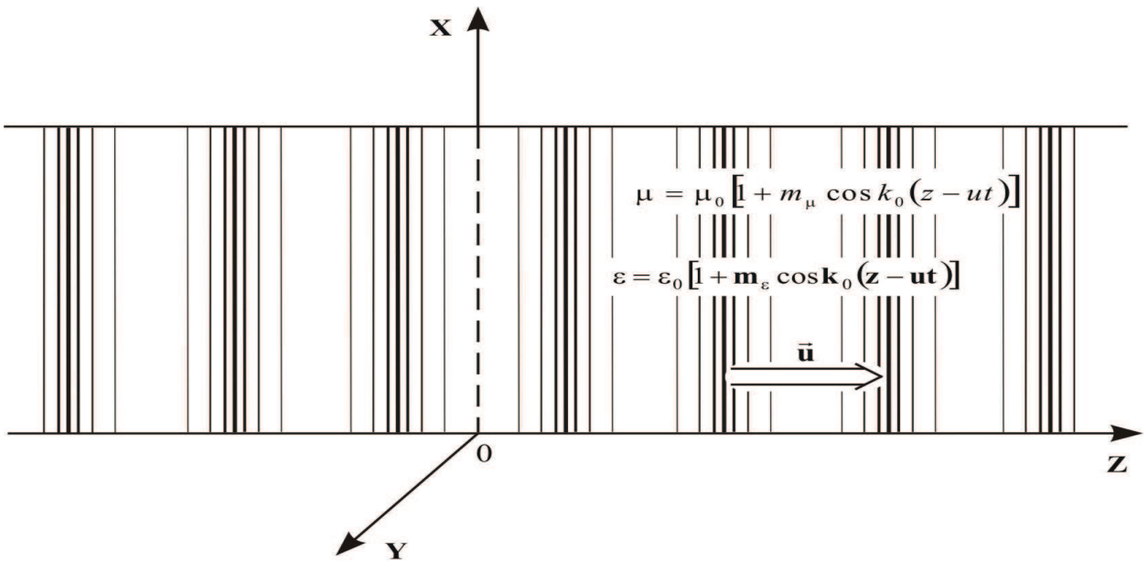


Fig. 1.1. Geometry of cross section of a waveguide with harmonically modulated filling.

propagates in a similar waveguide along their axis in the positive direction. Suppose that the signal wave doesn't change the quantities of ε and μ . It is mean that we have the approximation of small signals. The field in similar waveguide represents the superposition of transverse-electric (TE) and transverse-magnetic (TM) waves, which in this consideration are described with help of longitudinal components of magnetic (H_z) and electric (E_z) vectors. These components satisfy to partial differential equations with variable coefficients which are obtained from the Maxwell equations taking into account that the charge density and the current density are equal to zero. These wave equations have a form [23-25], [31]

TE field

$$\Delta_\perp H_z + \frac{\partial}{\partial z}\left(\frac{1}{\mu}\frac{\partial(\mu H_z)}{\partial z}\right) - \frac{1}{c^2}\frac{\partial}{\partial t}\left(\varepsilon\frac{\partial(\mu H_z)}{\partial t}\right) = 0\,, \tag{1.2}$$

TM field

$$\Delta_{\perp}E_z+\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial\left(\varepsilon E_z\right)}{\partial z}\right)-\frac{1}{c^2}\frac{\partial}{\partial t}\left(\mu\frac{\partial\left(\varepsilon E_z\right)}{\partial t}\right)=0\,, \tag{1.3}$$

where $\Delta_{\perp}=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the two-dimensional Laplace operator, c is the velocity of light in vacuum.

The solution of wave equations (1.2) and (1.3) we look for the form of decomposition by orthonormal eigenfunctions of the second and first boundary-value problems for the cross section of the waveguide $\left(\hat{\psi}_n(x,y),\psi_n(x,y)\right)$. These functions satisfy to the following Helmholtz equations with corresponding boundary conditions on the surface of the waveguide [23], [31]:

second boundary-value problem

$$\Delta_{\perp}\hat{\Psi}_n(x,y)+\hat{\lambda}_n^2\hat{\Psi}_n(x,y)=0,\quad \left.\frac{\partial\hat{\Psi}_n}{\partial\vec{n}}\right|_{\Sigma}=0\,, \tag{1.4}$$

first boundary-value problem

$$\Delta_{\perp}\Psi_n(x,y)+\lambda_n^2\Psi_n(x,y)=0,\quad \Psi_n(x,y)\big|_{\Sigma}=0\,, \tag{1.5}$$

where $\hat{\lambda}_n$ and λ_n are the eigenvalues of the second and first boundary-value problems for the transverse cross section of the waveguide, Σ is the contour of the waveguide's cross section, $\vec{n}$ is the normal to Σ. From Maxwell equations the transverse components of transverse-electric (TE) and transverse-magnetic (TM) fields can be represented in terms of

$$H_z(x,y,z,t)=\sum_{n=0}^{\infty}H_n(z,t)\cdot\hat{\Psi}_n(x,y),\ E_z(x,y,z,t)=\sum_{n=0}^{\infty}E_n(z,t)\cdot\Psi_n(x,y) \tag{1.6}$$

as follows [23]:

TE field

$$\vec{H}_{\tau}(x,y,z,t)=\frac{1}{\mu(z,t)}\sum_{n=0}^{\infty}\hat{\lambda}_n^{-2}\frac{\partial\left[\mu(z,t)H_n(z,t)\right]}{\partial z}\nabla\hat{\Psi}_n(x,y), \tag{1.7}$$

$$\vec{E}_{\tau}(x,y,z,t)=\frac{1}{c}\sum_{n=0}^{\infty}\hat{\lambda}_n^{-2}\frac{\partial\left[\mu(z,t)\,H_n(z,t)\right]}{\partial t}\left[\vec{z}_0\nabla\hat{\Psi}_n(x,y)\right], \tag{1.8}$$

TE field

$$\vec{H}_{\tau}(x,y,z,t)=-\frac{1}{c}\sum_{n=0}^{\infty}\lambda_n^{-2}\frac{\partial\left[\varepsilon(z.t)E_n(z,t)\right]}{\partial t}\left[\vec{z}_0\nabla\Psi_n(x,y)\right], \tag{1.9}$$

$$\vec{E}_{\tau}(x,y,z,t)=\frac{1}{\varepsilon(z,t)}\sum_{n=0}^{\infty}\lambda_n^{-2}\frac{\partial\left[\varepsilon(z,t)E_n(z,t)\right]}{\partial z}\nabla\Psi_n(x,y), \tag{1.10}$$

where $\nabla = \vec{i}\left(\partial / \partial x\right) + \vec{j}\left(\partial / \partial y\right)$ is the two-dimensional nabla operator.

If into the wave equations (1.2) and (1.3) of variables z and t to introduce the new quantities by the formulas

$$\tilde{H}_z = \mu H_z, \quad \tilde{E}_z = \varepsilon E_z \tag{1.11}$$

and to pass to the new variables ξ and η according to the formulas [22]

$$\xi = z - ut, \quad \eta = \frac{z}{u} - \frac{1}{u}\int_0^{\xi} \frac{d\xi}{1 - \beta^2 \dfrac{\varepsilon(\xi)\,\mu(\xi)}{\varepsilon_0\,\mu_0}}, \tag{1.12}$$

where $\beta^2 = u^2 \varepsilon_0 \mu_0 / c^2$ and when $u \to 0$ then $\xi \to z, \eta \to t$, and the solutions of received partial differential equations to look for the form [22]

$$\tilde{H}_z = \sum_{n=0}^{\infty} e^{i\gamma\eta} \tilde{H}_{nz}(\xi) \cdot \Psi_n(x,y), \; \tilde{E}_z = \sum_{n=0}^{\infty} e^{i\gamma\eta} \tilde{E}_{nz}(\xi) \cdot \Psi_n(x,y), \tag{1.13}$$

taking into account the orthonormalization of the eigenfunctions $\bar{\psi}_n(x,y)$ and $\psi_n(x,y)$ then we receive for $\tilde{H}_{nz}$ and $\tilde{E}_{nz}$ the following ordinary differential equations with variable coefficients:

$$\mu \frac{d}{d\xi}\left[\frac{1}{\mu}\left(1 - \beta^2 \frac{\varepsilon\,\mu}{\varepsilon_0\,\mu_0}\right)\frac{d\tilde{H}_{nz}}{d\xi}\right] + \frac{\hat{\chi}_n^2}{1 - \beta^2 \dfrac{\varepsilon\,\mu}{\varepsilon_0\,\mu_0}} \tilde{H}_{nz} = 0, \tag{1.14}$$

$$\varepsilon \frac{d}{d\xi}\left[\frac{1}{\varepsilon}\left(1 - \beta^2 \frac{\varepsilon\,\mu}{\varepsilon_0\,\mu_0}\right)\frac{d\tilde{E}_{nz}}{d\xi}\right] + \frac{\chi_n^2}{1 - \beta^2 \dfrac{\varepsilon\,\mu}{\varepsilon_0\,\mu_0}} \tilde{E}_{nz} = 0, \tag{1.15}$$

where

$$\hat{\chi}_n^2 = \frac{\gamma^2}{c^2}\varepsilon\,\mu - \hat{\lambda}_n^2\left(1 - \beta^2 \frac{\varepsilon\,\mu}{\varepsilon_0\,\mu_0}\right), \; \chi_n^2 = \frac{\gamma^2}{c^2}\varepsilon\,\mu - \lambda_n^2\left(1 - \beta^2 \frac{\varepsilon\,\mu}{\varepsilon_0\,\mu_0}\right). \tag{1.16}$$

In this investigation we are limited of small quantities of modulation indexes of the waveguide filling. It is explained that in real experiment the modulation indexes are very small and they can change from 10^{-4} to $4 \cdot 10^{-2}$ (the quantity $4 \cdot 10^{-2}$ is fixed in the chrome gelatin). Note that if the velocity of modulation wave satisfies the condition $u \le 0{,}8 \cdot \upsilon_{\text{ph}}$, where $\upsilon_{ph} = c/\sqrt{\varepsilon_0\,\mu_0}$ is the phase velocity in the non-disturbance medium, then side by side of modulation indexes is small the parameter $\ell = \left(m_\varepsilon + m_\mu\right)\beta^2 / b$ ($b = 1 - \beta^2$) too, that is $l << 1$.

Then with help of changed of variables

$$\hat{s}=\frac{k_0\left(1-\beta^2\right)}{2\mu_0}\int_0^{\xi}\frac{\mu\, d\xi}{1-\beta^2\dfrac{\varepsilon\,\mu}{\varepsilon_0\,\mu_0}},\ s=\frac{k_0\left(1-\beta^2\right)}{2\varepsilon_0}\int_0^{\xi}\frac{\varepsilon\, d\xi}{1-\beta^2\dfrac{\varepsilon\,\mu}{\varepsilon_0\,\mu_0}} \tag{1.17}$$

and taking into account that permittivity and permeability of the filling change by the harmonic law (1.1) the above received differential equations (1.14) and (1.15) on variables ξ and η are transformed to the differential equations with periodic coefficients of Mathie-Hill type [32]. In the first approximation on small modulation indexes they have a form [23]

$$\frac{d^2\tilde{H}_{nz}(\hat{s})}{d\hat{s}^2}+\sum_{k=-1}^{1}\hat{\theta}_k^n\, e^{2ik\hat{s}}\tilde{H}_{nz}(\hat{s})=0, \tag{1.18}$$

$$\frac{d^2\tilde{E}_{nz}(s)}{ds^2}+\sum_{k=-1}^{1}\theta_k^n\, e^{2ikS}\tilde{E}_{nz}(s)=0, \tag{1.19}$$

where

$$\hat{\theta}_0^n=\frac{4}{k_0^2b^2}\left(\hat{\chi}_0^n\right)^2,\ \theta_0^n=\frac{4}{k_0^2b^2}\left(\chi_0^n\right)^2, \tag{1.20}$$

$$\hat{\theta}_{\pm1}^n=\frac{2}{k_0^2b^2}\left(\frac{\gamma^2}{u^2}-\left(\hat{\chi}_0^n\right)^2\right)\ell-\frac{4}{k_0^2b^2}\left(\hat{\chi}_0^n\right)^2 m_\mu, \tag{1.21}$$

$$\theta_{\pm1}^n=\frac{2}{k_0^2b^2}\left(\frac{\gamma^2}{u^2}-\left(\chi_0^n\right)^2\right)\ell-\frac{4}{k_0^2b^2}\left(\chi_0^n\right)^2 m_\varepsilon, \tag{1.22}$$

$$\left(\hat{\chi}_0^n\right)^2=\frac{\gamma^2}{c^2}\varepsilon_0\,\mu_0-\hat{\lambda}_n^2\,b,\ \left(\chi_0^n\right)^2=\frac{\gamma^2}{c^2}\varepsilon_0\,\mu_0-\lambda_n^2\,b. \tag{1.23}$$

The solutions of the equations (1.18) and (1.19) we look for the form

$$\tilde{H}_{nz}(\hat{s})=e^{i\hat{\mu}_n\hat{s}}\sum_{k=-1}^{1}\hat{C}_k^n\cdot e^{2ik\hat{s}},\ \tilde{E}_{nz}(s)=e^{i\mu_n s}\sum_{k=-1}^{1}C_k^n\cdot e^{2iks}. \tag{1.24}$$

Substituting these expressions into Mathie-Hill equations (1.18) and (1.19) for determination of characteristic indexes $\hat{\mu}_n$ and μ_n we receive the following dispersion equations:

$$\hat{\mu}_n^2=\hat{\theta}_0^n+\frac{\left(\hat{\theta}_1^n\right)^2}{\left(\hat{\mu}_n-2\right)^2-\hat{\theta}_0^n}+\frac{\left(\hat{\theta}_1^n\right)^2}{\left(\hat{\mu}_n-2\right)^2-\hat{\theta}_0^n}, \tag{1.25}$$

$$\mu_n^2=\theta_0^n+\frac{\left(\theta_1^n\right)^2}{\left(\mu_n-2\right)^2-\theta_0^n}+\frac{\left(\theta_1^n\right)^2}{\left(\mu_n-2\right)^2-\theta_0^n}. \tag{1.26}$$

The analysis of these dispersion equations show that under the following conditions [33]

$$\left|1-\hat{\theta}_0^n\right| \geq \hat{\delta}_n \approx \ell, \quad \left|1-\theta_0^n\right| \geq \delta_n \approx \ell \tag{1.27}$$

we become to the region of weak interaction between the signal wave and the modulation wave where the characteristic indexes $\hat{\mu}_n$ and μ_n are real and then the Mathie-Hill equations have the stable solutions. With help of obtained solutions of dispersion equations

$$(\hat{\mu}_n)^2 = \hat{\theta}_0^n, \quad (\mu_n)^2 = \theta_0^n \tag{1.28}$$

and the expressions for the coefficients

$$\hat{C}_{\pm 1}^n = \frac{\hat{\theta}_1^n \cdot \hat{C}_0^n}{4\left(1 \pm \sqrt{\hat{\theta}_0^n}\right)}, \quad C_{\pm 1}^n = \frac{\theta_1^n \cdot C_0^n}{4\left(1 \pm \sqrt{\theta_0^n}\right)}, \tag{1.29}$$

where $\hat{C}_0^n$ and C_0^n are defined from the conditions of normalizing, we obtained the analytical expressions for the H_z and E_z of TE and TM fields in the waveguide in the region of weak interaction. They have a form [33]

$$H_z = \frac{1}{\mu_0}\sum_{n=0}^{\infty} \hat{\Psi}_n(x,y) e^{i\left(\hat{P}_0^n z - \omega_0 t\right)} \cdot \hat{C}_0^n \sum_{k=-1}^{1} \hat{V}_k^n \cdot e^{ikk_0(z-ut)}, \tag{1.30}$$

$$E_z = \frac{1}{\varepsilon_0}\sum_{n=0}^{\infty} \Psi_n(x,y) e^{i\left(P_0^n z - \omega_0 t\right)} \cdot C_0^n \sum_{k=-1}^{1} V_k^n \cdot e^{ikk_0(z-ut)}, \tag{1.31}$$

where

$$\hat{V}_k^n = \left(k \cdot \frac{\hat{\Delta}_0^n}{2} + \frac{\hat{C}_k^n}{\hat{C}_0^n} - \frac{m_\mu}{2}\right)^{|k|}, \; V_k^n = \left(k \cdot \frac{\Delta_0^n}{2} + \frac{C_k^n}{C_0^n} - \frac{m_\varepsilon}{2}\right)^{|k|}, \tag{1.32}$$

$$\hat{\Delta}_0^n = \frac{\hat{\mu}_n}{2} m_\mu + \frac{\omega_0}{k_0 u}\ell, \; \Delta_0^n = \frac{\mu_n}{2} m_\varepsilon + \frac{\omega_0}{k_0 u}\ell, \tag{1.33}$$

$$\hat{\theta}_{\pm 1}^n = \frac{2}{k_0^2 \beta^2 b}\left[\left(\tilde{\chi}_0^n\right)^2 + \tilde{\lambda}_n^2\right]\ell - \frac{4}{k_0^2 b^2}\left(\tilde{\chi}_0^n\right)^2 m_\mu, \tag{1.34}$$

$$\theta_{\pm 1}^n = \frac{2}{k_0^2 \beta^2 b}\left[\left(\chi_0^n\right)^2 + \lambda_n^2\right]\ell - \frac{4}{k_0^2 b^2}\left(\chi_0^n\right)^2 m_\varepsilon, \tag{1.35}$$

$$\left(\hat{P}_0^n\right)^2 = \frac{(\omega_0)^2}{c^2}\varepsilon_0\mu_0 - \hat{\lambda}_n^2, \left(P_0^n\right)^2 = \frac{(\omega_0)^2}{c^2}\varepsilon_0\mu_0 - \lambda_n^2. \tag{1.37}$$

As is seen from the expressions (1.30) and (1.31) TE and TM fields in the waveguide with modulated filling are represented as the set of space-time harmonics with different

amplitudes. At that time the amplitude of the zero (fundamental) harmonic are independent of small modulation indexes, while the amplitudes of the plus and minus first harmonics (side harmonics) are proportional to the small modulation indexes in the first degree.
At the realization the following condition [31], [33]

$$\left|1-\hat{\theta}_0^n\right| \le \hat{\delta}_n, \tag{1.38}$$

where

$$\hat{\delta}_n = \frac{\hat{\eta}_n^2 - \beta_\varepsilon^2}{4\sqrt{2}\beta_\varepsilon^2}\ell_\varepsilon,\ \hat{\eta}_n = \sqrt{1+\frac{4\hat{\lambda}_n^2}{k_0^2 b_\varepsilon}},\ \ell_\varepsilon = \frac{m_\varepsilon \beta_\varepsilon^2}{b_\varepsilon}, \tag{1.39}$$

$$\hat{\theta}_{\varepsilon_0}^n = \frac{4\left(u\sqrt{\frac{(\omega_0)^2}{C^2}\varepsilon_0 - \hat{\lambda}_n^2} - \beta_\varepsilon^2\,\omega_0\right)^2}{k_0^2 u^2 b_\varepsilon^2},\ \beta_\varepsilon = \frac{u}{c}\sqrt{\varepsilon_0}, b_\varepsilon = 1-\beta_\varepsilon^2 \tag{1.40}$$

(here are shown the results for the TE field when $\mu = 1$) the strong (resonance) interaction between the signal wave and the modulation wave takes place, when occurs the considerable energy exchange between them. The analytical expression for the frequency of strong interaction is found in the form

$$\omega_{0,s} - \Delta\omega_0 \le \omega_0 \le \omega_{0,s} + \Delta\omega_0,\ \omega_{0,s} = \frac{uk_0}{2\beta_\varepsilon}\left(\hat{\eta}_n + \beta_\varepsilon\right) \tag{1.41}$$

and is shown that the width of strong interaction is small and proportional to the modulation index in the first degree [31], [33]

$$\Delta\omega_0 = \frac{k_0 u}{8\sqrt{2}\beta_\varepsilon}\frac{\left(1+\hat{\eta}_n\beta_\varepsilon\right)\left(\hat{\eta}_n^2 - \beta_\varepsilon^2\right)}{\hat{\eta}_n}\ell_\varepsilon. \tag{1.42}$$

In the region of strong interaction the dispersion equation (1.25) has complex solutions in the following form

$$\tilde{\mu}_{\varepsilon,n} = 1 \pm i\frac{\sqrt{\left(\hat{\theta}_{\varepsilon 1}^n\right)^2 - \hat{\delta}_n^2}}{2},\quad \hat{\theta}_{\varepsilon,1}^n = 2\sqrt{2}\hat{\delta}_n. \tag{1.43}$$

(1.43) allow to receive the analytical expressions for the amplitudes of different harmonics in the form [31], [33]

$$\left|\hat{V}_{\varepsilon,-1}^n\right| \approx 1,\quad \left|\hat{V}_{\varepsilon,1}^n\right| \approx \frac{\left(\hat{\eta}_n+\beta_\varepsilon\right)\left(\hat{\eta}_n+3\beta_\varepsilon\right)}{16\beta_\varepsilon^2}\ell_\varepsilon. \tag{1.44}$$

The analysis of these expressions shows that in the case of forward modulation, when the directions of propagation of the signal wave and the modulation wave coincide, the

amplitude of minus first harmonic doesn't depend from the modulation index, while the amplitude of the plus first harmonic is proportional to the modulation index in the first degree. In other words in the region of strong interaction besides the fundamental harmonic the substantial role plays the minus first harmonic reflected from the periodic structure of the filling on the frequency

$$\omega_{-1,s} = \omega_{0,s} - k_0 u = \frac{k_0 u}{2\beta_\varepsilon}\left(\hat{\eta}_n - \beta_\varepsilon\right), \quad \hat{\eta}_n > \beta_\varepsilon . \tag{1.45}$$

In the backward modulation case, when the directions of propagation of the signal wave and the modulation wave don't coincide, the minus first and plus first harmonics change their roles.

The results received above admit the visual physical explanation of the effect of strong interaction between the signal wave and the modulation wave. Below the physical explanation we show by example of TE field in the case of forward modulation. The zero harmonic in the modulated filling of the waveguide is incident on the density maxima of the filling at the angle $\hat{\varphi}^n_{\varepsilon,0}$ and is reflected from them at the angle $\hat{\varphi}^n_{\varepsilon,1}$ (Fig.2). These angles are defined from the following correlations [33]

$$\cos\hat{\varphi}^n_{\varepsilon,0} = \frac{c}{\omega_0\sqrt{\varepsilon_0}}\sqrt{\frac{\omega_0^2}{c^2}\varepsilon_0 - \hat{\lambda}_n^2}, \quad \cos\hat{\varphi}^n_{\varepsilon,1} = \frac{\left(1+\beta_\varepsilon^2\right)\cos\hat{\varphi}^n_{\varepsilon_0} - 2\beta_\varepsilon}{1+\beta_\varepsilon^2 - 2\beta_\varepsilon\cos\hat{\varphi}^n_{\varepsilon,0}}. \tag{1.46}$$

At that time the incident and reflection angles are different because of the moving of the modulation wave of the filling and the frequencies of incident and reflected waves satisfy to the following correlation [33]

$$\omega_1 \cdot \sin\hat{\varphi}^n_{\varepsilon,1} = \omega_0 \cdot \sin\hat{\varphi}^n_{\varepsilon,0}. \tag{1.47}$$

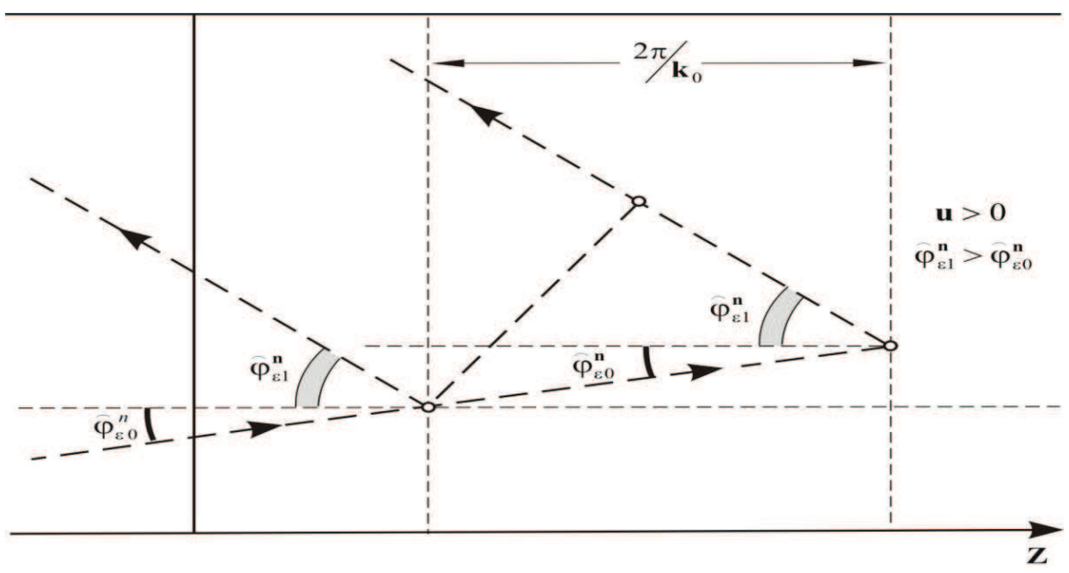


Fig. 2. The physical explanation of the effect of strong interaction.

If now we apply the first-order Wolf-Bragg condition, when the waves reflected from high-density points of the interference pattern are amplified, we obtain the following equation

$$\frac{2\sqrt{\varepsilon_0}\cdot\omega_0\left(\cos\bar{\varphi}^n_{\varepsilon,0}-\beta_\varepsilon\right)}{k_0c\left(1-\beta_\varepsilon^2\right)}=1. \tag{1.48}$$

It is not difficult to note, taking into account (1.46), that the solution of the equation (1.48) precisely coincides with the expression of the frequency of strong interaction (see (1.41)), received above.

3. Propagation of electromagnetic waves in a waveguide with a periodically modulated anisotropic insert

Consider a waveguide of arbitrary cross section with an anisotropic nonmagnetic $(\mu=1)$ modulated insert (modulated uniaxial crystal) the permittivity tensor of which has the form

$$\hat{\varepsilon}=\begin{pmatrix}\varepsilon_1(z,t) & 0 & 0\\ 0 & \varepsilon_1(z,t) & 0\\ 0 & 0 & \varepsilon_2(z,t)\end{pmatrix}, \tag{2.1}$$

where components $\varepsilon_1(z,t)$ and $\varepsilon_2(z,t)$ are modulated by the pumping wave in space and time according to the harmonic law

$$\varepsilon_1(z,t)=\varepsilon_1^0[1+m_1\cos k_0(z-ut)],\quad \varepsilon_2(z,t)=\varepsilon_2^0[1+m_2\cos k_0(z-ut)]. \tag{2.2}$$

Here, $\varepsilon_1{}^0$ and $\varepsilon_2{}^0$ are the permittivities in the absence of a modulating wave; m, and m_2 are the modulation indices; and k_0 and и are, respectively, the wavenumber and velocity of the modulating wave.

Consider the propagation of a signal electromagnetic wave at frequency ω_0 in this waveguide under the assumption that the modulation indices are small $(m_1<<1, m_2<<1, m_1\approx m_2)$. Note that, when the condition $\beta_1\le 0.8$ is satisfied, where $\beta_1=u\sqrt{\varepsilon_1^0}/c$, not only the modulation indices, but also parameter $l_1=m_1\beta_1/\left(1-\beta_1^2\right)$ are small $(l_1<<1)$.

As in my earlier works (see, e.g., [23], [31], [34-37]), transverse electric (TE) and transverse magnetic (TM) waves in the waveguide will be described through the longitudinal components of the magnetic (H_z) and electric (E_z) field. Then, bearing in mind that $D_x=\varepsilon_1(z,t)E_x, D_y=\varepsilon_1(z,t)E_y, D_z=\varepsilon_2(z,t)E_z$ and **B = H** and using the Maxwell equations, we obtain equations for H_z(x, y, z, f) and E,(x, y, z, 0; namely,

for the TE wave

$$\Delta_\perp H_z+\frac{\partial^2 H_z}{\partial z^2}-\frac{1}{c^2}\frac{\partial}{\partial t}\left[\varepsilon_1\frac{\partial H_z}{\partial t}\right]=0\,, \tag{2.3}$$

for the TM wave

$$\Delta_{\perp}\tilde{E}_z + \varepsilon_2 \frac{\partial}{\partial z}\left(\frac{1}{\varepsilon_1}\frac{\partial \tilde{E}_z}{\partial z}\right) - \frac{\varepsilon_2}{c^2}\frac{\partial^2 \tilde{E}_z}{\partial t^2} = 0\,, \tag{2.4}$$

where $\Delta_{\perp}$ is the two-dimensional Laplacian and $\tilde{E}_z = \varepsilon_2 E_z$.

It is easy to check in this case that the transverse components of the TE and TM fields can be expressed in terms of (1.6) as:

for TE wave

$$\vec{H}_{\tau} = \sum_{n=0}^{\infty} \hat{\lambda}_n^{-2} \frac{\partial H_n(z,t)}{\partial z} \nabla \hat{\Psi}_n(x,y), \tag{2.5}$$

$$\vec{E}_{\tau} = \frac{1}{c}\sum_{n=0}^{\infty} \hat{\lambda}_n^{-2} \frac{\partial H_n(z,t)}{\partial t}\left[\vec{z}_0 \nabla \hat{\Psi}_n(x,y)\right], \tag{2.6}$$

for TM wave

$$\vec{H}_{\tau} = -\frac{1}{c}\sum_{n=0}^{\infty} \lambda_n^{-2} \frac{\partial\left[\varepsilon_2 E_n(z,t)\right]}{\partial t}\left[\vec{z}_0 \nabla \Psi_n(x,y)\right], \tag{2.7}$$

$$\vec{E}_{\tau} = \frac{1}{\varepsilon_1}\sum_{n=0}^{\infty} \lambda_n^{-2} \frac{\partial\left[\varepsilon_2 E_n(z,t)\right]}{\partial z} \nabla \Psi_n(x,y)\,. \tag{2.8}$$

Let us introduce the new variables

$$\xi = z - ut,\ \eta = \frac{z}{u} - \frac{1}{u}\int_0^{\xi} \frac{d\xi}{1 - \beta_1^2 \varepsilon_1(\xi)/\varepsilon_1^0} \tag{2.9}$$

into equations (2.3) and (2.4) and seek for solutions to the above equations in the form

$$H_z = \sum_{n=0}^{\infty} e^{i\left(\tilde{P}_0^n u - \omega_0\right)\eta} H_{nz}(\xi)\hat{\Psi}_n(x,y), \tag{2.10}$$

$$\tilde{E}_z = \sum_{n=0}^{\infty} e^{i\left(P_0^n u - \omega_0\right)\eta} E_{nz}(\xi)\Psi_n(x,y)\,. \tag{2.11}$$

Taking into account that functions $\hat{\psi}_n(x,y)$ and $\psi_n(x,y)$ satisfy the Helmholtz equations (1.4) and (1.5), we get ordinary second-order differential equations in variable ξ to find $H_{nz}(\xi)$ and $E_{nz}(\xi)$,

$$\frac{d}{d\xi}\left[\left(1 - \beta_1^2 \frac{\varepsilon_1}{\varepsilon_1^0}\right)\frac{dH_{nz}}{d\xi}\right] + \frac{\tilde{\chi}_n^2}{1 - \beta_1^2 \dfrac{\varepsilon_1}{\varepsilon_1^0}} H_{nz} = 0\,, \tag{2.12}$$

$$\varepsilon_2 \frac{d}{d\xi}\left[\frac{1}{\varepsilon_1}\left(1-\beta_1^2\frac{\varepsilon_1}{\varepsilon_1^0}\right)\frac{d\tilde{E}_{nz}}{d\xi}\right]+\frac{\chi_n^2}{1-\beta_1^2\frac{\varepsilon_1}{\varepsilon_1^0}}\tilde{E}_{nz}=0\,. \tag{2.13}$$

Here,

$$\tilde{\chi}_n^2=\frac{\left(u\tilde{p}_0^n-\omega_0\right)^2}{c^2}\varepsilon_1-\hat{\lambda}_n^2\left(1-\beta_1^2\frac{\varepsilon_1}{\varepsilon_1^0}\right),\ \chi_n^2=\frac{\left(up_0^n-\omega_0\right)^2}{c^2}\varepsilon_1-\lambda_n^2\left(1-\beta_1^2\frac{\varepsilon_1}{\varepsilon_1^0}\right), \tag{2.14}$$

$$\left(\tilde{p}_0^n\right)^2=\frac{\omega_0^2}{c^2}\varepsilon_1^0-\hat{\lambda}_n^2\,,\ \left(p_0^n\right)^2=\frac{\varepsilon_2^0-\varepsilon_1^0\beta_1^2}{\varepsilon_2^0-\varepsilon_1^0\beta_2^2}\frac{\omega_0^2}{c^2}\varepsilon_1^0-\lambda_n^2 b_1\,,\ b_1=1-\beta_1^2\,,\ \beta_2=\frac{u\sqrt{\varepsilon_2^0}}{c}\,. \tag{2.15}$$

In terms of the new variables

$$s=\frac{k_0 b_1}{2}\int_0^{\xi}\frac{d\xi}{1-\beta_1^2\frac{\varepsilon_1}{\varepsilon_1^0}},,\ s=\frac{k_0 b_1}{2\varepsilon_1^0}\int_0^{\xi}\frac{\varepsilon_1 d\xi}{1-\beta_1^2\frac{\varepsilon_1}{\varepsilon_1^0}} \tag{2.16}$$

Equations (2.12) and (2.13) take the form of the Mathieu-Hill equations

$$\frac{d^2H_{nz}}{d\hat{s}^2}+\frac{4\tilde{\chi}_n^2}{k_0^2b_1^2}H_{nz}=0\,,\ \frac{d^2\tilde{E}_{nz}}{ds^2}+\frac{4\chi_n^2\left(\varepsilon_1^0\right)^2}{k_0^2b_1^2\varepsilon_1\varepsilon_2}\tilde{E}_{nz}=0\,. \tag{2.17}$$

Note that the frequency domain described by the conditions

$$\left|1-\hat{\theta}_0^n\right|>>\hat{\delta}_n\cong\frac{\hat{\theta}_1^n}{2\sqrt{2}}\,,\ \left|1-\theta_0^n\right|>>\delta_n\cong\frac{\theta_1^n}{2\sqrt{2}}\,, \tag{2.18}$$

where

$$\hat{\theta}_0^n=\frac{4}{k_0^2b_1^2}\left(\hat{p}_0^n-\frac{\omega_0\beta_1^2}{u}\right)^2,\ \theta_0^n=\frac{4}{k_0^2b_1^2}\left(p_0^n-\frac{\omega_0\beta_1^2}{u}\right)^2, \tag{2.19}$$

$$\hat{\theta}_1^n=\hat{\theta}_{-1}^n=\frac{2}{k_0^2b_1^2}\left[\frac{\left(u\tilde{p}_0^n-\omega_0\right)^2}{u^2}+\frac{k_0^2b_1^2}{4}\hat{\theta}_0^n\right]\ell_1\,,\ \theta_1^n=\theta_{-1}^n=\frac{2\varepsilon_1^0\lambda_n^2}{k_0^2b_1^2\varepsilon_2^0}m_2-\frac{\theta_0^n}{2}m_1\,, \tag{2.20}$$

is the domain of weak interaction between the signal wave and the wave that modulates the insert. Solving (2.17) by the method developed in [23], [31], [34-37] and discarding the terms proportional to the modulation indices in the first power, we obtain the following expressions for the TE and TM field in the frequency domain defined by formulas (2.18): [38]

for the TE wave

$$H_z = \sum_{n=0}^{\infty} \hat{\Psi}_n(x,y) e^{i\left(\hat{P}_0^n z - \omega_0 t\right)} \hat{c}_0^n \sum_{k=-1}^{1} \hat{V}_k^n e^{ikk_0(z-ut)}, \tag{2.21}$$

where

$$\hat{V}_k^n = \left(k \frac{\omega_0}{2uk_0} \ell_1 + \frac{\hat{c}_k^n}{\hat{c}_0^n} \right)^{|k|}, \quad \hat{c}_{\pm 1}^n = \frac{\hat{\theta}_1^n \hat{c}_0^n}{4\left(1 \pm \sqrt{\hat{\theta}_0^n}\right)}, \tag{2.22}$$

for the TM wave

$$E_z = \sum_{n=0}^{\infty} \Psi_n(x,y) e^{i\left(P_0^n z - \omega_0 t\right)} c_0^n \sum_{k=-1}^{1} V_k^n e^{ikk_0(z-ut)}, \tag{2.23}$$

where

$$V_k^n = \left[k \frac{\omega_0}{2uk_0} \ell_1 + \frac{c_k^n}{c_0^n} - \frac{1}{2}(m_1 + m_2) \right]^{|k|}, \quad c_{\pm 1}^n = \frac{\theta_1^n c_0^n}{4\left(1 \pm \sqrt{\theta_0^n}\right)}. \tag{2.24}$$

Note that quantities $\hat{c}_0^n$ and c_0^n in (2.21) and (2.23) are found from the normalization condition. As follows from (2.21) and (2.23), when an electromagnetic wave propagates in a waveguide with an insert harmonically modulated in space and time, the TE and TM fields represent a superposition of space-time harmonics of different amplitudes. In the domain of weak interaction between the signal and modulation waves, the amplitudes of harmonics +1 and -1 prove to be small (they are linearly related to the modulation indices) compared with the amplitude of the fundamental harmonic (which is independent of modulation indices).
It is known [21] that, when $\hat{\theta}_0^n$ and θ_0^n tend to unity, i.e., when the conditions

$$\left|1 - \hat{\theta}_0^n\right| \le \hat{\delta}_n, \quad \left|1 - \theta_0^n\right| \le \delta_n \tag{2.25}$$

are satisfied, the signal wave and the wave that modulates the insert strongly interact (the first-order Bragg condition for waves reflected from a high-density area is met) and vigorously exchange energy.
Condition (2.25) can be recast (for the TM field) as

$$\omega_{0,s} - \Delta\omega_0 \le \omega_0 \le \omega_{0,s} + \Delta\omega_0, \tag{2.26}$$

where $\omega_{0,s}$ given by

$$\omega_{0,s} = \frac{uk_0}{2\beta_1}\left(\beta_1 + \bar{\eta}_n\right), \bar{\eta}_n = \sqrt{1 + \frac{4\bar{\lambda}_n^2}{b_1 k_0^2}}, \bar{\lambda}_n^2 = \frac{\varepsilon_1^0}{\varepsilon_2^0}\lambda_n^2, \tag{2.27}$$

is the frequency near which the strong interaction takes place, and

$$\Delta\omega_0 = \frac{k_0 u\left(1+\beta_1\bar{\eta}_n\right)}{8\sqrt{2}\beta_1\bar{\eta}_n}\theta_1^n \tag{2.28}$$

is the width of the domain of strong interaction. Calculations show that

$$\left|V_{-1}^n\right| \sim 1\,,\ \left|V_1^n\right| \sim m_1, m_2 \tag{2.29}$$

in frequency domain (2.26). From relationships (2.29), it follows that the amplitude of reflected harmonic -1 is independent of modulation indices in the domain where the signal wave and the wave that modulates the anisotropic insert strongly interact. In other words, not only the zeroth harmonic of the signal, but also reflected harmonic -1 of frequency

$$\omega_{-1,s} = \frac{uk_0}{2\beta_1}\left(\bar{\eta}_n - \beta_1\right), \left(\bar{\eta}_n > \beta_1\right) \tag{2.30}$$

plays a significant role in this domain.

Note in conclusion that the results obtained here turn into those reported in [37] in the limit $m_1 \to 0$; in the limit $u \to 0$, one arrives at results for a waveguide with an inhomogeneous but stationary anisotropic insert.

4. Interaction of electromagnetic waves with space-time periodic anisotropic magneto-dielectric filling of a waveguide

Let the axis of a regular waveguide of an arbitrary cross section coincides with the OZ axis of a Certain Cartesian coordinate frame. Assume that the waveguide is filled with a periodically modulated anisotropic magneto- dielectric filling whose tensor permittivity and permeability are specified by the formulas

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_2\left(z,t\right) \end{pmatrix},\ \hat{\mu} = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_2\left(z,t\right) \end{pmatrix}. \tag{3.1}$$

In (3.1) $\varepsilon_1 = const, \mu_1 = const$ and the $\varepsilon_2\left(z,t\right)$ and $\mu_2\left(z,t\right)$ components are harmonic functions in space and time:

$$\varepsilon_2\left(z,t\right) = \varepsilon_2^0\left[1 + m_\varepsilon \cos\left(k_0 z - k_0 u t\right)\right], \tag{3.2}$$

$$\mu_2\left(z,t\right) = \mu_2^0\left[1 + m_\mu \cos\left(k_0 z - k_0 u t\right)\right], \tag{3.3}$$

where $m_\varepsilon << 1$ and $m_\mu << 1$ are small modulation indexes, $\varepsilon_2^0 = const$ and $\mu_2^0 = const$ are, respectively, the permittivity and permeability of the filling in the absence of a modulation wave.

Let a signal wave unit amplitude with frequency ω_0 propagates in such a waveguide in a positive Direction of an axis *OZ*. After some algebra, the wave equations for the longitudinal components $H_z\left(x,y,z,t\right)$ and $E_z\left(x,y,z,t\right)$ of TE and TM fields can be obtained from Maxwell equations

$$curl\vec{H} = \frac{\partial \vec{D}}{\partial t},\ curl\vec{E} = -\frac{\partial \vec{B}}{\partial t},\ \ div\vec{D} = 0,\ div\vec{B} = 0, \tag{3.4}$$

$$\vec{D} = \varepsilon_0 \hat{\varepsilon}\vec{E},\ \vec{B} = \mu_0 \hat{\mu}\vec{H},\ \varepsilon_0 = \left(1/4\pi \cdot 9 \cdot 10^9\right) F/m,\ \mu_0 = 4\pi \cdot 10^{-7}\, H/m \tag{3.5}$$

with allowance for the equalities

$$D_x = \varepsilon_0 \varepsilon_1 E_x,\ D_y = \varepsilon_0 \varepsilon_1 E_y,\ D_z = \varepsilon_0 \varepsilon_2 (z,t) E_z, \tag{3.6}$$

$$B_x = \mu_0 \mu_1 H_x,\ B_y = \mu_0 \mu_1 H_y,\ B_z = \mu_0 \mu_2 (z,t) H_z. \tag{3.7}$$

We arrive at the following equations:

for TE waves

$$\Delta_\perp \tilde{H}_z + \frac{\mu_2(z,t)}{\mu_1}\frac{\partial^2 \tilde{H}_z}{\partial z^2} = \varepsilon_0 \mu_0 \varepsilon_1 \mu_2 (z,t)\frac{\partial^2 \tilde{H}_z}{\partial t^2}, \tag{3.8}$$

for the TM waves

$$\Delta_\perp \tilde{E}_z + \frac{\varepsilon_2(z,t)}{\varepsilon_1}\frac{\partial^2 \tilde{E}_z}{\partial z^2} = \varepsilon_0 \mu_0 \mu_1 \varepsilon_2 (z,t)\frac{\partial^2 \tilde{E}_z}{\partial t^2}, \tag{3.9}$$

where

$$\tilde{H}_z = \mu_2 (z,t) H_z,\ \ \tilde{E}_z = \varepsilon_2 (z,t) E_z. \tag{3.10}$$

With the use of Maxwell equations (3.4) and (3.5), the transverse components of TE and TM fields can be represented in terms of

$$H_z = \sum_{n=0}^{\infty} H_n (z,t)\hat{\Psi}(x,y),\ E_z = \sum_{n=0}^{\infty} E_n (z,t)\Psi(x,y) \tag{3.11}$$

as follows:

for TE waves

$$\vec{H}_\tau = \frac{1}{\mu_1}\sum_{n=0}^{\infty} \hat{\lambda}_n^{-2}\frac{\partial\left[\mu_2 (z,t) H_n (z,t)\right]}{\partial z}\nabla\hat{\Psi}_n (x,y), \tag{3.12}$$

$$\vec{E}_\tau = \mu_0 \sum_{n=0}^{\infty} \hat{\lambda}_n^{-2}\frac{\partial\left[\mu_2 (z,t) H_n (z,t)\right]}{\partial t}\left[\vec{z}_0 \nabla\hat{\Psi}_n (x,y)\right], \tag{3.13}$$

for TM waves

$$\vec{H}_\tau = -\varepsilon_0 \sum_{n=0}^{\infty} \lambda_n^{-2}\frac{\partial\left[\varepsilon_2 (z,t) E_n (z,t)\right]}{\partial t}\left[\vec{z}_0 \nabla\Psi_n (x,y)\right], \tag{3.14}$$

$$\vec{E}_\tau = \frac{1}{\varepsilon_1}\sum_{n=0}^{\infty}\lambda_n^{-2}\frac{\partial\left[\varepsilon_2(z,t)E_n(z,t)\right]}{\partial z}\nabla\Psi_n(x,y)\,. \tag{3.15}$$

With the new variables

$$\xi = z - ut,\quad \eta = \frac{z}{u} - \frac{1}{u}\frac{z-ut}{1-\beta^2} \tag{3.16}$$

where $\beta^2 = u^2\varepsilon_0\mu_0\varepsilon_1\mu_1$, wave equations (3.8) and (3.9) can be modified to obtain

$$\Delta_\perp\tilde{H}_z + \frac{\mu_2(z,t)}{\mu_1}\left(1-\beta^2\right)\frac{\partial^2\tilde{H}_z}{\partial\xi^2} - \frac{\varepsilon_0\mu_0\varepsilon_1\mu_2(z,t)}{1-\beta^2}\frac{\partial^2\tilde{H}_z}{\partial\eta^2} = 0\,, \tag{3.17}$$

$$\Delta_\perp\tilde{E}_z + \frac{\varepsilon_2(z,t)}{\varepsilon_1}\left(1-\beta^2\right)\frac{\partial^2\tilde{E}_z}{\partial\xi^2} - \frac{\varepsilon_0\mu_0\mu_1\varepsilon_2(z,t)}{1-\beta^2}\frac{\partial^2\tilde{E}_z}{\partial\eta^2} = 0\,. \tag{3.18}$$

Let us seek solutions of equations (3.17) and (3.18) in the form (1.13). Then, taking into account (1.4) and (1.5), we obtain for $\tilde{H}_{nz}(\xi)$ and $\tilde{E}_{nz}(\xi)$ the following second-order ordinary differential equations with the periodic Mathieu-Hill coefficients:

$$\frac{d^2\tilde{H}_{nz}(\xi)}{d\xi^2} + \frac{\mu_1}{\mu_2(z,t)\left(1-\beta^2\right)}\left[\frac{\varepsilon_0\mu_0\varepsilon_1\mu_2(z,t)}{1-\beta^2}\gamma^2 - \hat{\lambda}_n^{\,2}\right]\tilde{H}_{nz}(\xi) = 0\,, \tag{3.19}$$

$$\frac{d^2\tilde{E}_{nz}(\xi)}{d\xi^2} + \frac{\varepsilon_1}{\varepsilon_2(z,t)\left(1-\beta^2\right)}\left[\frac{\varepsilon_0\mu_0\mu_1\varepsilon_2(z,t)}{1-\beta^2}\gamma^2 - \lambda_n^{\,2}\right]\tilde{E}_{nz}(\xi) = 0\,. \tag{3.20}$$

With the new variable $\zeta = k_0\xi/2$ equations (3.19) and (3.20) can be modified into the form

$$\frac{d^2\tilde{H}_{nz}(\zeta)}{d\zeta^2} + \sum_{k=-1}^{1}\hat{\theta}_k^n\exp(2ik\zeta)\tilde{H}_{nz}(\zeta) = 0\,, \tag{3.21}$$

$$\frac{d^2\tilde{E}_{nz}(\zeta)}{d\zeta^2} + \sum_{k=-1}^{1}\theta_k^n\exp(2ik\zeta)\tilde{E}_{nz}(\zeta) = 0\,, \tag{3.22}$$

where quantities $\hat{\theta}_k^n$ and θ_k^n are the coefficients of the Fourier decompositions of the expressions that appear before functions $\tilde{H}_{nz}(\zeta)$ and $\tilde{E}_{nz}(\zeta)$ entering equations (3.19) and (3.20). In the first approximation for small parameters m_ε and m_μ these coefficients are expressed according to the formulas

$$\hat{\theta}_0^n = \frac{4\mu_1}{b^2\mu_2^0k_0^2}\left(\mu_2^0\varepsilon_0\mu_0\varepsilon_1\gamma^2 - \hat{\lambda}_n^2 b\right),\ \hat{\theta}_{\pm1}^n = \frac{2\mu_1\hat{\lambda}_n^2}{b\mu_2^0k_0^2}m_\mu\,, \tag{3.23}$$

$$\theta_0^n = \frac{4\varepsilon_1}{b^2\varepsilon_2^0 k_0^2}\left(\varepsilon_2^0\varepsilon_0\mu_0\mu_1\gamma^2 - \lambda_n^2 b\right),\ \theta_{\pm1}^n = \frac{2\varepsilon_1\lambda_n^2}{b\varepsilon_2^0 k_0^2}m_\varepsilon,\ b = 1-\beta^2. \tag{3.24}$$

We seek solutions to equations (3.21) and (3.22) in the form

$$\tilde{H}_{nz}(\zeta) = e^{i\hat{\mu}_n\zeta}\sum_{k=-1}^{1}\hat{C}_k^n e^{2ik\zeta},\ \tilde{E}_{nz}(\zeta) = e^{i\mu_n\zeta}\sum_{k=-1}^{1}C_k^n e^{2ik\zeta}. \tag{3.25}$$

It is known [33] that, under the conditions (1.27), which provide for weak interaction between the signal wave and the wave of the waveguide-filling modulation, quantities $\hat{\mu}_n, \mu_n, \hat{C}_{\pm1}^n$ and $C_{\pm1}^n$ have the form (1.28) and (1.29) (accurate to within small parameters m_ε and m_μ inclusively). Taking into account (3.25), (1.28), (1.29) and changing to variables z and t, we obtain from (3.10) analytic expressions for H_z and E_z of TE and TM waves. These expressions correspond to the first approximation for m_ε and m_μ, are valid in the region of weak interaction between the signal wave and the wave of the waveguide-filling modulation, and have the form [39]

$$H_z = \frac{1}{\mu_2^0}\sum_{n=0}^{\infty}\hat{\Psi}_n(x,y)e^{i\left(\hat{P}_0^n z-\omega_0 t\right)}\hat{C}_0^n\sum_{k=-1}^{1}\hat{V}_k^n e^{ikk_0(z-ut)}, \tag{3.25}$$

$$E_z = \frac{1}{\varepsilon_2^0}\sum_{n=0}^{\infty}\Psi_n(x,y)e^{i\left(P_0^n z-\omega_0 t\right)}C_0^n\sum_{k=-1}^{1}V_k^n e^{ikk_0(z-ut)}, \tag{3.26}$$

where

$$\hat{V}_k^n = \left(\frac{\hat{C}_k^n}{\hat{C}_0^n} - \frac{m_\mu}{2}\right)^{|k|},\ V_k^n = \left(\frac{C_k^n}{C_0^n} - \frac{m_\varepsilon}{2}\right)^{|k|},\ \hat{\theta}_0^n = \frac{4}{k_0^2 b^2}\left(\sqrt{\frac{\beta^2}{u^2}\omega_0^2 - \frac{\mu_1}{\mu_2^0}\hat{\lambda}_n^2} - \frac{\beta^2\omega_0}{u}\right)^2, \tag{3.27}$$

$$\theta_0^n = \frac{4}{k_0^2 b^2}\left(\sqrt{\frac{\beta^2}{u^2}\omega_0^2 - \frac{\varepsilon_1}{\varepsilon_2^0}\lambda_n^2} - \frac{\beta^2\omega_0}{u}\right)^2$$

$$\hat{P}_0^n = \sqrt{\frac{\beta^2}{u^2}\omega_0^2 - \frac{\mu_1}{\mu_2^0}\hat{\lambda}_n^2},\qquad P_0^n = \sqrt{\frac{\beta^2}{u^2}\omega_0^2 - \frac{\varepsilon_1}{\varepsilon_2^0}\lambda_n^2}. \tag{3.28}$$

Note, that for the frequency and frequency width of the strong interaction region (see [31], [33]) the following expressions can easily be obtained from (2.25):

for TE waves

$$\omega_{0,c} = \frac{k_0 u}{2\beta}\left(\beta + \tilde{\hat{\eta}}_n\right),\ \tilde{\hat{\eta}}_n = \sqrt{1 + \frac{4\mu_1\hat{\lambda}_n^2}{\mu_2^0 b k_0^2}},\ \Delta\omega_{0,c} = \frac{k_0 u\left(1+\beta\hat{\eta}_n\right)}{4\beta\hat{\eta}_n}\hat{\delta}_n, \tag{3.29}$$

for TM waves

$$\omega_{0,c} = \frac{k_0 u}{2\beta}\left(\beta + \tilde{\eta}_n\right),\ \tilde{\eta}_n = \sqrt{1 + \frac{4\varepsilon_1\lambda_n^2}{\varepsilon_2^0 b k_0^2}},\ \Delta\omega_{0,c} = \frac{k_0 u\left(1+\beta\eta_n\right)}{4\beta\eta_n}\delta_n. \tag{3.30}$$

For the quantities $\widehat{V}_{\pm 1}$ and $V_{\pm 1}$ in this case we obtain

$$\left|\widehat{V}_{-1}^{n}\right| \cong 1\,,\ \left|\widehat{V}_{1}^{n}\right| \cong \frac{\mu_1 \widehat{\lambda}_n^2 - 2b\mu_2^0 k_0^2}{4b\mu_2^0 k_0^2} m_\mu\,, \tag{3.31}$$

$$\left|V_{-1}^{n}\right| \cong 1\,,\ \left|V_{1}^{n}\right| \cong \frac{\varepsilon_1 \lambda_n^2 - 2b\varepsilon_2^0 k_0^2}{4b\varepsilon_2^0 k_0^2} m_\varepsilon\,. \tag{3.32}$$

According to (3.31) and (3.32), in the strong- interaction region a substantial role is played not only by the fundamental harmonic but also by the reflected minus-first harmonic that exists at the frequency:

for TE waves

$$\omega_{-1} = \frac{k_0 u}{2\beta}\left(\tilde{\tilde{\eta}}_n - \beta\right),\ \tilde{\tilde{\eta}}_n > \beta\,, \tag{3.33}$$

for TM waves

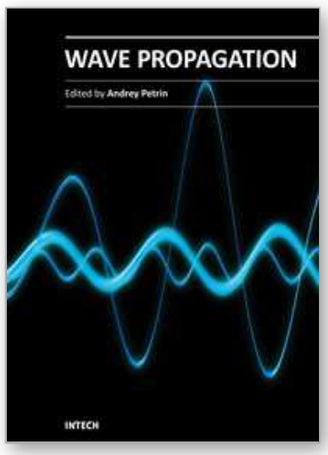
$$\omega_{-1} = \frac{k_0 u}{2\beta}\left(\tilde{\eta}_n - \beta\right),\ \tilde{\eta}_n > \beta\,. \tag{3.34}$$

Note that, in limiting case $u \to 0$ the above obtained relationships yield results for the stationary inhomogeneous anisotropic magneto-dielectric filling of a waveguide.

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