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Correlations between Hydraulic Conductivity and Selected Hydrogeological Properties of Rocks

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1. Introduction

Solving problems related to the occurrence, accumulation, discharge and flow of groundwaters requires knowledge of many rock properties. Their determination is laborious and costly. Therefore, for a long time scientists have been looking for and formulating relationships between particular parameters. Literature contains plenty of relations representing mutual correlations between different hydrogeological properties of rocks. This pertains in particular to determination of hydraulic conductivity (Bear, 1972; Arya et al., 1999; Kasenow, 2002). Formulae obtained empirically are the commonest, but there are also those based on an adopted soil model. In this case the range of application is usually broader, and application limitations are directly connected with the application range of the model.

The discussion presented in this chapter concerns the relationship between hydraulic conductivity and selected hydrogeological properties of rocks, based on a rock model in the shape of a bundle of tortuous capillaries, known in literature (Carman, 1956). The properties which have the paramount importance for this model are specific surface area and porosity. As specific surface area is very often determined based on gradation analysis, these issues have received more detailed attention. A broader discussion of these issues is also connected with a clear formulation of all the assumption and simplifications comprised in the used formulae.

The best known relation based on a rock model in the form of a bundle of tortuous capillaries is a formula known as Kozeny-Carman-equation (Olsen, 1960; Liszkowska, 1996; Mauran et al. 2001; Chapuis & Aubertin, 2003; Carrier, 2003). However, based on this model, one may look for correlations with other hydrogeological parameters too (Petersen et al. 1996).

This chapter presents the results of such investigations in relation to most parameters used in hydrogeological calculations. In particular, this refers to mutual relations between hydraulic conductivity, specific surface area, effective grain diameter, effective capillary diameter, specific yield, specific retention, porosity and capillary rise height. One should emphasize that a satisfactory attempt to present such relationships could contribute to a significant reduction in the range of necessary analyses connected with soil identification.

An important element of the described research is verification of theoretically determined correlations between different parameters. Therefore, the results of experimental

examinations will be presented and then compared with calculation results obtained from the derived relations.

2. Theoretical correlations between hydrogeological properties of rocks

Theoretical correlations between various properties of rocks always refer to a particular rock model. Therefore, it should be strictly defined. The model adopted in this work, presenting rock as a bundle of capillaries, is well-known and used to determine hydraulic conductivity. However, as its range of usefulness has been extended to include a possibility to define other rock properties, it will be discussed here more broadly.

2.1 Rock model

The discussion will cover a model presenting rock as undeformable material containing a bundle of tortuous capillaries with identical cross-sections. (Fig. 1).

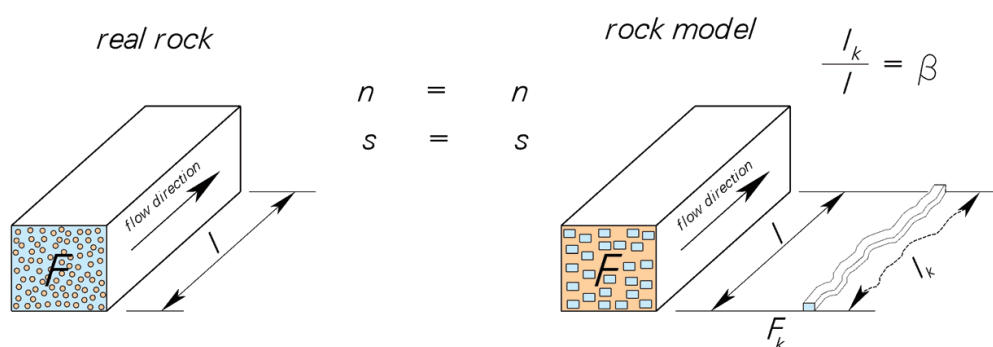


Fig. 1. Model of rock as a bundle of tortuous capillaries (s – specific surface area, n – porosity).

Capillaries are arranged in such a way that the line joining their beginnings and endings is parallel to a potential direction of a fluid flow in real soil. The horizontal cross-section of capillaries will be normally adopted as circle-shaped, although in some considerations concerning determination of hydraulic conductivity, any other shape can be adopted. Capillary tortuosity is characterised by the ratio of the length of a capillary along its axis to the length along a straight line between its beginning and ending. One can adopt different values of capillary tortuosity along different directions, thus allowing for anisotropic properties of soils.

The characteristic feature of the discussed rock model is the fact that its specific surface area s and porosity n are the same as specific surface area and porosity of real rock.

2.2 Specific surface area

Specific surface area is a very important parameter, on which the structure of the adopted rock model is based. For the needs of hydrogeology, it is very often determined based on soil gradation analysis, especially sieve analysis. Spherical grain shape is usually adopted then. If grain shape is more complex, specific surface area can be determined more precisely by considering three dimensions of the grain, i.e. the largest, the smallest and medium. These dimensions can be obtained by analysing grain shape in a small, randomly chosen sample or subjecting it to laser analysis using devices produced specially for this purpose. Hence, further considerations concerning determination of specific surface area based on

soil gradation analysis will be covered more extensively, with the assumption that we have more detailed knowledge of the shape of rock-building grains. Specific surface area s will refer to the ratio between the total area of grains and particles ΣF_z (the boundary surface of grain skeleton) contained in total rock volume V to this volume. For rock built of grains with identical sizes and shapes, specific surface area is

$$s = \frac{\sum F_z}{V} = \frac{N F_z}{V} = \frac{\frac{V_s}{V_z} F_z}{V} = \frac{V(1-n)}{V_z} \frac{F_z}{V} = \frac{(1-n)F_z}{V_z} = A \frac{(1-n)}{b} \tag{1}$$

where: N – number of grains in total rock volume V , F_z – area of a an individual grain, V_s – volume of grain skeleton in volume V , V_z – volume of an individual grain, n – porosity, A – grain shape factor, b – largest grain dimension. Shape factor can be determined from the boundary surface F_z of a grain, its volume V_z and the largest dimension b .

$$A = b \frac{F_z}{V_z} \tag{2}$$

For example, if grains have the shape of a sphere with diameter d , then $b = d$ and $A = 6$. The value of grain shape factor is also 6 if a grain has the shape of a cube with side b or a cylinder with base diameter b and height b . However, if grains are elongated or oblate, value A clearly rises (Fig. 2).

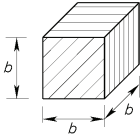
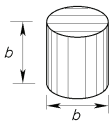
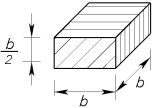
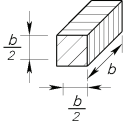
<i>sphere with diameter $b = d$</i>		$A = 6$
<i>cube with edge b</i>		$A = 6$
<i>cylinder with diameter b and height b</i>		$A = 6$
<i>cuboid with sides $b, b, b/2$</i>		$A = 8$
<i>cuboid with sides $b, b/2, b/2$</i>		$A = 10$

Fig. 2. Shape factor A for different grain shapes. Grains in sedimentary rocks are normally rounded. One can often assume that they have the shape of a sphere, a spheroid or another smooth solid. Fig. 3 illustrates shape factor relation

for different ellipsoid dimensions a and c , where c – the smallest dimension and b – the largest dimension.

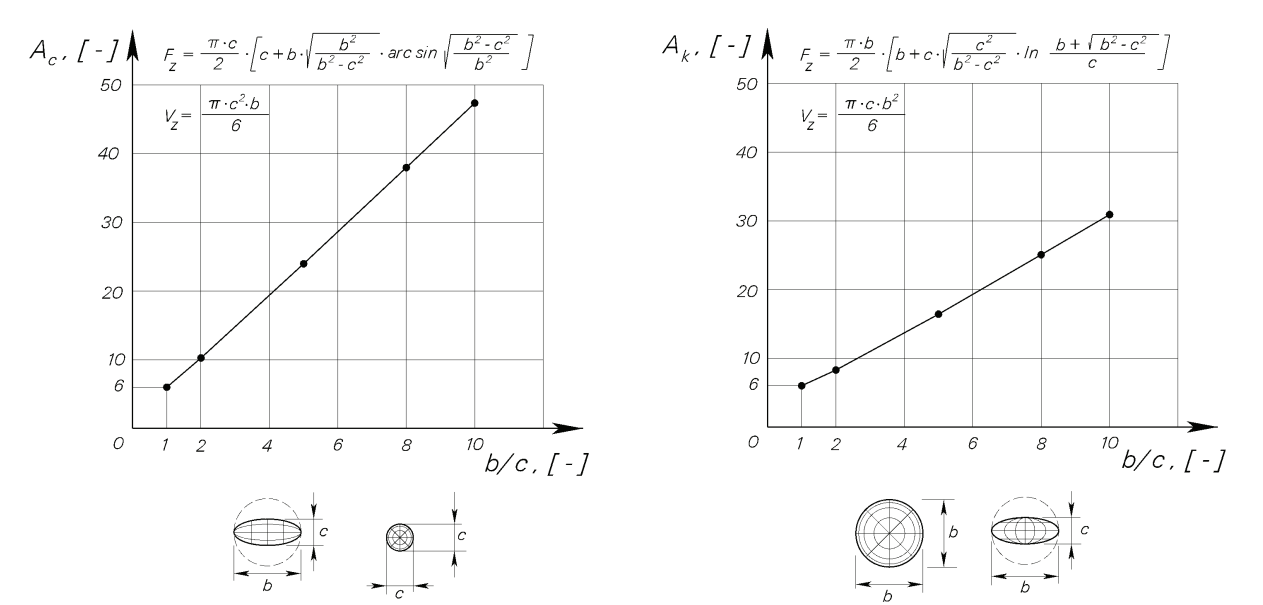


Fig. 3. Shape factor of a cigar-shaped (A_c) and disc-shaped (A_k) spheroid in relation to the ratio of its biggest to smallest dimension (b/c).

If a grain is rounded (its surface is smooth) and has three different dimensions a , b and c , then its volume V_z can be adopted as:

$$V_z = \frac{\pi a b c}{6} \tag{3}$$

The value of shape factor A for such shapes can be calculated with the assumption that A changes linearly from value A_k for a disc to value A_c for a cigar, with mean size a ranging from c to b . Then the shape factor is:

$$A = A_k + \frac{A_c - A_k}{\frac{b}{c} - 1} \left(\frac{b}{a} - 1 \right) \tag{4}$$

Changes in shape factor are illustrated by Fig. 4.

For a soil type with varied grain sizes, determination of specific surface area s depends on the chosen method of gradation analysis. In the case of measuring the mass of different fractions building a rock sample, specific surface area can be calculated from the formula:

$$s = \frac{\sum F_z}{V} = \frac{\sum (N_i F_{zi})}{V} = \frac{\sum \left(\frac{m_{si}}{\rho_{si} V_{zi}} F_{zi} \right)}{\frac{m_s}{\rho_s (1-n)}} = \rho_s (1-n) \sum_i \left(\frac{g_i A_i}{\rho_{si} b_{ei}} \right) = \sum_i s_i \tag{5}$$

where: N_i – number of i -sized grains, F_{zi} – area of an average i -sized grain, ρ_{si} – density of i -fraction soil skeleton, m_{si} – mass of i -fraction soil, V_{zi} – volume of an average i -sized grain, m_s

– total mass of soil used for sieve analysis, ρ_s – density of the soil skeleton of all the sample, A_i – shape factor of a typical i -sized grain, b_{ei} – the largest dimension of effective grain size of i -fraction, $g_i = m_{si}/m_s$ – mass proportion of i -fraction, s_i – specific surface area component resulting from the proportion of i -fraction, $g_i = m_{si}/m_s$ – proportion of i -fraction.

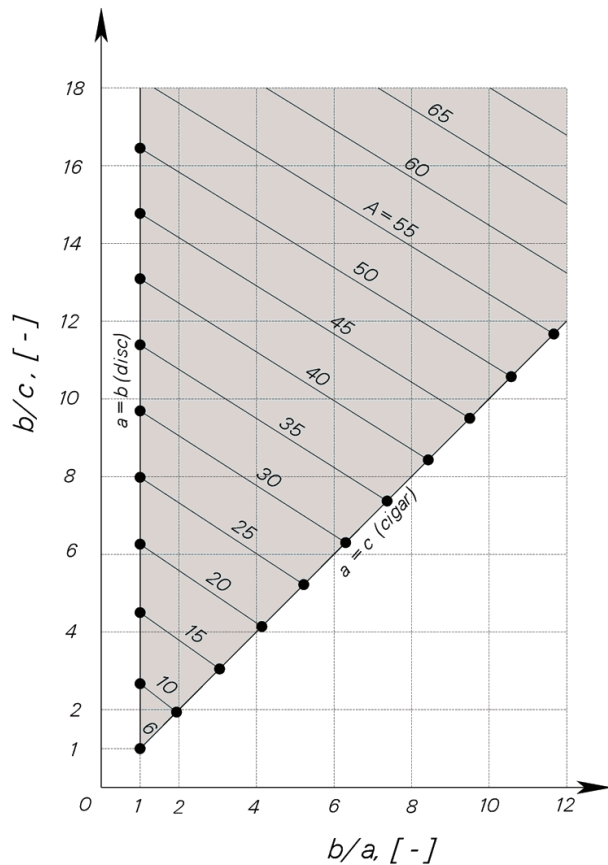


Fig. 4. Shape factor values for rounded grains, according to dimensions (a – medium, b – the largest and c – the smallest).

For a soil with spherical grains, whose soil skeleton density does not change with grain size, specific surface area, according to the above formula, will be

$$s = 6 (1 - n) \sum_i \frac{g_i}{d_{ei}} \tag{6}$$

where d_{ei} denotes the effective diameter of i - size. Another method of determining specific surface area is connected with determining frequency f_i of grain occurrence in a soil sample with specific dimensions (the largest b_i , the smallest c_i and medium a_i). The occurrence frequency is normally expressed in %.

$$f_i = \frac{N_i}{\sum_i N_i} 100 = \frac{N_i}{N} 100 \tag{7}$$

where: N_i – number of grains with specific size i ,
 N – number of all grains in the tested sample.

Given grain dimensions and occurrence frequency, specific surface area can be determined from the formula:

$$s = \frac{\sum F_z}{V} = \frac{\sum_i (N_i F_{zi})}{V} = \frac{\sum_i \left(\frac{f_i N}{100} F_{zi} \right)}{\frac{\sum_i m_i}{\rho_s (1-n)}} = \frac{\rho_s (1-n) \frac{N}{100} \sum_i (f_i F_{zi})}{\sum_i (N_i V_{zi} \rho_{si})} = \frac{\rho_s (1-n) \sum_i \left(f_i \frac{V_{zi} A_i}{b_i} \right)}{\sum_i (f_i V_{zi} \rho_{si})} \quad (8)$$

In the case of a mixture of spherical grains with identical thickness and different diameters d_i , the specific surface area is:

$$s = 6 (1-n) \frac{\sum_i (f_i d_i^2)}{\sum_i (f_i d_i^3)} \quad (9)$$

Analysing formulae (6) and (9), one can see that if soil is composed of spherical grains with identical diameters, both formulae assume the same form.

2.3 Effective grain size

Effective grain size b_e will be defined as the largest grain dimension in imaginary material built of grains with identical sizes and shapes, which has the same specific surface area and the same porosity as real soil. The grain shape in this material is the same as that of a typical grain in real soil.

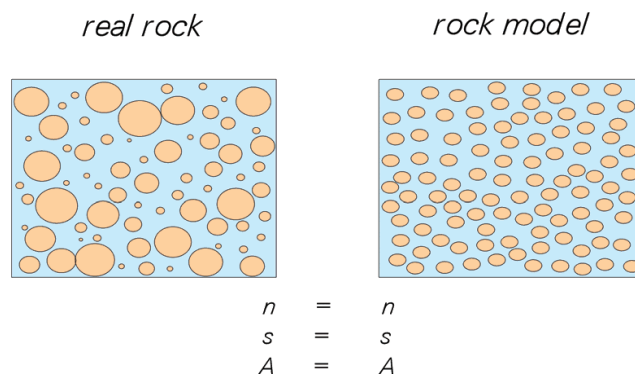


Fig. 5. Modelling of real rock as material composed of grains with identical shape and size (n – porosity, s – specific surface area, A – shape factor).

Comparing the specific surface area of well-graded real soil with that of its counterpart with uniform-size grains, one can define the effective grain size. When determining the mass of various sizes building a rock sample, we obtain:

$$\rho_s (1-n) \sum_i \left(\frac{g_i A_i}{\rho_{si} b_{ei}} \right) = \frac{A_e (1-n)}{b_e} \quad (10)$$

Assuming that

$$A_e = \sum_i g_i A_i \quad (11)$$

one can define effective grain size b_e .

$$b_e = \frac{\sum_i g_i A_i}{\rho_s \sum_i \left(\frac{g_i A_i}{\rho_{si} b_{ei}} \right)} \quad (12)$$

If all grains in a rock sample, regardless of their size, have similar shapes i.e. $A_1 = A_2 = \dots = A_e$ and the density of soil skeleton does not change either ($\rho_{s1} = \rho_{s2} = \dots = \rho_s$), then the effective grain size is

$$b_e = \frac{1}{\sum_i \frac{g_i}{b_{ei}}} \quad (13)$$

Certainly, for a soil whose grains are spherical, b_{ei} values are equal to the diameters of particular grains sizes d_{ei} , and b_e is equal to effective grain diameter d_e . Hence we get the commonly used relation

$$d_e = \frac{1}{\sum_i \frac{g_i}{d_{ei}}} \quad (14)$$

When it comes to determining the frequency f of grain occurrence in a soil sample with defined dimensions, what we get when comparing the specific surface area of well-graded rock with that of rock with uniform grade size is:

$$\frac{(1-n) \sum_i \left(f_i \frac{V_{zi} A_i}{b_i} \right)}{\sum_i (f_i V_{zi})} = \frac{A_e (1-n)}{b_e} \quad (15)$$

Assuming in this case that

$$A_e = \sum_i \left(\frac{f_i}{100} A_i \right) \quad (16)$$

the effective grain size will be

$$b_e = \frac{\sum_i \left(\frac{f_i}{100} A_i \right) \sum_i (f_i V_{zi})}{\sum_i \left(f_i \frac{V_{zi} A_i}{b_i} \right)} \quad (17)$$

If the shape of a grain does not depend on its size, i.e. $A_1 = A_2 = A_3 = \dots = A_N = A_e$, then

$$b_e = \frac{\sum_i (f_i V_{zi})}{\sum_i \left(f_i \frac{V_{zi}}{b_i} \right)} \quad (18)$$

For rounded grains, whose grain volume could be adopted in accordance with formula (3), we get:

$$b_e = \frac{\sum_i f_i a_i b_i c_i}{\sum_i f_i a_i c_i} \quad (19)$$

For rock whose grains are spherical, the values of b_i are equal to the diameters of particular grain sizes d_i and b_e is equal to effective grain diameter d_e . Hence we get:

$$d_e = \frac{\sum_i (f_i d_i^3)}{\sum_i (f_i d_i^2)} \quad (20)$$

One can see then that effective grain size will be defined according to different formulae, depending on the chosen method of rock gradation analysis.

2.3.1 Effective grain size in a particular fraction

The term rock fraction refers to the isolated part of mineral skeleton of rock whose grains are contained within a specified size range. For instance, in the case of sieve analysis, a fraction will be rock composed of grains that will pass through the upper sieve with mesh openings D_{i-1} , and will be retained on a sieve with openings D_i .

If grain size distribution curve (Fig. 6) is known, then the value adopted as effective fraction value will be the one enabling correct determination of its specific surface area.

Therefore, if b_{ei} denotes the effective size of fraction i , then specific surface area component resulting from the proportion of this fraction, according to formula (5), is

$$s_i = g_i \frac{A_i (1-n)}{b_{ei}} \quad (21)$$

The specific surface area component calculated in this way should be equal to the specific surface area of this fraction treated as well-graded material, which it actually is.

In order to calculate this component of specific surface area, one must adopt specific grain size distribution. To achieve this aim, it was accepted that gradation graph between equivalent grain diameters D_{i-1} and D_i has the shape of a straight line segment (Fig. 6). One should emphasize here that with sufficiently dense breakdown into fractions, the gradation curve can be represented as a curve composed of straight line segments. Moreover, it was assumed that soil mass in the range between D_{i-1} and D_i depends on the mean dimension of the grain. Then, dividing any line segment i into N equal sections (Fig. 7) and assuming that

density, grain shape and the ratio of the largest to the mean grain dimension $\frac{b_{i,j}}{a_{i,j}} = \lambda_i$ within each fraction do not change, one can determine the specific surface area component, in accordance with formula, (3) in the form

$$s_i = A_i (1-n) \sum_{j=1}^N \frac{g_{i,j}}{b_{i,j}} = \frac{A_i}{\lambda_i} (1-n) \sum_{j=1}^N \frac{g_{i,j}}{a_{i,j}} \quad (22)$$

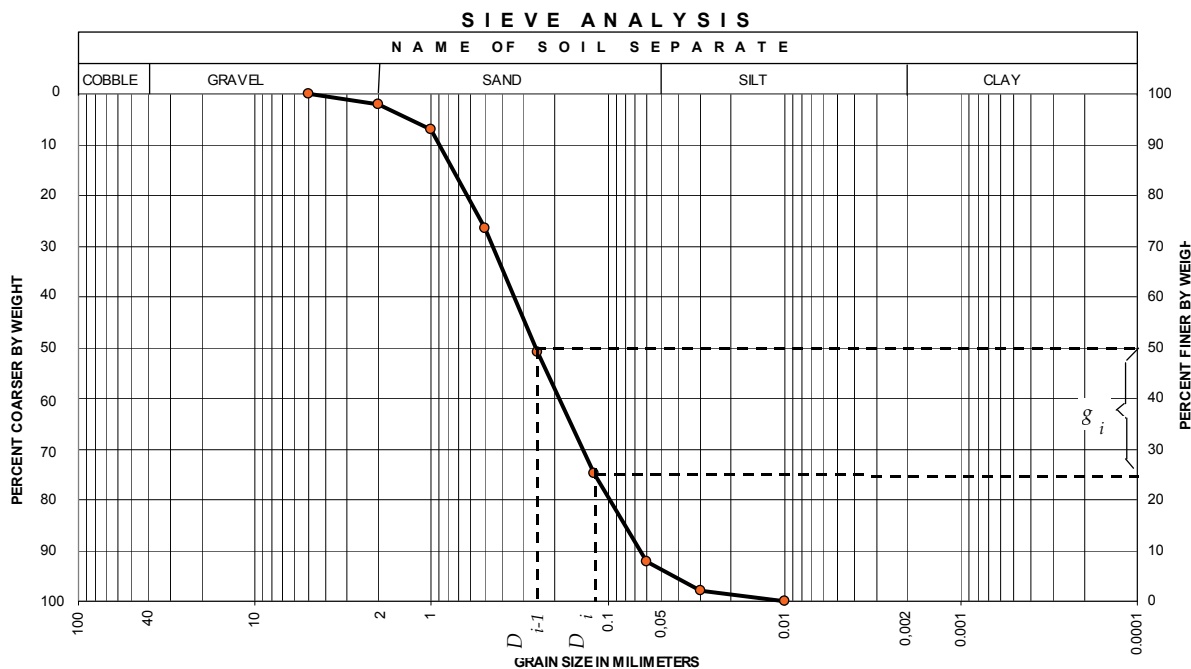


Fig. 6. Typical grain size distribution graph.

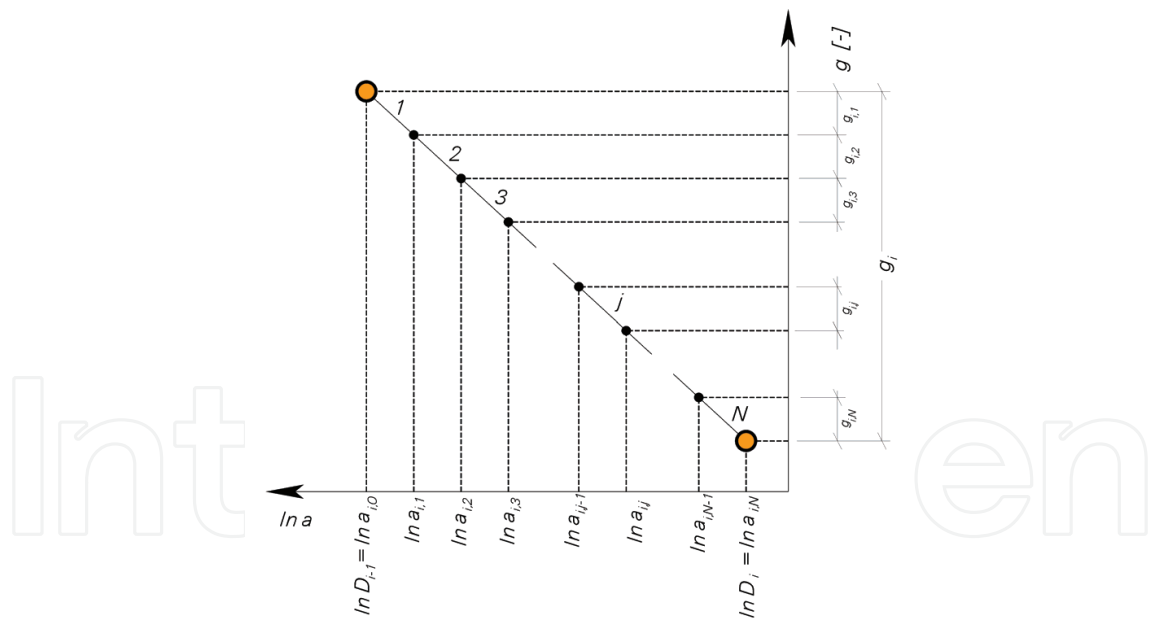


Fig. 7. A fragment of grain size distribution curve.
where $b_{i,j}$ – effective size of part j of the fraction, and

$$g_{i,j} = \frac{g_i}{N} \tag{23}$$

Assuming that division figure N of line segment i approaches infinity, the sum in equation (22) can be replaced by an integral and then

$$s_i = \frac{A_i}{\lambda_i} (1-n) \int_{D_i}^{D_{i-1}} \frac{dg}{a} \quad (24)$$

Since we have assumed that g is a linear function (Fig. 7),

$$g = \alpha x + \beta \quad (25)$$

And

$$dg = \frac{dg}{dx} dx = \alpha dx \quad (26)$$

where α is the slope of a line

$$\alpha = \frac{g_i}{\ln(D_{i-1}) - \ln(D_i)} \quad (27)$$

The x -axis on the gradation curve is in logarithmic coordinates, so $x = \ln a$ and

$$dx = \frac{1}{a} da.$$

Hence, after substituting appropriate data in equation (24), we obtain

$$s_i = \frac{A_i}{\lambda_i} (1-n) \int_{D_i}^{D_{i-1}} \frac{dg}{a} = \frac{A_i}{\lambda_i} (1-n) \int_{D_i}^{D_{i-1}} \frac{\alpha}{a^2} da = \frac{A_i}{\lambda_i} (1-n) g_i \frac{\frac{1}{D_i} - \frac{1}{D_{i-1}}}{\ln \frac{D_{i-1}}{D_i}} \quad (28)$$

Comparing formulae (21) and (28), one can determine the effective size of fraction b_{ei}

$$b_{ei} = \lambda_i \frac{\ln \frac{D_{i-1}}{D_i}}{\frac{1}{D_i} - \frac{1}{D_{i-1}}} \quad (29)$$

It should be emphasized that effective grain size b_{ei} calculated from formula (29) refers to the situation when the mass of grains contained between D_{i-1} and D_i depends on the mean grain dimension. Such a case should occur if the gradation curve is constructed based on aerometric analysis. If, on the other hand, sieve analysis is performed, then it is mostly the largest grain dimension that determines what is left on a particular sieve. Consequently, for results obtained from sieve analysis, one should adopt $\lambda_i = 1$.

With the assumption that grains are spherical, b_{ei} is equal to the effective diameter of fraction, d_{ei} .

2.4 Effective capillary diameter

Effective capillary diameter will be defined as the size of capillaries in material with the same specific surface area and the same porosity as real soil, but with all capillaries having

the same constant cross-section. In order to define the size of these capillaries, let us cut out, theoretically, a soil cuboid with volume $V = F \cdot l$ (Fig. 1). Let us assume that instead of soil we have the same volume of material containing sinuous capillaries with any, but identical, cross-section F_k . Let us designate the cross-section circumference of an individual capillary as U_k . The boundary surface of capillaries inside volume V of such material is

$$s \ V = U_k \ \Sigma l_k, \quad (30)$$

where Σl_k denotes the total length of all capillaries, and l_k – the length of an individual capillary. On the other hand, the volume of capillaries inside volume V is

$$n \ V = F_k \ \Sigma l_k. \quad (31)$$

Dividing equations (31) and (30) by sides, we get

$$\frac{n}{s} = \frac{F_k}{U_k} \quad (32)$$

The ratio F_k/U_k is defined as hydraulic radius R_h . If we accept that capillaries have circular cross-section, then the cross-section diameter will be defined as effective Φ_e and, according to the above equation, it will be

$$\Phi_e = \frac{4 \ n}{s} \quad (33)$$

Taking specific surface area into account, we obtain a correlation between effective grain size b_e , and effective capillary diameter Φ_e

$$\Phi_e = \frac{4 \ n}{A_e (1 - n)} b_e \quad (34)$$

If we assume that grains are spherical, then b_e will be equal to sphere diameter d_e and $A_e = 6$, and

$$\Phi_e = \frac{2 \ n}{3(1 - n)} d_e \quad (35)$$

In this way, known correlations between effective capillary diameter, effective grain diameter, specific surface area and porosity have been derived.

2.5 Capillary rise height

If we know the effective diameter of capillaries in rock, we can determine capillary rise height h_k .

$$h_k = \frac{4 \ \sigma \ \cos \theta}{\Phi_e \ \rho \ g} \cong \frac{4 \ \sigma}{\Phi_e \ \rho \ g} \quad (36)$$

where: σ – surface tension, ρ – density, g – gravitational acceleration, θ – contact angle.

Taking into account the above relationships between the effective capillary diameter and the specific surface area of a grain, we get

$$h_k = \frac{4 \sigma}{\Phi_e \rho g} = \frac{\sigma s}{n \rho g} = \frac{6(1-n) \sigma}{n \rho g d_e} \quad (37)$$

After substitution of appropriate data for rock composed of quartz sand, the rise height h_k of water at the temperature of 10°C becomes

$$h_k = 0,462 \frac{1-n}{n} \frac{1}{d_e} \quad (38)$$

where the values of h_k and d_e are expressed in cm.

An analogical relationship was presented by Polubarinova-Kochina (1962)

$$h_k = 0,45 \frac{1-n}{n} \frac{1}{d_{10}} \quad (39)$$

In this case, d_{10} stands for grain diameter below which grains make 10% of soil mass, and is also expressed in cm.

Apparently, the numerical coefficients in formulae (38) and (39) are very comparable. A bigger difference in capillary rise values could be related to the method of determining d_e and d_{10} .

2.6 Hydraulic conductivity

The hydraulic conductivity of soil will be defined with the assumption that a flow in soil is equivalent to a flow through a bundle of capillaries with a constant cross-section. (Fig. 8).

If the length of filtration path is l , then the length of an individual capillary, owing to its tortuousness, is higher and amounts to $l_k = l \cdot \beta$, where coefficient β expresses capillary tortuousness. Volumetric flow rate Q_k of an individual capillary, with the assumption of a flow of incompressible fluid, can be expressed by the formula Hagen-Poiseuille

$$Q_k = F_k v_{sr} = F_k \frac{R_h^2}{\alpha \eta} \frac{\Delta p}{l_k} = F_k \frac{\rho g \left(\frac{F_k}{U_k} \right)^2}{\alpha \eta \beta} \frac{\Delta H}{l} \quad (40)$$

where: v_{sr} – mean capillary flow velocity, R_h – hydraulic radius of a capillary, α – coefficient related to capillary shape, η – dynamic viscosity, Δp – pressure difference at flow path length l , ΔH – hydraulic head difference at flow path length l .

By defining the number of capillaries N contained in a material

$$N = \frac{\sum l_k}{l_k} = \frac{s V}{U_k l \beta} = \frac{s F}{U_k \beta} \quad (41)$$

one can define the total volumetric flow rate Q through a bundle of capillaries

$$Q = Q_k N = \frac{F_k s F \rho g \left(\frac{F_k}{U_k} \right)^2}{U_k \alpha \eta \beta^2} \frac{\Delta H}{l} = F \frac{\rho g n^3}{\alpha \beta^2 \eta s^2} \frac{\Delta H}{l} \quad (42)$$

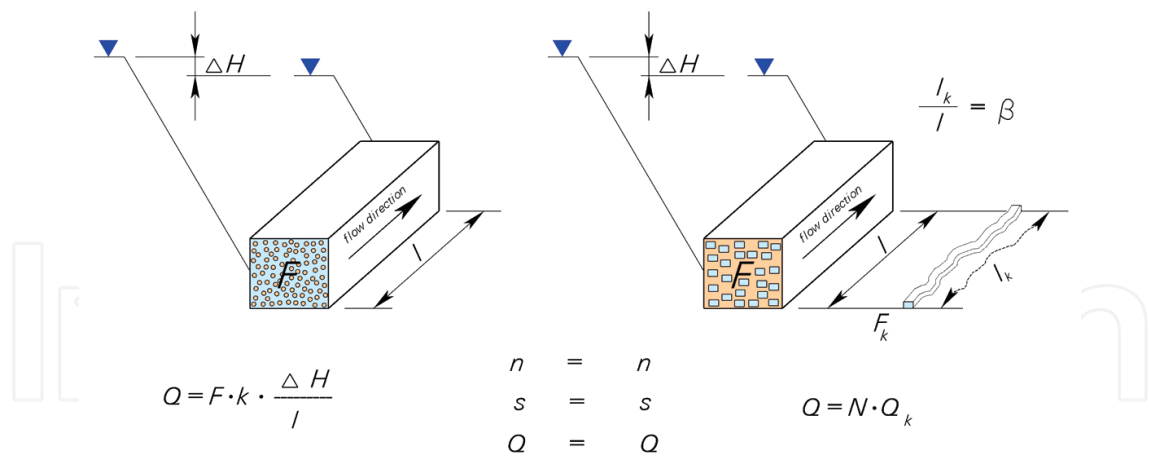


Fig. 8. Assumptions for a flow through real soil and a soil model (n – porosity , s – specific surface area, Q - volumetric flow rate).

Comparing the above relation to volumetric flow rate defined according to Darcy's law, we get a formula for hydraulic conductivity k

$$k = \frac{\rho g n^3}{\alpha \beta^2 \eta s^2} \tag{43}$$

The performed tests have revealed that the value of product $\alpha' = \alpha \beta^2$ is equal to c. 5 (Carman, 1956) and the extreme variation interval of this product ranges from 3,5 to 5,8 (Richardson et al., 1991).

Consequently, if the shape of soil grains is close to a sphere, then after substituting value 5 for α and expressing specific surface area s by effective soil diameter, we obtain

$$k = \frac{\rho g n^3}{5 \eta s^2} = \frac{\rho g n}{80 \eta} \Phi_e^2 = \frac{1}{180} \frac{n^3}{(1-n)^2} \frac{\rho g}{\eta} d_e^2 \tag{44}$$

The above formula is known in literature as Carman - Kozeny formula (Olsen, 1960; Liszkowska, 1996; Mauran et al. 2001; Chapuis & Aubertin, 2003; Carrier, 2003).

2.7 Specific yield

As a result of drainage process, soil retains bound waters, which are held by forces stronger than gravitation. Bound waters can form a film around soil grains and particles. As capillaries have different diameters, additional amounts of water can be held at grain contact. The idea of determining specific yield consists of adopting the mean thickness of bound water layer present on the surface of capillaries formed by pores in soil. Assuming that capillaries have circular cross-section with effective diameter Φ_e , and the mean thickness of bound waters on the boundary surface of soil equals δ , one can define specific yield from the formula

$$\mu = \frac{V_w}{V} = \frac{\pi (\Phi_e - 2 \delta)^2 \sum l_k}{4 V} = \frac{n (\Phi_e - 2 \delta)^2}{\Phi_e^2} \tag{45}$$

where V_w stands for the value of water drained by gravity off the total volume V of rock fully saturated with water, and $\sum l_k$ - the total length of capillaries in volume V .

Equation (45) contains the notion of bound water thickness related to a capillary with circular cross-section. In order to define it, the author used the results of thorough and reliable research conducted by other scholars, including the results of relationships between hydraulic conductivity and specific yield (Drainage Manual, 1984). Based on them, the relationship between effective capillary diameter Φ_e [m], and bound water thickness δ [m] was defined. It has the following form:

$$\delta = 7.54 \cdot 10^{-7} + \frac{8.67 \cdot 10^{-6} \cdot \Phi_e}{0.000032 + \Phi_e} \quad (46)$$

The author suggests employing this equation for natural soils containing dust and clay particles on grain surface.

For sorted washed soils, the author used the research results produced by C.F. Tolman, defining the relationship between grain diameter and specific yield (Fig. 9).

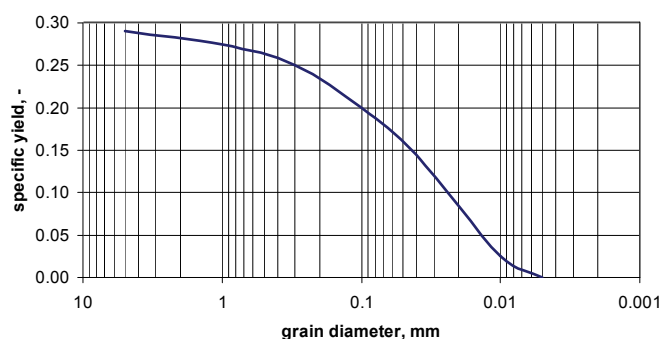


Fig. 9. Correlation between specific yield and grain diameter in non-cohesive soils with porosity $n=0.3$ (Tolman 1937).

For these soils, the relationship between effective capillary diameter Φ_e [m] and thickness of bound water film δ [m] was obtained in the form:

$$\delta = 8.88 \cdot 10^{-7} + \frac{7.15 \cdot 10^{-6} \cdot \Phi_e}{0.0001 + \Phi_e} \quad (47)$$

Certainly, both in formula (46) and (47), condition $2\delta \geq \Phi_e$ must be met, so the highest film thickness may not be higher than half the diameter of a capillary. Assuming that $\delta = 0.5 \Phi_e$, one can calculate the effective capillary diameter below which soil pores are completely filled with bound water, and water drainage from the soil will not take place. For homogeneous sorted soils, this is $2.99 \mu\text{m}$, and for others – $2.07 \mu\text{m}$. Consequently, film thickness in a capillary with circular cross-section would be $1.5 \mu\text{m}$ and $1.04 \mu\text{m}$ respectively. Comparing this thickness to film thickness on a flat surface, we can observe that it will be exactly twice as thinner and will come to $0.75 \mu\text{m}$ or $0.52 \mu\text{m}$. This results from the balance of mass contained in pores, as equation $\pi \Phi_e \delta = \frac{\pi}{4} \Phi_e^2$ should be fulfilled.

In compliance with relations (46) and (47), the mean thickness of bound water film δ remaining in capillaries with circular cross-section may reach the maximum value of c. 8 - 9.5 micrometres. The pattern of these changes is shown in Fig. 10.

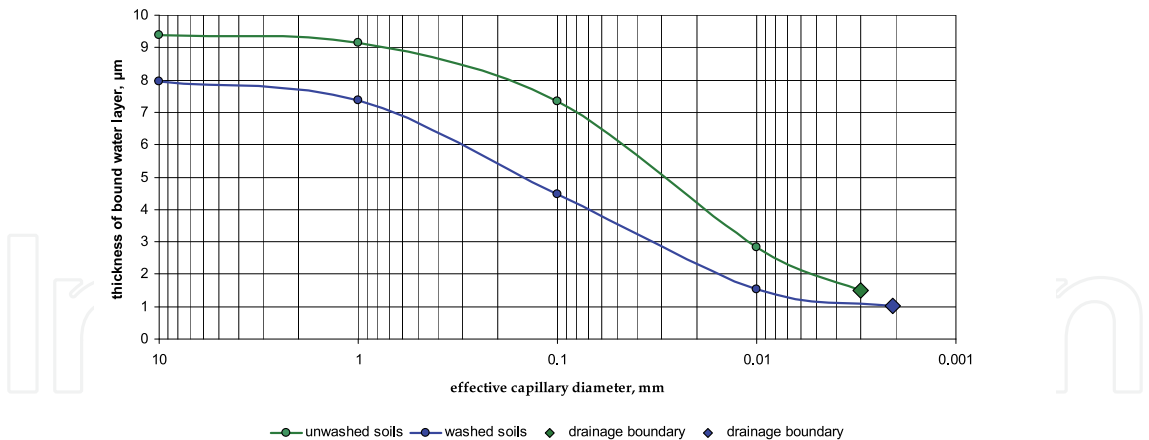


Fig. 10. Bound water layer thickness depending on effective capillary diameter.

It should be emphasized here that both curves are well-matched to measurement points. The point is that specific yields calculated by means of formula (45), with the assumed mean thickness of water layer, according to formulae (46) and (47), are very consistent with the results described by Tolman and Dranaige Manual.

One should add here that adopting the described rock model (a bundle of capillaries with identical diameters) for describing drainage process is a result of broader investigations undertaken by the author. Many variants have been considered, in which a layer of bound waters remains on the surface of grains and soil particles after drainage process. It has been assumed that its thickness on a flat surface is the same as that on the surface of spherical and ellipsoid-shaped grains. It has also been assumed to form a film on the surface of variously-shaped capillaries. In particular, it has been assumed that capillaries have the shape of adjoining balls or the shape of a void formed when spherical soil grains are arranged in such a way that lines joining the centres of these balls form cubes and rhombi. For all these cases, the relation between the thickness of this layer and specific yield was determined theoretically in a way analogical to formula (45). Then, as before, specific yields calculated based on these formulas were compared with the results of the above tests. Based on this comparison, the mean thickness of bound waters was determined. In all the discussed cases, it turned out that film thickness δ was lower for small grains and higher for large ones. Therefore, the simplest geometry of the capillaries was adopted for further investigations, i.e. that with constant circular-shaped cross-section. Adopting such capillary geometry enables employing a uniform rock model.

2.8 Specific retention

Specific retention s_r is defined as the ratio of volume V_z of bound water i.e. water retained in rock by a force higher than gravity, to total rock volume V .

$$s_r = \frac{V_z}{V} \tag{48}$$

Volume V_z is the volume of water which remains in rock after the drainage process. When it comes to drainage of rock fully saturated with water, the following relation between specific retention s_r , specific yield μ and porosity n takes place:

$$s_r = n - \mu \quad (49)$$

Noticeably, specific retention, as well as other parameters discussed before, can be determined from two other known rock parameters.

3. Description of laboratory tests

The aim of the tests was to verify correlations between rock properties that had been obtained based on a rock model in the shape of a bundle of capillaries. Some of them, as already mentioned, had been investigated before, had been accepted and are now often used. This is true about correlations between hydraulic conductivity, porosity, specific surface area and effective grain diameter known as Carman- Kozeny formulas. Therefore in his research, the author focused mainly on verifying mutual correlations between hydraulic conductivity, specific yield and other rock parameters.

The tests were conducted on washed quartz sand with well-rounded grains. Based on microscope observation of a studied soil sample, the mean values of the ratios of the highest to the mean grain dimension and of the largest to the smallest dimension were determined. These values come to 1.1 and 1.2 respectively. The calculated shape factor for such grains, according to equation (4) is 6.3. The sands used for investigations were characterised by grain sizes in the ranges 1.5-1.02 mm, 1.02-0.5 mm, 0.5-0.25 mm, 0.35-0.06 mm and 2.0-0.06 mm. The first three sand fractions were specially selected so that all the grains composing them did not vary a lot in diameter. This was to ensure good homogeneousness of the drained soil. Sands with grain sizes 0.35-0.06 mm and 2.0-0.06 mm corresponded to well-graded soil. The gradation curve was determined for these sands.

Each fraction was subjected to preliminary research, which covered grain skeleton density ρ_s and capillarity (capillary rise height) h_k . Capillarity was defined as the height of a water column allowing the air to break through a soil sample placed in a glass funnel.

The analyses of hydraulic conductivity and specific yield were conducted by placing soil in high columns. The soil was first washed, then dried at the temperature of 105-110°C, and cooled in a dessicator. After cooling it down to the temperature of c. 21°C, the soil was placed and thickened in batches in a thermally insulated column with the height of 1.75 m and diameter of 23.23 cm². The amount of soil placed in the column was weighed in order to determine its porosity. The porosity was calculated from the formula

$$n = \frac{V - \frac{m_s}{\rho_s}}{V} \quad (50)$$

where: V - column volume, m_s – soil mass in the column and ρ_s - soil skeleton density.

Then, very slowly, de-aerated water with the temperature of 21±1°C was fed from the bottom. After saturation with water, the flow of de-aerated water through the column was realised with the assigned hydraulic head drop of c. 0.2. The flow was carried out until gas bubbles were dissolved in pores and constant volumetric flow rate was achieved. Based on the volumetric flow rate of the flowing water, hydraulic conductivity was determined. After defining the hydraulic conductivity, the determination of specific yield was started. At the initial stage of the investigation, constant volumetric flow rate was forced at the column base until the air broke through the soil sample.

Then, gravitational outflow of water was enabled. During the drainage process, the mass of drained-off water was measured. The research lasted from 2 months to three years. Stable temperature was maintained for all this time. The devices mounted at the base of the column enabled the analysis and determination of suction height Ψ_s . The results of laboratory tests are shown in Tab. 1.

Rock No	Gradation range	n	h_k	Ψ_s	k	μ
	mm	-	cm	cm	cm/s	-
1	1.5 – 1.02	0.352	4.5	5.2 do 7.6	0.765	0.337
2	1.02 – 0.5	0.317	9.0	9.8 do 14.7	0.180	0.285
		0.329			0.206	0.298
3	0.5 – 0.25	0.315	23.0	23.9 do 34.0	0.0340	0.278
		0.334			0.0502	0.298
4	0.35 – 0.06	0.297	34.0	34.7 do 53.6	0.0160	0.259
		0.308			0.0183	0.271
5	2.0 – 0.06	0.289	25.4	25.0 do 35.3	0.0405	0.245

Table 1. Results of laboratory tests of hydrogeological properties of rocks.

As suction height Ψ_s changed during the fall of water table, Table 1 presents its change range.

4. Comparison of calculation results and laboratory tests

All the formulas presented in the article, concerning determination of various hydrogeological properties of rocks, are interrelated. The analysis of these formulas demonstrates that if any two properties are known, then any remaining one can be determined.

In the first stage of the comparative analysis it was assumed that laboratory results of sieve analysis and porosity n are known. Based on them, seven different rock parameters, namely effective grain diameter, specific surface area, effective capillary diameter, capillary rise, hydraulic conductivity, specific yield and specific retention were calculated. For rocks No. 1, 2 and 3, it was assumed that the gradation curve (Fig. 5) is composed of one straight line segment. Consequently, the effective diameter was calculated for them directly from formula (29). For rock designated as 5, the effective grain diameter was determined from formula (14). As the gradation curves for all the studied soils were obtained as a result of sieving, in formula (29) $\lambda=1$ was adopted. The specific surface area was calculated from formula (5), adopting grain shape factor equal to 6.3. The effective capillary diameter, capillary rise height and specific yield were calculated from formulas (33), (36) and (45) respectively. The thickness of bound water layer found in formula (45) was adopted as for washed soils, i.e. from formula (47). The specific retention was determined from formula (49), and the results of all calculations were compiled in Table 2.

When comparing the corresponding results in Tables 1 and 2, one can observe good accordance of specific yield and hydraulic conductivity. The conformity of suction height with capillary rise height can be also regarded as high. Slightly bigger differences occur in relation to passive capillarity. However, one should remember that passive capillarity has

been determined for small rock samples placed in a funnel, i.e. for poorly condensed samples. Moreover, determining passive capillarity is influenced by increased rock porosity next to the funnel walls.

Tab. 3., in turn, presents properties of sandy soil calculated based on the knowledge of hydraulic conductivity and porosity.

Rock No	Data		Calculation results						
	Gradation range	n	s	b_e	Φ_e	h_k	k	μ	s_r
	mm	-	mm ⁻¹	mm	mm	cm	cm/s	-	-
1	1.5 – 1.02	0.352	3.32	1.23	0.424	7.0	0.781	0.333	0.019
2	1.02 – 0.5	0.317	6.15	0.699	0.206	14.4	0.166	0.288	0.029
		0.329	6.05	0.699	0.218	13.6	0.193	0.300	0.029
3	0.5 – 0.25	0.315	12.45	0.347	0.101	29.3	0.0398	0.271	0.044
		0.334	12.11	0.347	0.110	26.9	0.0502	0.289	0.045
5	2.0 – 0.06	0.289	10.64	0.421	0.109	27.3	0.0421	0.250	0.039

Table 2. Rock properties determined based on sieve analysis and porosity.

Rock No	Gradation range	Data		Calculation results					
		k	n	s	b_e	Φ_e	h_k	μ	s_r
	mm	cm/s	-	mm ⁻¹	mm	mm	cm	-	-
4	0.35 – 0.06	0.0160	0.297	17.99	0.246	0.0660	44.9	0.247	0.0504
		0.0183	0.308	17.76	0.245	0.0694	42.8	0.257	0.0513

Table 3. Rock properties calculated based on hydraulic conductivity and porosity.

In his case, the specific surface area was calculated from formula (44), the effective capillary diameter – from formula (33), and the effective grain parameter – from formula (34). The capillary rise height, specific yield, and specific retention were calculated analogically by means of the same relations as those shown in Table 2. One can observe that also in this case, i.e. for rock No. 4 with non-uniform gradation, high consistence of the calculated parameters with parameters obtained from direct laboratory tests was obtained.

5. Conclusions

In this chapter, the author has presented mutual relationships between hydrogeological properties of rocks. They are related to hydraulic conductivity, specific surface area, porosity, effective grain diameter, effective capillary diameter, capillary rise, specific yield and specific retention. All the relations were derived based on a rock model in the form of a bundle of tortuous capillaries with constant circular-shaped cross-section. The analysis of thus obtained relations demonstrates that if any two rock properties are known, one can define all the remaining ones. Hence, knowing two properties, one can determine six or seven other properties of rock. Some of the formulas included in the chapter have already been known, verified and widely used for practical problems for a long time. This pertains in particular to Carman-Kozeny formulas. Except for formerly known formulas, a new one, related to determining specific yield, has been proposed. It is based on adopting the mean

thickness of bound water in capillaries with circular cross-section. The presented formula for bound water thickness in homogeneous sorted soils is based on research results from Tolman (Tolman, 1937), and in other soils – on results published in Drainage Manual (Kasenow, 2002). Its form also enables determining boundary effective capillary diameter, below which only bound waters occur and water is no longer drained off rock. Introducing a formula for specific yield also has enabled linking rock properties with specific retention.

Determining different rock parameters requires different amounts of work. One of the simplest tests, almost always performed when recognising soils, is gradation analysis. Therefore a lot of attention has been paid to using its results for determining other hydrogeological properties of rocks. Complete derivation of formulas enabling determination, based on gradation analysis, of specific surface area and effective grain size, considering complex size of soil-building grains and non-uniform density of soil skeleton has been presented.

A very important element of this research is verification of presented relations between different rock properties. To achieve this aim, long-term laboratory tests were launched. The key issue was the investigation of specific yield and hydraulic conductivity. They were determined on samples placed in high columns. Such a method produces good results but determining specific yield is very time-consuming. As a result of many years of investigations, few results were obtained. This is why investigations connected with verification of presented relations should be continued.

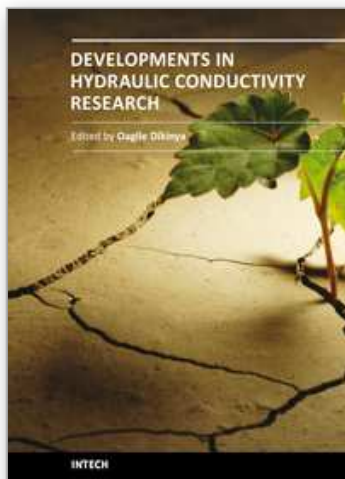
Notwithstanding, it should be emphasised that the presented results have demonstrated satisfactory accuracy of the described relations. In particular, positive results connected with using new formulas for determining specific yield and its relations with hydraulic conductivity have been obtained.

6. References

- Arya, L.M.; Leij F.J.; Shouse, P.J. & Genuchten, M. Th. (1999). Relationship between the hydraulic conductivity function and the particle-size distribution. *Soil Sci.Soc. Am. J.* Vol. 63, (September-October 1999), pp. (1063-1070), ISSN 0361-5995
- Bear, J. (1972). *Dynamics of fluids in porous media*, Dover Publications, ISBN 0-486-65675-6, New York USA
- Carman, P.C. (1956). *Flow of Gases through Porous Media*. Butterworths Scientific Publications, London.
- Carrier, W.D. (2003). Goodbye, Hazen; Hello, Kozeny-Carman. *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 129, No 11 (November 2003), pp1054-1056, ISSN 1090-0241
- Chapuis, R.P. & Aubertin, M. (2003). On the use of the Kozeny-Carman equation to predict the hydraulic conductivity of soils. *Can. J. Geotech./Rev. Can. Geotech.* Vol. 40 No 3 pp. 616-628, ISSN 1208-6010
- Drainage Manual (1984). US Department of the Interior. U.S. Government Printing Office, Denver, Colorado
- Richardson, J. F.; Harker, J. H. & Backhurst J. R. (1991). *Coulson and Richardson's Chemical Engineering. Volume 2. Particle Technology and Separation Processes*, Butterworth Heineman, ISBN 0-7506-4445-1 Oxford
- Kasenow, M. (2002). *Determination of hydraulic conductivity from grain size analysis*, Water Resources Publications, ISBN 1-887201-31-9, Colorado USA

- Liszkowska, E. (1996). Wzór Carmana-Kozeny uniwersalnym wzorem na obliczanie współczynnika filtracji, *Geologos*, No 1 (1996) (193-202)
- Mauran, S.; Rigaud, L. & Coudeville O. (2001). Application of the Carman-Kozeny correlation to high-porosity and anisotropic consolidated medium: The compressed expanded natural graphite, *Transport in Porous Media*, Vol. 43, pp. 355-376, ISSN 0169-3913
- Olsen, H.W. (1960). Hydraulic flow through saturated clays, *Clays and Clay Minerals*, Vol. 9, No 1, pp. 131-161, ISSN 00009-8604
- Petersen L.W.; Moldrup P.; Jacobsen O.H. & Rolston D.E. (1996). Relations between specific surface area and soil physical and chemical properties, *Soil Science* Vol. 161, No 1, (January 1996) pp. 9-21, ISSN 0038-075X
- Polubarinova-Kochina P. (1962). *Theory of Ground Water Movement*, Princeton Univ. Pr. ISBN 0691080488, New Jersey USA
- Tolman C.F. (1937). *Ground Water*, Mc Graw-Hill, New York.

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This book provides the state of the art of the investigation and the in-depth analysis of hydraulic conductivity from the theoretical to semi-empirical models perspective as well as policy development associated with management of land resources emanating from drainage-problem soils. A group of international experts contributed to the development of this book. It is envisaged that this thought provoking book will excite and appeal to academics, engineers, researchers and University students who seek to explore the breadth and in-depth knowledge about hydraulic conductivity. Investigation into hydraulic conductivity is important to the understanding of the movement of solutes and water in the terrestrial environment. Transport of these fluids has various implications on the ecology and quality of environment and subsequently sustenance of livelihoods of the increasing world population. In particular, water flow in the vadose zone is of fundamental importance to geoscientists, soil scientists, hydrogeologists and hydrologists and allied professionals.

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