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# Integral Optimization of the Container Loading Problem

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## 1. Introduction

The rapid globalization of the world economy has led to the development of ample and quickly growing (aerial, maritime, terrestrial) networks for merchandise distribution in containers [Wang et al., 2008]. The transport costs afforded by the specialized companies operating in this sector are directly related to appropriate loading and efficient use of space [Xue and Lai, 1997a]. The efficient loading of a set of containers can be done technically by solving the Container Loading Problem (CLP).

CLPs are NP-Hard problems that basically consist in placing a series of rectangular boxes inside a rectangular container of known dimensions, seeking to optimize volume utilization [Pisinger, 2002], and taking into consideration the basic constraints enounced by Wäscher et al. (2007): (i) all the boxes must be totally accommodated inside the container, and (ii) boxes should not overlap. Notwithstanding, the solving of actual container loading problems can be limited or rendered inappropriate if only these two constraints are considered [Bischoff and Ratcliff, 1995; Bortfeldt and Gehring, 2001; Eley 2002].

In this sense, Bischoff and Ratcliff (1995) enounced a series of practical restrictions that are applicable to real situations: orientation and handling constraints, load stability, grouping, separation and load bearing strength of items within a container, multi-drop situations, complete shipment of certain item groups, shipment priorities, complexity of the loading arrangement, container weight limit and weight distribution within the container. According to the literature on the topic, these considerations have not been included in many of the existing approaches to the CLP problem. Some of these criteria are difficult to quantify [ibidem] due to their qualitative nature. The traditional optimization approaches, which cardinalize qualitative aspects, tend to cause loss of important criterion information. For this reason, more natural treatments such as those resulting from ordinal approaches are advisable [García et al., 2009].

The CLP has a natural correspondence with the integral optimization concept, which includes qualitative and quantitative criteria within an optimization problem [ibidem]. The CLP solving approach treated here not only considers the fundamental quantitative criteria stated by Wäscher et al. (2007), but two other important ones contributed by Bischoff and Ratcliff (1995) as well: i) not exceeding the container's weight transportation limit, and ii) once the container has been loaded, its center of gravity (COG) should be close to the geometrical center of its base (weight distribution within a container). In turn, the

qualitative criterion is the fragility of the elements packed inside the boxes. Finally, the stochastic consideration has to do with the load bearing strength of items, which results from the fragility or structural features of their contents or other reasons.
The current chapter uses the Integral Analysis Method (IAM) [García et al., 2009] to optimize the CLP. IAM adapts well to stochastic optimization problems, thus allowing the development of more complex and natural models, which are therefore closer to actual problem contexts. In section 2, the current chapter includes an analysis of the background of the problem, including the list of restrictions contributed by Bischoff and Ratcliff (1995). Section 3 develops IAM: item 3.1 introduces the quantitative assessment, which includes the mathematical model and heuristic solution to the problem, as well as the analysis of the computational results; item 3.2 addresses the qualitative analysis, and item 3.3, the integrated analysis. Finally, section 4 presents conclusions and recommendations.

## 2. Background

The CLP has been studied since the beginning of the sixties [Pisinger, 2002]. Our literature review, which is detailed in table 1, allowed identifying heuristic and metaheuristic methodologies as the most common approaches to solving the problem. Albeit less frequent, other approaches have made use of Mixed Integer Programming (MIP), Nonlinear Programming (NLP) and Approximation Algorithm (AA) models.
The solving technique and set of constraints considered in each of these studies can be found in tables 1 and 2, respectively. The methodologies used to treat the constraints identified by Bischoff and Ratcliff (1995) are presented in table 3. Regarding the constraints taken into consideration in the referred studies, those defined by Wäscher et al. (2007) are the most common ones: not exceeding the volume of the container and not allowing box overlapping. Few studies have addressed the constraints contributed by Bischoff and Ratcliff (1995).
The works of Eley (2002), Bortfeldt and Gehring (2001), Davies and Bischoff (1999), Xue and Lai (1997b) and Chen et al. (1995) include the most considerations, although none of them reaches the complexity treated here. These studies solve the CLP by trying to minimize the wasted space in the container. It is worthwhile mentioning that all the criteria modeled in the reviewed CLP versions were treated quantitatively, even those that could be more naturally treated in a qualitative way. Examples of these criteria are separation of items within a container, shipment priorities or loading arrangement.

## 3. Integral optimization of the problem

The quantitative analysis proposes a mathematical programming model and a heuristic method to solve the CLP. In the qualitative and integration analysis we applied the developments contributed by IAM.

### 3.1 Quantitative analysis

The works of Chen et al. (1995) and of Xue and Lai (1997b) developed MIP models which include the set of restrictions contemplated in the current work. The model detailed in section 3.1.2 is proposed for homogeneous load (all the boxes have the same dimensions when they are not bearing anything on top) and includes the stochastic consideration defined in section 3.1.1

| Author | Container filling strategy | | | | | Exact methods | | AA models | | Heuristics and metaheuristics | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Wall Building | Block Building | Column Building | Multi-Faced Building | Caving Degree | NLP | MIP | Next Fit | First Fit | Local Search | Greedy | Evolutionary algorithm | Genetic algorithms | Tabu Search | Tree Search | Proper of the author |
| Huang and He (2009b) | | | | | ✓ | | | | | | ✓ | | | | | |
| Huang and He (2009a) | | | | | ✓ | | | | | | | | | | | ✓ |
| Chien et al. (2009) | ✓ | | | | | | | | | | | | | | | ✓ |
| Soak et al. (2008) | | | | | | | | | | ✓ | | ✓ | | | | |
| Wang et al. (2008) | | ✓ | | | | | | | | | | | | | | ✓ |
| Birgin et al. (2005) | | | | | | ✓ | | | | | | | | | | |
| Chien and Deng (2004) | ✓ | | | | | | | | | | | | | | | ✓ |
| Lewis et al. (2004) | | | | | | | | | | | | | ✓ | | | |
| Bortfeldt et al. (2003) | | ✓ | | | | | | | | | | | | ✓ | | |
| Lim et al. (2003) | | | | ✓ | | | | | | | | | | | | ✓ |
| Miyazawa and Wakabayashi (2003) | | | | | | | | ✓ | ✓ | | | | | | | |
| Eley (2002) | | ✓ | | | | | | | | | ✓ | | | | ✓ | |
| Pisinger (2002) | ✓ | | | | | | | | | | | | | | ✓ | |
| Bortfeldt and Gehring (2001) | ✓ | | | | | | | | | | ✓ | | ✓ | | | |
| Teng et al. (2001) | | | | | | | | | | | | | | | | ✓ |
| Davies and Bischoff (1999) | ✓ | ✓ | ✓ | | | | | | | | | | | | | ✓ |
| Xue and Lai (1997b) | | | | | | | ✓ | | | | | | | | | |
| Xue and Lai (1997a) | ✓ | | | | | | | | | | | | | | | ✓ |
| Bischoff and Ratcliff (1995) | | | ✓ | | | | | | | | | | | | | ✓ |
| Chen et al. (1995) | | | | | | | ✓ | | | | | | | | | |

Table 1. CLP solving methods

| Author | Constraints | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Huang and He (2009b) | ✓ | ✓ | | | | | | | |
| Huang and He (2009a) | ✓ | ✓ | | | | | | | |
| Chien et al. (2009) | ✓ | ✓ | | | ✓ | | | | |
| Soak et al. (2008) | ✓ | ✓ | | | | | | | |
| Wang et al. (2008) | ✓ | ✓ | | | | | | | |
| Birgin et al. (2005) | ✓ | ✓ | | | | | | | |
| Chien and Deng (2004) | ✓ | ✓ | | | | | | | |
| Lewis et al. (2004) | ✓ | ✓ | | | | | | | |
| Bortfeldt et al. (2003) | ✓ | ✓ | | | ✓ | | | | |
| Lim et al. (2003) | ✓ | ✓ | | | | | | | |
| Miyazawa and Wakabayashi (2003) | ✓ | ✓ | | | | | | | |
| Eley (2002) | ✓ | ✓ | ✓ | | | | | | |
| Pisinger (2002) | ✓ | ✓ | | | | | | | |
| Bortfeldt and Gehring (2001) | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | ✓ |
| Teng et al. (2001) | ✓ | ✓ | ✓ | | ✓ | | | | |
| Davies and Bischoff (1999) | ✓ | ✓ | ✓ | | | | | | |
| Xue and Lai (1997b) | ✓ | ✓ | | | | | | ✓ | |
| Xue and Lai (1997a) | ✓ | ✓ | | | | | ✓ | | |
| Bischoff and Ratcliff (1995) | ✓ | ✓ | | | ✓ | ✓ | | | |
| Chen et al. (1995) | ✓ | ✓ | ✓ | ✓ | | | | | |

1. Container volume
2. Boxes cannot overlap
3. Weight distribution within a container
4. Orientation constraints
5. Load stability
6. Multi-drop situations
7. Shipment frequencies
8. Container weight limit
9. Stacking of boxes

Table 2. Constraints addressed in CLP studies

### 3.1.1 Defining the stochastic consideration

Boxes can be vertically compressed depending on the load they bear on top. Such deformation may depend on box content itself and on its structural features. In order to include this consideration in our MIP model we have made the following assumptions:

- Boxes might (or might not) be deformed.
- Only affecting height, deformation is homogeneous on the upper side, which bears the load.
- Boxes might be made of different materials and have diverse contents.
- The maximum load a box can support is a known feature.
- Boxes have a deformation limit

It is assumed that the deformation experimented by a box is directly proportional to the weight it bears on top. That is to say, the higher the weight, the more deformed the box will be. In this way the box reaches its maximum deformation when it is bearing the maximum permitted load. Additionally, we have included a stochastic factor that models the deformation that is not explained by the mentioned relation. The deterministic behavior of the deformation process is described in Figure 2.

| Practical constraint | Quantitative nature | Qualitative nature | Authors that have included it | Applied methodology |
|---|---|---|---|---|
| **Orientation constraints:** One simple example of this constraint is the warning "This way up" that appears in certain boxes. | ✓ | | Bortfeldt et al. (2003) | A Tabu Search metaheuristic is applied as a solution, making use of a Block Building approach which groups the boxes according to their orientation constraints. |
| | | | Bortfeldt and Gehring (2001) | Possible box rotations are handled through modifications of the wall filling method of the proposed greedy heuristic. |
| | | | Chen et al. (1995) | The model is modified according to the need for orientation constraints. |
| **Load bearing strength of items:** Depending on its structural features and on the fragility of its contents, a box may or may not tolerate the placing of weight on top. | ✓ | ✓ | Bortfeldt and Gehring (2001) | The proposed greedy heuristic's wall filling method is quantitatively modified to prevent the creation of empty spaces above boxes with weight bearing restrictions. Excessive waste of space resulting from this constraint is prevented through the incorporation of additional rules. |
| **Handling constraints:** According to the size and weight of the boxes, and to the necessary tools to store them in the container, the bigger elements may need to be placed on the floor of the container, or the heavier ones may not be allowed above a certain height. | ✓ | | NA | NA |
| **Load stability:** If, for example, the merchandise is prone to get damaged inside the container, it might be necessary to restrict its movement beyond significant limits during transportation. | ✓ | | Bortfeldt et al. (2003) | The blocks are built so that their base is entirely supported by another block or by the base of the container. |
| | | | Eley (2002) | Each block is built with identical elements in order to prevent the formation of empty spaces among them. |
| | | | Teng et al. (2001) | Mathematical equations are applied to minimize the system's inertial momentum. |
| | | | Bortfeldt and Gehring (2001) | The proposed wall filling method of the greedy heuristic is modified to avoid placing a box on top of another that is not supporting its bottom in its entirety. |

| Practical constraint | Quantitative nature | Qualitative nature | Authors that have included it | Applied methodology |
|---|---|---|---|---|
| | | | Bischoff and Ratcliff (1995) | A Column Building based heuristic solution is presented. Stability is increased through columns built with boxes of the same type, so that none of them lacks base support. |
| **Grouping of items:** Load checking and operation may be rendered easier if similar items are placed as close to one another as possible. | ✓ | | NA | NA |
| **Multi-drop situations:** If the container is scheduled to stop several times on the way, it might result practical to group together those items having the same destiny. | ✓ | | Bischoff and Ratcliff (1995) | A heuristic that checks all available spaces in the container before placing a box is introduced. Additional stability rules make sure all the boxes have their bases entirely supported by other boxes beneath. |
| **Separation of items within a container:** If, for example, the container is carrying both chemical and food products, the loading arrangement must prevent them from having any contact. | | ✓ | NA | NA |
| **Complete shipment of certain item groups:** A particular shipment may include several boxes. If the decision is made to store one of them, the others might also need to be stored together. | ✓ | | NA | NA |
| **Shipment priorities:** The shipping of certain elements might be more important than that of some other ones. | ✓ | ✓ | NA | NA |
| **Complexity of the loading arrangement:** Depending on the resulting load arrangement, special technology to unload the container (clamp or | | ✓ | NA | NA |

| Practical constraint | Quantitative nature | Qualitative nature | Authors that have included it | Applied methodology |
|---|---|---|---|---|
| forklift trucks) might result necessary instead of manual labor. However, if the task has technical limitations, the loading arrangement must adapt to them. | | | | |
| **Container weight limit:** The container may have a maximum capacity which cannot be exceeded. | ✓ | | Bortfeldt and Gehring (2001) | While executing the greedy heuristic, the accumulated weight that has been loaded into the container is continuously checked. When an additional box leads to exceeding the container's weight limit, it is not stored. |
| | | | Xue and Lai (1997b) | This constraint is included in the mathematical programming section. |
| **Weight distribution within a container:** From the standpoint of the operation of a loaded container, its center of gravity should not be far from the geometrical center of its base; otherwise certain maneuvers may be impossible. | ✓ | | Eley (2002) | The length of the container is divided in equal sections that are filled up according to each of the proposed heuristics. The sections are then exchanged in order to attain an optimum weight distribution. |
| | | | Teng et al. (2001) | During the second phase of the heuristic, the elements are tentatively swapped in order to drive the center of gravity of the system close to that of the container. |
| | | | Bortfeldt and Gehring (2001) | The load balance is handled through the greedy heuristic as follows: along the length of the container through exchanging the walls that have been built; and along the width of the container through reflecting the load arrangement of each wall. |
| | | | Davies and Bischoff (1999) | A heuristic that combines the Column, Wall and Block Building approaches is introduced. Load balance is sought by exchanging and rotating the different blocks resulting in the load arrangement. |
| | | | Chen et al. (1995) | The model is modified as to include two restrictions aimed at preventing the load balance along the container from exceeding a certain limit. |

Table 3. Practical constraints defined by Bischoff and Ratcliff (1995)

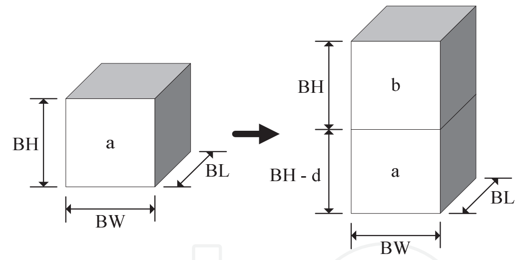


Fig. 1. Box height reduction due to top load.

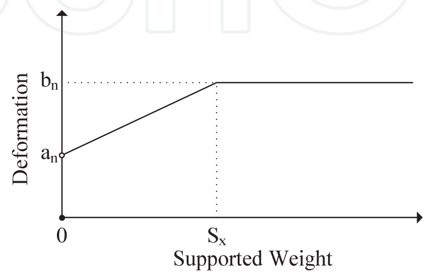


Fig. 2. Supported weight – Deformation

Deformation is modeled as follows:

$$d_{xn} = \begin{cases} 0 & \Leftrightarrow c_x = 0 \lor n = n_{max} \\ a_n + \dfrac{c_x}{S_x}(b_n - a_n) + \varepsilon_{xn} & \text{otherwise} \end{cases} \tag{1}$$

Where

$n$: level where box $x$ is located; $n \in \{1,2,\dots,n_{max} = \lfloor CH/BH \rfloor\}$
$d_{xn}$: deformation undergone by box $x$ at level $n$, $d_{xn} \geq 0$.
$c_x$: weight supported by box $x$, equaling $\sum_{i=n+1}^{n_{max}} P_{yi}$, where $P_{yi}$ is the weight exerted by box $y$ at level $i$, with $y \neq x$.
$S_x$: maximum weight bearable by box $x$, with $S_x > 0$.
$a_n$: minimum deterministic deformation experimented at level $n$.
$b_n$: maximum deterministic deformation experimented at level $n$.
$\varepsilon_{xn}$: stochastic parameter explaining the deformation that is not attributable to the functional relation of box $x$ at level $n$. This parameter associates a different probability density function to each $n$, $\varepsilon_{xn} \in \mathbb{R}$ and $\theta_n^{min} \leq \varepsilon_{xn} \leq \theta_n^{max}$.
This way of modeling the deformation facilitates the simulation of instances in which one box can be more deformed than another, even when they are bearing the same weight and number of boxes. This might be the case of, for example, the different structural features of the boxes or of their contents. In sum, as a result of unknown reasons that cannot be attributed to the described function.

### 3.1.2 MIP model

Given that the boxes have the same dimensions, the container can be divided in multiple cells of box dimensions (Figure 3). As the model does not allow rotating the boxes, all their sides remain parallel to their corresponding container homologues. In this context, a hypothetical container can be conceived so that the boxes fit its width and length perfectly well because in practice the empty space (dotted zone in Figure 3) can be completed with filling material.

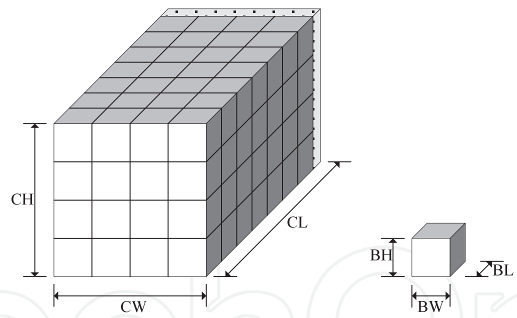


Fig. 3. Inner division of the container

The model includes the following parameters:

- $I$: number of boxes to be stored.
- $(CL, CW, CH)$: dimensions of the container (length, width, height).
- $(BL, BW, BH)$: dimensions of the boxes (length, width, height). It is assumed that the COG of each box coincides with its geometric center.
- $(J, K, L)$: number of boxes that can be accommodated in the container along its length, width and height, respectively; where $J = \{1, \dots, j_{max} = \lfloor CL/BL \rfloor\}$, $K = \{1, \dots, k_{max} = \lfloor CW/BW \rfloor\}$ and $L = \{1, \dots, l_{max} = \lfloor CH/BH \rfloor\}$.
- $P_i$: weight of box $i$.
- $S_i$: maximum weight bearable by box $i$, being $S_i > 0$.
- $P_C$: maximum load capacity of the container as measured in weight.
- $G$: the distance between the COG of the loaded container and its base is restricted to a predetermined value $(G)$. This distance is only measured along the length of the container (Figure 4).

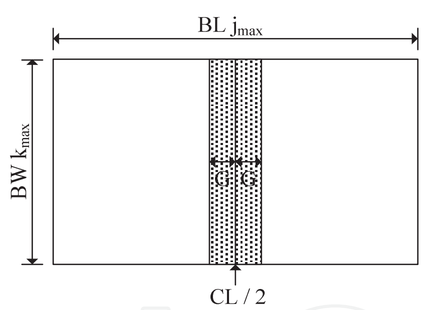


Fig. 4. Top view of the container

- $A_n$: minimum deterministic deformation experimented at level $n$ of the container.
- $B_n$: maximum deterministic deformation experimented at level $n$ of the container.
- $f_n$: probability density function that determines the stochastic deformation experimented by a box at level $n$ of the container.
- $\theta_n^{min}$: minimum possible value of the stochastic deformation parameter for boxes located at level $n$ of the container.
- $\theta_n^{max}$: maximum possible value of the stochastic deformation parameter for boxes located at level $n$ of the container.

The model uses the following variables:

- $x_{ijkl} = \begin{cases} 1 & \text{If box } i \text{ is in cell}(j,k,l) \\ 0 & \text{Otherwise} \end{cases}$
- $c_{ijkl}$: total load supported by box $i$ in cell $(j, k, l)$.
- $\varepsilon_{il}$: stochastic deformation experimented by box $i$ at level $l$.

– $d_{ijkl}$: total deformation experimented by box $i$ in cell $(j,k,l)$.
The model has the following constraints:
(R1) Volume capacity: the number of boxes stored in the container must not exceed the number of cells available in it:

$$\sum_{i=1}^{I}\sum_{j=1}^{j_{max}}\sum_{k=1}^{k_{max}}\sum_{l=1}^{l_{max}} x_{ijkl} \leq j_{max}k_{max}l_{max} \tag{2}$$

(R2) No box shall occupy more than one cell:

$$\sum_{j=1}^{j_{max}}\sum_{k=1}^{k_{max}}\sum_{l=1}^{l_{max}} x_{ijkl} \leq 1 \quad \forall i \in \{1,\dots,I\} \tag{3}$$

(R3) Each cell shall only be assigned to one box:

$$\sum_{i=1}^{I} x_{ijkl} \leq 1 \quad \forall j \in J; \forall k \in K; \forall l \in L \tag{4}$$

(R4) All the boxes that are not in contact with the base of the container must be supported by other boxes beneath them:

$$x_{ijkl} \leq \frac{1}{l-1}\sum_{n=1}^{l-1}\sum_{m=1,m\neq i}^{I} x_{mjkn} \quad \forall i \in \{1,\dots,I\}; \forall j \in J; \forall k \in K; l \in \{2,\dots,l_{max}\} \tag{5}$$

(R5) The total stored weight of the boxes cannot exceed the load limit of the container:

$$\sum_{i=1}^{I} P_i \sum_{j=1}^{j_{max}}\sum_{k=1}^{k_{max}}\sum_{l=1}^{l_{max}} x_{ijkl} \leq P_C \tag{6}$$

(R6) Weight distribution within the container: once the container has been loaded, its COG is calculated along its length because its stability is more compromised along its largest dimension. The distance between this point and $\frac{CL}{2}$ must not be larger than $G$ (Figure 4). To calculate the COG of the container, it is divided in $j_{max}$ walls of dimensions $BL, BW k_{max}, BH l_{max}$, each of them with weight $m_j$ which is assumed to be exerted at the middle point of its base; that is, at $BL/2$ (Figure 5).

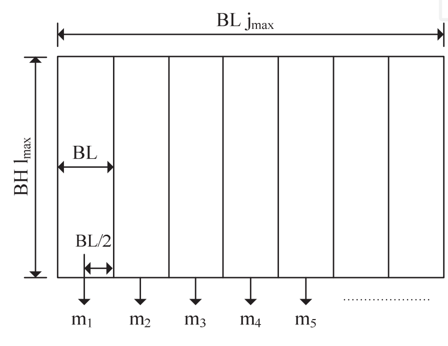


Fig. 5. Side view of the container

As a reference, we take the lower left corner as the origin of axis $X$. The force diagram on the base of the container is the following:

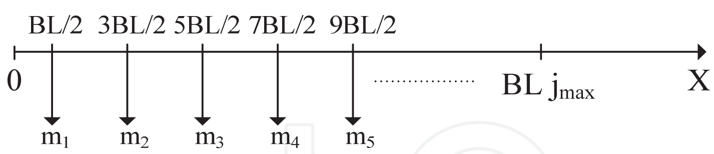


Fig. 6. Force diagram on the base of the container

Applying the equation to calculate the COG we obtain:

$$COG_X = \frac{\sum_{j=1}^{j_{max}} m_j o_j}{\sum_{j=1}^{j_{max}} m_j} \tag{7}$$

Where $o_j$ is the distance from the center of the base of wall $j$ to the origin, and $m_j$ is the weight of wall $j$.

The distance between $COG_X$ and $\frac{CL}{2}$ cannot be larger than $G$. This constraint is expressed as:

$$-G \leq \frac{\sum_{j=1}^{j_{max}} m_j o_j}{\sum_{j=1}^{j_{max}} m_j} - \frac{CL}{2} \leq G \tag{8}$$

The weight of wall $j$ is given by the sum of the box weights stored in it:

$$m_j = \sum_{i=1}^{I} \sum_{k=1}^{k_{max}} \sum_{l=1}^{l_{max}} x_{ijkl} P_i \tag{9}$$

The distance from the center of wall $j$ to the origin is given by:

$$o_j = BL \frac{2j-1}{2} \tag{10}$$

Replacing $m_j$ and $o_j$ in the constraint we obtain:

$$-G \leq \frac{\sum_{i=1}^{I} \sum_{j=1}^{j_{max}} \sum_{k=1}^{k_{max}} \sum_{l=1}^{l_{max}} \frac{BL(2j-1)}{2} x_{ijkl} P_i}{\sum_{i=1}^{I} \sum_{j=1}^{j_{max}} \sum_{k=1}^{k_{max}} \sum_{l=1}^{l_{max}} x_{ijkl} P_i} - \frac{CL}{2} \leq G \tag{11}$$

Which can be redistributed as:

$$-2G \leq BL \frac{\sum_{i=1}^{I} \sum_{j=1}^{j_{max}} \sum_{k=1}^{k_{max}} \sum_{l=1}^{l_{max}} (2j-1) x_{ijkl} P_i}{\sum_{i=1}^{I} \sum_{j=1}^{j_{max}} \sum_{k=1}^{k_{max}} \sum_{l=1}^{l_{max}} x_{ijkl} P_i} - CL \leq 2G \tag{12}$$

(R7) The weight supported by box $i$ in cell $(j, k, l)$ is given by the overall sum of the box weights it is bearing, that is, in those $(j, k, f)$ cells satisfying the condition $l < f \leq l_{max}$. Said weight must not exceed the box's load bearing limit:

$$c_{ijkl} = \sum_{m=1, m\neq i}^{I} \sum_{n=l+1}^{l_{max}} x_{mjkn} P_m \quad \forall i \in \{1, \dots, I\}; \forall j \in J; \forall k \in K; \forall l \in L \tag{13}$$

$$c_{ijkl} \leq S_i \quad \forall i \in \{1, \dots, I\}; \forall j \in J; \forall k \in K; \forall l \in L \tag{14}$$

$$c_{ijkl} = 0 \quad \forall i \in \{1, \dots, I\}; \forall j \in J; \forall k \in K; l = l_{max} \tag{15}$$

As it can be seen in the constraint above, the weight supported by the boxes at the top level is zero.

(R8) Deformation of box $i$ is calculated from the deterministic deformation range $[A_l, B_l]$ for level $l$, the ratio of the supported weight $(c_{ijkl}/S_i)$ and the probability function $(f_l(\theta_l^{min}, \theta_l^{max}))$ corresponding to level $l$ where the box is located. The deformation of the boxes found at the uppermost level of the container is made equal to zero:

$$\varepsilon_{il} = f_l(\theta_l^{min}, \theta_l^{max}) \quad \forall i \in \{1, \dots, I\}; \forall l \in L \tag{16}$$

$$d_{ijkl} = A_l + \frac{(B_l - A_l)}{S_i} c_{ijkl} + \varepsilon_{il} \quad \forall i \in I; \forall j \in J; \forall k \in K; \forall l \in \{1, \dots, l_{max} - 1\} \tag{17}$$

$$d_{ijkl} = 0 \quad \forall i \in I; \forall j \in J; \forall k \in K; l = l_{max} \tag{18}$$

Finally, the objective function minimizes the empty space inside the container:

$$\min z = -BL \cdot BW \cdot \sum_{i=1}^{I} \sum_{j=1}^{j_{max}} \sum_{k=1}^{k_{max}} \sum_{l=1}^{l_{max}} (BH - d_{ijkl}) x_{ijkl} \tag{19}$$

### 3.1.3 Heuristic method

Although the literature review does not report the application of the GRASP (Greedy Randomized Adaptive Search Procedure) metaheuristic to solve three dimensional packing problems, it has shown very good results in combinatorial problems raised in production programming [Vega-Mejía and Caballero-Villalobos, 2010; Binato et al., 2002] and supply chain [Carreto and Baker, 2002; Delmaire et al., 1999] studies, among other research areas. In sum, the evidence of good performance of this metaheuristic for solving combinatorial problems led to its application in the current problem.

#### 3.1.3.1 GRASP Metaheuristic

The procedure consists in an iterative process comprising two phases, namely construction and local search. In the constructive phase a feasible solution whose neighborhood is examined until reaching a local minimum is generated. At the end, the most feasible solution found is retained as the final solution of the problem [Glover et al., 2003].

In conducting the constructive phase it is necessary to define a utility function for the specific problem. Said function allows evaluating each of the elements that might be part of the initial feasible solution. When all the elements have been evaluated, a Restricted Candidate List (RCL) is elaborated with those exhibiting the best utility function. That is to say:

$$RCL = \{x | L \leq f_c(x) \leq L + \alpha(U - L)\} \quad (20)$$

Where:

- $f_c(x)$ is the utility function of element $x$
- $\alpha$ is a random number between 0 and 1.
- In case there is a problem of minimization, $L$ is the lowest value found in the utility function, whereas $U$ is the greatest one.

The pseudo-code proposed by Resende and González (2003) is the following:

```
PROCEDURE Constructive Phase – V
PARAMETERS
       α: numeric value between 0 and 1
       E: problem data
       c(·): utility function
VARIABLES
       x: initial solution
       C: copy of problem data
BEGIN PROCEDURE
       x ← ∅
       C ← E
       Evaluate utility function c(e),∀e ∈ C
       WHILE C ≠ ∅
              c₊ ← min{c(e)|e ∈ C}
              c* ← max{c(e)|e ∈ C}
              RCL ← {e ∈ C | c(e) ≤ c₊ + α(c* − c₊)}
              Choose from the RCL a random element s that maintains solution feasibility
              x ← x ∪ {s}
              Remove element s from C
              Evaluate utility function c(e),∀e ∈ C
       END WHILE
       RETURN x
END PROCEDURE
```

Fig. 7. GRASP – Constructive phase

This phase chooses an RCL candidate at random to add it to the initial solution, and then it empties the RCL. The process of filling and emptying the RCL is repeated until a feasible solution is obtained. Thus, the clearest advantage of the process is that the initial solution is attained step by step without affecting the feasibility of the result [Glover and Kochenberger, 2003].

The second phase of GRASP uses a local search method that improves the value of the solution found for the objective function during the constructive phase, through simple swapping of the elements of the initial solution. Said procedure is analogue to conducting searches in the close vicinity of the initial solution within the problem's solving space [Ibidem]. The local search pseudo-code is the following [Resende and González, 2003]:

```
PROCEDURE Local Search Phase
PARAMETERS
       x⁰: current solution
       N(·): neighborhood of x⁰
       f(·): objective function
VARIABLES
       x: improved solution
       y: solution in the vicinity of x
BEGIN PROCEDURE
       x ← x⁰
       WHILE x is not a locally optimal in N(x)
              Find y ∈ N(x) such that f(y) < f(x) and y is a feasible solution
              x ← y
       END WHILE
       RETURN x
END PROCEDURE
```

Fig. 8. GRASP – Local Search

Finally, the pseudo-code for GRASP is:

```
PROCEDURE GRASP
PARAMETERS
       I: number of iterations
       α: numeric value between 0 and 1
       f(·): objective function
       c(·): utility function
VARIABLES
       E: problem data
       x: solution
       s: best solution
       u: objective function value
BEGIN PROCEDURE
       E ← Read problem data
       u = ∞
       i = 1
       WHILE i ≤ I
              x ← Constructive Phase - V (α,E,c(·))
              x ← Local Search Phase (x,N(x),f(·))
              IF f(x) < u THEN
                     u = f(x)
                     s ← x
              END IF
              i = i + 1
       END WHILE
       RETURN s
END PROCEDURE
```

Fig. 9. GRASP

### 3.1.3.2 Implementation of GRASP

During the constructive phase, our version of GRASP solves a relaxation of the problem at the COG constraint (12) by considering the feasibility of the latter in the local search phase, guaranteeing in this way the feasibility of the solution.
In the constructive phase of GRASP the utility function is used to find the best candidates to be placed at a given position in the container, which is filled up from bottom to top. For each available position, the utility function is defined as:

$$f_c(i,j,k,l) = \begin{cases} b_1 + \varepsilon_{i1} & \text{si } l = 1 \\ b_l + \varepsilon_{il} + \sum_{n=1}^{l-1} \sum_{m=1, m\neq i}^{I} d_{mn} & \text{si } l > 1 \end{cases} \tag{21}$$

Where:
- $i$: represents the box under evaluation for cell $(j,k,l)$ of the container.
- $b_l$: is the maximum possible deterministic deformation experimented by a box at level $l$.
- $\varepsilon_{il}$: is the stochastic deformation that can be experimented by box $i$ at level $l$.
- $d_{mn}$: is the deformation experimented by box $m$ at cell $(j,k,n)$ of the container.

When $l = 1$, that is, when the positions at the base of the container are under examination, the RCL is elaborated with those boxes that would undergo the least deformation under the maximum possible weight they can support. Considering equation (1), the utility function (21) applies the following evaluation strategy: the heaviest boxes are preferably located at the bottom level, so that the weight loaded on top of a box $i$ ($c_i$), located in cell $(j,k,1)$ is equal to the maximum weight bearable by ($S_i$). In this way, equation (1) is reduced to $b_1 + \varepsilon_{i1}$. For the rest of the levels of the container ($l > 1$), the RCL is elaborated with those boxes that, for one thing, may be least deformed when bearing on top the maximum load they can be assigned $(b_l + \varepsilon_{il})$ at level $l$, and for another thing, may induce the least deformation on the boxes supporting them $(\sum_{n=1}^{l-1} \sum_{m=1,m\neq i}^{I} d_{mn})$. In this way the algorithm makes sure that each new assignation is both good and feasible for the relaxation of the problem at the load distribution constraint.

The local search phase of GRASP was conceived to minimize stored box deformation and improve weight distribution. The latter is achieved by approximating the COG of the loaded container to its geometric center at the base level, as calculated along its length as its largest dimension. The local search comprises four stages. In the first one box pairs are swapped as to reduce total deformation and therefore minimize unoccupied volume. Said swapping is carried out according to a 2-Optimal algorithm [Croes, 1958] whose pseudo-code is the following:

```
PROCEDURE Box Swapping
PARAMETERS
        x^0: current solution
        f(·): objective function
VARIABLES
        x*: improved solution
        s*: objective function value of the improved solution
BEGIN PROCEDURE
        x* ← x^0
        s* = f(x^0)
        i = 1
        WHILE i is less than the number of elements in x^0
                j = i + 1
                WHILE j is less or equal to the number of elements in x^0
                        x ← x*
                        Swap box i with box j in x
                        IF f(x) < s* AND x is a feasible solution THEN
                                x* ← x
                                s* = f(x)
                        END IF
                        j = j + 1
                END WHILE
                i = i + 1
        END WHILE
        RETURN x*, s*
END PROCEDURE
```

Fig. 10. 2-Optimal box swapping

If, at this stage, total deformation can be reduced, we will have reached a better distribution of the boxes in the container, which would eventually constitute a better utilization of total available space. The second stage is intended to check whether there is room for additional boxes. If at least one more box can be added, stage 1 is repeated. Periodically, the procedure checks compliance with the problem relaxation constraints. It finishes when all available positions in the container have been checked. The pseudo-code of the second stage is:

```
PROCEDURE Add Boxes
PARAMETERS
        E: problem data
        x^0: current solution
        f(·): objective function
VARIABLES
        result: boolean variable that determines if any boxes were added to the solution
        s^0: objective function value of the current solution
BEGIN PROCEDURE
        result = false
        WHILE there are empty positions inside the container
                FOR EACH e IN E AND NOT IN x^0
                        IF adding e to x^0 does not exceed the container's weight limit
                        AND e can be supported by the boxes below THEN
```

```
                                            x⁰ ← x⁰ ∪ {e}
                                            s⁰ = f(x⁰)
                                            Remove e from E
                                            result = true
                                 END IF
                        END FOR EACH
         END WHILE
         RETURN x⁰,s⁰, result
END PROCEDURE
```

Fig. 11. Adding boxes to the solution

The third and fourth stages of the local search improve weight distribution within the container. In this respect, given that the container is divided in equal cells, the latter can be grouped in $j_{max}$ walls of dimensions $BL, BW k_{max}, BH l_{max}$, as illustrated in Figure 12.

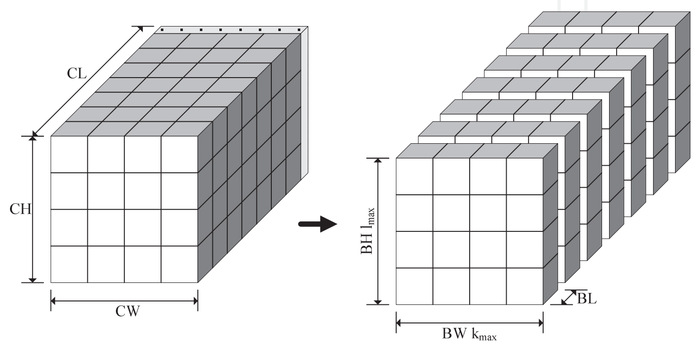


Fig. 12. Inner division of the container at stage 3 of the local search

In the third stage, these walls are swapped by the 2-Optimal algorithm, selecting for those modifications that allow driving the COG of the container to its medium length point. The pseudo-code goes as follows:

```
PROCEDURE Wall Swapping
PARAMETERS
         x⁰: current solution
         CL: container length
         j_max: number of possible walls alongside the container length
         cog(·): function that calculates the COG of the loaded container
VARIABLES
         x*: improved solution
         g*: COG of the improved solution
BEGIN PROCEDURE
         x* ← x⁰
         g* = cog(x⁰)
         i = 1
         WHILE i is less than the number of possible walls (j_max)
                        j = i + 1
                        WHILE j is less or equal to the number of possible walls (j_max)
                                 x ← x*
                                 Swap boxes in wall i with those in wall j
                                 g = cog(x)
                                 IF g is closer to CL/2 than g* THEN
                                            x* ← x
                                            g* = g
                                 END IF
                                 j = j + 1
                        END WHILE
                        i = i + 1
         END WHILE
         RETURN x*,g*
END PROCEDURE
```

Fig. 13. Wall swapping

The fourth stage initiates once the 2-Optimal wall swapping has been finished. In this stage, the container is divided in $k_{max}$ walls of dimensions $BLj_{max}, BW, BHl_{max}$, as illustrated in Figure 14.

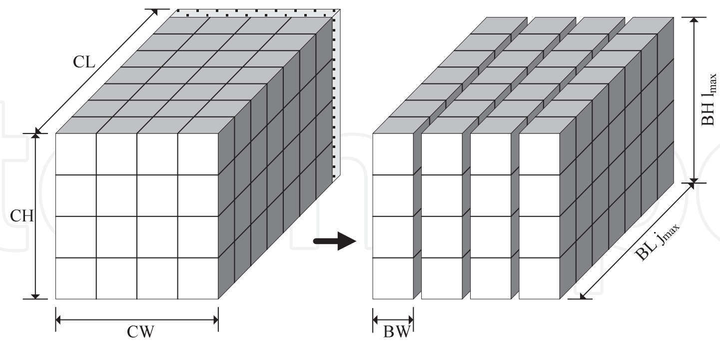


Fig. 14. Inner division of the container at stage four of the local search.

As the COG of the container is sought only along its length, the swapping of these walls is discarded because it would have no effect on the task. The incidence of these walls on the COG is analyzed by putting them back to front (reflection) as illustrated in Figure 15.

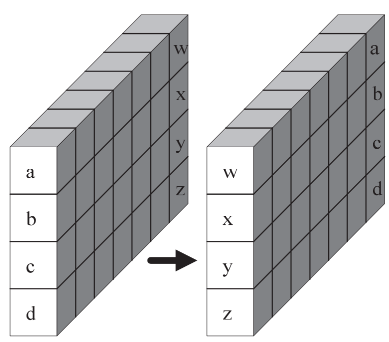


Fig. 15. Wall reflection.

The pseudo-code that is applied for this task is presented below:

```
PROCEDURE Reflect Wall
PARAMETERS
       x^0: current solution
       j_max: maximum number of available cells in the container along its length
       k: wall to reflect
VARIABLES
       W_k: set of boxes within wall k
       w^j: cell j along the container length in which box w has been placed
BEGIN PROCEDURE
       Assign to W_k all boxes that had been placed in wall k of x^0
       FOR EACH w IN W_k
              w^j = j_max - w^j + 1
       END FOR EACH
       RETURN x^0
END PROCEDURE
```

Fig. 16. Wall reflection

If at least one of these wall reflection movements drives the COG closer to the midpoint of the container's length, the third stage of the local search must be executed again. Otherwise, the local search is finished.
The pseudo-code for the local search phase is:

```
PROCEDURE Local Search Phase
PARAMETERS
        E: problem data
        x^0: current solution
        f(·): objective function
        CL: container length
        j_max: number of possible walls alongside the container's length
        k_max: number of possible walls alongside the container's width
        cog(·): function that calculates the COG of the loaded container
VARIABLES
        x*: improved solution
        g*: COG of the improved solution
BEGIN PROCEDURE
        x* ← Box Swapping (x^0, f(·))
        WHILE Add Boxes (E, x*, f(·)) = true
                x* ← Box Swapping (x*, f(·))
        END WHILE
        (x*, g*) ← Wall Swapping (x*, j_max, cog(·))
        FOR k = 1 TO k_max
                x ← x*
                x ← Reflect Wall (x, j_max, k)
                g = cog(x)
                IF g is closer to CL/2 than g* THEN
                        x* ← x
                        (x*, g*) ← Wall Swapping (x*, j_max, cog(·))
                        Set k = 0 to restart fourth stage
                END IF
        INCREMENT k
        RETURN x*
END PROCEDURE
```

Fig. 17. Local Search

Finally, the pseudo-code of the proposed GRASP metaheuristic is:

```
PROCEDURE GRASP
PARAMETERS
        I: number of iterations
        α: numeric value between 0 and 1
        f(·): objective function
        c(·): utility function
        cog(·): function that calculates the COG of the loaded container
        j_max: number of possible walls alongside the container's length
        k_max: number of possible walls alongside the container's width
VARIABLES
        E: problem data
        x: solution
        s: best solution
        u: objective function value of the best solution
BEGIN PROCEDURE
        E ← Read problem data
        u = ∞
        i = 1
        WHILE i ≤ I
                x ← Constructive Phase - V (α, E, c(·))
                x ← Local Search Phase (E, x, f(·), j_max, k_max, cog(·))
                IF f(x) < u THEN
                        u = f(x)
                        s ← x
                END IF
                i = i + 1
        END WHILE
        RETURN s
END PROCEDURE
```

Fig. 18. CLP solving GRASP

### 3.1.4 Computational results

The problem instances used to test the proposed heuristic procedure were generated as follows:

- The length, width and height of the container were set at 587cm, 233cm, and 220cm, respectively; corresponding to those used in previous works [Eley, 2002; Davies and Bischoff, 1999; Bischoff and Ratcliff, 1995].
- The dimensions of the boxes (length, width and height, in centimeters) were arbitrarily set at (293, 77, 55) and (72, 72, 72). The number of boxes $I$ of each problem is:

$$I = \left\lfloor \frac{CL}{BL} \right\rfloor \cdot \left\lfloor \frac{CW}{BW} \right\rfloor \cdot \left\lfloor \frac{CH}{BH} \right\rfloor \tag{22}$$

- The weights of the $I$ boxes as measured in kg were generated by means of a uniform distribution with parameters $a = 5$ and $b = 10$.
- Each of the $I$ boxes' bearable weight (in kg) was estimated by multiplying its weight by a random number between 1 and 3.
- The weight limit (in kg) that can be loaded into the container was established arbitrarily as 90% of the total weight of the $I$ boxes.
- One problem instance was generated for each box size configuration in order to perform the qualitative and integration analysis only on two problems: a 24 box one and a 72 box one.
- $\alpha$ values of 0.05; 0.10; 0.15 and 0.20 were used in elaborating the RCL. The realized implementation was executed 1,000 times for every instance and value of $\alpha$. Each execution comprised 500 GRASP iterations.

Considering equation (1):

- For all tested instances, the parameter for maximum deterministic deformation $b_n$ (expressed in cm) is calculated arbitrarily as shown below:

$$b_n = \frac{l_{max} - n}{\left\lceil \frac{l_{max}}{2} \right\rceil} \tag{23}$$

- For all tested instances, the parameter for minimum deterministic deformation $a_n$ (expressed in cm) is calculated arbitrarily as shown below:

$$a_n = b_n - \frac{l_{max}}{10} \tag{24}$$

- Stochastic deformation $\varepsilon_{xn}$ (expressed in cm) is a random number in the interval $(-a_n, a_n)$

The proposed GRASP was implemented on C# using MS Visual Studio 2005. All tests were run on a 2 GHz Dual Core processor with 3.49 GB RAM loaded with Windows XP.

The tables below summarize the results obtained in testing the instances resulting from the different values of $\alpha$. The registered parameters are: percentage of container space utilized; total load weight with respect to the weight limit of the container; distance from the COG of the cargo to the center of the base of the container alongside its length; and average length of time spent in executing the solution.

| Number of boxes | α | Average duration of a 500 iteration execution (sec) | Utilization of the container's space (%) | | | Utilization of the weight capacity of the container (%) | | | Distance from the COG of the loaded container to the center of the base of the container (cm) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Min | Max | Avg | Min | Max | Avg | Min | Max | Avg |
| 24 | 0.05 | 16.53 | 89.82 | 90.15 | 89.96 | 98.66 | 99.95 | 99.57 | 0.01 | 16.30 | 2.85 |
| | 0.10 | 13.48 | 89.82 | 90.14 | 89.98 | 98.66 | 99.95 | 99.50 | 0.00 | 18.52 | 2.89 |
| | 0.15 | 12.12 | 89.86 | 90.17 | 89.98 | 98.66 | 99.95 | 99.56 | 0.01 | 12.79 | 2.92 |
| | 0.20 | 12.33 | 89.84 | 90.15 | 89.98 | 98.66 | 99.95 | 99.51 | 0.02 | 16.69 | 2.41 |
| 72 | 0.05 | 53.18 | 81.32 | 81.54 | 81.44 | 99.18 | 100.00 | 99.83 | 0.00 | 1.40 | 0.13 |
| | 0.10 | 48.51 | 81.36 | 81.53 | 81.43 | 99.23 | 100.00 | 99.87 | 0.00 | 1.66 | 0.15 |
| | 0.15 | 47.72 | 81.34 | 81.54 | 81.43 | 99.24 | 100.00 | 99.86 | 0.00 | 4.85 | 0.13 |
| | 0.20 | 48.45 | 81.34 | 81.54 | 81.43 | 99.44 | 100.00 | 99.89 | 0.00 | 1.45 | 0.18 |

Table 4. Summary of results

The qualitative analysis only made use of those results whose $\alpha$ value allowed an optimal utilization of the space inside the container. Such value was determined through a One-Way ANOVA applied to the results of every instance.
The 24 box problem was assessed through Levene's test conducted in Minitab, which showed no statistical evidence of homogeneity between the variances of the different values of $\alpha$ at a 95% confidence level. This led to applying Tamhane's test to analyze differences between means. The results, as obtained in SPSS are:

| (I) alpha | (J) alpha | Mean Difference (I-J) | Std Error | Sig | 95% Confidence Interval | |
|---|---|---|---|---|---|---|
| | | | | | Lower Bound | Upper Bound |
| 0.05 | 0.10 | 5.1794E+03 | 719.2948 | 0.0000 | 3,285.0517 | 7,073.8021 |
| | 0.15 | 5.6447E+03 | 760.5627 | 0.0000 | 3,641.6754 | 7,647.7988 |
| | 0.20 | 3.9670E+03 | 748.0209 | 0.0000 | 1,997.0231 | 5,937.0744 |
| 0.10 | 0.05 | -5.1794E+03 | 719.2948 | 0.0000 | -7,073.8021 | -3,285.0517 |
| | 0.15 | 4.6531E+02 | 742.2614 | 0.9890 | -1,489.5696 | 2,420.1900 |
| | 0.20 | -1.2124E+03 | 729.4050 | 0.4570 | -3,133.3868 | 708.6305 |
| 0.15 | 0.05 | -5.6447E+03 | 760.5627 | 0.0000 | -7,647.7988 | -3,641.6754 |
| | 0.10 | -4.6531E+02 | 742.2614 | 0.9890 | -2,420.1900 | 1,489.5696 |
| | 0.20 | -1.6777E+03 | 770.1314 | 0.1640 | -3,705.9459 | 350.5692 |
| 0.20 | 0.05 | -3.9670E+03 | 748.0209 | 0.0000 | -5,937.0744 | -1,997.0231 |
| | 0.10 | 1.2124E+03 | 729.4050 | 0.4570 | -708.6305 | 3,133.3868 |
| | 0.15 | 1.6777E+03 | 770.1314 | 0.1640 | -350.5692 | 3,705.9459 |

Table 5. Objective function mean differences for the 24 box problem

At a 95% confidence interval, it can be concluded that, for the 24 box problem, the best objective function value is obtained with $\alpha = 0.15$. Applying the same procedure to the 72 box problem, Tamhane's test gave the following results:

| (I) alpha | (J) alpha | Mean Difference (I-J) | Std, Error | Sig, | 95% Confidence Interval | |
|---|---|---|---|---|---|---|
| | | | | | Lower Bound | Upper Bound |
| 0.05 | 0.10 | -2.8474E+03 | 439.2716 | 0.0000 | -4,004.2949 | -1,690.5171 |
| | 0.15 | -3.5134E+03 | 436.5531 | 0.0000 | -4,663.1334 | -2,363.6766 |
| | 0.20 | -2.9394E+03 | 427.8336 | 0.0000 | -4,066.1716 | -1,812.6444 |
| 0.10 | 0.05 | 2.8474E+03 | 439.2716 | 0.0000 | 1,690.5171 | 4,004.2949 |
| | 0.15 | -6.6600E+02 | 445.3927 | 0.5810 | -1,839.0072 | 507.0092 |

| (I) alpha | (J) alpha | Mean Difference (I-J) | Std, Error | Sig, | 95% Confidence Interval | |
|---|---|---|---|---|---|---|
| | | | | | Lower Bound | Upper Bound |
| | 0.20 | -9.2002E+01 | 436.8497 | 1.0000 | -1,242.5138 | 1,058.5098 |
| 0.15 | 0.05 | 3.5134E+03 | 436.5531 | 0.0000 | 2,363.6766 | 4,663.1334 |
| | 0.10 | 6.6600E+02 | 445.3927 | 0.5810 | -507.0092 | 1,839.0072 |
| | 0.20 | 5.7400E+02 | 434.1161 | 0.7100 | -569.3141 | 1,717.3081 |
| 0.20 | 0.05 | 2.9394E+03 | 427.8336 | 0.0000 | 1,812.6444 | 4,066.1716 |
| | 0.10 | 9.2002E+01 | 436.8497 | 1.0000 | -1,058.5098 | 1,242.5138 |
| | 0.15 | -5.7400E+02 | 434.1161 | 0.7100 | -1,717.3081 | 569.3141 |

Table 6. Objective function mean differences for the 72 box problem

With a 95% confidence interval, it can be concluded that, for the 72 box problem, the best objective function value is obtained with $\alpha = 0.05$.

Pareto analysis was applied to the solutions obtained for each of the two problems. Sixty six and sixty eight percent of the solutions of the 24 and 72 box problems were respectively analyzed, representing 20 alternatives of each problem. Their adjusted probabilities, as well as the expected values of the objective function and their standard deviations, all of them specified for IAM, are shown in Table **7**. The load arrangement of each of the selected alternatives is shown in Appendix 1.

| i | 24 boxes | | | | 72 boxes | | | |
|---|---|---|---|---|---|---|---|---|
| | Frequency | $P(i), \alpha$ | $E(z^i)$ | $\sigma^{Z_i}$ | Frequency | $P(i), \beta$ | $E(z^i)$ | $\sigma^{Z_i}$ |
| 1 | 50 | 0.0765 | -27,130,328.4503 | 0 | 45 | 0.0667 | -24,497,492.7011 | 0 |
| 2 | 49 | 0.0749 | -27,069,022.1712 | 0 | 39 | 0.0578 | -24,520,440.0663 | 0 |
| 3 | 47 | 0.0719 | -27,079,711.8588 | 0 | 39 | 0.0578 | -24,512,652.0699 | 0 |
| 4 | 37 | 0.0566 | -27,071,467.0888 | 0 | 38 | 0.0563 | -24,516,202.0138 | 0 |
| 5 | 36 | 0.0550 | -27,066,716.6023 | 0 | 38 | 0.0563 | -24,502,754.7084 | 0 |
| 6 | 35 | 0.0535 | -27,068,471.0632 | 0 | 38 | 0.0563 | -24,496,098.7389 | 0 |
| 7 | 35 | 0.0535 | -27,088,277.5107 | 0 | 37 | 0.0548 | -24,504,513.1559 | 0 |
| 8 | 34 | 0.0520 | -27,072,118.7352 | 0 | 36 | 0.0533 | -24,506,201.4630 | 0 |
| 9 | 31 | 0.0474 | -27,052,647.8558 | 0 | 35 | 0.0519 | -24,515,697.0110 | 0 |
| 10 | 30 | 0.0459 | -27,066,675.3657 | 0 | 35 | 0.0519 | -24,503,437.4409 | 0 |
| 11 | 30 | 0.0459 | -27,093,102.3737 | 0 | 33 | 0.0489 | -24,499,385.3670 | 0 |
| 12 | 29 | 0.0443 | -27,075,607.8937 | 0 | 33 | 0.0489 | -24,493,609.8728 | 0 |
| 13 | 29 | 0.0443 | -27,070,443.4751 | 0 | 32 | 0.0474 | -24,504,895.3574 | 0 |
| 14 | 28 | 0.0428 | -27,075,537.4827 | 0 | 31 | 0.0459 | -24,491,334.7171 | 0 |
| 15 | 27 | 0.0413 | -27,065,936.3019 | 0 | 30 | 0.0444 | -24,496,901.1626 | 0 |
| 16 | 27 | 0.0413 | -27,059,497.8935 | 0 | 29 | 0.0430 | -24,515,483.4574 | 0 |
| 17 | 26 | 0.0398 | -27,070,016.2422 | 0 | 29 | 0.0430 | -24,507,811.1248 | 0 |
| 18 | 25 | 0.0382 | -27,091,519.0548 | 0 | 27 | 0.0400 | -24,526,575.7616 | 0 |
| 19 | 25 | 0.0382 | -27,071,515.1479 | 0 | 26 | 0.0385 | -24,511,051.1289 | 0 |
| 20 | 24 | 0.0367 | -27,052,190.1312 | 0 | 25 | 0.0370 | -24,495,802.2597 | 0 |

α and β correspond to the joint integral index values of the alternatives associated to the cardinal result variable shown in table 11.

Table 7. Frequency, probability, expected value and deviation of the selected alternatives

### 3.2 Qualitative analysis

The qualitative stage is based on stochastic multicriteria acceptability analysis with ordinal SMAA-O data (Lahdelma et al., 2003). SMAA-O has been developed to support public decision making processes. According to IAM, the use of SMAA-O is restricted to ordinal variables and to the alternatives resulting from the cardinal analysis. This phase is particularly complex because of the difficulties that usually arise when defining the matrix of typical relative values that will be used as input. IAM's ordinal stage allows identifying the set of favorable weights that support each of the alternatives in a particular ranking. The most important resulting variable (indicator) featuring this analysis is the ordinal acceptability index ($b_r^t$), which defines its probability of acceptation of each alternative and indicates the ordinal ranking ($r$). Nevertheless, this indicator might prove insufficient to support the decision making process. For this reason, the technique provides two additional indicators: range values and central weight vectors, which establish the bounds of each alternative's favorable weight set and its associated centroid, respectively. In using this method, for every cardinal variable ($p(t)$), it is necessary to qualify each alternative's set of ordinal variables. Likert tables can be used to convert particular qualitative aspects into ordinal variables (Albaum, 1997). In this case, each of the original binary variables of an optimization problem has several ordinal associated variables, that are defined by the decision-makers, and that altogether allow building up the ordinal value associated to each alternative. For it to be efficient, the procedure applies the class concept (represented through index ($a$) in Table 10), which refers to a set of alternatives with identical utilities for all their associated ordinal variables. In the present work we have only considered one qualitative criterion, and consequently, one single analysis ranking ($r = 1$). The particular features of IAM's ordinal stage, which are explained below, are detailed in García et al. (2009).

The qualitative variable was defined as the fragility of the elements packed inside the boxes, which were then classified according to a scale ranging from 1 to 3, in which 3 indicated fragile contents, and 1, resistant ones; while 2 was assigned to boxes containing medium resistance materials.

The load arrangement of each of the alternatives corresponding to the 24 and 72 box problems was qualified according to table 8, which penalizes the boxes according to the fragility of their content and the level of the container at which they have been placed.

| **Box location penalization for the 24 box problem** | | | | **Box location penalization for the 72 box problem** | | | |
|---|---|---|---|---|---|---|---|
| **Levels of the container** | **Box content resistance** | | | **Levels of the container** | **Box content resistance** | | |
| | **1** | **2** | **3** | | **1** | **2** | **3** |
| **1** | 1 | 3 | 3 | **1** | 1 | 2 | 3 |
| **2** | 2 | 1 | 3 | **2** | 2 | 1 | 2 |
| **3** | 3 | 1 | 2 | **3** | 3 | 3 | 1 |
| **4** | 3 | 2 | 1 | | | | |

Table 8. Box location penalization

For each alternative we summed up all the penalization scores assigned to the stored boxes according to their content resistance value and location. The results were classified in four categories according to (Likert) table 9, which shows the ordinal values assigned to the different alternatives.

Table 10 shows the input of IAM's qualitative stage and its associated index of acceptability. For the two problems treated in the current work, the results show that all the weights support the alternatives corresponding to class 1 ($a = 1$) for acceptability ranking 1.

| Likert table for the 24 box problem | | Likert table for the 72 box problem | |
|---|---|---|---|
| **Criterion** | **Ordinal value** | **Criterion** | **Ordinal value** |
| If total < 37 | 1 | If total < 109 | 1 |
| If 37 ≤ total < 49 | 2 | If 109 ≤ total < 145 | 2 |
| If 49 ≤ total < 61 | 3 | If 145 ≤ total < 181 | 3 |
| If 61 ≤ total | 4 | If 181 ≤ total | 4 |

Table 9. Likert tables for the 24 and 72 box problems

| Ordinal parameters and indicators of the 24 box problem | | | | Ordinal parameters and indicators of the 72 box problem | | | |
|---|---|---|---|---|---|---|---|
| *a* | *F(a)* | *Fragility, j: 1* | $b_1^a$ | *a* | *F(a)* | *Fragility, j: 1* | $b_1^a$ |
| 1 | t: {4, 5, 9, 10, 11, 12, 13, 14, 16, 18, 19} | 1 | 1 | 1 | t:{12, 17} | 2 | 1 |
| 2 | t: {1, 2, 3, 6, 7, 8, 9, 15, 17, 20} | 2 | 0 | 2 | t:{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 18, 19, 20} | 3 | 0 |

Table 10. Ordinal parameters and indicators

### 3.3 Integration analysis

The cardinal and ordinal analyses help determine a set of results that support the decision-making process significantly. At the same time, these results are the input of the integration procedure, which provides the indicators that are going to facilitate the analysis of the problem in a broader context. The integration analysis stage analyses what kind of valuations would make each alternative the preferred one in a particular ordinal ranking. The integration ranking ($o$) of each alternative is conditioned by the ordinal analysis ranking because each optimal cardinal solution may have a different ordinal ranking status. In the present case we have focused on ordinal ranking 1 ($r = 1$). The input of the deterministic SMAA (Lahdelma and Salminen, 2001) applied to complete the integration analysis stage of IAM is a utility matrix composed of $m$ alternatives and two result variables (cardinal and ordinal). Thus, the process is simplified, allowing the obtention of a series of 2-dimensional central weight vectors with two ranges of mutually complementing favorable convex weights each, and of the integral acceptability indexes of each alternative. The particular features of IAM's integration analysis stage, which are explained below, are detailed in García et al. (2009).

For the integration analysis, the joint integral index is defined as $p_r(e^t)$. This value provides a comprehensive assessment of each alternative's ordinal ranking. Assuming that both cardinal and ordinal variables (listed in tables 7 and 10, respectively) are independent, the index is calculated as:

$$p_r(e^t) = p(t)b_1^t \tag{25}$$

Similar to the ordinal phase, the integration phase has a comparable set of indicators supporting the decision making process: the integral acceptability index and the weight of the result variable (qualitative and quantitative). As in the ordinal phase, in the integration phase, we have only used the acceptability index as a support indicator. The results of the integration phase are shown below.

| Integration indicators of the 24 box problem | | | Integration indicators of the 72 box problem | | |
|---|---|---|---|---|---|
| $t$ | $p_1(e^t)$ | $d_1^t$ | $t$ | $p_1(e^t)$ | $d_1^t$ |
| 1 | 0.0765 | 0.0109 | 1 | 0.0667 | 0.0088 |
| 4 | 0.0566 | 0.9891 | 12 | 0.0489 | 0.9912 |
| 5, 9, 10, 11, 12, 13, 14, 16, 18, 19. | α | 0 | 17 | β | 0 |
| 2, 3, 6, 7, 8, 9, 15, 17, 20. | 0 | 0 | 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 18, 19, 20. | 0 | 0 |

α and β correspond to the joint integral index values of the alternatives associated to the cardinal result variable.

Table 11. Results of the integration phase

The results indicate that, from the standpoint of the considerations addressed in the present analysis, alternatives 4 and 12 constitute the most favorable load arrangements for the 24 and 72 box problems, respectively.

## 4. Conclusions

The present work addresses CLP optimization in an integrated and actual context, including several restrictions which had not been worked out altogether in previous works. In addition, it introduces the modeling of SKU deformation and fragility content for the first time. New research perspectives have to do with the inclusion of additional considerations such as the complexity of the loading arrangement that ultimately facilitates unloading, and the management of client priority issues.

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## Appendix

| Box | Weight | Supported weight | Fragility | Box | Weight | Supported weight | Fragility | Box | Weight | Supported weight | Fragility |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6.24 | 15.77 | 3 | 9 | 6.63 | 7.84 | 1 | 17 | 5.94 | 10.57 | 2 |
| 2 | 6.18 | 11.37 | 2 | 10 | 9.98 | 22.04 | 2 | 18 | 8.14 | 22.29 | 3 |
| 3 | 5.44 | 8.7 | 1 | 11 | 7.05 | 15.65 | 2 | 19 | 8.67 | 24.47 | 3 |
| 4 | 6.49 | 13.73 | 2 | 12 | 5.19 | 6.05 | 1 | 20 | 5.68 | 11.83 | 2 |
| 5 | 8.98 | 21.1 | 3 | 13 | 6.05 | 13.36 | 2 | 21 | 9.69 | 25.69 | 3 |
| 6 | 5.18 | 14.72 | 3 | 14 | 8.48 | 10.74 | 1 | 22 | 6.63 | 16.52 | 3 |
| 7 | 9.07 | 11.9 | 1 | 15 | 6.7 | 7.4 | 1 | 23 | 9.81 | 23.04 | 3 |
| 8 | 6.04 | 10.5 | 2 | 16 | 9.48 | 19.46 | 2 | 24 | 8.89 | 25.51 | 3 |

Table 12. Twenty four box problem

| Box | Weight | Supported weight | Fragility | Box | Weight | Supported weight | Fragility | Box | Weight | Supported weight | Fragility |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 9.45 | 26.35 | 3 | 25 | 6.81 | 8.74 | 1 | 49 | 6.97 | 17.12 | 3 |
| 2 | 8.77 | 18.68 | 2 | 26 | 8.32 | 22.18 | 3 | 50 | 8.33 | 15.06 | 2 |
| 3 | 8.34 | 9.21 | 1 | 27 | 10 | 28.72 | 3 | 51 | 5.93 | 10.85 | 2 |
| 4 | 5.15 | 15 | 3 | 28 | 6.18 | 8.02 | 1 | 52 | 5.45 | 13.7 | 3 |
| 5 | 7.85 | 11.42 | 1 | 29 | 8.34 | 22.98 | 3 | 53 | 7.14 | 17.8 | 3 |
| 6 | 9.1 | 25.15 | 3 | 30 | 6.63 | 11.93 | 2 | 54 | 7.74 | 11.19 | 1 |
| 7 | 8.03 | 20.94 | 3 | 31 | 7.84 | 23.09 | 3 | 55 | 6.06 | 7.73 | 1 |
| 8 | 5.76 | 13.36 | 2 | 32 | 8.9 | 12.9 | 1 | 56 | 6.81 | 16.73 | 3 |
| 9 | 6.09 | 17.38 | 3 | 33 | 8.11 | 8.4 | 1 | 57 | 7.12 | 17.03 | 3 |
| 10 | 6.47 | 10.9 | 2 | 34 | 7.11 | 19.14 | 3 | 58 | 6.96 | 12.48 | 2 |
| 11 | 6.36 | 15.77 | 3 | 35 | 6.12 | 13.51 | 2 | 59 | 6.39 | 15.9 | 3 |
| 12 | 6.13 | 16.88 | 3 | 36 | 8.57 | 24.31 | 3 | 60 | 7.72 | 20.83 | 3 |
| 13 | 6.2 | 9.52 | 1 | 37 | 6.42 | 16.4 | 3 | 61 | 6.08 | 17.02 | 3 |
| 14 | 7.42 | 8.26 | 1 | 38 | 8.63 | 10.3 | 1 | 62 | 6.6 | 17.06 | 3 |
| 15 | 5.89 | 9.96 | 2 | 39 | 8.94 | 25.73 | 3 | 63 | 8.62 | 19.75 | 2 |
| 16 | 8.03 | 20.89 | 3 | 40 | 7.27 | 16.43 | 2 | 64 | 9.72 | 10.55 | 1 |
| 17 | 7.3 | 18.41 | 3 | 41 | 5.04 | 6.77 | 1 | 65 | 9.13 | 10.62 | 1 |
| 18 | 7.9 | 16.57 | 2 | 42 | 8.48 | 9.08 | 1 | 66 | 8.51 | 9.55 | 1 |
| 19 | 9.73 | 15.56 | 1 | 43 | 8.4 | 21.39 | 3 | 67 | 6.37 | 7 | 1 |
| 20 | 8.21 | 16.72 | 2 | 44 | 9.01 | 18.43 | 2 | 68 | 7.22 | 15.79 | 2 |
| 21 | 8.08 | 10.91 | 1 | 45 | 7.63 | 14.2 | 2 | 69 | 5.41 | 15.55 | 3 |
| 22 | 9.76 | 15.37 | 1 | 46 | 9.4 | 10.67 | 1 | 70 | 8.04 | 23.06 | 3 |
| 23 | 7.11 | 15.77 | 2 | 47 | 9.87 | 14.65 | 1 | 71 | 6.12 | 13.22 | 2 |
| 24 | 6.3 | 10.25 | 1 | 48 | 9.75 | 22.31 | 2 | 72 | 6.56 | 11.65 | 2 |

Table 13. Seventy two box problem

| Position representation (X,Y,Z) | Position representation (X,Y,Z) | Position representation (X,Y,Z) | Position representation (X,Y,Z) |
|---|---|---|---|
| 1,1,1 = 1 | 1,1,2 = 7 | 1,1,3 = 13 | 1,1,4 = 19 |
| 1,2,1 = 2 | 1,2,2 = 8 | 1,2,3 = 14 | 1,2,4 = 20 |
| 1,3,1 = 3 | 1,3,2 = 9 | 1,3,3 = 15 | 1,3,4 = 21 |
| 2,1,1 = 4 | 2,1,2 = 10 | 2,1,3 = 16 | 2,1,4 = 22 |
| 2,2,1 = 5 | 2,2,2 = 11 | 2,2,3 = 17 | 2,2,4 = 23 |
| 2,3,1 = 6 | 2,3,2 = 12 | 2,3,3 = 18 | 2,3,4 = 24 |

Table 14. List of positions inside the container for 24 boxes

| Position representation (X,Y,Z) | Position representation (X,Y,Z) | Position representation (X,Y,Z) | Position representation (X,Y,Z) |
|---|---|---|---|
| 1,1,1 = 1 | 3,3,1 = 19 | 5,2,2 = 37 | 7,1,3 = 55 |
| 2,1,1 = 2 | 4,3,1 = 20 | 6,2,2 = 38 | 8,1,3 = 56 |
| 3,1,1 = 3 | 5,3,1 = 21 | 7,2,2 = 39 | 1,2,3 = 57 |
| 4,1,1 = 4 | 6,3,1 = 22 | 8,2,2 = 40 | 2,2,3 = 58 |
| 5,1,1 = 5 | 7,3,1 = 23 | 1,3,2 = 41 | 3,2,3 = 59 |
| 6,1,1 = 6 | 8,3,1 = 24 | 2,3,2 = 42 | 4,2,3 = 60 |
| 7,1,1 = 7 | 1,1,2 = 25 | 3,3,2 = 43 | 5,2,3 = 61 |
| 8,1,1 = 8 | 2,1,2 = 26 | 4,3,2 = 44 | 6,2,3 = 62 |
| 1,2,1 = 9 | 3,1,2 = 27 | 5,3,2 = 45 | 7,2,3 = 63 |
| 2,2,1 = 10 | 4,1,2 = 28 | 6,3,2 = 46 | 8,2,3 = 64 |
| 3,2,1 = 11 | 5,1,2 = 29 | 7,3,2 = 47 | 1,3,3 = 65 |
| 4,2,1 = 12 | 6,1,2 = 30 | 8,3,2 = 48 | 2,3,3 = 66 |
| 5,2,1 = 13 | 7,1,2 = 31 | 1,1,3 = 49 | 3,3,3 = 67 |
| 6,2,1 = 14 | 8,1,2 = 32 | 2,1,3 = 50 | 4,3,3 = 68 |
| 7,2,1 = 15 | 1,2,2 = 33 | 3,1,3 = 51 | 5,3,3 = 69 |
| 8,2,1 = 16 | 2,2,2 = 34 | 4,1,3 = 52 | 6,3,3 = 70 |
| 1,3,1 = 17 | 3,2,2 = 35 | 5,1,3 = 53 | 7,3,3 = 71 |
| 2,3,1 = 18 | 4,2,2 = 36 | 6,1,3 = 54 | 8,3,3 = 72 |

Table 15. List of positions inside the container for 72 boxes

| i | Load arrangement (position - box) | Ordinal criterion total |
|---|---|---|
| 1 | 1-24; 2-11; 3-5; 4-18; 5-13; 6-21; 7-10; 8-2; 9-6; 10-19; 11-9; 12-23; 13-17; 14-7; 15-20; 16-1; 17-12; 18-22; 19-15; 20-0; 21-4; 22-8; 23-0; 24-3 | 39 |
| 2 | 1-18; 2-21; 3-24; 4-16; 5-4; 6-23; 7-2; 8-19; 9-14; 10-1; 11-11; 12-13; 13-20; 14-8; 15-15; 16-9; 17-22; 18-3; 19-6; 20-5; 21-0; 22-12; 23-0; 24-17 | 40 |
| 3 | 1-21; 2-23; 3-24; 4-11; 5-19; 6-10; 7-18; 8-2; 9-4; 10-9; 11-5; 12-13; 13-7; 14-15; 15-20; 16-6; 17-8; 18-17; 19-12; 20-0; 21-1; 22-0; 23-22; 24-3 | 38 |
| 4 | 1-10; 2-11; 3-19; 4-24; 5-21; 6-5; 7-22; 8-6; 9-18; 10-16; 11-13; 12-12; 13-17; 14-15; 15-9; 16-2; 17-8; 18-20; 19-7; 20-0; 21-3; 22-1; 23-4; 24-0 | 36 |
| 5 | 1-18; 2-19; 3-10; 4-6; 5-23; 6-21; 7-11; 8-24; 9-20; 10-17; 11-13; 12-16; 13-15; 14-3; 15-22; 16-14; 17-4; 18-8; 19-1; 20-9; 21-0; 22-0; 23-12; 24-2 | 33 |
| 6 | 1-10; 2-23; 3-18; 4-7; 5-24; 6-16; 7-1; 8-20; 9-11; 10-12; 11-13; 12-4; 13-19; 14-5; 15-9; 16-8; 17-15; 18-22; 19-17; 20-0; 21-2; 22-0; 23-6; 24-3 | 43 |
| 7 | 1-19; 2-13; 3-23; 4-5; 5-21; 6-22; 7-24; 8-8; 9-11; 10-7; 11-18; 12-1; 13-14; 14-2; 15-3; 16-17; | 37 |

| i | Load arrangement (position - box) | Ordinal criterion total |
|---|---|---|
| | 17-15; 18-12; 19-20; 20-0; 21-4; 22-6; 23-9; 24-0 | |
| 8 | 1-18; 2-24; 3-21; 4-4; 5-19; 6-23; 7-5; 8-7; 9-15; 10-22; 11-10; 12-1; 13-13; 14-12; 15-3; 16-17; 17-2; 18-6; 19-20; 20-8; 21-0; 22-0; 23-11; 24-9 | 38 |
| 9 | 1-21; 2-19; 3-22; 4-5; 5-11; 6-24; 7-16; 8-4; 9-17; 10-13; 11-6; 12-10; 13-7; 14-20; 15-12; 16-2; 17-15; 18-8; 19-1; 20-9; 21-0; 22-3; 23-0; 24-14 | 30 |
| 10 | 1-19; 2-21; 3-1; 4-24; 5-16; 6-18; 7-5; 8-22; 9-9; 10-7; 11-4; 12-11; 13-2; 14-15; 15-8; 16-12; 17-17; 18-10; 19-13; 20-3; 21-0; 22-6; 23-20; 24-0 | 36 |
| 11 | 1-21; 2-6; 3-19; 4-18; 5-10; 6-24; 7-4; 8-3; 9-23; 10-2; 11-5; 12-22; 13-8; 14-14; 15-17; 16-9; 17-20; 18-11; 19-13; 20-0; 21-1; 22-0; 23-12; 24-15 | 34 |
| 12 | 1-18; 2-24; 3-23; 4-5; 5-19; 6-4; 7-6; 8-8; 9-21; 10-7; 11-11; 12-3; 13-2; 14-16; 15-17; 16-15; 17-1; 18-20; 19-9; 20-0; 21-13; 22-12; 23-22; 24-0 | 36 |
| 13 | 1-21; 2-18; 3-10; 4-5; 5-23; 6-22; 7-11; 8-20; 9-1; 10-4; 11-19; 12-24; 13-6; 14-3; 15-8; 16-13; 17-15; 18-12; 19-9; 20-0; 21-14; 22-2; 23-17; 24-0 | 34 |
| 14 | 1-23; 2-5; 3-10; 4-21; 5-19; 6-18; 7-6; 8-14; 9-24; 10-4; 11-22; 12-1; 13-15; 14-20; 15-3; 16-9; 17-2; 18-8; 19-13; 20-0; 21-11; 22-12; 23-17; 24-0 | 36 |
| 15 | 1-16; 2-11; 3-10; 4-5; 5-23; 6-24; 7-6; 8-13; 9-19; 10-1; 11-22; 12-14; 13-9; 14-3; 15-2; 16-20; 17-7; 18-12; 19-17; 20-0; 21-15; 22-8; 23-4; 24-0 | 41 |
| 16 | 1-18; 2-10; 3-22; 4-24; 5-1; 6-19; 7-11; 8-13; 9-21; 10-23; 11-9; 12-6; 13-14; 14-20; 15-17; 16-8; 17-4; 18-15; 19-3; 20-12; 21-0; 22-5; 23-0; 24-2 | 34 |
| 17 | 1-20; 2-24; 3-21; 4-10; 5-23; 6-19; 7-22; 8-5; 9-13; 10-6; 11-11; 12-14; 13-12; 14-8; 15-2; 16-18; 17-9; 18-4; 19-0; 20-15; 21-17; 22-3; 23-1; 24-0 | 38 |
| 18 | 1-24; 2-21; 3-1; 4-5; 5-22; 6-23; 7-13; 8-10; 9-18; 10-11; 11-16; 12-4; 13-9; 14-14; 15-20; 16-12; 17-6; 18-17; 19-3; 20-2; 21-0; 22-8; 23-0; 24-15 | 30 |
| 19 | 1-23; 2-7; 3-19; 4-18; 5-16; 6-5; 7-24; 8-17; 9-4; 10-8; 11-22; 12-10; 13-20; 14-12; 15-2; 16-11; 17-15; 18-3; 19-1; 20-0; 21-9; 22-0; 23-13; 24-6 | 36 |
| 20 | 1-23; 2-18; 3-16; 4-24; 5-10; 6-21; 7-22; 8-13; 9-14; 10-4; 11-19; 12-1; 13-6; 14-17; 15-11; 16-9; 17-3; 18-20; 19-8; 20-2; 21-0; 22-12; 23-15; 24-0 | 37 |

Table 16. Twenty four box problem alternatives selected for IAM

| i | Load arrangement (position - box) | Ordinal criterion total |
|---|---|---|
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| i | Load arrangement (position - box) | Ordinal criterion total |
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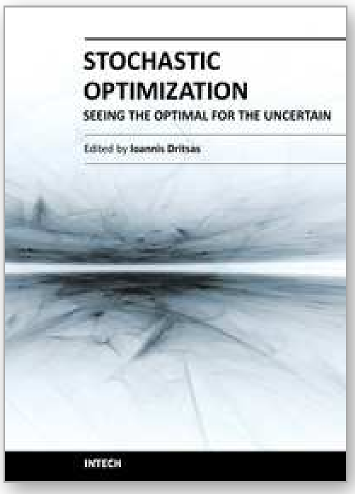
Table 17. Seventy two box problem selected alternatives for IAM

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Stochastic Optimization Algorithms have become essential tools in solving a wide range of difficult and critical optimization problems. Such methods are able to find the optimum solution of a problem with uncertain elements or to algorithmically incorporate uncertainty to solve a deterministic problem. They even succeed in â€œfighting uncertainty with uncertaintyâ€. This book discusses theoretical aspects of many such algorithms and covers their application in various scientific fields.



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