

# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

185,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index  
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?  
Contact [book.department@intechopen.com](mailto:book.department@intechopen.com)

Numbers displayed above are based on latest data collected.  
For more information visit [www.intechopen.com](http://www.intechopen.com)



# Dynamical Analysis of a Biped Locomotion CPG Modelled by Means of Oscillators

Armando Carlos de Pina Filho and Max Suell Dutra  
Federal University of Rio de Janeiro – UFRJ  
Brazil

## 1. Introduction

The study of mechanisms like mechanical members intends not only to build autonomous robots, but also to help in the rehabilitation of human being. The study of locomotion to take part in this context, and has been intensively studied since the second half of 20th century. An ample vision of the state of the technique up to 1990 can be found in works as Raibert (1986) and Vukobratovic et al. (1990).

Year after year, from technological advances, based on theoretical and experimental researches, the man tries to copy or to imitate some systems of the human body. It is the case, for example, of the central pattern generator (CPG), responsible for the production of rhythmic movements. Modelling of this CPG can be made by means of coupled oscillators, and this system generates patterns similar to human CPG, becoming possible the human gait simulation. There are some significant works about the locomotion of vertebrates controlled by central pattern generators: Grillner (1985), and Pearson (1993).

From a model of two-dimensional locomotor, oscillators with integer relation of frequency can be used for simulating the behaviour of the hip angle and of the knees angles. Each oscillator has its own parameters and the link to the other oscillators is made through coupling terms. We intend to evaluate a system with coupled van der Pol oscillators. Some previous works about CPGs using nonlinear oscillators, applied in the human gait simulation, can be seen in Bay & Hemami (1987), Zielinska (1996), Dutra et al. (2003), Pina Filho (2005), and Pina Filho (2008).

The objective of this work is to analyze the dynamics of this coupled oscillators system by means of bifurcation diagrams and Poincaré maps. From the analysis and graphs generated in MATLAB, it was possible to evaluate some characteristics of the system, such as: sensitivity to the initial conditions, presence of strange attractors and other phenomena of the chaos.

## 2. Central Pattern Generator - CPG

The first indications that the spinal marrow could contain the basic nervous system necessary to generate locomotion date back to the early 20th century. According to Mackay-Lyons (2002), the existence of nets of nervous cells that produce specific rhythmic movements for a great number of vertebrates is something unquestionable. Nervous nets in the spinal marrow are capable of producing rhythmic movements, such as: swimming, jumping, and walking, when isolated from the brain and sensorial entrances. These

specialized nervous systems are known as nervous oscillators or central pattern generators (CPGs). The human locomotion is controlled, in part, by a central pattern generator, which is evidenced in the works as Calancie et al. (1994), and Dimitrijevic et al. (1998).

The choice of an appropriate pattern of locomotion depends on the combination of a central programming and sensorial data, as well as of the instruction for one determined motor condition. This information determines the way of organisation of the muscular synergy, which is planned for adequate multiple conditions of posture and gait (Horak & Nashner, 1986).

Figure 1 presents a scheme of the control system of the human locomotion, controlled by the central nervous system, which the central pattern generator supplies a series of pattern curves for each part of the locomotor. This information is passed to the muscles by means of a network of motoneurons, and the conjoined muscular activity performs the locomotion. Sensorial information about the conditions of the environment or some disturbance are supplied as feedback of the system, providing a fast action proceeding from the central pattern generator, which adapts the gait to the new situation.

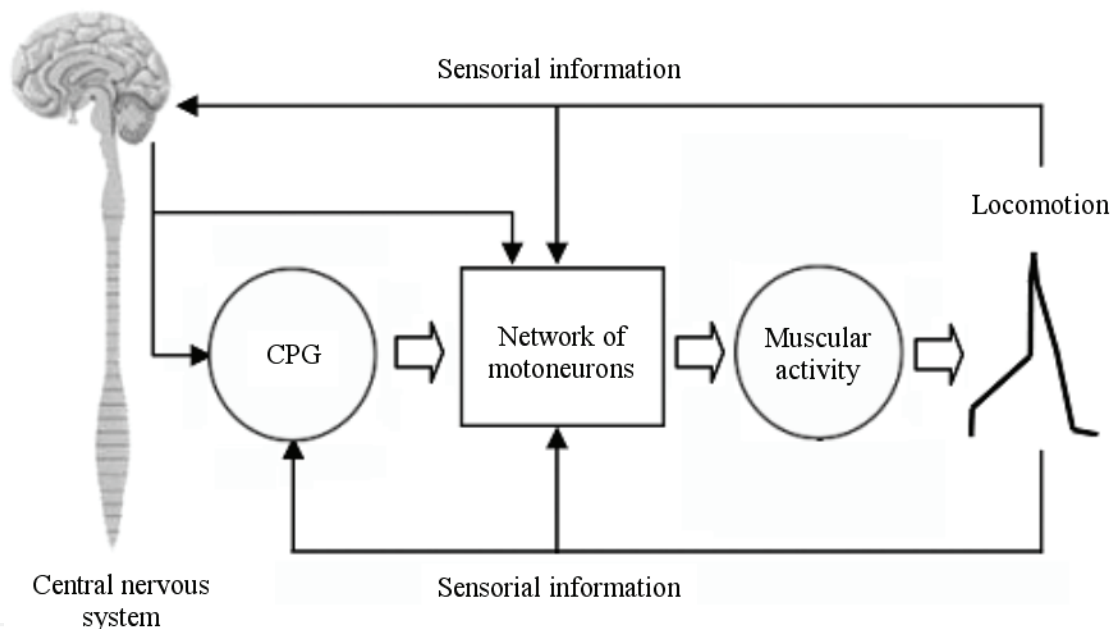


Fig. 1. Control system of the human locomotion.

Despite the people not walk in completely identical way, some characteristics in the gait can be considered universal, and these similar points serve as base for description of patterns of the kinematics, dynamics and muscular activity in the locomotion.

In the study to be presented here, the greater interest is related to the patterns of the hip and knee angles. From the knowledge of these patterns of behaviour, the simulation of the central pattern generator using the system of coupled oscillators becomes possible.

Considering the anatomical planes of movement, we need to know the behaviour of hip and knee in sagittal plane. Figure 2 presents the movements of flexion and extension of the articulation of hip and knee in sagittal plane.

According Pina Filho et al. (2006), figures 3 and 4 present the graphs of angular displacement and phase space of the hip in the sagittal plane, related to the movements of flexion and extension, while figures 5 and 6 present the graphs of angular displacement and phase space of the knee, related to the movements of flexion and extension.

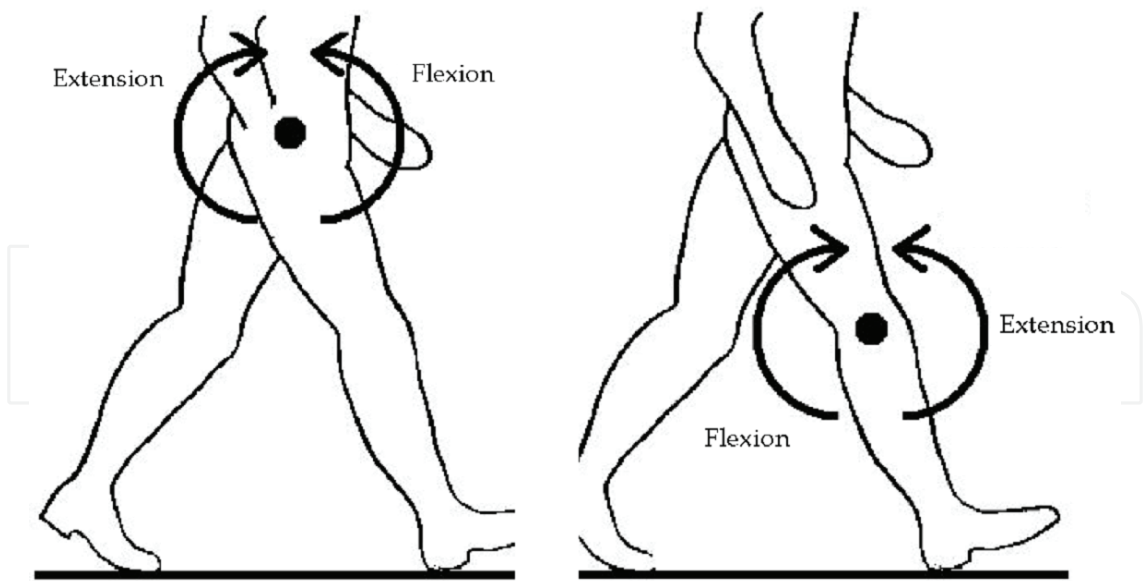


Fig. 2. Movements of the hip and knee: flexion and extension.

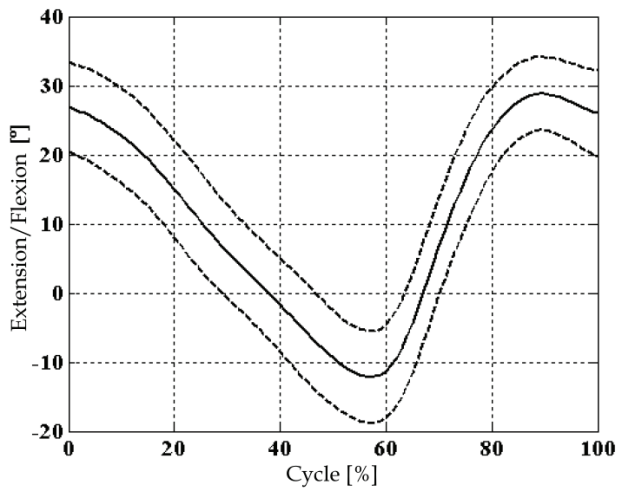


Fig. 3. Angular displacement of the hip in the sagittal plane (mean  $\pm$  deviation).

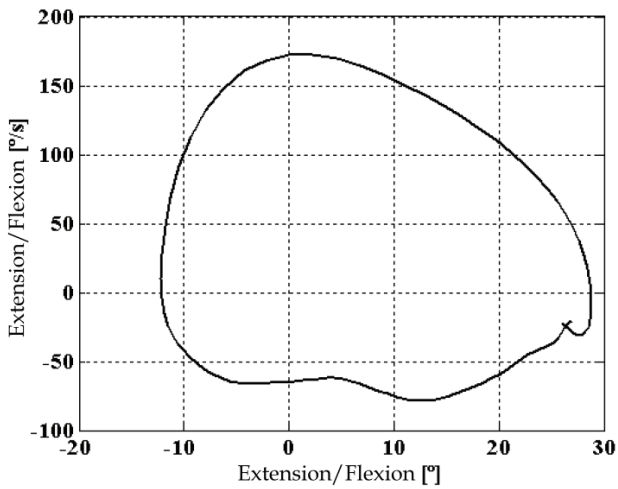


Fig. 4. Phase space of the hip in the sagittal plane.

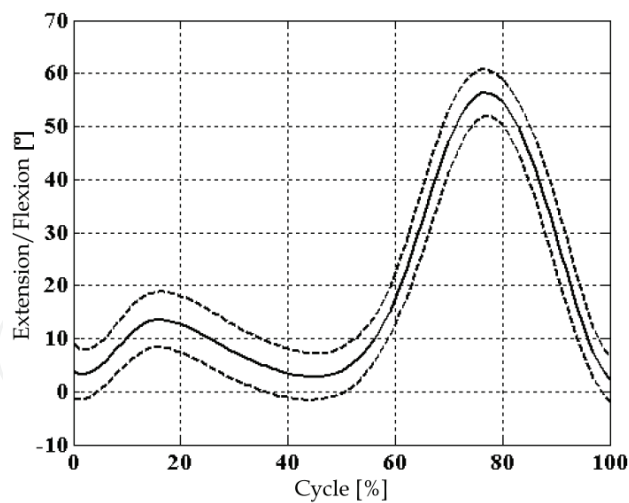


Fig. 5. Angular displacement of the knee in the sagittal plane (mean  $\pm$  deviation).

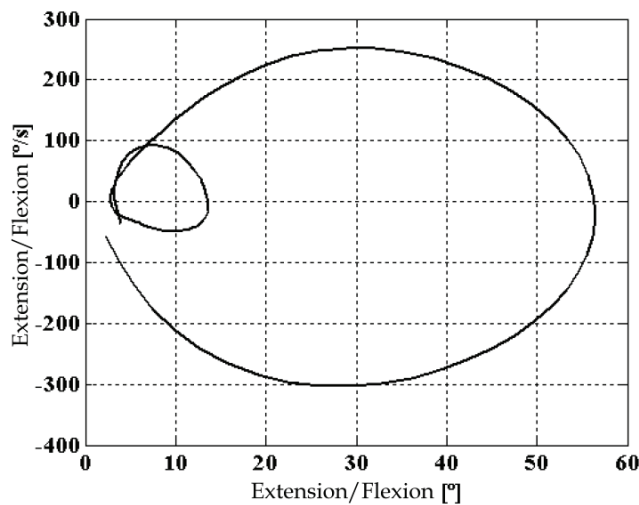


Fig. 6. Phase space of the knee in the sagittal plane.

3. Biped locomotor model

Before a model of CPG can be applied to a physical system, the desired characteristics of the system must be completely determined, such as: the movement of the leg or another rhythmic behaviour of the locomotor. Some works with description of the rhythmic movement of animals include Eberhart (1976), Winter (1983) and McMahon (1984), this last one presenting an ample study about the particularities of the human locomotion. To specify the model to be studied is important to know some concepts related to the bipedal gait, such as the determinants of gait.

The modelling of natural biped locomotion is made more feasible by reducing the number of degrees of freedom, since there are more than 200 degrees of freedom involved in legged locomotion. According to Saunders et al. (1953), the most important determinants of gait are: 1) the compass gait that is performed with stiff legs like an inverted pendulum. The pathway of the centre of gravity is a series of arcs; 2) pelvic rotation about a vertical axis. The influence of this determinant flattens the arcs of the pathway of the centre of gravity; 3)

pelvic tilt, the effects on the non-weight-bearing side further flatten the arc of translation of the centre of gravity; 4) knee flexion of the stance leg. The effects of this determinant combined with pelvic rotation and pelvic tilt achieve minimal vertical displacement of the centre of gravity; 5) plantar flexion of the stance ankle. The effects of the arcs of foot and knee rotation smooth out the abrupt inflexions at the intersection of the arcs of translation of the centre of gravity; 6) lateral displacement of the pelvis.

Figure 7 presents a 3D model with 15 degrees of freedom, and the six determinants of gait. The kinematical analysis, using the characteristic pair of joints method is presented in Saunders et al. (1953).

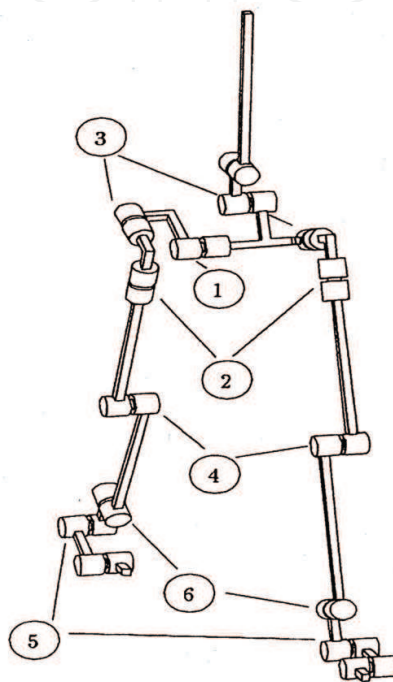


Fig. 7. Three-dimensional model with the six determinants of gait.

In order to simplify the investigation, a 2D model that performs motions parallel only to the sagittal plane will be considered. This model, showed in Fig. 8, characterises the three most important determinants of gait, determinants 1 (the compass gait), 4 (knee flexion of the stance leg), and 5 (plantar flexion of the stance ankle). The model does not take into account the motion of the joints necessary for the lateral displacement of the pelvis, for the pelvic rotation, and for the pelvic tilt.

Figure 8 presents too the angles and lengths of the model, where:  $\ell_s$  is the length of foot responsible for the support (toes),  $\ell_p$  is the length of foot that raises up the ground (sole),  $\ell_t$  is the length of tibia, and  $\ell_f$  is the length of femur. The angle of the hip  $\theta_4$  and the angles of the knees  $\theta_3$  and  $\theta_5$  will be determined by a coupled oscillators system, representing the CPG, while the other angles are calculated by the kinematical analysis of the mechanism. In this work we will not present details of this analysis, which can be seen in Pina Filho (2005).

This model must be capable to show clearly the phenomena occurred in the course of the motion, and works with the hypothesis of the rigid body, where the natural structural movements of the skin and muscles, as well as bone deformities, are disregarded. The

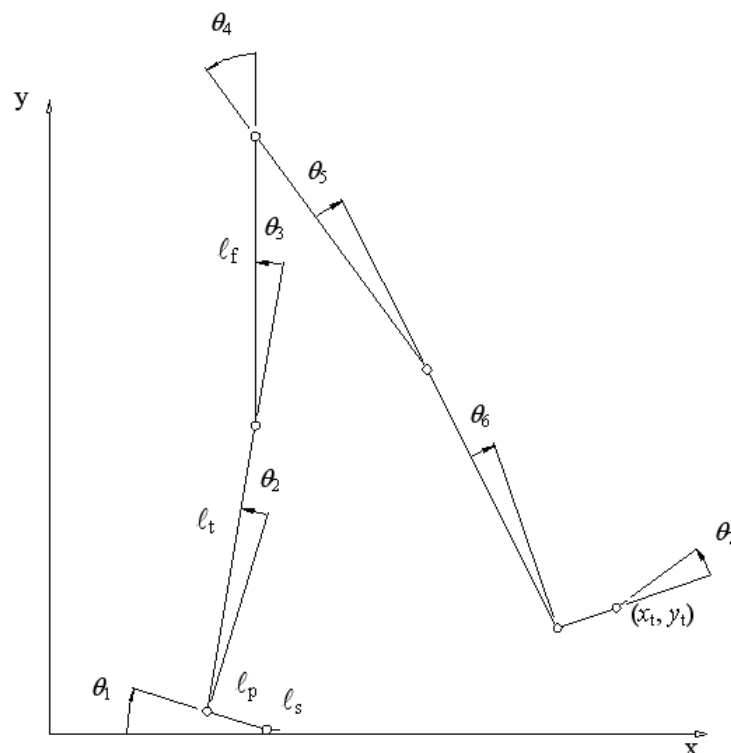


Fig. 8. Two-dimensional model with the determinants of gait, angles and lengths.

locomotion cycle can be divided in two intervals: double support phase, with the two feet on the ground, and single support phase, with only one foot touching the ground, and one of the legs performs a balance movement (the extremity of the support leg is assumed as not sliding).

From this model, we can now to study the CPG, simulated by means of nonlinear oscillators, which can be used in control systems of locomotion, providing the approach trajectories of the legs. The CPG is composed of a set of oscillators, where each oscillator, with own amplitude, frequency and parameters, generates angular signals of reference for the movement of the legs, as we will see in the next section.

#### 4. Modelling of the oscillators system

Coupled oscillators systems have been extensively used in studies of physiological and biochemical modelling. Since the years of 1960, many researchers have studied the case of coupling between two oscillators, because this study is the basis to understand the phenomenon in a great number of coupled oscillators. One of the types of oscillators that can be used in coupled systems is the auto-excited ones, which have a stable limit cycle without external forces. The van der Pol oscillator is an example of this type of oscillator, and it will be used in this work. Then, considering a system of  $n$  coupled van der Pol oscillators, from van der Pol equation:

$$\ddot{x} - \varepsilon(1 - p(x - x_0)^2)\dot{x} + \Omega^2(x - x_0) = 0 \quad \varepsilon, p \geq 0 \quad (1)$$

where  $\varepsilon$ ,  $p$  and  $\Omega$  correspond to the parameters of the oscillator, and adding coupling terms that relate the oscillators velocities, we have:

$$\ddot{\theta}_i - \varepsilon_i \left[ 1 - p_i (\theta_i - \theta_{i0})^2 \right] \dot{\theta}_i + \Omega_i^2 (\theta_i - \theta_{i0}) - \sum_{j=1}^n c_{i,j} (\dot{\theta}_i - \dot{\theta}_j) = 0 \quad i = 1, 2, \dots, n \quad (2)$$

which represents coupling between oscillators with the same frequency, where  $\theta$  corresponds to the system degrees of freedom. In the case of coupling between oscillators with integer relation of frequency, the equation would be:

$$\ddot{\theta}_h - \varepsilon_h \left[ 1 - p_h (\theta_h - \theta_{h0})^2 \right] \dot{\theta}_h + \Omega_h^2 (\theta_h - \theta_{h0}) - \sum_{i=1}^m c_{h,i} [\dot{\theta}_i (\theta_i - \theta_{i0})] - \sum_{k=1}^n c_{h,k} (\dot{\theta}_h - \dot{\theta}_k) = 0 \quad (3)$$

where  $c_{h,i} [\dot{\theta}_i (\theta_i - \theta_{i0})]$  is responsible for the coupling between oscillators with different frequencies, while  $c_{h,k} (\dot{\theta}_h - \dot{\theta}_k)$ , also seen in Eq. (2), effects the coupling between oscillators with the same frequency. Both terms were defined by Dutra (1995).

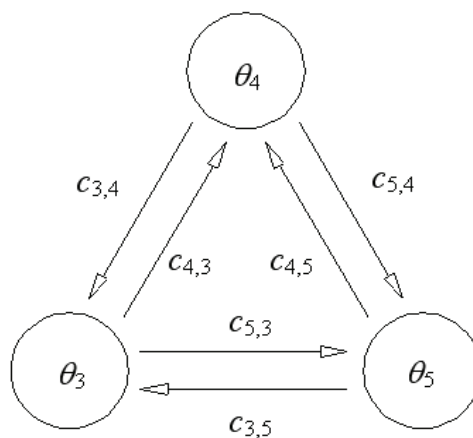


Fig. 9. Structure of coupling oscillators.

Experimental studies of human locomotion (Braune & Fischer, 1987) and Fourier analysis of these data (Dutra, 1995) show that the movements of  $\theta_3$ ,  $\theta_4$  and  $\theta_5$  (see Fig. 8) can be described very precisely by their fundamental harmonic, whether the biped in single or double support phase.

To generate the angles  $\theta_3$ ,  $\theta_4$  and  $\theta_5$  as a periodic attractor of a nonlinear net, three coupled van der Pol oscillators were used. These oscillators are mutually coupled by terms that determine the influence of one oscillator on the others (Fig. 9).

Applying Eq. (2) and (3) to the proposed problem, knowing that the frequency of  $\theta_3$  and  $\theta_5$  (knee angles) is double of  $\theta_4$  (hip angle), we have the following equations:

$$\ddot{\theta}_3 - \varepsilon_3 [1 - p_3 (\theta_3 - \theta_{30})^2] \dot{\theta}_3 + \Omega_3^2 (\theta_3 - \theta_{30}) - c_{3,4} [\dot{\theta}_4 (\theta_4 - \theta_{40})] - c_{3,5} (\dot{\theta}_3 - \dot{\theta}_5) = 0 \quad (4)$$

$$\ddot{\theta}_4 - \varepsilon_4 [1 - p_4 (\theta_4 - \theta_{40})^2] \dot{\theta}_4 + \Omega_4^2 (\theta_4 - \theta_{40}) - c_{4,3} [\dot{\theta}_3 (\theta_3 - \theta_{30})] - c_{4,5} [\dot{\theta}_5 (\theta_5 - \theta_{50})] = 0 \quad (5)$$

$$\ddot{\theta}_5 - \varepsilon_5 [1 - p_5 (\theta_5 - \theta_{50})^2] \dot{\theta}_5 + \Omega_5^2 (\theta_5 - \theta_{50}) - c_{5,4} [\dot{\theta}_4 (\theta_4 - \theta_{40})] - c_{5,3} (\dot{\theta}_5 - \dot{\theta}_3) = 0 \quad (6)$$

From Eq. (4)-(6), using the parameters shown in Table 1 together with values supplied by Pina Filho (2005), the graphs were generated in MATLAB as shown in Fig. 10 and 11, which

present, respectively, the behaviour of  $\theta_3$ ,  $\theta_4$  and  $\theta_5$  as a function of time and stable limit cycles of oscillators.

$c_{3,4}$	$c_{4,3}$	$c_{3,5}$	$c_{5,3}$	$c_{4,5}$	$c_{5,4}$	$\varepsilon_3$	$\varepsilon_4$	$\varepsilon_5$
0.001	0.001	0.1	0.1	0.001	0.001	0.01	0.1	0.01

Table 1. Parameters of van der Pol oscillators.

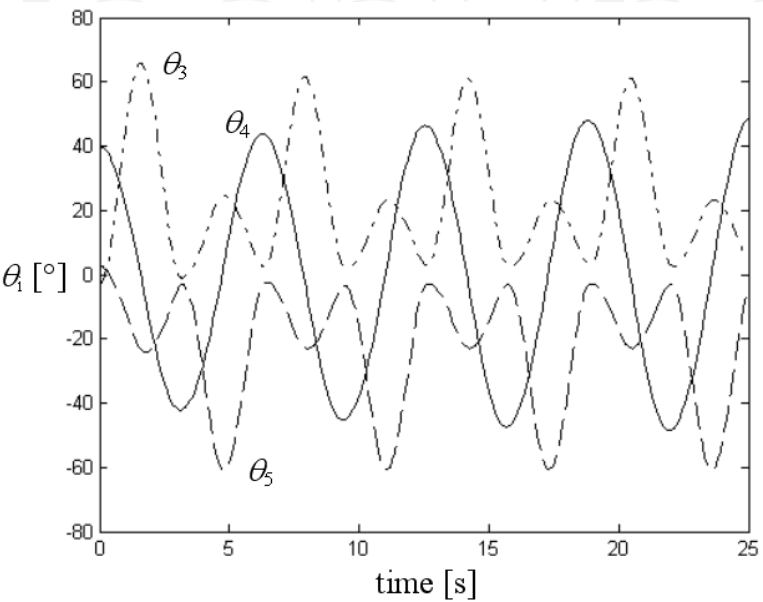


Fig. 10. Angles as a function of time.

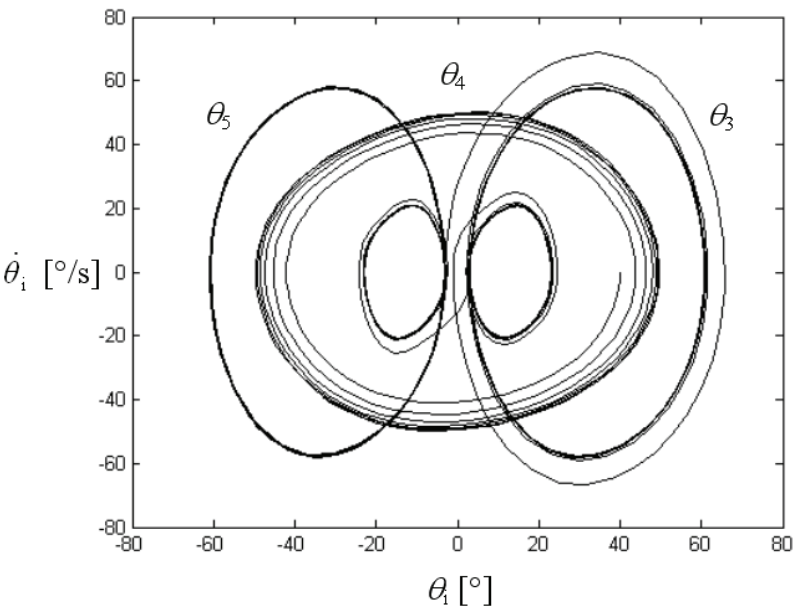


Fig. 11. Trajectories in the phase space.

Comparing Fig. 10 and 11 with the experimental results presented in Section 2 (Fig. 3, 4, 5, 6), it is verified that the coupling system supplies similar results, what confirms the possibility of use of mutually coupled van der Pol oscillators in the modelling of the CPG.

Figure 12 shows, with a stick figure, the gait with a step length of 0.63 m. Figure 13 shows the gait with a step length of 0.38 m. Dimensions used in the model can be seen in Table 2.

Thumb	Foot	Leg (below the knee)	Thigh
0.03 m	0.11 m	0.37 m	0.37 m

Table 2. Model dimensions.

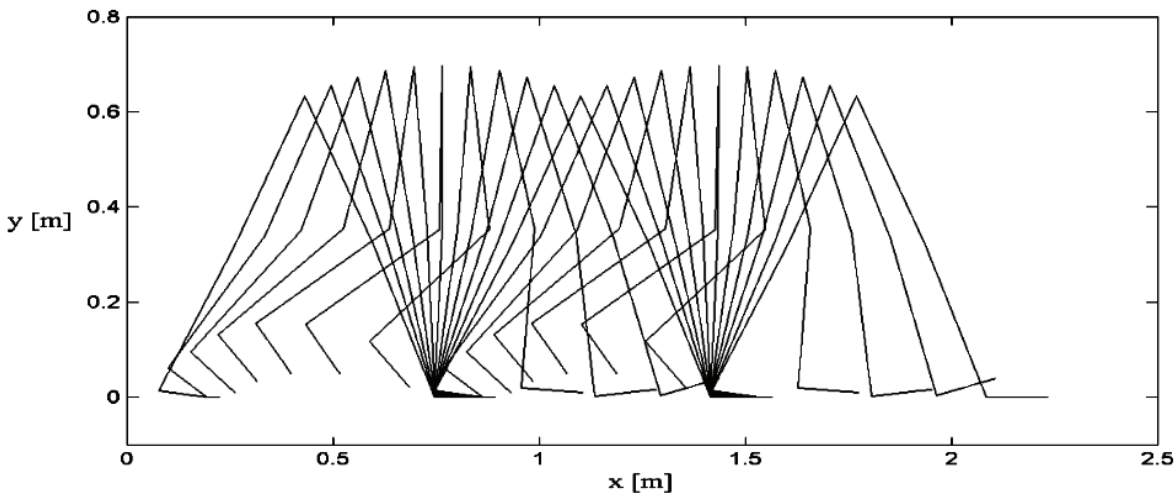


Fig. 12. Stick figure showing the gait with a step length of 0.63 m.

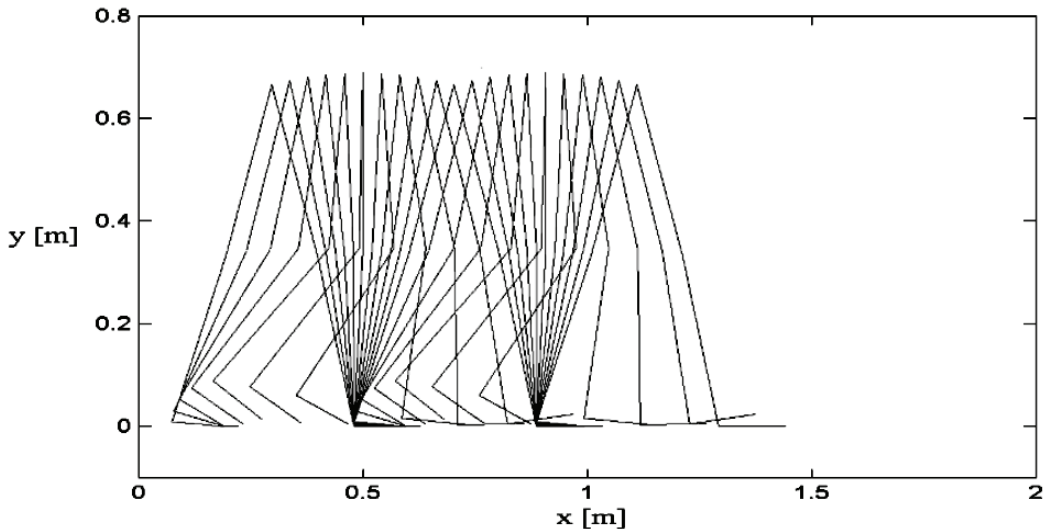


Fig. 13. Stick figure showing the gait with a step length of 0.38 m.

5. Dynamical analysis of the oscillators system

The nonlinear dynamical analysis of the system presented here requires the definition of some usual concepts. Usually, for some values of parameters, the system behaviour is periodic, and for other values the behaviour is chaotic. A periodic system returns to its state after a finite time  $t$ . The trajectory in the phase space is represented by a closed curve. The chaotic system presents trajectories of non-closed orbits that are generated by the solution of a deterministic set of ordinary differential equations.

Two conditions must be satisfied to make possible that a system presents chaotic behaviour: the equations of motion must include a nonlinear term; and the system must have at least three independent dynamic variables. The main consequence associated with the chaos is the sensitivity to the initial conditions. In chaotic systems, a small change in the initial conditions results in a drastic change in the system behaviour. More details about the Chaos theory and its characteristics can be found in many works, such as: Strogatz (1994) and Baker & Gollub (1996).

The existence of bifurcation in a system is related with the existence of chaos. In all chaotic system, it is possible to observe the bifurcation phenomenon, however, not all system that presents bifurcation necessarily presents a chaotic response. The influence of some parameter in the system response can be identified by means of bifurcation diagrams, which present the stroboscopic distribution of the system response from slow variation of a parameter (Thompson & Stewart, 1986). This method was applied here, which implies to simulate different parameter values that we want to analyze, evaluating the response in Poincaré maps.

The Poincaré map consists in the reduction of continuous systems in time (flows) in discrete systems in time (maps). Then, a Poincaré map allows that system dynamics to be represented in a space with lesser dimension than original system, reducing a  $n$ -dimensional space for  $n-1$  dimensions. The Poincaré map is obtained from the phase space diagram by observing this “stroboscopically”, i.e., sample points in the phase space in regular intervals.

Then, considering different values for the parameters  $\varepsilon_3$ ,  $\varepsilon_4$  and  $\varepsilon_5$ , the tests have been performed using MATLAB to generate the bifurcation diagrams and Poincaré maps. In principle, keeping values of  $\varepsilon_4 = 0.1$  and  $\varepsilon_5 = 0.01$ , the value of  $\varepsilon_3$  was varied from 0 to 2. Other values of the system have been kept. Figure 14 presents the bifurcation diagram showing the behaviour of knee oscillator  $\theta_3$  with variation of parameter  $\varepsilon_3$ , which represents the damping term related with this oscillator.

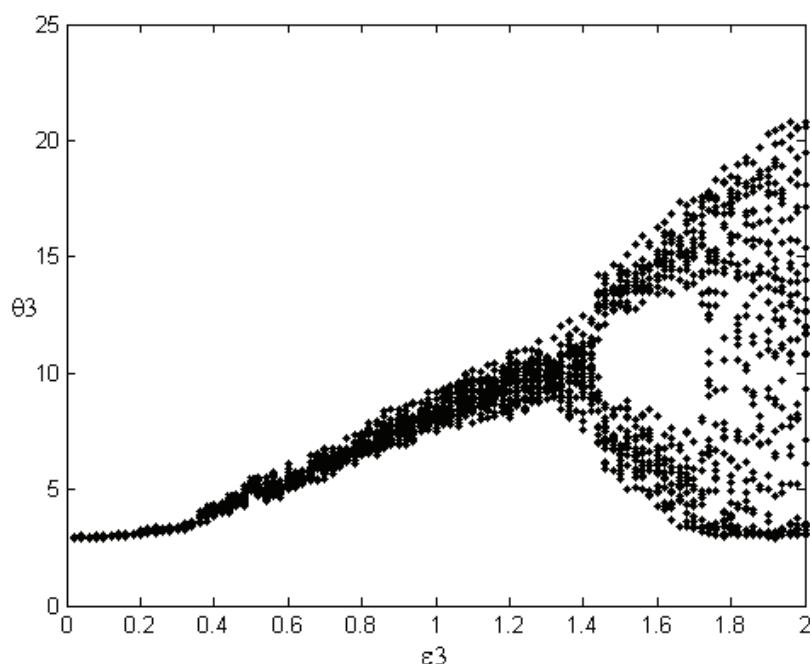


Fig. 14. Bifurcation diagram for  $\theta_3$  with variation of  $\varepsilon_3$ .

The diagram of Fig. 14 does not represent the bifurcation as simple curves, which normally happens in dynamical analysis of a system, but with a cloud of points. Considering the complexity of coupled oscillators system, this fact can be explained by relation between coupling terms or by quasiperiodic response of the system.

According to Santos et al. (2004), a great variation between coupling terms, with one of them approaching to zero, makes the system presents practically a unidirectional coupling, and consequently the response in bifurcation diagram is represented by a cloud of points, characterizing not only the presence of periodic and chaotic orbits, as also pseudo-trajectories. More details about this subject can be seen in Grebogi et al. (2002).

In relation to system behaviour, with small values of damping term, below 0.1, the system presents a periodic response (Fig. 15). With the increase of damping term, the system starts to present a quasiperiodic response and later chaotic response, as presented in Fig. 16 and 17, respectively.

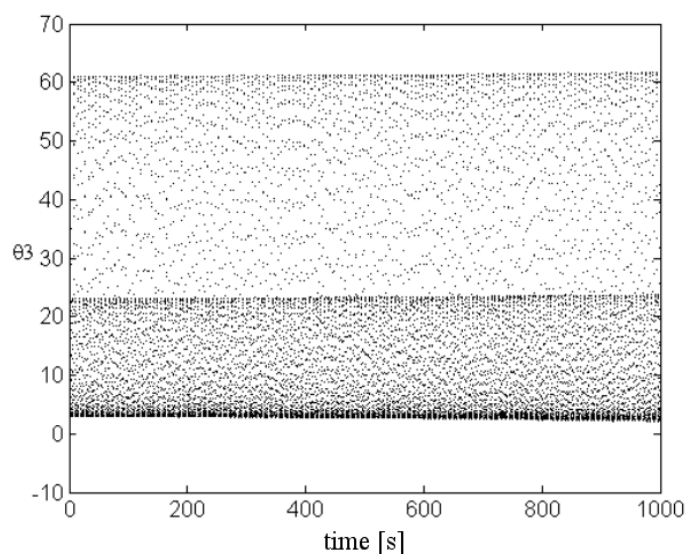


Fig. 15. Periodic response:  $\varepsilon_3 = 0.01$ .

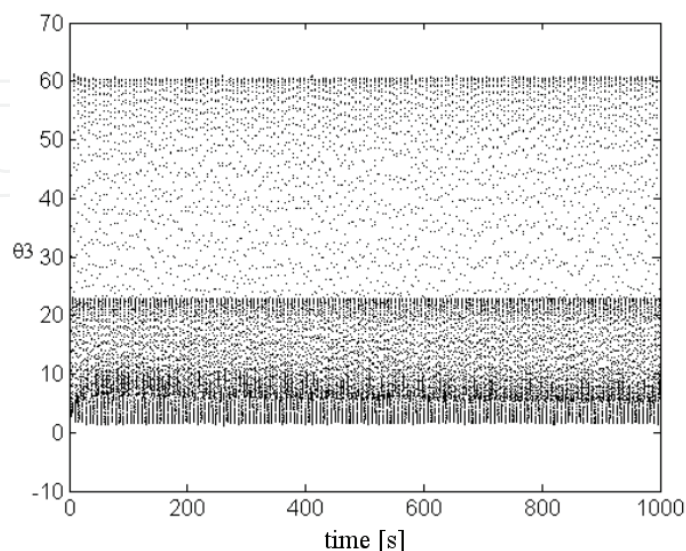


Fig. 16. Quasiperiodic response:  $\varepsilon_3 = 1$ .

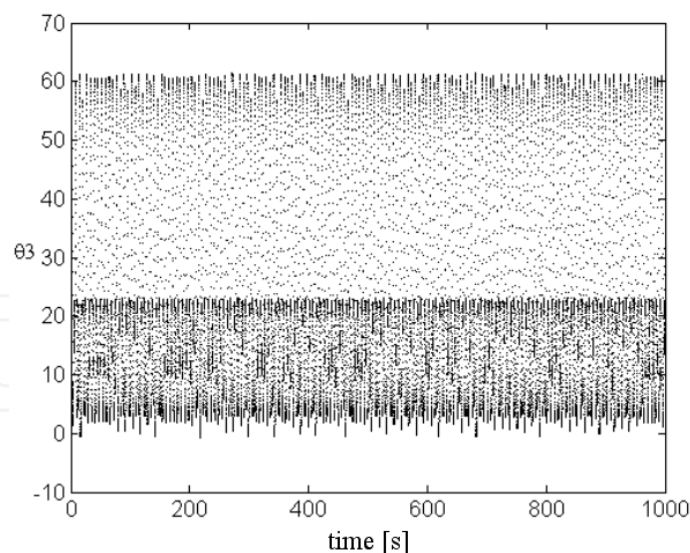


Fig. 17. Chaotic response:  $\varepsilon_3 = 2$ .

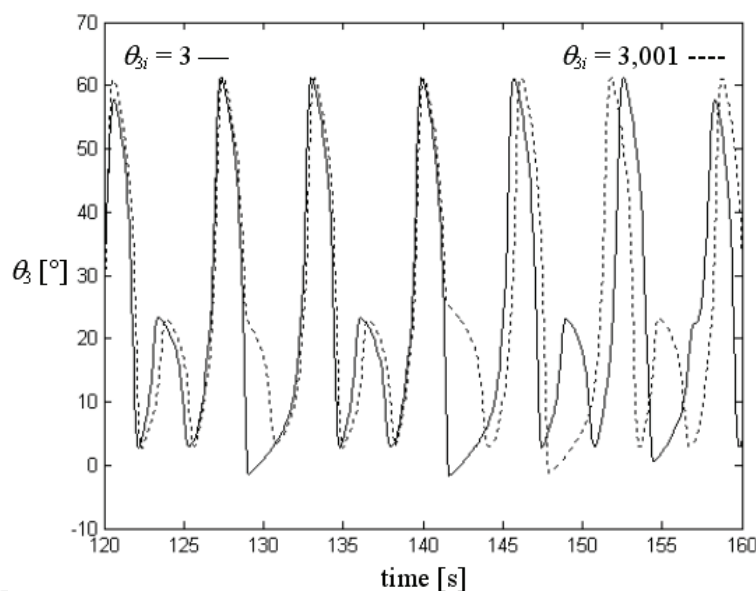


Fig. 18. Sensitivity to the initial conditions in chaotic response.

From Fig. 14 and 17, we observed the configuration of chaotic regime when  $\varepsilon_3 = 2$ . More details about transition between quasiperiodic and chaotic response are presented by Yoshinaga & Kawakami (1994), Yang (2000) and Pazó et al. (2001).

Sensitivity to the initial conditions can be verified considering two simulations with different conditions, for example, with  $\varepsilon_3 = 3$  (chaotic regime), choosing initial values for the angles:  $\theta_3 = 3^\circ$ ,  $\theta_4 = 50^\circ$ ,  $\theta_5 = -3^\circ$ , and changing  $\theta_3 = 3.001^\circ$ , we observed the influence of initial conditions in the system response (Fig. 18).

Another interesting point of the chaos analysis is the presence of strange attractors, which can be observed in Poincaré map. In dissipative systems the Poincaré map presents a set of points disposed in an organized form, with a preferential region in phase space that attracts the states of dynamical system. Figure 19 showing the strange attractor generated in the analysis of knee oscillator  $\theta_3$ .

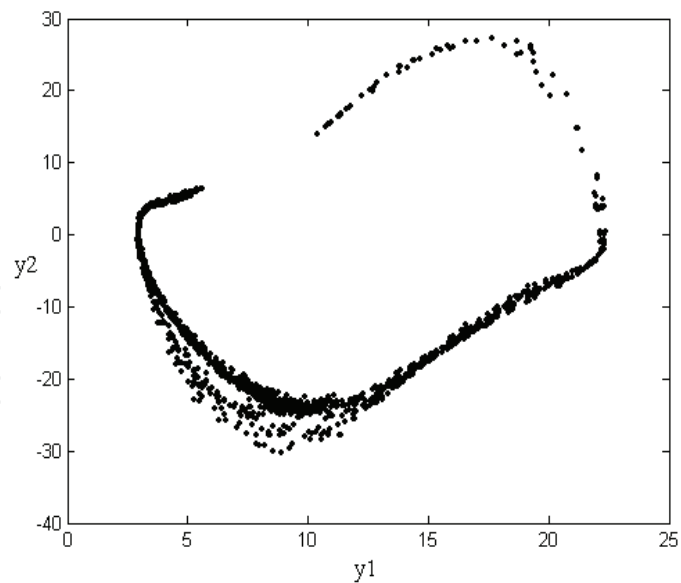


Fig. 19. Strange attractor for  $\theta_3$ .

Considering the coupling oscillators, the degree of influence between them is defined by the coupling term. Then, a change of oscillator parameters must influence the behaviour of other oscillators. Figure 20 presents the bifurcation diagram showing the behaviour of knee oscillator  $\theta_5$  with variation of parameter  $\varepsilon_3$ .

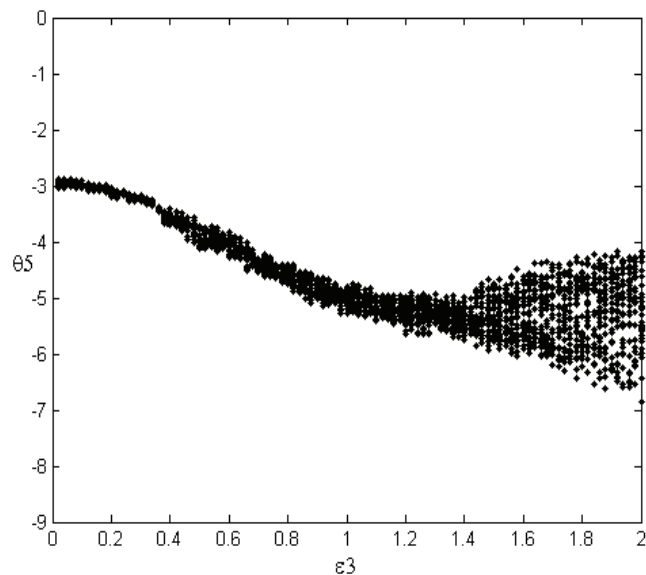


Fig. 20. Bifurcation diagram for  $\theta_5$  with variation of  $\varepsilon_3$ .

In relation to the hip angle, the influence of knee oscillator  $\theta_3$  on the hip is small, therefore the behaviour of  $\theta_4$  does not show many alterations. This occurs due to small value adopted for the coupling term between the oscillators ( $c_{34} = c_{43} = 0.001$ ). In relation to the knees, the coupling term is greater ( $c_{35} = c_{53} = 0.1$ ), configuring a more significant influence. Similarly to analysis of  $\varepsilon_3$ , the system response can be analyzed by varying the values of  $\varepsilon_4$  (from 0 to 2) and keeping the other values fixed. Figure 21 presents the bifurcation diagram showing the behaviour of hip oscillator  $\theta_4$  with variation of parameter  $\varepsilon_4$ , which represents

the damping term related with this oscillator. Figure 22 showing the strange attractor generated in the analysis of this oscillator.

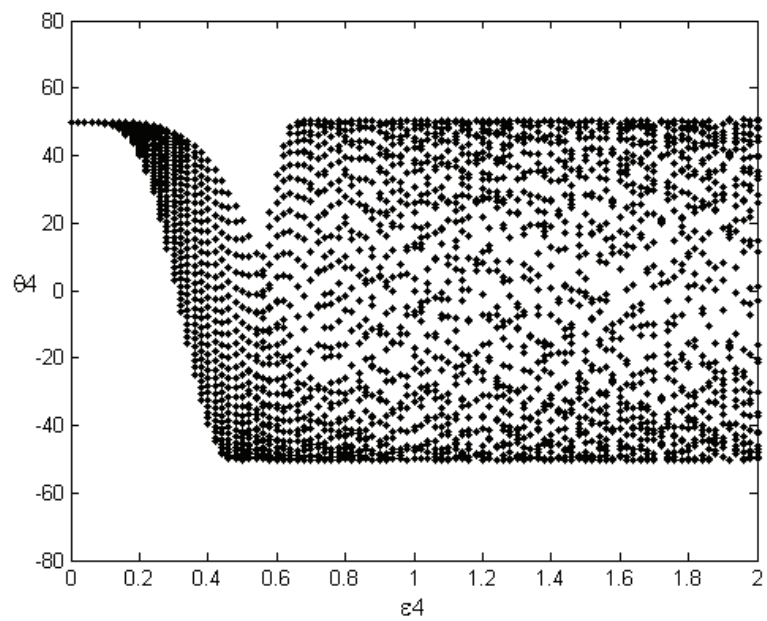


Fig. 21. Bifurcation diagram for  $\theta_4$  with variation of  $\epsilon_4$ .

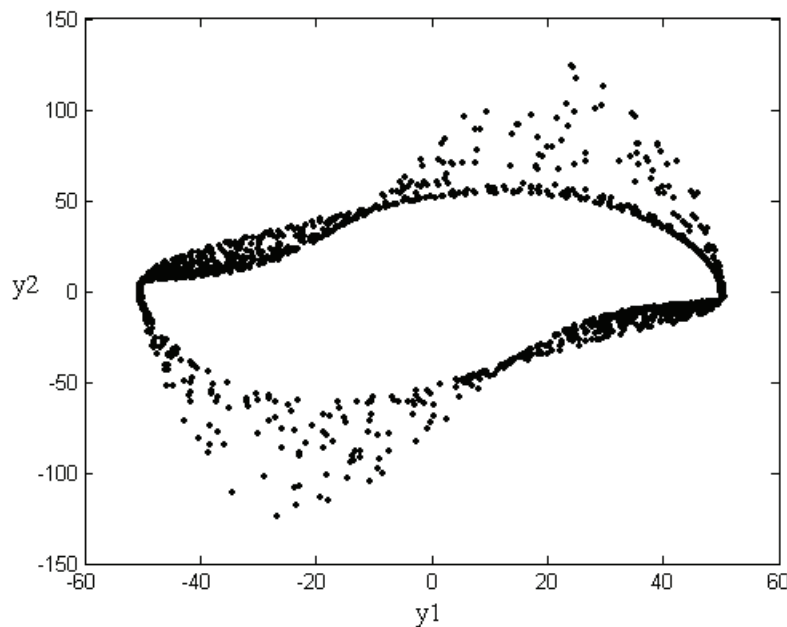


Fig. 22. Strange attractor for  $\theta_4$ .

As seen previously in the analysis of  $\epsilon_3$ , the influence of hip on the knees is small, then a variation of  $\epsilon_4$  does not cause great changes in  $\theta_3$  and  $\theta_5$ . Finally, the system response can be analyzed by varying the values of  $\epsilon_5$  (from 0 to 2) and keeping the other system values fixed. Figure 23 presents the bifurcation diagram showing the behaviour of knee oscillator  $\theta_5$  with variation of the parameter  $\epsilon_5$ , which represents the

damping term related with this oscillator. Figure 24 showing the strange attractor generated in the analysis of this oscillator.

Figure 25 presents the bifurcation diagram showing the behaviour of knee oscillator  $\theta_3$  with variation of the parameter  $\varepsilon_5$ . In relation to the hip, the knee oscillator  $\theta_5$  presents small influence on the hip angle  $\theta_4$ .

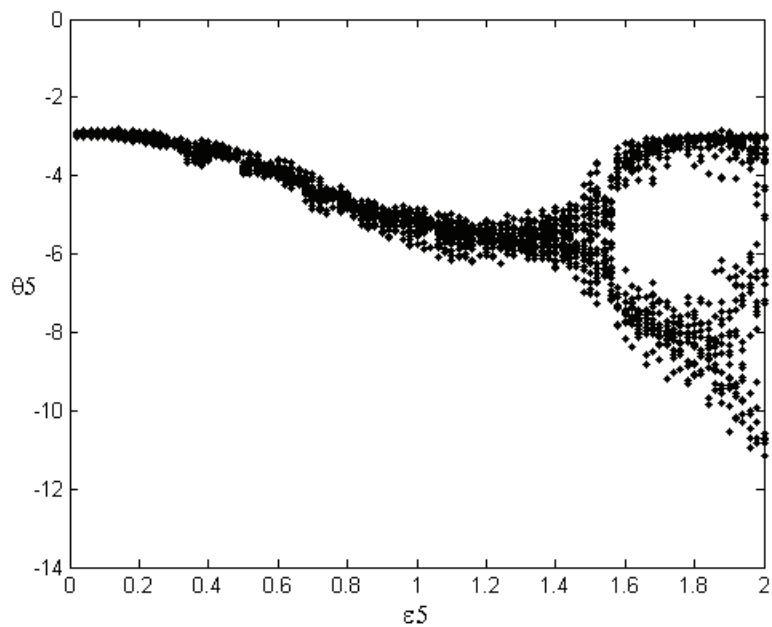


Fig. 23. Bifurcation diagram for  $\theta_5$  with variation of  $\varepsilon_5$ .

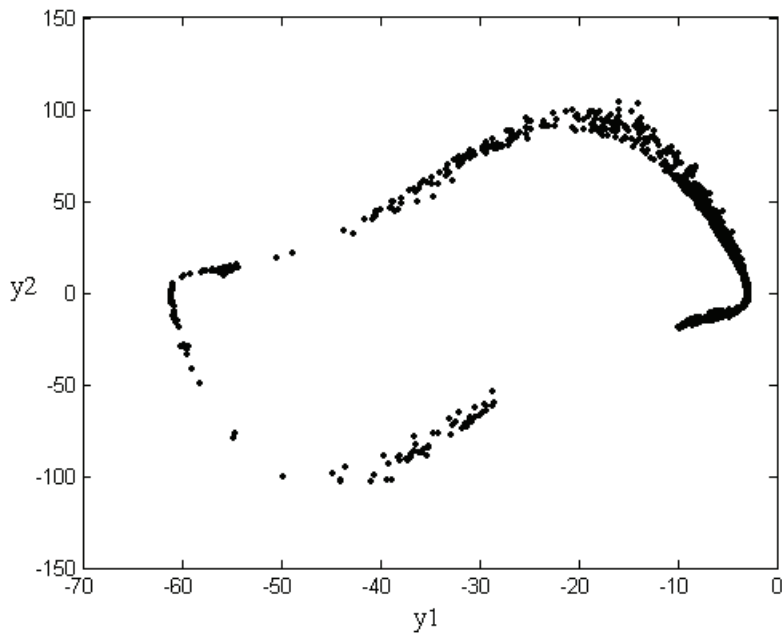


Fig. 24. Strange attractor for  $\theta_5$ .

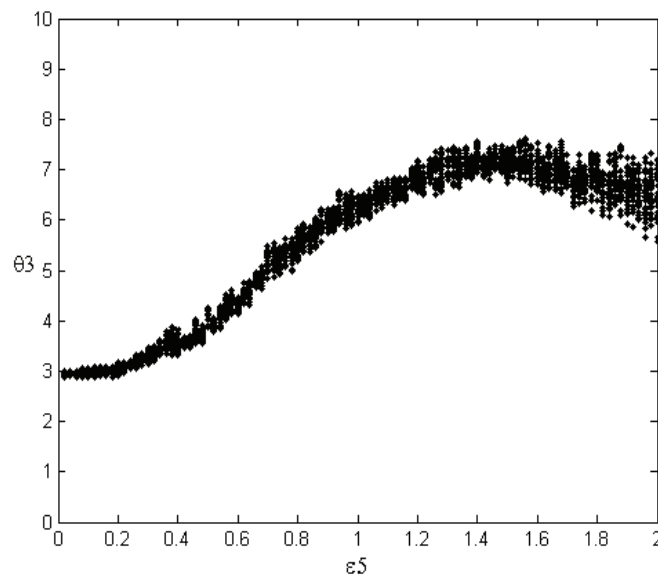


Fig. 25. Bifurcation diagram for  $\theta_3$  with variation of  $\varepsilon_5$ .

## 6. Conclusion

In this chapter, we present the study of a biped locomotor with a CPG formed by a system of coupled van der Pol oscillators. A biped locomotor model with three of the six most important determinants of human gait was used in the analyses. After the modelling of the oscillators system, a dynamical analysis was performed to verify the performance of the system, in particular, aspects related to the chaos. From presented results and discussion, we come to the following conclusions: the use of mutually coupled nonlinear oscillators of van der Pol can represent an excellent way to generate locomotion pattern signals, allowing its application for the control of a biped by the synchronization and coordination of the legs, once the choice of parameters is correct, which must be made from the data supplied by the analysis of bifurcation and chaos. Through the dynamical analysis it was possible to evidence a weak point of coupling systems. The influence of the knee oscillators on the hip, and vice versa, is very small, what can harm the functionality of the system. The solution for this problem seems immediate: to increase the value of the coupling term between the hip and knees. However, this can make the system unstable. Then, it is necessary a more refined study of the problem, which will be made in future works, as well as a study of the behaviour of the ankles, and simulation of the hip and knees in the other anatomical planes, increasing the network of coupled oscillators, and consequently simulating with more details the human locomotion CPG. This study has great application in the project of autonomous robots and in the rehabilitation technology, not only in the project of prosthesis and orthosis, but also in the searching of procedures that help to recuperate motor functions of human beings.

## 7. Acknowledgment

The authors would like to express their gratitude to InTech - Open Access Publisher, for the invitation to participate in this book project.

## 8. References

- Baker, L.G. & Gollub, J.P. (1996). *Chaotic Dynamics, an Introduction*, 2nd edition, Cambridge University Press
- Bay, J.S. & Hemami, H. (1987). Modelling of a neural pattern generator with coupled nonlinear oscillators. *IEEE Trans. Biomed. Eng.* 34, pp. 297-306
- Braune, W. & Fischer, O. (1987). *The Human Gait*, Springer Verlag, New York
- Calancie, B.; Needham-Shropshire, B.; Jacobs, P.; Willer, K.; Zych, G. & Green, B.A. (1994). Involuntary stepping after chronic spinal cord injury. Evidence for a central rhythm generator for locomotion in man. *Brain* 117(Pt 5), pp. 1143-1159
- Dimitrijevic, M.R.; Gerasimenko, Y. & Pinter, M.M. (1998). Evidence for a spinal central pattern generator in humans. *Annals of the New York Academy of Sciences* 860, pp. 360-376
- Dutra, M.S. (1995). *Bewegungskoordination und Steuerung einer zweibeinigen Gehmaschine*, Shaker Verlag, Germany
- Dutra, M.S.; Pina Filho, A.C.de & Romano, V.F. (2003). Modelling of a Bipedal Locomotor Using Coupled Nonlinear Oscillators of Van der Pol. *Biological Cybernetics* 88(4), pp. 286-292
- Eberhart, H.D. (1976). Physical principles for locomotion, In: *Neural Control of Locomotion*, R. M. Herman et al., Plenum, USA
- Grebogi, C.; Poon, L.; Sauer, T.; Yorke, J.A. & Auerbach, D. (2002). *Handbook of Dynamical Systems*, Vol. 2, Chap. 7, Elsevier, Amsterdam
- Grillner, S. (1985). Neurobiological bases of rhythmic motor acts in vertebrates. *Science* 228, pp. 143-149
- Horak, F.B. & Nashner, L.M. (1986). Central programming of postural movements adaptation to altered support-surface configurations. *Journal of Neurophysiology* 55, pp. 1369-1381
- Mackay-Lyons, M. (2002). Central Pattern Generation of Locomotion: A Review of the Evidence. *Physical Therapy* 82(1)
- McMahon, T.A. (1984). *Muscles, Reflexes and Locomotion*, Princeton University Press
- Pazó, D.; Sánchez, E. & Matías, M.A. (2001). Transition to High-Dimensional Chaos Through Quasiperiodic Motion. *International Journal of Bifurcation and Chaos*, Vol. 11, No. 10, pp. 2683-2688
- Pearson, K.G. (1993). Common principles of motor control in vertebrates and invertebrates. *Annu. Rev. Neurosci.* 16, pp. 265-297
- Pina Filho, A.C.de (2005). *Study of Mutually Coupled Nonlinear Oscillators Applied in the Locomotion of a Bipedal Robot*, D.Sc. Thesis, PEM/COPPE/UFRJ, Brazil
- Pina Filho, A.C.de (2008). Simulating the Hip and Knee Behaviour of a Biped by Means of Nonlinear Oscillators. *The Open Cybernetics & Systemics Journal*, Vol. 2, pp. 185-191
- Pina Filho, A.C.de; Dutra, M.S. & Raptopoulos, L.S.C. (2006). Modelling of Bipedal Robots Using Coupled Nonlinear Oscillators, In: *Mobile Robots, Toward New Applications*, Aleksandar Lazinica, (Ed.), Pro Literatur Verlag, Germany/ARS, Austria, pp. 57-78
- Raibert, M.H. (1986). Legged Robots, *Communications of the ACM* 29(6), pp. 499-514
- Santos, A.M.dos; Lopes, S.R. & Viana, R.L. (2004). Rhythm Synchronization and Chaotic Modulation of Coupled van der Pol Oscillators in a Model for the Heartbeat. *Physica A* 338, pp. 335-355

- Saunders, J.B.; Inman, V. & Eberhart, H.D. (1953). The Major Determinants in Normal and Pathological Gait. *J. Bone Jt. Surgery* 35A
- Strogatz, S. (1994). *Nonlinear Dynamics and Chaos*, Addison-Wesley
- Thompson, J.M.T. & Stewart, H.B. (1986). *Nonlinear Dynamics and Chaos*, John Wiley & Sons, Chichester
- Vukobratovic, M.; Borovac, B.; Surla, D. & Stovic, D. (1990). *Biped Locomotion*, Springer Verlag, Berlin
- Winter, D. (1983). Biomechanical motor patterns in normal walking. *J. Motor Behav.* 15(4), pp. 302-330
- Yang, J. (2000). Quasiperiodicity and transition to chaos. *Physical Review E* 61, pp. 6521-6526
- Yoshinaga, T. & Kawakami, H. (1994). Bifurcations of Quasi-periodic Responses in Coupled van der Pol Oscillators with External Force. *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, Vol. E77-A, No. 11, pp. 1783-1787
- Zielinska, T. (1996). Coupled oscillators utilised as gait rhythm generators of a two-legged walking machine. *Biological Cybernetics* 74, pp. 263-273

IntechOpen



## **Biped Robots**

Edited by Prof. Armando Carlos Pina Filho

ISBN 978-953-307-216-6

Hard cover, 322 pages

**Publisher** InTech

**Published online** 04, February, 2011

**Published in print edition** February, 2011

Biped robots represent a very interesting research subject, with several particularities and scope topics, such as: mechanical design, gait simulation, patterns generation, kinematics, dynamics, equilibrium, stability, kinds of control, adaptability, biomechanics, cybernetics, and rehabilitation technologies. We have diverse problems related to these topics, making the study of biped robots a very complex subject, and many times the results of researches are not totally satisfactory. However, with scientific and technological advances, based on theoretical and experimental works, many researchers have collaborated in the evolution of the biped robots design, looking for to develop autonomous systems, as well as to help in rehabilitation technologies of human beings. Thus, this book intends to present some works related to the study of biped robots, developed by researchers worldwide.

### **How to reference**

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Armando Carlos de Pina Filho and Max Suell Dutra (2011). Dynamical Analysis of a Biped Locomotion CPG Modelled by Means of Oscillators, Biped Robots, Prof. Armando Carlos Pina Filho (Ed.), ISBN: 978-953-307-216-6, InTech, Available from: <http://www.intechopen.com/books/biped-robots/dynamical-analysis-of-a-biped-locomotion-cpg-modelled-by-means-of-oscillators>

**INTECH**  
open science | open minds

### **InTech Europe**

University Campus STeP Ri  
Slavka Krautzeka 83/A  
51000 Rijeka, Croatia  
Phone: +385 (51) 770 447  
Fax: +385 (51) 686 166  
[www.intechopen.com](http://www.intechopen.com)

### **InTech China**

Unit 405, Office Block, Hotel Equatorial Shanghai  
No.65, Yan An Road (West), Shanghai, 200040, China  
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元  
Phone: +86-21-62489820  
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen