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# Rendezvous Between Two Active Spacecraft with Continuous Low Thrust

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## 1. Introduction

Rendezvous problems between spacecraft have attracted a major research interest due to important applications in actual space missions. Rendezvous problems may be categorized into two groups depending on how the rendezvous maneuver is performed by the participating spacecraft for a given mission: active-passive and cooperative maneuvers. In an active-passive rendezvous problem involving two vehicles, the first vehicle, known as the target vehicle, does not apply any control force while following its trajectory. The second vehicle, which serves as the active vehicle, is controlled in such a way as to meet the target vehicle at a later time. The control objective for the active vehicle is to match the position and the velocity of the target vehicle. On the other hand, in a cooperative rendezvous problem, both vehicles are controlled and guided to match the same position and velocity. Therefore, solutions to either rendezvous problem consist of a sequence of control maneuvers or guidance laws, designed to bring the vehicles to the same states, i.e., position and velocity. Based on the overall mission requirements, the control objective of the rendezvous can include additional constraints such as the total amount of propellant for each vehicle or the mission time from beginning to end.

An optimal control problem consists of finding the control sequences as a function of time, while achieving the control objectives and satisfying the given constraints. In the rendezvous problem, the performance index to be minimized can be the duration of the maneuver or the amount of fuel for each vehicle. The control objective is to match the final states of each vehicle. The vehicle dynamics are defined by a set of ordinary differential equations.

The solution methods for the optimal control problem consist of indirect and direct methods. Indirect methods are founded on the methods of the calculus of variation and its extension to the maximum principle of Pontryagin and use a Hamiltonian formulation. They require the solution of a two-point boundary value problem (TPBVP) for the system state variables and a set of adjoint variables. The state variables are subject to the initial conditions and the adjoint variables to the final conditions (constraints). The TPBVP is derived from the original optimal control problem and its dimension is higher than the dimension of the original problem. In addition, a TPBVP is more difficult to solve than an initial value problem (IVP) [Shampine & Thompson (2003)]. For this reason, direct methods have been developed in order to avoid the development of the Hamiltonian formulation from the optimal control problem.

Direct search methods include direct collocation nonlinear programming (DCNLP) [Hargraves & Paris (1987)], genetic algorithms (GA) [Goldberg (1989); Michalewicz (1994)], particle swarm optimization (PSO) [Venter & Sobieszczanski-Sobieski (2002)], simulated

annealing (SA) [Kirkpatrick & Vecchi (1983); van Laarhoven & Aarts (1987); Davis (1987)], etc. In the DCNLP method, the system of equations and the constraints are satisfied at a finite set of specific points, known as the collocation points. On the other hand, evolutionary methods such as the GA, the PSO and SA, mimic either some biological behavior or some physical phenomenon. For example, the PSO algorithm is inspired by the social behavior of a swarm of birds or insects [Crispin (2005)].

Orbital transfer problems and cooperative rendezvous problems using an optimal control formulation have been studied by many authors [Pourtakdous & Jalali (1995); Marinescu (1976); Park & Guibout (2006); Jezewski (1992); Rauwolf & Coverstone-Carroll (1996); Carpenter & Jackson (2003); Kim & Spencer (2002); Olsen & Fowler (2005)]. A Hamiltonian formulation was used as a solution searching method by Marinescu [Marinescu (1976)] and Pourtakdous [Pourtakdous & Jalali (1995)]. Another method based on generating functions has been proposed by Park [Park & Guibout (2006)], where generating functions were used to find the optimal solution, treating the TPBVP as a canonical transformation. Rouwolf and Coverstone-Carroll [Rauwolf & Coverstone-Carroll (1996)] proposed the use of a GA for a low-thrust orbital transfer problem. A chaser-target rendezvous problem was solved by Carpenter [Carpenter & Jackson (2003)] using a GA. They used the Clohessy-Wiltshire equations as a linear approximation for preliminary mission planning. Another chaser-target type problem is studied by Kim and Spencer [Kim & Spencer (2002)] with minimum fuel consumption as the objective function. Olsen and Fowler [Olsen & Fowler (2005)] also adopted the GA to generate a near optimal solution to a rendezvous problem using elliptic orbits.

Crispin [Crispin (2006; 2007)] obtained GA based solutions to rendezvous problems as nonlinear discrete or continuous time optimal control problems with terminal constraints. Using the GA has the advantage of completely eliminating the need for a TPBVP reformulation. Instead, GA utilizes a stochastic search method that can explore a large region of the solution space. In addition, GA requires no derivatives, which simplifies the computation of the actual algorithm. However, it does not guarantee finding the global optimum, because of the stochastic behavior. Several methods have been proposed in order to prevent GA's from converging to a local optimum [Schraudolph & Belew (1992); Fogel (1995)]. In this paper, we treat the rendezvous problem using GA's and simulated annealing in order to find nearly optimal solutions. We consider a cooperative rendezvous problem where both vehicles are active and the mission time duration is taken as a terminal constraint. The spacecraft dynamics are based on a continuous low-thrust assumption and are not restricted to a specific orbital shape. This is different from the Clohessy-Wiltshire approximation, which assumes the trajectories remain in the vicinity of a given orbit, usually circular or elliptic. Furthermore, the near optimal solutions obtained by the GA or the SA are used as an initial guess in a collocation method, in order to obtain more accurate solutions.

The paper is organized as follows. In the following Section 2, a detailed problem formulation is presented and the equations of motion for both spacecraft are derived. Then, it is followed by Section 3, which contains numerical simulation results in order to demonstrate the points given in the previous section. Finally, concluding remarks and future research directions are given in Section 4.

## 2. Problem formulation

This section describes the formulation of the optimal control problem for the given rendezvous mission with both spacecraft active. The rendezvous mission time is prescribed

a priori. If no solution can be found for the prescribed time duration, it usually means that the time is too short to achieve rendezvous. Therefore, the mission time is increased and the optimal problem is solved again. Fig. 1 shows the frame of reference and the nomenclature for the rendezvous problem.

It is assumed that the initial orbit of each spacecraft is circular. Here  $\nu$  is the true anomaly measured counter-clockwise from the  $+x$  axis direction. Initially, the two spacecraft are located at different positions so that their radial locations and true anomalies values are different at the initial time  $t = t_0$ . Each spacecraft is equipped with a low thrust rocket engine, which is assumed to have a constant thrust force  $T$ . This assumption simplifies the optimal control problem because the steering angle  $\theta$  of the thrust vector can be used as a single control variable. The measurement directions for each quantity are depicted in Fig. 1. It is also assumed that there are no gravity perturbations due to the oblateness of the main attracting body, the existence of another attracting body in the vicinity of the spacecraft and the mutual attraction between the spacecraft. We restrict our problem to two identical spacecraft with the same initial masses as well as the same constant thrust force and the same propellant mass flow rate. Therefore, the dynamics of each spacecraft are given by

$$m \left( \frac{d^2r}{dt^2} - r \left( \frac{dv}{dt} \right)^2 \right) = -\frac{m\mu}{r^2} + T \sin \theta \tag{1}$$

$$m \left( r \frac{d^2\nu}{dt^2} + 2 \left( \frac{dr}{dt} \right) \left( \frac{d\nu}{dt} \right) \right) = T \cos \theta \tag{2}$$

where  $\mu = GM$  is the gravitational constant of the main attracting body, which is located at the origin  $O$  in Fig. 1,  $M$  is the corresponding mass of the attracting body, and  $G$  is the universal gravitational constant. As seen in Eq. (1) and Eq. (2), the motion of each spacecraft is decomposed into the radial and the transverse direction. Accordingly, the radial and transverse components of the velocity vector are given by

$$u = \frac{dr}{dt}, \quad v = r \frac{d\nu}{dt} \tag{3}$$

Using the velocity components given in Eq. (3), we may now rewrite Eq. (1) and Eq. (2) as first-order differential equations as follows

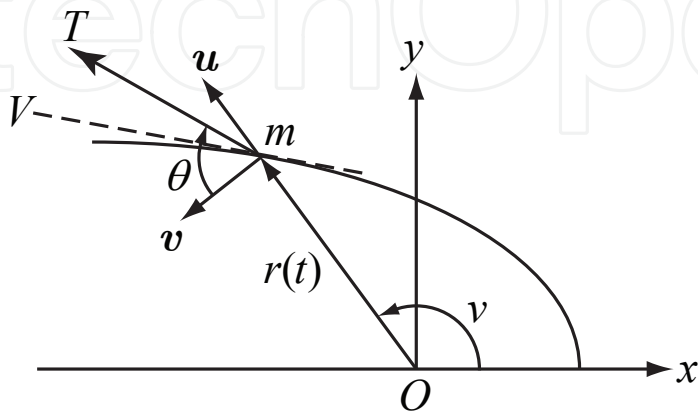


Fig. 1. Spacecraft trajectory for the rendezvous mission. The center of attraction is at the origin of the  $xy$ -frame.  $u$  and  $v$  are the components of the velocity  $V$  in polar coordinates

$$m \left( \frac{du}{dt} - \frac{v^2}{r} \right) = -\frac{m\mu}{r^2} + T \sin \theta \quad (4)$$

$$m \left( \frac{dv}{dt} + \frac{uv}{r} \right) = T \cos \theta \quad (5)$$

Therefore, the system of first-order differential equations consisting of Eq. (3), Eq. (4), and Eq. (5) describes the dynamics of the spacecraft. We now introduce the following characteristic parameters (time, speed, and thrust force) in order to non-dimensionalize the system of equations.

$$t^* = \sqrt{\frac{r_o^3}{\mu}}, \quad V_o = \sqrt{\frac{\mu}{r_o}}, \quad T_o = \frac{\mu m_o}{r_o^2} \quad (6)$$

where  $r_o$  is the radius of the initial orbit;  $m_o$  is the initial mass of each spacecraft; and  $V_o$  is the initial velocity of each spacecraft. Since each spacecraft starts its motion on a circular orbit before the start of the rendezvous mission,  $V_o$  has only a transverse velocity component, which is identical to the orbital velocity of the spacecraft in its circular orbit. The definitions of the non-dimensional variables are given by

$$\begin{aligned} \bar{r} &= r/r_o & \bar{m} &= m/m_o & \bar{t} &= t/t^* = t/\sqrt{r_o^3/\mu} \\ \bar{u} &= u/V_o = u/\sqrt{\mu/r_o} & \bar{v} &= v/V_o = v/\sqrt{\mu/r_o} & \tau &= T/T_o = T/(\mu m_o/r_o^2) \end{aligned} \quad (7)$$

where the bar denotes non-dimensional variables and the symbol  $\tau$  is chosen for the non-dimensional thrust in order to avoid confusion with  $\bar{t}$ , which is the normalized time. Since the mass of the spacecraft varies during the rendezvous mission, the mass  $m$  is a function of  $\bar{t}$  and is given by

$$m(t) = m_o + \dot{m}t \quad (8)$$

where  $\dot{m}$  is negative, since the mass of the vehicle decreases as fuel is consumed. Thus,  $\bar{m}$  is obtained by

$$\bar{m} = 1 - \frac{|\dot{m}|}{m_o} \sqrt{\frac{r_o^3}{\mu}} \bar{t} = 1 - B\bar{t} \quad (9)$$

where  $B = (|\dot{m}|/m_o) \sqrt{r_o^3/\mu}$  is introduced for notational simplicity. The corresponding non-dimensionalized system of equations (Eq. (3), Eq. (4), and Eq. (5)) are obtained as follows

$$\frac{d\bar{r}}{d\bar{t}} = \bar{u} \quad (10)$$

$$\frac{d\bar{v}}{d\bar{t}} = \bar{v} \quad (11)$$

$$\frac{d\bar{u}}{d\bar{t}} = \frac{\bar{v}^2}{\bar{r}} - \frac{1}{\bar{r}^2} + \frac{\tau \sin \theta}{(1 - B\bar{t})} \quad (12)$$

$$\frac{d\bar{v}}{d\bar{t}} = -\frac{\bar{u} \cdot \bar{v}}{\bar{r}} + \frac{\tau \cos \theta}{(1 - B\bar{t})} \quad (13)$$

The non-dimensional initial conditions are

$$\bar{r}(0) = 1, \quad \bar{u}(0) = 0, \quad \bar{v}(0) = 1, \quad \nu(0) = \nu_0 \quad (14)$$

In order to achieve the rendezvous, the following final conditions must be satisfied by each spacecraft

$$\bar{r}(\bar{t}_f) = \bar{r}_f, \quad \bar{u}(\bar{t}_f) = 0, \quad \bar{v}(\bar{t}_f) = \bar{v}_f = \frac{1}{\sqrt{\bar{r}_f}}, \quad \nu(\bar{t}_f) = \nu_f \quad (15)$$

From Eq. (15), we see that the rendezvous point is located on the final circular orbit with an arbitrary true anomaly value. As mentioned before,  $\bar{r}_f$  and  $\bar{t}_f$  are prescribed parameters of the optimal control problem. The final  $\bar{t}_f$  can be reduced if it is required to find a shorter time for the mission duration.

The objective function to be minimized for the given rendezvous problem is chosen as

$$J = \max\{w_1 \cdot [\bar{r}(\bar{t}_f) - \bar{r}_f]^2, w_2 \cdot [\bar{u}(\bar{t}_f) - \bar{u}_f]^2, w_3 \cdot [\bar{v}(\bar{t}_f) - \bar{v}_f]^2, w_4 \cdot [\nu(\bar{t}_f) - \nu_f]^2\} \quad (16)$$

where  $w_i$  is a weight determining the contribution of each error and can be adjusted based on the mission scenario. In fact, in our simulations,  $w_4$  is set to zero for a free final true anomaly value at the rendezvous point. The choice of  $J$  is not unique and the reason for the choice of  $J$  in Eq. (16) is to improve the simulation speed based on several  $J$  trials. In the following section, the rendezvous between two spacecraft is considered. If we denote the objective function for the  $i$ th spacecraft as  $J_i$  from Eq. (16), the objective function for a rendezvous mission consisting of  $N$  spacecraft (e.g.,  $N = 2$  in our simulations) is given by

$$J_{\text{tot}} = \max \bigcup_{i=1}^N J_i \quad (17)$$

### 3. Numerical results

The rendezvous simulations are performed using canonical units. The characteristic distance  $r_0$  is in astronomical unit (AU) which is the average distance between the Earth and the Sun ( $r_0 = 1\text{AU} \approx 1.4960 \times 10^8$  km). The gravitational constant  $\mu$  for the sun is about  $1.3271 \times 10^{11}$  km<sup>3</sup>/s<sup>2</sup>. The mass of the spacecraft [Bryson (1999)] is  $m_0 = 4536$  kg. The characteristic time is obtained by Eq. (6) and set to  $t^* = 58.13$  days. Similarly,  $V_0 = 29.785$  km/s and  $T_0 = 26.9$  N. The constant thrust force for the spacecraft [Bryson (1999)] is  $T = 3.778$  N and leads to the non-dimensionalized thrust force  $\tau = 0.1405$ . Finally, the mass consumption rate estimated from the spacecraft model [Bryson (1999)] is  $\dot{m} = 6.7564 \times 10^{-5}$  kg/s and corresponding  $B$  for Eq. (9) is about 0.0748. As a reference mission, we consider the rendezvous mission at Mars orbit ( $\bar{r}_f = 1.5237$ ). The velocity constraints at the final position are  $\bar{u}_f = 0$  and  $\bar{v}_f = 0.8101$ . The prescribed mission time  $\bar{t}_f = 5.5$  to ensure the fast convergence to near optimal solutions by the GA and SA. In the following discussion, we drop the bar for the non-dimensional variables to simplify the notation.

#### 3.1 Rendezvous between two spacecraft using genetic algorithms and simulated annealing

Two cases are discussed in the following simulations. First, both spacecraft start from the orbit of the Earth. However, the difference between the initial values of the true anomaly of the two spacecraft is larger than  $\pi/2$ , in order to study the case of relatively large initial angular differences between the locations of the two vehicles. The second case is about the

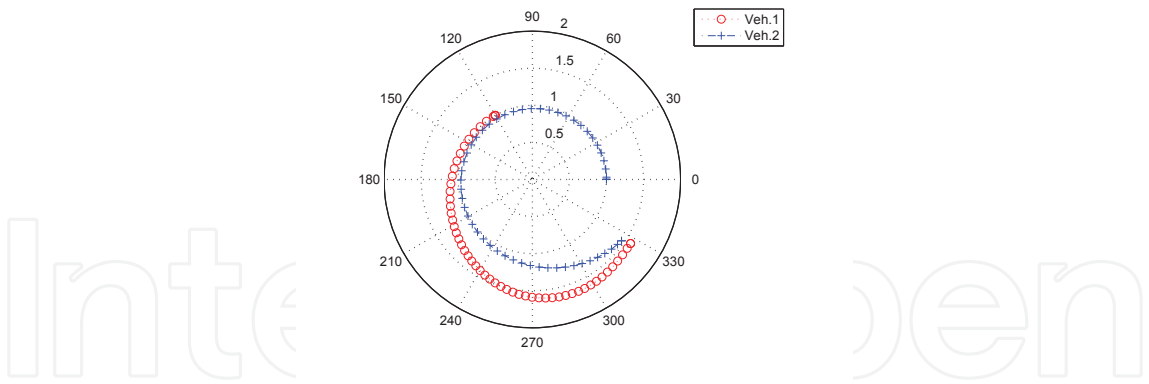


rendezvous of spacecraft starting from two different orbits such as the orbit of Mars and orbit of the Earth, meeting at an intermediate orbit. In both cases, the final value of the true anomaly of both spacecraft is free (is not prescribed a priori). In both cases, the GA and SA methods are used and the results are compared.

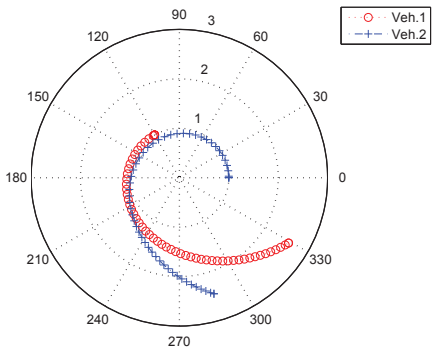
The numerical integration of the differential equations describing the spacecraft dynamics is performed using a fourth order Runge-Kutta method. The time duration  $t_f = 5.5$  is divided into  $N_t$  time steps each of  $\Delta t = t_f / N_t$ . The discrete time is  $t_i = i\Delta t$ . The corresponding control function  $\theta(t)$  is also discretized to  $\theta^i = \theta(t_i)$  based on the number of time steps  $N_t$ . In the simulations, the number of time steps  $N_t$  is fixed. The control function  $\theta(t)$  is smoothed by fitting a third order polynomial to the discrete values of  $\theta^i$  from  $i = 1$  to  $i = 41$ . A population size of 50 members was used for the GA. All the members of the initial population  $\theta^i = \theta(t_i)$  are set to zero. The chosen crossover fraction is 0.8. For SA, the re-annealing interval is 100 and the initial temperature is 100. The maximum iteration number for the GA is 100 and for the SA it is 300. The objective function tolerance is set to 0.001 for both the GA and the SA.

The rate of convergence may vary dramatically when running the same case many times, because both the GA and SA operations are stochastic processes. Furthermore, the number of iterations for the same order of magnitude of the objective function is another aspect that distinguishes the GA from the SA. Fig. 2 shows the simulation results using the GA and the SA. Both methods failed to satisfy the tolerance condition (less than 0.001) for the objective function and were stopped after 100 iterations. Spacecraft 1 starts from  $r_{1o} = 1$  with  $\theta_{1o} = 0$  and Spacecraft 2 starts from  $r_{2o} = 1$  with  $\theta_{2o} = 2\pi/3$ . The final radius for both spacecrafts is  $r_f = 1.528$ , corresponding to the orbit of Mars ( $r_f = 1.528au$ ). The final true anomaly  $\theta_f$  is free as mentioned earlier. Fig. 2(a) represents the result when the objective function value is 0.043 and Fig. 2(b) corresponds to 11.046 for the same objective function. The errors in Fig. 2(b) are due to a large value of the final radius being greater than 2. However, the calculation time for the SA is much less than that of the GA for the same number of iterations. The final true anomaly values for Spacecraft 1 and 2 are  $\nu_{1f} \approx 325.5^\circ$  and  $\nu_{2f} \approx 326.9^\circ$  when using the GA;  $\nu_{1f} \approx 286.6^\circ$  and  $\nu_{2f} \approx 329.1^\circ$  when using the SA. Fig. 2(c) and Fig. 2(d) show the control history obtained by the GA and the SA. Since we use third order polynomials to smooth the control function based on  $\theta_1^i$  and  $\theta_2^i$  with  $i \in [1, 41]$ , both  $\theta_1(t)$  for Spacecraft 1 and  $\theta_2(t)$  for Spacecraft 2 look similar to each other except for the direction of the curvatures.

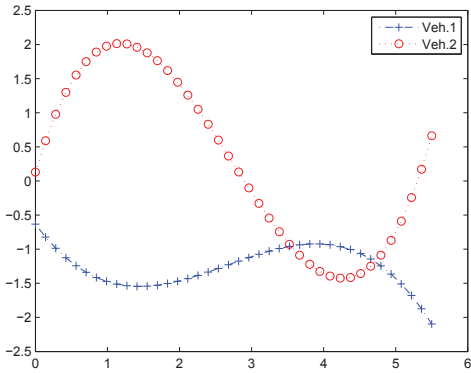
The second case we consider is when each spacecraft starts from a different orbit (the orbits of Mars and Earth, respectively) and rendezvous at an intermediate orbit. Spacecraft 1 starts from a point on the Earth orbit ( $r_{1o} = 1, \theta_{1o} = 2\pi/3$ ) and Spacecraft 2 from a point located on the Mars orbit ( $r_{2o} = 1.528, \theta_{2o} = 2\pi/3$ ). The rendezvous is at the intermediate orbit ( $r_f = 1.2$ ) between Earth and Mars orbits and the final true anomaly is free (not prescribed). The final time is the same as in the previous simulations ( $t_f = 5.5$ ). The maximum numbers of iterations for the GA and the SA are also the same as in the previous case. Fig. 3 shows the simulation results for the GA and the SA. The objective function value obtained using the SA Fig. 3(a) is about 0.06 and for the GA in Fig. 3(b) it is about 3.64. Although the number of iterations of the SA is larger than the number of generations of the GA, the actual CPU time for the SA is shorter than that of the GA. The final true anomalies obtained are  $\nu_{1f} \approx 23.4^\circ$  and  $\nu_{2f} = 22.2^\circ$  in the case of the GA; and  $\nu_{1f} \approx 37.9^\circ$  and  $\nu_{2f} \approx 44.4^\circ$  in the case of the SA. Fig. 3(c) and Fig. 3(d) show the corresponding control histories.



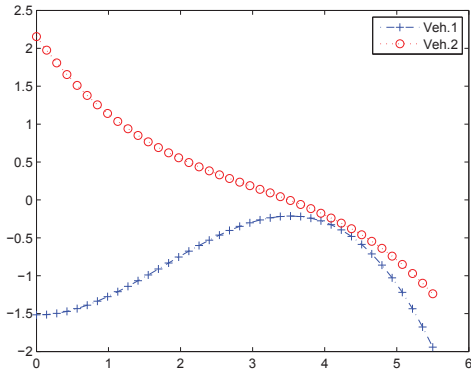
(a) Trajectories for a rendezvous between two spacecraft (circles and crosses) obtained by the GA



(b) Trajectories for a rendezvous between two spacecraft (circles and crosses) obtained by the SA



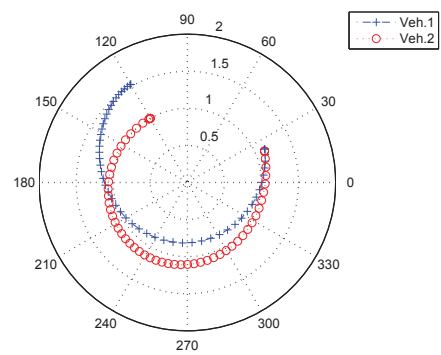
(c) Steering angles  $\theta_1(t)$  (circles) and  $\theta_2(t)$  (crosses) for a rendezvous between two spacecraft obtained by GA



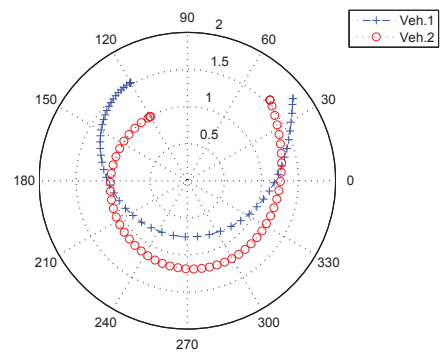
(d) Steering angles  $\theta_1(t)$  (circles) and  $\theta_2(t)$  (crosses) for a rendezvous between two spacecraft obtained by SA

Fig. 2. Trajectories generated by the GA and the SA direct search methods. The number of iterations is 100 for the GA and 300 for the SA. The radial distances and the angles are in AU and degrees, respectively.

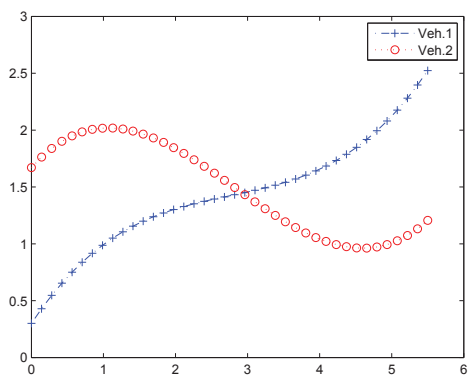




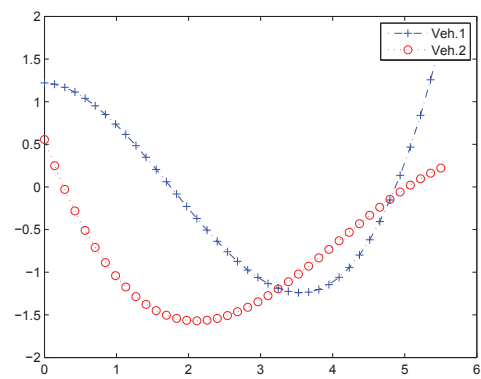
(a) Trajectories for a rendezvous between two spacecraft (circles and crosses) obtained by the GA



(b) Trajectories for a rendezvous between two spacecrafts (circles and crosses) obtained by the SA



(c) Steering angles  $\theta_1(t)$  (circles) and  $\theta_2(t)$  (crosses) for a rendezvous between two spacecraft obtained by GA



(d) Steering angles of  $\theta_1(t)$  (circles) and  $\theta_2(t)$  (crosses) for a rendezvous between two spacecraft obtained by the SA

Fig. 3. Optimal control trajectories generated by the GA and the SA direct search methods. The number of iterations for the GA is 100 and 300 for the SA. The units for the radius and angles are AU and degrees, respectively

### 3.2 Using GA and SA near-optimal solutions as initial guesses for a collocation method

Direct search methods like the GA and the SA may not generate solutions accurate enough to satisfy the final conditions because of the stochastic behavior. On the other hand, numerical methods for the solution of TPBVP's are more accurate than stochastic methods, but they require the knowledge of initial solutions (initial guesses) for starting the solution. Noting that the GA/SA methods can provide approximate trajectories, we can use them as initial guesses in a numerical method for solving TPBVP's, based on the collocation method. In this way, we can attempt to combine the advantages of the stochastic method and of the more accurate collocation method for TPBVP's. The nearly optimal initial solutions are obtained without solving optimal control problems with adjoint variables and we solve a TPBVP of reduced dimensions without adjoint variables, which simplifies the original optimal control problem significantly. Solving optimal control problems directly by TPBVP is not easy and numerical solutions are very sensitive to the initial guesses for the solutions [Bailey & Waltman (1968); Shampine & Thompson (2003)]. For this purpose, we combine the SA method with a collocation method [Kierzenka (1998); Shampine & Thompson (2003)]. Fig. 4 shows the simulation results where SA results are used as an initial guess.

We use the solutions obtained using the SA method in Section 3.1. We parameterize the steering control as follows

$$\theta_1(t) = \sum_{i=0}^{N_1} A_i t^{N_1-i}, \quad \theta_2(t) = \sum_{i=0}^{N_2} B_i t^{N_2-i} \quad (18)$$

where the subscript  $i$  refers to the  $i$ th spacecraft in the rendezvous mission. By adopting the parametrization for the control inputs, the spacecraft dynamics become

$$\frac{d\mathbf{y}}{dt} = \mathbf{f}(t, \mathbf{y}, \mathbf{A}) \quad (19)$$

where the parameter vector  $\mathbf{A}$  for the collocation method is defined by

$$\mathbf{A} = [A_0, A_1, \dots, A_{N_1}, B_0, B_1, \dots, B_{N_2}]^T \quad (20)$$

and the state vector is

$$\mathbf{y} = [r_1, u_1, v_1, \nu_1, r_2, u_2, v_2, \nu_2]^T \quad (21)$$

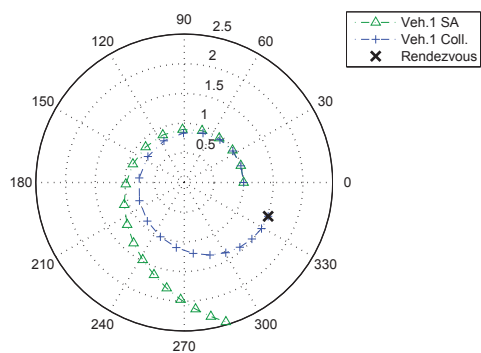
The nonlinear vector field  $\mathbf{f}$  in Eq. (19) refers to the system of equations Eq. (10), Eq. (12), Eq. (13), and Eq. (11) in an order of components in  $\mathbf{y}$ . We use polynomials of degree 3 for each spacecraft ( $N_1 = N_2 = 3$ ). The initial value of  $\mathbf{A}$  using SA is given by

$$\mathbf{A} = [-0.0613, 0.3264, -0.0196, -1.5158, -0.0365, 0.3279, -1.3149, 2.1545]^T \quad (22)$$

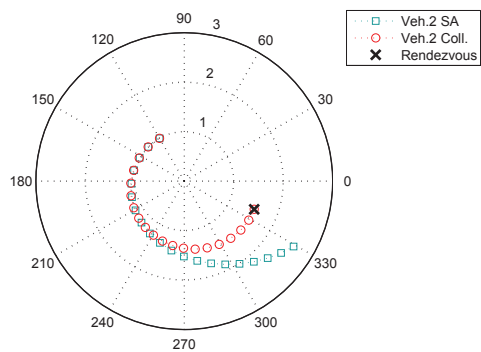
The results obtained by the collocation method for  $\mathbf{A}$  are

$$\mathbf{A} = [0.0291, -0.3704, 1.3080, -2.4064, -0.0080, -0.1863, 0.4425, 1.5668]^T \quad (23)$$

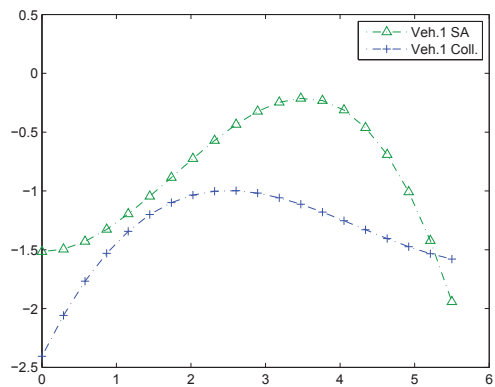
As we can see from Eq. (22) and Eq. (23), the solution of the TPBVP by the collocation method gives results which satisfy the final conditions of Eq. (15). A comparison of the control functions and the trajectories for each spacecraft is presented in Fig. 4.



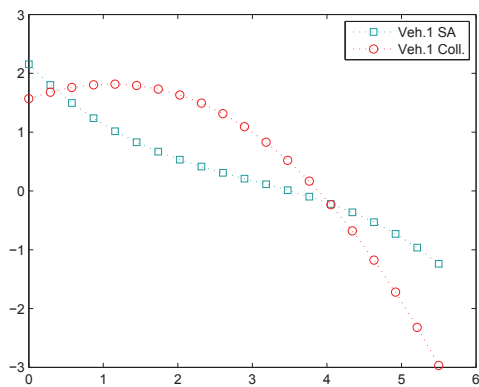
(a) Trajectories of the first spacecraft obtained by SA and the collocation method



(b) Trajectories of the second spacecraft obtained by SA and the collocation method



(c) Steering angles  $\theta_1(t)$  by SA and the collocation method



(d) Steering angles  $\theta_2(t)$  by SA and the collocation method

Fig. 4. Optimal trajectories generated by a combination of SA and a collocation method. The units for the radius and the angle are AU and degrees, respectively

#### 4. Conclusion

The rendezvous problem between two spacecraft using low thrust continuous propulsion systems has been formulated as an optimal control problem. Instead of using a Hamiltonian formulation, the optimal control problem is solved by direct search methods such as GA's and SA. Since SA is faster than the GA for the same number of iterations, SA is combined with the collocation method to overcome the stochastic behavior of SA (i.e., to match the final constraints). Simulations of a rendezvous mission between two spacecraft are performed in order to demonstrate the proposed methodology. The SA and the collocation method have been used successfully as complementary methods in order to achieve improved solutions to the original optimal control problem.

#### 5. References

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## **Advances in Spacecraft Technologies**

Edited by Dr Jason Hall

ISBN 978-953-307-551-8

Hard cover, 596 pages

**Publisher** InTech

**Published online** 14, February, 2011

**Published in print edition** February, 2011

The development and launch of the first artificial satellite Sputnik more than five decades ago propelled both the scientific and engineering communities to new heights as they worked together to develop novel solutions to the challenges of spacecraft system design. This symbiotic relationship has brought significant technological advances that have enabled the design of systems that can withstand the rigors of space while providing valuable space-based services. With its 26 chapters divided into three sections, this book brings together critical contributions from renowned international researchers to provide an outstanding survey of recent advances in spacecraft technologies. The first section includes nine chapters that focus on innovative hardware technologies while the next section is comprised of seven chapters that center on cutting-edge state estimation techniques. The final section contains eleven chapters that present a series of novel control methods for spacecraft orbit and attitude control.

### **How to reference**

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Yechiel Crispin and Dongeun Seo (2011). Rendezvous between Two Active Spacecraft with Continuous Low Thrust, *Advances in Spacecraft Technologies*, Dr Jason Hall (Ed.), ISBN: 978-953-307-551-8, InTech, Available from: <http://www.intechopen.com/books/advances-in-spacecraft-technologies/rendezvous-between-two-active-spacecraft-with-continuous-low-thrust>

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