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Estimation of Tsunami Initial Displacement of Water Surface Using Inversion Method with a priori Information

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1. Introduction

Up to now, because the tsunami initial displacement of water surface cannot be observed directly, the approach of presuming it from the fault model of the earthquake has been taken. Thanks to Yamashita and Sato (1974), it is established that an amount of the water level change in the surface of the sea is equal to the amount of the perpendicular ground change in the bottom of the sea caused by the earthquake. And, thanks to Aida (1969), the style of a present tsunami numerical calculation is established. The analytical solution proposed by Mansinha and Smylie (1971) is widely used, in order to presume the amount of the perpendicular ground change in bottom of the sea from the fault model. Still, for the risk evaluation of the tsunami, magnitude scales of earthquakes and those locations are assumed, the earthquake fault model is presumed, and a tsunami numerical calculation is done by the method as the above-mentioned.

On the other hand, the research on the estimation process of the tsunami initial displacement of water surface, that is the initial-value problem of the tsunami, includes pioneering research of Aida (1969). Here, the coefficient that becomes each amount of the change is requested by the least squares method, when setting up the unit source of wave in the sea area where the initial water level distribution is expected. However, it is necessary to note that the location of the unit source of wave is assumed from earthquake information beforehand. In the research on the tsunami inversion, the research of Satake (1986) is typical. Here the tsunami record of the tidal station is used by the research. However, Satake only used the tsunami record of the tidal station, not to have done the tsunami inversion in order to presume the tsunami initial displacement of water surface, but in order to examine no homogeneity of the fault movement. Actually, the amount of slip of each small fault that divides the fault model into some models is chosen as an unknown parameter of the inverse problem. Therefore, it can be said that the style that essentially assumes the fault model and calculates the numerical value of the tsunami will not have changed.

However, Koike et al. (2003) proposed a new method. Here, the tsunami initial displacement of water surface is presumed directly by the tsunami inversion. It is devised to reduce the number of the unknown parameters by using the base of wavelet in order to evade non-appropriateness of the inverse problem, and to solve the inverse problem by the least squares method uniquely. But, at the present stage theoretically setting the standard of the

selection of the base of wavelet is extremely difficult, and the theoretical consideration concerning the presumption accuracy becomes impossible.
In this study, we propose the method of solving the inverse problem by giving new a priori information. This inverse problem is that the unknown parameter is the tsunami initial displacement of water surface. This has seemed to be impossible in the tsunami inversion that has increased too much up to now the unknown parameter and uses the tsunami waveform at the tidal station. That is, it is clarified to be able to solve the inverse problem uniquely by newly giving a priori information, though the inverse problem cannot be uniquely solved by the least squares method without a priori information because there are a lot of unknown parameters.

2. Method

2.1 Superposition principle

When the linear long wave theory consists in the numerical calculation of a tsunami, the superposition principle of the output from the numerical calculation consists also. When the tsunami initial displacement of water surface assumes $\mathbf{e}_0$, this can be developed as follows, as the superposition of basis vector $\mathbf{e}_k$(k=1, 2,..., n).

$$\mathbf{e}_0 = c_1\mathbf{e}_1 + c_2\mathbf{e}_2 + \ldots + c_n\mathbf{e}_n \tag{1}$$

Here, c_k (k=1, 2,..., n) is a development coefficient.
It is $a_k(t)$ (k=1, 2,..., n: t is time) respectively as for the output of assuming basis vector $\mathbf{e}_k$ (k=1,2,...,n) to be an initial condition and calculating the numerical value. The numerical calculation output a(t) in an observation station to the former tsunami initial displacement of water surface $\mathbf{e}_0$, can be shown by the superposition principle as follows.

$$a(t) = c_1 a_1(t) + c_2 a_2(t) + \ldots + c_n a_n(t) \tag{2}$$

Thus, the superposition principle consists strictly theoretically. However, it is realistic in an actual numerical calculation to select the some among bases developed as Eq. (1) and to apply the superposition principle of Eq. (2). The problem of the selection of the base is important for the tsunami inversion. Actually, the number of unknown parameters that should be presumed, changes by the way how to select the base.

2.2 Formulation of tsunami inversion

Here, a tsunami inversion is formulated based on Eq. (2). A tsunami inversion presumes the tsunami initial displacement of water surface from the observation waveform of the tidal stations. In Eq. (2), the numerical calculation output a(t) thinks of the observation waveform d(t) in the tidal stations and development coefficient c_k(k=1, 2,..., n) thinks of unknown development coefficient m_k(k=1 ,2 ,..., n). A tsunami inversion can be formulated as follows.

$$\mathbf{Gm}=\mathbf{d} \tag{3}$$

$$G = \begin{bmatrix} a_1(1) & a_2(1) & \ldots & a_n(1) \\ a_1(2) & a_2(2) & \ldots & a_n(2) \\ \ldots & \ldots & \ldots & \ldots \\ a_1(t) & a_2(t) & \ldots & a_n(t) \end{bmatrix}$$

$$\mathbf{m} = [\, m_1, m_2, ..., m_n \,]^T$$

$$\mathbf{d} = [\, d(1), d(2), ..., d(t) \,]^T$$

When there are two or more numbers of tidal stations, it only has to increase the number of t of the observation waveform d(t). And, this inverse problem is able to be solved by some methods. We assume that unknown development coefficient m_k (k= 1, 2,..., n) are able to be presumed as follows.

$$\mathbf{m}^{est}=\mathbf{G}^{-g}\mathbf{d} \tag{4}$$

Here, $\mathbf{G}^{-g}$ is called as general inverse matrix, but isn't ordinary inverse matrix.
When the unknown development coefficient m_k^{est}(k= 1, 2,..., n) presumed thus are substituted for the development coefficient of the base that Eq. 1 corresponds, the estimation of the tsunami initial displacement of water surface $\mathbf{e}_0^{est}$ is finally requested as follows.

$$\mathbf{e}_0{}^{est}=m_1^{est}\,\mathbf{e}_1+m_2^{est}\,\mathbf{e}_2+...+\,m_n^{est}\,\mathbf{e}_n \tag{5}$$

2.3 Least squares method

Eq. (3) is the simplest equation in the inverse problems, and it is called the linear inverse problem. The linear inverse problem is often solved by the least squares method that generally minimizes the prediction error (ex., Menke, 1989). Here, we think how the linear inverse problem of Eq. (3) is solved by the least squares method. First of all, error E can be shown in the linear inverse problem as follows.

$$E=\mathbf{e}^T\mathbf{e}=(\mathbf{d}-\mathbf{G}\mathbf{m})^T(\mathbf{d}-\mathbf{G}\mathbf{m}) \tag{6}$$

And, each of partial differential of error E by m_k (k= 1, 2,..., n) are assumed to be 0. The following equation obtains.

$$\mathbf{G}^T\mathbf{G}\mathbf{m}-\mathbf{G}^T\mathbf{d}=0 \tag{7}$$

Here, if $[\mathbf{G}^T\mathbf{G}]^{-1}$ assumes to exist, the solution of the linear inverse problem by the least squares method can be requested as follows.

$$\mathbf{m}^{est}=[\mathbf{G}^T\mathbf{G}]^{-1}\mathbf{G}^T\mathbf{d} \tag{8}$$

Here, the inverse matrix $[\mathbf{G}^T\mathbf{G}]^{-1}$ assumes to exist, the solution by the least squares method has been derived. Because this assumption is an important problem, we examine it later.

2.4 Uniqueness of solution and selection of base

In Eq. (8), when the inverse matrix $[\mathbf{G}^T\mathbf{G}]^{-1}$ exists, the solution of the inverse problem has uniqueness. Therefore, the existence of the inverse matrix becomes important. This inverse matrix depends on only the data nucleus **G**, but not on the observed value **d**. And, as being able to understand from Eq. (3), the data nucleus **G** are the sets of Green's functions $a_k(t)$ (k= 1, 2,..., n). Therefore, the existence of the inverse matrix $[\mathbf{G}^T\mathbf{G}]^{-1}$ depends on Green's functions. In addition, it also depends on the selection of base which becomes the origin.
The study of Koike et al. (2003) had selected the base of wavelet as a base before. The method used in that study was the method derived for the inverse matrix $[\mathbf{G}^T\mathbf{G}]^{-1}$ to exist, in order to solve the tsunami inversion by least squares method with uniqueness. However, when the base by wavelet is used, because calculating all Green's functions in Eq. (2) is

impossible, they should actually discontinue the number n of bases in a suitable point. But, it is a current state that it is difficult to derive the standard theoretically.
In this study therefore, we will adopt the base in which only the 10 times 10 meshes have 1, but all the remainder have 0, that understands easily and knows by intuition. Though this is not a basis vector precisely, all the discussions of the above-mentioned consist, because the superposition principle consists even in this case. This also will be called a base in this study. Actually, if we will select such base, the character of data nucleus **G** becomes extremely worse and the inverse matrix $[\mathbf{G}^T\mathbf{G}]^{-1}$ doesn't exist. Therefore, the inverse problem cannot be uniquely solved by the least squares method. Then, it is necessary to give the condition in addition.

2.5 A priori information

If in the inverse problem the character of data nucleus **G** is bad and the inverse matrix $[\mathbf{G}^T\mathbf{G}]^{-1}$ doesn't exist, this problem is called as inferior decision. The inferior decision happens when many solutions whose prediction error E becomes 0 exist. Usually, when the number of unknown parameters is more than that of data, inferior decision problem is caused. To obtain the solution $\mathbf{m}^{est}$ to the inverse problem, the method of choosing only one appropriate solution among the infinite solutions whose prediction error becomes 0 is needed. For that, it is necessary to add other information not included to the inverse problem in Eq. (3). This additional information is called as a priori information (Jackson, 1979). That is quantitatively the expressions of the feature expectation of the solution beforehand, but no one is based on actual data.
In this study, it decides to be taken that the solution is smooth as a priori information. That is, tsunami initial displacement of water surface is expected to change smoothly. This a priori information can be formulated as follows. The smoothness of the continuous function of the space can be quantified with norm of the first derived function. It only has to use the difference between unknown development coefficients that are physically adjacent as the approximation of the differentiation for a discrete, unknown development coefficient. That is, the smoothness **I** of unknown development coefficient **m** is shown as follows (For one dimension).

$$\mathbf{I}=\begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \cdots & & \\ & & & -1 & 1 \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \\ \cdots \\ m_n \end{bmatrix} = \mathbf{Dm} \tag{9}$$

Here, **D** is the matrix who gives smoothness. The smoothness of the solution as the whole is shown as follows in the form of norm L.

$$L = \mathbf{I}^T\mathbf{I} = [\mathbf{Dm}]^T[\mathbf{Dm}]$$

$$= \mathbf{m}^T\mathbf{D}^T\mathbf{Dm} = \mathbf{m}^T\mathbf{W}_m\mathbf{m} \tag{10}$$

Here, the matrix $\mathbf{W}_m=\mathbf{D}^T\mathbf{D}$ can be interpreted as the weight coefficient, when norm of vector **m** is calculated.
And, the solution of the inverse problem when a priori information is given to the least squares method can be solved as follows by the minimization of $E+p^2L$ that is the combination of the prediction error and norm of the solution.

$$\mathbf{m}^{est}=[\mathbf{G}^T\mathbf{G}+p^2\mathbf{W}_m]^{-1}\mathbf{G}^T\mathbf{d} \quad (11)$$

Here, p is the weight coefficient. That is a coefficient by which a relative ratio of the importance is decided which the prediction error E and norm L of the solution are to put the emphasis. When p is large enough, the inferior decision part of the solution is minimized. However, the superior decision part of the solution tends also to be minimized at the same time. Consequentially, the solution doesn't become to minimize the prediction error E, and not become good presumption too much about the parameter of a true development coefficient. Oppositely, though the prediction error is minimized when putting p to 0, a priori information will not be considered at all, to decide only one parameter of the development coefficient of the inferior decision. Thus, when this equation regards as dumping least squares method, the theoretical method to decide the value of p isn't at present (Menke). However, from the viewpoint of the theory of probability idea, this weight coefficient p is not only to stabilize the solution, but also to decide that value by the ratio of the dispersion of the error in observational data and a priori data (Franklin, 1970). Here, Eq. (11) is regarded as dumping least squares method.
Thus, if the 10 times 10 mesh base is taken as a base even, the inverse problem can be solved by adding a priori information. However, when to doing the discussion related to uniqueness of the estimation $\mathbf{m}^{est}$ in Eq. (11), it becomes extremely important whether a priori assumption that the solution is smooth is appropriate

3. Results

Here, we will plainly explain the results of the theory in the preceding chapter through a concrete numerical calculation example of the tsunami inversion.

3.1 Location of computational domain and tidal stations

As the numerical calculation area where the tsunami inversion is done, the sea area off the coast of Japan shown in Fig.1 is taken. In addition, the tidal staions of nine points that actually exist as shown in Fig.1 are chosen.

3.2 10 times 10 mesh base and Green's function

When the 10 times 10 mesh bases (10km times 10km) shown in Fig.2 in which 10 times 10 meshes only take the value of 1 but remainder is all 0 are the initial condition, the waveforms (Green's function) in the tidal stations of nine points are calculated by the usual linear tsunami numerical calculation. In the computational domain of Fig.1, because the number of total meshes is 400 times 400, 10 times 10 mesh bases will be 40 times 40 = 1,600 pieces, and we have to calculate a tsunami numerical simulation 1,600 times to request all Green's functions. Actually, the number of calculating is fewer than 1,600, because we may not calculate the part in the land shown in Fig.1. However, here, the development coefficient (actually, tsunami initial displacement of water surface in 10 times 10 meshes) to 40 times 40 = 1,600 bases including the part in the land will be taken as unknown parameters.

3.3 A priori information

In the preceding chapter, when an unknown development coefficient is one dimension, the matrix which gives a priori information on the smoothness of the solution is shown. However, here, when an actual, unknown development coefficient is two dimensions, the matrix which gives the smoothness of the solution will be shown.

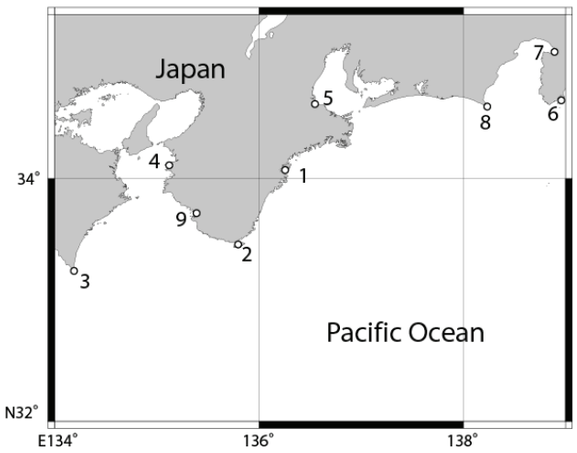


Fig. 1. Location of computational area and tidal stations

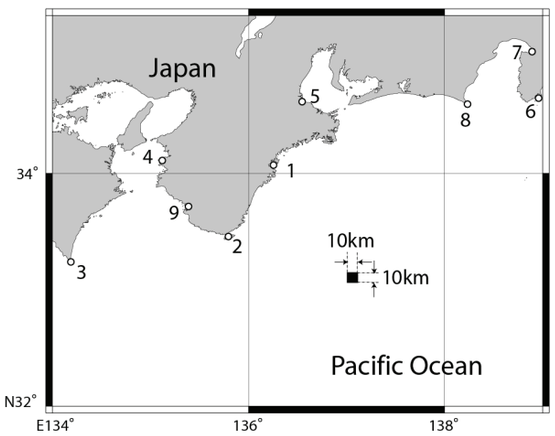


Fig. 2. The 10 times 10 mesh base

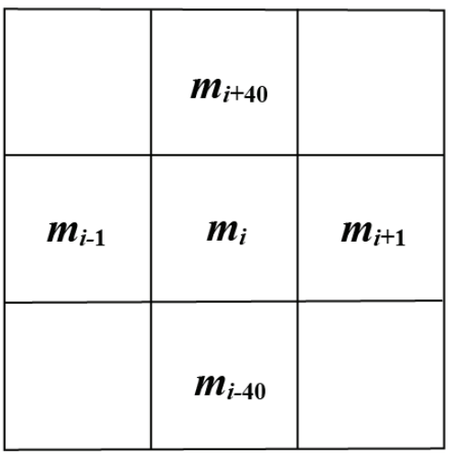


Fig. 3. Two dimension's arrangement of unknown development coefficient

Though Fig.3 is two dimension's arrangement of unknown development coefficient of 10 times 10 mesh bases, we think about the condition concerning development coefficient m_i.
Though the development coefficient m_i is adjacent to the development coefficients m_{i+1}, m_{i-1}, m_{i+40}, and m_{i-40}, we has to think only m_{i+1} and m_{i+40}, because we think about m_{i-1} and m_{i-40} when each of m_{i-1} and m_{i-40} is the centre of Fig.3. The smoothness of the solution can be quantified with norm of the first derived function as described in the preceding chapter, and because we only has to use the difference instead of the derivation, the row corresponding to the development coefficient m_i of the matrix **D** in Eq. (9) is as follows.

$$[\ -2 \quad 1 \quad\quad 1\ ] \tag{12}$$

Here, -2 is in i column, and just right 1 is in i+1 column, and in addition right 1 is in i+40 column.
In addition, a priori information that the unknown development coefficient located in the land is 0 is given. Though the development coefficient located in the land do not have to be put in the unknown parameter of the inversion originally, giving the matrix becomes difficult if only the part of the land will be excluded when the matrix which gives the above-mentioned smoothness of the solution is considered. So, a priori information that the value is sure to become 0 is decided to be programmed by putting the development coefficient located in the land into the unknown number here. The condition of unknown development coefficient m_i located in the land being equal to 0 can be shown as follows.

$$\begin{bmatrix} 0 \dots 0\ 1\ 0 \dots 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \dots \\ m_i \\ \dots \\ m_n \end{bmatrix} = 0 \tag{13}$$

In order to put in such a constraint condition, we should include this equation of constraint condition in the row of the inverse problem $\mathbf{Gm}=\mathbf{d}$. And, as putting of infinitely large weight compared with other equations, we only has to adjust the weight matrix $\mathbf{W}_e$ of the prediction error $E=\mathbf{e}^T\mathbf{W}_e\mathbf{e}$ that considers weight (Lawson and Hanson, 1974).
Finally, when these a priori information is given to the least squares method, the solution of the inverse problem $\mathbf{Gm}=\mathbf{d}$ can be solved as follows by the minimization of $E+p^2L=\mathbf{e}^T\mathbf{W}_e\mathbf{e}+p^2\mathbf{m}^T\mathbf{W}_m\mathbf{m}$ that is the combination of the prediction error and norm of the solution. That is, the combination of the prediction error and norm of the solution is shown as follows.

$$E+p^2L=(\mathbf{d}-\mathbf{Gm})^T\mathbf{W}_e(\mathbf{d}-\mathbf{Gm}) +p^2\mathbf{m}^T\mathbf{W}_m\mathbf{m} \tag{14}$$

And, each of partial differential of this equation by m_k (k= 1, 2,..., n) are assumed to be 0. The following equation obtains.

$$[\mathbf{G}^T\mathbf{W}_e\mathbf{G}+p^2\mathbf{W}_m\ ]\mathbf{m}-\mathbf{G}^T\mathbf{W}_e\mathbf{d}=0 \tag{15}$$

And, if the inverse matrix $[\mathbf{G}^T\mathbf{W}_e\mathbf{G}+p^2\mathbf{W}_m]^{-1}$ exists, the solution is required as follows.

$$\mathbf{m}^{est}=[\mathbf{G}^T\mathbf{W}_e\mathbf{G}+p^2\mathbf{W}_m\ ]^{-1}\mathbf{G}^T\mathbf{W}_e\mathbf{d} \tag{16}$$

3.4 Artificial observation data

Actually, here, we try to test numerically whether tsunami initial displacement of water surface can be presumed by the tsunami inversion using this method. That is, we apply this method to the artificial observation data, but not to the tsunami record in actual tidal stations. That is, we apply to the artificial observation data calculated from artificial tsunami initial displacement of water surface, and how much former tsunami initial displacement of water surface can be reproduced is examined.

First of all, the tsunami initial displacement of water surface (Fig.4) calculated from the fault model of the Tonankai earthquake in 1944 by the method of Mansinha and Smylie (1971) is used. And, this is assumed to be an initial condition, the usual linear tsunami numerical calculation is calculated, and the calculation waveform in the nine tidal stations of Fig.1 is assumed to be artificial observation data.

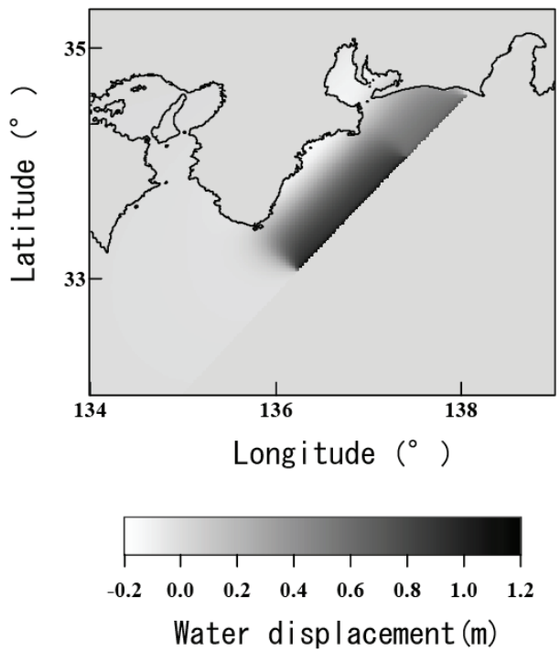


Fig. 4. The artificial tsunami initial displacement of water surface

3.5 Tsunami inversion

Here, we executed the tsunami inversion based on the artificial observation data of every 20 seconds until after two hours from the earthquake generation. Moreover, the number of tidal stations used all of the nine points. In addition, the value of p in Eq. (16) was set to 1.0. The tsunami initial displacement of water surface as shown in Fig.5 can be presumed by doing the tsunami inversion on the above-mentioned condition. When this is compared with former artificial tsunami initial displacement of water surface (Fig.4) that is the correct answer, it is understood to be corresponding by considerable accuracy if the notched part of 10 times 10 meshes that cannot be avoided because of the setting of an unknown development coefficient is excluded. If Root Mean Square Error (RMSE) is actually calculated, it becomes 0.070m.

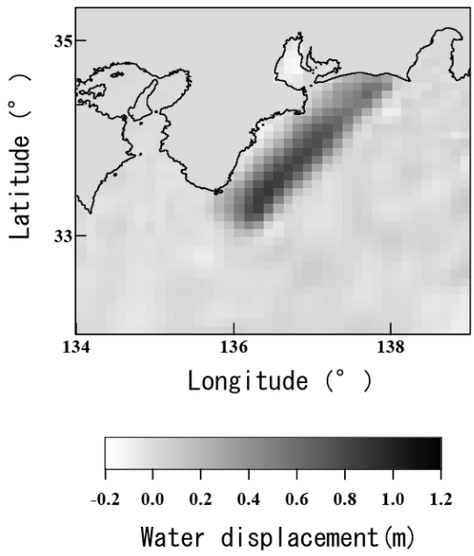


Fig. 5. Estimation result of tsunami initial displacement of water surface by tsunami inversion

4. Consideration about estimation error

When the tsunami inversion is done, it often becomes a problem which tide stations' waveform records should be used and how much observation time should be used. Because the influence of the observation error and the nonlinearity of a tsunami are included in the tide record of a tsunami in the real problem, consideration of the record of tide stations is necessary. However, here, uniqueness of the solution and the estimation error clarified when thinking about the tsunami inversion as an ideal linear inverse problem that the observation error is not included will be considered.

4.1 Model resolution matrix and uniqueness of solution

First of all, it is assumed that true development coefficient $\mathbf{m}^{true}$ of $\mathbf{Gm}^{true}=\mathbf{d}^{obs}$ exists. And, we examines whether the estimation of an unknown development coefficient is how much close to this true solution. When it substitutes the equation of $\mathbf{Gm}^{true}=\mathbf{d}^{obs}$ to the observation value for the equation of $\mathbf{m}^{est}=\mathbf{G}^{-g}\mathbf{d}^{obs}$ that shows the estimation of an unknown development coefficient, it becomes the following.

$$\mathbf{m}^{est}=\mathbf{G}^{-g}\mathbf{d}^{obs}=\mathbf{G}^{-g}[\mathbf{Gm}^{true}]$$

$$=[\mathbf{G}^{-g}\mathbf{G}]\mathbf{m}^{true}=\mathbf{Rm}^{true} \tag{17}$$

Here, the matrix of $\mathbf{R}=\mathbf{G}^{-g}\mathbf{G}$ is called as the model resolution matrix of n times n (n is a number of unknown development coefficients) (Wiggins, 1972). In this method, because the generalized inverse matrix $\mathbf{G}^{-g}$ can be regarded as $\mathbf{G}^{-g}=[\mathbf{G}^{T}\mathbf{W}_{e}\mathbf{G}+p^{2}\mathbf{W}_{m}]^{-1}\mathbf{G}^{T}\mathbf{W}_{e}$ from Eq. (16), the model resolution matrix is requested as follows.

$$\mathbf{R}=[\mathbf{G}^{T}\mathbf{W}_{e}\mathbf{G}+p^{2}\mathbf{W}_{m}]^{-1}\mathbf{G}^{T}\mathbf{W}_{e}\mathbf{G} \tag{18}$$

If $\mathbf{R}=\mathbf{I}$ ($\mathbf{I}$ is the identity matrix) is true, the value of each unknown development coefficient can be uniquely decided (uniqueness of the solution). If $\mathbf{R}$ is not the identity matrix, the

estimation of an unknown development coefficient will actually show the average with the weight of a true development coefficient. The model resolution matrix is a function of a priori information added to data nucleus **G** and the problem. The model resolution matrix is irrelevant to an actual value of the observation data.

4.2 Introduction of parameter resolution

By the foregoing paragraph, it has been understood to be able to discuss uniqueness of the solution by examining the model resolution matrix. That is, when a row of the model resolution matrix **R** is taken out, if only the value of a certain column is 1 and the value of the column of the remainder is all 0, a corresponding unknown development coefficient is uniquely decided. However, when it is not so, an unknown development coefficient cannot be uniquely decided. But, if the value of the column corresponding to an unknown development coefficient is close to 1 and the difference of the value of the row is small, it can be interpreted that it is case when the unknown development coefficient is decided almost uniquely and the character as the inverse problem is good. Therefore, if the difference condition of the value of the row corresponding to an unknown development coefficient in the model resolution matrix **R** can be quantified, it is convenient to do a discussion about uniqueness of the solution. So, we think introduction of the index of the parameter resolution **C**. First of all, from the value (r_1, r_2, ..., r_i, ..., r_n) of the row corresponding to an unknown development coefficient m_i of the model resolution matrix **R** of Eq. (17), the estimation m_i^{est} of an unknown development coefficient can be shown as follows.

$$m_i^{est}=r_1m_1^{true}+r_2m_2^{true}+...$$

$$+r_im_i^{true}+...+r_nm_n^{true} \tag{19}$$

Here, if r_i=1 is and r of the remainder is all 0, unknown development coefficient m_i is uniquely decided. Moreover, when r_i is close to 1, only the one that the value of other r is close to m_i is large and the difference condition is small, it is thought that the estimation is also very close to the true value. In order to express this quantitatively, it only has to think about the weighed mean average, by giving big weight to the value of the development coefficient to which the distance is close, and by giving only small weight to a distant development coefficient. Then, parameter resolution C_i to estimation m_i^{est} of an unknown development coefficient will be shown as follows.

$$C_i = \sum_{l=1}^{n}\left(r_l \times \frac{1}{1+\sqrt{\Delta x_{l-i}^2 + \Delta y_{l-i}^2}} \right) \tag{20}$$

Here, $\Delta x_{l\text{-}i}$ is the distance in the direction of x of development coefficient m_i and m_l, and $\Delta y_{l\text{-}i}$ is the distance in the direction of y. For instance, when an unknown development coefficient is arranged shown as Fig.3, the distance of m_i and m_{i+1} is 1, and the distance of m_i and m_{i+41} is $2^{1/2}$. The parameter resolution C_i becomes 1 when the value of $0 < C_i < 1$ is taken and an unknown development coefficient is decided uniquely, and the more approaches 0, the more the difference condition grows and the character worsens. Therefore, it is thought that the estimation error is small when resolution C_i is close to 1, but the estimation error is big when it is close to 0.

Though there is Dirichlet spread function based on L_2 norm of the difference of the resolution matrix and the identity matrix as an amount that shows the goodness of the

character of the entire of the parameter resolution matrix. This is different from the parameter resolution introduced in this chapter. The parameter resolution is an index that pays attention to one parameter (development coefficient).

4.3 Relationship between parameter resolution and estimation error

Here, the influence that the parameter resolution C_i affects the estimation error will be seen through the concrete example. The estimation error is defined as follows.

$$\text{estimation error (m)} = |\text{true value (m)} - \text{estimation value(m)}|$$

Fig.6 shows the estimation result that executes the tsunami inversion used of tide stations No.1, 2 in Fig.1 as well as Chapter 3, and Fig.7 is used of No.2, 3. The value of root mean square error RMSE is 0.092m, and 0.119m respectively. Compared with Fig.4 that is the correct answer, it is understood that estimation error of Fig.6 is small than that of Fig.7. In these two cases, the condition of inversion is the same, which is 20 seconds at observation intervals, is two hours at observed time and is p=1.0, but the location of the observation stations is only different. Where is the difference of this estimation error caused from? By examining the parameter resolution, the difference is possible to explain to some degree.

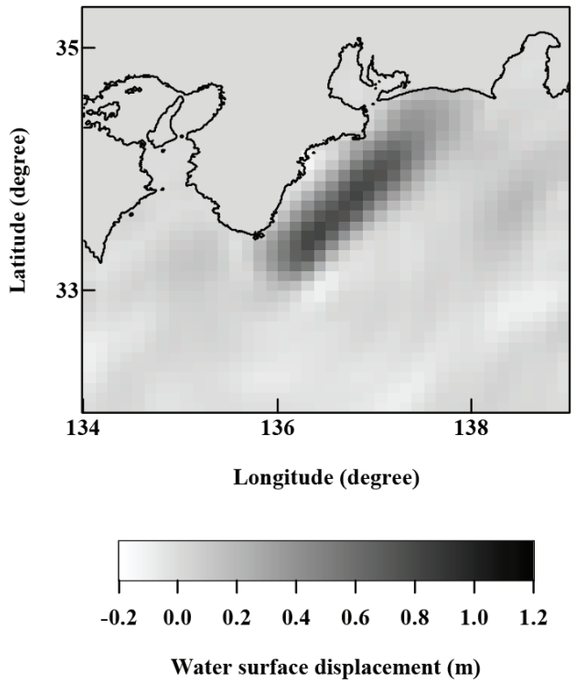


Fig. 6. Estimation of tsunami initial displacement of water surface (in the case used of No.1, 2 stations)

Fig.8 and Fig.9 are figures drawn by arranging the parameter resolution, when the tsunami inversion is done by using the combination of observation stations No.1, 2 and No.2, 3, at the location of an unknown development coefficient. From these figures, while in Fig.8 the area where the parameter resolution is 0.5 or more covers the area of tsunami initial displacement of water surface of the correct answer of Fig.4, in Fig.9 the area where the parameter resolution is 0.5 or more moves to the west side and the parameter resolution has

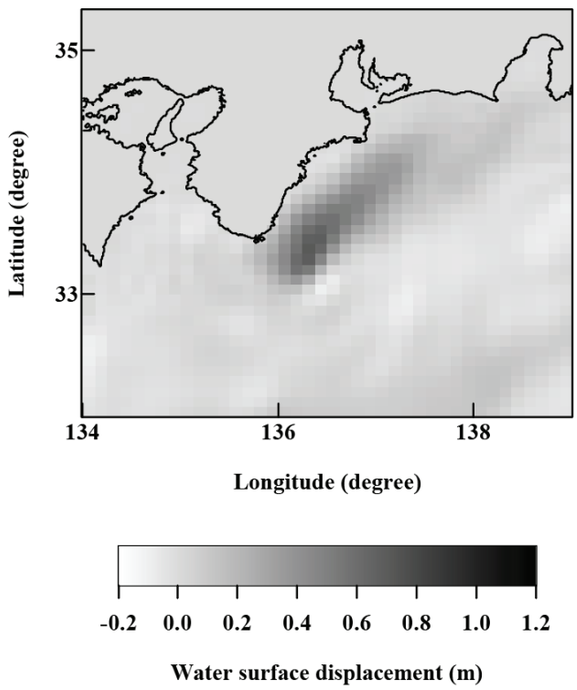


Fig. 7. Estimation of tsunami initial displacement of water surface (in the case used No.2, 3 stations)

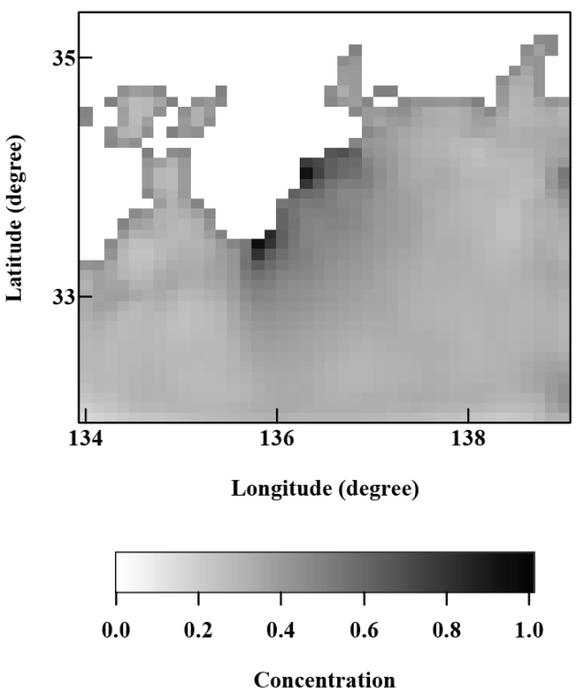


Fig. 8. Parameter resolution when the tsunami inversion is done (in the case used of No.1, 2 stations)

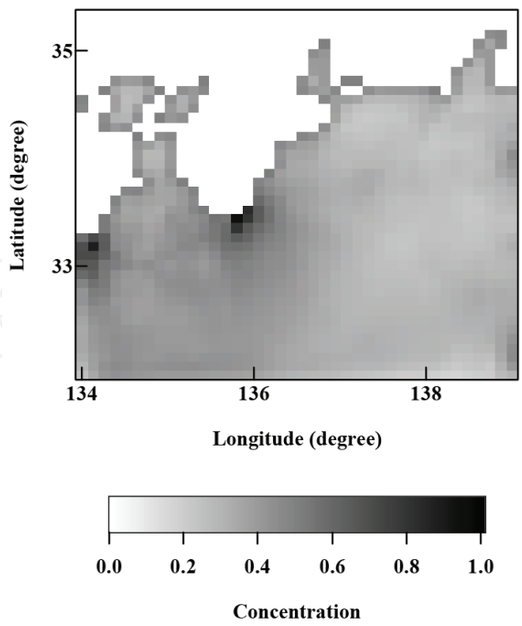


Fig. 9. Parameter resolution when the tsunami inversion is done (in the case used of No.2, 3 stations)

lowered in half an east side of the area of tsunami initial displacement of water surface of the correct answer of Fig.4. When Fig.8 and Fig.9 of distribution of these parameter resolutions are made to correspond to Fig.6 and Fig.7 respectively, it is understood that the estimation accuracy has lowered in the area where the parameter resolution is low, though the estimation accuracy is high in the area where the parameter resolution is high. Thus, the estimation error of an unknown development coefficient understands that the expectation is roughly applied by examining the parameter resolution.

5. Conclusion

Here, the condition that the solution is smooth as a priori information is taken to the tsunami inversion, and we proposed a new method of presuming the tsunami initial displacement of water surface directly from the observed waveforms of a tsunami in the tidal stations. And, when the artificial observation data is set and the condition of the tsunami inversion is changed, it was considered how the estimation error became. The conclusion obtained in this study is as follows.

1. When the tsunami initial displacement of water surface is taken as an unknown parameter, the number of the unknown parameters increases more than the number of observation data and the unknown parameters cannot be specified by the tsunami inversion method that uses only a usual least squares method. Then, we proposed a new method of specifying an unknown development coefficient by taking to the inversion a priori information which the development coefficients of the adjoined unknown parameters are smooth. And, it was shown that the tsunami initial displacement of water surface can be estimated by using this method due to the error with a small.

2. In order to discuss uniqueness of the unknown development coefficient requested by the tsunami inversion, the index of parameter resolution based on the model resolution matrix was introduced. Though uniqueness of an unknown development coefficient consists only when the parameter resolution is 1, when the parameter resolution is 0.5 or more, it showed that the estimation error lowered to 0.1m or less from consideration when the artificial observation data was set.
3. The parameter resolution is an index that doesn't depend on the observation data, only depending on the condition of Green's function and the inversion. Therefore, when it tries to presume the tsunami initial displacement of water surface that actually happens, it can be discussed how accuracy the tsunami initial displacement of water surface can be presumed by firstly examining the distribution of the parameter resolution. Thus, the index of parameter resolution is an index very useful for the discussion about the estimation error of the tsunami inversion.
4. As an application of this study, the tsunami initial displacement of water surface is presumed by using the data of the tsunami recorder set up offshore, and the tsunami numerical value forecast is that the height of tsunamis of various places in the coast part is forecast by the numerical calculation. The installation plan of the tsunami recorder is able to discuss by examining the introduced parameter resolution in this study, for example, where it is necessary to set up the tsunami recorders of how much number.

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The Tsunami Threat - Research and Technology

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Submarine earthquakes, submarine slides and impacts may set large water volumes in motion characterized by very long wavelengths and a very high speed of lateral displacement, when reaching shallower water the wave breaks in over land - often with disastrous effects. This natural phenomenon is known as a tsunami event. By December 26, 2004, an event in the Indian Ocean, this word suddenly became known to the public. The effects were indeed disastrous and 227,898 people were killed. Tsunami events are a natural part of the Earth's geophysical system. There have been numerous events in the past and they will continue to be a threat to humanity; even more so today, when the coastal zone is occupied by so much more human activity and many more people. Therefore, tsunamis pose a very serious threat to humanity. The only way for us to face this threat is by increased knowledge so that we can meet future events by efficient warning systems and aid organizations. This book offers extensive and new information on tsunamis; their origin, history, effects, monitoring, hazards assessment and proposed handling with respect to precaution. Only through knowledge do we know how to behave in a wise manner. This book should be a well of tsunami knowledge for a long time, we hope.

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