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Stabilization of Networked Control Systems with Input Saturation

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1. Introduction

The technologies on multi-accessible communication networks have been recently received considerable attention in the area of networked control systems (NCSs) since the multi-accessibility feature leads to flexibility, cost reduction, and distributiveness. However, since the multi-accessibility sometimes causes random network-induced delays that deteriorate the stability and control performance of closed-loop control systems, it is necessary to always handle the delay problem when implementing a feedback control loop closed through multi-accessible communication networks. As a result, numerous investigation and research efforts are underway to deal with the delays (see *e.g.,* Kim *et al.* (2003), Tipsuwan and Chow (2003), Kim *et al.* (2004), Seiler and Sengupta (2005), Yue *et al.* (2005), Yang (2006), and references therein), recent results of which have focused on the problem of attenuating not only the effect of the network-induced delay but also that of unknown disturbances. According to the trend, Kim *et al.* (2004) designed a network-delay-dependent $\mathcal{H}_\infty$ controller for discrete-time systems over communication networks using a deterministic approach. And Seiler and Sengupta (2005) established necessary and sufficient linear matrix inequality (LMI) conditions for the synthesis of the $\mathcal{H}_\infty$ controller for discrete-time systems with Markovian jumping parameters. And Yue *et al.* (2005) designed a robust $\mathcal{H}_\infty$ controller for uncertain continuous-time NCSs with the effects both of the network-induced delay and of data dropout.
In addition to the networked-induced delay and the external disturbance problem, this paper is interested in addressing the input saturation problem since every physical actuator is subject to saturation, and moreover the input saturation deteriorates the stability of the feedback control. Already for the traditional point-to-point communication network, various research results of explicitly handling the input saturation have been published in the system and control literature. Of them, several important results of directly handling the input saturation have recently appeared well in Nguyen and Jabbari (2000), Gomes da Silva *et al.* (2001), Hu and Lin (2001), Hu *et al.* (2002), and references therein. However, to the best of our knowledge, there has been almost no results of considering the input saturation as a constraint in designing the NCSs. Hence, in this paper, we intend to design an $\mathcal{H}_\infty$ control capable of reducing the adverse effects both of the input saturation as well as of the network-induced delay.
The goal of this paper is to design a networked-delay-dependent switching controller for systems with input saturation using the deterministic information from the current

timestamp. The structural assumption of the networked system follows most of Kim *et al.* (2004). Particulary, we are to employ a reliable transport protocol that guarantees data delivery and supports that transmitted data have their *time-stamp* information. However, taking a step forward in Kim *et al.* (2004), we insert a saturator between the controller and the communication network, and extract built-in memory from the controller for clarity of explanation (see Fig. 1). Based on the modified structure, we propose a

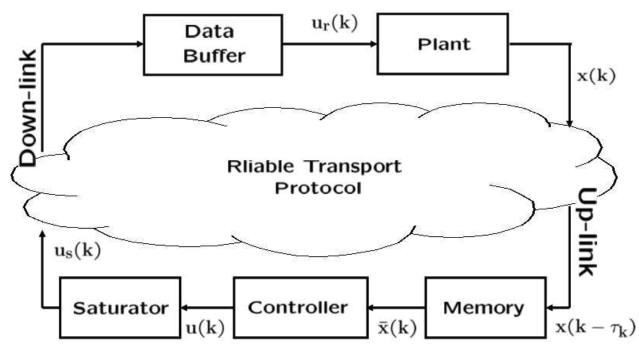

Fig. 1. Networked control system.

systematic method for designing a network-delay-dependent switching controller that achieves the $\mathcal{H}_\infty$ disturbance-rejection performance under an $\mathcal{L}_\infty$ performance representing componentwise input saturation. Methodologically, we first construct a suitable networked control system (NCS) with asymmetric path-delay, and build up the conditions for set invariance, involved in local stabilization, and then incorporate these conditions in the synthesis of a network-delay-dependent $\mathcal{H}_\infty$ controller. In the procedure, we use the polytopic representation method, proposed by Hu and Lin (2001), as one way to deal with the saturation nonlinearity, and employ a quasi-linear parameter-varying (QLPV) structure for the control system as in Wu *et al.* (2005). Then the resultant convex solvability conditions are expressed as a finite number of linear matrix inequalities (LMIs), whose solutions are applied to the construction of the switching controller depending on the previous and current modes characterized by network traffic.

The chapter is organized as follows: Section 2 states target systems and assumptions. Section 3 proposes the condition for set invariance, and explains the procedure of constructing an $\mathcal{H}_\infty$ dynamic state-feedback controller. Section 4 illustrates the performance of the proposed algorithm through an example. Finally, in Section 5, concluding remarks are made.

Notation: Notations in this paper are fairly standard. For $x \in \mathcal{R}^n$, $||x||$ is taken to be the standard Euclidian norm., *i.e.*, $||x|| = (x^T x)^{1/2}$. The Lebesgue space $\mathcal{L}_{2+} = \mathcal{L}_2[0,\infty)$ consists of square-integrable functions on $[0,\infty)$. $\mathcal{L}_\infty$ denotes the space of bounded vector sequences $u(k)$, equipped with the norm $||u||_\infty = \sup_i\{\sup_k |u_i(k)|\}$, and $\mathcal{L}_{\infty,e}$ denotes the space of bounded vector sequences $w(k)$, with the norm $||w||_{\infty,e} = \sup_k\{w^T(k)w(k)\}$. The notation $X \geq Y$ and $X > Y$ means that $X - Y$ is positive semi-definite and positive definite, respectively. Inequalities between vectors mean componentwise inequalities, $\mathrm{svd}(\cdot)$ means a singular value decomposition function, and the saturation function $\mathrm{sat}(u,\bar{u}) : \mathcal{R}^m \to \mathcal{R}^m$ denotes

$$\mathrm{sat}(u,\bar{u}) = [s_1\ s_2\ \cdots\ s_m]^T,\ s_i = \mathrm{sign}(u_i) \cdot \min\{\bar{u}_i, |u_i|\}, \tag{1}$$

where $\mathrm{sign}(\cdot)$ returns the signs of the corresponding argument, and u_i and $\bar{u}_i$ stand for the i-the element of $u \in \mathcal{R}^m$ and $\bar{u} \in \mathcal{R}^m$, respectively. For a matrix $V \in \mathcal{R}^{r\times s}$, $\mathcal{L}(V)$ denotes a linear region as defined in

$$\mathcal{L}(V) \overset{\triangle}{=} \{x \in \mathcal{R}^s | -\sigma \leq Vx \leq \sigma\}, \tag{2}$$

where $\sigma \in \mathcal{R}^r$ and $x \in \mathcal{R}^s$. Finally, in symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry.

2. Mathematical representation

Consider the following linear time-invariant (LTI) plant of the form

$$x(k+1) = Ax(k) + B_1 w(k) + B_2 u_r(k), \tag{3}$$
$$z(k) = Cx(k) + D_1 w(k) + D_2 u_r(k), \tag{4}$$

where $x(k) \in \mathcal{R}^n$, $u_r(k) \in \mathcal{R}^m$, $w(k) \in \mathcal{R}^p$ and $z(k) \in \mathcal{R}^q$ denote the state, the input, the disturbance and the performance output, respectively. Throughout this paper, we assume that the disturbance $w(k)$ is unknown but belongs to a known bounded set $\mathcal{W}$ defined as:

$$\mathcal{W} \overset{\triangle}{=} \left\{w \in \mathcal{R}^p \mid ||w(k)||^2 \leq \bar{w},\ \bar{w} \geq 0,\ \forall k \geq 0\right\}, \tag{5}$$

which implies $w(k) \in \mathcal{L}_{\infty,e}$ for all $k \geq 0$. In addition, we recall some assumptions from Kim *et al.* (2004):

(A1) Network-induced delay is composed of the down-link delay τ_k^d and the up-link delay τ_k^u which are random but bounded as $0 \leq \tau_k^d \leq \bar{\tau}$, $0 \leq \tau_k^u \leq \bar{\tau}$.

(A2) Multiple data can be received at the same time.

(A3) Time-stamp information is appended to the transmitted data between the plant and a controller, which plays an important role in configuring a networked-control system (NCS) for the plant (4).

Fig. 1 shows an NCS subject to our needs, where the difference from that of Kim *et al.* (2004) is that a saturator is inserted between the controller and the communication network, i.e., $u_s(k) = \mathrm{sat}(u(k), \bar{u})$, and a built-in memory is extracted from the controller for clarity of explanation. Except for the difference, the remaining framework follows that of Kim *et al.* (2004), that is, the down-link delay τ_k^d is fixed into its bound value $\bar{\tau}$ through the data buffer, and the up-link delay τ_k^u is confirmed by the real-time information on the up-link delay sequence delivered to the controller.
With the above settings, the system model in the controller-saturator point of view is given as

$$\tilde{x}(k+1) = \tilde{A}\,\tilde{x}(k) + \tilde{B}_1\,w(k) + \tilde{B}_2\,u_s(k), \tag{6}$$
$$z(k) = \tilde{C}\,\tilde{x}(k) + \tilde{D}\,w(k), \tag{7}$$

where $\tilde{x}(k) \triangleq [x^T(k) \mid x^T(k-1) \cdots x^T(k-\bar{\tau}) \mid u_s^T(k-1) \cdots u_s^T(k-\bar{\tau})]^T \in \mathcal{R}^{n+(n+m)\bar{\tau}}$ denotes the augmented state and the matrices are defined as

$$\left[\begin{array}{c|c|c} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \hline \tilde{C} & \tilde{D} & 0 \end{array}\right] \triangleq \left[\begin{array}{c|cccc|cccc|c|c} A & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & B_2 & B_1 & 0 \\ \hline I & 0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 \\ 0 & I & 0 & & \vdots & \vdots & & & \vdots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & & & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & I & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 \\ \hline 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & I \\ 0 & \vdots & & & \vdots & I & 0 & & \vdots & \vdots & 0 \\ \vdots & \vdots & & & \vdots & 0 & I & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & & & \vdots & \vdots & \ddots & \ddots & 0 & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & I & 0 & 0 \\ \hline C & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & D_2 & D_1 & 0 \end{array}\right]. \tag{8}$$

And the state available in the controller, *say*, $\bar{x}(k)$, is given as

$$\bar{x}(k) = \tilde{E}_{m(k)} \tilde{x}(k), \tag{9}$$

where

$$\tilde{E}_{m(k)} \triangleq \left[\begin{array}{c|ccccc|ccc} \Phi_{k0} & 0 & \cdots & \cdots & 0 & & 0 & \cdots & 0 \\ \hline 0 & \Phi_{k1} & 0 & & \vdots & & \vdots & & 0 \\ \vdots & 0 & \Phi_{k2} & \ddots & \vdots & & \vdots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \Phi_{k\bar{\tau}} & & 0 & \cdots & 0 \end{array}\right], \tag{10}$$

$$\Phi_{kr} \triangleq \begin{cases} I & \text{if } x(k-r) \text{ is avilable at time } k, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Here, $m(k)$ denotes a mode corresponding to each status of $\tilde{E}_{m(k)}$ with $(2^{\bar{\tau}+1} - 1)$ different cases, and the unique number assigned to the mode $m(k)$ can be expressed as $m(k) = (b_0 b_1 \cdots b_{\bar{\tau}})_2$, where $(\cdot)_2$ means the binary representation of $m(k)$, and the r-th bit b_r is set to 1 if the r-delayed state, $x(k-r)$, is available at time k, otherwise, the bit is set to 0. From this binary representaion, we can know that the mode $m(k)$ belongs to a set $\mathcal{M} \triangleq \{m \in \mathcal{R} \mid m = 1, 2, \cdots, 2^{\bar{\tau}+1} - 1\}$. Besides, under the assumption (A2), we can uniquely determine a set of transitions, $\mathcal{S}$, only if $\bar{\tau}$ is determined:

$$\begin{aligned} \mathcal{S} \triangleq \{(m(k), m(k-1)) \mid & \text{ all possible transition pairs yielding (A2)} \\ & \text{for } m(k) \in \mathcal{M},\ m(k-1) \in \mathcal{M},\ \forall k\}. \end{aligned} \tag{12}$$

For each case of $\bar{\tau} = 1, 2$, all possible modes and transitions has already been described by Kim *et al.* (2004), where if the initial state set $\{x(0), x(1), \cdots, x(-\bar{\tau})\}$ is given, and the packet loss

does not exist, then there is no necessity for considering all transitions as in Kim *et al.* (2004) since the assumption (A1) ensures $b_{\bar{\tau}} = 1$.
The following lemma presents the polytopic representation method for the input saturation, proposed by Hu and Lin (2001).

Lemma 2.1 *(Hu and Lin (2001)) Let $\mathcal{D}$ be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. Suppose that $|v_r| \le \bar{u}_r$ for all $r = 1, \cdots, m$, where v_r and $\bar{u}_r$ denote the r-th element of $v \in \mathcal{R}^m$ and $\bar{u} \in \mathcal{R}^m$, respectively. Then*

$$\mathrm{sat}(u, \bar{u}) = \sum_{\ell=1}^{2^m} \theta_\ell \left(D_\ell u + D_\ell^- v \right), \ \sum_{\ell=1}^{2^m} \theta_\ell = 1, \ \theta_\ell \ge 0, \tag{13}$$

where D_ℓ denote all elements of $\mathcal{D}$, and $D_\ell^- = I - D_\ell$.

In the following, we present a previous mode (PM)-dependent dynamic quasi-linear parameter varying (QLPV) control law which switches itself depending on its previous and current modes:

$$x_c(k+1) = F_{ji}(\Theta_k) x_c(k) + G_{ji}(\Theta_k) \bar{x}(k), \tag{14}$$

$$u(k) = H_{ji} x_c(k) + J_{ji} \bar{x}(k), \tag{15}$$

$$v(k) = K_{ji} x_c(k) + L_{ji} \bar{x}(k), \tag{16}$$

subject to

$$\left[F_{ji}(\Theta_k) \ G_{ji}(\Theta_k) \right] = \sum_{\ell=1}^{2^m} \theta_\ell(k) \left[F_{ji}^\ell \ G_{ji}^\ell \right], \tag{17}$$

where $v(k)$ is an auxiliary control input, $\Theta_k \in \mathcal{R}^{2m}$ denotes a vector consisting of the time-varying interpolation coefficients $\theta_\ell(k)$ at time k, and the subscripts j and i stand for $m(k)$ and $m(k-1)$, respectively.
Consequently, by Lemma 2.1, the closed-loop system subject to $\hat{x}(k) \in \mathcal{L}([L_{ji}\tilde{E}_j \ \ K_{ji}])$, for all $k \ge 0$, is given as

$$\hat{x}(k+1) = \hat{A}_{ji}(\Theta_k) \hat{x}(k) + \hat{B} w(k), \ w(k) \in \mathcal{W}_\delta, \tag{18}$$

$$z(k) = \hat{C} \hat{x}(k) + \hat{D} w(k), \tag{19}$$

where $\hat{A}_{ji}(\Theta_k) = \sum_{\ell=1}^{2^m} \theta_\ell(k) \hat{A}_{ji}^\ell$, $\hat{B}^T = \left[\tilde{B}_1^T \ \ 0 \right]^T$, $\hat{C} = \left[\tilde{C} \ 0 \right]$, $\hat{D} = \tilde{D}$ and

$$\hat{A}_{ji}^\ell = \begin{bmatrix} \tilde{A} + \tilde{B}_2 \left(D_\ell J_{ji} + D_\ell^- L_{ji} \right) \tilde{E}_j & \tilde{B}_2 \left(D_\ell H_{ji} + D_\ell^- K_{ji} \right) \\ G_{ji}^\ell \tilde{E}_j & F_{ji}^\ell \end{bmatrix}. \tag{20}$$

3. Main results

This section is explained in three steps:
- invariant ellipsoid property,
- $\mathcal{H}_\infty$ problem description,
- linear matrix inequality (LMI) formulation.

3.1 Invariant ellipsoid property

Before designing a controller, we shall first derive the conditions for obtaining the ellipsoidal sets $\mathcal{E}(P_i)$ such that, for all $k \geq 0$,

$$\psi(k,\hat{x}(0),w) \in \mathcal{E}(P_i),\ \forall \hat{x}(0) \in \mathcal{E}(P_i),\ i \in \mathcal{M},\ w \in \mathcal{W}, \tag{21}$$

where $\psi(\cdot)$ denotes the state trajectory of the closed-loop system, and $\mathcal{E}(P_i)$ denote PM-dependent ellipsoidal sets defined as

$$\mathcal{E}(P_i) \triangleq \left\{ \hat{x} \in \mathcal{R}^{2(n+(n+m)\bar{\tau})} \mid \hat{x}^T P_i \hat{x} \leq 1,\ P_i > 0 \right\},\ \forall i \in \mathcal{M}. \tag{22}$$

The following lemma presents the conditions for obtaining the ellipsoidal sets $\mathcal{E}(P_i)$ with the property (21).

Lemma 3.1 *Let $\bar{w} \geq 0$ be given. Suppose that there exist $0 \leq \lambda_1 \leq 1$ and $\bar{P}_i > 0$ such that*

$$0 \leq \begin{bmatrix} \lambda_1 P_i & 0 & (*) \\ 0 & (1/\bar{w})(1-\lambda_1)I & (*) \\ \hat{A}_{ji}^{\ell} & \hat{B} & \bar{P}_j \end{bmatrix},\ \forall (j,i) \in \mathcal{S},\ \ell \in [1,2^m], \tag{23}$$

$$\mathcal{E}(P_i) \subset \mathcal{L}(V_{ji}),\ \forall (j,i) \in \mathcal{S}, \tag{24}$$

where $\bar{P}_j \triangleq P_j^{-1}$. Then there exist the ellipsoidal sets $\mathcal{E}(P_i)$ with the property (21).

Proof: The property (21) can be altered as follows: for $k \geq 0$, $(j,i) \in \mathcal{S}$,

$$\hat{x}(k+1) \in \mathcal{E}(P_j) \text{ subject to } \hat{x}(k) \in \mathcal{E}(P_i),\ w^T(k)w(k) \leq \bar{w}. \tag{25}$$

At time k, let $\hat{x}(k) \in \mathcal{E}(P_i)$ and $w^T(k)w(k) \leq \bar{w}$, that is,

$$0 \leq 1 - \hat{x}^T(k)P_i\hat{x}(k) \ \text{ and } \ 0 \leq \bar{w} - w^T(k)w(k). \tag{26}$$

Then, by the condition (24), the transition of the state $\hat{x}(k)$ is determined by the closed-loop system (18), and hence $\hat{x}(k+1) \in \mathcal{E}(P_j)$ becomes

$$0 \leq \begin{bmatrix} \hat{x}(k) \\ w(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -\hat{A}_{ji}^T(\Theta_k)P_j\hat{A}_{ji}(\Theta_k) & (*) & 0 \\ -\hat{B}^T P_j \hat{A}_{ji}(\Theta_k) & -\hat{B}^T P_j \hat{B} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}(k) \\ w(k) \\ 1 \end{bmatrix} \forall (j,i) \in \mathcal{S}. \tag{27}$$

To convert, based on (26) and (27), the condition (25) into a matrix inequality. we employ the S-procedure as a constraint-elimination method for the conditions (26), which yields

$$0 \leq \left[\begin{array}{cc|c} -\hat{A}_{ji}^T(\Theta_k)P_j\hat{A}_{ji}(\Theta_k) + \lambda_1 P_i & (*) & 0 \\ -\hat{B}^T P_j \hat{A}_{ji}(\Theta_k) & -\hat{B}P_j\hat{B} + \lambda_2 I & 0 \\ \hline 0 & 0 & 1 - \lambda_1 - \lambda_2 \bar{w} \end{array} \right], \tag{28}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. And, from (28), it is straightforward

$$0 \leq \begin{bmatrix} -\hat{A}_{ji}^T(\Theta_k)P_j\hat{A}_{ji}(\Theta_k) + \lambda_1 P_i & (*) \\ -\hat{B}^T P_j \hat{A}_{ji}(\Theta_k) & -\hat{B}P_j\hat{B} + \lambda_2 I \end{bmatrix},\ \forall (j,i) \in \mathcal{S}, \tag{29}$$

$$0 \leq \lambda_2 \leq (1/\bar{w})(1-\lambda_1). \tag{30}$$

Here, note that $\lambda_2 = (1/\varpi)(1-\lambda_1)$ since, for any given λ_1, the feasibility of P_i increases as λ_2 increases. Thus, with the help of Schur complements, the conditions (29) and (30) becomes

$$0 \leq \begin{bmatrix} \lambda_1 P_i & 0 & (*) \\ 0 & (1/\varpi)(1-\lambda_1)I & (*) \\ \hat{A}_{ji}(\Theta_k) & \hat{B} & P_j^{-1} \end{bmatrix}, \ \forall (j,i) \in \mathcal{S}. \tag{31}$$

Furthermore, since multiplying (23) by $\theta_\ell(k)$ and summing it from $\ell = 1$ to $\ell = 2^m$ yields (31), it is clear that the condition (31) also holds if the condition (23) holds. ∎

3.2 $\mathcal{H}_\infty$ Problem description

Now let us consider the PM-dependent Lyapunov candidate $V_i(\hat{x}(k))$ given as

$$V_i(\hat{x}(k)) = \hat{x}^T(k)P_i\hat{x}(k), \ \ P_i > 0, \ \ \forall i \in \mathcal{M}. \tag{32}$$

Then the following two statements are equivalent:

- The closed-loop system (18) is stable with the $\mathcal{H}_\infty$ performance γ.
- There exist P_i such that (23), (24),

$$0 \leq \begin{bmatrix} P_i & 0 & (*) & (*) \\ 0 & \gamma^2 I & (*) & (*) \\ \hat{A}_{ji}(\Theta_k) & \hat{B} & P_j^{-1} & 0 \\ \hat{C} & \hat{D} & 0 & I \end{bmatrix}, \ \forall (j,i) \in \mathcal{S}. \tag{33}$$

In this equivalence, the condition (33) is directly derived by

$$V_i(\bar{x}(k+1)) - V_i(\bar{x}(k)) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) \leq 0, \ \forall i \in \mathcal{M}, \tag{34}$$

where since the conditions (23) and (24) make the state trajectories remain inside $\mathcal{E}(P_i) \subset \mathcal{L}(V_{ji})$, the transition of the state $\hat{x}(k)$ is always determined by the closed-loop system (18). Consequently, we shall solve the following minimization problem to construct a PM-dependent dynamic QLPV controller which achieves the maximal disturbance rejection capability:

$$\min \gamma \text{ subject to } (23), (24), \text{ and } (33). \tag{35}$$

3.3 LMI formulation

With the help of the replacement method, we shall formulate the conditions in the optimization problem (35) in terms of LMIs. To this end, let us first partition matrices P_i and $\bar{P}_i$ in the form:

$$P_i \triangleq \begin{bmatrix} X_i & Y_i \\ Y_i^T & Z_i \end{bmatrix}, \ \bar{P}_i = P_i^{-1} \triangleq \begin{bmatrix} \bar{X}_i & \bar{Y}_i \\ \bar{Y}_i^T & \bar{Z}_i \end{bmatrix}. \tag{36}$$

Theorem 3.1 *Let $\varpi \geq 0$ be given. Suppose, for a prescribed value $0 \leq \lambda_1 \leq 1$, that there exist matrices X_i, $\bar{X}_i$, $\Psi_{1,ji}^{\ell}$, $\Psi_{2,ji}$, $\Psi_{3,ji}$, Π_{ji}^{ℓ}, J_{ji}, L_{ji}, and Γ that are solutions of the following optimization problem:*

$$\gamma^* = \min \gamma \tag{37}$$

subject to, for all $(j,i) \in \mathcal{S}$,

$$0 \leq \begin{bmatrix} \lambda_1 X_i & (*) & (*) & (*) & (*) \\ \lambda_1 I & \lambda_1 \bar{X}_i & (*) & (*) & (*) \\ 0 & 0 & (1/\bar{w})(1-\lambda_1)I & (*) & (*) \\ \Delta_{ji}^{\ell}(1,1) & \Pi_{ji}^{\ell} & X_j \tilde{B}_1 & X_j & (*) \\ \Delta_{ji}^{\ell}(2,1) & \Delta_{ji}^{\ell}(2,2) & \tilde{B}_1 & I & \bar{X}_j \end{bmatrix}, \ \ell \in [1,2^m], \tag{38}$$

$$0 \leq \begin{bmatrix} \Gamma & L_{ji}\tilde{E}_j & \Psi_{3,ji} \\ (*) & X_i & I \\ (*) & (*) & \bar{X}_i \end{bmatrix}, \ \Gamma_{rr} \leq \bar{u}_r^2, \ r \in [1,m], \tag{39}$$

$$0 \leq \begin{bmatrix} X_i & (*) & (*) & (*) & (*) & (*) \\ I & \bar{X}_i & (*) & (*) & (*) & (*) \\ 0 & 0 & \gamma^2 I & (*) & (*) & (*) \\ \Delta_{ji}^{\ell}(1,1) & \Pi_{ji}^{\ell} & X_j \tilde{B}_1 & X_j & (*) & (*) \\ \Delta_{ji}^{\ell}(2,1) & \Delta_{ji}^{\ell}(2,2) & \tilde{B}_1 & I & \bar{X}_j & (*) \\ \tilde{C} & \tilde{C}\bar{X}_i & \tilde{D} & 0 & 0 & I \end{bmatrix}, \ \ell \in [1,2^m], \tag{40}$$

where Γ_{rr} denotes the r-th diagonal element of Γ,

$$\Delta_{ji}^{\ell}(1,1) \stackrel{\triangle}{=} X_j \tilde{A} + \Psi_{1,ji}^{\ell} \tilde{E}_j, \tag{41}$$

$$\Delta_{ji}^{\ell}(2,1) \stackrel{\triangle}{=} \tilde{A} + \tilde{B}_2 \left(D_\ell J_{ji} + D_\ell^- L_{ji} \right) \tilde{E}_j, \tag{42}$$

$$\Delta_{ji}^{\ell}(2,2) \stackrel{\triangle}{=} \tilde{A}\bar{X}_i + \tilde{B}_2 \left(D_\ell \Psi_{2,ji} + D_\ell^- \Psi_{3,ji} \right). \tag{43}$$

Then closed-loop system (18) is asymptotically stable in the absence of disturbances, and $||z(k)||_2 \leq \gamma^* ||w(k)||_2$ holds in the presence of disturbances. Moreover, based on the solutions X_i, $\bar{X}_i$, $\Psi_{1,ji}^{\ell}$, $\Psi_{2,ji}$, $\Psi_{3,ji}$, Π_{ji}^{ℓ}, J_{ji} and L_{ji}, the PM-dependent dynamic QLPV control gains $(F_{ji}(\Theta_k), G_{ji}(\Theta_k), H_{ji}, J_{ji})$ can be obtained by the following procedure: :

- **Off-line procedure**
  - *(i) obtain Y_i and Z_i from $\mathrm{svd}(X_i - \bar{X}_i^{-1}) = Y_i Z_i^{-1} Y_i^T$, and then obtain $\bar{Y}_i = -\bar{X}_i Y Z_i^{-1}$.*
  - *(ii) reconstruct H_{ji}, K_{ji}, G_{ji}^{ℓ}, and F_{ji}^{ℓ}:*

$$H_{ji} = \left(\Psi_{2,ji} - J_{ji}\tilde{E}_j \bar{X}_i \right) \bar{Y}_i^{-T}, \tag{44}$$

$$K_{ji} = \left(\Psi_{3,ji} - L_{ji}\tilde{E}_j \bar{X}_i \right) \bar{Y}_i^{-T}, \tag{45}$$

$$G_{ji}^{\ell} = Y_j^{-1} \left(\Psi_{1,ji}^{\ell} - X_j \tilde{B}_2 \left(D_\ell J_{ji} + D_\ell^- L_{ji} \right) \right), \tag{46}$$

$$F_{ji}^{\ell} = Y_j^{-1} \left(\Pi_{ji}^{\ell} - X_j \tilde{A} \bar{X}_i - \Psi_{1,ji}^{\ell} \tilde{E}_j \bar{X}_i - X_j \tilde{B}_2 \left(D_\ell H_{ji} + D_\ell^- K_{ji} \right) \bar{Y}_i^T \right) \bar{Y}_i^{-T}. \tag{47}$$

- **On-line procedure**
  - *(i) obtain $u(k)$ and $v(k)$ on-line from (15) and (16), respectively, and then calculate the interpolation coefficient vector Θ_k from (13).*
  - *(ii) update, based on (17), $F_{ji}(\Theta_k)$ and $G_{ji}(\Theta_k)$ at discrete time instance.*

Proof: Before formulating the conditions (23), (24), and (33) in terms of LMIs, let us define matrices W_i as

$$W_i \triangleq \begin{bmatrix} X_i & I \\ Y_i^T & 0 \end{bmatrix}, \ \forall i \in \mathcal{M}, \tag{48}$$

and, without loss of generality, assume that the matrices Y_i are full rank. And then, to convert the condition (23) into (38), we pre- and post-multiply T_{1ji}^T and $T_{1ji} = \text{blockdiag}(\bar{P}_i W_i, I, W_j)$ on the right-hand side of the inequality (23), respectively, which yields

$$0 \leq \begin{bmatrix} \lambda_1 W_i^T \bar{P}_i W_i & 0 & (*) \\ 0 & (1/\delta)(1-\lambda_1) I & \bar{B}^T \\ W_j^T \hat{A}_{ji}^\ell \bar{P}_i W_i & W_j^T \hat{B} & W_j^T \bar{P}_j W_j \end{bmatrix}, \tag{49}$$

where

$$W_i^T \bar{P}_i W_i = \begin{bmatrix} X_i & I \\ I & \bar{X}_i \end{bmatrix}, \ W_j^T \hat{B} = \begin{bmatrix} X_j \tilde{B}_1 \\ \tilde{B}_1 \end{bmatrix}, \tag{50}$$

$$W_j^T \hat{A}_{ji}^\ell \bar{P}_i W_i = \begin{bmatrix} \Delta_{ji}^\ell(1,1) & \Pi_{ji}^\ell \\ \Delta_{ji}^\ell(2,1) & \Delta_{ji}^\ell(2,2) \end{bmatrix}, \tag{51}$$

$$\Delta_{ji}^\ell(1,1), \Delta_{ji}^\ell(2,1), \text{ and } \Delta_{ji}^\ell(2,2) \text{ in (41)–(43)}, \tag{52}$$

$$\Psi_{1,ji}^\ell \triangleq X_j \tilde{B}_2 \left(D_\ell J_{ji} + D_\ell^- L_{ji} \right) + Y_j G_{ji}^\ell, \tag{53}$$

$$\Psi_{2,ji} \triangleq J_{ji} \tilde{E}_j \bar{X}_i + H_{ji} \bar{Y}_i^T, \tag{54}$$

$$\Psi_{3,ji} \triangleq L_{ji} \tilde{E}_j \bar{X}_i + K_{ji} \bar{Y}_i^T, \tag{55}$$

$$\Pi_{ji}^\ell \triangleq X_j \tilde{A} \bar{X}_i + \Psi_{1,ji}^\ell \tilde{E}_j \bar{X}_i + X_j \tilde{B}_2 \left(D_\ell H_{ji} + D_\ell^- K_{ji} \right) \bar{Y}_i^T + Y_j F_{ji}^\ell \bar{Y}_i^T. \tag{56}$$

Here, from (53)–(56), it follows (44)–(47). Next, to convert the condition $\mathcal{E}(P_i) \subset \mathcal{L}(V_{ji})$, that is,

$$0 \leq \begin{bmatrix} \Gamma & V_{ji} \\ (*) & P_i \end{bmatrix}, \ \Gamma_{rr} \leq \bar{u}_r^2, \ \forall r \in [1, m], \tag{57}$$

into (39), we pre- and post-multiply T_{2i}^T and $T_{2i} = \text{blockdiag}(I, \bar{P}_i W_i)$ on the right-hand side of (57), respectively, which yields

$$0 \leq \begin{bmatrix} \Gamma & V_{ji} \bar{P}_i W_i \\ (*) & W_i^T P_i W_i \end{bmatrix}, \ \Gamma_{rr} \leq \bar{u}_r^2, \ \forall r \in [1, m], \tag{58}$$

where $V_{ji} \bar{P}_i W_i = \begin{bmatrix} L_{ji} \tilde{E}_j & \Psi_{3,ji} \end{bmatrix}$. Finally, by pre- and post-multiplying T_{3ji}^T and $T_{3ji} = \text{blockdiag}(\bar{P}_i W_i, I, W_j, I)$ on the right-hand side of the inequality (33), respectively, and then by substituting (50), (51) and $\hat{C} \bar{P}_i W_i = \begin{bmatrix} \tilde{C} & \tilde{C} \bar{X}_i \end{bmatrix}$, we can obtain (40). ∎

Remark 3.1 *Theorem 3.1 can be applied to the case of the delay-free communication network* $\bar{\tau} = 0$ *by substituting* (A, B_1, B_2, C, I) *for* $(\tilde{A}, \tilde{B}_1, \tilde{B}_2, \tilde{C}, \tilde{E}_j)$*,*

Remark 3.2 *The greatest disturbance rejection capability, i.e., the smallest* γ^**, can be obtained by tuning the prescribed value* λ_1 *between 0 and 1.*

4. Numerical example

$\bar{\tau}$	$\bar{u}$	γ^*	$\bar{u}$	γ^*
1	1	$0.5005\ (\lambda_1 = 0.91)$	3	$0.2910\ (\lambda_1 = 0.94)$
2	1	$0.5284\ (\lambda_1 = 0.90)$	3	$0.3128\ (\lambda_1 = 0.91)$

Table 1. Minimized $\mathcal{H}_\infty$ γ-performance

To verify the performance of the proposed control algorithm, we consider a classical angular positioning system, adapted from Kwakernaak and Sivan (1972), with the sampling time $0.2s$:

$$\left[\begin{array}{c|c|c} A & B_1 & B_2 \\ \hline C & D_1 & D_2 \end{array}\right] = \left[\begin{array}{cc|c|c} 1.0 & 0.20 & 0.02 & 0.0000 \\ 0.0 & 0.08 & 0.12 & 0.1574 \\ \hline 1.0 & 0.00 & 0.15 & 0.1000 \end{array}\right], \tag{59}$$

where the control problem is to supply the input voltage (u_r volts) to the motor in order to rotate the antenna toward the desired direction. To demonstrate the effectiveness of our

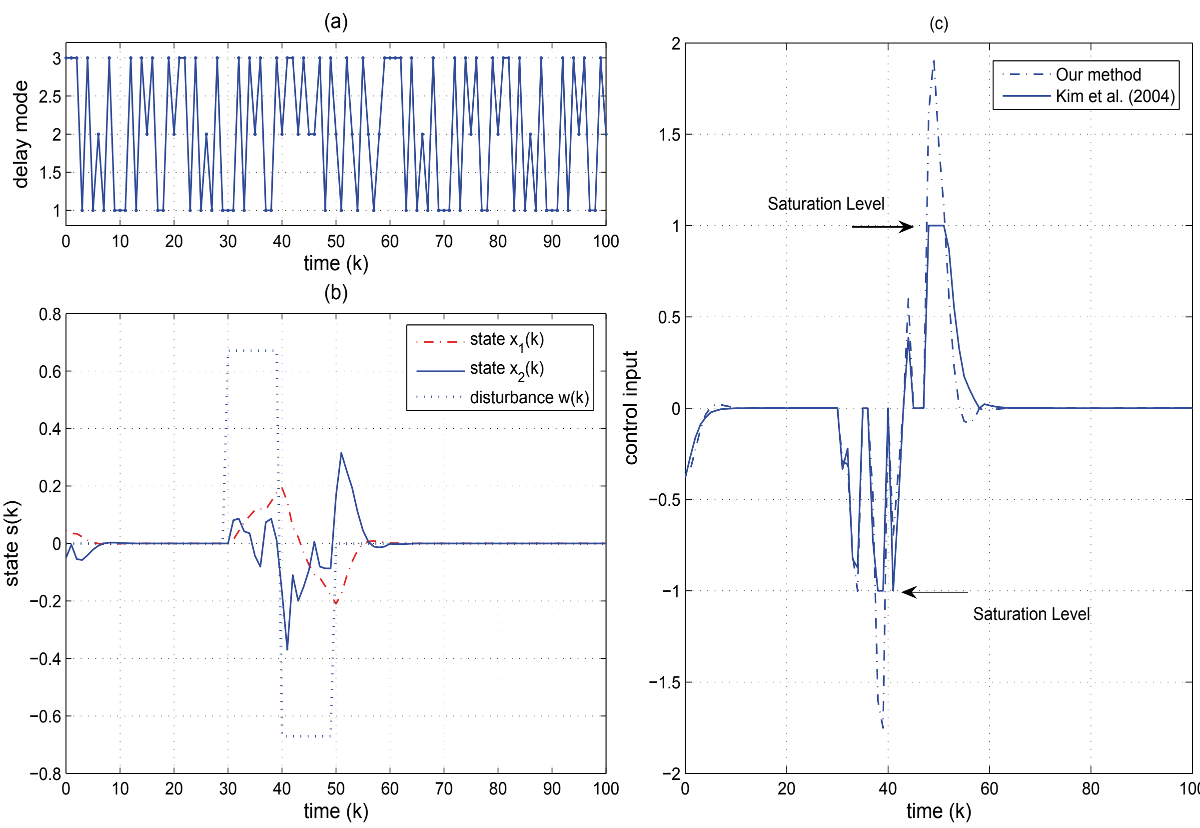

Fig. 2. (a) Delay mode $m(k)$; (b) external disturbance $w(k)$, and state responds $x_1(k)$ and $x_2(k)$; (d) control inputs for our method and Kim *et al.* (2004).

method for the control problem, we assume that the impressed voltage is constrained to be within the limits $-\bar{u}$ to $\bar{u}$ and that a multi-accessible communication network is employed for the transmission of the information between the system and the controller. Based on the setting, for $\bar{\tau} = 1, 2$, we first solve the optimization problem in Theorem 3.1 with $\bar{w}^{1/2} = 0.6325$ and $\bar{u} = 1$ (or $\bar{u} = 3$) to obtain the minimized $\mathcal{H}_\infty$ performance γ^*. Table 1 shows the minimized $\mathcal{H}_\infty$ performance γ^* for respective cases, from which we can observe that the disturbance rejection capability increases as the saturation level $\bar{u}$ increases. Next, for the initial conditions $x(0) = [0.45 \;\; -0.5]^T$ and $x(-1) = [0.0 \;\; 0.0]^T$, we simulate the behaviors of the closed-loop systems under the PM-dependent $\mathcal{H}_\infty$ controller corresponding to $\bar{\tau} = 1$ and $\bar{u} = 1$, where the external disturbances $w(k)$ are generated in the form of two-phase pulse with amplitude $\bar{w}^{1/2} = 0.6708$, and the delay sequences $m(k)$ are generated as random integers between 1 and 3. Fig. 2-(b) and (c) show the state and control input profiles, respectively, when $\gamma^* = 0.5084$. Particularly, Fig. 2-(c) depicts the comparison of the control inputs generated by our method and Kim *et al.* (2004), where the dotted line corresponds to the result of Kim *et al.* (2004) with $\gamma = 0.5084$, and the solid line corresponds to our result. As shown in Fig. 2-(c), contrary to Kim *et al.* (2004), our input voltage does never exceed the saturation level of the motor, $\bar{u} = 1$.

5. Concluding remarks

In this paper, we addressed the problem of designing an $\mathcal{H}_\infty$ control for networked control systems (NCSs) with the effects of both the input saturation as well as the network-induced delay. Based on a PM-dependent dynamic QLPV control law, we first found the conditions for set invariance, involved in the local stabilization, and then incorporated these conditions in the synthesis of dynamic state-feedback $\mathcal{H}_\infty$ control. The resultant convex solvability conditions have been expressed as a finite number of LMIs.

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Ferroelectrics
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Ferroelectric materials exhibit a wide spectrum of functional properties, including switchable polarization, piezoelectricity, high non-linear optical activity, pyroelectricity, and non-linear dielectric behaviour. These properties are crucial for application in electronic devices such as sensors, microactuators, infrared detectors, microwave phase filters and, non-volatile memories. This unique combination of properties of ferroelectric materials has attracted researchers and engineers for a long time. This book reviews a wide range of diverse topics related to the phenomenon of ferroelectricity (in the bulk as well as thin film form) and provides a forum for scientists, engineers, and students working in this field. The present book containing 24 chapters is a result of contributions of experts from international scientific community working in different aspects of ferroelectricity related to experimental and theoretical work aimed at the understanding of ferroelectricity and their utilization in devices. It provides an up-to-date insightful coverage to the recent advances in the synthesis, characterization, functional properties and potential device applications in specialized areas.

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