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## Plant identification by relay method

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### 1. Introduction

Plant identification is very important in control design. The relay method belongs among special kinds of experimental frequency response identification. It was originally used for plant identification by Rotach (Rotač, 1964) and lately also for controller autotuning, see reference (Åström & Wittenmark, 1989; Åström, 1993; Åström & Hägglund, 1995; Bi et al., 1997; Hang et al., 1993; Lu et al., 1998; Shen et al., 1999; Tan et al., 1996; Wang et al., 1999). Autotuning of controllers for MIMO plants is described in references (Halevi et al., 1997; Loh & Vasnani, 1994; Semino & Scali, 1998). At present more publications exist which are devoted to identification or controller autotuning on the basis of the relay experiment, e.g. (Besançon-Voda & Roux-Buisson, 1997; Hang et al., 2002; Huang et al. 2005; Johnson & Moradi, 2005; Kaya & Atherton, 2001; Lee et al., 1995; Leva & Donida, 2009; Leva & Maggio, 2009; Liu & Gao, 2009; Majhi, 2005; Majhi, 2007; Panda & Yu, 2003; Panda & Yu, 2005; Prokop et al., 2010; Sung et al., 2009; Tan et al., 2001; Tan et al., 2002; Vítěčková & Vítěček, 2005; Vivek & Chidambaram, 2005).

The aim of this paper is to summarily describe and show the basic modifications of the relay methods from the viewpoint of experimental plant identification and to bring out the computational formulas for simple plants. Two-position symmetric relays without and with hysteresis and with the integrator in front of the relay and behind of the relay are considered.

### 2. Relay Method without Integrator

In experimental plant identification using the relay method without the integrator it is assumed that the relay is plugged into the closed-loop system in lieu of a controller in accordance with Fig. 1, where:  $e$ ,  $w$ ,  $u$  and  $y$  are the control error, desired, manipulated and controlled variables,  $G_p(s)$  – the plant transfer function,  $N(a)$  – the describing function of the relay (Fig. 2),  $s$  – the complex variable in L-transform,  $a$  – the harmonic oscillation amplitude.

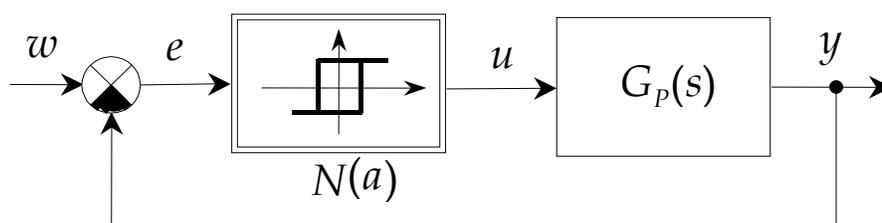


Fig. 1. Closed-loop system with two-position relay

The relay role is to effect stable oscillations of the closed-loop system in Fig. 1, i.e. to cause rise of the stable limit cycle. The describing function method is suitable to verify the limit cycle rise (Vukić et al., 2003). The describing function of the relay  $N(a)$  can be considered as the complex gain which depends on the harmonic oscillation amplitude  $a$  with the angular frequency  $\omega$

$$e(t) = a \sin \omega t \quad (1)$$

in the relay input and therefore it is possible to work with it like a common transfer function.

The condition of the limit cycle rise of the non-linear closed-loop system in Fig. 1 has the simple form

$$G_p(j\omega) = -\frac{1}{N(a)} \quad (2)$$

$$G_p(j\omega) = A_p(\omega)e^{j\varphi_p(\omega)} \quad (3)$$

where  $G_p(j\omega)$  is the plant transfer function in the frequency domain,  $A_p(\omega)$  - the plant transfer function magnitude,  $\varphi_p(\omega)$  - the plant transfer function phase.

The relation (2) is in analogy with the stability boundary for the linear control system

$$G_o(j\omega) = -1 \quad (4)$$

where  $G_o(j\omega)$  is the open-loop control system transfer function in the frequency domain.

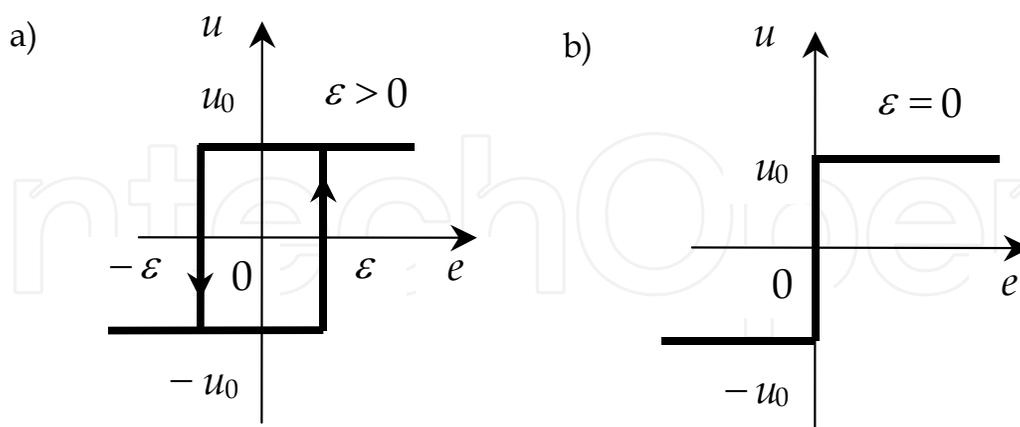


Fig. 2. Two-position symmetric relay: a) with hysteresis, b) without hysteresis

From comparison of the relations (2) and (4) it is obvious that the term  $-1/N(a)$  has the same role as the critical point  $-1$  for the linear control systems and therefore it is called a critical characteristic.

For the two-position symmetric relay with hysteresis ( $\varepsilon > 0$ , Fig. 2a) or without hysteresis ( $\varepsilon = 0$ , Fig. 2b) the describing function and the corresponding critical characteristic have the forms (Vukić et al., 2003)

$$N(a) = \begin{cases} \frac{4u_0}{\pi i} \left[ \sqrt{1 - \left(\frac{\varepsilon}{a}\right)^2} - j\frac{\varepsilon}{a} \right] & \text{for } 0 \leq \varepsilon < a \\ 0 & \text{for } 0 \leq a < \varepsilon \end{cases} \quad (5)$$

$$-\frac{1}{N(a)} = \begin{cases} A_N(a) e^{j\varphi_N(a)} & \text{for } 0 \leq \varepsilon < a \\ -\infty & \text{for } 0 \leq a < \varepsilon \end{cases} \quad (6a)$$

$$A_N(a) = \frac{\pi i}{4u_0} \quad (6b)$$

$$\varphi_N(a) = -\pi + \operatorname{arctg} \frac{\varepsilon}{\sqrt{a^2 - \varepsilon^2}} \quad (6c)$$

where  $2\varepsilon$  is the hysteresis width,  $u_0$  - the relay amplitude (maximum value of the manipulated variable  $u$ ),  $A_N(a)$  - the critical characteristic magnitude,  $\varphi_N(a)$  - the critical characteristic phase.

Because the describing function (5) uses only the fundamental harmonic component of the oscillation at the relay input, therefore the describing function method is the approximate method, which gives more accurate results if the behaviour of the plant with the transfer function  $G_p(s)$  is close to behaviour of a low-pass filter (Vukić et al., 2003).

The condition represents the complex equation (2) can be substituted by two generally nonlinear equations

$$\begin{aligned} A_p(\omega) &= A_N(a) \\ \varphi_p(\omega) &= \varphi_N(a) \end{aligned} \quad (7)$$

By solving of (7) the amplitude  $a_M$  and the angular frequency  $\omega_M$  can be obtained. If the obtained values  $a_M$  and  $\omega_M$  are positive and real, then in the closed-loop system in Fig. 1, the stable limit cycle rises with the oscillation amplitude  $a_M$  at the relay input and with the angular frequency  $\omega_M$  (Vukić et al., 2003).

The geometric interpretation of the solution of the complex equation (2) or the two real equations (7) is given in Fig. 3. The arrows of the curve  $G_p(j\omega)$  and the critical characteristic (6) show the directions of the growth of the angular frequency  $\omega$  and the harmonic oscillation amplitude  $a$  at the relay input (1).

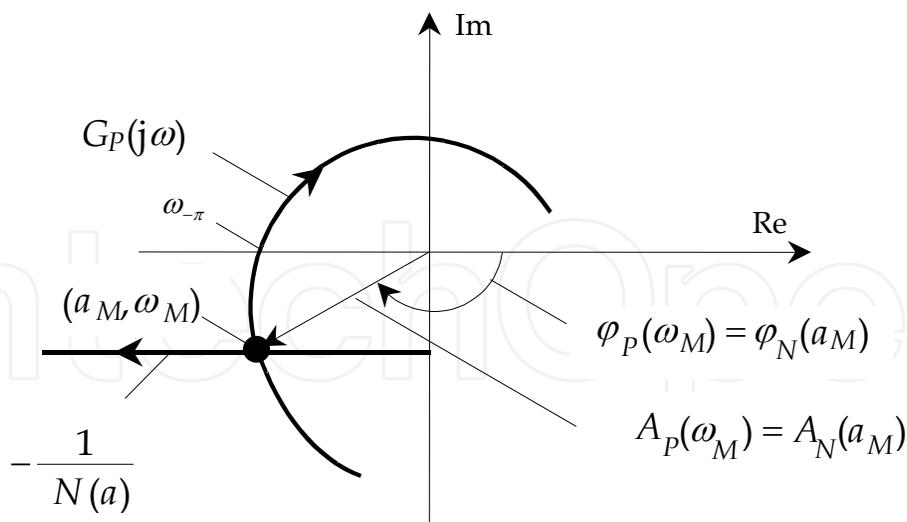


Fig. 3. Geometric interpretation of relay method without integrator

If in the closed-loop system in Fig. 1 the stable limit cycle rises, then from the measured values  $a_M$  and  $\omega_M$  on the basis of the equations (7) is possible to obtain two unknown plant parameters, see Fig. 4.

For  $w(t) = 0$  the plant output variable  $y(t)$  (except for sign) is the relay input variable  $e(t)$ , the equality

$$a_M = a_y \quad (8)$$

holds and the angular frequency  $\omega_M$  is the same for all closed-loop system variables and it can be determined from the formula

$$\omega_M = \frac{2\pi}{T_y} \quad (9)$$

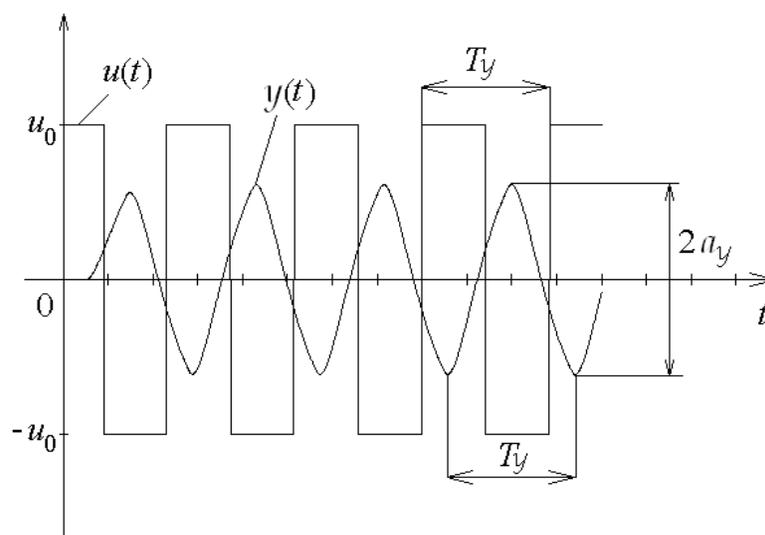


Fig. 4. Courses of relay output variable  $u(t)$  and plant output variable  $y(t)$  in the case of stable limit cycle rise

An application of the relay with hysteresis is useful in the case of existence of noise. In this case it is recommended so as to hysteresis width  $2\varepsilon$  was greater then double noise amplitude and the relay amplitude  $u_0$  should be such so as to output plant variable amplitude  $a_y$  was at least triple noise amplitude (Hang et al., 1993). Between the output oscillation amplitude  $a_y$  and the relay amplitude  $u_0$  a direct proportion holds.

For the relay with hysteresis the angular frequency (9) is lower then for the relay without hysteresis.

From Fig. 3 it is obvious that the relay method without the integrator is suitable for proportional and integral plants with time delay. The plant gain can be determined from the steady state or by the other corresponding way. For the relay without hysteresis the equality  $\omega_M = \omega_{-\pi}$  holds.

### Example 2.1

For the proportional plant with transfer function

$$G_P(s) = \frac{k_1}{(T_i s + 1)^i} e^{-T_{di}s} \quad (10)$$

it is necessary to determine the plant time constant  $T_i$  and plant time delay  $T_{di}$  on condition that the plant gain  $k_1$  and plant order  $i$  are known, using the relay method without integrator.

#### Solution:

For plant (10) the relations

$$G_P(j\omega) = \frac{k_1}{(jT_i\omega + 1)^i} e^{-jT_{di}\omega} = A_P(\omega) e^{i\varphi_P(\omega)} \quad (11a)$$

$$A_P(\omega) = \frac{k_1}{(1 + \omega^2 T_i^2)^{\frac{i}{2}}} \quad (11b)$$

$$\varphi_P(\omega) = -[\omega T_{di} + i \arctg(\omega T_i)] \quad (11c)$$

hold.

From experimentally obtained periodic course of the output variable  $y(t)$  the oscillation amplitude  $a_y$  and period  $T_y$  can be measured (see Fig. 4), and than on the basis of the relations (6b), (6c), (7), (8), (9), (11b) and (11c) for  $a = a_M = a_y$  and  $\omega = \omega_M$  the formulas

$$T_i = \frac{T_y}{2\pi} \sqrt[2]{\sqrt[2]{\frac{16k_1^2 u_0^2}{\pi^2 a_y^2} - 1}} \quad (12a)$$

$$T_{di} = \frac{T_y}{2\pi} \left[ \pi - i \operatorname{arctg} \frac{2\pi T_i}{T_y} - \operatorname{arctg} \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (12b)$$

can be obtained.

E.g. for a proportional (first order plus time delay) plant with the transfer function

$$G_p(s) = \frac{k_1}{T_1 s + 1} e^{-T_{d1}s} \quad (13a)$$

from formulas (12) for  $i = 1$  can be obtained

$$T_1 = \frac{T_y}{2\pi} \sqrt{\frac{16k_1^2 u_0^2}{\pi^2 a_y^2} - 1} \quad (13b)$$

$$T_{d1} = \frac{T_y}{2\pi} \left[ \pi - \operatorname{arctg} \frac{2\pi T_1}{T_y} - \operatorname{arctg} \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (13c)$$

Likewise for a proportional (second order plus time delay) plant with the transfer function

$$G_p(s) = \frac{k_1}{(T_2 s + 1)^2} e^{-T_{d2}s} \quad (14a)$$

from formulas (12) for  $i = 2$  can be obtained

$$T_2 = \frac{T_y}{2\pi} \sqrt{\frac{4k_1 u_0}{\pi a_y} - 1} \quad (14b)$$

$$T_{d2} = \frac{T_y}{2\pi} \left[ \pi - 2 \operatorname{arctg} \frac{2\pi T_2}{T_y} - \operatorname{arctg} \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (14c)$$

**Notice:**

The plant (10) parameters  $T_i$  and  $T_{di}$  for known  $k_1$  and  $i$  can also be determined from the ultimate (critical) gain of a proportional controller  $k_{pc}$  and the ultimate (critical) period  $T_c$ , it holds

$$T_i = \frac{T_c}{2\pi} \sqrt{i \sqrt{k_{pc}^2 k_1^2} - 1} \quad (15a)$$

$$T_{di} = \frac{T_c}{2\pi} \left( \pi - i \operatorname{arctg} \frac{2\pi T_i}{T_c} \right) \quad (15b)$$

The ultimate controller gain  $k_{pc}$  and the corresponding ultimate period  $T_c$ , must be determined iteratively. In this case formulas (15) are exact and therefore the accuracy of the determined parameters  $T_i$  and  $T_{di}$  is considerably higher.

### Example 2.2

Likewise in the example 2.1 on the basis of the relay method (without the integrator) for the integral plant with the transfer function

$$G_p(s) = \frac{k_1}{s(T_i s + 1)^i} e^{-T_{di}s} \quad (16)$$

it is necessary to determine the plant time constant  $T_i$  and plant time delay  $T_{di}$  on condition that the plant gain  $k_1$  and order  $i$  are known.

### Solution:

For the plant (16) on the basis of the relations (11) it can be written directly

$$A_p(\omega) = \frac{k_1}{\omega(1 + \omega^2 T_i^2)^{\frac{i}{2}}} \quad (17a)$$

$$\varphi_p(\omega) = -\left[\frac{\pi}{2} + \omega T_{di} + i \arctg(\omega T_i)\right] \quad (17b)$$

For experimentally obtained plant oscillation amplitude  $a_y$  and period  $T_y$  (see Fig. 4), and than on the basis of the relations (6b), (6c), (7), (8), (9), (17a) and (17b) for  $a = a_M = a_y$  and  $\omega = \omega_M$  the formulas

$$T_i = \frac{T_y}{2\pi} \sqrt[i]{\frac{4k_1^2 T_y^2 u_0^2}{\pi^4 a_y^2} - 1} \quad (18a)$$

$$T_{di} = \frac{T_y}{2\pi} \left[ \frac{\pi}{2} - i \arctg \frac{2\pi T_i}{T_y} - \arctg \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (18b)$$

can be obtained.

E.g. for the integral (integral plus first order plus time delay) plant with the transfer function

$$G_p(s) = \frac{k_1}{s(T_1 s + 1)} e^{-T_{d1}s} \quad (19a)$$

from the formulas (18) for  $i = 1$  can be obtained

$$T_1 = \frac{T_y}{2\pi} \sqrt{\frac{4k_1^2 T_y^2 u_0^2}{\pi^4 a_y^2} - 1} \quad (19b)$$

$$T_{d1} = \frac{T_y}{2\pi} \left[ \frac{\pi}{2} - \operatorname{arctg} \frac{2\pi T_1}{T_y} - \operatorname{arctg} \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (19c)$$

Likewise for the integral (integral plus second order plus time delay) plant with the transfer function

$$G_p(s) = \frac{k_1}{s(T_2 s + 1)^2} e^{-T_{d2}s} \quad (20a)$$

from the formula (18) for  $i = 2$  can be obtained

$$T_2 = \frac{T_y}{2\pi} \sqrt{\frac{2k_1 T_y u_0}{\pi^2 a_y} - 1} \quad (20b)$$

$$T_{d2} = \frac{T_y}{2\pi} \left[ \frac{\pi}{2} - 2 \operatorname{arctg} \frac{2\pi T_2}{T_y} - \operatorname{arctg} \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (20c)$$

**Notice:**

As well in this case the plant (16) parameters  $T_i$  and  $T_{di}$  for known  $k_1$  and  $i$  can be determined from the ultimate controller gain  $k_{pc}$  and the ultimate period  $T_c$ , it holds that

$$T_i = \frac{T_c}{2\pi} \sqrt{i \sqrt{\frac{k_{pc}^2 k_1^2 T_c^2}{4\pi^2}} - 1} \quad (21a)$$

$$T_{di} = \frac{T_c}{2\pi} \left( \frac{\pi}{2} - i \operatorname{arctg} \frac{2\pi T_i}{T_c} \right) \quad (21b)$$

The formulas (21) are exact, but the ultimate controller gain  $k_{pc}$  and the corresponding ultimate period  $T_c$  must be determined iteratively.

### 3. Relay Method with Integrator

The relay method with integrator is extending the previous approach (Åström & Hägglund 1995). It can be used in two alternatives: with the integrator behind of the relay (Fig. 5a) and with the integrator in front of the relay (Fig. 5b). Both these alternatives must be strictly differentiated. For both alternatives in Fig. 5 the condition of the stable limit rise has a form [compare with (2)]

$$\frac{1}{j\omega} G_p(j\omega) = -\frac{1}{N(a)} \quad (22)$$

and can be substituted by two equations [compare with (7)]

$$\begin{aligned} \frac{1}{\omega} A_P(\omega) &= A_N(a) \\ \varphi_P(\omega) - \frac{\pi}{2} &= \varphi_N(a) \end{aligned} \tag{23}$$

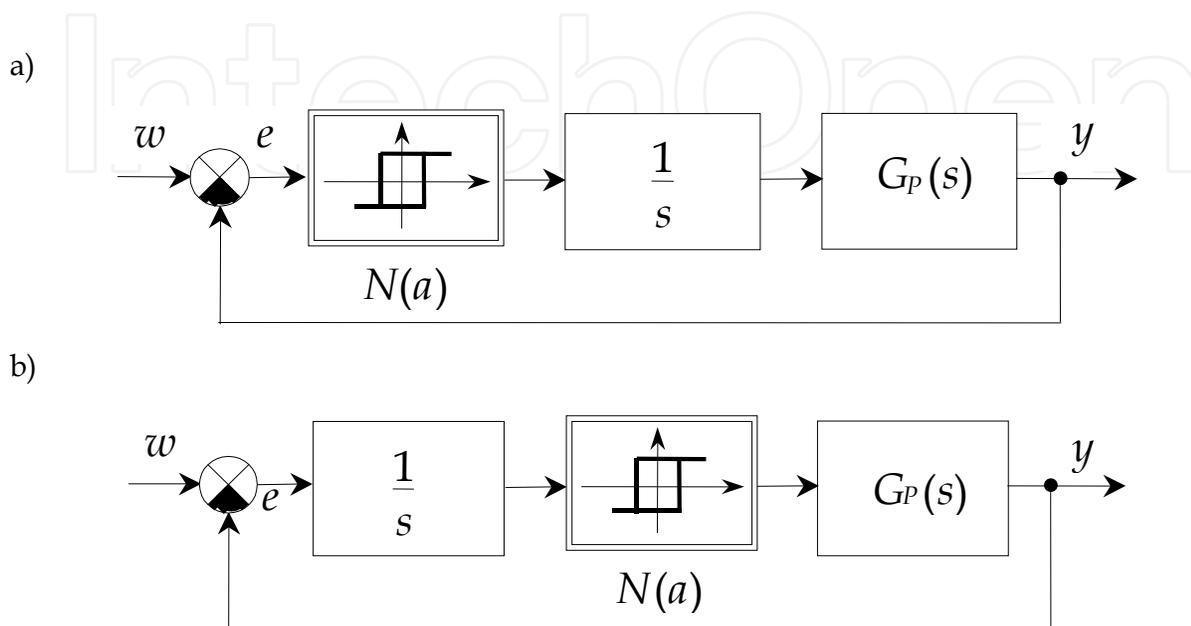


Fig. 5. Closed-loop system with relay with integrator: a) behind of relay, b) in front of relay

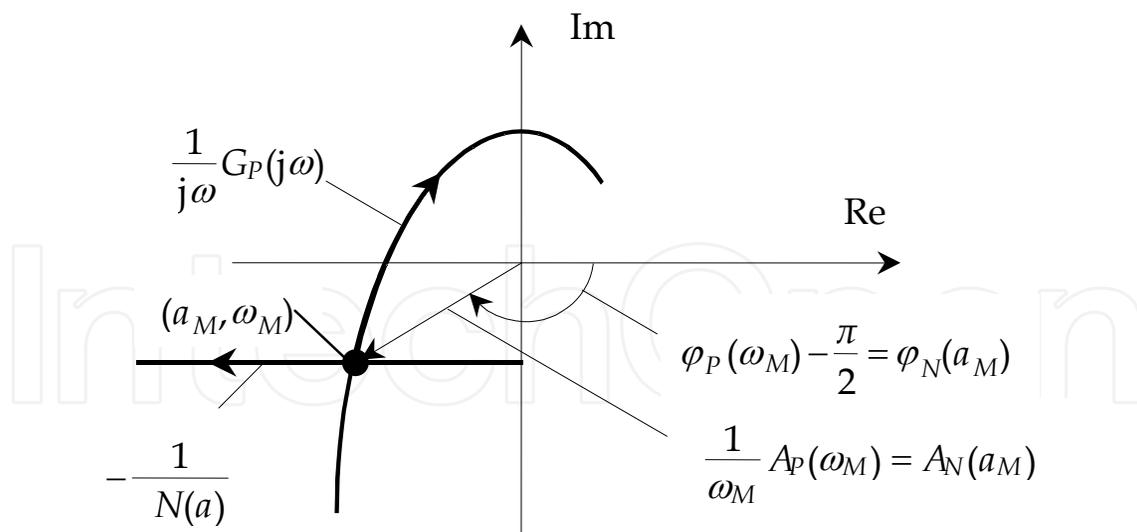


Fig. 6. Geometric interpretation of relay method with integrator

If the solution, i.e. the values  $a = a_M$  and  $\omega = \omega_M$ , are positive and real (Fig. 6), then in the closed-loop systems in Fig. 5 the stable limit cycle rises with the oscillation amplitude  $a_M$  and the angular frequency  $\omega_M$  at the relay input.

From Fig. 6 it is obvious that the relay method with the integrator is applicable only for proportional plants. For the relay without hysteresis with the integrator the relation  $\omega_M = \omega_{-\pi/2}$  holds.

### a) Integrator is behind the relay

In this case for  $w(t) = 0$  the output plant variable  $y(t)$  (except for sign) is the input relay variable  $e(t)$ , and therefore (8) holds, i.e.  $a_M = a_y$ .

#### Example 3.1

For the plant (10) from the example 2.1 it is necessary when using the relay method with the integrator behind the relay, to determine the plant time constant  $T_i$  and plant time delay  $T_{di}$  on condition that the plant gain  $k_1$  and its order  $i$  are known.

#### Solution:

From the experimental obtained periodic course of the output variable  $y(t)$  the amplitude  $a_y$  and period  $T_y$  were obtained (Fig. 4). On the basis of the relations (6b), (6c), (8), (9), (11b), (11c) and (23) for  $a = a_M = a_y$  and  $\omega = \omega_M$  can be obtained the relations (18), which are the same like in the example 2.2.

#### Example 3.2

On the basis of the relay method without hysteresis and with the integrator behind the relay for the second order oscillatory plant

$$G_p(s) = \frac{k_1}{T_0^2 s^2 + 2\xi_0 T_0 s + 1} \quad (24)$$

it is necessary to determine the plant time constant  $T_0$  and plant damping coefficient  $\xi_0$  on condition that the plant gain  $k_1$  is known.

#### Solution:

For the plant (24) holds

$$G_p(j\omega) = A_p(\omega) e^{j\varphi_p(\omega)} \quad (25a)$$

$$A_p(\omega) = \frac{k_1}{\sqrt{(1 - \omega^2 T_0^2)^2 + 4\xi_0^2 \omega^2 T_0^2}} \quad (25b)$$

$$\varphi_p(\omega) = \begin{cases} -\operatorname{arctg} \frac{2\xi_0 \omega T_0}{1 - \omega^2 T_0^2} & \text{for } 0 \leq \omega < \frac{1}{T_0} \\ -\frac{\pi}{2} & \text{for } \omega = \frac{1}{T_0} \\ -\pi + \operatorname{arctg} \frac{2\xi_0 \omega T_0}{\omega^2 T_0^2 - 1} & \text{for } \omega > \frac{1}{T_0} \end{cases} \quad (25c)$$

For the relay without hysteresis with the integrator the relation

$$\omega_M = \omega_{-\pi/2} = \frac{1}{T_0} \Rightarrow T_0 = \frac{T_y}{2\pi} \quad (26a)$$

holds.

On the basis of the relations (6b), (25b), (26a) and the first equation in (23) can be obtained

$$\xi_0 = \frac{k_1 T_y u_0}{\pi^2 a_y} \quad (26b)$$

From the experimental obtained values  $a_y$  and  $T_y$  (see Fig. 4) by means of (26) the unknown parameters  $T_0$  and  $\xi_0$  can be determined.

### b) Integrator is in front of relay

In this case for  $w(t) = 0$  the input relay variable is given

$$\int e(t) dt = -\int a_y \sin(\omega_M t + \varphi) dt = \frac{a_y}{\omega_M} \cos(\omega_M t + \varphi) \quad (27)$$

and therefore the relation

$$a_M = \frac{a_y}{\omega_M} = \frac{a_y T_y}{2\pi} \quad (28)$$

holds.

With respect to high frequency damping by the integrator, an accuracy of the relay method with the integrator in front of the relay is less than the accuracy of the relay method with the integrator behind the relay.

### Example 3.3

For proportional plant (10) from example 2.1 it is necessary by the relay method with the integrator in front of the relay to determine plant time constant  $T_i$  and plant time delay  $T_{di}$  on condition that the plant gain  $k_1$  and plant order  $i$  are known.

### Solution:

On the basis of the relations (6b), (6c), (9), (11b), (11c), (23) and (28) for  $a = a_M$  and  $\omega = \omega_M$  can be obtained

$$T_i = \frac{T_y}{2\pi} \sqrt[2i]{\frac{16k_1^2 u_0^2}{\pi^2 a_y^2} - 1} \quad (29a)$$

$$T_{di} = \frac{T_y}{2\pi} \left( \frac{\pi}{2} - i \operatorname{arctg} \frac{2\pi T_i}{T_y} - \operatorname{arctg} \frac{\varepsilon}{\sqrt{\frac{a_y^2 T_y^2}{4\pi^2} - \varepsilon^2}} \right) \quad (29b)$$

The formula (29a) is the same as the formula (12a).

E.g. for proportional plant (13a) on the basis of the formulas (29) for  $i = 1$  can be obtained

$$T_{d1} = \frac{T_y}{2\pi} \left( \frac{\pi}{2} - \operatorname{arctg} \frac{2\pi T_1}{T_y} - \operatorname{arctg} \frac{\varepsilon}{\sqrt{\frac{a_y^2 T_y^2}{4\pi^2} - \varepsilon^2}} \right) \quad (30)$$

The formula for  $T_1$  is the same as the formula (13b).

Likewise for proportional plant with the transfer function (14a) on the basis of the formulas (29) for  $i = 2$  can be obtained

$$T_{d2} = \frac{T_y}{2\pi} \left( \frac{\pi}{2} - 2 \operatorname{arctg} \frac{2\pi T_2}{T_y} - \operatorname{arctg} \frac{\varepsilon}{\sqrt{\frac{a_y^2 T_y^2}{4\pi^2} - \varepsilon^2}} \right) \quad (31)$$

The formula for  $T_2$  is the same as the formula (14b).

### Example 3.4

On the basis of the relay method without hysteresis and with the integrator in front of the relay for the second order oscillatory plant (24) from the example 3.2 it is necessary to determine the plant time constant  $T_0$  and plant damping coefficient  $\xi_0$  on condition that the plant gain  $k_1$  is known.

#### Solution:

Likewise in the example 3.2 for the relay without hysteresis with the integrator (in front of or behind the relay) (26a) holds. Further on the basis of the relations (6b), (25b), (26a), (28) and the first equation in (23) can be obtained

$$\xi_0 = \frac{2k_1 u_0}{\pi a_y} \quad (32)$$

From the periodic course of the output variable  $y(t)$  the amplitude  $a_y$  and period  $T_y$  of the oscillation and on the basis of the formulas (26a) and (32) parameters  $T_0$  and  $\xi_0$  can be determined.

**Notice:**

The plant transfer function (24) can be expressed in terms of the damping coefficient value  $\xi_0$  in forms

$$G_p(s) = \begin{cases} \frac{k_1}{T_0^2 s^2 + 2\xi_0 T_0 s + 1} & \text{for } 0 < \xi_0 < 1 \end{cases} \quad (33a)$$

$$G_p(s) = \begin{cases} \frac{k_1}{(T_0 s + 1)^2} & \text{for } \xi_0 = 1 \end{cases} \quad (33b)$$

$$G_p(s) = \begin{cases} \frac{k_1}{(T_1 s + 1)(T_2 s + 1)} & \text{for } \xi_0 > 1 \end{cases} \quad (33c)$$

where  $T_1 > T_2$  are the different plant time constants, which can be determined on the basis of the formulas

$$T_1 = T_0 \left( \xi_0 + \sqrt{\xi_0^2 - 1} \right), \quad T_2 = T_0 \left( \xi_0 - \sqrt{\xi_0^2 - 1} \right) \quad (34)$$

The plant parameters  $T_0$  and  $\xi_0$  can be also determined by means of the integral controller with the transfer function  $1/T_i s$ , which is plugged into the closed-loop system in Fig. 1 in lieu of the relay or the proportional controller with the transfer function  $k_p$ , which is plugged into the closed-loop systems in Fig. 5 in lieu of the relay and causing the stable oscillation. Then from the measured ultimate period  $T_c$  any closed-loop system variable and from the ultimate controller gain  $k_{pc}$  or the ultimate integral time  $T_{ic} = 1/k_{pc}$  the above mentioned parameters can be determined on the basis of the formulas

$$T_0 = \frac{T_c}{2\pi} \quad (35a)$$

$$\xi_0 = \frac{k_1 k_{pc} T_c}{4\pi} = \frac{k_1 T_c}{4\pi T_{ic}} \quad (35b)$$

Likewise in the cases of the plants (10) and (16) the values of the ultimate parameters  $T_c$ ,  $k_{pc}$  or  $T_{ic}$  must be determined iteratively but on the other side the formulas (35) are exact.

#### 4. Conclusions

The book chapter describes the use of the relay method with and without hysteresis and further more with and without integrator, which is plugged in behind or in front of the relay for experimental identification of the simple plants. The relay method without the integrator is suitable for proportional and integral plants and the relay method with the integrator is suitable only for proportional plants. For proportional and integral plants with multiple time constants with time delay and for second order oscillatory proportional plant the general formulas for computation of their two parameters are derived. Experimentally it is possible to obtain one point of the plant frequency response, i.e. two values of the plant

parameters, it is possible by means of several experiments (with or without hysteresis, without or with the integrator, with the integrator in front of or behind the relay) to obtain more values of the plant parameters or take out the average of these values etc. It is obvious that the relay methods can be used only for the plants, which can oscillate or which cannot be destroyed by oscillation.

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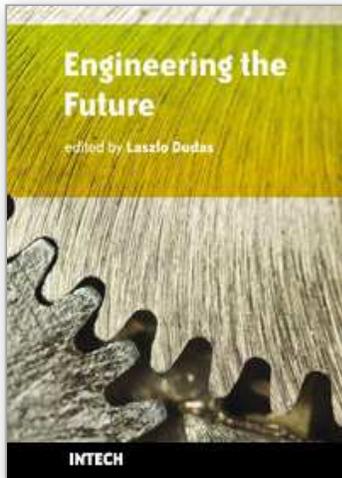
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